Large-N Gauge Theory
and
"Strange Metallic" Transport

Based on e.g.

- S. Hartnoll, J. Polchinski, ES, D. Tong '09
  (5.1, 5.3, 7.3)
- K. Jensen, S. Kachru, A. Karch, J. Polchinski, ES '11

& background material

Outline

- Field Theory examples giving

$$\rho \propto T$$

- Comments on (A)dS/CMT

"Strange Metal" Phenomenology

poses an interesting strong coupling problem.

Scales (not to scale):

Λ_{UV} (e.g. $1/a_{lattice}$)

$E_{gap} \gtrsim \mu$

melting

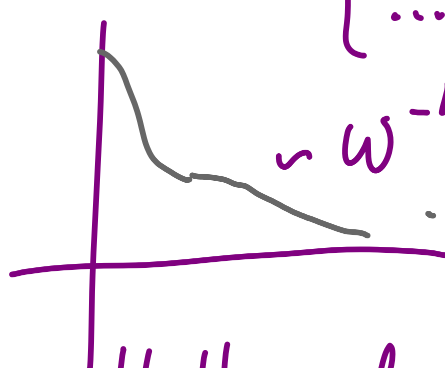
$\gtrsim 3$ orders of magnitude with non-Fermi-liquid behaviors: e.g.

T

- - - - ← e.g. SC transition

$$\rho_{\omega=0} \propto T^{\nu_1} \begin{cases} \nu_1 = 2 & \text{FL} \\ 1 & \text{cuprates} \\ & \text{heory F} \\ \ldots & \end{cases}$$

$\sigma(\omega)$



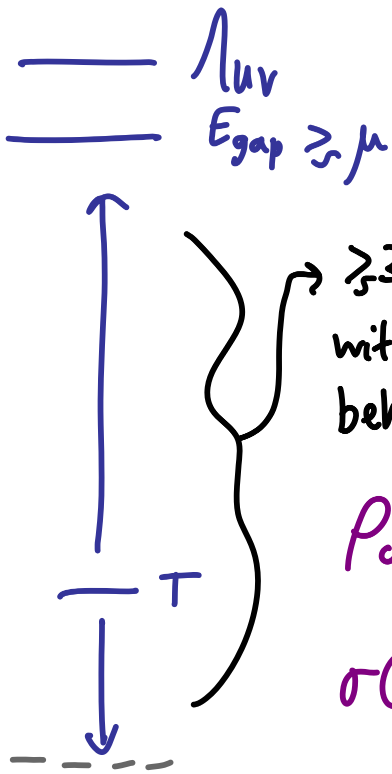
$$\sim \omega^{-\nu_2} \begin{cases} 1 & \text{FL} \\ 0.65 & \text{SM} \end{cases}$$

(?)

anomalous Hall conductivity

...

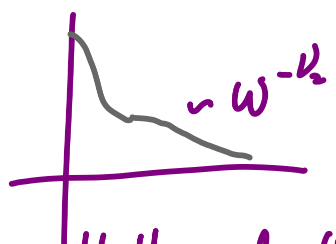
A more proximate goal is to find specific controlled QFT(s) exhibiting these NFL features.



$\gtrsim 3$ orders of magnitude with non-Fermi-liquid behavio(u)rs: e.g.

$\rho_{\omega=0} \propto T^{\nu_1}$ $\quad \nu_1 \neq 2$

$\sigma(\omega)$



$\nu_2 \neq 1$

anomalous Hall conductivity

...

→ Question: what does it take to model this in $\underbrace{\text{AdS/CFT (i.e. holographically)}}_{\text{(albeit unrealistic: large } N_c\text{, near-SUSY, ...)}}$?

