Focus week introduction: Criticality and instabilities in holography

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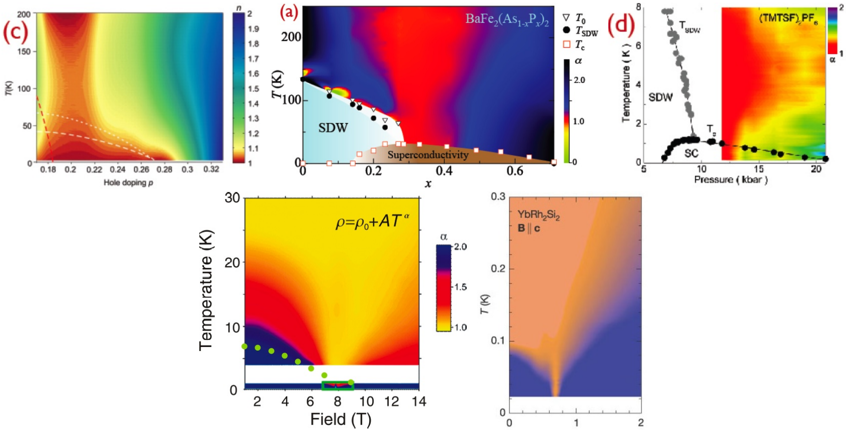
Critical metals and their instabilities

Cuprates, Pnictides, Organics, Ruthenates, Heavy fermions



Zeroth order observations

- Breathtaking regularity across diverse compounds.
- Recurring themes:
  - Bad metallic regimes (poor conductance) bounded by Fermi liquids.
  - Proximity of competing orders (e.g. magnetism, nematicity).
  - Putative quantum critical points hidden by ordered phases.
  - Absence of Landau quasiparticles.
- Difficult to get a theoretical handle on.

Focus week

- Three strands:
  1. Experimental status of critical metals and their instabilities (Mackenzie + Lonzarich)
  2. Perspectives on critical metals from condensed matter theory (Kivelson + Senthil)
  3. Holographic perspectives on critical metals (Silverstein + McGreevy)

- I will introduce how various of the physical ingredients for this discussion appear holographically.

Holography and criticality

- To be a bad metal, current must be dissipated more efficiently than in a Fermi liquid + lattice.
- Presumably requires additional (collective) critical excitations.
- A condensed matter system must work hard to achieve additional critical modes:
  1. Tune to a quantum critical point.
  2. Constraint (e.g. no double occupancy) leading to emergent Gauss law and gauge fields.
- In holography, the critical modes are put in from the start.

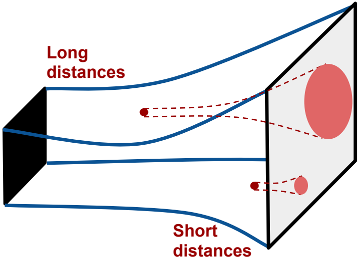
Holography and criticality

- Typical structure of the UV action

$$\mathcal{L} \sim \mathrm{tr}\left(F^2 + (\partial\Phi)^2 + i\bar{\Psi}\Gamma\cdot\partial\Psi + [\Phi,\Phi]^2 + i\bar{\Psi}[\Phi,\Psi]\right) \, .$$

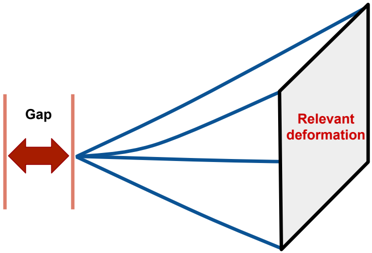
- Gapless bosons and fermions. Gauge symmetry. Conformal invariance.

$$ds^2 = \frac{dr^2}{r^2} + \frac{-dt^2 + dx^2 + dy^2}{r^2}$$



Holography and criticality

- Need to work to get rid of all these gapless modes.
  - Deform by relevant operators: Confinement + Higgs phases.

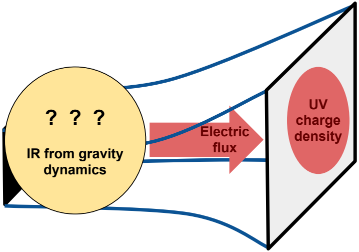


- Gapping kills all the interesting things that we put in in the UV.

