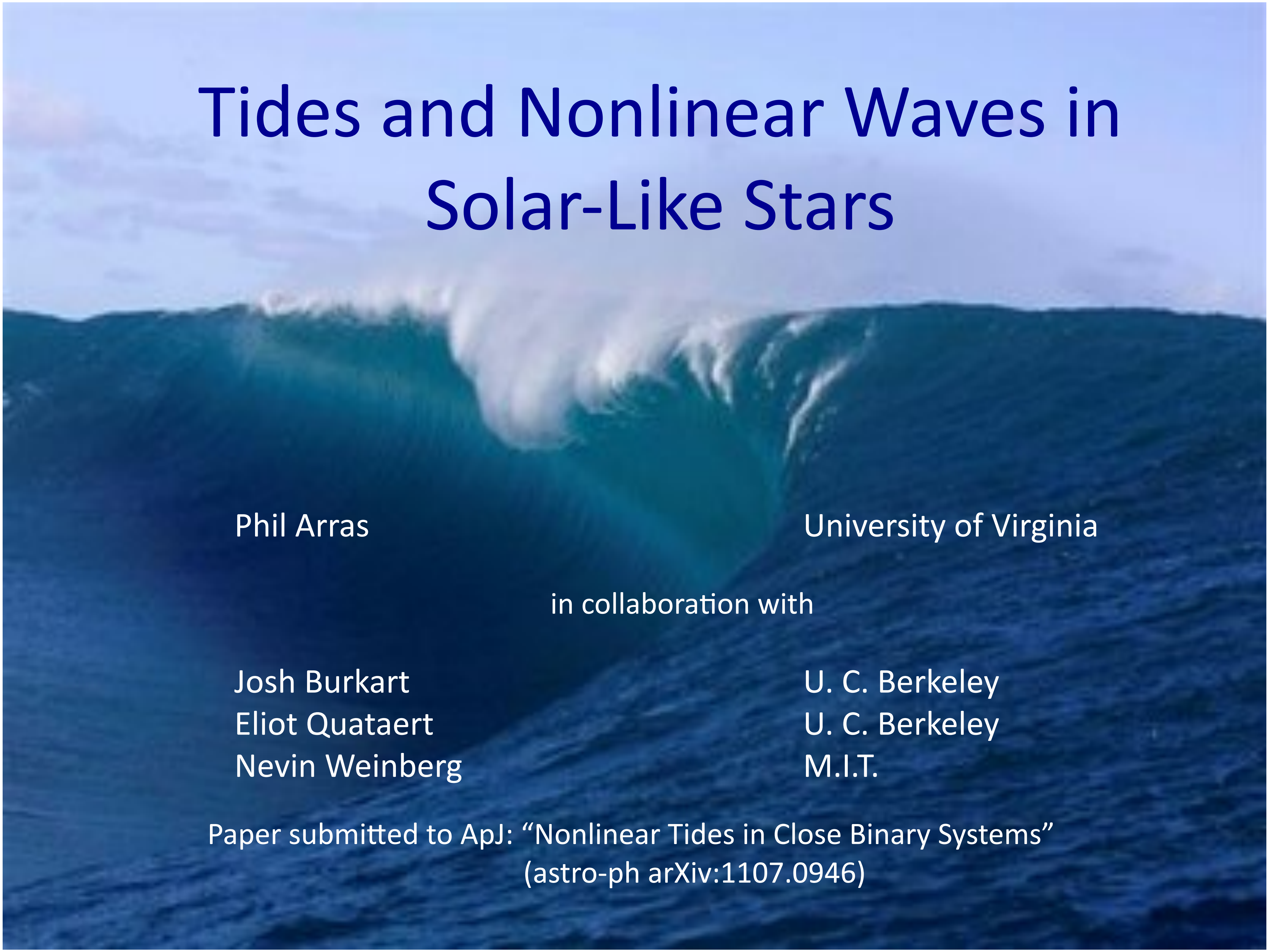


Tides and Nonlinear Waves in Solar-Like Stars

Phil Arras — University of Virginia

in collaboration with

Josh Burkart — U. C. Berkeley
Eliot Quataert — U. C. Berkeley
Nevin Weinberg — M.I.T.

Paper submitted to ApJ: “Nonlinear Tides in Close Binary Systems”
(astro-ph arXiv:1107.0946)

Outline

- Tidal evolution review
- Observations of tidal evolution
- Linear calculations
- Nonlinear effects
- Summary

caveat

- In the spirit of this conference, want to discuss how to use tides to constrain stellar interiors,

  but ….

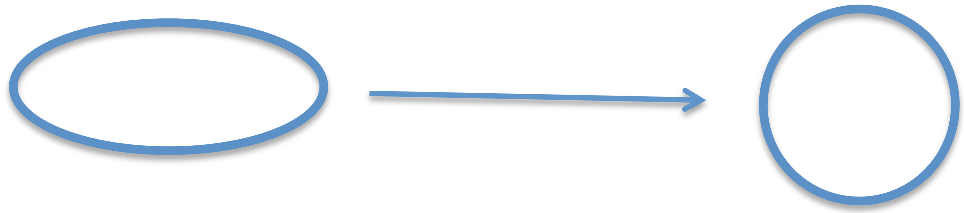
- There is uncertain wave physics, both at the linear and nonlinear level, and this depends in detail on what type of star.

- Focus here: tidal excitation and damping of waves,

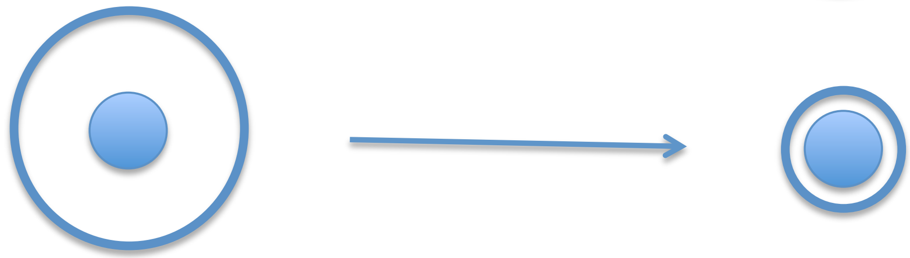
  and dependence on gross properties of stellar structure.

Tidal evolution effects

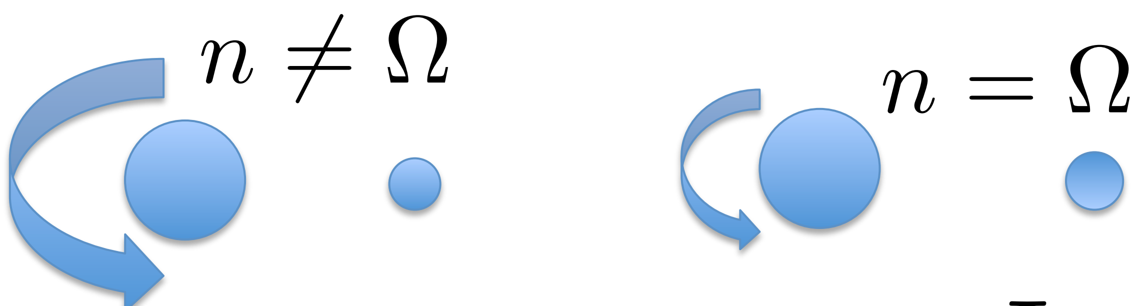
Circularization of orbits:



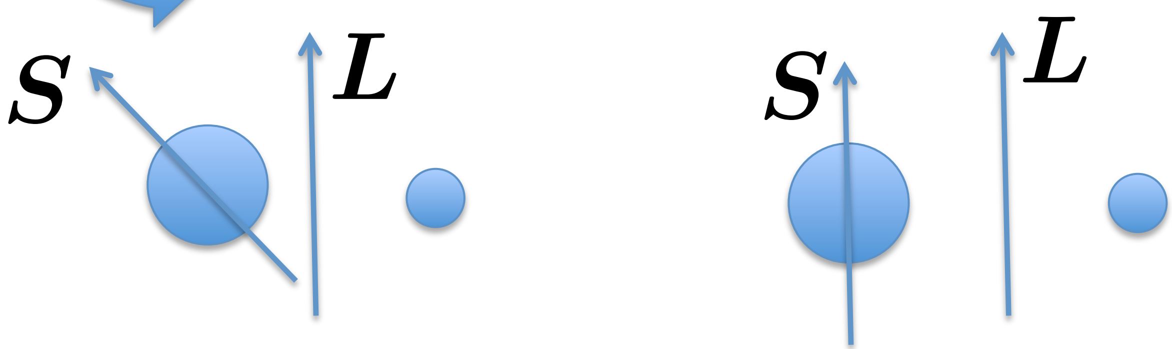
Orbital decay



Synchronization of spin



Alignment of spin



Origin of Secular Tidal Evolution

tidal potential
due to companion:

$$U \sim \epsilon\ e^{-i\omega t}$$

$\epsilon =$ strength of tide

density perturbation
response to tide:

$$\delta\rho \sim \epsilon\ e^{i(\delta - \omega t)}$$

lag time $= \delta/\omega$

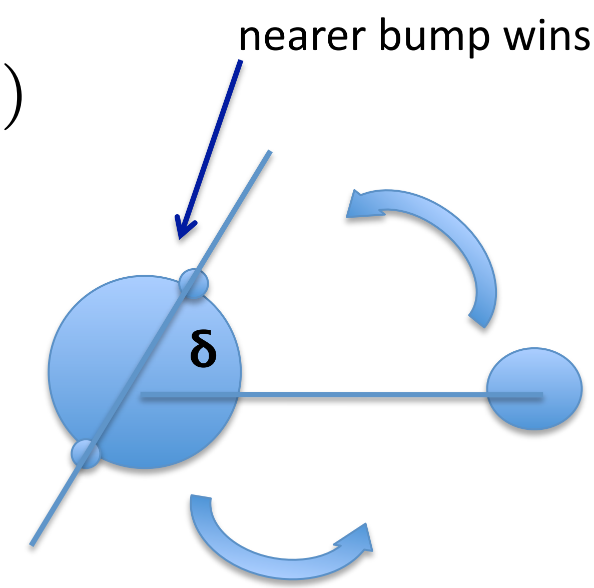
due to dissipation

external potential
from this density
perturbation acts
back on companion:

