# Aspects of probe branes in AdS/CFT: Conformal phase transition

Andrei Parnachev

Leiden University

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# Introduction

Will be interested in models which exhibit phase transitions of the the BKT (or conformal, CPT) type:

$$M \simeq \Lambda \exp(-\frac{c}{\sqrt{\alpha - \alpha_c}})$$

In general, such separation of scales is a result of "walking behavior" when the system sits close to RG fixed point for a long RG time

$$\beta(g;\alpha) = (\alpha_* - \alpha) - (g - g_*)^2$$

## Outline

## D3-D7 system

- D3-D7 at strong coupling
- D3-D7: field theory

## D3-Dp system: BKT limit

- Field theory
- AdS/CFT
- Spectrum
- Finite $T, \mu$

# D3-D7 system

Consider N D3 branes stretched along 0123 directions and a D7 brane stretched along 012 45678. The low energy dynamics is described by defect fermions coupled to CFT:

$$\mathcal{S} = \mathcal{S}_{N=4} + \int d^3x (i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi + g\bar{\psi}\phi^9\psi)$$

Giving vev to $\phi^9$ corresponds to giving a bare mass to $\psi$. We will consider situation where the bare mass vanishes. There might be dynamical mass generation if in the vacuum $\phi^9 \neq 0$; at small 't Hooft coupling this does not happen.

# DBI action and EOM

At large 't Hooft coupling $\lambda \gg 1$ we are instructed to consider $AdS_5 \times S^5$ space $[r^2 = \rho^2 + (x^9)^2]$

$$ds^2 = r^2 dx_\mu dx^\mu + r^{-2} \left( d\rho^2 + \rho^2 d\Omega_4^2 + (dx^9)^2 \right)$$

The D7 brane embedding is specified by $x^9 = f(\rho)$, giving the DBI action

$$S_{D7} \sim \int d^3x \int d\rho \frac{\rho^4}{\rho^2 + f(\rho)^2} \sqrt{1 + f'(\rho)^2}$$

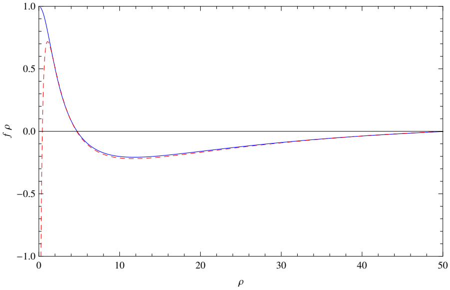
EOM for $f(\rho)$ are second order, non-linear differential equation. In the UV region, it gets linearized and can be solved analytically.

# Solutions of EOM

Namely, for $\rho \gg f(\rho)$,

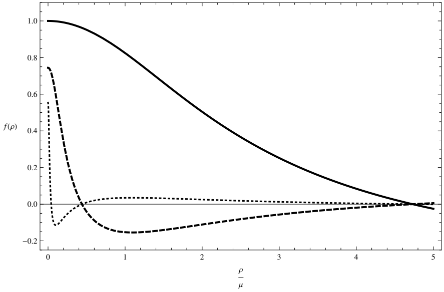
$$f(\rho) \approx A\mu^{3/2}\rho^{-1/2}\sin(\frac{\sqrt{7}}{2}\log\rho/\mu + \varphi)$$

One can also solve the full EOM numerically starting from $f'(0) = 0$.



# Solutions of EOM

To proceed, impose Dirichlet boundary conditions at the UV cutoff $\Lambda$. Solutions are labeled by the number of nodes in $(0, \Lambda)$.




In particular, $f_0(0) \sim \Lambda$. One can study spectrum of excitations around $f_n(\rho)$. There are $n$ tachyons. Solution $f_0(\rho)$ is energetically preferred and does not have tachyons living on it. Masses $\sim \Lambda$.

# Comments

- In the large momenta regime the lagrangian for $f(\rho)$ is that for the tachyon (significantly) below the BF bound. We follow its condensation.

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- Can study the system at finite temperature and chemical potential. Observe first order phase transition at $T, \mu \sim \Lambda$
- It is desirable to separate the scale of meson masses from $\Lambda$. See below.

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- Can study the system at finite temperature and chemical potential. Observe first order phase transition at $T, \mu \sim \Lambda$
- It is desirable to separate the scale of meson masses from $\Lambda$. See below.
- There is a phase transition as $\lambda$ is varied from 0 to infinity.

# D3-D7: field theory

The gauge-invariant measure of symmetry breaking is

$$OW(x,y) = \bar{\psi}(x)\mathcal{P}\exp\left[ig\int_x^y A\cdot dl\right]\psi(y)$$

Can use OPE to study the large momentum behavior of its Fourier transform:

$$\langle OW(p)\rangle \sim p^{\Delta(\lambda)-4}\left(\not{p} + M(p) + \cdots\right)$$

where

$$M(p) = m\left(\frac{p}{\mu}\right)^{\gamma} + \frac{\langle\bar{\psi}\psi\rangle}{p}\left(\frac{\mu}{p}\right)^{\gamma}$$

and $\gamma$ is anomalous dimension of $\bar{\psi}\psi$, $\Delta[\bar{\psi}\psi] = 2+\gamma$.