- Maybe useful for understanding dynamics of confinement, probably not for novel phases of matter.

Finite density holography

- We also want to put in the charge density.
- This is a relevant deformation – μJ^t – it will change the IR.

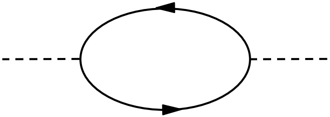


- The presence of the charge carriers shows up in ways that connect both the UV and IR. E.g. for $\omega \ll \mu$:

$$\sigma(\omega) \sim \frac{\rho}{\mu}\left(\frac{i}{\omega} + \delta(\omega)\right) .$$

Finite density holography

- What a charge density typically does to the critical sector



- Landau damping of boson dispersion: $\omega \sim k \;\Rightarrow\; \omega \sim k^z$
- Dynamical critical exponent $z > 1$.

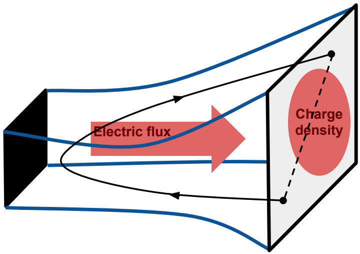
- Holographically, something analogous to this effect occurs

- The spacetime in the IR is often found to take the form

$$ds^2 = \frac{dr^2}{r^2} - \frac{dr^2}{r^{2z}} + \frac{dx^2 + dy^2}{r^2}$$

Finite density holography

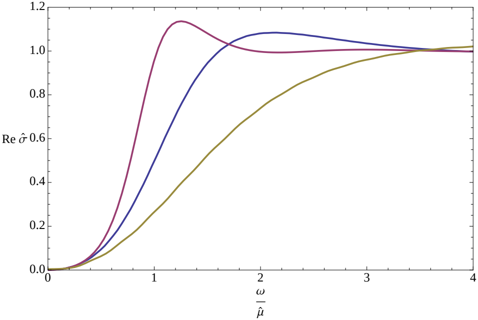
- Scaling in the IR tells us that the system has not been gapped (e.g. $s \sim T^{2/z}$ in 2+1 dimensions).
- How do charge carriers dissipate into the critical sector?
- Compute e.g. $\text{Im}\, G^R_{J_x J_x}(\omega)$
- Dissipation into the critical sector: falling through an event horizon



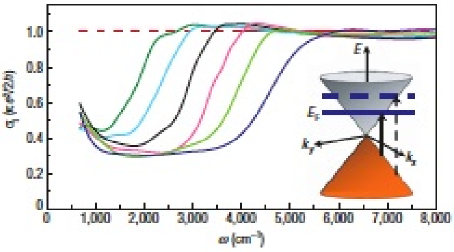
Finite density holography

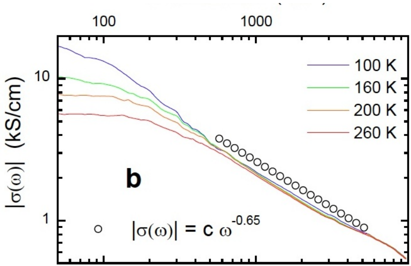
- Geometrization of dissipation: irreversibility implemented by GR modification of the causal structure of spacetime.
  (deepest and most powerful aspect of holography?)

- Result: more like graphene than a high temperature superconductor



Finite density holography





Finite density holography

- There is an interestingly universal $\mathrm{Re}\,\sigma \sim \omega^2$ at $\omega \ll \mu$. (Independent of z).
- The physics of the 'generalized Drude peak' is missing – more ingredients needed (cf. Silverstein talk).
- The physics of the DC conductivity (and associated Drude-like peak) and low ω optical conductivity are in some senses orthogonal.
- Low ω, $T \sim 0$, optical conductivity a clean + robust probe of charge carrier/criticality interactions if it can be isolated from Drude peak?

