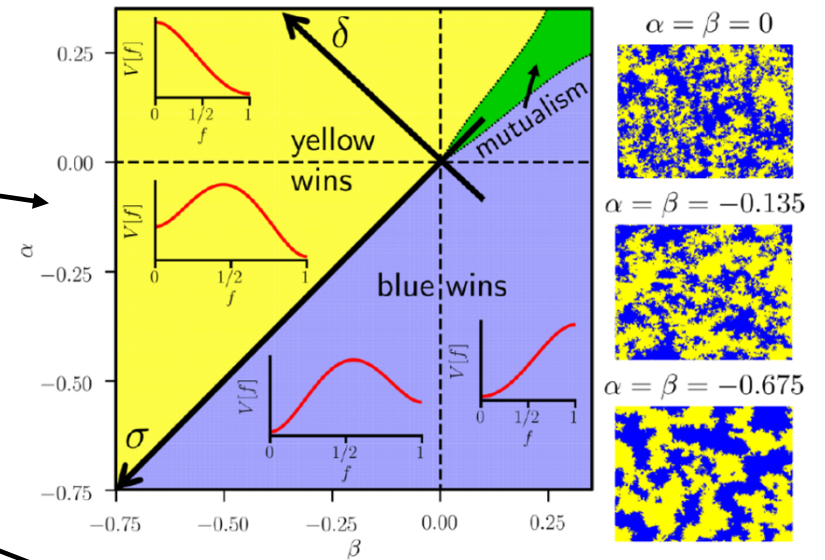
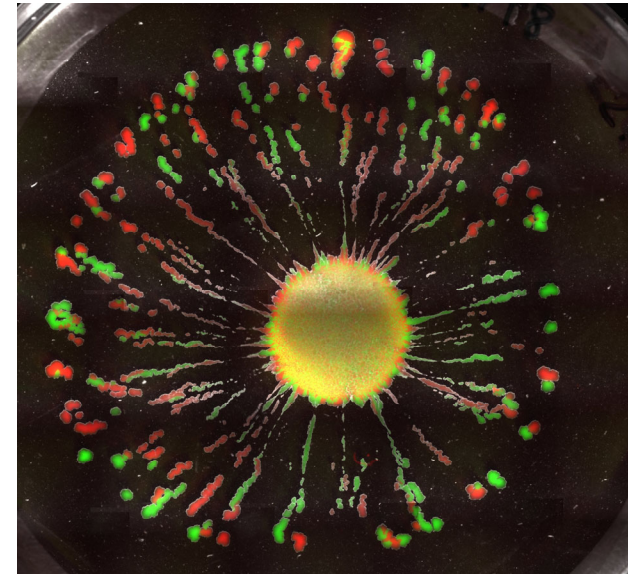
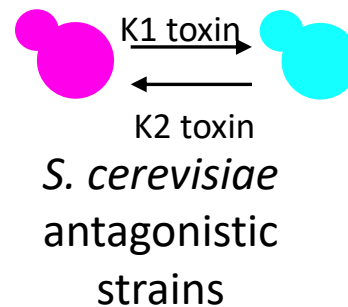
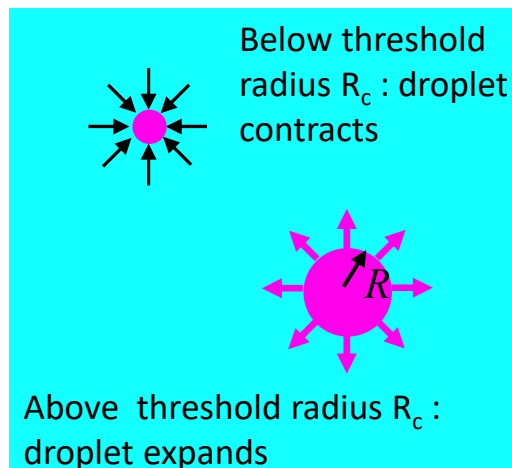


# Active Antagonism: Reproducing Microorganisms and Fluid Flow

- Spatial population genetics of antagonistic organisms
- Fluid mechanics
- Active dynamics, fluid flows & antagonism: spinodal decomposition and nucleation in "Model A"



Simple Darwinian expectations fail...



Severine Atis et al. "Microbial range expansions on liquid substrates." *Physical Review X* 9.2 (2019): 021058.

*“Clash of Genomes”*: Range expansions with neutrality, selective advantage, cooperation or competition

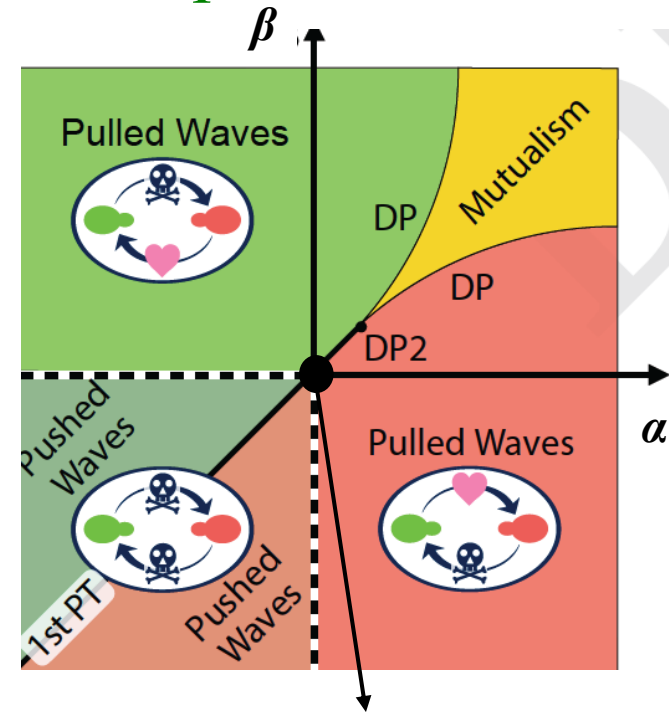
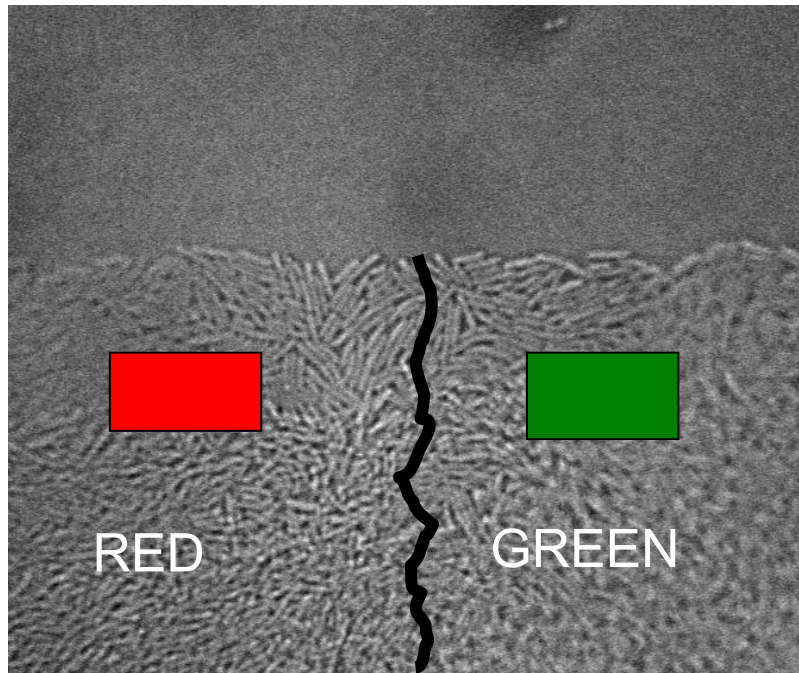
What happens at the interface?



*Collision of two neutral E. coli strains with a heritable genetic markers on a hard agar plate....*

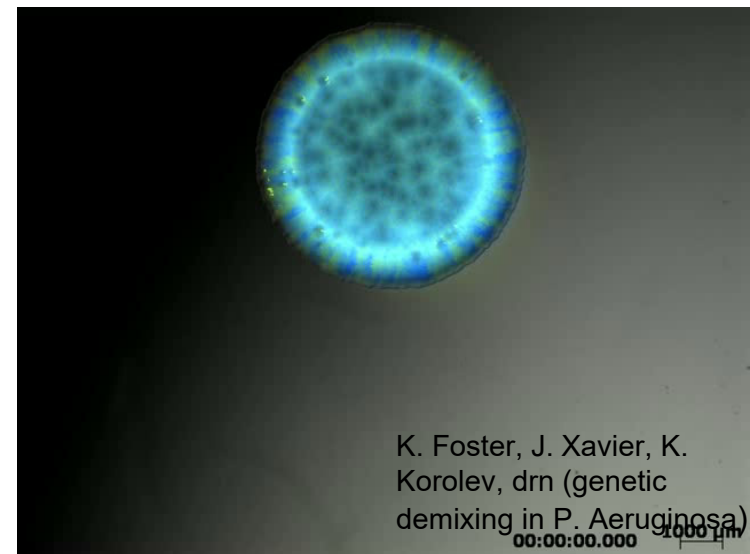
Hernan Garcia,  
Rob Phillips & drn

## Range expansions with competition cooperation, etc.



### Red and Green Bacterial Strains....

1. Could be neutral
2. Could have different doubling times
3. One or both could secrete amino acids useful to the other (mutualism)
4. One or both could secrete toxins that impede the other (competitive exclusion)



# Selective Advantage

assume  $W_G > W_R$  so green as a selective advantage

\* Let  $W_G = \#$  of green offspring in each time step  
 $W_R = \#$  of red offspring in each time step

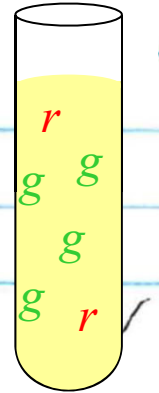
$f_G(t) =$  green fraction  
 $1 - f_G(t) =$  red fraction

$$f_G(t + \tau_g) - f_G(t) = \frac{W_G f_G}{W_G f_G + W_R (1 - f_G)} - f_G = \frac{(W_G - W_R) f_G (1 - f_G)}{\frac{1}{2}(W_G + W_R) + \frac{1}{2}(W_G - W_R)(2f_G - 1)}$$

\* assume  $W$

\* If  $|W_G - W_R| \ll \frac{1}{2}(W_G + W_R)$  (small selective advantage), then

$$\frac{df_G}{dt} \approx s f_G (1 - f_G), \quad s = \frac{2(W_G - W_R)}{\tau_g (W_G + W_R)}$$



In a well-mixed environment: Fisher equation for a pulled genetic wave

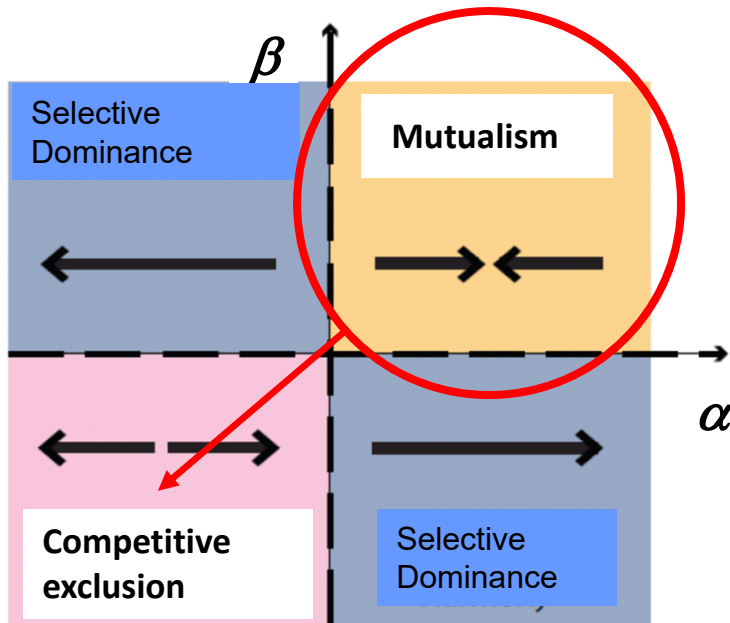
$$f_G(t) = \frac{f_G(0)e^{st}}{1 + f_G(0)(e^{st} - 1)}$$

$$\frac{\partial f_G(x, t)}{\partial t} = D \frac{\partial^2 f_G}{\partial x^2} + s f_G (1 - f_G)$$

# Frequency-Dependent Selection for Mutualists

If  $w_B$  and  $w_Y$  are the number of blue and yellow offspring produced during one generation at a given point on the frontier...

selective advantage  $s \approx 2 \frac{w_B - w_Y}{w_B + w_Y}$



*M. Nowak et al., Nature (2004)*

*J. Gore et al. Nature (2009)*

*E. Frey et al., Phys. Rev. Lett. (2010)*

If  $f(x,t)$  is the yellow fraction,

describe mutualism by...

$$w_Y(x,t) = g + \beta(1 - f(x,t))$$

$$w_B(x,t) = g + \alpha f(x,t)$$

$$s(f) \approx s_0(f^* - f)$$

$$s_0 = (\alpha + \beta) / g$$

$$f^* = \beta / (\alpha + \beta)$$

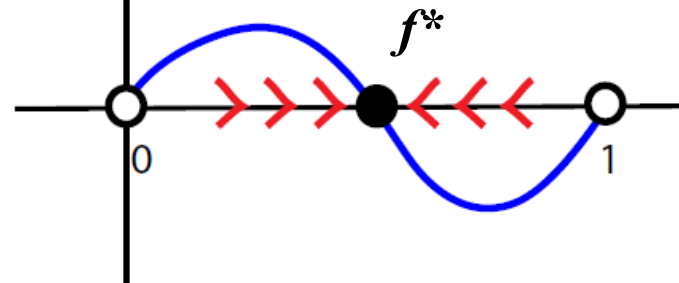
$$\frac{\partial f(x,t)}{\partial t} = D \frac{\partial^2 f(x,t)}{\partial x^2} + s(f)f(1-f)$$

$\alpha, \beta > 0 \rightarrow$  Mutualism,  $s(f) \approx s_0(f^* - f)$

frequency dependent selection:

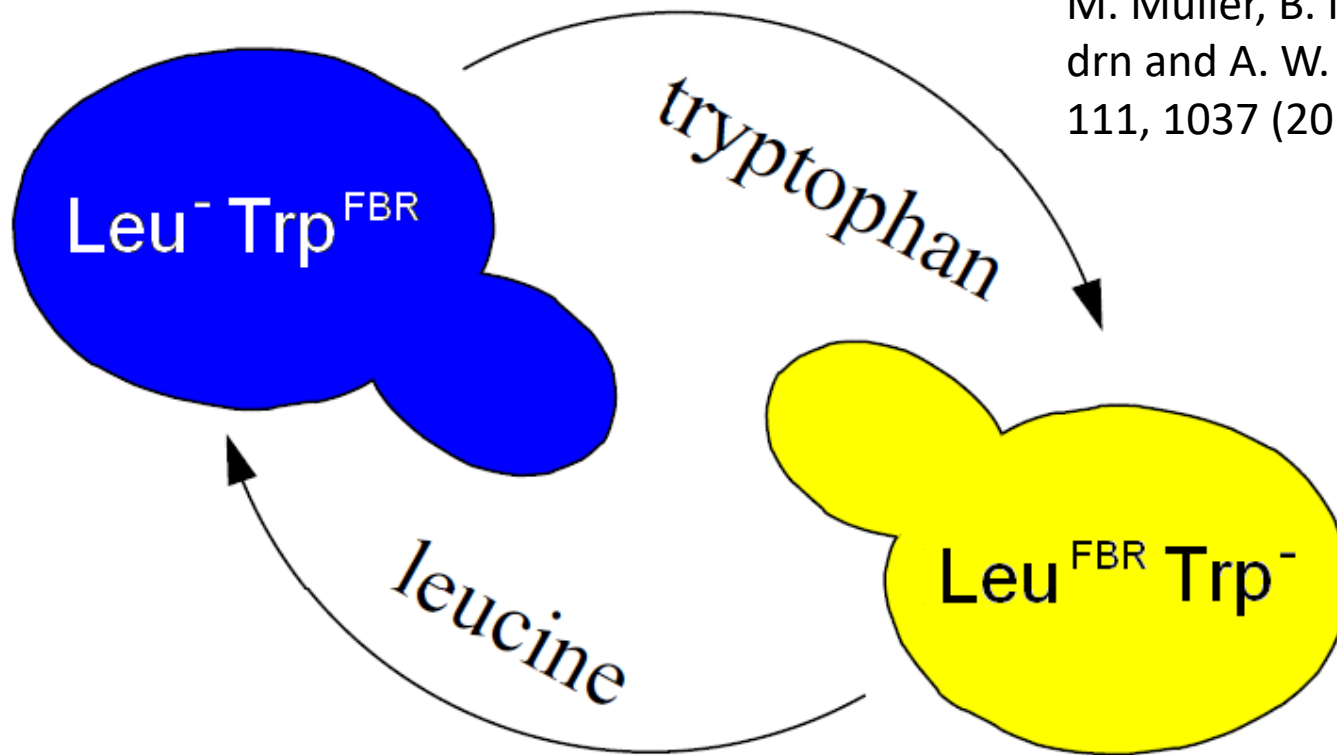
$$\frac{df(t)}{dt} = s_0(f^* - f)f(1-f)$$