(2) QFTs

Let's consider 2 explicit examples in 2+1 dimensions

(1) Yang-Mills + flavors + SUSY

(2) Chern-Simons + SUSY

1)
$$S = \int d^3x \left\{ \frac{1}{g^2} \overbrace{\mathrm{Tr}}^{SU(N_c)} F^2 + \sum_{f=1}^{N_f} \psi_f^\dagger D \psi_f + \text{SUSY completion} \right\}$$

weak coupling

$$\left.\frac{1}{g^2}\right|_{1\text{-loop}} = \frac{1}{g_0^2} + \frac{N_f}{|q|}$$

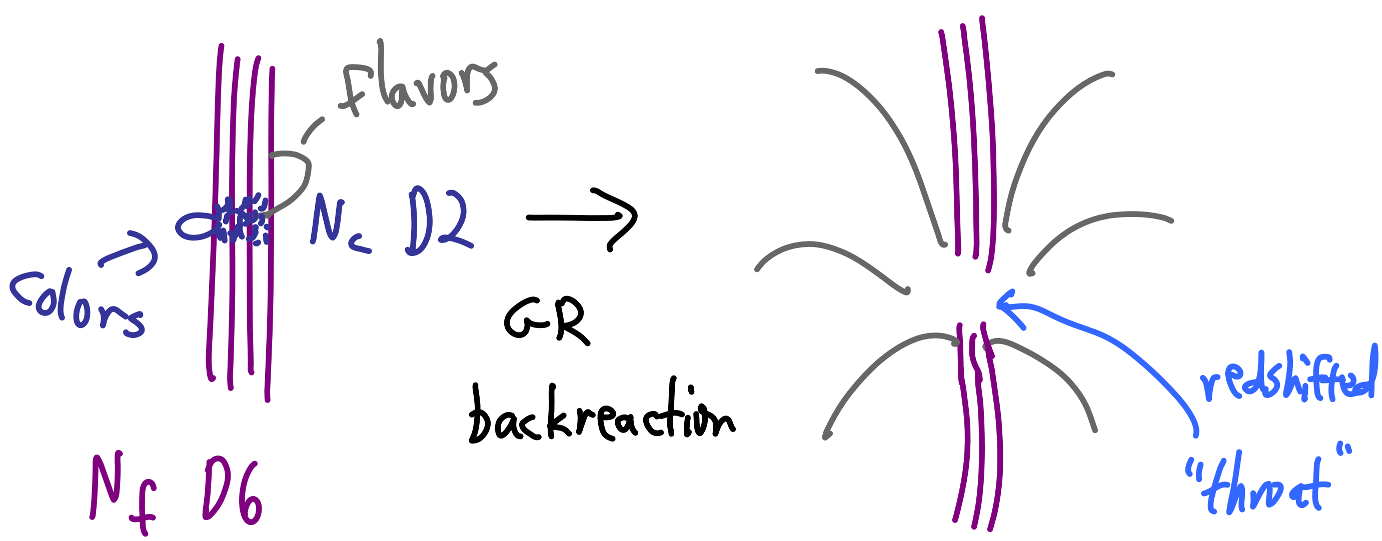
IR fixed pt $\frac{1}{g^2} \to N_f$ (independent of SUSY)

(so far at zero density)