$$\delta\phi \sim \epsilon\ e^{i(\delta - \omega t)}$$

out-of-phase
force on companion
leading to secular
evolution:

$$f \sim \epsilon^2 \sin(\delta)$$

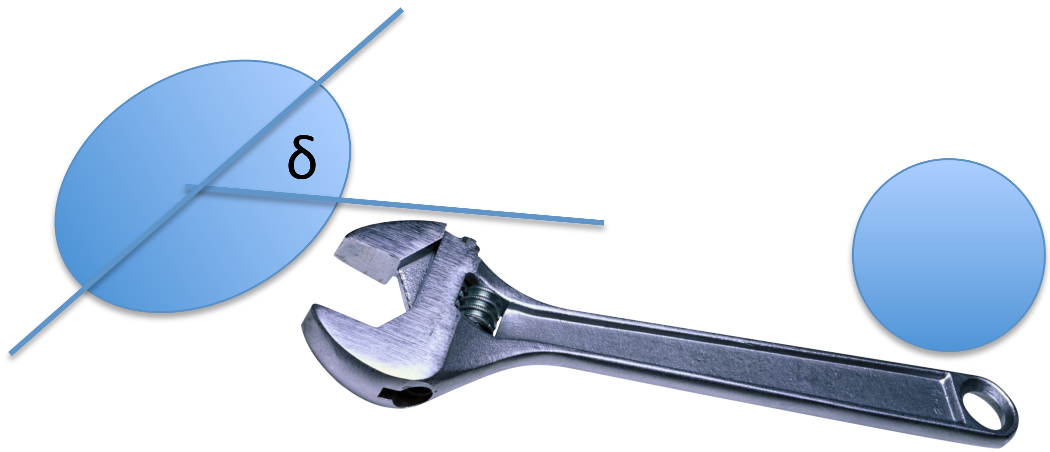


tidal Q

By analogy with the ``quality" of a damped, driven oscillator:

$$Q^{-1} = \frac{\oint (-\dot{E})\, dt}{2\pi E_{\rm tide}} = \tan(2\delta)$$

Q is related to the lag angle δ of the tidal bulge



(G. Darwin)

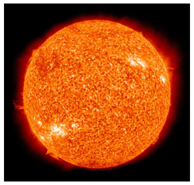
<u>standard operating procedure:</u>

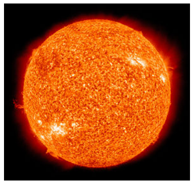
- Q = constant
- independent of frequency and amplitude
- calibrated from one observation and applied to another

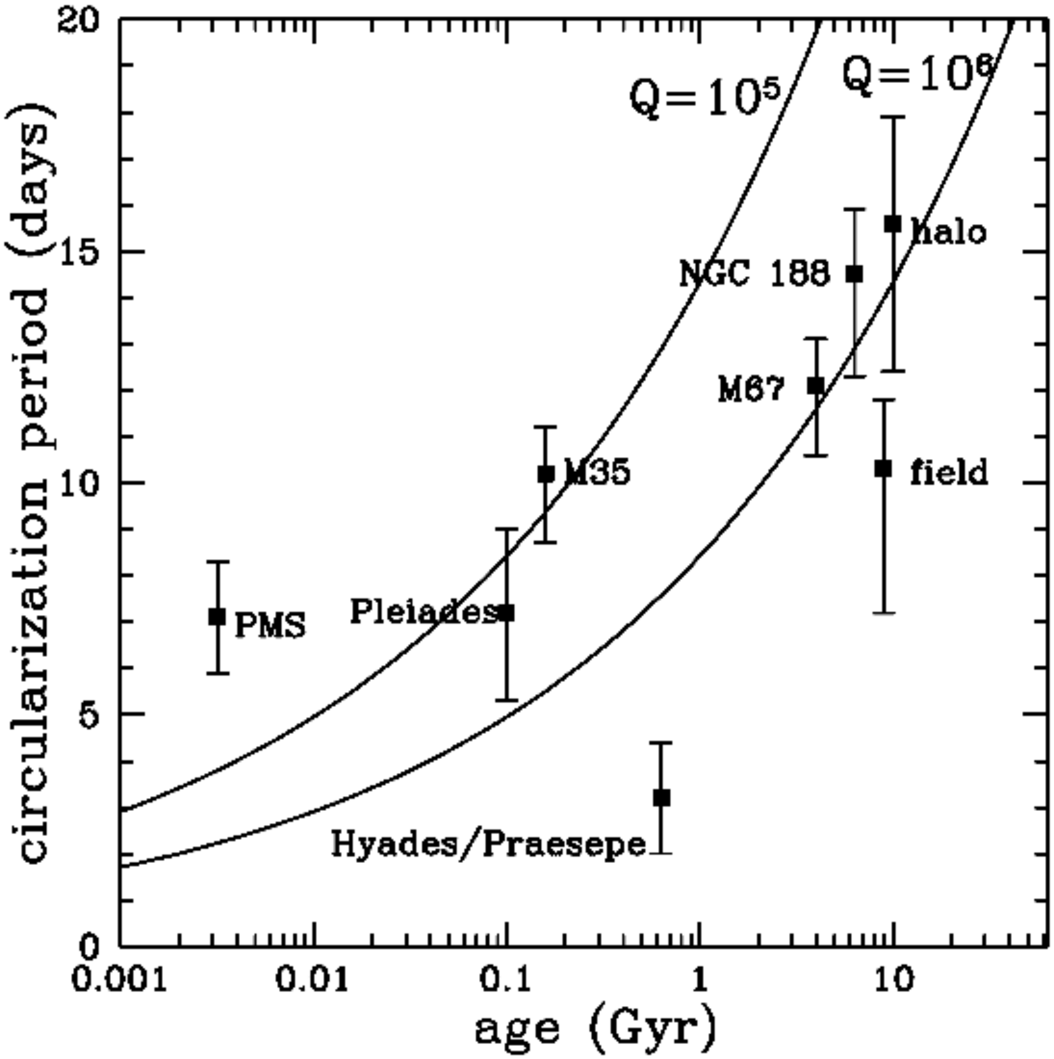
data

- Circularization of binaries with two solar-type stars.
- Orbital decay of planets.
- Alignment of stellar spin to planet orbit.

Circularization of solar-type binaries







(data from Meibom and Mathieu 2005)

Calibration of Q using the observed *circularization of binaries with two solar-type stars:*

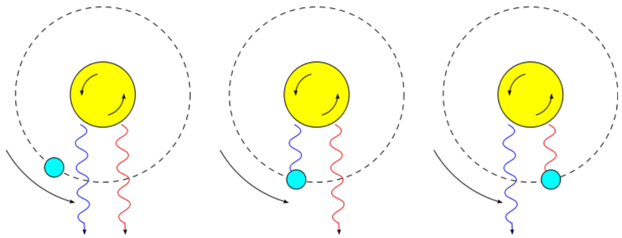
$$Q_{\mathrm{solar-type\ star}} \sim 10^5 - 10^6$$

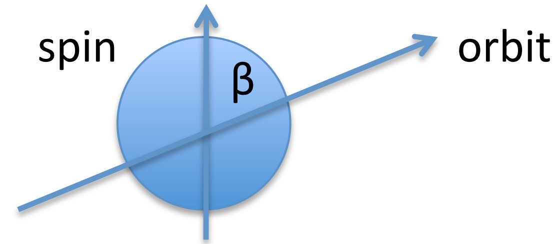
Forcing periods: ~ 10-15 days (planets: 1-7 days)

But this observed Q is not well explained by theory!

Theory <u>underpredicts</u> tidal dissipation rate.

(Goodman and Dickson, Terquem et al, Savonije and Witte, Ogilvie and Lin)





HOT STARS WITH HOT JUPITERS HAVE HIGH OBLIQUITIES

JOSHUA N. WINN[1], DANIEL FABRYCKY[2,4], SIMON ALBRECHT[1], AND JOHN ASHER JOHNSON[3]

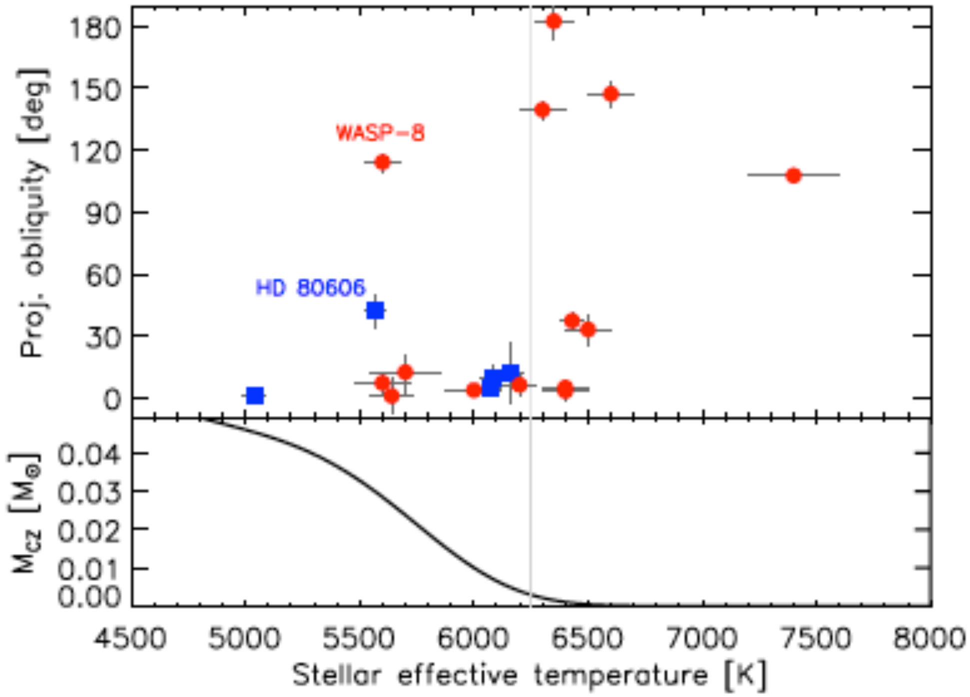


Figure 2. Misaligned systems have hotter stars. Top: the projected obliquity is plotted against the effective temperature of the host star. A transition from mainly aligned to mainly misaligned seems to occur at $T_{\rm eff} \approx 6250$ K. The two strongest exceptions are labeled. Symbol colors and shapes have the same meaning as in Figure 1. Bottom: the mass of the convective zone of a main-sequence star as a function of $T_{\rm eff}$, from Pinsonneault et al. (2001). It is suggestive that 6250 K is approximately the temperature at which the mass of the convective zone has bottomed out.

