

Heavy fermions : condensed matter theory perspective

C. Pépin (IPhT, CEA-Saclay + IIP, Natal)

M. Norman (Argonne)

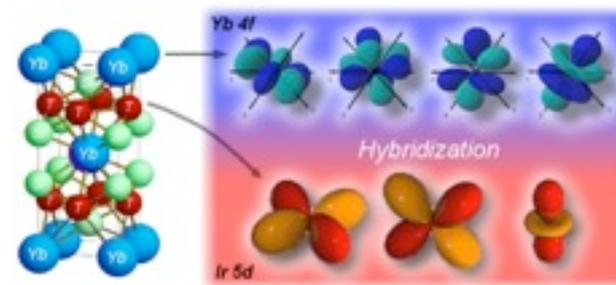
I. Paul (U. Paris VI)

A. Benlagra (Dresden)

K-S. Kim (Pohang, Korea)



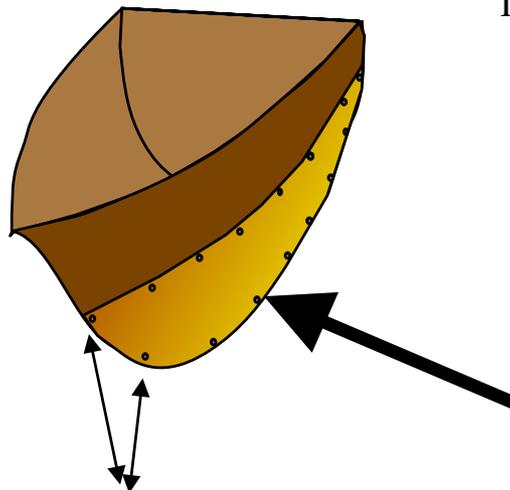
KITP, Oct. 13th 2011



Materials in the Revolutionary Wars.



1779: end of war of independence.

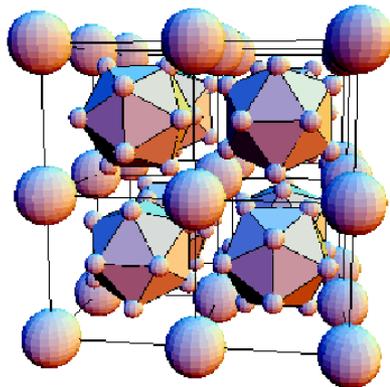
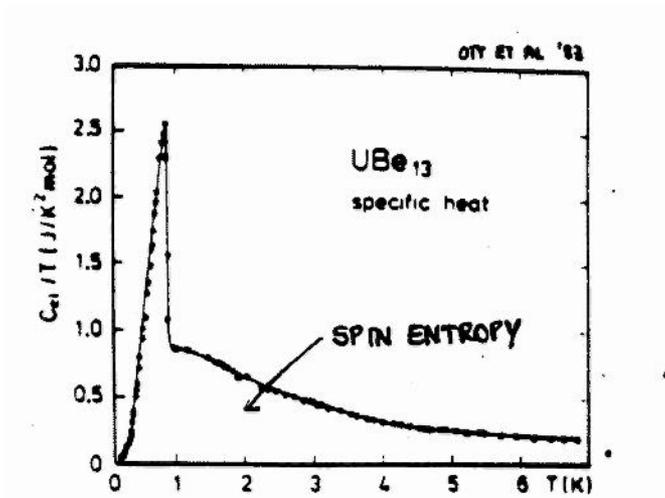


"For God's sake and our countries- send copper bottomed ships to relieve the foul and crippled ones."

Copper plated Hull

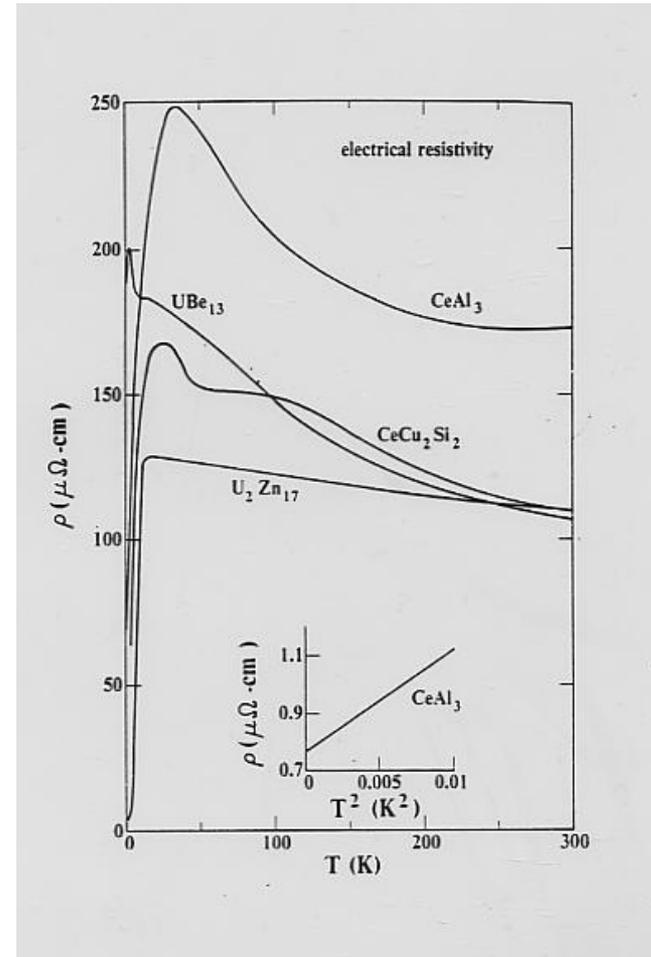
IRON BOLTS : RAPIDLY CORRODED

Heavy Fermion Metals: Extreme Limit of Mass Renormalization.



UBe₁₃

$$\frac{m^*}{m} \sim 1000$$



Ce : $4f^1$

Yb : $4f^{13}$

U : $5f^2$

S=1/2 L=3

S=1/2 L=3

S=1 L=3+2

Spin Orbit : $J=|L-S|=5/2$

S O : $J=|L+S|=7/2$

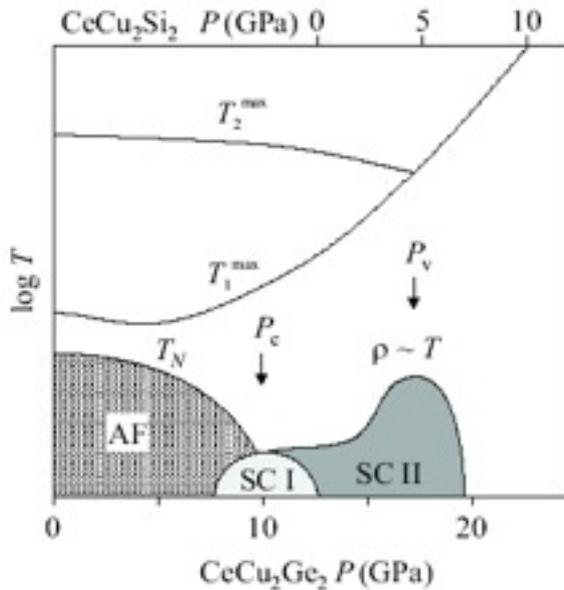
S O : $J=|L-S|=4$

Crystal Electric Field effects split the big moments and compete with Hunds rules

- Ferromagnetic fluctuations
- valence fluctuations
- multiple stage screening ?