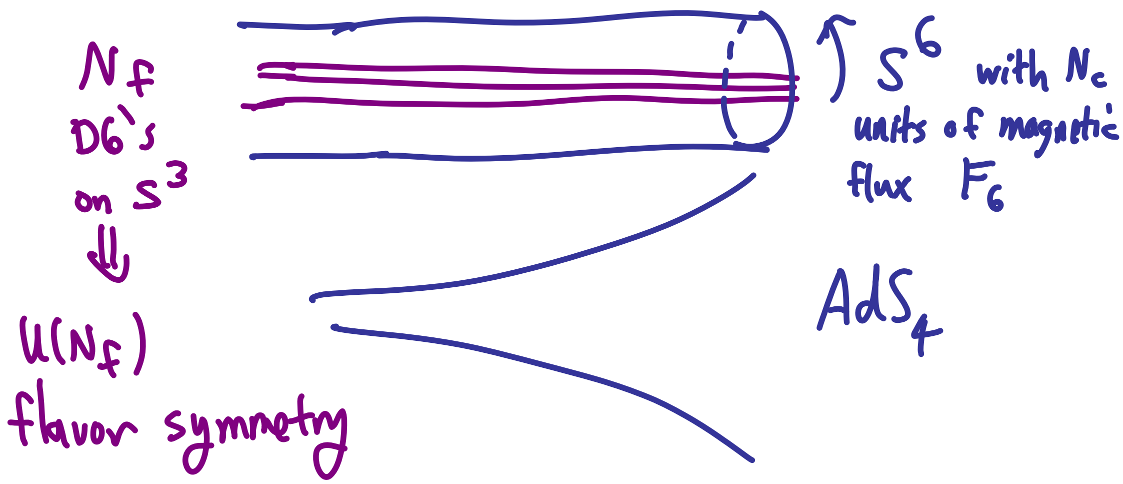
Also holds at strong coupling (with all this SUSY, $\frac{1}{g^2}$ is protected).

$$\lambda_{\text{'t Hooft}} = \bar{g}^2 N_c = \frac{N_c}{N_f}$$

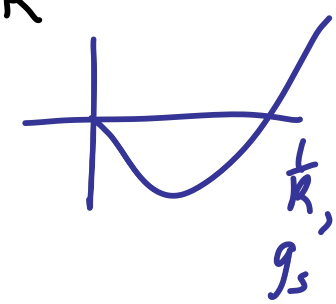
For $\lambda_{\text{'t Hooft}} \gg 1$, there is a large-radius gravity description, again the flavors are intrinsic to the fixed point (= AdS_4 solution)



Near-horizon geometry



Effective potential for the size R of S^6 and the string coupling



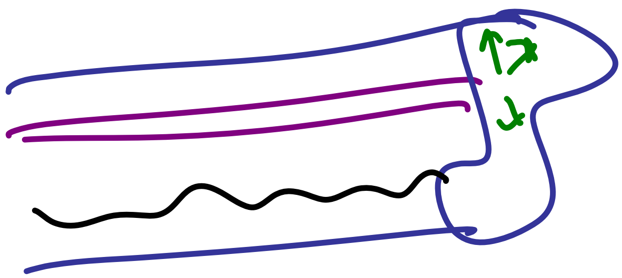
$$\mathcal{U} \sim \left(\frac{g_s^4}{R^6}\right)\left(-\frac{1}{g_s^2 R^2} + \frac{N_f}{g_s R^3} + \frac{N_c^2}{R^{12}}\right)$$

has a minimum with

$$R_{AdS} \sim R \sim \left(\frac{N_c}{N_f}\right)^{\frac{1}{4}} ; \; g_s \sim \left(\frac{N_c}{N_f^5}\right)^{\frac{1}{4}}$$

- Here flavors are not "probes"; they are intrinsic to the solution.
- Fixed point of β ftn $\Leftrightarrow$ "moduli"-fixing

(Aside: there are <u>many</u> string theory compactifications



and stress-energy sources

→ strong argument for rich "landscape" of (A)dS solutions.)

Now consider finite density: on the gravity side this corresponds to putting in a boundary condition

$$A_0 \Rightarrow \mu$$

on a bulk gauge field, corresponding to a flavor symmetry in the QFT. Then solve for charge density J^t

e.g. • "RR" $U(1)$ $\quad C^{(1)}_\mu$

• $U(N_f) \to U(1) \times U(N_f - 1)$

$(\cdots)$

On the GR side, certain particles (D0-branes) are charged under $U(1)_{RR}$. So solve for

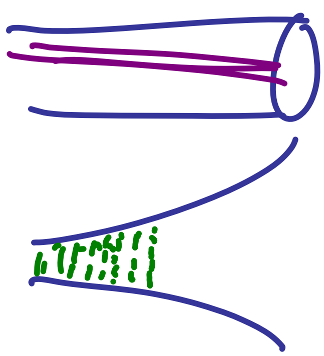
ρ_{D0}, $A_0(r)$, and geometry.