The time dependence of hot Jupiters' orbital inclinations

A. H. M. J. Triaud

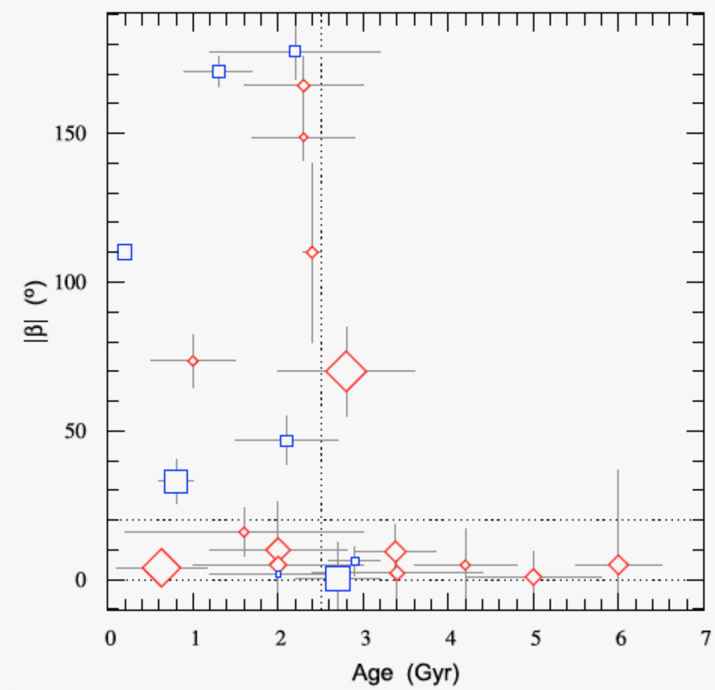
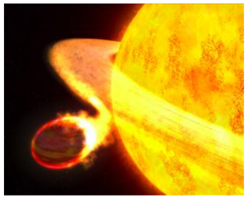


Fig. 2. Secure |β| against stellar age (in Gyr), for stars with $M_\star \geq 1.2\,M_\odot$. Size of the symbols scales with planet mass. Blue squares indicate stars with $M_\star \geq 1.3\,M_\odot$; red diamonds, stars with $1.3 > M_\star \geq 1.2\,M_\odot$. Horizontal dotted line show where aligned systems are. Vertical dotted line shows the age at which misaligned planets start to disappear.

(see theory papers Dong Lai, Barker and Ogilvie)

Orbital decay of exoplanets?



(e.g. Jackson et al 2009)

- Sun-like stars rotate slow compared to planet orbit ($\Omega_\star \ll \Omega_{\rm orb}$).
- Tide raised in star by planet spins star up.
- Orbit must decay inward to conserve total angular momentum.
- Insufficient J in orbit to synchronize star inside P_{orb} ≈1 week.

$$\frac{\dot a}{a} = -\frac{9}{2}\left(\frac{G}{M_\star}\right)^{1/2}\left(\frac{R_\star^5 M_p}{Q'_\star}\right) a^{-13/2} \sim \Omega_{\rm orb} \ \left(\frac{R_\star}{a}\right)^2 \left(\frac{{\rm ImQ}_{\ell={\rm m}=2}}{M_\star R_\star^2}\right)$$

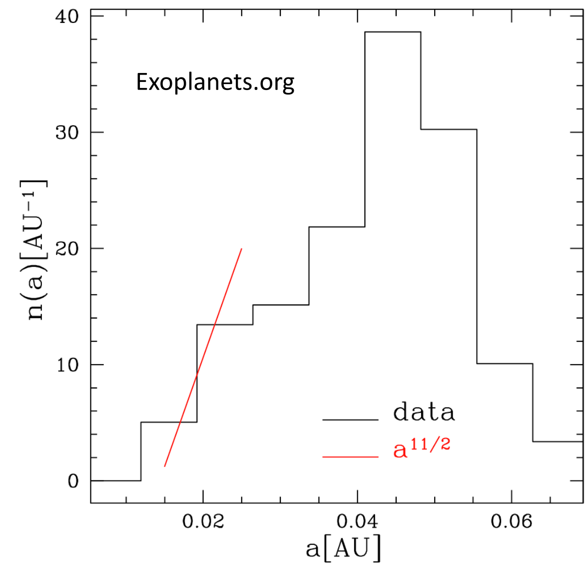
Example: WASP 18-b has $M_p = 10 M_{\rm Jup}$ and $a = 0.02$ AU . For $Q'_\star = 10^6$

$$t_{\rm decay} \simeq 1 \ {\rm Myr} \simeq 10^{-3} \ {\rm age}$$

If true, orbit decay is ongoing, even for Gyr old planets.
we happen to be able to see planets "just before they fall in".
Possible explanations:
(a) We're lucky. (b) Feed from outside. (c) Q wrong.

Orbital decay of a population of planets

(In preparation.With Uva undergrads Meredith Nelson and Sarah Peacock)

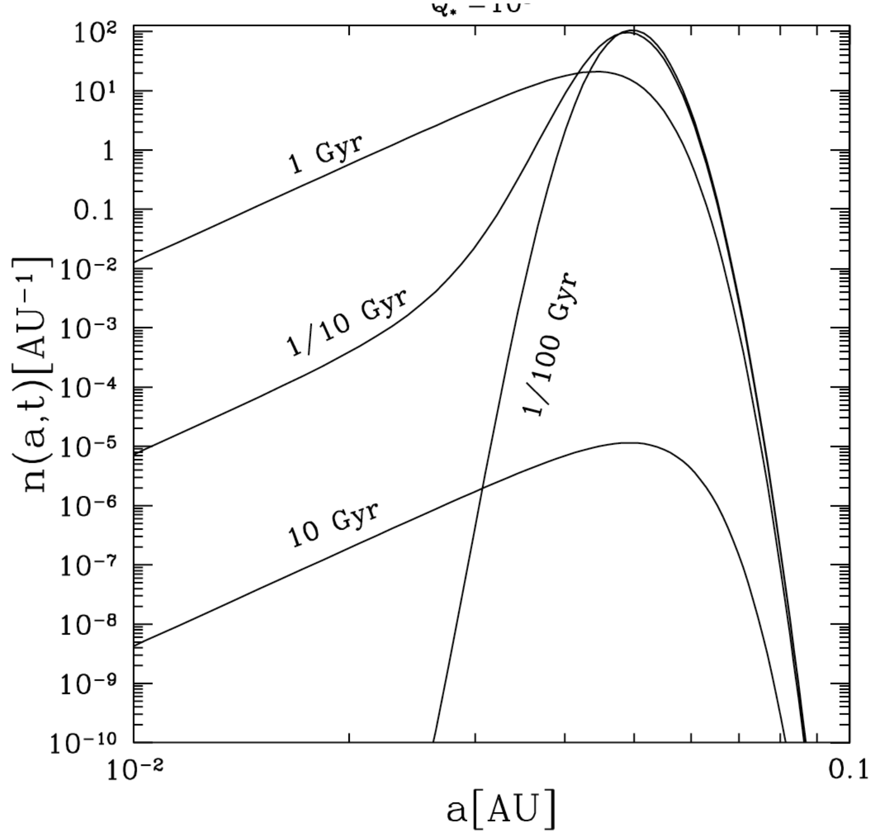


Constrain tidal dissipation from the distribution of semi-major axes of planets (e.g. Jackson et al 2009)

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial a}\left(\dot{a}n\right) = 0$$

When decay time << age, flux constant and observed distribution should reflect the orbital decay law:

$$n(a,t) = \frac{\text{constant}}{\dot{a}(a)}$$



If decay times really short,
you should be able to measure the frequency dependence of the tidal Q from the observed distribution.

linear theory

Tidal excitation: linear, adiabatic

Companion orbits at position $(r,\theta,\phi) = (D, \pi/2, \Phi)$ in spin-orbit aligned case.

Momentum: $$\underbrace{\rho\ddot{\boldsymbol{\xi}}}_{\text{inertia}} + \overbrace{2\rho\boldsymbol{\Omega}\times\dot{\boldsymbol{\xi}}}^{\text{Coriolis}} = -\underbrace{\boldsymbol{\nabla}\delta p}_{\text{pressure}} + \overbrace{\boldsymbol{g}\delta\rho}^{\text{buoyancy}} - \underbrace{\rho\boldsymbol{\nabla}U}_{\text{tidal forcing}}$$

Mass: $$\delta\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{\xi}) = 0$$

Energy: $$\delta\rho = \frac{\delta p}{c^2} + \rho\frac{N^2}{g}\xi_r$$

Boundary conditions: regular at r=0, $\Delta p = 0$ at surface.

