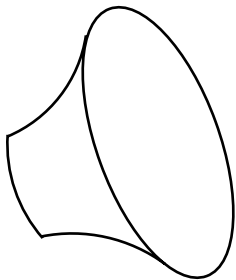


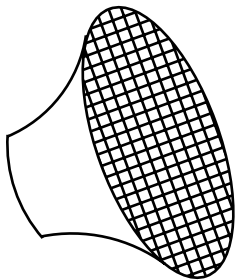
*String theory duals  
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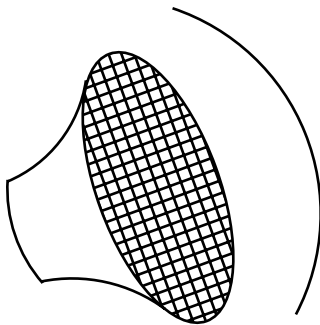
**Koushik Balasubramanian, John McGreevy**

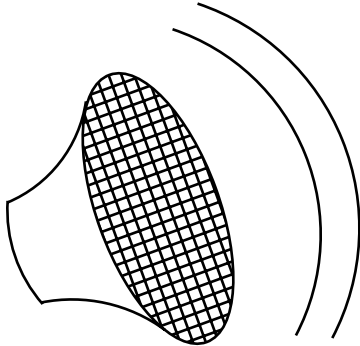
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Based on arXiv:1111.0634[hep-th]

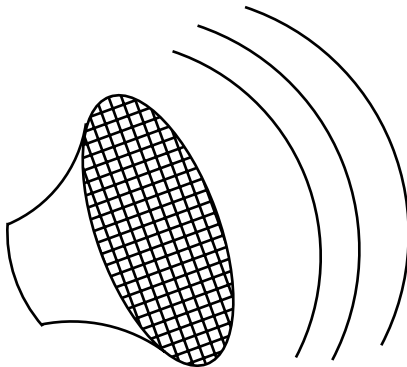








Please let me know if I am not audible



# Introduction

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Gravity in  $AdS_5 \times S^5 \iff 3 + 1 \text{ D } \mathcal{N} = 4 \text{ SYM theory}$

Can we find examples of field theories that are holographically dual to gravity in Lifshitz spacetime?

Gravity in  $Lif_{d+1}^z \text{ "x" } \mathcal{M} \iff \boxed{?}_{z,d,\mathcal{M}}$

(1005.3291, 1008.2062, 1009.3445)

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- $z = 2$  Non-abelian Chern-Simons gauge theories studied by Kachru, Mulligan and Nayak (and generalizations of KMN).
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A confining solution with asymptotic  $z = 2$  Lifshitz symmetry – dual to pure gauge theory. KK modes decouple from low energy dynamics. Somewhat unusual!

Holographic dictionary  $\rightarrow$  arguments to support Claim # 1.

$2 + 1$  D  $z = 2$  LCS theories (with or without adjoint matter) can be realized as deformations of  $3 + 1$  D  $\mathcal{N} = 4$  SYM theory.

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1. Solves type IIB supergravity equations of motion.
  2. Approaches  $Lif_{z=2}^{d=2}$  as  $r \rightarrow 0$  and ends at  $r = r_\star = r_0^2 \Gamma$ .
  3. Regular... No conical singularity at  $r_\star$ ! Fermions satisfy APBC around  $x_3$ . What determines  $r_0$ ?
  4.  $r_0$  is related to a parameter specifying boundary conditions on the metric!<sup>hep-th/990215</sup>
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## Identification of the dual field theory

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Conformal boundary:  $ds^2 = 2dx_3dt + d\vec{x}^2$ . Boundary theory is a deformation of DLCQ  $\mathcal{N} = 4$  SYM theory.

RR-axion ( $C_0$ )  $\iff$   $\theta$ -angle of  $\mathcal{N} = 4$  theory.

$$\begin{aligned}\int \theta \text{Tr}(F \wedge F) &= \int d\theta \wedge \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \\ &= \frac{Q}{L_3} \int dx_3 \wedge \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)\end{aligned}$$

Reducing along  $x_3$

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Asymptotic metric is invariant under  $t \rightarrow \lambda^2 t$ ,  $\vec{x} \rightarrow \lambda \vec{x} \Rightarrow$  dual field theory has  $z = 2$  scale invariance.

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We can identify operators dual to bulk deformations according to scaling dimension. Supersymmetry to the rescue!  $S^5$ -Sidekick!

Operators that are irrelevant in the relativistic theory can become marginal in the DLCQ theory.

$E_i = F_{3i}$  appears as an auxiliary field in the DLCQ theory and  $[E_i]_{DLCQ} = 1 \Rightarrow$  Terms like  $\text{tr}(F_{3i} F_{3i} F_{3j} F_{3j})$  cannot be ignored.

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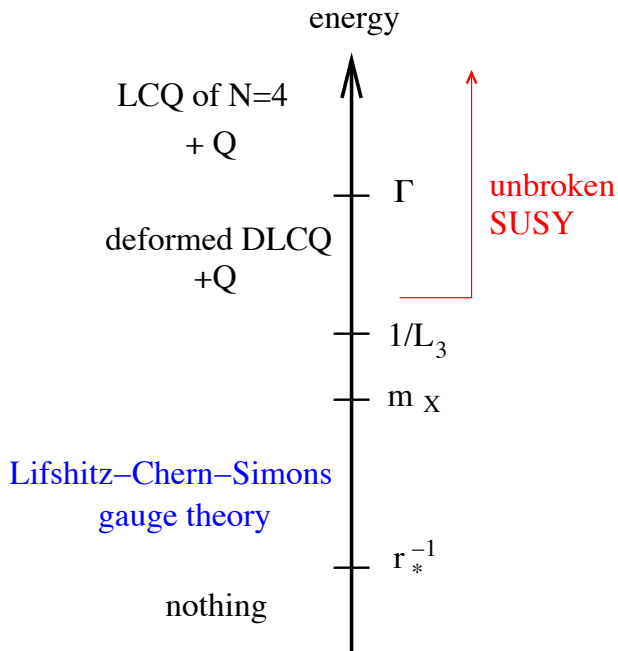
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## At Last!

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No Fermion zero modes. Scalar zero modes lifted by the mass deformation  $\text{tr}(X^2)$ .

Scalar mass deformation dual to an excited string state. Effect felt through non-trivial boundary conditions on SUGRA fields. This determines  $r_0$  [hep-th/990215] NOT PRECISE!

Zero modes of gauge field organize themselves into LCS gauge theory.

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# Dual Field Theory

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$$S_{2+1} = \int dt d^2x \left[ \frac{1}{2g_1^2} \text{tr} \left( \tilde{E}_i D_t \tilde{A}_i + \tilde{A}_t D_i \tilde{E}_i \right) + \frac{1}{4g'^2} \text{tr} \left( \tilde{F}_{ij} \tilde{F}^{ij} \right) + \lambda_1' \text{tr} \left( [\tilde{E}_i, \tilde{E}_j]^2 \right) + \right. \\ \left. i\kappa' \text{tr} [\tilde{E}_i, \tilde{E}_j] \tilde{F}^{ij} \right] + \frac{1}{2g_3'^2} \int d^2x dt \text{tr} \left( \tilde{E}_3^2 \right) + \frac{1}{2\alpha^2} \int d^2x dt \text{tr} \left( \left( D_i \tilde{E}_j \right)^2 \right) + Q \int \text{tr} \left( \tilde{A} \wedge \tilde{F} \right) \\ + \text{irrelevant terms}$$

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Kachru, Mulligan, Nayak (to appear)

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*Thank you!*