Quantum matter & black hole ringing

Sean Hartnoll

Harvard University

Work in collaboration with

Chris Herzog and Gary Horowitz :	0801.1693, 0810.1563.
Frederik Denef :	0901.1160.
Frederik Denef and Subir Sachdev :	0907.????.

AdS/CMT – July 09 – KITP

Plan of talk

Motivation – unconventional phases at finite density

1. Low temperature and finite density (bosons and fermions)
2. Two uses of magnetic fields
3. Experimental examples
4. Free fermions and bosons

Strongly coupled theories with gravity duals

1. The normal state
2. Large N magnetic susceptibility
3. Black hole instabilities (superconducting instabilities)
4. 1/N corrections to the free energy
5. Black hole ringing
6. Quantum oscillations

Motivation – unconventional phases at finite density

1. Low temperature and finite density (bosons and fermions)
2. Two uses of magnetic fields
3. Quantum oscillations in High - T_c superconductors
4. Quantum criticality under the dome in High - T_c superconductors
5. Free fermions
6. Free bosons

Low temperature and finite density

- Effective field theories in condensed matter physics often have a finite charge density.
- Weak coupling intuition at low temperatures and finite density:
  - Charged fermions: Fermi surface is built up.
  - Charged bosons: condensation instabilities (e.g. superconductivity).
- Weakly interacting low energy excitations about a condensate or Fermi surface are very well characterised.
- There seem to be materials where these descriptions do not work.
- Perspective of this talk: AdS/CFT gives a tractable theory with an exotic finite density ground state.

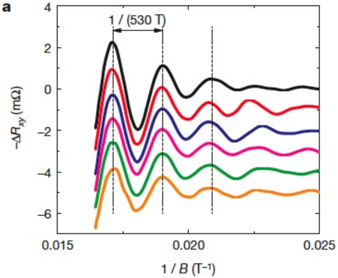
Two uses of magnetic fields

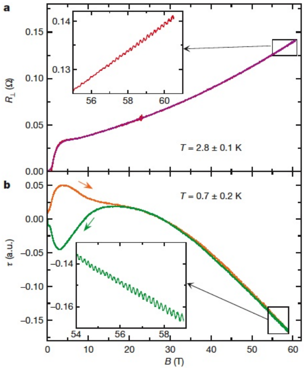
- Magnetic fields useful for probing both fermions and bosons.
- de Haas - van Alphen effect: a Fermi surface leads to oscillations in the magnetic susceptibility as a function of $1/B$.
  - In a magnetic field
  $$[P_x, P_y] \sim iB \quad \Rightarrow \quad \oint P_x dP_y \sim 2\pi(\ell + \tfrac{1}{2})B \,.$$
  - When the area of the orbit is a cross section of the Fermi surface there is a sharp response. I.e. at $1/B \sim \ell/A_F \sim \ell/k_F^2 \sim \ell/\mu^2$.
- Large magnetic field will suppress superconducting instabilities.

Quantum oscillations in High - T_c superconductors

Doiron-Leyraud et al. 2007 (Nature), Vignolle et al. 2008 (Nature).

- de Haas - van Alphen oscillations in underdoped and overdoped cuprates.



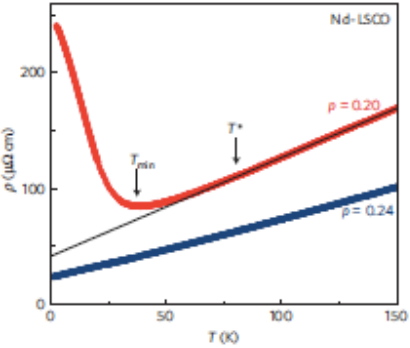


- In underdoped region, carrier density much lower than naïve expectation: "small Fermi surface".

Criticality under the dome in High - T_c superconductors

Daou et al. 2008 (Nature Physics)

- Resistivity in 'normal phase' linear in temperature (anomalous).
- Applying a large magnetic field shows persistance down to $T = 0$ at critical doping.