Note that $M(p)$ satisfies the same equation as $f(\rho)$ at large $\rho$,

$$\frac{d}{dp}(p^2 M'(p)) + C(\lambda)M(p) = 0$$

where

$$\gamma(\lambda) = -\frac{1}{2} + \sqrt{\frac{1}{4} - C(\lambda)}$$

At weak coupling $C(\lambda) \sim \lambda + \ldots$.

As long as $C(\lambda) < 1/4$, the trivial solution is favored. But we know that $C(\lambda = \infty) > 1/4$ from AdS/CFT, and at $\lambda = \lambda_c \sim 1$ there is a dynamical mass generation. As $\kappa \equiv C(\lambda) - \frac{1}{4} \sim (\lambda - \lambda_c) \to 0$, the solution becomes

$$M(p) = A\mu \left(\frac{\mu}{p}\right)^{\frac{1}{2}} \sin\left(\sqrt{\kappa} \ln \frac{p}{\mu} + \phi\right)$$

Dirichlet boundary conditions at $p = \Lambda$ imply

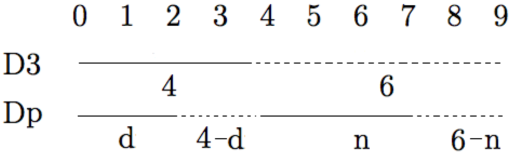
$$\mu \simeq \Lambda \exp\left(-\frac{\pi}{\sqrt{\kappa}}\right)$$

Note that $\kappa = 0$ is associated with the "double trace" operator $(\bar{\psi}\psi)^2$ becoming marginal. This is familiar from AdS/CFT: for mass slightly above the BF bound, there are two fixed points which correspond to alternative and standard quantizations. One can flow between the two fixed points by turning on the "double trace" operator. The two points merge when this operator becomes marginal.

Computing the exact value of $\lambda_c$ is beyond the regime of applicability of either AdS/CFT or perturbation theory. In the following we will change the parameters of the system to push it into the regimes where one can maintain control.

# The setup

Consider Dp-brane which shares $d$ spacetime dimensions with the D3-branes, and is extended in $n = p + 1 - d$ additional spatial directions:




The nonsupersymmetric D3-D7 corresponds to $(d, n) = (3, 5)$.
Supersymmetric D3-D7 is $(d, n) = (4, 4)$.

# Field theory

In the large momenta regime the equation of motion is

$$\frac{d}{dp}\left(p^{d-1}M'(p)\right) + C(\lambda)p^{d-3}M(p) = 0$$

and the dynamics depends on the sign of $\kappa(\lambda) = C(\lambda) - \left(\frac{d-2}{2}\right)^2$ (For $\kappa < 0$ the solution is $M(p) = 0$, while for $\kappa > 0$ the solution is nontrivial). For $d = 2 + \epsilon$ the perturbative expansion is reliable, since $C(\lambda) \sim \lambda \sim \epsilon^2$.

This is not surprising, since $\Delta[\bar{\psi}\psi] = 1$ in $d = 2$. The phase transition is a CPT since $M \sim \Lambda \exp(-\pi/\sqrt{\kappa})$

# AdS/CFT

Consider strong coupling regime, $\lambda \gg 1$; set $d = 3$ and write $AdS_5 \times S^5$ metric as

$$ds^2 = \left(\frac{r}{L}\right)^2 dx_\mu dx^\mu + \left(\frac{L}{r}\right)^2 (d\rho^2 + \rho^2 d\Omega_{n-1}^2 + df^2 + f^2 d\Omega_{5-n}^2)$$

The resulting DBI action is

$$S_{Dp} = \int d^d x \int d\rho \left(\frac{\rho^2 + f^2}{L^2}\right)^{\frac{d-n}{2}} \rho^{n-1} \sqrt{1 + f'^2}$$

Again, the EOM are nonlinear, but linearize in the large momentum limit.

# AdS/CFT

The $n = 5$ case was studied before; there was a tachyon well below the BF bound and no separation of scales. The tachyon mass is controlled by

$$\kappa_\infty = \lim_{\lambda\to\infty} \kappa(\lambda) = n - d - \left(\frac{d-2}{2}\right)^2$$

In $d = 3$ case the critical value of $n$ is $n_c = 13/4$. Tune $n = n_c + \epsilon$ so that $\kappa \ll 1$. In the large momentum regime

$$f(\rho) = A\mu \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}} \sin\left(\sqrt{\kappa}\ln\frac{\rho}{\mu} + \phi\right)$$

# AdS/CFT

The dynamically generated scale is

$$f(0) = \mu \simeq \Lambda \exp\left(-\frac{\pi}{\sqrt{\kappa}}\right)$$

Note that for $\Lambda \gg \rho \gg \mu$

$$f(\rho) = \mu \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}} \left(C_1 \ln \frac{\rho}{\mu} + C_2\right)$$

One can find $C_1, C_2$ numerically by integrating full EOM at $\kappa = 0$. The resulting values of $A, \phi$ are obtained via

$$A = \frac{C_1}{\sqrt{\kappa}}, \qquad \phi = \frac{C_2}{C_1}\sqrt{\kappa}$$

## Comments

We focus on the BKT limit ($\kappa \to 0$) where there is a parametric separation between the UV cutoff and the physical scale.