Tidal potential:

$$\begin{aligned} U(\boldsymbol{x},t) &= -\frac{GM'}{|\boldsymbol{x}-\boldsymbol{D}|} - (\text{dipole term}) \\ &= -GM' \sum_{\ell\geq 2,m} \frac{4\pi}{2\ell+1}\frac{r^\ell}{D^{\ell+1}} Y^*_{\ell m}(\pi/2,\Phi) Y_{\ell m}(\theta,\phi) \end{aligned}$$

General features of solution

- Equations inhomogeneous => "wave" + "non-wave" parts, also called "equilibrium" and "dynamical" tides.

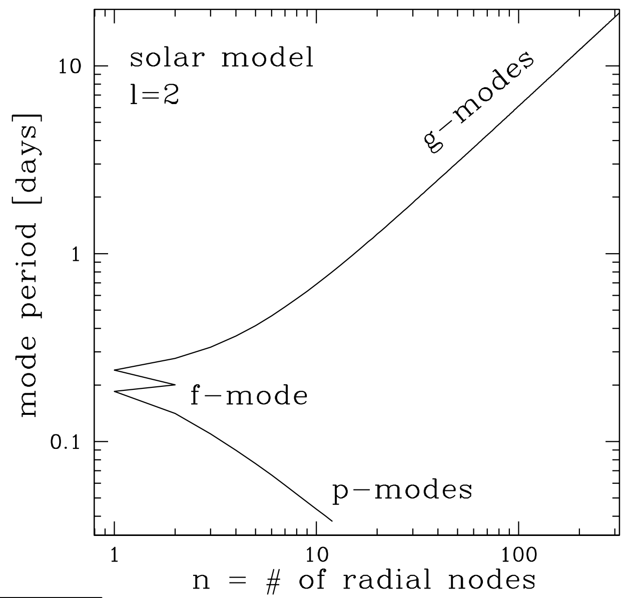
- Non-wave part often approximated with the analytic "equilibrium tide" found by setting fluid inertia to zero => instantaneous response, follow equipotentials:

$$\xi_r \simeq -\frac{U}{g} \qquad \text{and} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{\xi} \simeq 0$$

- Wave ("dynamical") part composed of standing p, g, f & inertial modes with Lorentzian amplitudes when no dissipation, leaky boundaries, nonlinear effects.

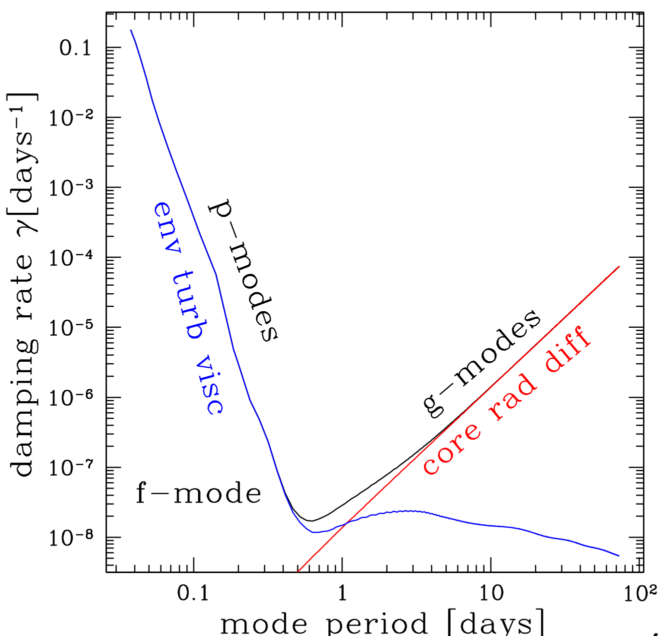
Linear theory: stellar input data

G-modes can be resonant with the tide, but have small overlaps and weak damping.

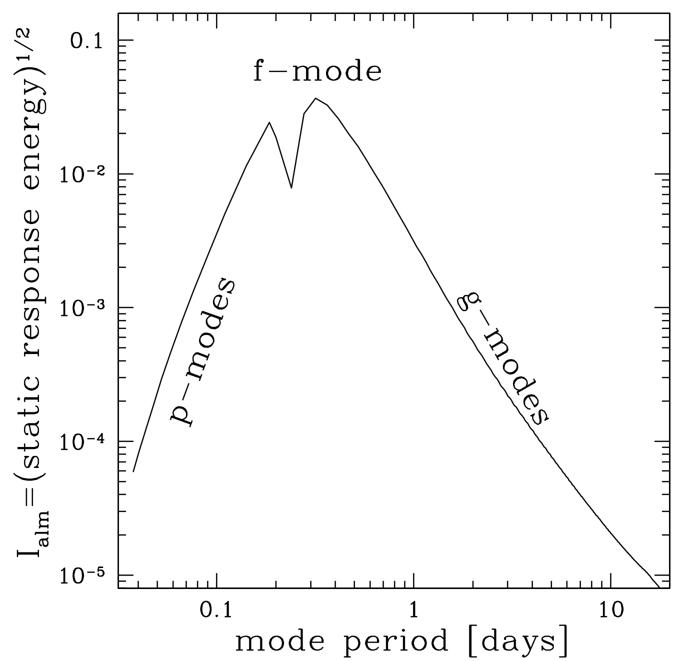


f-modes: large overlap, weak damping, not resonant

p-modes: small overlap, large damping, not resonant



(turb visc from Penev et al)



Linear theory for tidal Q: non-rotating

$$Q^{-1} = \frac{12\pi}{10} \sum_{n=1}^{\infty} I_n^2 \sigma_n \gamma_n \left[\frac{1}{(\sigma_n - 2\Omega)^2 + \gamma_n^2} - \frac{1}{(\sigma_n + 2\Omega)^2 + \gamma_n^2} \right]$$

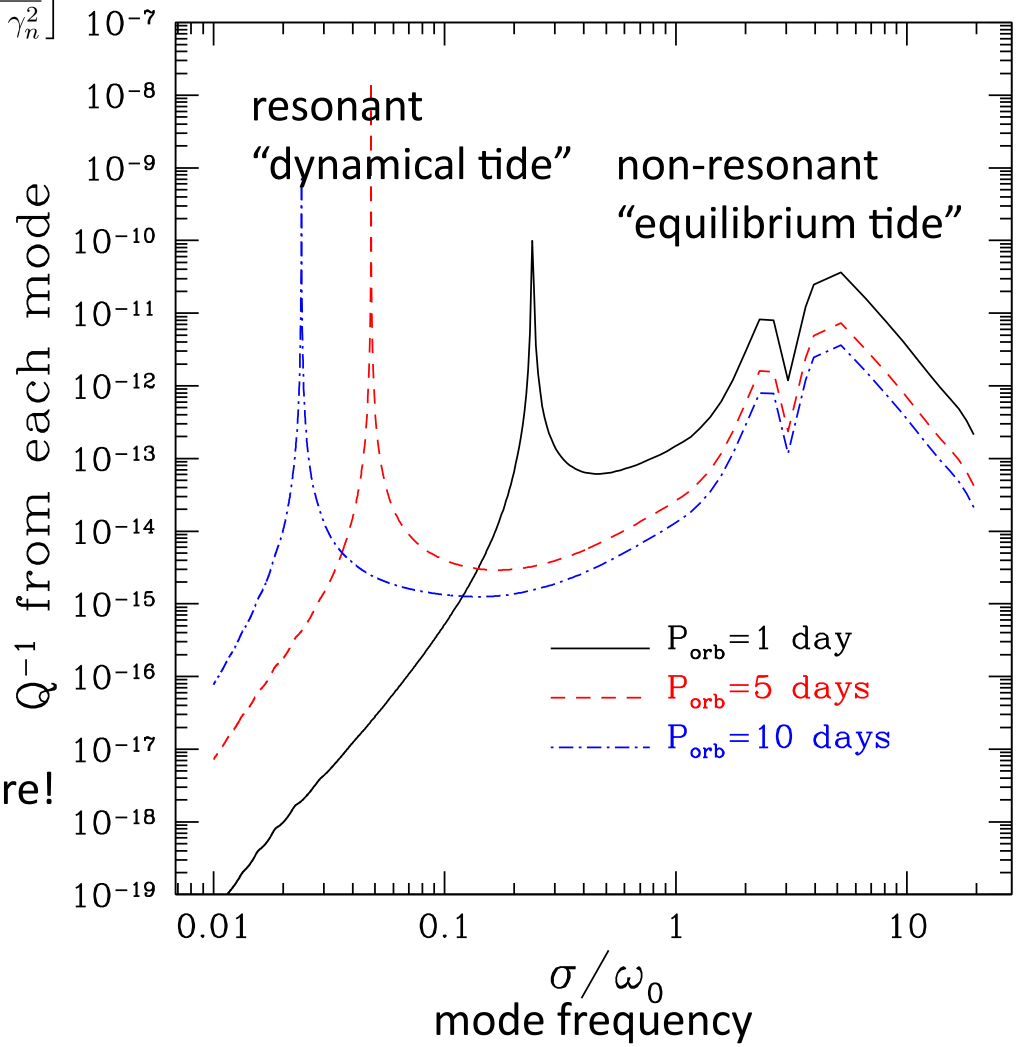
$$Q_{\mathrm{eq\ tide}} \sim 10^{10} \left(\frac{P_{\mathrm{orb}}}{1\ \mathrm{day}} \right)$$

$$Q_{\mathrm{dyn\ tide}} \sim 5 \times 10^{10} \left(\frac{1\ \mathrm{day}}{P_{\mathrm{orb}}} \right)^{4/3}$$