Free fermions

- Free bosons or fermions in magnetic fields have Landau levels

$$\varepsilon_\ell = \sqrt{m^2 + 2|qB|(\ell + \tfrac{1}{2})} \,.$$

- Free energy for fermions (D=2+1)

$$\Omega = -\frac{|qB|AT}{2\pi} \sum_\ell \sum_\pm \log\left(1 + e^{-(\varepsilon_\ell \pm q\mu)/T}\right) \,.$$

- Zero temperature limit

$$\lim_{T\to 0} \Omega = -\frac{|qB|A}{2\pi} \sum_\ell (q\mu - \varepsilon_\ell)\theta(q\mu - \varepsilon_\ell) \,.$$

- Magnetic susceptibility has oscillations

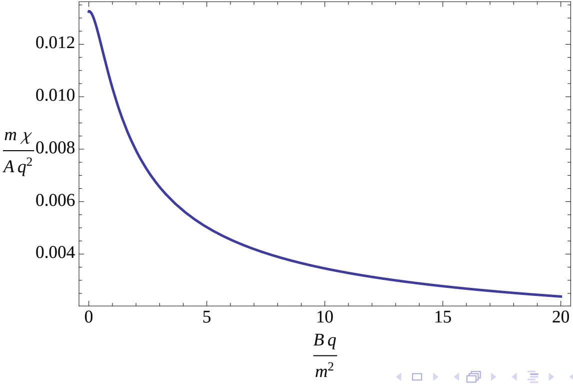
$$\chi \equiv \frac{\partial^2 \Omega}{\partial B^2} = \frac{|qB|A}{2\pi} \sum_\ell \frac{q^2(\ell + \frac{1}{2})^2}{\varepsilon_\ell^2} \delta(q\mu - \varepsilon_\ell) + \cdots \,,$$

Free bosons

- Free energy for bosons – unstable if $\varepsilon_0 < |q\mu|$

$$\Omega = \frac{|qB|A}{2\pi} \sum_{\ell} \sum_{\pm} \log\left(1 - e^{-(\varepsilon_\ell \pm q\mu)/T}\right) + \Omega|_{T=0} \,.$$

- Magnetic susceptibility at T=0 if stable (Hurwitz zeta function)



Strongly coupled theories with gravity duals

1. The normal state
2. Large N magnetic susceptibility
3. Black hole instabilities (superconducting instabilities)
4. 1/N corrections to the free energy
5. Black hole ringing
6. Quantum oscillations

The normal state

- The minimal ingredient is Einstein-Maxwell theory

$$S_E[A,g] = \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)+\frac{1}{4g^2}F^2\right] .$$

- The 'normal state' is dual to a dyonic black hole

$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2+\frac{dr^2}{f(r)}+dx^idx^i\right) ,$$

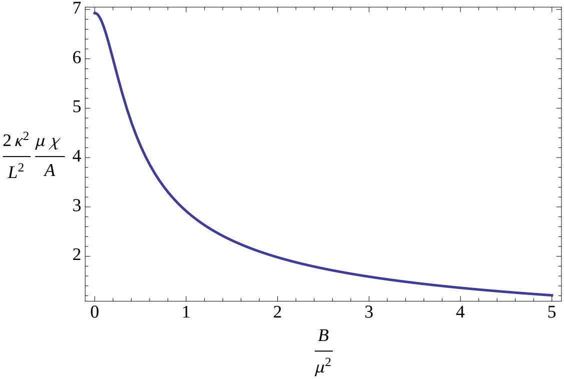
$$A = i\mu\left[1-\frac{r}{r_+}\right]d\tau + B\,x\,dy .$$

- Free energy is the action evaluated on shell

$$\Omega_0 = -\frac{AL^2}{2\kappa^2 r_+^3}\left(1+\frac{r_+^2\mu^2}{\gamma^2}-\frac{3r_+^4B^2}{\gamma^2}\right) .$$

Large N magnetic susceptibility

- Easy to compute $\chi \equiv \frac{\partial^2 \Omega_0}{\partial B^2}$
- Plot result:



- Looks just like free bosons.... (but massless!)