“Evolutionarily Stable Strategy”



# Range expansions for mutualists ( $\alpha$ & $\beta > 0$ )

M. Müller, B. I. Neugeboren, drn and A. W. Murray, PNAS 111, 1037 (2014).



*FBR = "feed back resistant"*

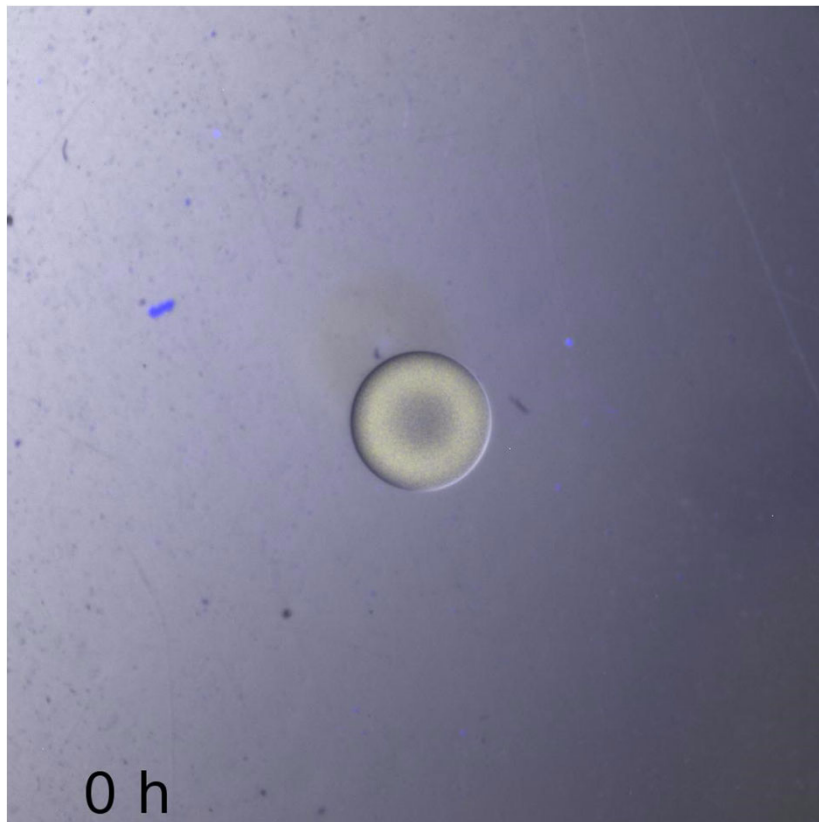
***Mutualism:*** survival requires exchanging amino acids, if leucine and tryptophan are not already present in the natural environment

# Mutualists on various substrates

*CSM (Complete Synthetic Medium;  
abundant Leucine & Trptophan)*

→ *Mutualism unimportant*

$$\alpha = \beta = 0$$

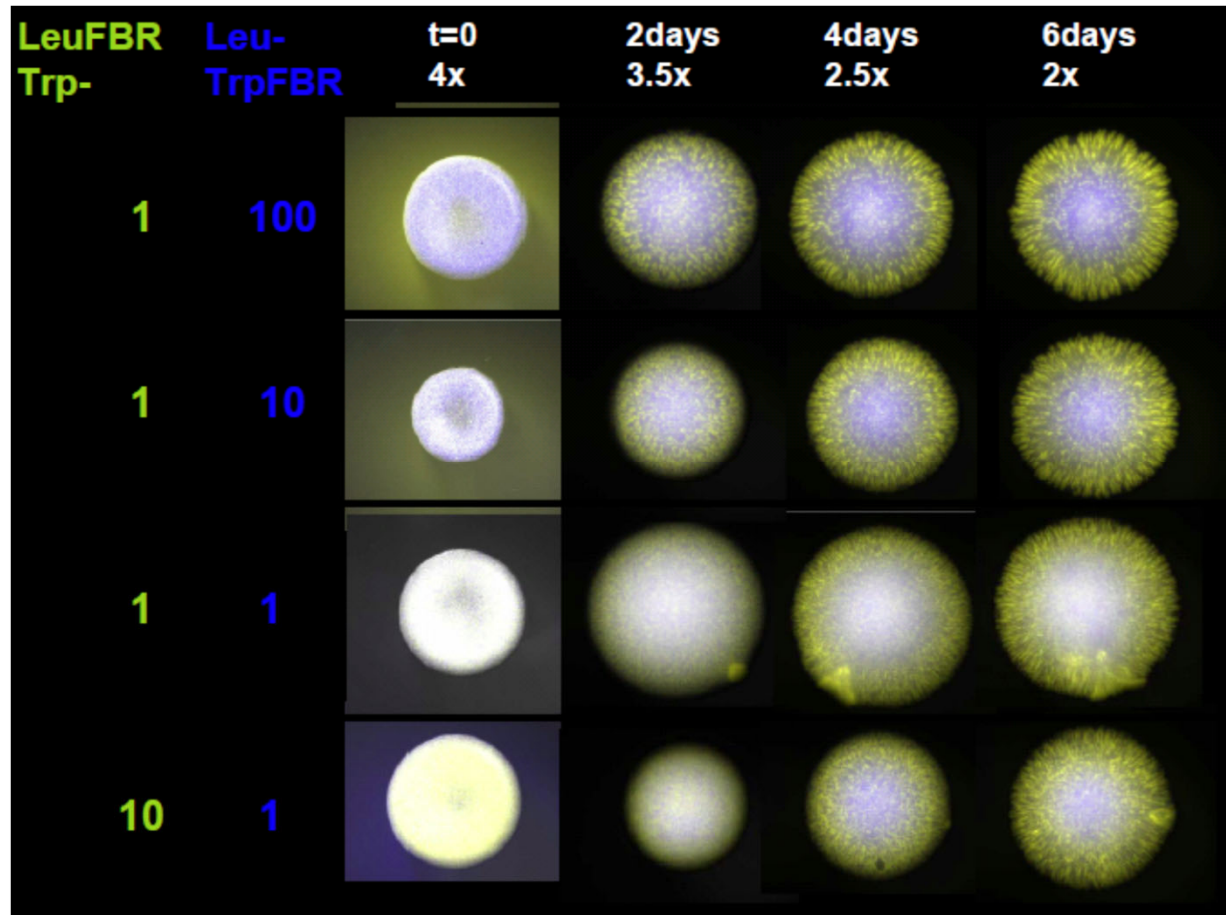


*CSM-Leu-Trp (Leu, Trp  
missing) → obligate mutualism*

$$\alpha, \beta > 0$$



*Experiments reveal that an “evolutionary stable strategy” is indeed reached independent of the initial condition...*



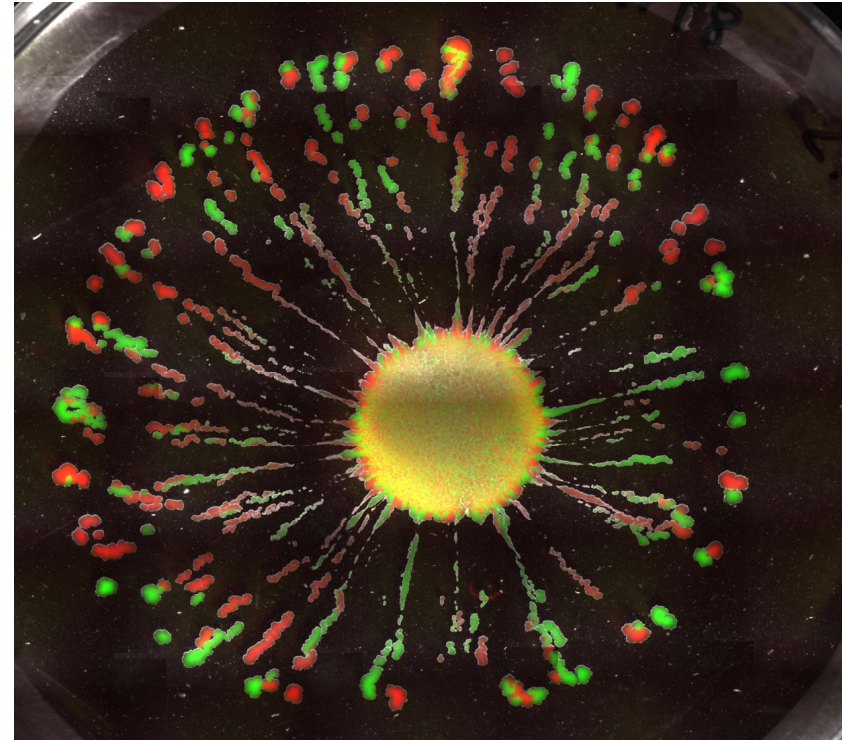
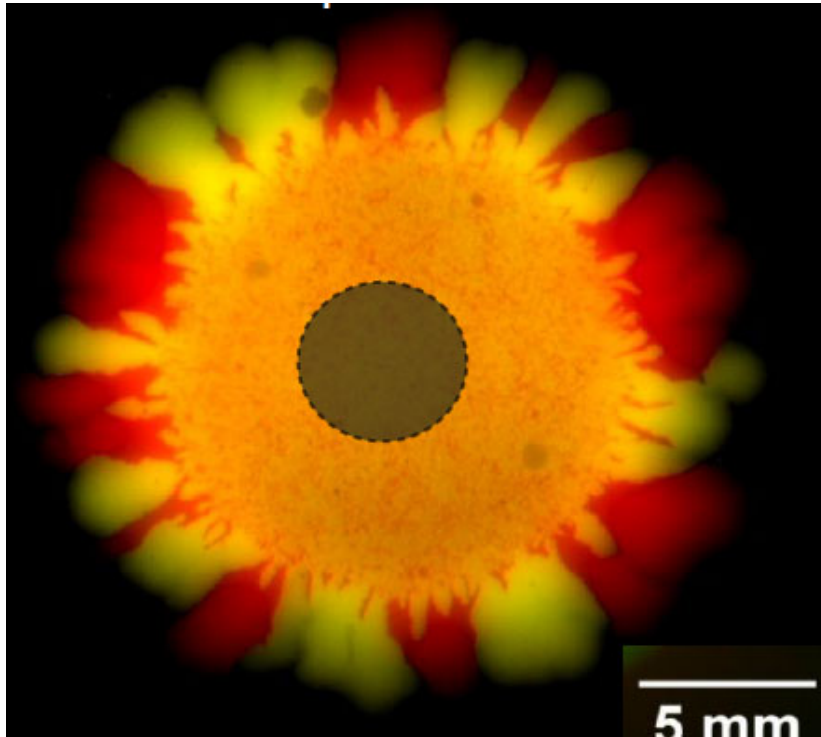
Dynamics at the frontier described by an inflating version of Model A: *K. Korolev, and drn, PRL 10, 88103 (2011)*

$$\frac{\partial f(x,t)}{\partial t} = D \frac{\partial^2 f(x,t)}{\partial x^2} - s_0 f(1-f)(f-f^*) + \sqrt{f(1-f)/N_{\text{eff}}} \zeta(x,t); \quad f^* = \frac{\beta}{\alpha + \beta}, \quad s_0 = (\alpha + \beta) / g$$

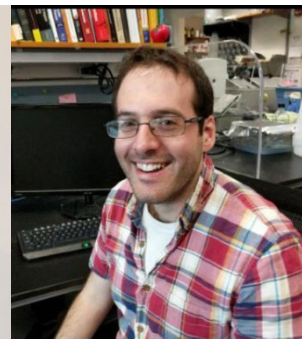
$$\frac{\partial f(x,t)}{\partial t} \approx D \frac{\partial^2 f(x,t)}{\partial x^2} - s_0 \Delta (f - f^*) + \sqrt{\Delta / N_{\text{eff}}} \zeta(x,t), \quad \Delta = f^*(1-f^*), \quad \sim \text{Model A, } T > T_c$$



Now add fluid mechanics: Microorganisms on *liquid* substrates

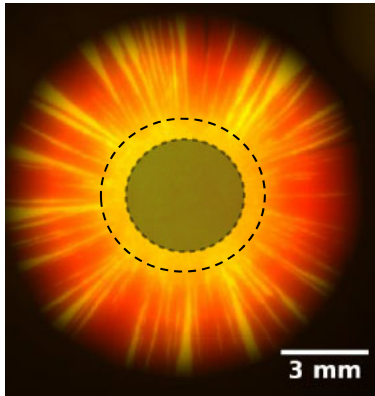


Severine Atis  
Bryan Weinstein  
Andrew Murray



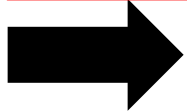
*Microorganisms grown on liquid but highly viscous substrates create their own flows (without pumps and syringes...)*

Hard Agar

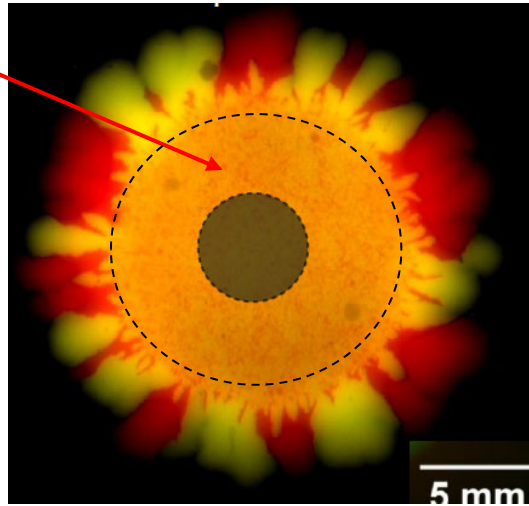


*Genetic demixing of yeast on a 1% hard agar YPD plate (viscosity  $\eta = \infty$ )*

*Epoch of genetic demixing stretched out....*



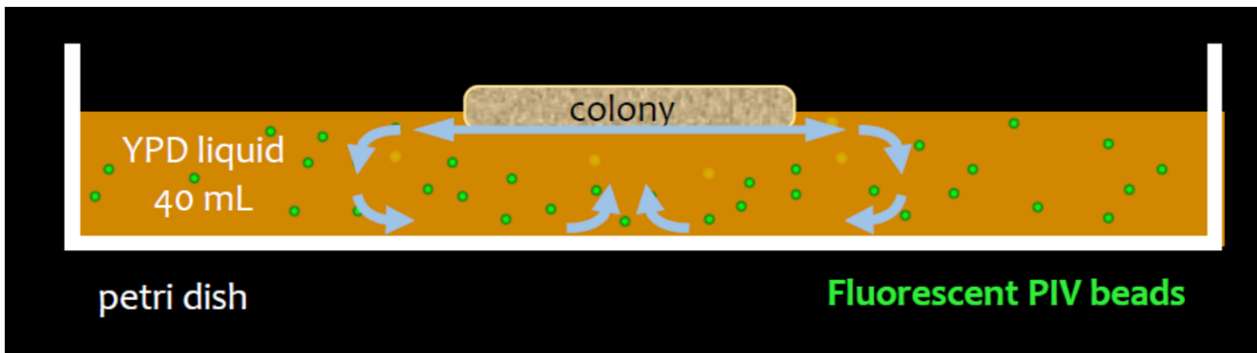
Liquid Media



*Yeast on a liquid but highly viscous YPD (yeast/peptone/dextrose) media with 3% cellulose ( $\eta \approx 600 \text{ Pa-s}$ )*