- The ground state is given by integrating EOM at $\kappa = 0$ starting from $f(0) = \mu; f'(0) = 0$. The action is negative compared to that of the trivial $f = 0$ solution.

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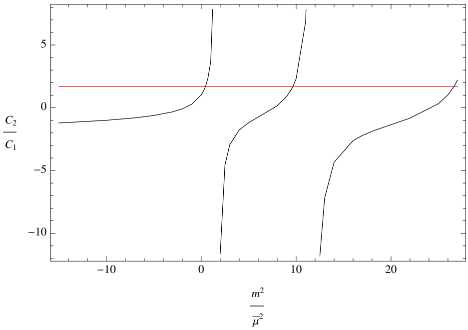
- The ground state is given by integrating EOM at $\kappa = 0$ starting from $f(0) = \mu$; $f'(0) = 0$. The action is negative compared to that of the trivial $f = 0$ solution.
- At finite $\kappa$ there were other solutions labeled by a number of nodes. They disappear in the BKT limit.

# Spectrum

We can study the spectrum of mesonic excitations by considering small fluctuations around the background solution. The scalar mesons come from the fluctuations of $f(x^\mu, \rho)$. Boundary conditions at the UV cutoff fix the ratio of $C_1/C_2$.



# Spectrum

The spectrum of the scalar mesons is given by

$$m^2/\mu^2 \approx 0.44, 9.65, 26.63, 51.35, 84, \cdots$$

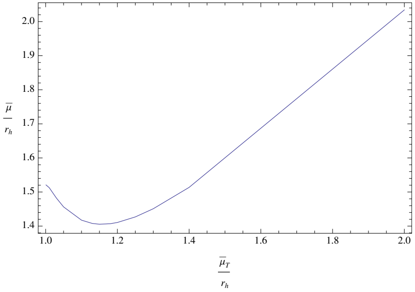
While that of the vector mesons is

$$m^2/\mu^2 \approx 3.08, 15.12, 34.87, 62.32, 97.46, 140.31, \ldots$$

Note that there are no tachyons. There is a light technidilaton, but it is not parametrically light.

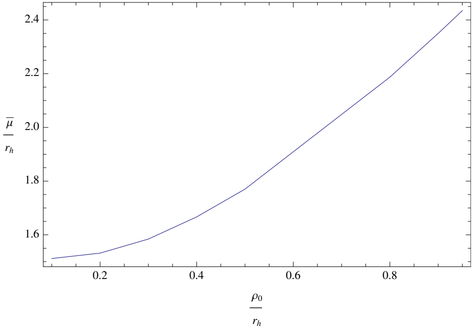
# Finite temperature

One can study the system at finite temperature and chemical potential in the usual way. At strong coupling one can study Dp brane propagating in the black brane geometry with $r_h \sim T$. For $\rho \gg T, \mu$ the asymptotic behavior must be unchanged; One can relate $\mu_T = f(0)$ at $T > 0$ with $\mu = f(0)$ at $T = 0$. "Minkowski embeddings" only exist for $T < T_c \sim \mu$.



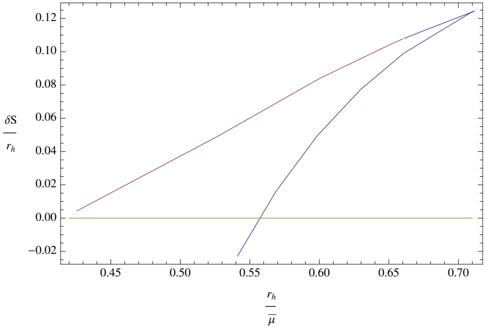
# Finite temperature

There are 2 solutions for generic $\mu$. On the other hand, there is a unique black hole embedding in a finite interval of temperatures:



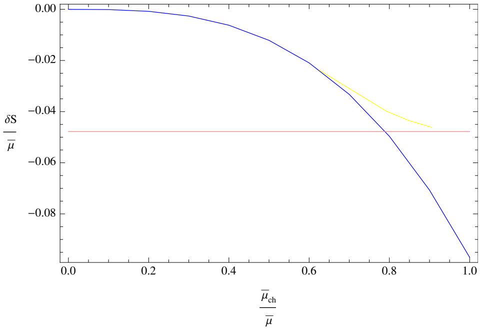
# Finite temperature

The resulting phase structure is given by a diagram which is typical for the first order phase transitions



# Finite chemical potential

Using standard techniques one can construct phase diagram for the finite chemical potential case.



# Phase structure: summary

To summarize, there is a line of first order phase transitions in the $\mu_{chem}$, $T$ plane which separates phases with broken and unbroken conformal symmetry. The characteristic scale of $T$ is given by the meson mass $\mu$ while that of chemical potential is given by the consituent fermion mass $\sim \mu\sqrt{\lambda}$.

This is familiar from other brane systems. As usual, the mesons are tightly bound.

# Thank you!