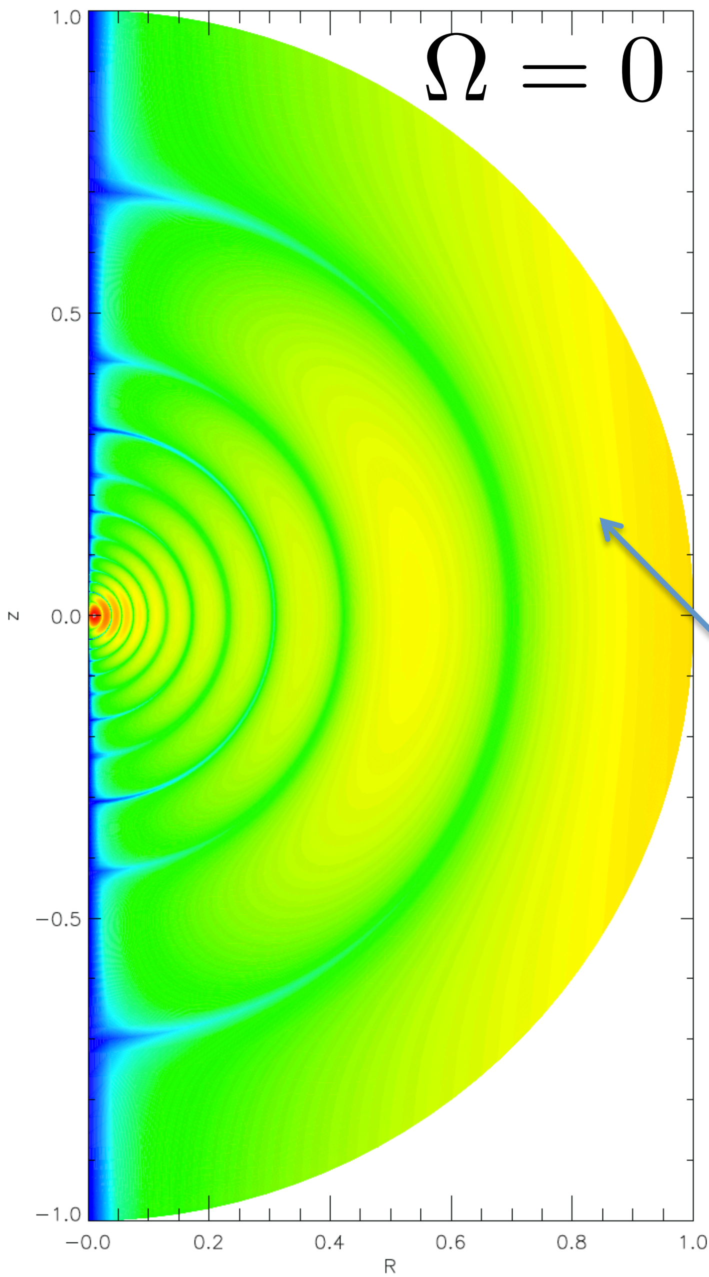
$$t_{\mathrm{decay,dyn}} > 100\ \mathrm{Gyr} \left(\frac{\mathrm{M_J}}{\mathrm{M_p}} \right)$$

Orbital decay time very long, but…. beware!
This same theory under-predicts
rate for solar-type binaries.

Solar model.
Radiative diffusion + turbulent viscosity.



Inertial waves in solar-type star: 2d code results



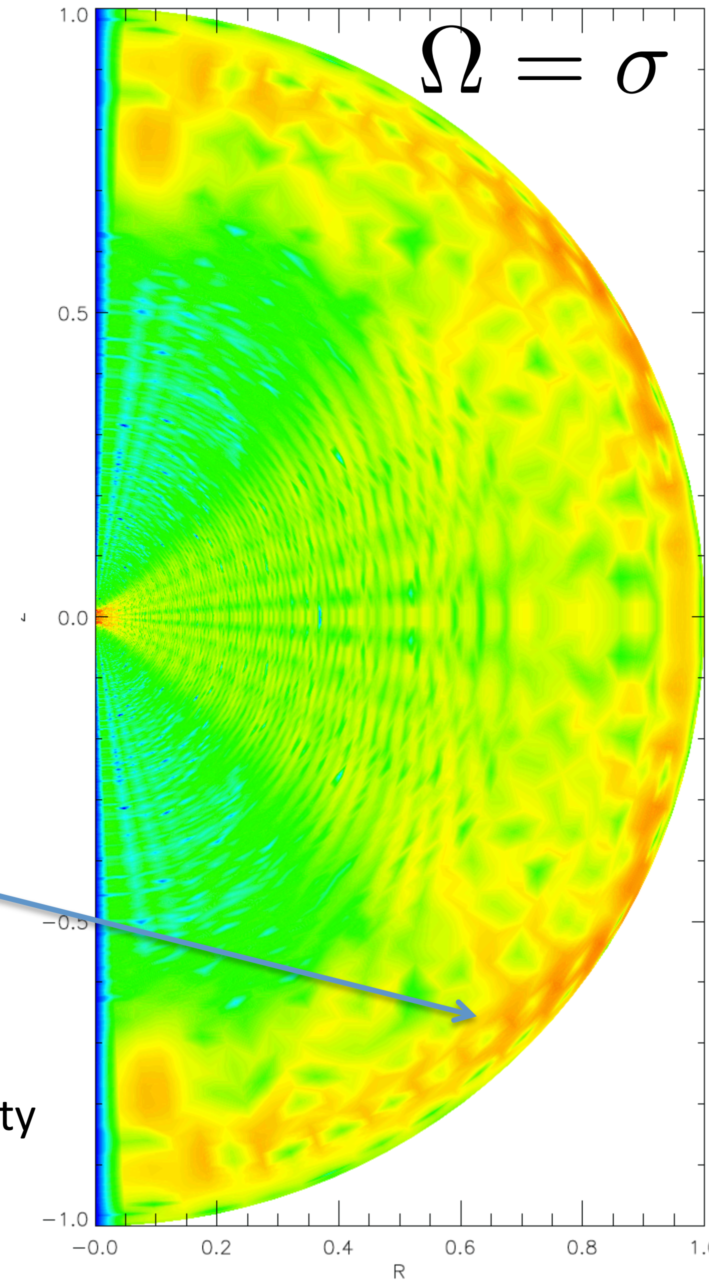
Plotted: response to l=m=2 tide

$$\log_{10} |\xi_r - \xi_{r,eq}|$$

Red = big
Blue = small

g-modes: small shear in convection zone.

inertial waves: large shear in convection zone => turbulent viscosity damping.



2 main linear damping mechanisms

- Radiative diffusion for g-modes in the core

$$\gamma t_{\rm group} \simeq \left(\frac{P_{\rm orb}}{12 \ {\rm days}} \right)^4$$

(Goodman & Dickson)

- Turbulent viscosity for inertial waves in convective envelope

$$\gamma t_{\rm group} \simeq \left(\frac{n_{\|,\perp}}{10} \right)^3 \left(\frac{P_{\rm spin}}{1 \ {\rm day}} \right)$$

(un-reduced viscosity vH/3)

($n_{\|,\perp}$ denotes nodes in cz)

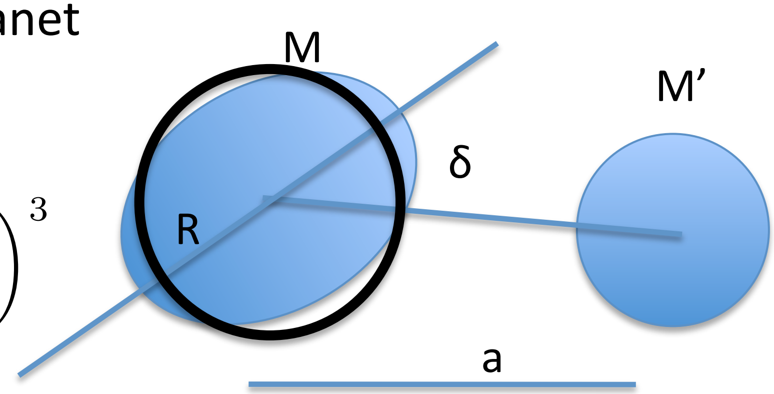
Need far fewer nodes in surface cz to get strong damping.

some nonlinear effects

why do nonlinear fluid effects matter?

size of the "equilibrium" tide for a Jupiter-like planet around a Sun-like star:

$$\frac{\delta R}{R} \equiv \epsilon = \frac{M'}{M}\left(\frac{R}{a}\right)^3 = 10^{-5}\left(\frac{M'}{10^{-3}M}\right)\left(\frac{4.6R}{a}\right)^3$$



Over most of the star, the wave amplitude is small, and the linear approximation to fluid dynamics is good. But…

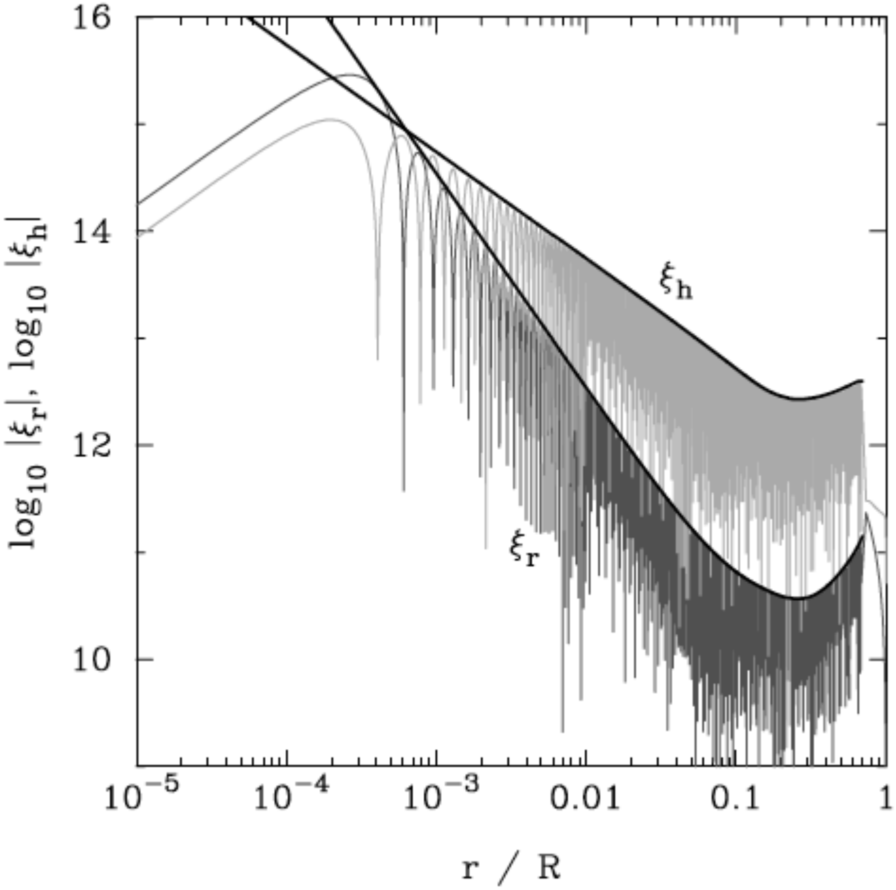
there can be small regions in stars where wave amplitudes become large, and nonlinear fluid processes become important.