$$S = \int d^4x \left[\sqrt{-g} \left(\frac{1}{2\kappa^2} \left[\mathcal{R} + \frac{6}{L^2} \right] - \mathcal{F}(|F|^2) \right) + \rho \left(-M\sqrt{-g_{tt}} + A_t \right) \right].$$

with ansatz $$ds^2 = L^2 \left(-e^{2\gamma_t(v)} dt^2 + e^{2\gamma_x(v)} (dx^2 + dy^2) + \frac{dv^2}{v^2} \right)$$

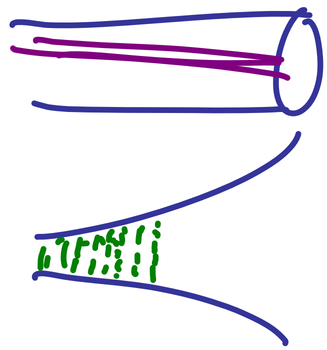
(allowing for Lifshitz dynamical scaling)

Result:



$z = 2$ Lifshitz

cf KLM

Result:  $z = 2$ Lifshitz

That is, we have a set of flavor fields charged under an $SU(N_c)$ gauge symmetry, which at finite density develops IR $z = 2$ Lifshitz scaling.

Next step: transport

Now consider $U(N_f) \to \underbrace{U(1)_f}_{} \times SU(N_f)$

$f_{or} \leftarrow$ put at finite density

electric field on a flavor brane

$$S_{\substack{\text{flavor}\\\text{brane}\\(DBI)}} = -\int \frac{1}{g_s}\sqrt{\det\left[G_{MN}\,\partial_\alpha X^M \partial_\beta X^N + f_{\alpha\beta}\right]}$$

neutral sector "statistical" $SU(N_c)$

By solving equations of motion with boundary conditions $A_0^f \to \mu \Rightarrow J^t$

cf K o'B adopted to $z \neq 1$ $\quad A_x \to Et$

we derive $\boxed{\rho = \frac{E_x}{J_x} = \frac{T^{2/z}}{J^t} \propto T}$

over a wide, but intermediate, range of scales.

No lattice, but:

- Do density evidently breaks translations, again an $\mathcal{O}(1)$ contribution to the solution

(is this enough to preserve ρ at finite N_c?)

- Large $\frac{N_c}{N_f}$ $\Rightarrow$ charged flavors $N_f N_c$

cf KÖB

uncharged (dark) N_c^2

$\Rightarrow$ don't see effect on dark sector until time $\sim \mathcal{O}(N_c)$

Optical conductivity:

$$\vec{E}_x(\omega) \to \vec{J}(\omega)$$

Result for $T \ll \omega \ll J, t; E_{gap}$

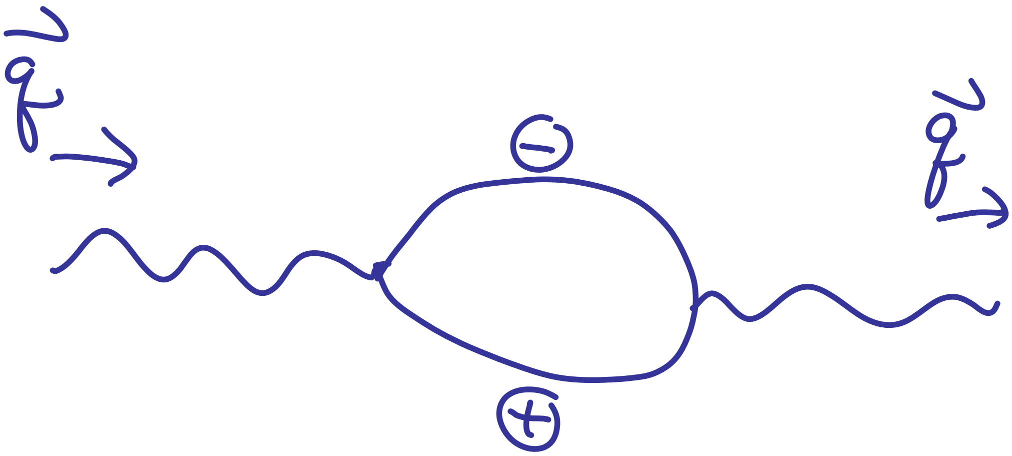
$$\sigma(\omega) \propto \begin{cases} \omega^{-1} & z<2 \\ \dfrac{1}{\omega \log \omega} & z=2 \\ \omega^{-2/z} & z>2 \end{cases}$$