Cellulose % (w/v)	Viscosity (Pa·s)
1.8	$22 \pm 3$
2.0	$51 \pm 6$
2.2	$81 \pm 9$
2.4	$120 \pm 10$
2.6	$340 \pm 50$

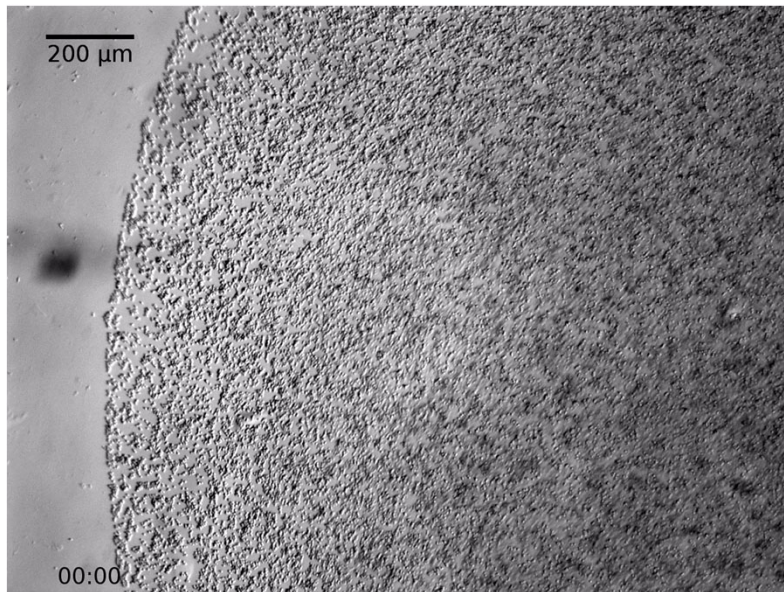
*(the viscosity of water is  $\eta \approx 10^{-3} \text{ Pa-s}$ ; our viscosities are  $10^4 - 10^5$  times larger)*



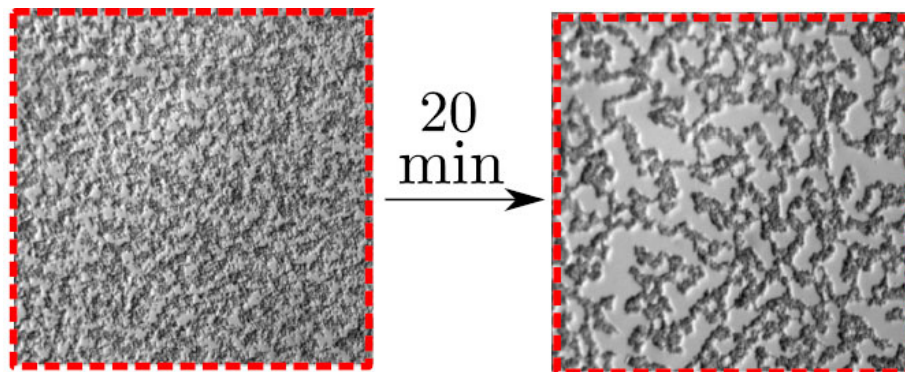
*The colony metabolism generates flows that dilate the growing cell mass radially!*

*After inoculation, reproducing microorganisms on liquid substrates can behave like gases, liquids or solids....*

*At very early times, the yeast cells exhibit gas-liquid phase separation*



*D. Vella and L. Mahadevan, American Journal of Physics 73.9 (2005): 817-825.*

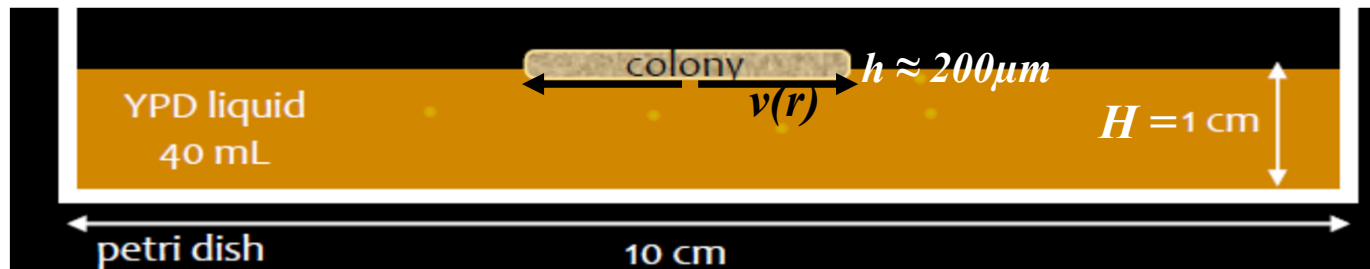


*Coarsening or “spinodal decomposition”....*

*Deformations of features inside colony in a liquid-like regime consistent with a dilational flow ( $\eta = 600 \text{ Pa}\cdot\text{sec}$ )*



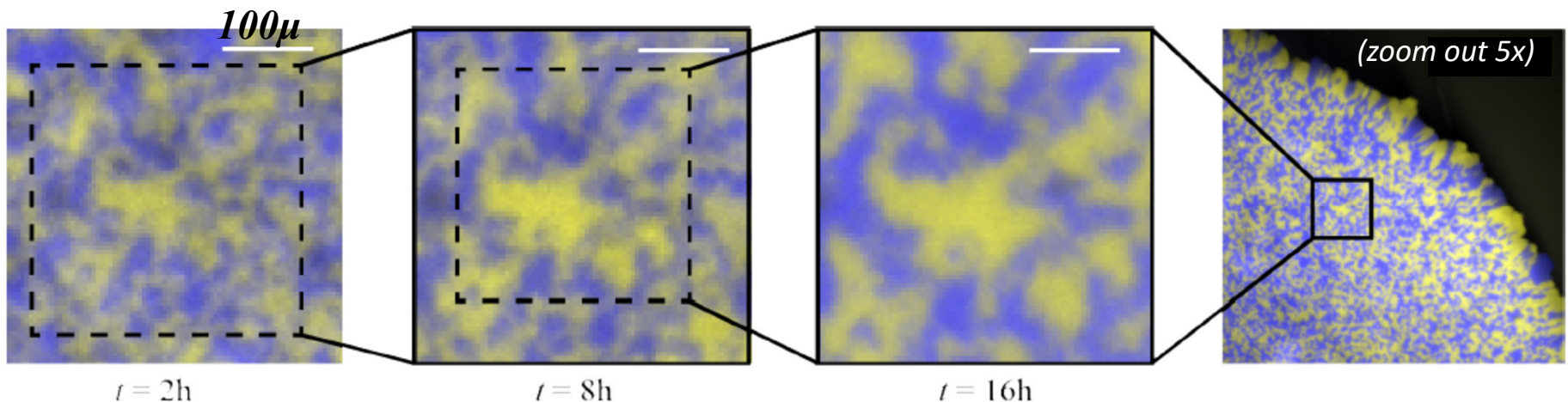
# Colony features dilate as if inscribed on an inflating balloon....



Simple model of 2d colony dynamics:  $\frac{\partial \rho_{2d}}{\partial t} + \vec{\nabla} \cdot (\rho_{2d} \vec{v}_{2d}) = \alpha_1 \rho_{2d}$ ,  $\rho_{2d}$  = cell density

$\alpha_1$  = growth rate  $\rightarrow \vec{\nabla} \cdot \vec{v}_{2d}(r) = \alpha_1$ ; assume overdamped liquid-like colony dynamics:

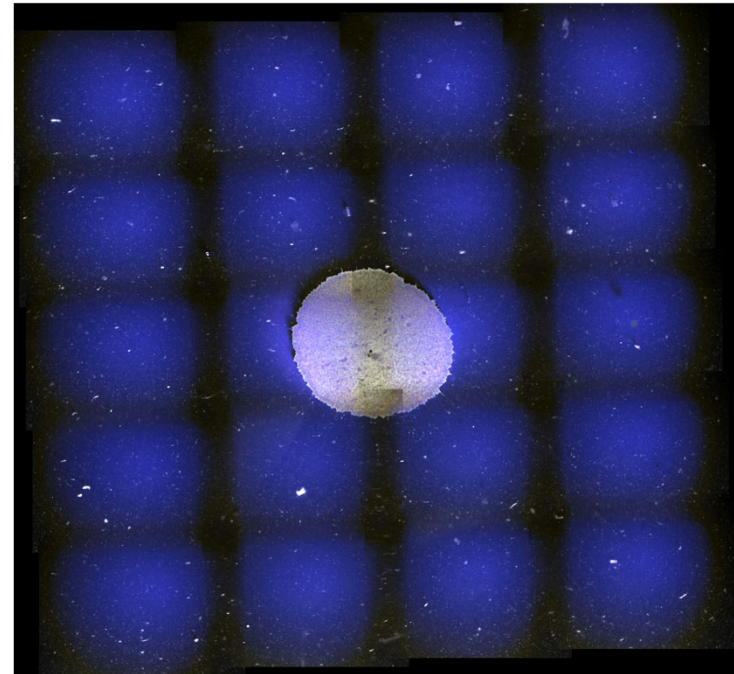
$0 \approx -\vec{\nabla} p_{2d} - \gamma \vec{v}(\vec{r})$ ;  $\gamma = \eta_s / hH$ ;  $\rightarrow \boxed{\vec{v}_{2d}(\vec{r}) \approx \alpha_1 r \hat{r} / 2}$  dilational velocity field



The first three images have the same scale bar = 100  $\mu\text{m}$ . The final picture, with scale bar 500  $\mu\text{m}$ , shows the same feature at the much larger colony scale

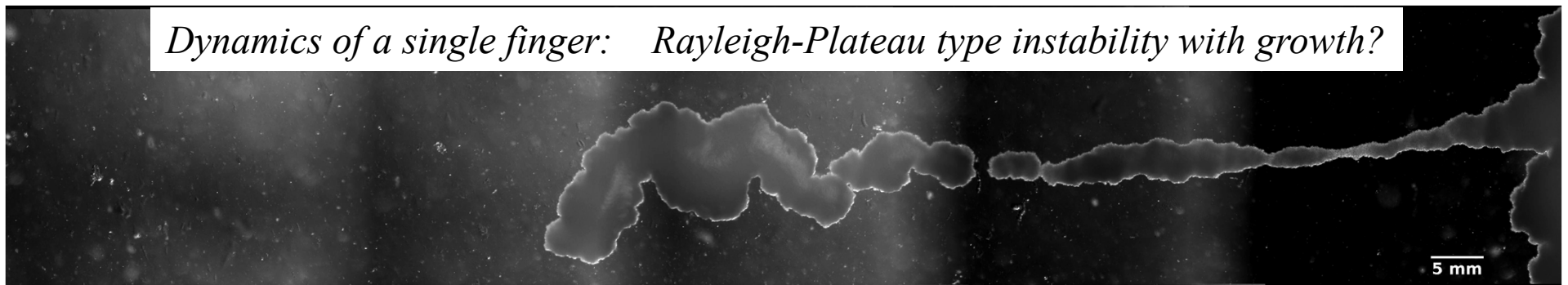
*Enhancing the radial flow field...*

*(moderate substrate  
viscosity  $\eta \approx 450 \text{ Pa}\cdot\text{s}$ )*



*Liquid-like fingering instabilities*

*Dynamics of a single finger: Rayleigh-Plateau type instability with growth?*



# Now add antagonism: Pushed genetic waves generated by antagonistic interactions



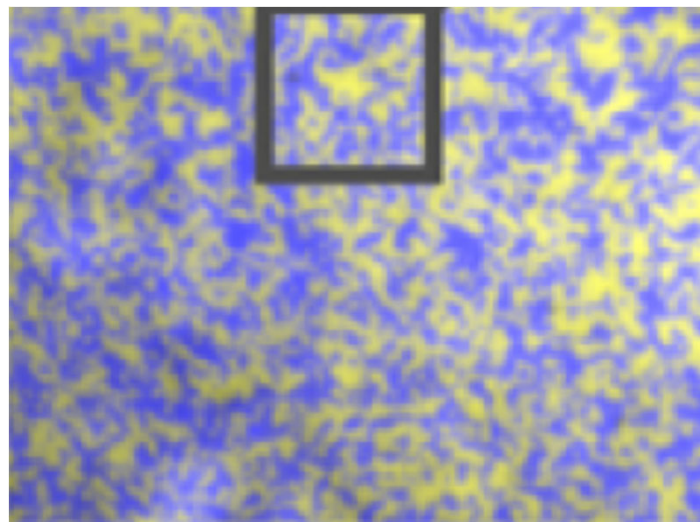
Andrea Giometto

- *For pushed genetic waves, a “critical nucleus” is required to excite the wave to get it started*
- *Can we detect the existence of a critical nucleus with killer yeast or bacteria strains?*



Andrew Murray

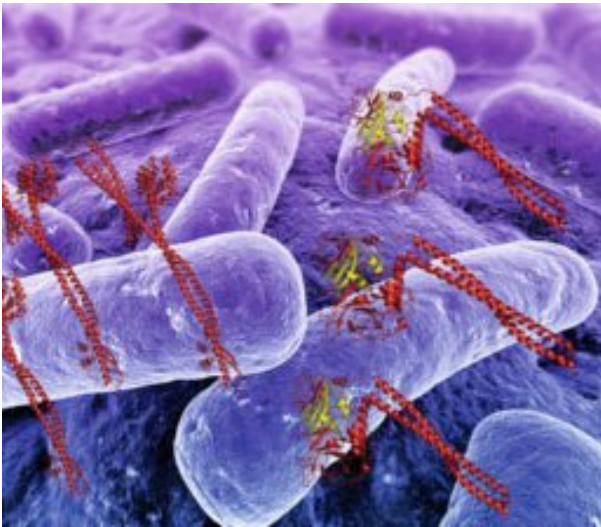
- *Spinodal decomposition of antagonists?*



Max Lavrentovich  
Univ. of Tennessee

# Antagonistic microbial interactions

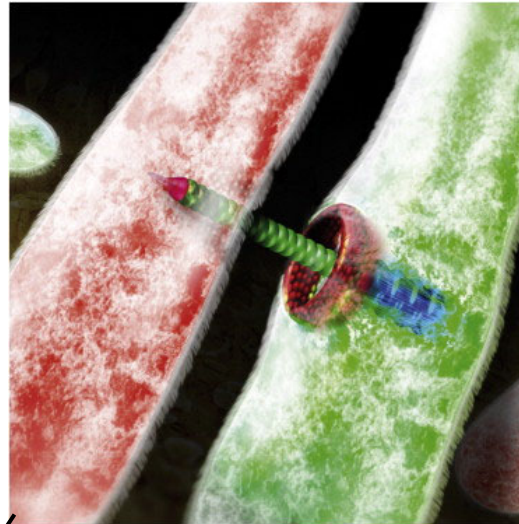
Diffusive antagonism



**Bacteriocins or peptidic toxins**

Image from <https://geneticliteracyproject.org>

Local antagonism



**Type VI secretion systems**

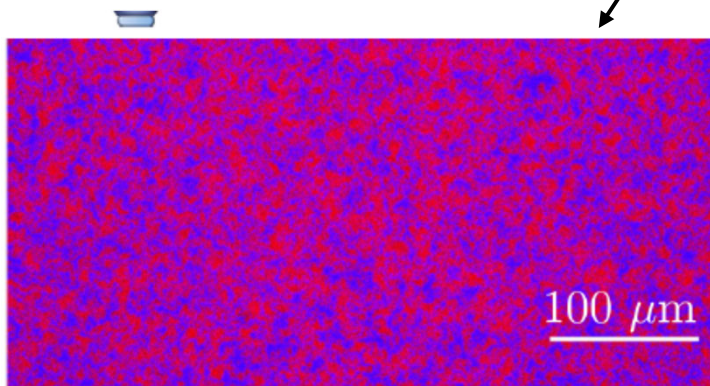
Image from Brunet et al (2013), Cell Reports 3(1)