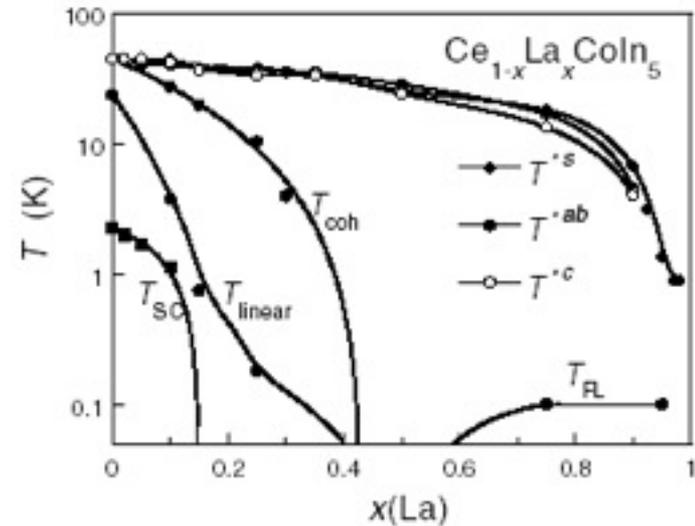
Miyake 99-04

Valence fluctuations at pc



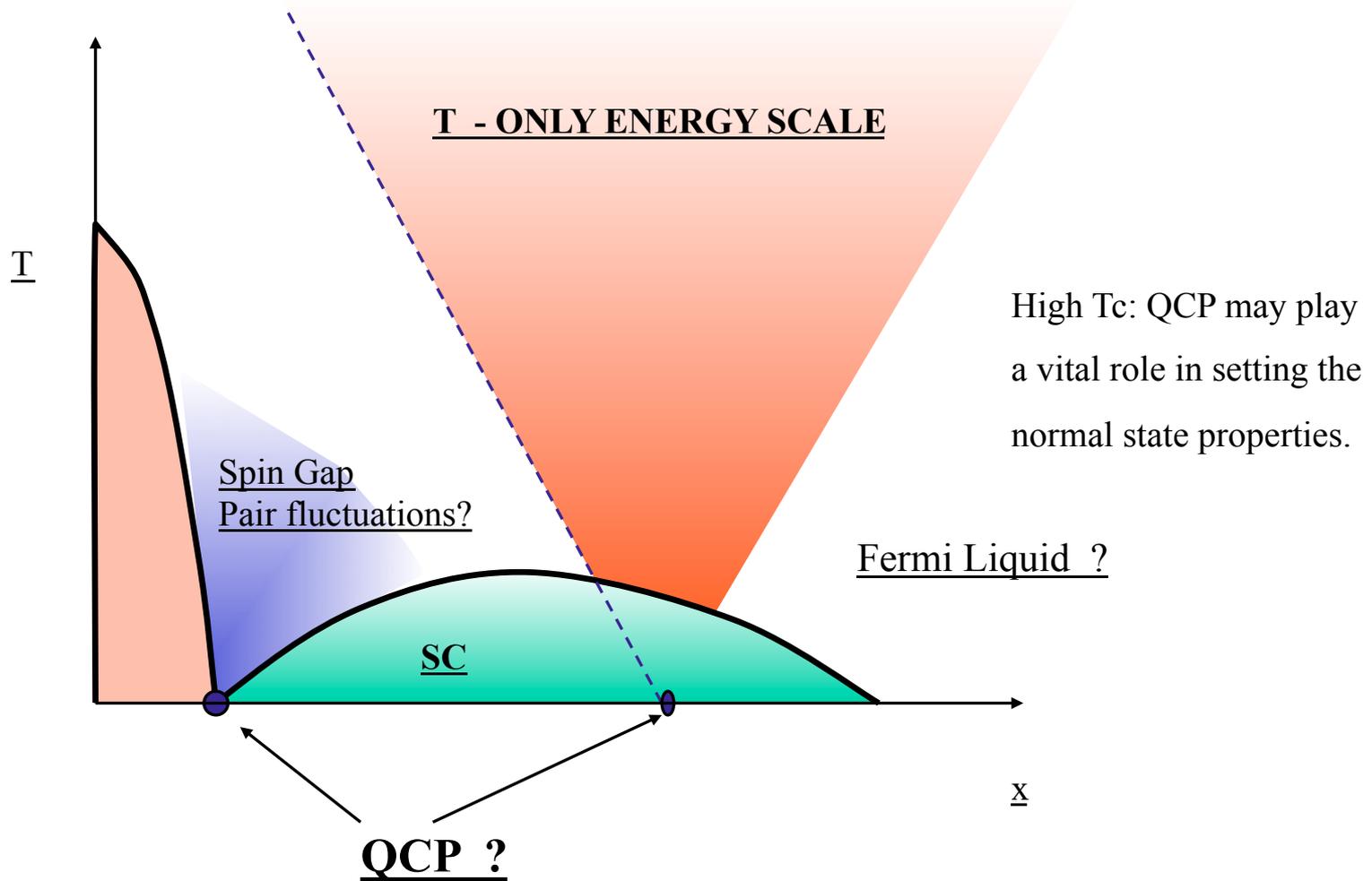
Jaccard 03

Multiple Kondo screening



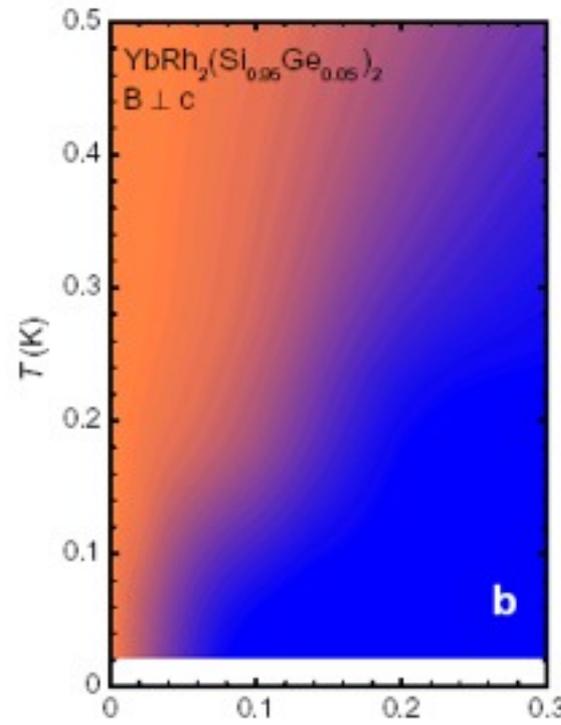
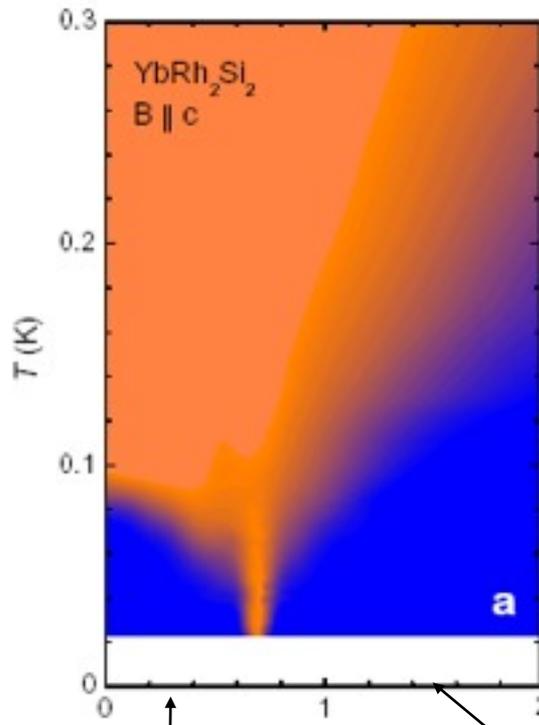
Nakatsuji 03

Quantum criticality



High T_c : QCP may play a vital role in setting the normal state properties.

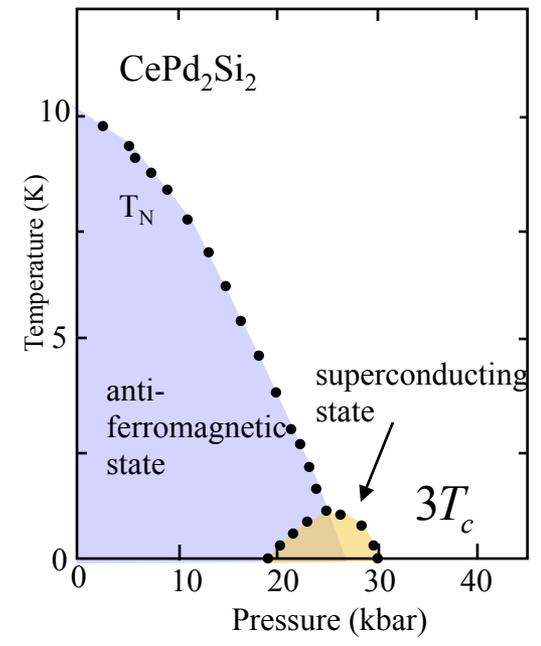
Do we understand QCPs where the order parameter is uncontraversial?



magnetic order

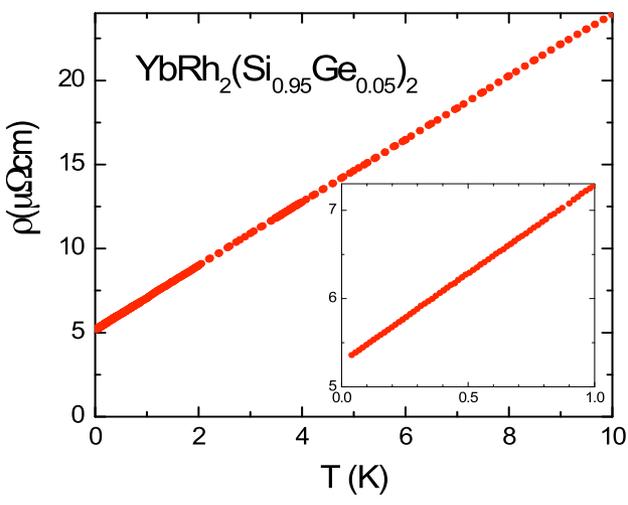
heavy Fermi liquid

Lhoneysen, (1999)



Custers et al (CP), Nature 2003

Resistivity linear !



What is critical and what is not ?