(solar literature: Press, Dziembowski, etc)

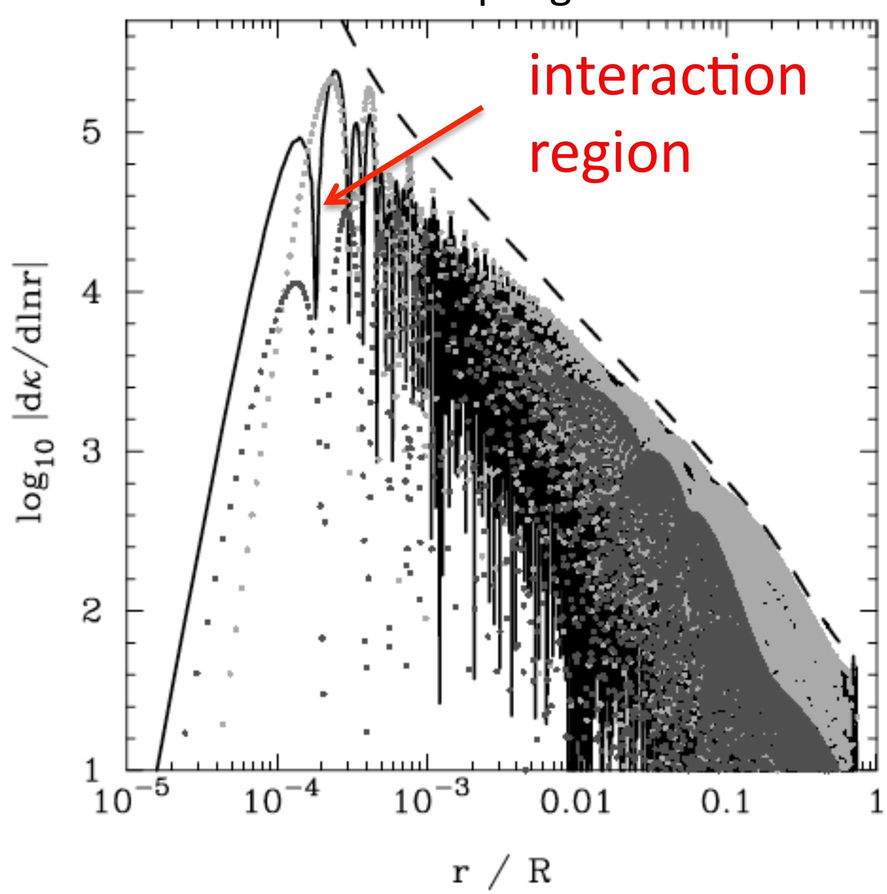
Steepening of g-modes near the center of solar-type stars

(Press, Dziembowski, Ogilvie Barker Lin, Weinberg et al)

eigenfunctions



3-wave coupling coefficient



Wave breaking at the center of stars with radiative cores

(Zahn, Press, Goodman and Dickson, Ogilvie Lin Barker)

Inward-going gravity wave launched from rcb. If central amplitude high enough, wave can overturn the stratification and not reflect.

Wave luminosity:

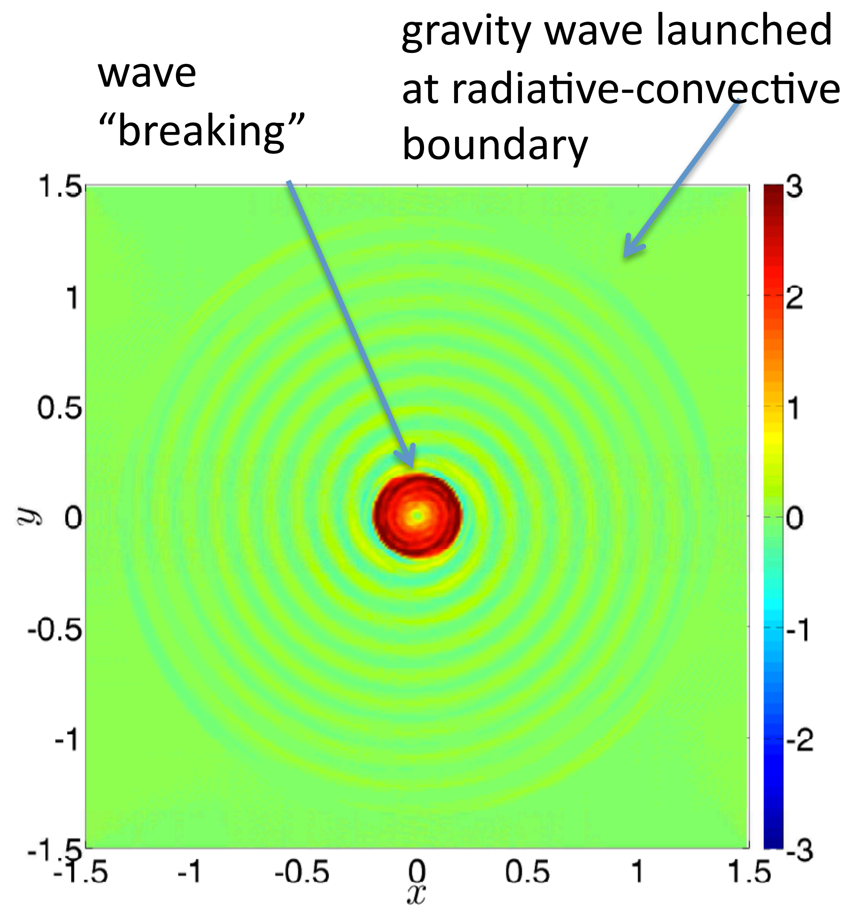
$$L_{\rm wave} \sim (\rho r^2 \lambda)_{\rm rcb}\, v_{\rm eq}^2 \Omega_{\rm orb}$$

$$\frac{\lambda}{H_p} \sim \left(\frac{\Omega_{\rm orb}}{\rm Lamb}\right)^{2/3}$$

At the center, measure of nonlinearity is

$$\frac{\xi_r}{\lambda_r} \simeq \left(\frac{M'}{M_{\rm Jup}}\right)\left(\frac{P_{\rm orb}}{1\ \rm day}\right)^{1/6}$$

So waves break near the center of the star *outside* a critical orbital separation.



(Barker 2011)

Prediction: planets decay and then park. (Observed?)

consequences of wave breaking

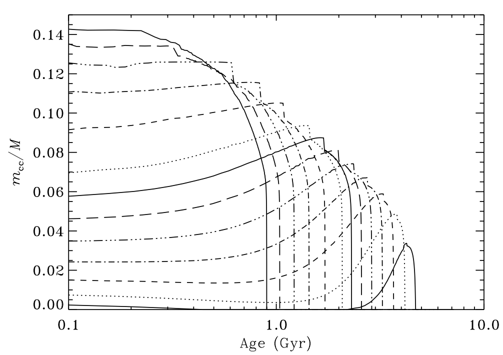
Large dissipation rate (small Q).

$$Q_{GD98} \sim 10^6 \left(\frac{P_{\rm orb}}{1\ {\rm day}}\right)^{8/3}$$

Important for planet orbital decay, but too small by ~1000 to explain circularization of solar-type binaries.

Also, deposit 10^{-5} L_{sun} in 10^{-10} M_{sun} !

For M>1.1-1.2M_{sun}, central convection zones shut wave breaking off.



(Aerts et al, *Asteroseismology*)

Dependence of wave lum on star:

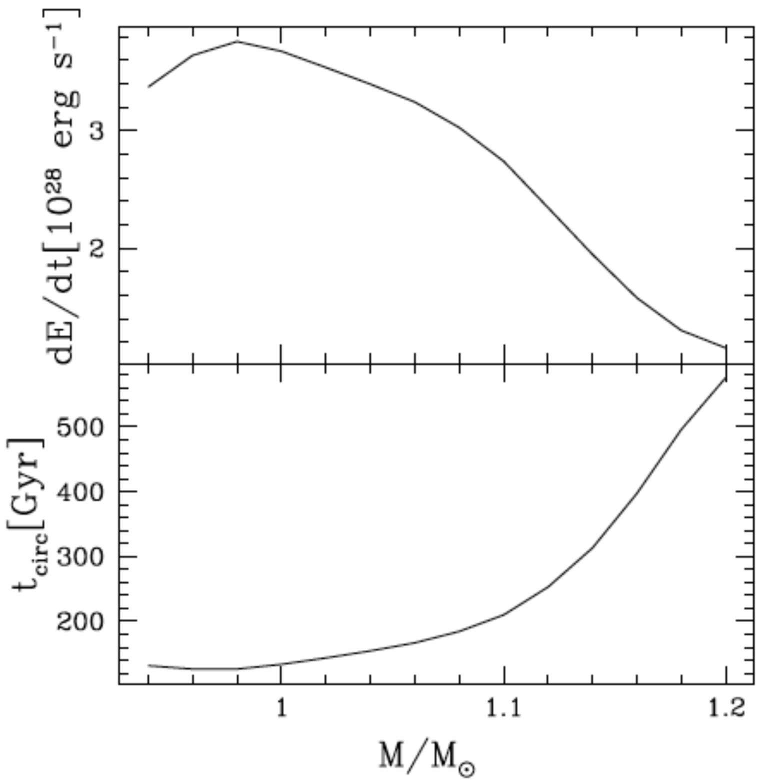


FIG. 2.— Power input to waves and circularization time (at small e) as a function of stellar mass, for equal mass binaries at orbital period $P_{\rm orb}$ = 10 days. To scale the results to

What if waves don't break?