Diffusive antagonism



**Killer yeast viruses**

*Saccharomyces cerevisiae*

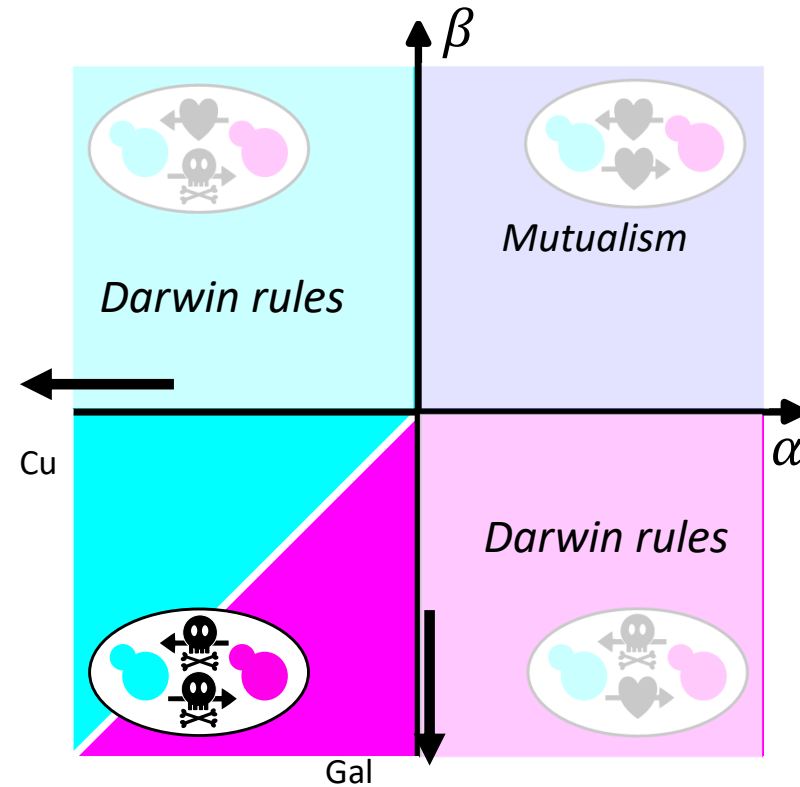
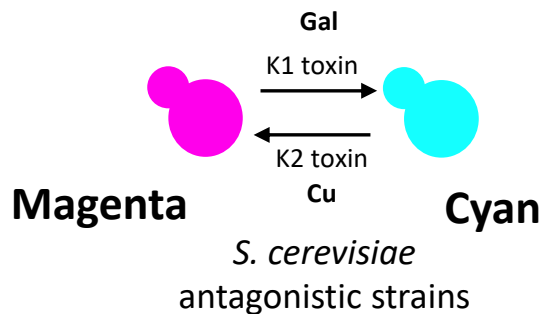
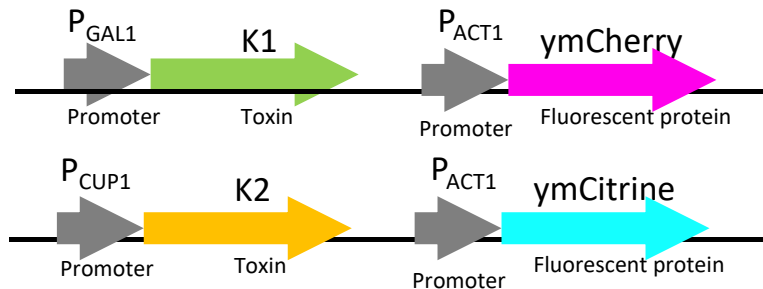
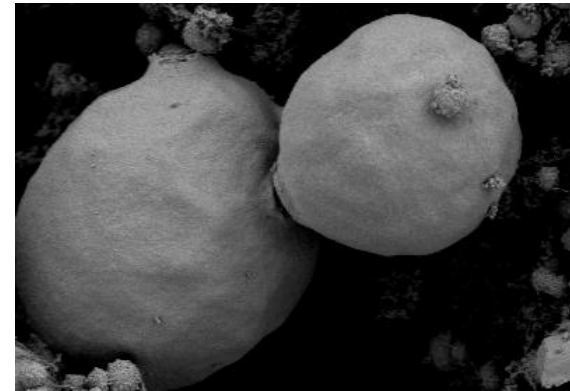


*L. McNally, et al. Nature Communications 8, 1 (2017) (Approximately modelled using a kinetic Ising model....)*



# Killer yeast (*S. cerevisiae*) genetic engineering

Express the toxin-encoding gene from the genome of *S. cerevisiae* under the control of titratable, inducible promoters, add fluorescent proteins to distinguish the two strains



(Galactose metabolic pathway knocked out; galactose acts only as a inducer....) 17

# A course-grained description:

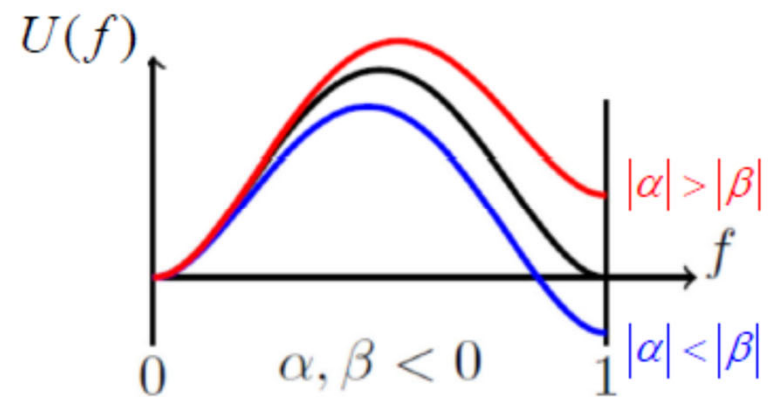
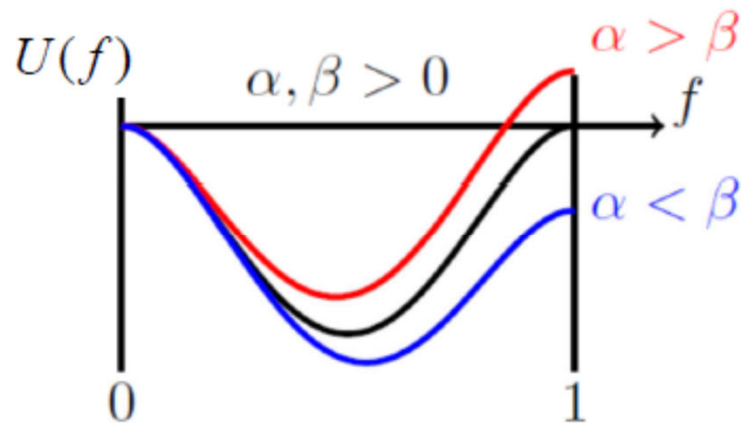
# Model A'

Consider an allele fraction  $f \equiv f(\mathbf{x}, t)$  at position  $\mathbf{x}$  and time  $t$ . Then:

$$\partial_t f = D_s \nabla_{\mathbf{x}}^2 f + \frac{f(1-f)}{\tau_g} \left[ (\alpha + \beta) \left( \frac{1}{2} - f \right) + \frac{\alpha - \beta}{2} \right] + \sqrt{\frac{2f(1-f)}{N\tau_g}}$$

$$= D_s \nabla_{\mathbf{x}}^2 f - \frac{1}{\tau_g} \frac{dU(f)}{df} + \sqrt{D_g f(1-f)} \xi \quad \text{with number fluctuations:}$$

$$\langle \xi(\mathbf{x}, t) \rangle = 0 \quad \text{and} \quad \langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle = \delta(t - t') \delta(\mathbf{x} - \mathbf{x}').$$



$D_s$  = spatial diffusion constant  $\sim$  motility

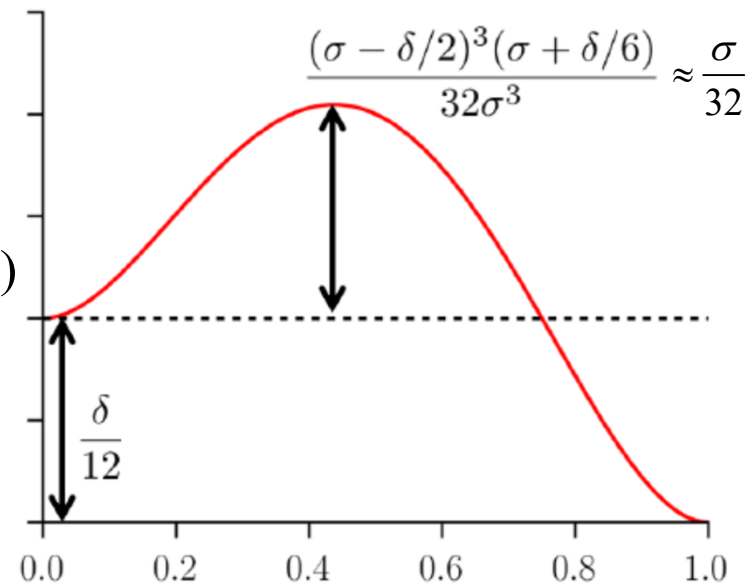
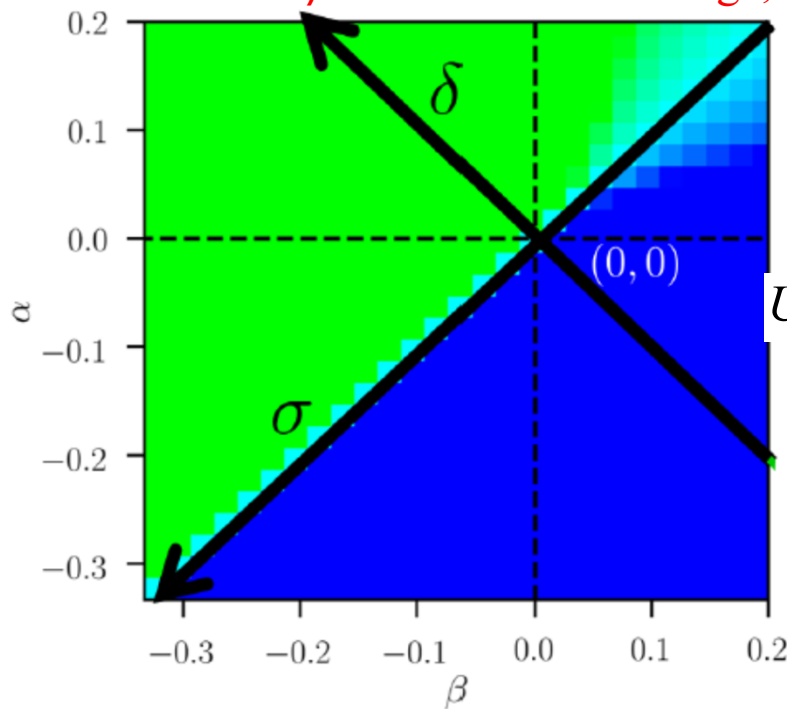
*Note double well potential with minima at  $f = 0$  and  $f = 1$*

# A convenient reparameterization (for competitive exclusion)

To study the antagonistic interactions in the third quadrant, we rotate our coordinates to  $\sigma = -(\alpha + \beta)/2$  and  $\delta = \alpha - \beta$ :

$$\partial_t f = D_s \nabla^2 f + f(1-f) \left[ \sigma(2f-1) + \frac{\delta}{2} \right] + \sqrt{D_g f(1-f)} \xi \quad (1)$$

$\delta = \alpha - \beta =$  selective advantage;  $\sigma = (\alpha + \beta)/2 =$  "antagonism"

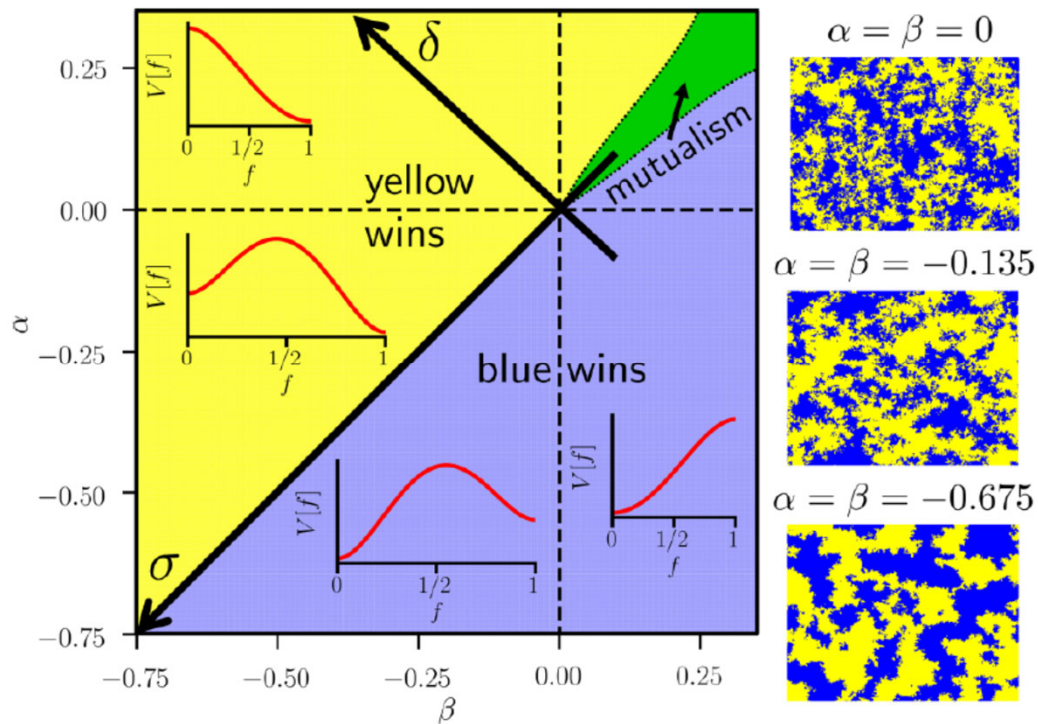


**Model A' = Model A with nonlinear noise....**

# Spinodal decomposition for model A'

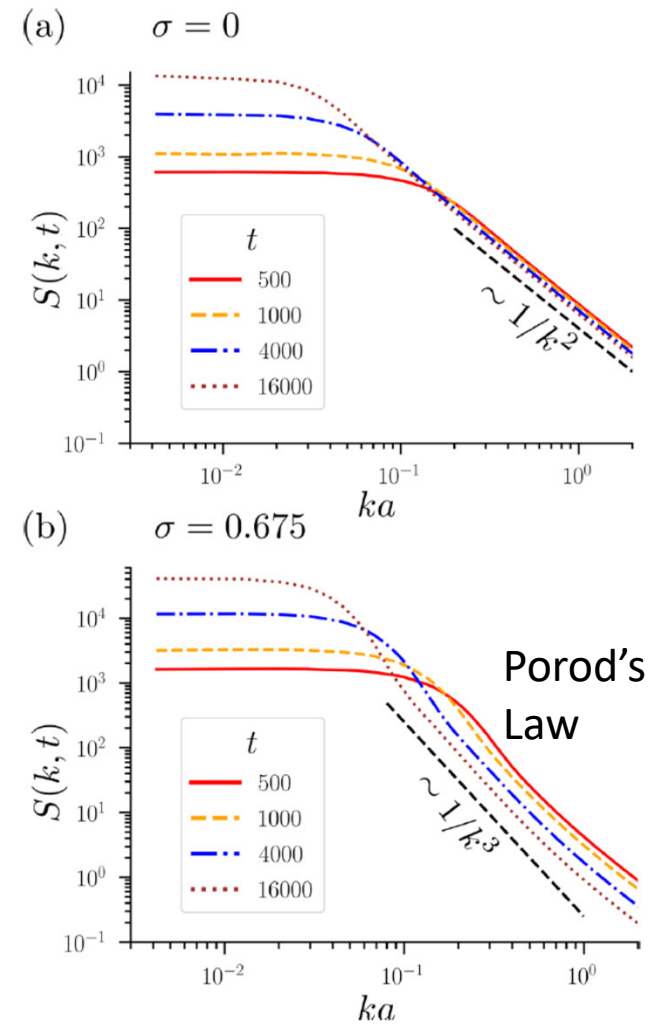
$$S(k, t) = 4 \sum_{\vec{r}'} \left\langle \left( f(\vec{r}, t) - \frac{1}{2} \right) \left( f(\vec{r} + \vec{r}', t) - \frac{1}{2} \right) \right\rangle_{\vec{r}} e^{i\vec{k} \cdot \vec{r}'}$$