Clear NFL in transport and specific heat

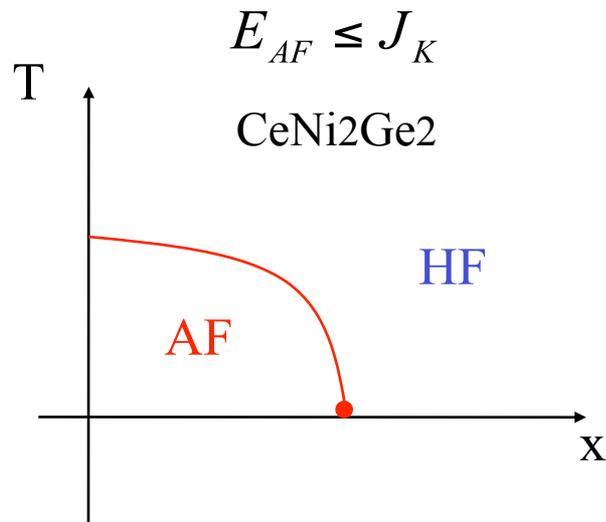
Explained by the standard theory of itinerant magnetism ?

Is the anomaly due to Quantum Criticality ?

Compound	$H_c/P_c/x_c$	$\frac{C_v}{T} \rightarrow \infty?$	$\rho \sim T^a$	Reference
$YbRh_2(Si_{1-x}Ge_x)_2$	$x_c = 0.05$ $H_c^{\parallel c} = 0.66T$ $H_c^{\perp c} = 0.06T$	$T^{-0.34}$	T	Dresden, Grenoble
$CeCoIn_5$	$H_c = 5T$	$T^{-\alpha}$	T	Los Alamos, Grenoble
$Ce(Cu_{1-x}Au_x)_6$	$x_c = 0.016$	$\text{Log} \left(\frac{T_0}{T} \right)$	T	Karlsruhe
$CeCu_{6-x}Ag_x$	$x_c = 0.2$	$\text{Log} \left(\frac{T_0}{T} \right)$	$T^{1.1}$	Gainesville
$CeNi_2Ge_2$	$P_c = 0$	$\text{Log} \left(\frac{T_0}{T} \right)$	$T^{1.4}$	Karlsruhe, Cambridge
U_2Pt_2In	$P_c = 0$	$\text{Log} \left(\frac{T_0}{T} \right)$	T	Leiden
$CeCu_2Si_2$	$P_c = 0$	$\text{Log} \left(\frac{T_0}{T} \right)$	$T^{1.5}$	Dresden, Grenoble
$Ce(Ni_{1-x}Pd_x)Ge_2$	$x = 0.065$	$\gamma_0 - T^{1/2}$	$\rho_0 + T^{3/2}$	Los Alamos
$YbAgGe$	$H = 4T$	$\text{Log} \left(\frac{T_0}{T} \right)$	T	Ames, Grenoble
$CeIn_{3-x}Sn_x$	$p_c = 26 \text{ kbar}$?	$T^{1.6}$	Dresden
U_2Pd_2In	$P_c < 0$?	T	Leiden
$CePd_2Si_2$	$P_c > 0$?	$T^{1.2}$	Karlsruhe, Dresden
$CeRhIn_5$	$P_c \sim 1.6 \text{ GPa}$?	T	Los Alamos, Grenoble
$CeIn_3$	$P_c > 0$?	$T^{1.5}$	Dresden
$Ce_{1-x}La_xRu_2Si_2$	$x_c = 0.1$	no	?	Grenoble
$U_3Ni_3Sn_4$	$P_c > 0$	no	?	Leiden

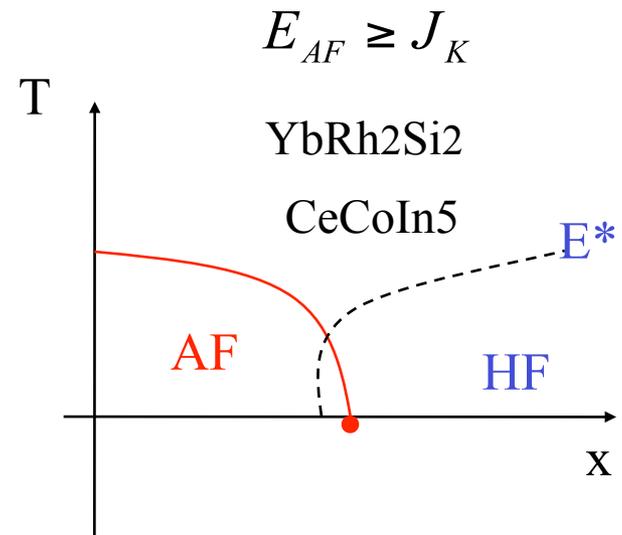
Two scenarios

Spin Density Wave



SDW scenario:
big Fermi surface at the QCP

Kondo Breakdown



QCP with fractionalization

Theoretical approaches

Low energy properties \longleftrightarrow **Universality**

UV

High energy physics – microscopic hamiltonian



Integrate out the “fast”
degrees of freedom

IR

Low energy, slow, universal part

Universality



Low energy properties \longleftrightarrow **Universality**

High energy physics – microscopic hamiltonian

Integrate out the “fast” degrees of freedom

Low energy, slow, universal part

UV

IR

Universality



What is observed around some QCP in heavy fermions

$$\rho(T) \sim T ,$$

$$\chi(T) \sim T^{-\alpha} , \text{ with } \alpha \leq 1$$

$$\gamma_p(T) = \frac{C_P}{T} \sim T^{-\beta} , \text{ with } \beta \leq 1 .$$

Universal Too!

Landau Fermi liquid theory verified by “all” conductors above 1D

$$\rho(T) \sim T^2 ,$$

$$\chi(T) \sim \mu_0^2 \rho(\epsilon_F) ,$$

$$\gamma_p(T) = \frac{C_P}{T} \sim \frac{k_B^2 \pi^2}{3} \rho(\epsilon_F) .$$

Universal exponents

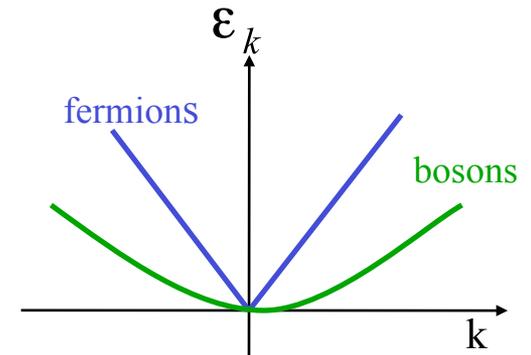
Can we integrate the fermions out of the partition function?

↪ φ^4 effective bosonic theory

For example $z=2$

$$D^{-1}(q, \Omega) = \frac{|\Omega|}{E_F} + \frac{q^2}{k_F^2}$$

fermions are mass-less but fast compared to bosons?



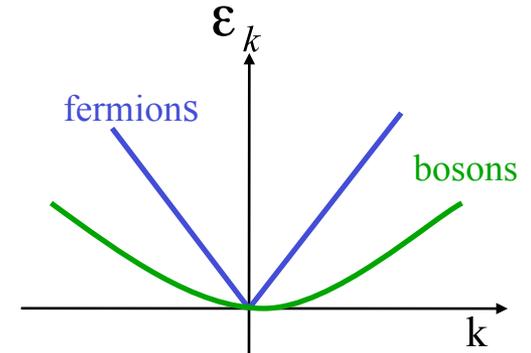
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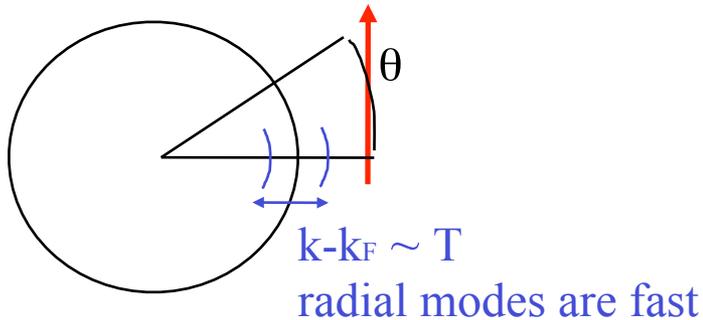
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transverse modes are slow!
(controlled by boson fluctuations)



Two types of modes cannot be separated at the level of the action

$$q_{\text{radial}} \sim T$$

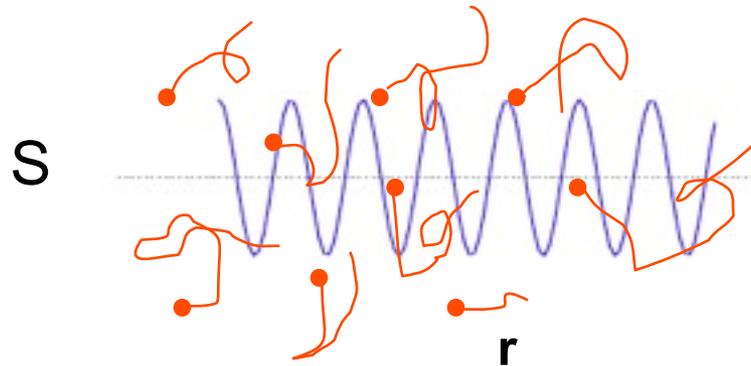
$$q_{\text{transverse}} \sim \sqrt{T}$$

The Spin Density Wave Scenario



John A. Hertz.