Black hole superconducting instabilities

- Add some matter to the bulk to make things more interesting...
- Charged bosons:

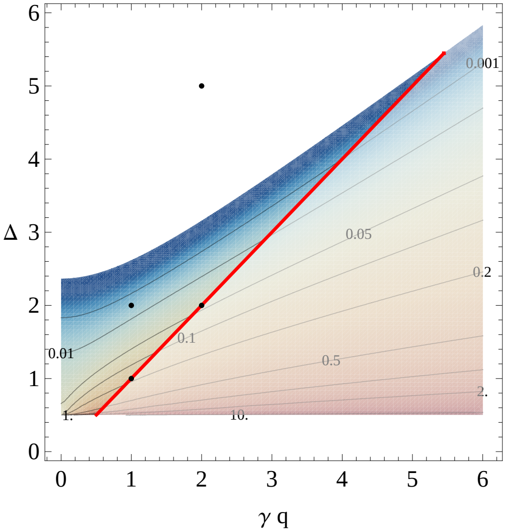
$$S_E[\phi] = \int d^4x \sqrt{g} \left[|\nabla\phi - iqA\phi|^2 + m^2|\phi|^2 \right] .$$

- Charged fermions:

$$S_E[\psi] = \int d^4x \sqrt{g} \left[\bar{\psi}\Gamma \cdot \left(\partial + \frac{1}{4}\omega_{ab}\Gamma^{ab} - iqA \right) \psi + m\bar{\psi}\psi \right] .$$

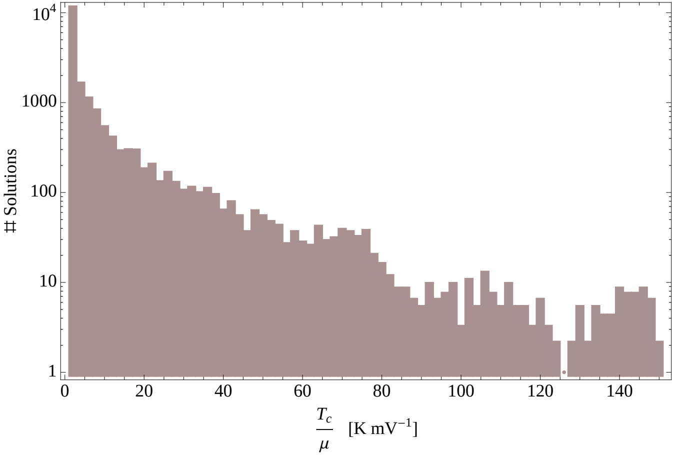
- Bosons: Criterion for homogeneous instability at $T = 0$

$$q^2\gamma^2 \geq 3 + 2m^2 , \qquad \gamma^2 = \frac{2g^2L^2}{\kappa^2} .$$



[Denef-SAH '09]

Landscape of superconducting membranes



[Denef-SAH '09]

1/N corrections to the free energy

- Suppose no superconducting instability (or suppress with B)
- Nontrivial Landau-level structure subleading in $1/N$?

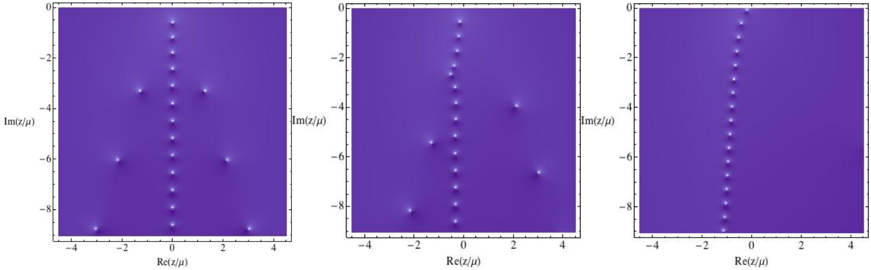
  $\Rightarrow$ Quantum contribution from charged matter:

$$\Omega_{\text{1-loop}} = T\,\mathrm{tr}\log\left[-\hat{\nabla}^2 + m^2\right] - T\,\mathrm{tr}\log\left[\Gamma\cdot\hat{D} + m\right] + \cdots .$$