= structure function



$\delta = \alpha - \beta =$  selective advantage

$\sigma = -(\alpha + \beta) / 2 =$  "antagonism"

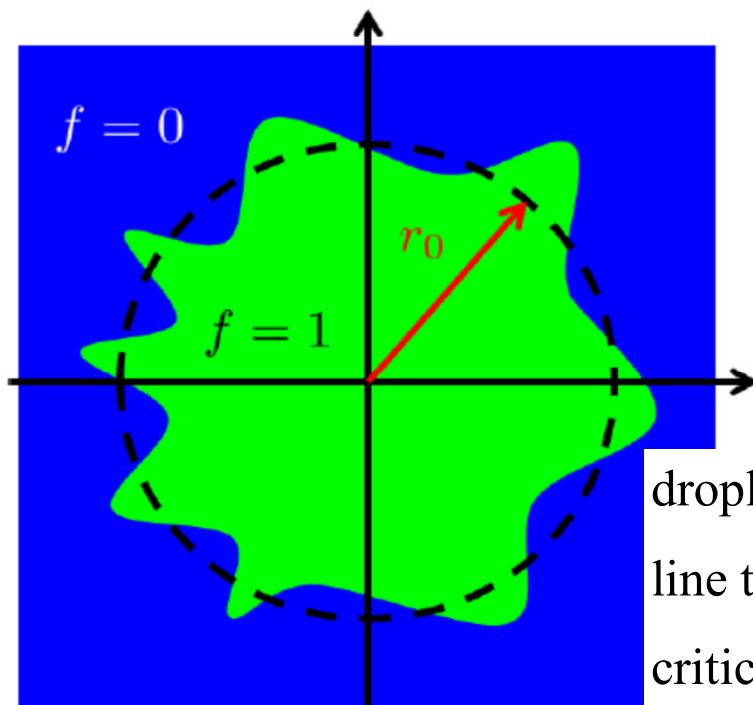


## Nucleation and growth for $\delta > 0, \sigma \geq 0$

Suppose we look at droplets with  $0 < \delta \ll \sigma$ . The “energy” of such a drop is

$$E[f] = \int d\mathbf{x} \left[ \frac{D_s}{2} (\nabla f)^2 + U(f) \right]$$

We can look for minimal  $E[f]$  solutions that look like droplets:



$$\nabla^2 f = -\frac{\sigma}{D_s} f(1-f)(2f-1)$$



$$f(r) = \frac{1}{2} - \frac{1}{2} \tanh \left[ \sqrt{\frac{\sigma}{4D_s}} (r - r_0) \right]$$



droplet energy  $E(r_0) = 2\pi\gamma r_0 - \pi c r_0^2$ ,

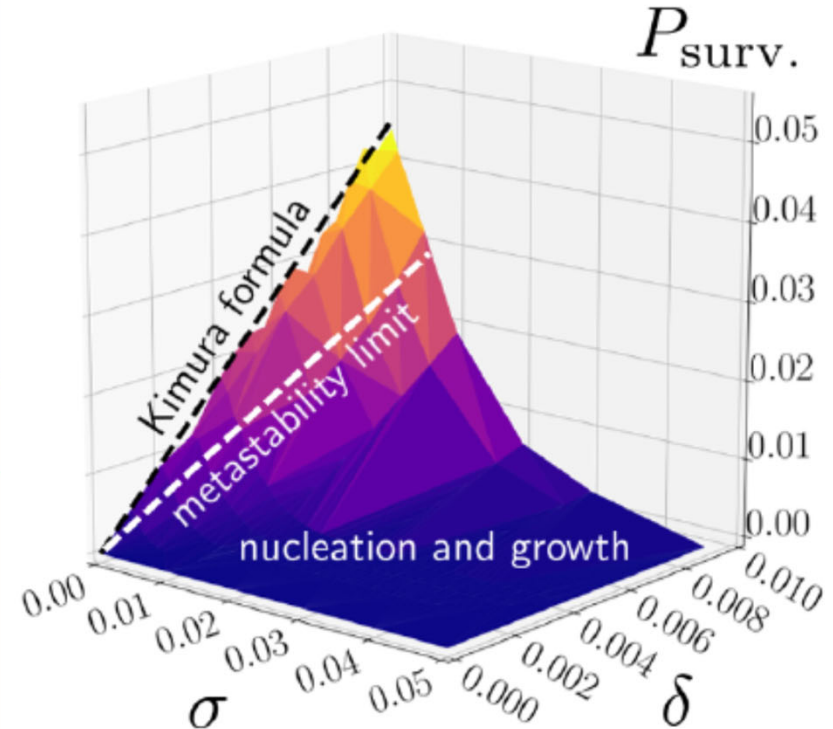
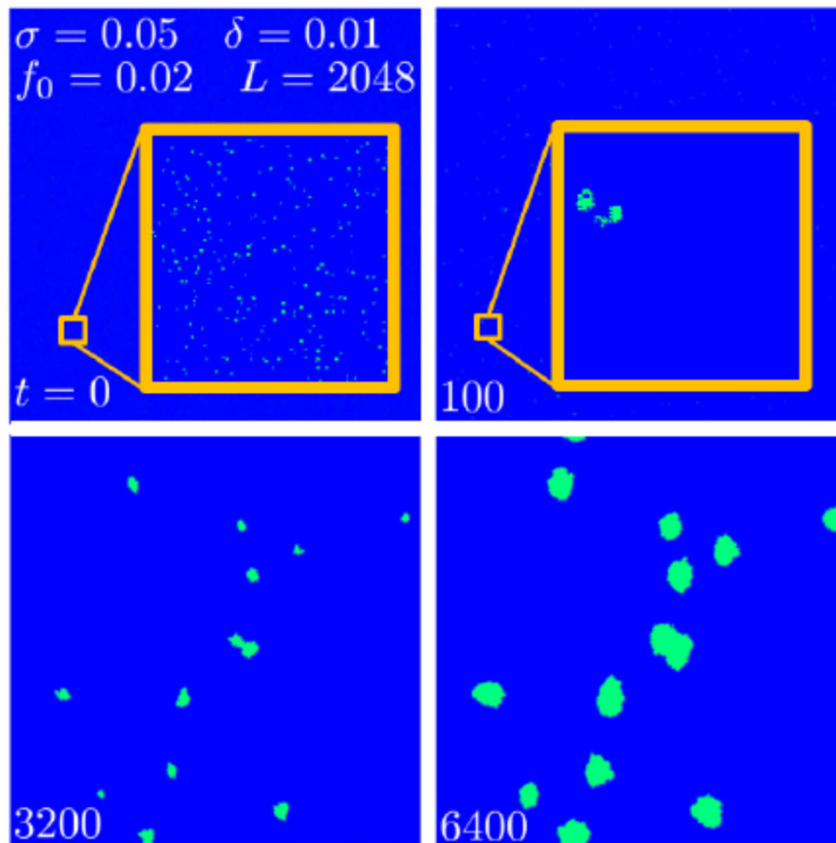
line tension  $\gamma = (2/3)\sqrt{\sigma/D}$ , cond. energy  $c = \delta/3D$

critical droplet size  $= R_c = \gamma/c = (2/\delta)\sqrt{D\sigma}$

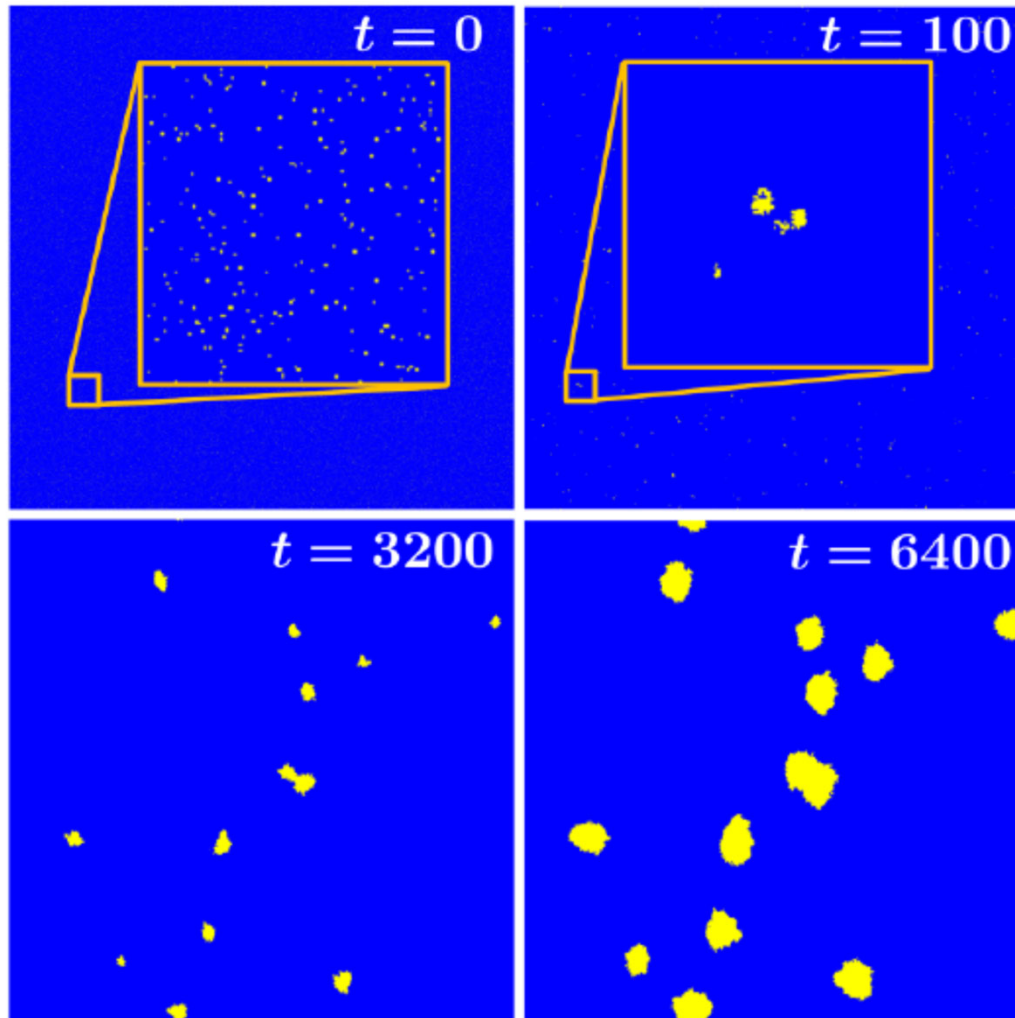
# Green cells must overcome line tension to grow

$$\sigma = -(\alpha + \beta)/2, \quad \delta = \alpha - \beta, \quad \delta \ll \sigma \ll 1$$

*Nucleation theory ideas can be used to compute the survival probability in this limit*

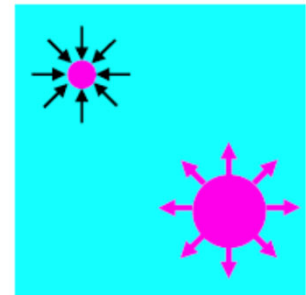
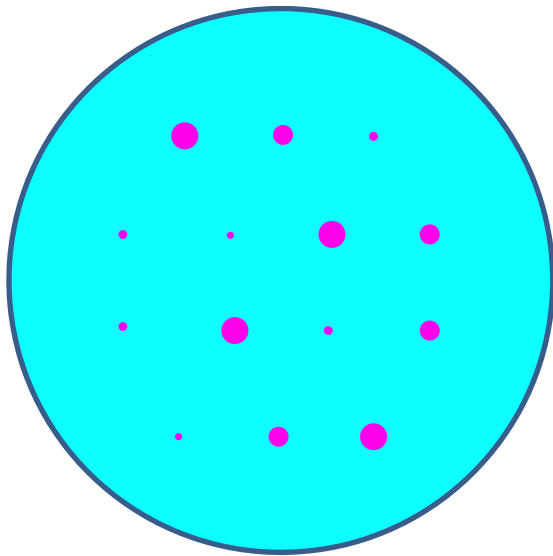
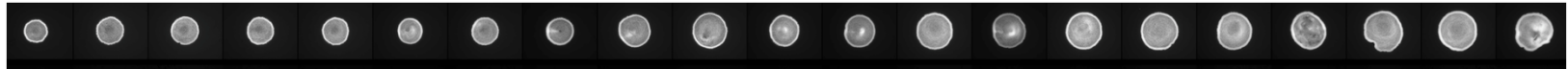


**Yellow cells enjoy a selective advantage, but must exploit number fluctuations and then overcome a line tension to grow....**



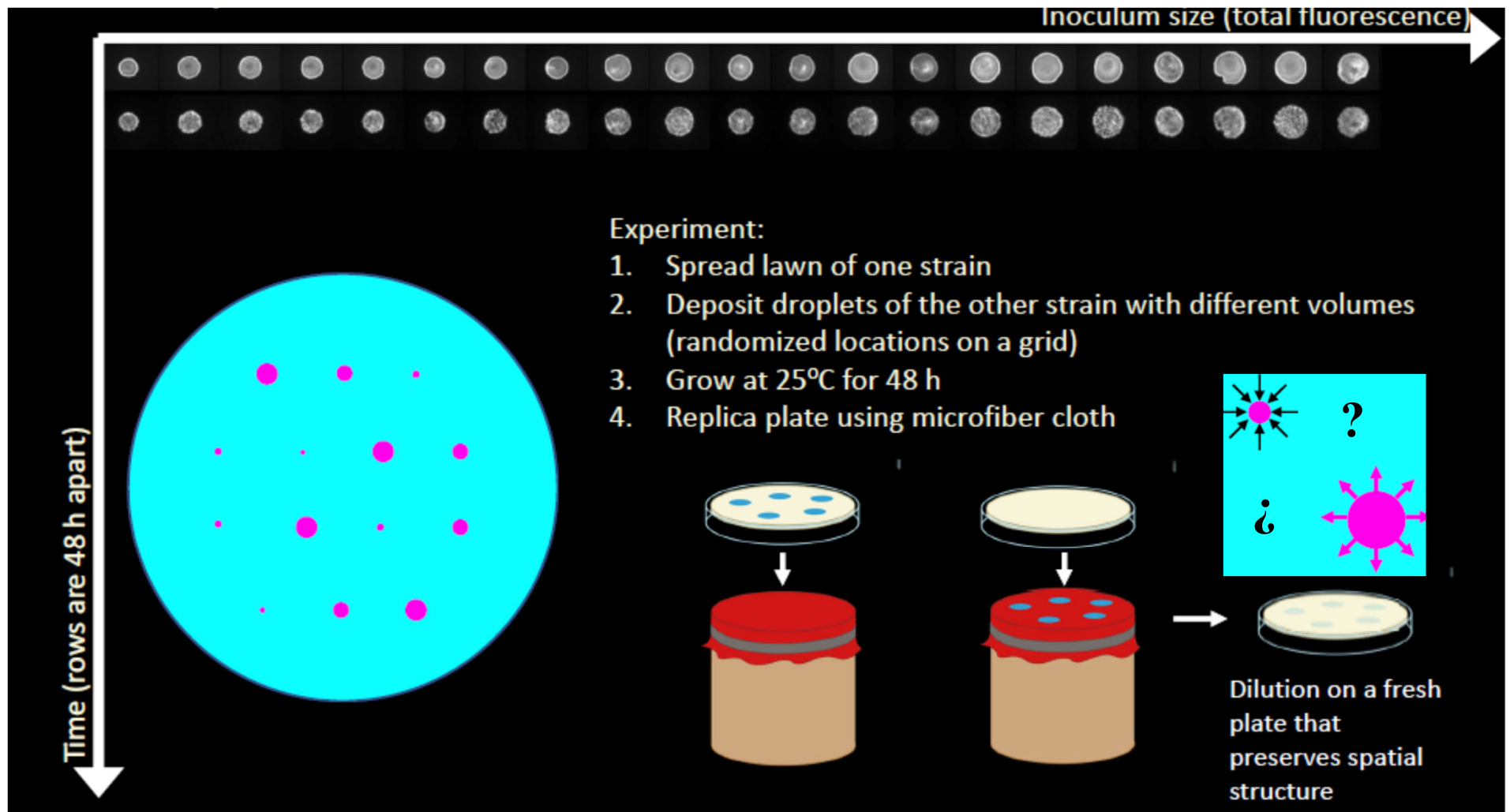
“The fate of  
the false  
vacuum”