Hertz-Millis-Moryia-Beal-Monod



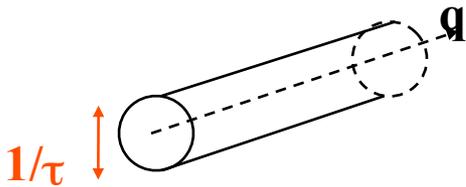
$$F = M \left(i\alpha\omega + \xi^2 q^2 + \delta \right) M + bM^4 .$$

$\delta = a(T - T_c)$ and the QCP occurs at $T_c = 0$.

$$d_{eff} = d + z$$

$z = 3$ for the ferromagnet ($\alpha = c/q$)

$z = 2$ for the anti-ferromagnet ($\alpha = c$)



The spin-fermion model

A Abanov, A. Chubukov, RMP 2003
 Belitz, Kirkpatrick, Vojta, RMP 05
 J Rech, CP, A Chubukov.

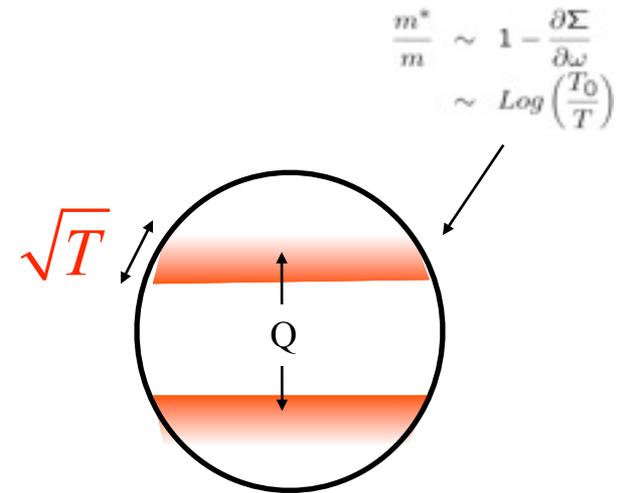
•3D Spin Density Wave

Rosch, '98

Ex: CeNi₂Ge₂ ...

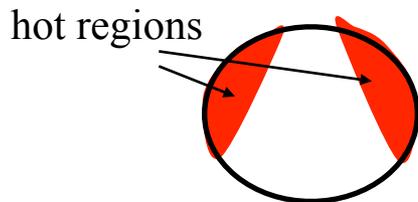
$$\Delta\rho(T) \sim T^{3/2}$$

$$\gamma_p(T) \sim Cst - \sqrt{T}$$



•2D Spin Density Wave into 3D metal

Rosch ('98), Georges, Kotliar, Paul ('03)



Ex: CeCu₆Au, or CeCu₆Ag ...

$$\Delta\rho(T) \sim T$$

$$\gamma_p(T) \sim \text{Log} \left(\frac{T_0}{T} \right)$$

**No anomalous exponent
 in spin susceptibility**

Eliashberg theory around itinerant ferromagnetism, or U(1) gauge theory coupled to matter

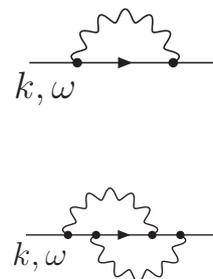
$$H_{sf} = \sum_{k,\alpha} \epsilon_k c_{k,\alpha}^\dagger c_{k,\alpha} + \sum_q \chi_{s,0}^{-1}(q) \mathbf{S}_q \mathbf{S}_{-q} \\ + g \sum_{k,q,\alpha,\beta} c_{k,\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{k+q,\beta} \cdot \mathbf{S}_q,$$

$$\chi_{s,0}(q, \Omega) = \frac{\chi_0}{\xi^{-2} + q^2 + A\Omega^2 + O(q^4, \Omega^4)}.$$

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d=2

$$\begin{array}{ccc} \omega_0^{1/3} & \omega^{2/3} & \\ & \downarrow & \left(\frac{\omega_0}{\omega}\right)^{1/3} \\ \omega_0^{2/3} & \omega^{1/3} & \end{array}$$

Rech, CP, Chubukov (06)

The bare power counting diverges in $d \leq 3$

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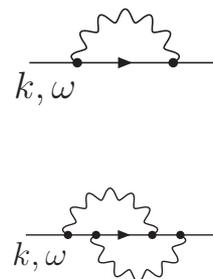
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- neglect vertex corrections
- dressed propagators (self-energy)

$$\alpha \sim \frac{\bar{g}^2}{\gamma v_F^3} \sim \frac{\bar{g}}{N E_F} \ll 1 \quad \beta \sim \frac{m \bar{g}}{\gamma v_F} \sim \frac{m_B}{N m} \ll 1.$$

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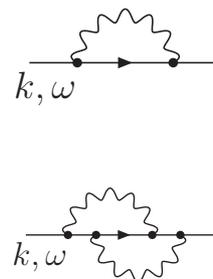
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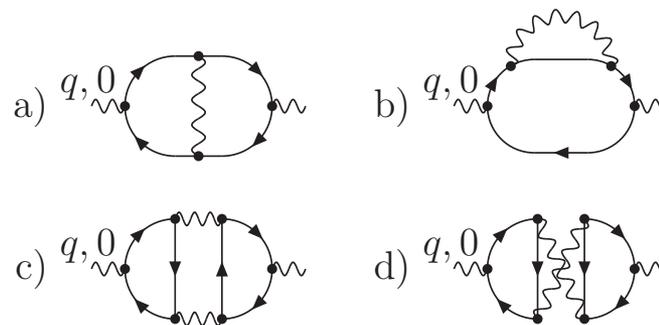
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Rech, CP, Chubukov (06)

The bare power counting diverges in $d \leq 3$



BKV type singularity

Belitz, Vojta, Kirkpatrick(03),
Chubukov, Maslov (07)
Green, ben Simon(11)



And the culprit is ...

$2k_F$ - scattering processes the back -scattering

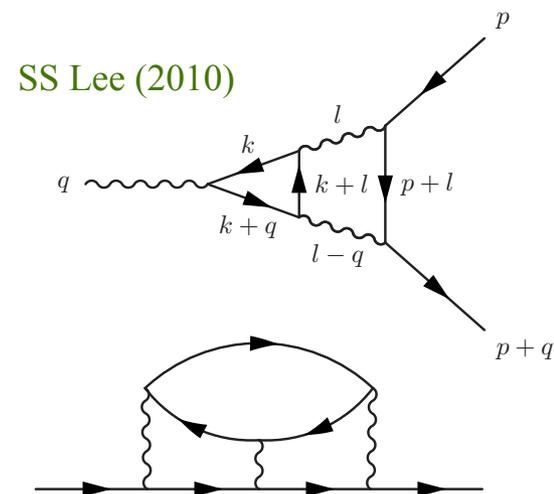
affecting AFM, nematic, and Ferro

- FS deformed at the hot spots
- anomalous exponents

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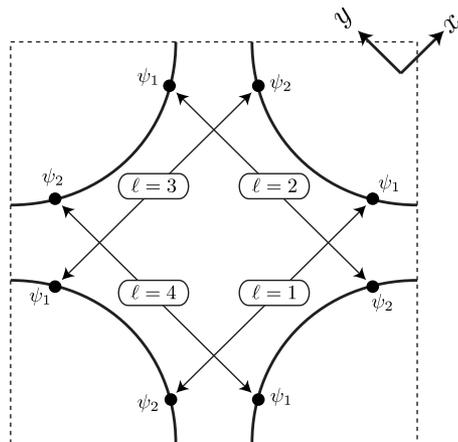
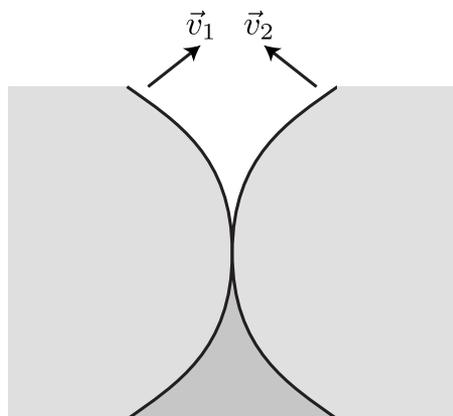
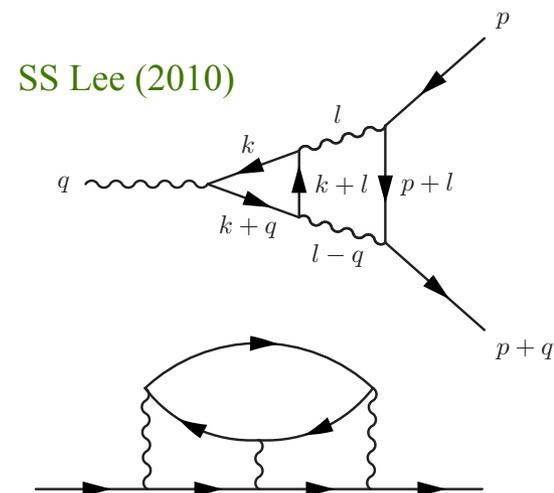


