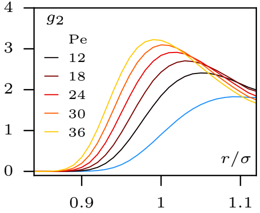
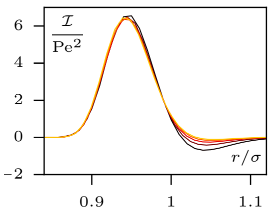
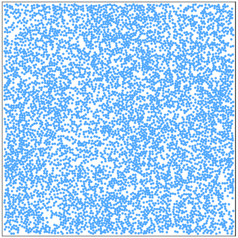
Dissipation in active matter

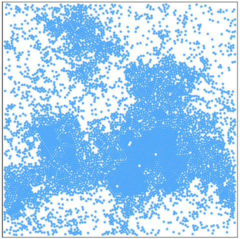
1. Dissipation and structure





2. Dissipation and phase transitions





Controlling dissipation – Which order emerges?

Thermodynamic observables and emerging order

- Equilibrium thermodynamics

Energy

- Nonequilibrium thermodynamics

Phase separation?

Collective motion?

Disordered state

Phase separation?

Collective motion?

Dissipation

Controlling dissipation – Which order emerges?

Controlling observables with conjugate parameter

Equilibrium	**Nonequilibrium**
Energy (U)	Dissipation ($\mathcal{J}$)
Temperature (T)	Unknown (s)
Configuration space $P\big(\{\mathbf{r}_i, \theta_i\} \,\big|\, U\big) \sim \mathrm{e}^{-U/T}$	Trajectory space $\mathcal{P}\big(\{\mathbf{r}_i, \theta_i\}_0^\tau\big)$

Trajectories conditioned by dissipation

$$\mathcal{P}_{\rm con}\big(\{\mathbf{r}_i, \theta_i\}_0^\tau \,\big|\, \mathcal{J}\big)$$

- What is the dynamics at a given dissipation?

Controlling dissipation – Which order emerges?

Large deviation theory

Distance between conditioned and original dynamics

$$\mathcal{D}\big[\mathcal{P}_{\rm con},\,\mathcal{P}\big] = \lim_{\tau\to\infty}\frac{1}{\tau}\left\langle \log\frac{\mathcal{P}_{\rm con}\big(\{\mathbf{r}_i,\theta_i\}_0^\tau\,\big|\,\mathcal{J}\big)}{\mathcal{P}\big(\{\mathbf{r}_i,\theta_i\}_0^\tau\big)}\right\rangle$$

$$\boxed{\mathcal{D}\big[\mathcal{P}_{\rm con},\,\mathcal{P}\big] \geq \mathcal{I}(\mathcal{J})}$$

Probability of atypical $\mathcal{J}$

$$p(\mathcal{J}) \asymp \exp\big(-\tau\,\mathcal{I}(\mathcal{J})\big)$$

- Optimal conditioning given by **biased ensemble**

$$\boxed{\mathcal{P}_{\rm opt} \sim \mathcal{P}\,\mathrm{e}^{-sN\tau\mathcal{J}}}$$

Jack, EPJB **93**, 74 (2020)

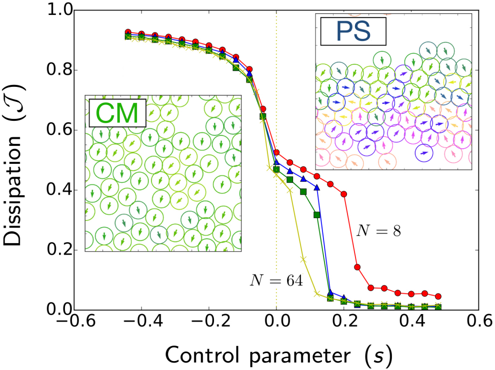
Cagnetta *et al*, PRL **119**, 158002 (2017)
Whitelam *et al*, JCP **148**, 154902 (2018)
Nemoto *et al*, PRE **99**, 022605 (2019)
Tociu *et al*, PRX **9**, 041026 (2019)
Gradenigo *et al*, J Stat Mech, 053206 (2019)
Mallmin *et al*, J Phys A **52**, 425002 (2019)
EF *et al*, NJP **22**, 013052 (2020)
Cagnetta *et al*, PRE **101**, 022130 (2020)

Controlling dissipation – Which order emerges?