More general exponents with more general Lifshitz + radially-rolling-scalar solutions.

cf Kachru, Trivedi et al

Remarks

1) May be possible to check for $2k_F$ singularity in $\langle J^t J^t \rangle =$



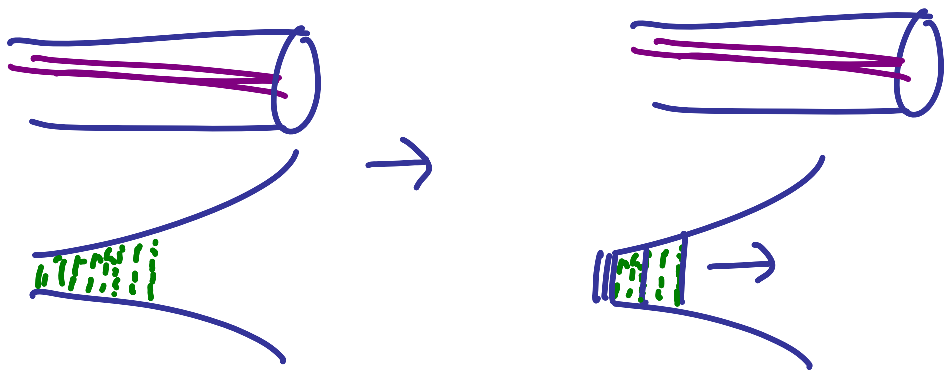
string tension $\to 0$ in radial direction in deep IR $\Rightarrow$ room for new effects

cf FL^*?

Bose metals?

2) As usual, control from extreme large-N_c: would like small-N analysis, again solving for z, cf Max talk

Fermi Seasickness



$U(N_c)$ scalar eigenvalues pulled up the warped throat, but

- takes forever at $N_c \to \infty$
- finite-T metastabilizes.

ABJM

2) $S = k \int \mathrm{Tr}\, \epsilon_{\mu\nu\lambda} \left(A^\mu \partial^\nu A^\lambda + \frac{2}{3} A^\mu A^\nu A^\lambda \right)$ + SUSY bifundamental flavors (A, B, F partners)