$2k_F$ - scattering processes

the back -scattering

affecting AFM, nematic, and Ferro

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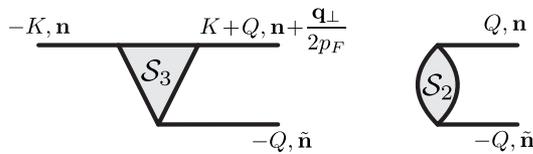
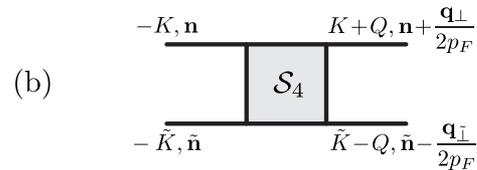


Metlitski, Sachdev (2010)

Recent susy-bosonization in high dimensions

Hendrik Meier, CP, Efetov

(a) $g_{\mathbf{n}}(K) = \frac{K, \mathbf{n}}{\quad}$



- Re-summation of the BS processes
- Curvature effects : charge and spin channels are coupled
- Re-summation of all non analyticities for the FL theory

$$\delta\Omega = \frac{\zeta(3)T^3}{\pi v_F^2} \left\{ \frac{\ln^2(1 + \gamma_{\pi}^I L)}{L^2} + 3 \frac{\ln^2(1 + \gamma_{\pi}^{II} L)}{L^2} \right\}$$

$$\gamma_I = \gamma_c - 3\gamma_s$$

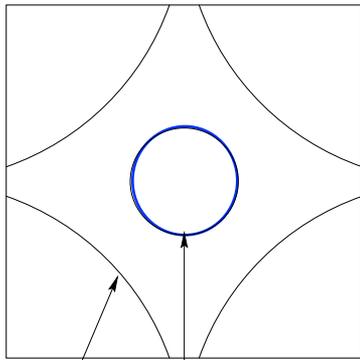
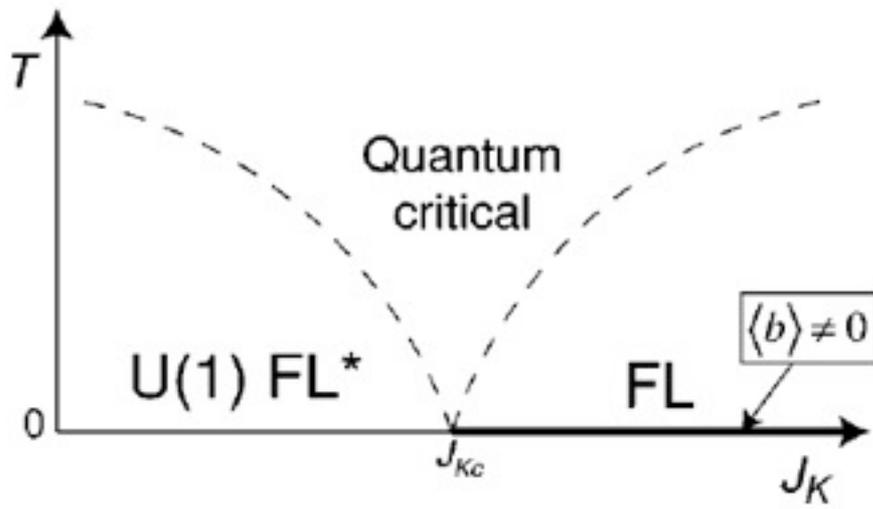
$$\gamma_{II} = \gamma_c + \gamma_s$$

* K.B. Efetov, C. Pepin, H. Meier,

Exact bosonization for an interacting Fermi gas in arbitrary dimensions

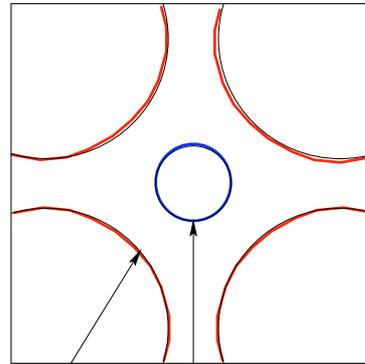
Phys. Rev. Lett. 103,186403 (2009); PRB 82,235120 (2010), preprint 2011

Strong(er) coupling



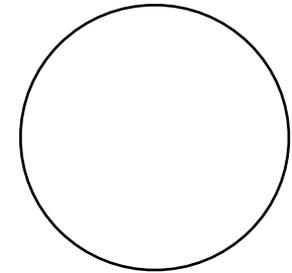
Spinon Fermi surface Small electron fermi surface

Spin liquid



Hot Fermi surface Cold Fermi surface

Quantum Critical



Heavy fermi liquid

Senthil, Sachdev, Vojta -PRL2003 PRB 2004

Entropic considerations

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$$\text{Yb} : 4f^{13}$$

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$$S=1/2 \quad L=3$$

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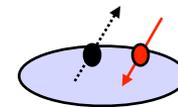
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Kondo screening

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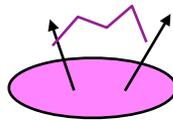
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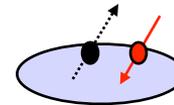
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Spin Liquid



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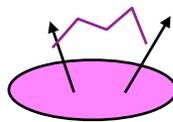
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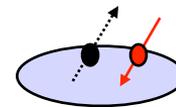
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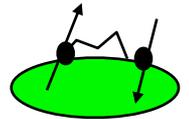
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Spin Liquid



Kondo screening



Cooper pairs

Entropic considerations

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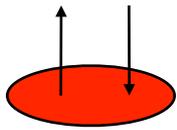
$$\text{S O} : J = |L+S| = 7/2$$