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Horizons

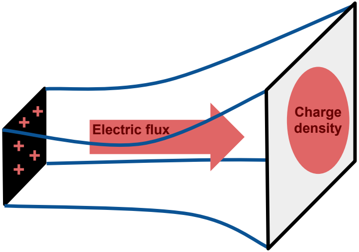
- Dissipation via horizons is possible because of the special structure of theories with gravity duals (cf. McGreevy talk).
- The UV ingredients were $N \times N$ matrices.
- Loosely speaking: a hierarchy between the eigenvalues and the off diagonal modes of these matrices.
- Off diagonal modes become parametrically heavy. C.f.

$$\lambda[\Phi, \Phi]^2 \to \lambda\langle(\Phi_i - \Phi_j)^2\rangle(\delta\Phi_{ij})^2$$

- Classical gravity cannot distinguish the many 'stringy' off diagonal modes $\Rightarrow$ effective ensemble description as an event horizon.

Horizons and fractionalization

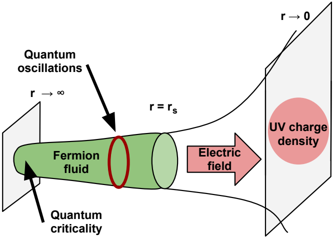
- The critical sector is manifested in an event horizon.
- The charge can be either behind or outside the horizon



- Corresponds to the charge dynamics being in either the 'stringy' or the 'gravity' sector.

Horizons and fractionalization

- Charge outside horizon essentially conventional (still interesting!)
  - Obeys Luttinger theorem (Hartnoll-Hofman-Vegh; Iqbal-Liu-Mezei)
  - Equivalent to semi(non?)-holographic system of explicit fermions + critical sector (MIT; Faulkner-Polchinski; McGreevy talk).
- Can look unconventional. E.g. quantum oscillations may see a missing charge (Hartnoll-Hofman-Tavanfar).

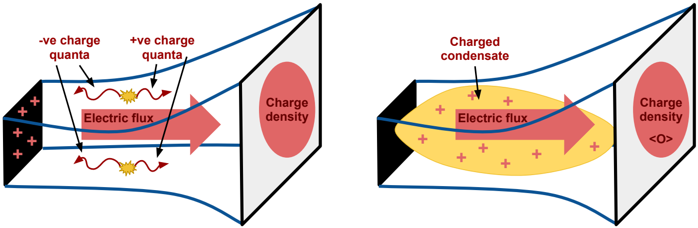


Horizons and fractionalization

- Charge behind the horizon remains mysterious.
- Presumably describes ‘fractionalized’ phases.
  (Started with the gauge-charged ‘quarks’, so fractionalized phases expected, unless there is some dynamics that ‘confines’ the charges).
- To date, no non-analytic momentum space structure (or quantum oscillations) has been exhibited in these cases.
- No obvious connection to ‘patch’ descriptions of critical Fermi surfaces.
- May be necessary to consider stringy physics in the bulk.

Instabilities of the critical phase

- Holographic frameworks come equipped with a natural mechanism for superconductivity.
- Electromagnetic charges screen external fields, but gravitating mass antiscreens (i.e. it clumps).
- If screening wins, then it is favorable for charged fields to condense.



Two comments about holographic superconductivity

1. Suppose $T_c \ll \mu$ is in the IR critical regime.
   - The operator that condenses will typically have a scaling dimension distinct from that of a BCS Cooper pair.
   - Leiden group have suggested this could be measured from the pairing susceptibility just above T_c.
2. At $T \ll T_c$ it appears that
   - Critical sector is often not totally gapped out. E.g. $s \sim T^{2/z'}$.
   - Charged sector also not totally gapped. E.g. $\text{Re}\,\sigma \sim \omega^{\#}$. (Horowitz-Roberts).

Summary

- Holographic systems start with a large number of critical modes.
- Add charge density to get critical metals.
- Generic features:
  1. Effect of charge on criticality: Landau damping.
  2. Effect of criticality on charges: $\text{Re}\,\sigma \sim \omega^2$.
- Things to be understood better:
  1. The generalized Drude peak.
  2. How to think about fractionalized phases.
- Instabilities require overcoming gravitational antiscreening.
- Criticality survives the onset of ordering (cf. Mackenzie talk??)