Sampling of biased ensemble

$$\dot{\mathbf{r}}_i = -\mu\nabla_i U + v\,\mathbf{e}_i + \sqrt{2D_\mathrm{t}}\,\boldsymbol{\xi}_i$$
$$\dot{\theta}_i = \sqrt{2D_\mathrm{r}}\,\eta_i$$

$$\mathcal{J} = \frac{1}{v^2 N\tau}\sum_{i=1}^{N}\int_0^\tau \dot{\mathbf{r}}_i \cdot v\mathbf{e}_i\,\mathrm{d}t$$



Dynamical transitions
- $s > 0$ small $\mathcal{J}$ **Phase separation** (PS)
- $s < 0$ large $\mathcal{J}$ **Collective motion** (CM)

Nemoto, EF, Cates, Jack, Tailleur, PRE **99**, 022605 (2019)

Controlling dissipation – Which order emerges?

Dynamics equivalent to biased ensemble

Conditioning trajectories yields **effective interactions**

$$\dot{\mathbf{r}}_i = -\mu\nabla_i U + v\,\mathbf{e}_i + \sqrt{2D_\mathrm{t}}\,\boldsymbol{\xi}_i$$
$$\dot{\theta}_i = -\frac{\partial}{\partial\theta_i} V\big(\{\mathbf{r}_i, \theta_i\}\big) + \sqrt{2D_\mathrm{r}}\,\eta_i$$

Comparing effective dynamics and biased ensemble

$$\boxed{\mathcal{D}\big[\mathcal{P}_\mathrm{eff}, \mathcal{P}\big] \geq \mathcal{D}\big[\mathcal{P}_\mathrm{opt}, \mathcal{P}\big]}$$

- Which interactions reproduce biased ensemble?

Chetrite, Touchette, PRL **111**, 120601 (2013)

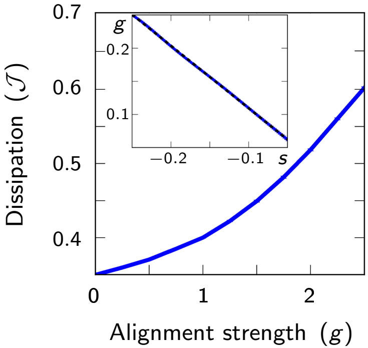
Jack, Sollich, EPJST **224**, 2351 (2015)

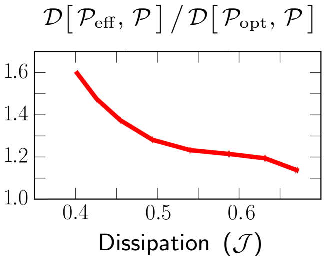
Controlling dissipation – Which order emerges?

Collective motion at large dissipation

$$\dot{\mathbf{r}}_i = -\mu\nabla_i U + v\,\mathbf{e}_i + \sqrt{2D_\mathrm{t}}\,\boldsymbol{\xi}_i$$

$$\dot{\theta}_i = \frac{g}{N}\sum_{j=1}^{N}\sin(\theta_i - \theta_j) + \sqrt{2D_\mathrm{r}}\,\eta_i$$





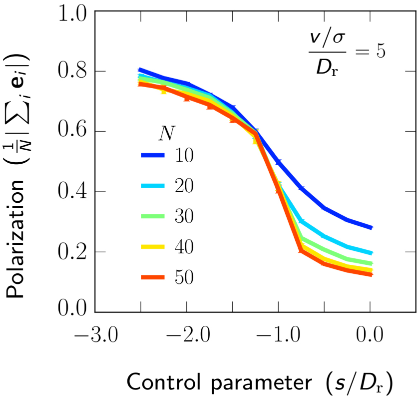
- **Effective alignment** captures biased ensemble dynamics

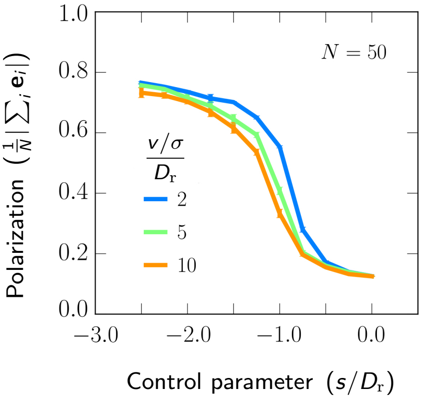
Controlling dissipation – Which order emerges?

Collective motion at large dissipation

Transition driven by entropy-energy competition

Critical parameter $g_c \sim D_r \longrightarrow s_c \sim -D_r$





Controlling dissipation – Which order emerges?

Collective motion at large dissipation

Transition driven by entropy-energy competition

Critical parameter $g_{\mathrm{c}} \sim D_{\mathrm{r}} \longrightarrow s_{\mathrm{c}} \sim -D_{\mathrm{r}}$

Entropic effect

Minimizing distance from original dynamics

Avoiding order

Extensive cost

Every particle aligns

Energetic effect

Achieving large dissipation

Promoting order

Extensive gain

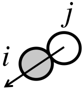
$\mathcal{P}_{\mathrm{opt}} \sim \mathcal{P}\, \mathrm{e}^{-s N \tau \mathcal{J}}$

Controlling dissipation – Which order emerges?