$$\text{U} : 5f^2$$

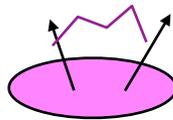
$$S=1$$

$$L=3+2$$

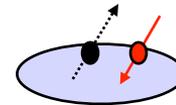
$$\text{S O} : J = |L-S| = 4$$



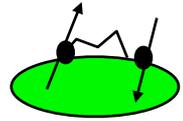
AF singlets



Spin Liquid



Kondo screening



Cooper pairs

Entropic considerations

Ce : $4f^1$

Yb : $4f^{13}$

U : $5f^2$

S=1/2 L=3

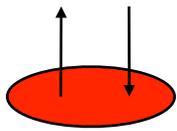
S=1/2 L=3

S=1 L=3+2

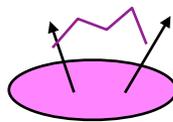
Spin Orbit : $J = |L-S| = 5/2$

S O : $J = |L+S| = 7/2$

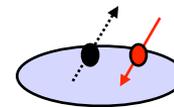
S O : $J = |L-S| = 4$



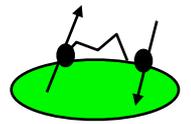
AF singlets



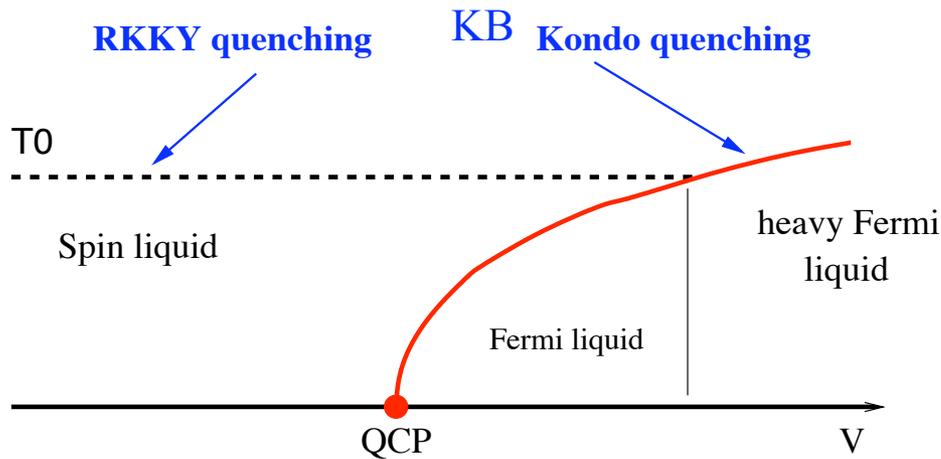
Spin Liquid



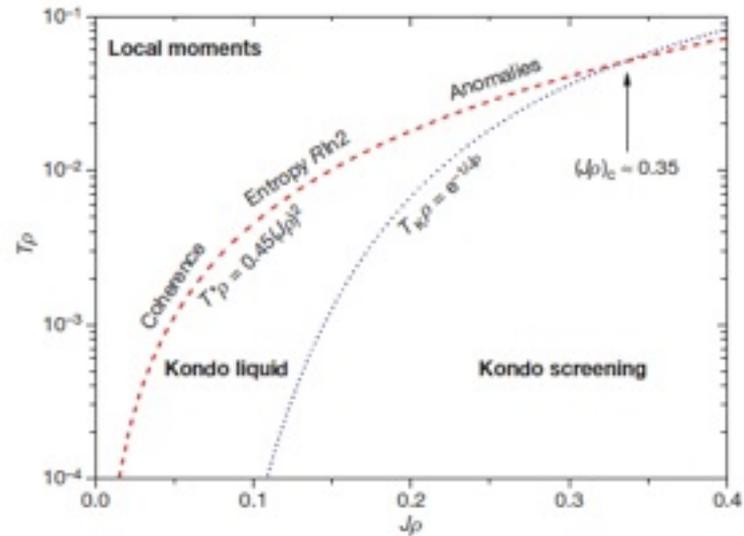
Kondo screening



Cooper pairs



Yang, Pines, Fisk, Thompson, 2008



Competition between Coulomb and Kinetic energy

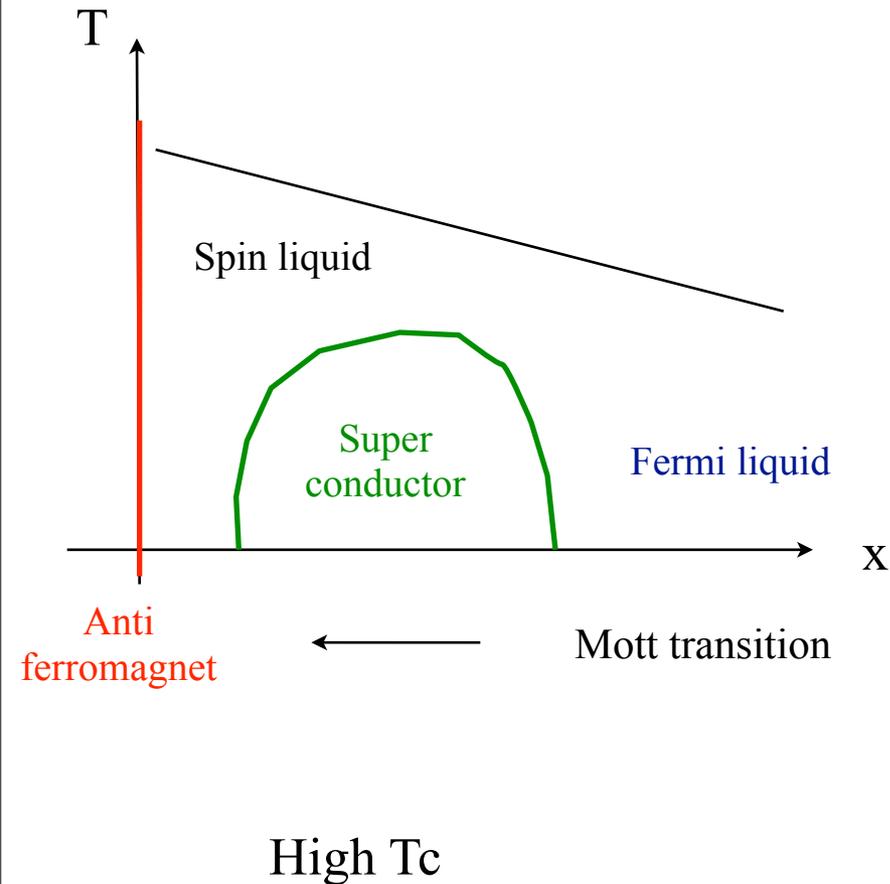
$$H_{Coulomb} = U \sum_i n_{f\uparrow} n_{f\downarrow}$$

$$H_{Kinetic} = - \sum_{\langle ij \rangle} f_i^\dagger t_{ij} f_j$$

Competition between Coulomb and Kinetic energy

$$H_{Coulomb} = U \sum_i n_{f\uparrow} n_{f\downarrow}$$

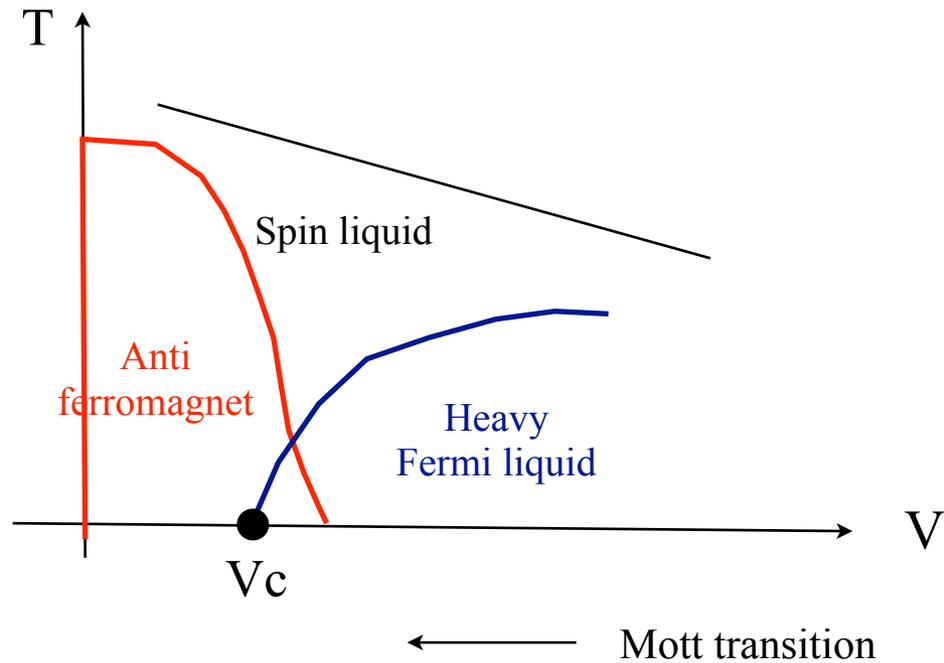
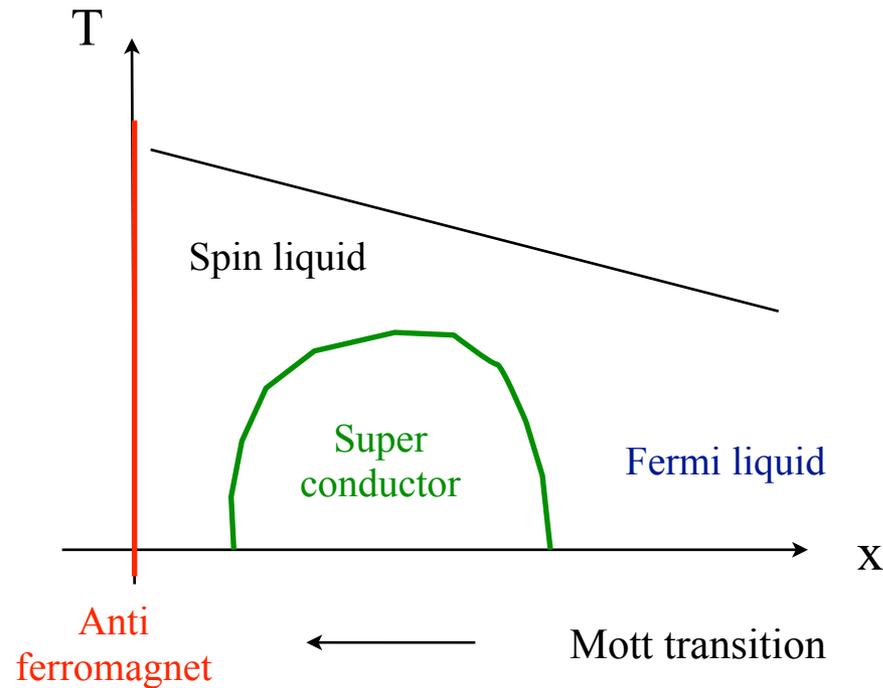
$$H_{Kinetic} = - \sum_{\langle ij \rangle} f_i^\dagger t_{ij} f_j$$



Competition between Coulomb and Kinetic energy

$$H_{Coulomb} = U \sum_i n_{f\uparrow} n_{f\downarrow}$$

$$H_{Kinetic} = - \sum_{\langle ij \rangle} f_i^\dagger t_{ij} f_j$$



Breakdown of the Kondo effect associated with a Mott transition on the f-electrons