# Experimental test of nucleation dynamics for killer yeast (A. Giometto, A. Murray)





# Experimental test of nucleation dynamics for killer yeast (A. Giometto, A. Murray)

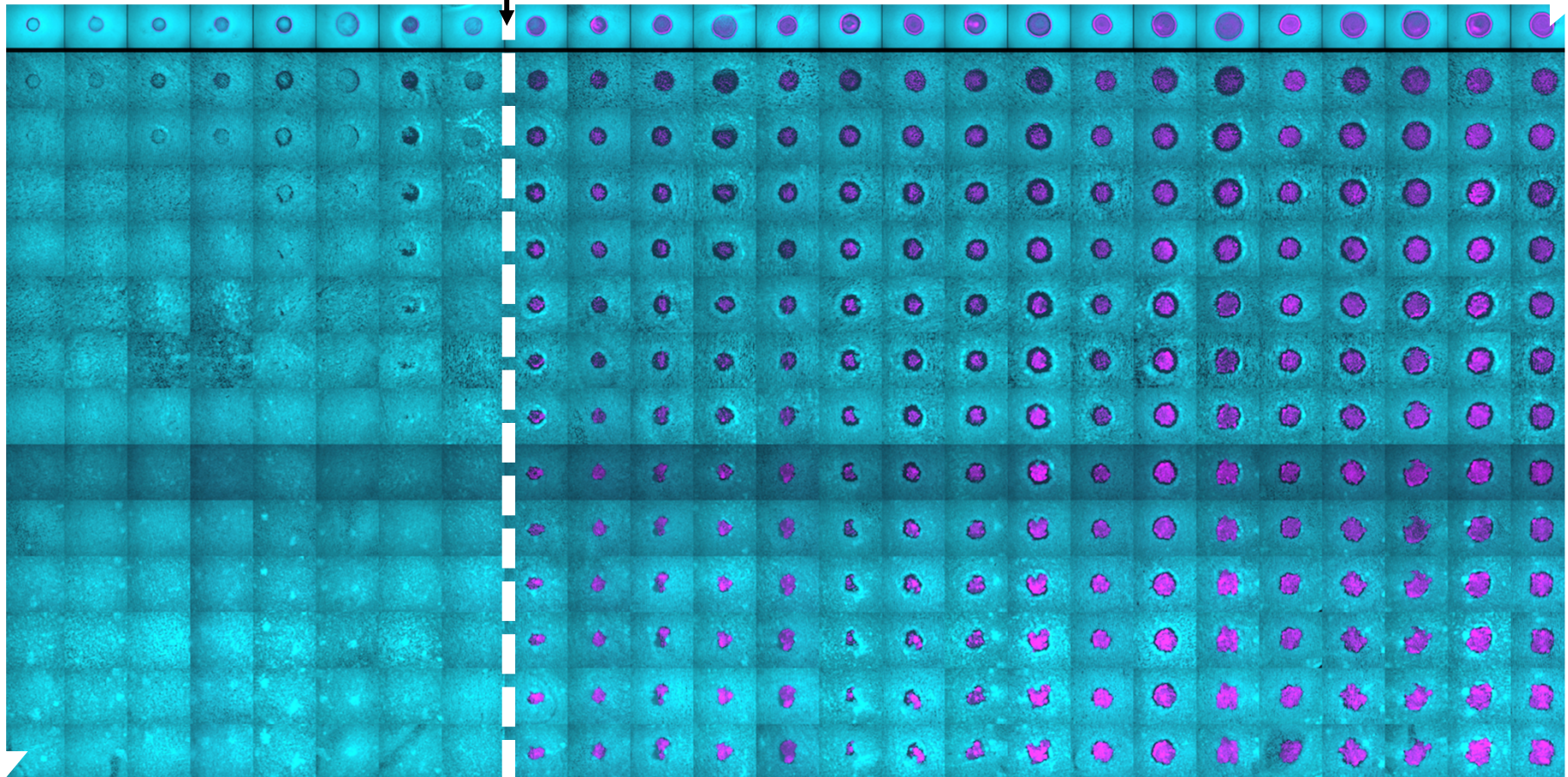


# Experimental test of nucleation dynamics for killer yeast (A. Giometto, A. Murray)

Stronger killer

Critical droplet radius

weaker killer lawm



*What happens when Model A' dynamics is subjected to fluid flow?*

# Nucleation in a flow: Simplified reaction-diffusion model of antagonistic organisms

(Xaiojue Zhu, R. Benzi, drn and F. Toschi)

$$\varepsilon_A \leftrightarrow \alpha; \quad \varepsilon_B \leftrightarrow \beta$$

Governing equations

*go to "board"*

$$\frac{\partial c_A}{\partial t} + \nabla \cdot (\mathbf{u}c_A) = D\nabla^2 c_A + c_A(1 - c_A - c_B + \epsilon_A c_B)$$

$$\frac{\partial c_B}{\partial t} + \nabla \cdot (\mathbf{u}c_B) = D\nabla^2 c_B + c_B(1 - c_B - c_A + \epsilon_B c_A)$$

**Flow**

$$u_x(x, y) = F[\alpha \sin(2\pi x/L) + (1 - \alpha) \sin(2\pi y/L)]$$

$$u_y(x, y) = F[\alpha \sin(2\pi y/L) + (1 - \alpha) \sin(2\pi x/L)]$$

**Parameters**

$$D = 10^{-4}, L = 1, \alpha = 0, \epsilon_A = -0.2, \epsilon_B = -0.3$$

# Nucleation in a flow: Simplified reaction-diffusion model of antagonistic organisms

(Xaiojue Zhu, R. Benzi, drn and F. Toschi)

$$\varepsilon_A \leftrightarrow \alpha; \quad \varepsilon_B \leftrightarrow \beta$$

$$\frac{\partial c_A}{\partial t} + \nabla \cdot (\mathbf{u}c_A) = D\nabla^2 c_A + c_A(1 - c_A - c_B + \varepsilon_A c_B)$$

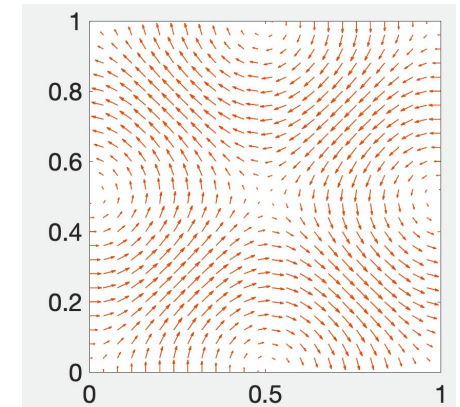
$$\frac{\partial c_B}{\partial t} + \nabla \cdot (\mathbf{u}c_B) = D\nabla^2 c_B + c_B(1 - c_B - c_A + \varepsilon_B c_A)$$

$$u_x(x, y) = F[\alpha \sin(2\pi x/L) + (1 - \alpha) \sin(2\pi y/L)]$$

$$u_y(x, y) = F[\alpha \sin(2\pi y/L) + (1 - \alpha) \sin(2\pi x/L)]$$

$$\sigma = -(\varepsilon_A + \varepsilon_B) / 2 = \text{antagonism}$$

$$\delta = \varepsilon_A - \varepsilon_B = \text{selective advantage}$$



$$\alpha = 0$$

# DYNAMICS OF TOTAL $c_T$ & A-FRACTION $f$

①

Scalar

①  $\frac{\partial c_A}{\partial t} + \nabla \cdot (u c_A) = D \nabla^2 c_A + c_A(1 - c_A - c_B + \epsilon_{ACB})$

②  $\frac{\partial c_B}{\partial t} + \nabla \cdot (u c_B) = D \nabla^2 c_B + c_B(1 - c_B - c_A + \epsilon_{BCA})$

Flow

$u_x(x, y) = F[\alpha \sin(2\pi x/L) + (1 - \alpha) \sin(2\pi y/L)]$

$u_y(x, y) = F[\alpha \sin(2\pi y/L) + (1 - \alpha) \sin(2\pi x/L)]$

Parameters

$D = 10^{-4}, L = 1, \alpha = 0, \epsilon_A = -0.2, \epsilon_B = -0.3$

$\epsilon_A, \epsilon_B < 0$

\* Change of variables, let  $c_T = c_A + c_B, f = \frac{c_A}{c_A + c_B} = \frac{c_A}{c_T}$   
 add ① & ②  $\Rightarrow \frac{\partial}{\partial t} (c_A + c_B) + \vec{\nabla} \cdot (\vec{u} c_T) = D \nabla^2 c_T + c_T(1 - c_T) + (\epsilon_A + \epsilon_B) c_A c_B$   $\frac{c_B}{c_T} = 1 - f$

$\frac{\partial}{\partial t} (c_T) + \vec{\nabla} \cdot (\vec{u} c_T) = D \nabla^2 c_T + c_T(1 - c_T) + (\epsilon_A + \epsilon_B) c_T^2 f(1 - f)$

$\sigma = -\frac{\epsilon_A + \epsilon_B}{2}$

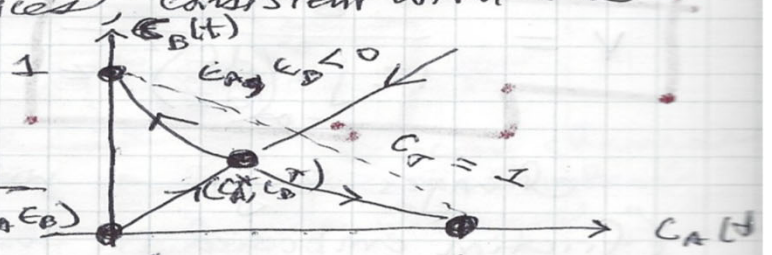
So... even when  $c_T \approx 1$ ,  $\frac{\partial c_T}{\partial t} \approx -2\sigma f(1 - f)$

So the antagonism parameter  $\sigma$  decreases the total population at A/B interfaces, consistent with the well mixed dynamics associated

with Eqs ① & ②  
 (see Pigolotti et al.)

Hyperbolic fixed point at  $(c_A^*, c_B^*) = \frac{(\epsilon_A, \epsilon_B)}{(\epsilon_A + \epsilon_B - \epsilon_A \epsilon_B)}$

but  $c_T \approx 1$ , provided  $|\epsilon_A + \epsilon_B| \ll |\epsilon_A \epsilon_B|$



(Pigolotti et al. Fig. 2)

$\Rightarrow$  at the fixed point  $c_A^* + c_B^* = 1 + \frac{\epsilon_A \epsilon_B}{|\epsilon_A + \epsilon_B| - \epsilon_A \epsilon_B} = \mathcal{O}(\epsilon_A, \epsilon_B)$

\* What is the dynamics of  $f(\vec{x}, t)$ ? First we study the well-mixed case... (2)

$$\frac{df}{dt} = \frac{1}{c_T} \frac{dc_A}{dt} = \frac{c_A}{c_T^2} \frac{dc_T}{dt} (1-f)c_T$$

$$= (1 - c_A - c_B + \epsilon_A c_B) f - \frac{f}{c_T} \left[ \cancel{c_T} (c_T) + (\epsilon_A + \epsilon_B) c_T^2 f(1-f) \right]$$

\* Everything simplifies provided  $\epsilon_A, \epsilon_B \ll 1 \Leftrightarrow c_T \approx 1$

$$\frac{df}{dt} = \epsilon_A f(1-f) - [\epsilon_A + \epsilon_B] f f(1-f), \text{ or}$$

$$\frac{df}{dt} = f(1-f) \left[ \epsilon_A - (\epsilon_A + \epsilon_B) f \right] \begin{cases} \epsilon_A - \epsilon_B = \delta \\ \frac{\epsilon_A + \epsilon_B}{2} = -\sigma \end{cases} \Leftrightarrow \begin{cases} \epsilon_A = \frac{\delta}{2} - \sigma \\ \epsilon_B = -\frac{\delta}{2} - \sigma \end{cases}$$

$$\therefore \epsilon_A - (\epsilon_A + \epsilon_B) f = \left( \frac{\delta}{2} - \sigma \right) + 2\sigma f$$

$$= \frac{\delta}{2} + \sigma(2f - 1) \quad \&$$

$$\frac{df}{dt} = f(1-f) \left[ \frac{\delta}{2} + \sigma(2f - 1) \right]$$

Same equation as Lavrentovich/dra draft paper

\* Additional terms when there are spatial gradients...

$$\frac{\partial f}{\partial t} = \frac{-1}{c_T} \vec{\nabla} \cdot (\vec{u} c_A) + \frac{D}{c_T} \nabla^2 c_A - \frac{c_A}{c_T^2} \left[ -\vec{\nabla} \cdot (\vec{u} c_T) + D \nabla^2 c_T \right] + \dots$$

$$\approx -\vec{\nabla} \cdot (\vec{u} c_A) + D \nabla^2 c_A + c_A (\vec{\nabla} \cdot \vec{u}) + \dots, \text{ if } c_T \approx 1$$

$$= -(\vec{u} \cdot \vec{\nabla}) f - f \vec{\nabla} \cdot \vec{u} + D \nabla^2 f + f (\vec{\nabla} \cdot \vec{u}) + \dots, \text{ if } c_T \approx 1$$

$$\Rightarrow \frac{\partial f}{\partial t} + (\vec{u} \cdot \vec{\nabla}) f = D \nabla^2 f + f(1-f) \left[ \frac{\delta}{2} + \sigma(2f - 1) \right]$$

deterministic generalization of Model A to include flow

# Test of nucleation theory in two dimensions

Xiaojue Zhu,

R. Benzi,

F. Toschi & drn

The dynamics of the droplet radius  $R(t)$  is given by

$$\frac{dR(t)}{dt} = -\frac{D}{R(t)} + \frac{\delta}{2} \sqrt{\frac{D}{\sigma\tau_g}} \quad (\text{require } R(t) \gg w = \text{interface width})$$

→ critical droplet radius  $R_c = \gamma / c = (2 / \delta) \sqrt{D\sigma}$

→ dying droplets should vanish with a square root singularity,

$$R(t) = \sqrt{R_0^2 - 2D(t - t_0)},$$

where  $R_0$  is the radius of a dying droplet has well below the maximum  $R_c$  at time  $t_0$

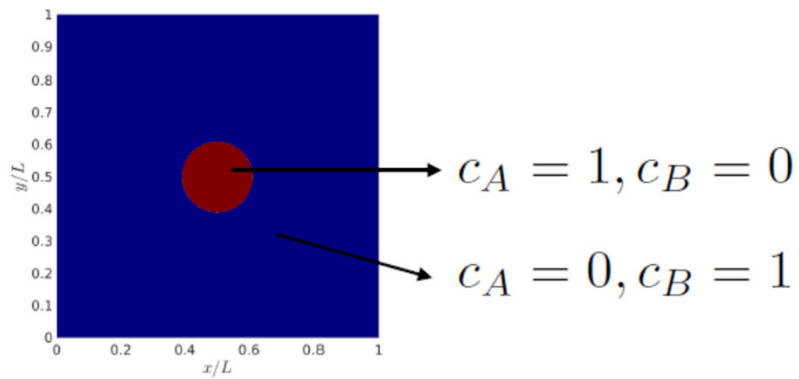
→ Once the droplet is above the maximum, we should eventually have a

circular, expanding pushed wave with  $R(t) \approx vt, v = (\delta / 2) \sqrt{D / \sigma\tau_g}$