$\underbrace{}_{U(N) \times U(N)}$

\+ defect lattice

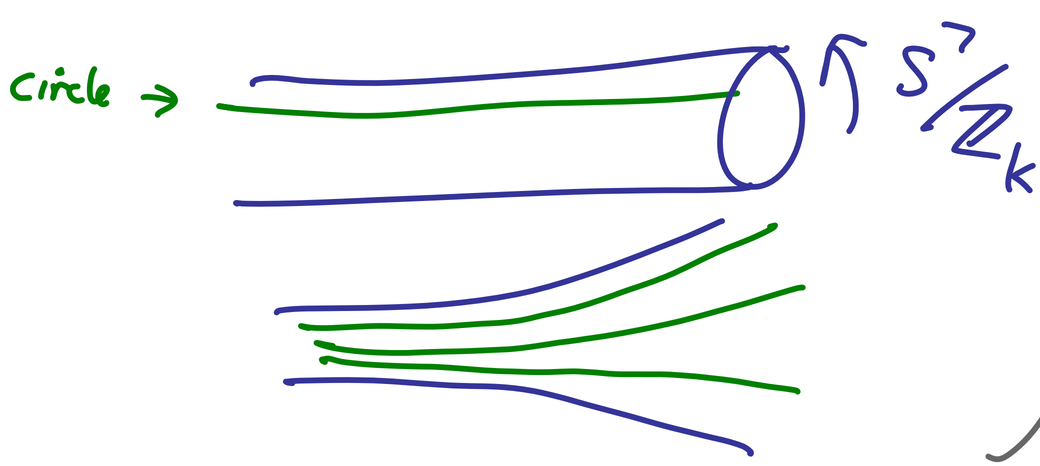
KKY
AEM O'B W

$$\mathcal{L}_{defect} = \text{kinetic} + |Q\chi|^2 + \psi \chi \psi$$

AB   AB

$\overset{?}{\equiv}$

M theory



preserves $SU(2) \times U(1)^2 \times \mathbb{Z}_4$

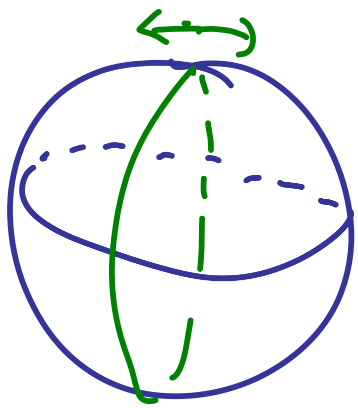
At large N and λ, the defects are independent $(0+1)$-dimensional systems which lie along an AdS_2 slice inside an AdS_4

$\Rightarrow$ inherit (trivially in gravity, but non-trivially in QFT) Locally Critical scaling for gauge-invariant ops:

$$\overline{\Psi}_\Delta(t)\,\Psi_\Delta(t') \sim \frac{1}{|t-t'|^{2\Delta}}$$

with Δ determined by gravity-side mass of localized field on defect.

(k=1)



$$m_B^2 = \Delta(\Delta - 1)$$

$$\Delta_B = \frac{1}{2} + \frac{\ell}{2}$$

$$\Rightarrow \Delta_F = \underbrace{1}_{\downarrow} + \frac{\ell}{2}$$

lowest-dimension operator transforming under SO(6), invariant under $\mathbb{Z}_4$
(k=1)

, These are consistent with the extrapolation of ops $\partial_t \hat{\tilde{Q}}_1 A \chi_2$ ← bifundamental , ...

$U(N)_1 \times U(N)_2$

to strong coupling (susy may protect these.)

• Δ <u>independent</u> of spatial momentum

To apply to original question: take FP interpretation of MIT/Leiden scenario for MFL.

$$S_{Total} = S_{LC}(A, B, Q, \tilde{Q})$$
$$+ \sum_{J,J'} \int dt\, c_J^\dagger \left(i\delta_{JJ'}\partial_t + \mu\delta_{JJ'} + t_{JJ'}\right) c_{J'}$$
$$+ g \sum_J \int dt \left(c_J^\dagger \Psi_J^F + h.c.\right)$$

$$\rightarrow \quad G_c = \frac{1}{\omega - v|\vec{k} - \vec{k}_F| - g^2 c\, \omega \log \omega}$$

Comments/Questions on (A)dS/CMT

Hard to say what AdS/CMT will ultimately contribute to CM

Some thoughts:

- should be able to test folk theorems about QFT, possibly spin off structures to study in themselves

cf particle phys (SUSY, YM as string theory, ...)
cosmology (systematics of NG, GW UV sensitivity...)

CMT-rich set of examples!

- Fresh eyes on old problems

Awareness is the first step...

$N_s = 10^3$ "String theorists"

$N_c = 10^4$ "CM physicists"

$$\frac{N_s}{N_c} \ll 1$$ Probe approximation

- String theory has its own difficult strong-coupling directions, like fully upgrading AdS/CFT to realistic ($\approx$ dS, FRW) spacetimes