P. Coleman (Schroder 2000)
**deconfinement,
fractionalization**

Zhu, Martin, PNP (09)
**modulations in Kondo
breakdown**

Q. Si, Nature (02-)
S. Kirchner (06,08)
**locally quantum
critical**

Burdin, Grepel, Georges
(98)
**breakdown by
exhaustion**



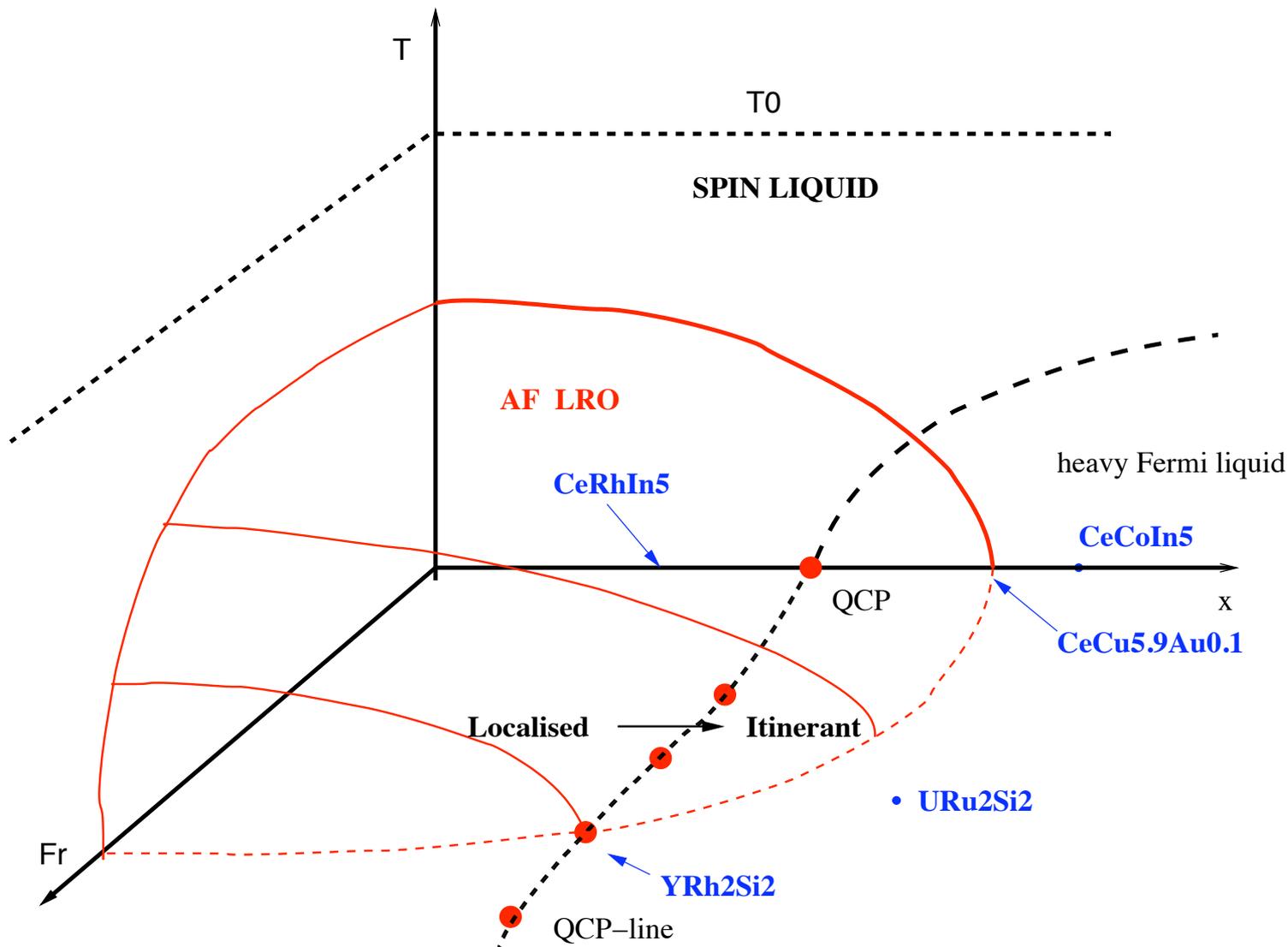
Pines, Zhang, Fisk (08)
S. Kirchner (06,08)
two fluids model

B. Jones (2010)
**RG on Kondo
Breakdown**

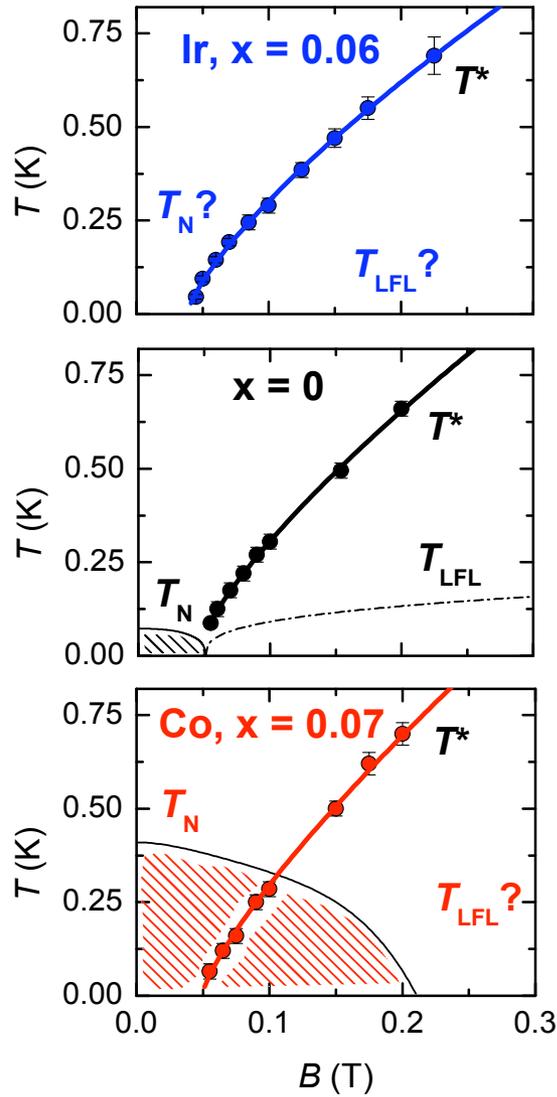
Continentino (09) Vekhter +
Seo+ CP (10) Paul ,Norman
(10)
**SC quantum critical
point**

CP, Norman ,Paul (07)
**selective Mott
transition, $z=3$ regime of
fluctuations**

Senthil, Sachdev ,Vojta (04)
**model for
fractionalization, spin
liquid**



Doping plays the role of pressure

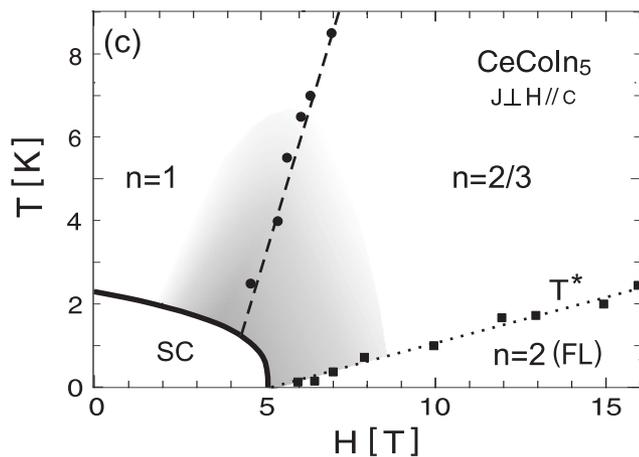
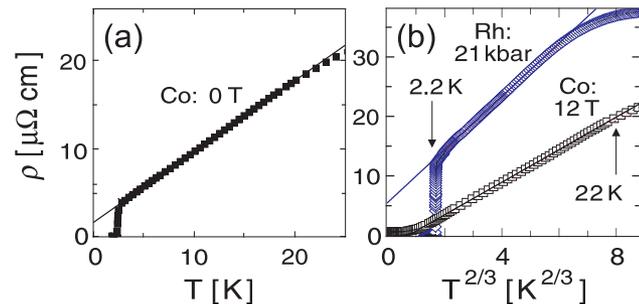
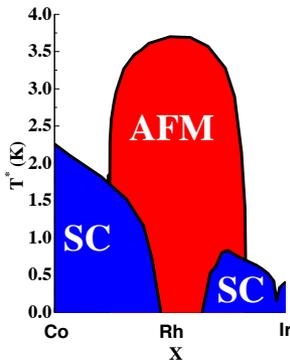


Dresden group (10)

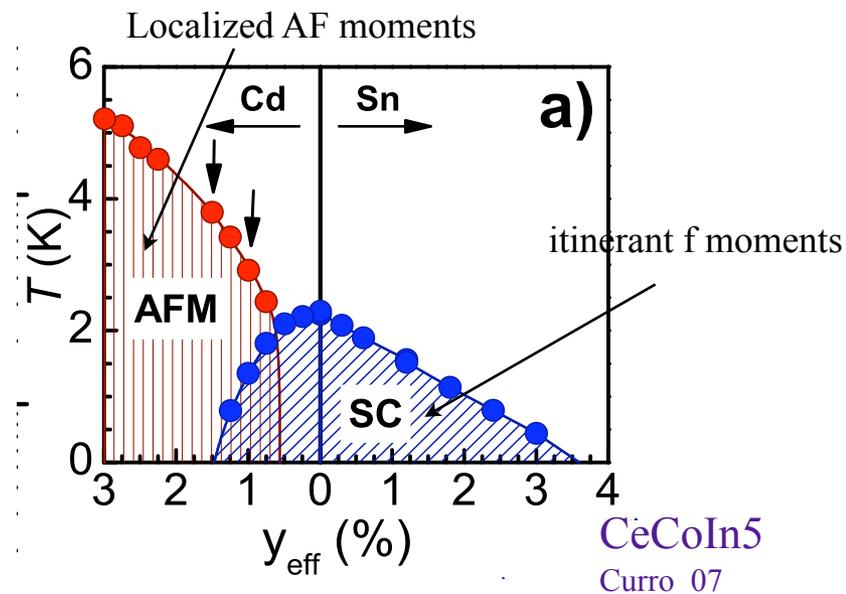
Is there a spin liquid on the left of T^* ?