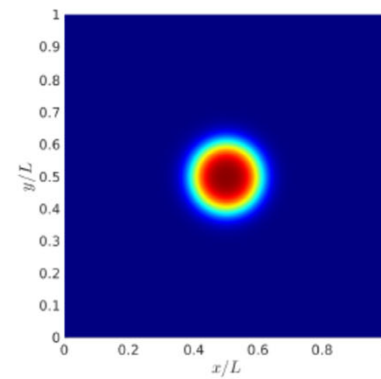
simulations: selective advantage =  $\delta = \varepsilon_A - \varepsilon_B = 0.1$

antagonism =  $\sigma = -(\varepsilon_A + \varepsilon_B) / 2 = 0.25$

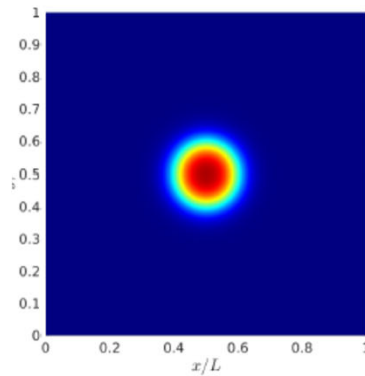
# 1. Initial radius=0.11 without flow



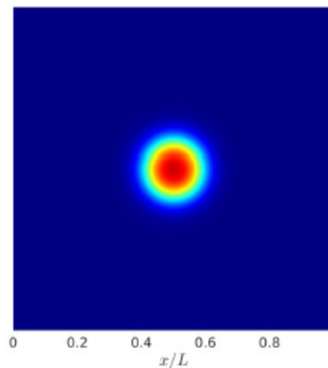
**t=0**



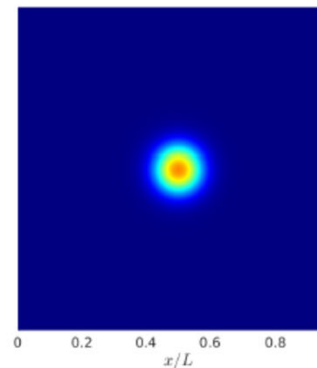
**t=10**



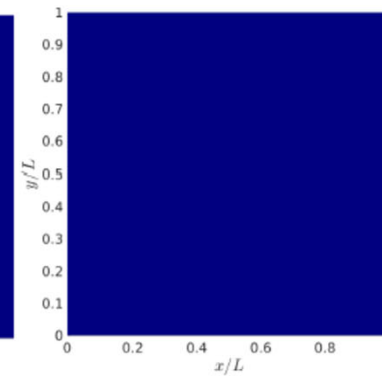
**t=100**



**t=150**



**t=200**

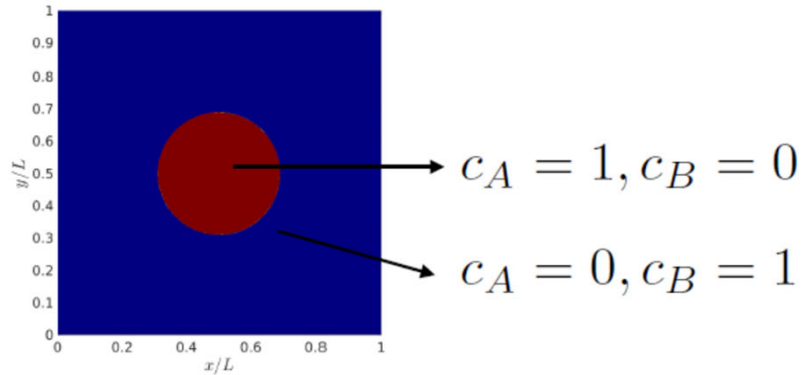


**t=300**

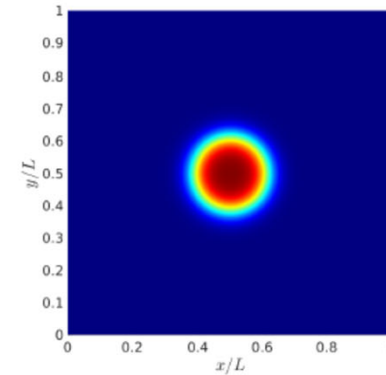
$$R < R_c$$



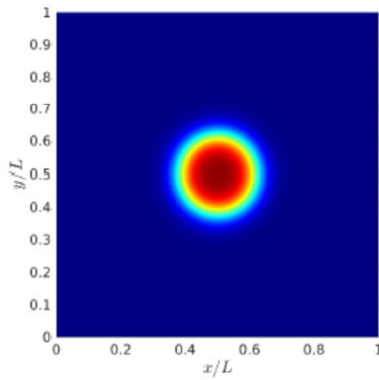
## 2. Initial radius=0.12 without flow



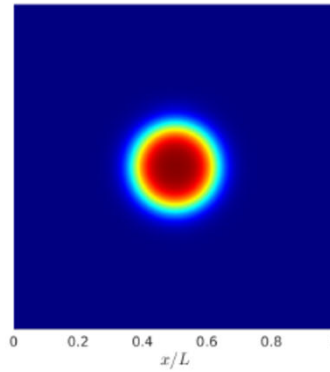
**t=0**



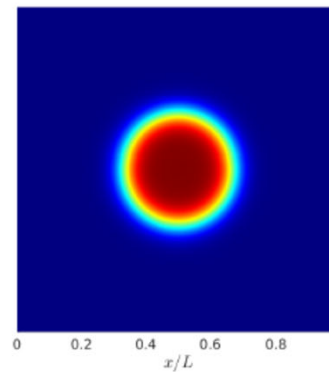
**t=10**



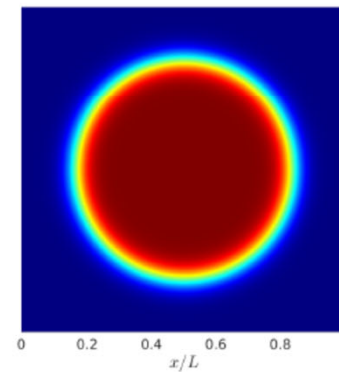
**t=100**



**t=200**



**t=400**

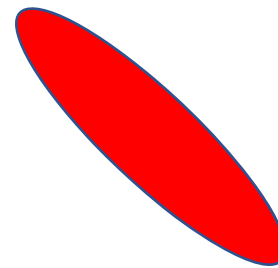
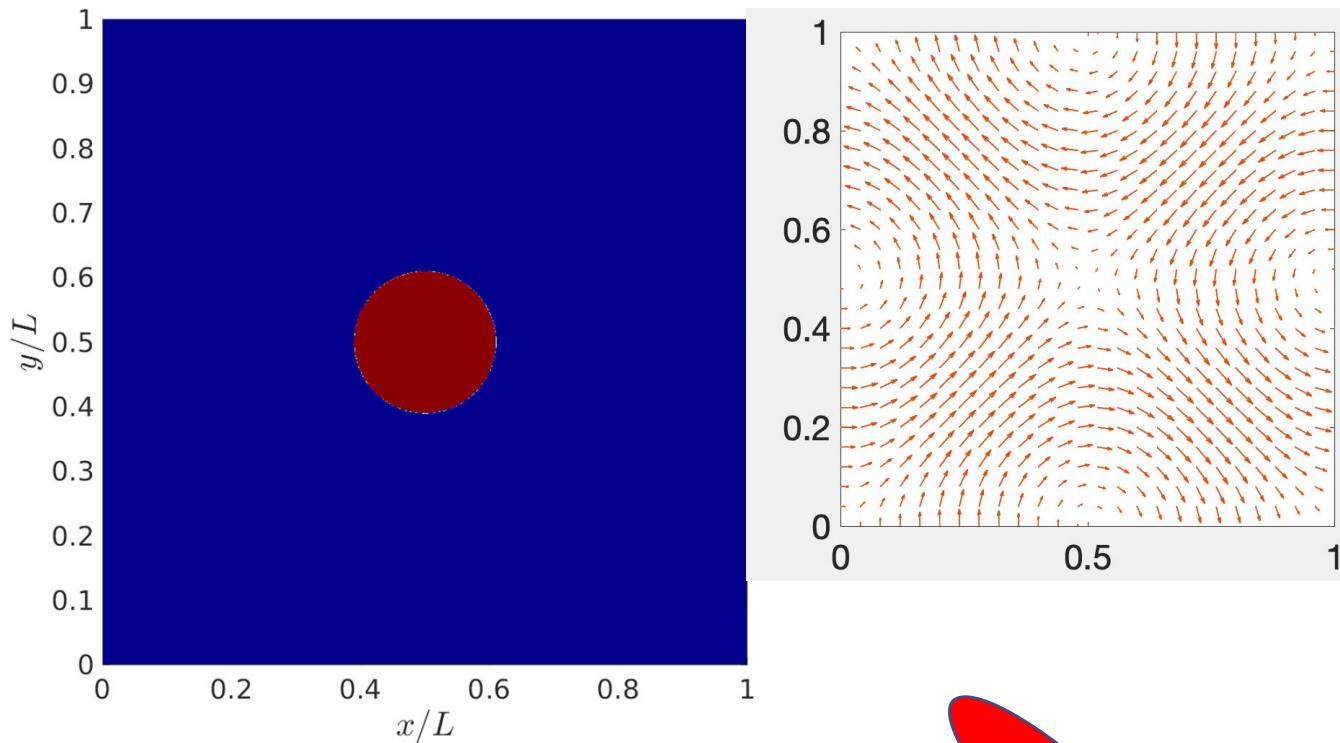


**t=756**

$$R > R_c$$

# *The effect of a saddle flow on a (slightly) subcritical droplet of a selectively favored species.*

Initial radius=0.11,  $F=0.0025$



•  $R(t=0) < R_c$  without flow.

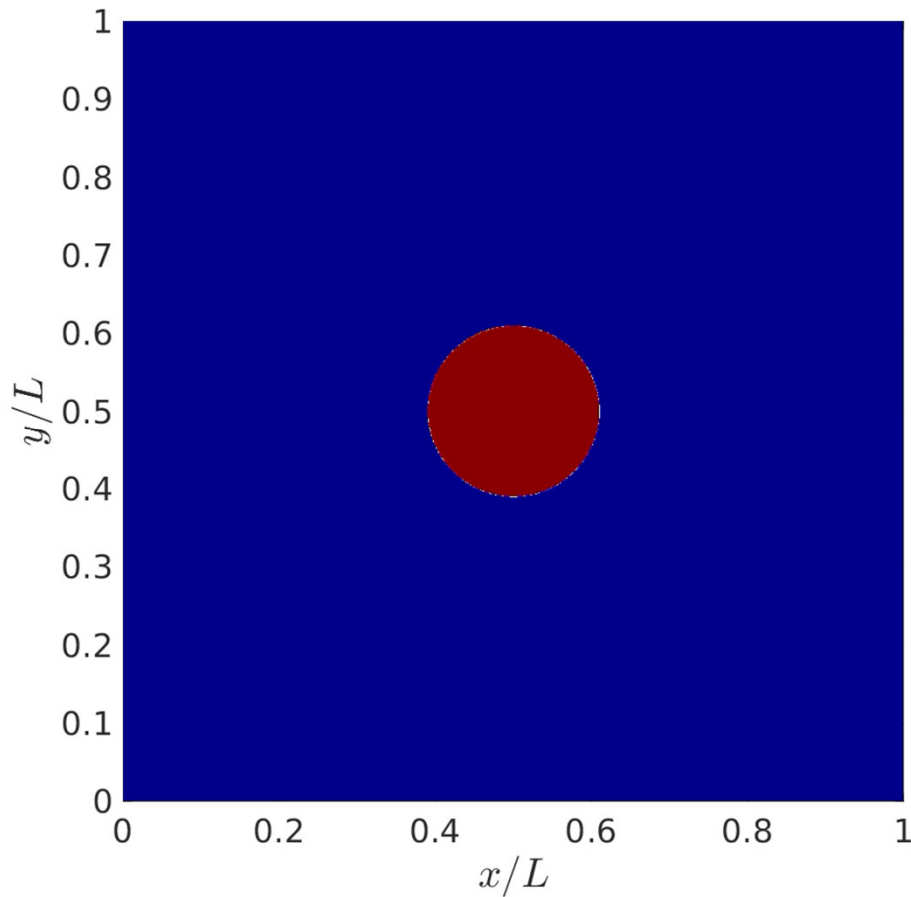
• *The saddle flow elongates the droplet, and resulting flat regions are relatively free from the confining effects of line tension.*

• *Although there is a selective advantage, the inward flows due to the saddle are larger than the outward pushed wave velocity due to the selective advantage.*

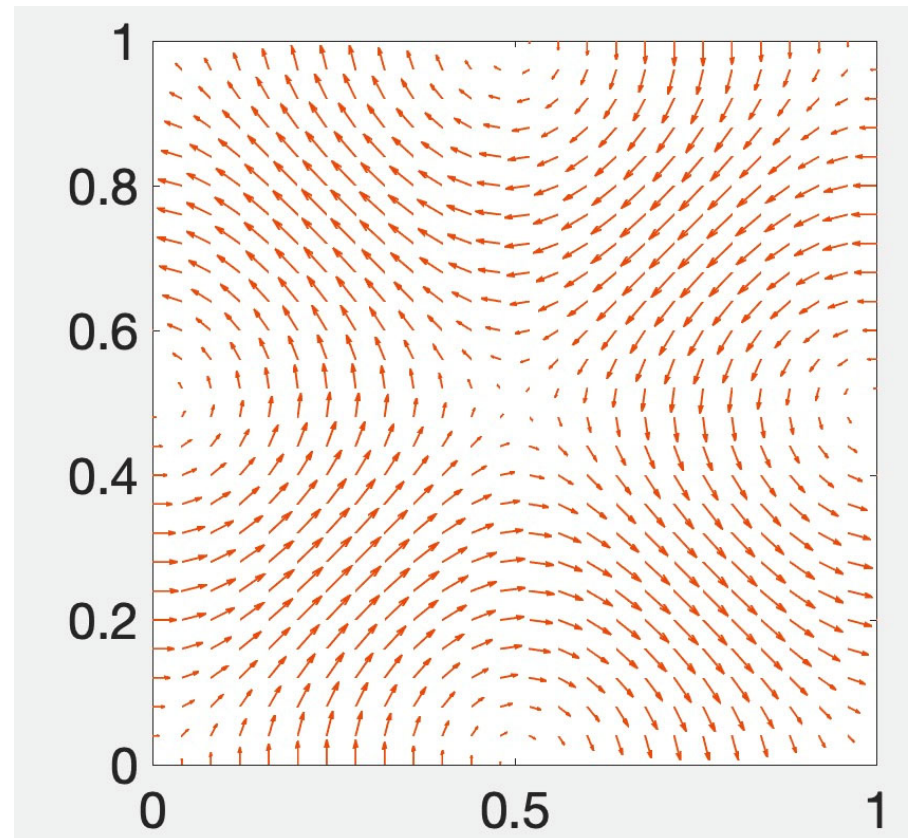
• *The net effect is to produce a shorter extinction time.*