The parametric instability

coupling the fluid to the orbit

Time evolution of standing f, p, g modes:

$$\ddot{A}_\alpha + \omega_\alpha^2 A_\alpha = -\gamma_\alpha \dot{A}_\alpha + U_\alpha(t) + \sum_\beta U_{\alpha\beta}(t) A_\beta + \sum_{\beta\gamma} \kappa_{\alpha\beta\gamma} A_\beta A_\gamma$$

damping

linear tide driving

nonlinear tide driving

3-wave coupling

orbit forcing waves

internal redistribution

Time evolution of the orbit:

waves force orbit

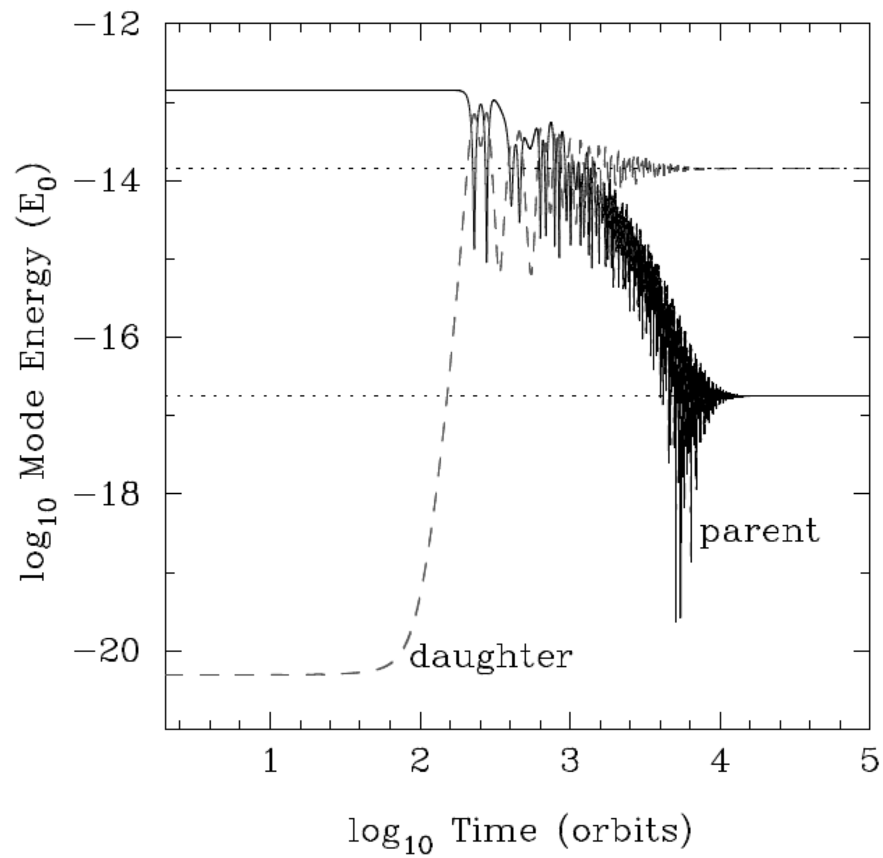
$$\ddot{D} - D\dot{\Phi}^2 = a_D \sim \mathcal{O}(A) + \mathcal{O}(A^2)$$

$$\frac{d}{dt}\left(D^2\dot{\Phi}\right) = Da_\Phi \sim \mathcal{O}(A) + \mathcal{O}(A^2)$$

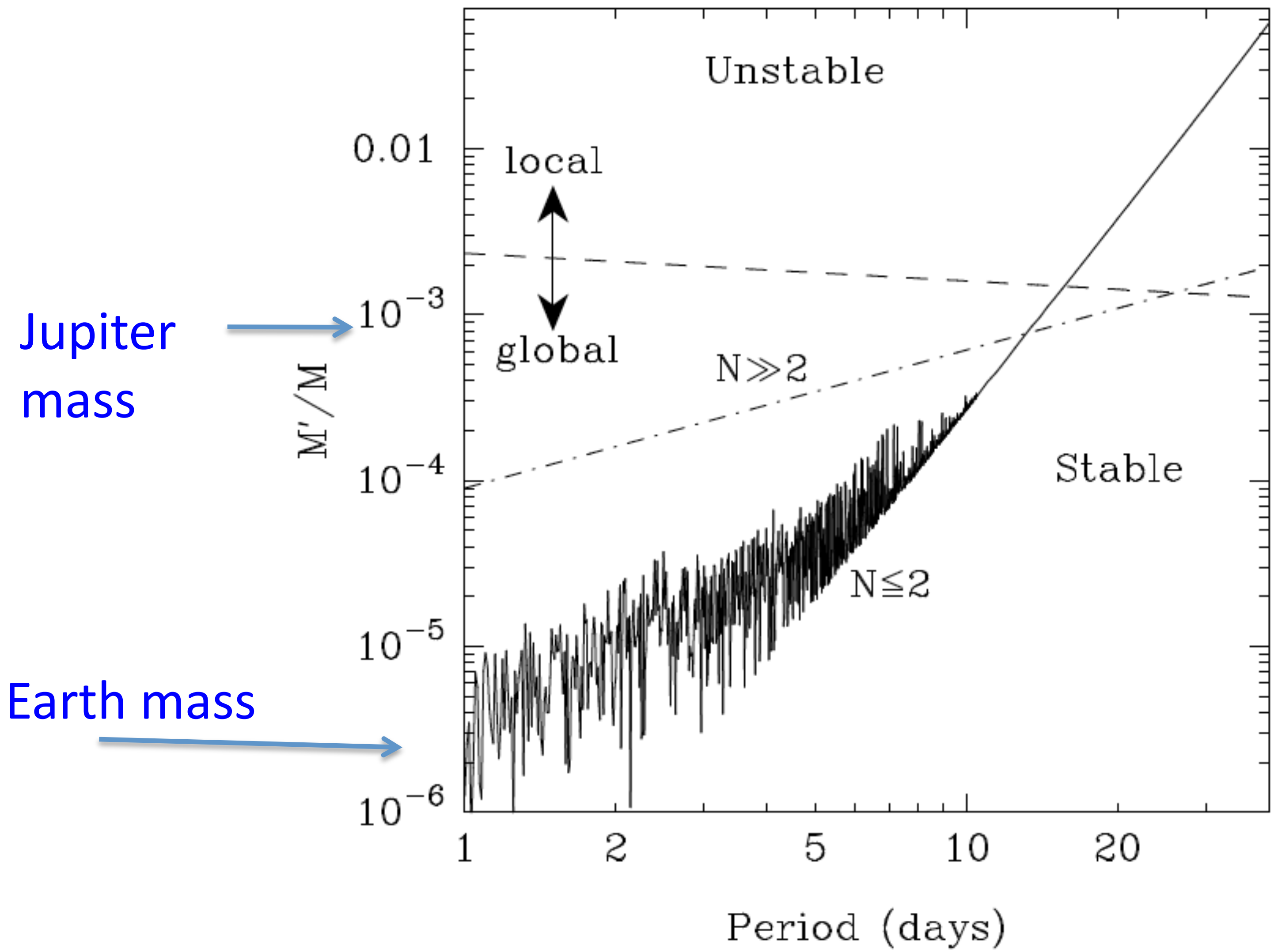
stability of the linear tide?

The linearly driven tide acts as a time-dependent background on which other waves propagate.

If this time-dependent background resonates with two daughter waves, then above a threshold amplitude they can undergo the ``parametric instability" and grow exponentially, eventually reaching a nonlinear equilibrium.