Differentiate the scenarios where the KB is tight to AFM transition (Si et al.) from the ones where the KB is alone

Evidence for localization in 115 .



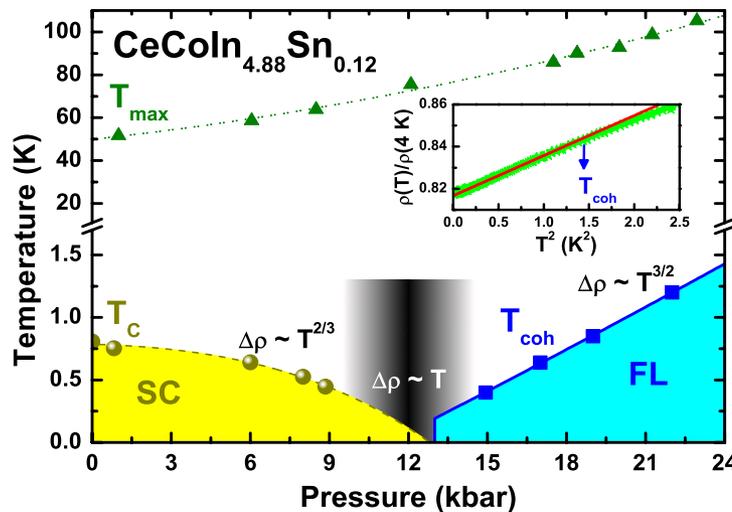
Paglione 04



CeCoIn5
Curro 07

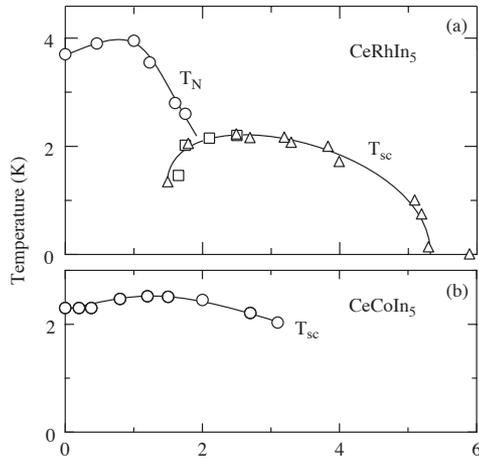
KB + Cross-over 3D \rightarrow 2 D

sub-linear exponents

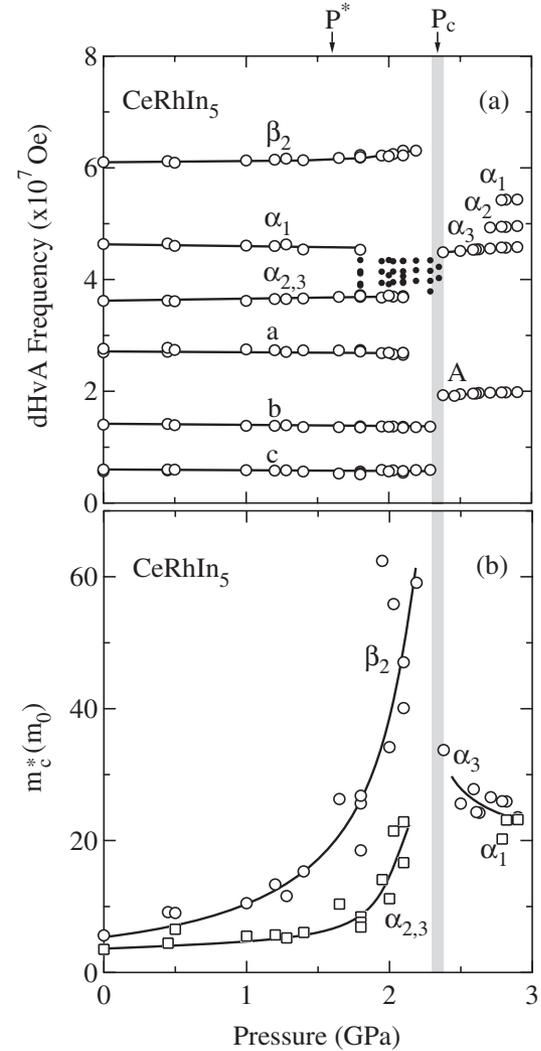


Paglione
Saitovich
Continentino
(10)

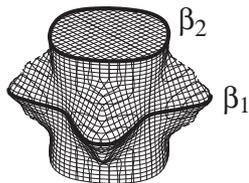
Pressure induced superconductivity in 115 series



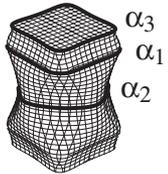
similar to UGe2
(Lonzarich's stalk)
but AFM vs FM



(a) LaRhIn_5
(CeRhIn_5)

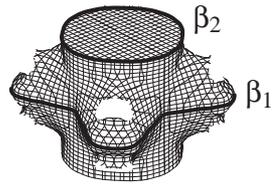


band 14 - electron

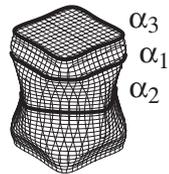


band 15 - electron

(b) CeCoIn_5



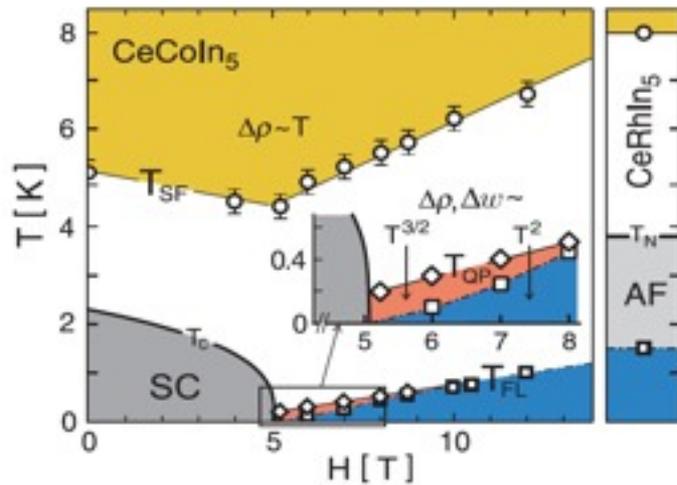
band 14 - electron



band 15 - electron

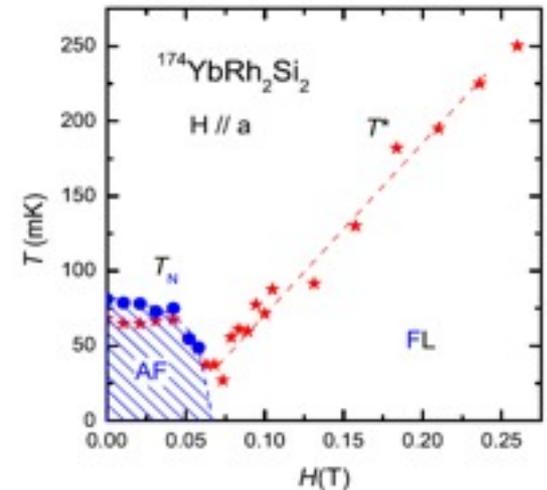
Onuki's group (05)

Multiple Energy Scales in Quantum Critical Regime



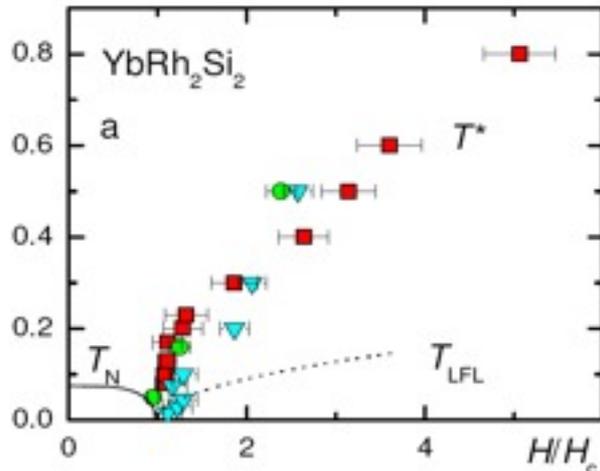
Finite energy scales T_{SF} , T_{QP} in QC regime.

J. Paolone *et al.* cond-mat/0605124



Finite T_{FL} at QCP from resistivity.

Courtesy J. Flouquet (unpublished)



Gegenwart *et al.*, cond-mat/0604571

Finite low-energy scale near Kondo breakdown QCP

Conclusions

- Strong experimental evidence for anomalous quantum criticality in HF compounds
- Breakdown of the conventional techniques which integrate out the fermions for (almost all?) models below $d=3$.
- Fractionalization-deconfinement and emerging spin liquid represent the state of the art to explain the data
- Better theories (and methods) needed ... for example
Ads/CMT or ... a new bosonization technique ?