- It is difficult to compute determinants in black hole backgrounds and it is hardly ever done...
- Reformulate the problem using quasinormal modes.

Black hole ringing

- Late times: a perturbed black hole 'rings' with characteristic frequencies.
- Quasinormal modes: poles of the retarded Green's function (bulk or boundary).
- Some typical quasinormal for charged AdS black holes at low temperature (not easy to make these plots!)



The free energy and quasinormal modes

- We derived (new to my knowledge) formulae for the determinant as a sum over quasinormal modes $z_\star(\ell)$ of the black hole

$$\Omega_{\text{1-loop, B}} = \frac{|qB|AT}{2\pi} \sum_{\ell} \sum_{z_\star(\ell)} \log \left(\frac{|z_\star(\ell)|}{2\pi T} \left| \Gamma \left(\frac{i z_\star(\ell)}{2\pi T} \right) \right|^2 \right) .$$

$$\Omega_{\text{1-loop, F}} = -\frac{|qB|AT}{2\pi} \sum_{\ell} \sum_{z_\star(\ell)} \log \left(\left| \Gamma \left(\frac{i z_\star(\ell)}{2\pi T} + \frac{1}{2} \right) \right|^2 \right) .$$

- For the BTZ black hole we did the sum explicitly and checked agreement with the known result.

Quantum oscillations

- The power of these formulae is that if an individual quasinormal mode does something non-analytic, then this is directly identified.
- Faulkner-Liu-McGreevy-Vegh have shown that at zero temperature there is a fermion quasinormal mode whose trajectory in the complex frequency plane bounces off the real axis at $k = k_F$.
- At a finite magnetic field, this gives a bounce when $2B\ell = k_F^2$.
- The contribution of this mode to the free energy is

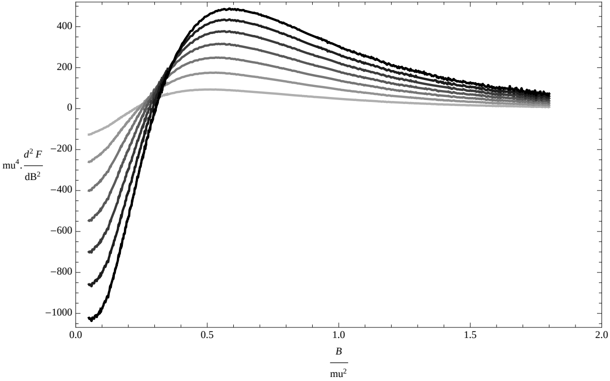
$$\Omega = -\frac{qBA}{2\pi^2} \log \frac{\mu}{T} \operatorname{Im} z_\star \,.$$

- Taking two derivatives, delta functions in the magnetic susceptibility with period

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi q}{A_F} \,.$$

Preliminary comments on bosons

- It looks possible that stable bosons can also give oscillations if a zero temperature bosonic quasinormal mode crosses a branch cut.
- Summing numerically over the bosonic modes does not converge very quickly but may indicate some structure



Conclusions

- There exist systems with finite charge density that are described as neither conventional Fermi liquids or superfluids.
- AdS/CFT provides model exotic stable finite density systems.
- Magnetic fields are an essential experimental and theoretical tool for probing such systems.
- There is interesting structure at $1/N$ in AdS/CFT related to Landau levels for fermions and bosons.
- Found a method for computing determinants about black holes using quasinormal modes.
- Fermionic loops are shown to give de Haas - van Alphen oscillations.