Initial radius=0.11,  
 $F=0.025$

*(Fluid driving force  $F$   
at the saddles is now  
a factor of 10 bigger.)*

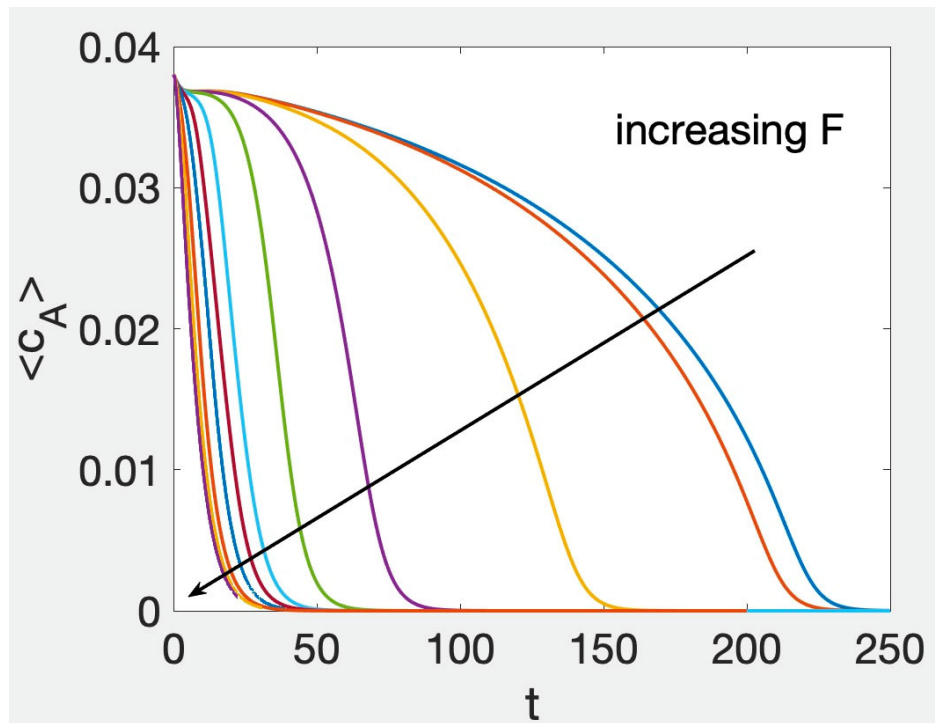


*Red droplet of the selectively favored  
phase dies even more rapidly...*



# Time series for initial radius=0.11, increasing flow strength F at the saddle

$$c_A(t) = \pi R^2(t); \quad R(t) = \sqrt{R_0^2 - 2D(t-t_0)}$$



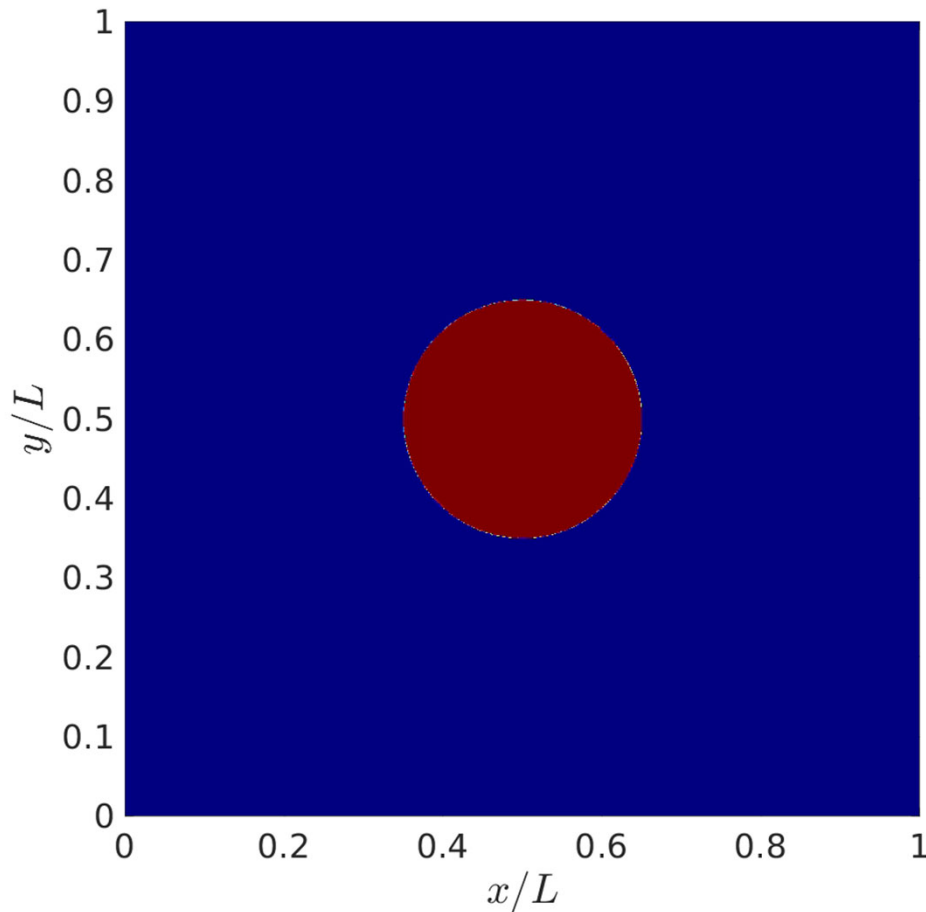
*The predicted linear vanishing of  $c_A(t)$  is rounded into a foot, due to the smoothing effect of diffusion?*

*The selectively favored droplet dies even more rapidly when born on a saddle point*

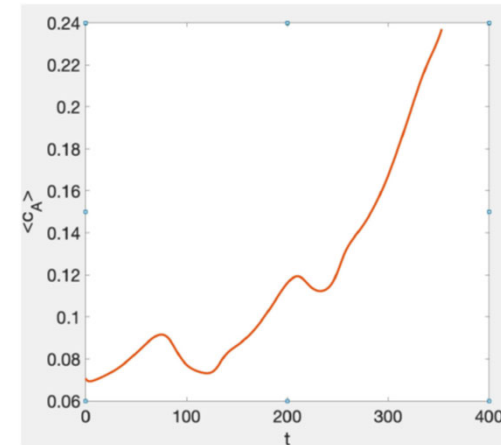
Extinction time  $T_E \approx T_0 - AF^2$ ;  $T_E(F)$  must be an even function of  $F$ .

Hence,  $T_E(F) \approx T_E(0) - AF^2$  for small  $F$ ....

Larger droplets can be strongly influenced by periodic boundary conditions!!

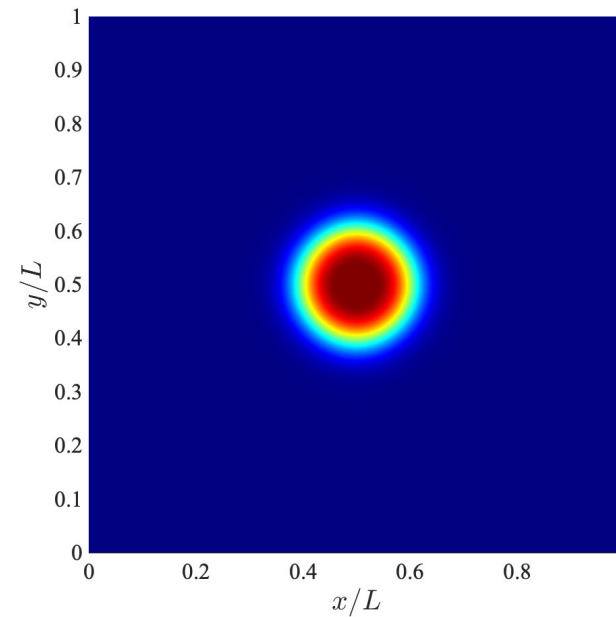
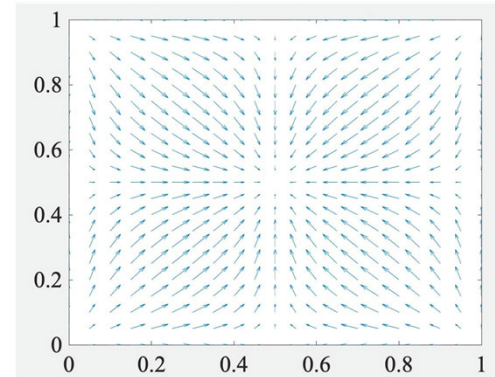
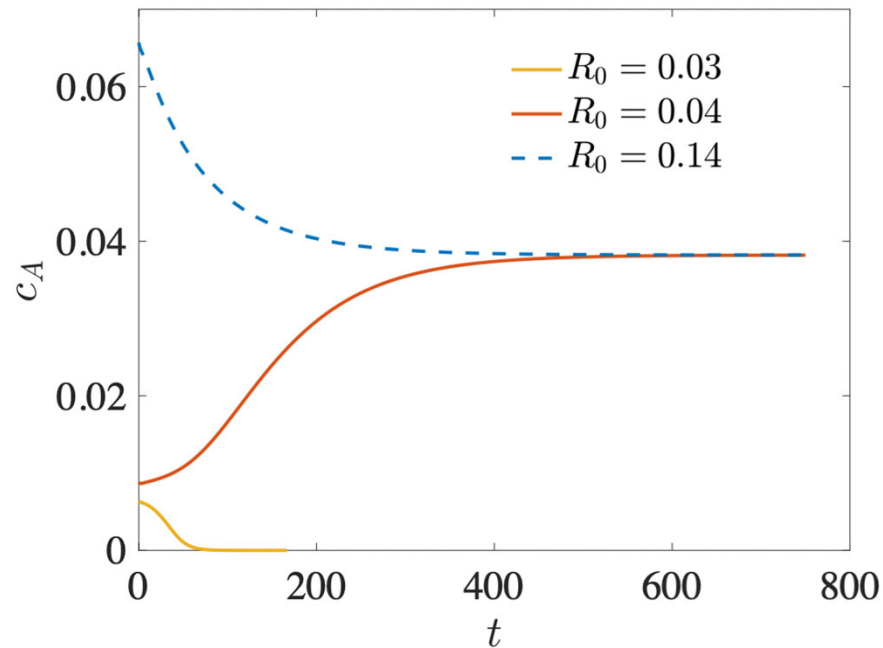


*Saddle flow with very small  $F$*



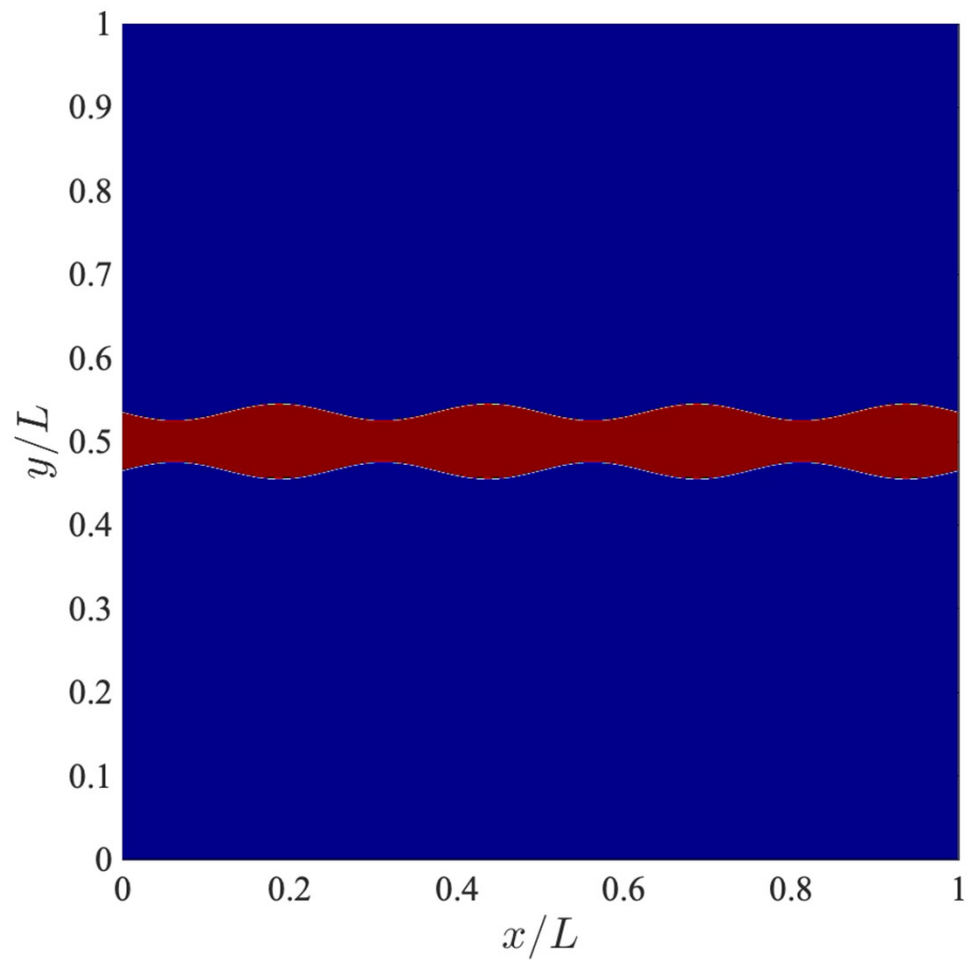
*Red variable species is initially at the saddle point.  $\langle c_A \rangle$  grows non-monotonically. Also as time goes by,  $A$  splits up and reconnects.*

A stable radius is possible when droplet  
with a selective advantage is born on a sink



# Effects of spatially modulation with line sink and a selective advantage

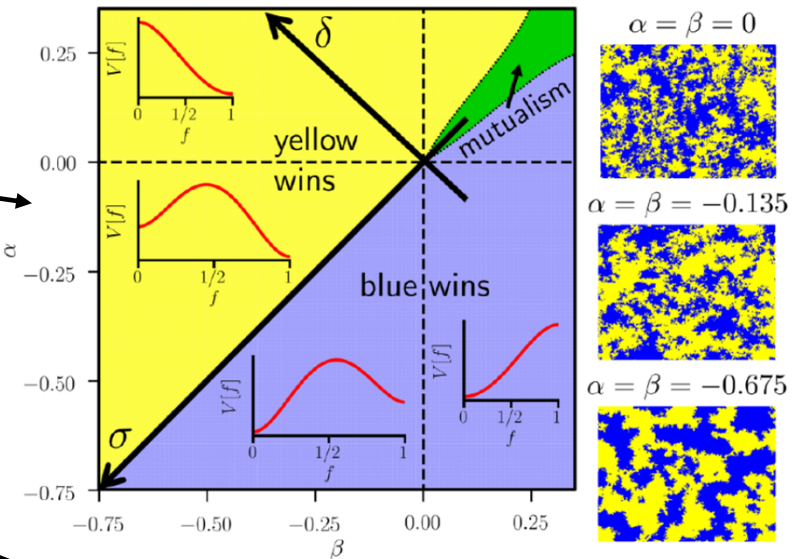
A layer of Species A is put in the middle of domain with width 0.07.  
A perturbation of  $0.01 \cdot \sin(2 \cdot \pi \cdot 4 \cdot x)$  is added to the interface



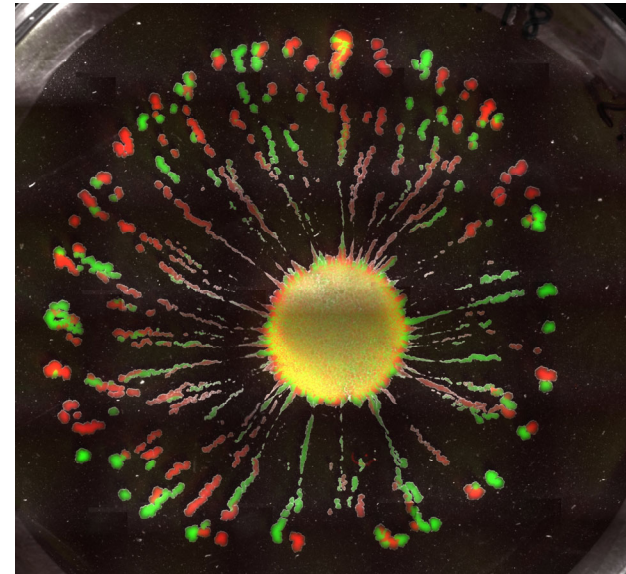
# Active Antagonism: Reproducing Microorganisms and Fluid Flow

- Spatial population genetics of antagonistic organisms
- Fluid mechanics
- Active dynamics & antagonism: spinodal decomposition and nucleation in Model A'

*Thank you!*



<http://streetanatomy.com>



Severine Atis et al. "Microbial range expansions on liquid substrates." *Physical Review X* 9.2 (2019): 021058.