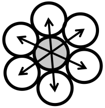
Phase separation at small dissipation

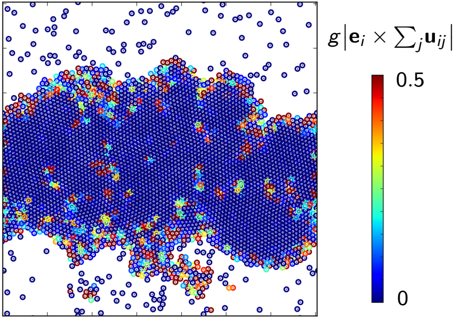
$$\dot{\mathbf{r}}_i = -\mu\nabla_i U + v\,\mathbf{e}_i + \sqrt{2D_\mathrm{t}}\,\boldsymbol{\xi}_i$$
$$\dot{\theta}_i = -g\,\mathbf{e}_i \times \sum_{j \wedge i} \mathbf{u}_{ij} + \sqrt{2D_\mathrm{r}}\,\eta_i$$

Unit vector $\mathbf{u}_{ij}$



Dense phase $\sum_j \mathbf{u}_{ij} = \mathbf{0}$





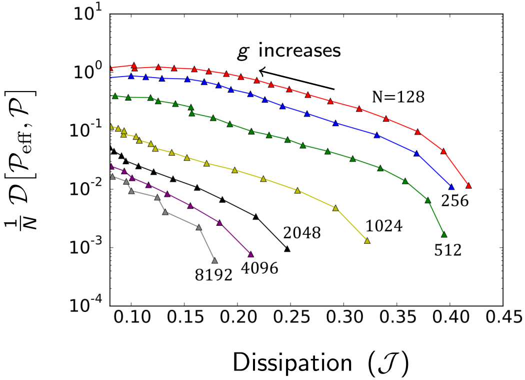
- Stabilizing clusters only requires **forces at boundaries**

Controlling dissipation – Which order emerges?

Phase separation at small dissipation

Sub-extensive distance from original dynamics

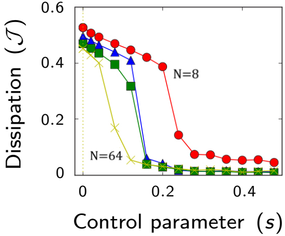
$$\mathcal{D}\big[\,\mathcal{P}_{\rm eff},\,\mathcal{P}\,\big] \;\geq\; \mathcal{D}\big[\,\mathcal{P}_{\rm opt},\,\mathcal{P}\,\big]$$



Controlling dissipation – Which order emerges?

Phase separation at small dissipation

Critical parameter $s_{\rm c} \xrightarrow[N\gg1]{} 0$



Entropic effect

Minimizing distance from original dynamics

Avoiding order

Sub-extensive cost

Leaders compress clusters

Energetic effect

Achieving small dissipation

Promoting order

Extensive gain

$\mathcal{P}_{\rm opt} \sim \mathcal{P}\, {\rm e}^{-s\,N\tau\,\mathcal{J}}$

Summary and outlook

Phase transitions induced by dissipation

Phase separation — Disordered state — Collective motion → Dissipation

How to **polarize isotropic particles**

Give instructions of avoiding collisions

Bird flocks

Bialek *et al*,
PNAS **109**, 4786 (2012)

Fish schools

Marchetti *et al*,
RMP **85**, 1143 (2013)

Human crowds

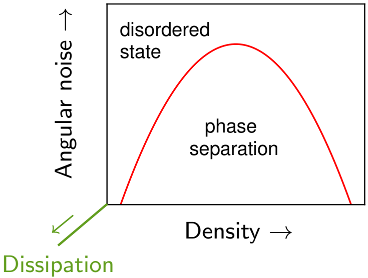
Bain, Bartolo,
Science **363**, 6422 (2019)

Summary and outlook

Classification of active systems

Orientational order even for isotropic particles

Isotropic particles

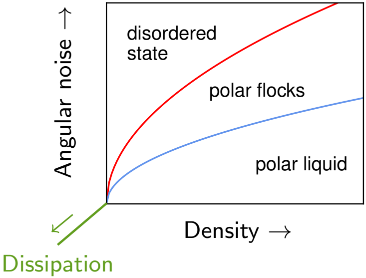


Cates, Tailleur, Annu Rev CMP **6**, 219 (2015)

Nemoto *et al*, PRE **99**, 022605 (2019)

EF *et al*, NJP **22**, 013052 (2020)

Aligning particles



Chaté, Annu Rev CMP **11**, 189 (2020)

Tociu *et al*, PRX **9**, 041032 (2019)





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