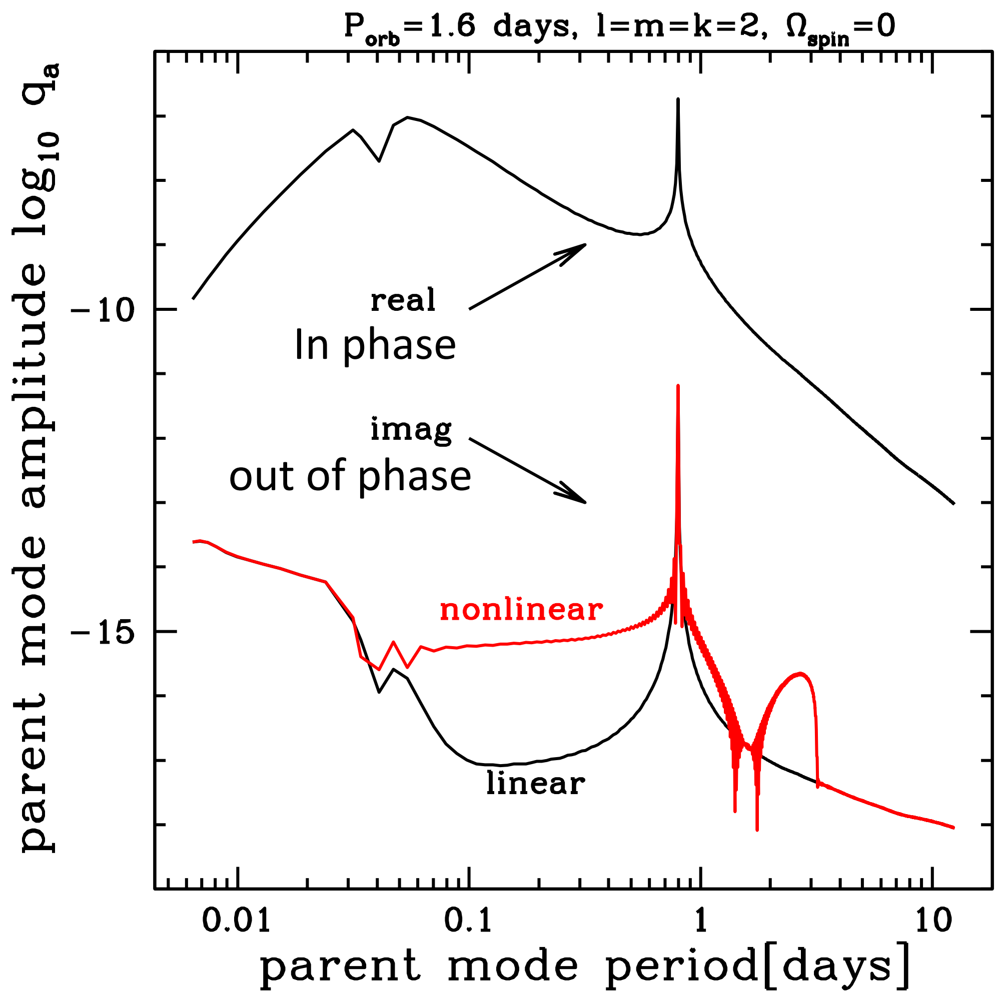


the linear tide is unstable, even for low mass companions



amplitudes just above threshold

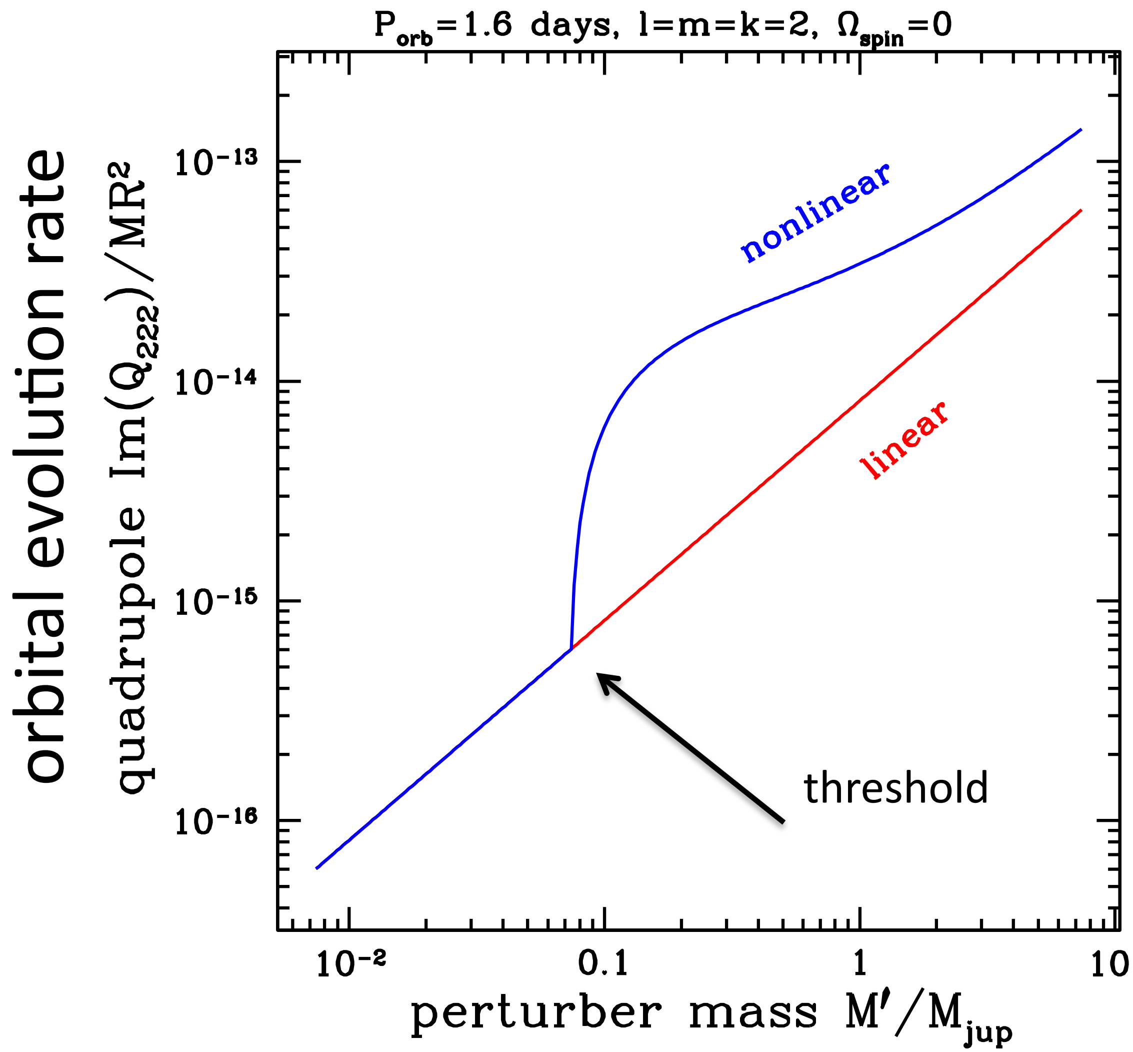
Analytic model for nonlinear equilibrium amplitudes of N>>1 parents and 1 daughter pair.



orbital evolution rate just above threshold

Even for just one daughter pair, orbital evolution can be increased by a factor of 10 or more.

For Jupiter mass planets, there can be thousands of daughter pairs excited, requiring numerical simulations to understand the orbital evolution rate.



Simulation of
many coupled
g-modes in
solar-type star.

Initial conditions:
linear tide

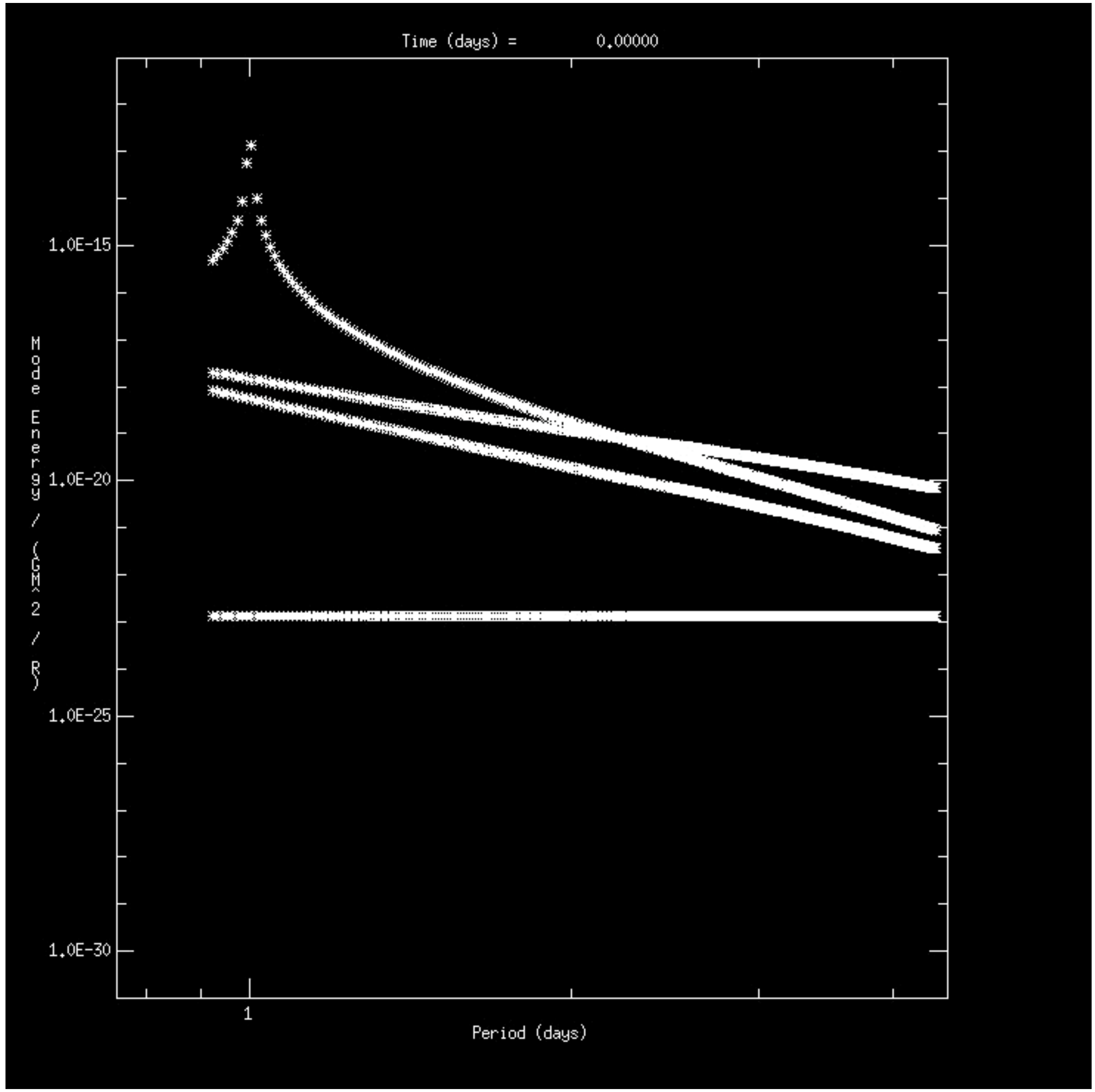
Porb=2 days.

parent = 1 day (m=2)

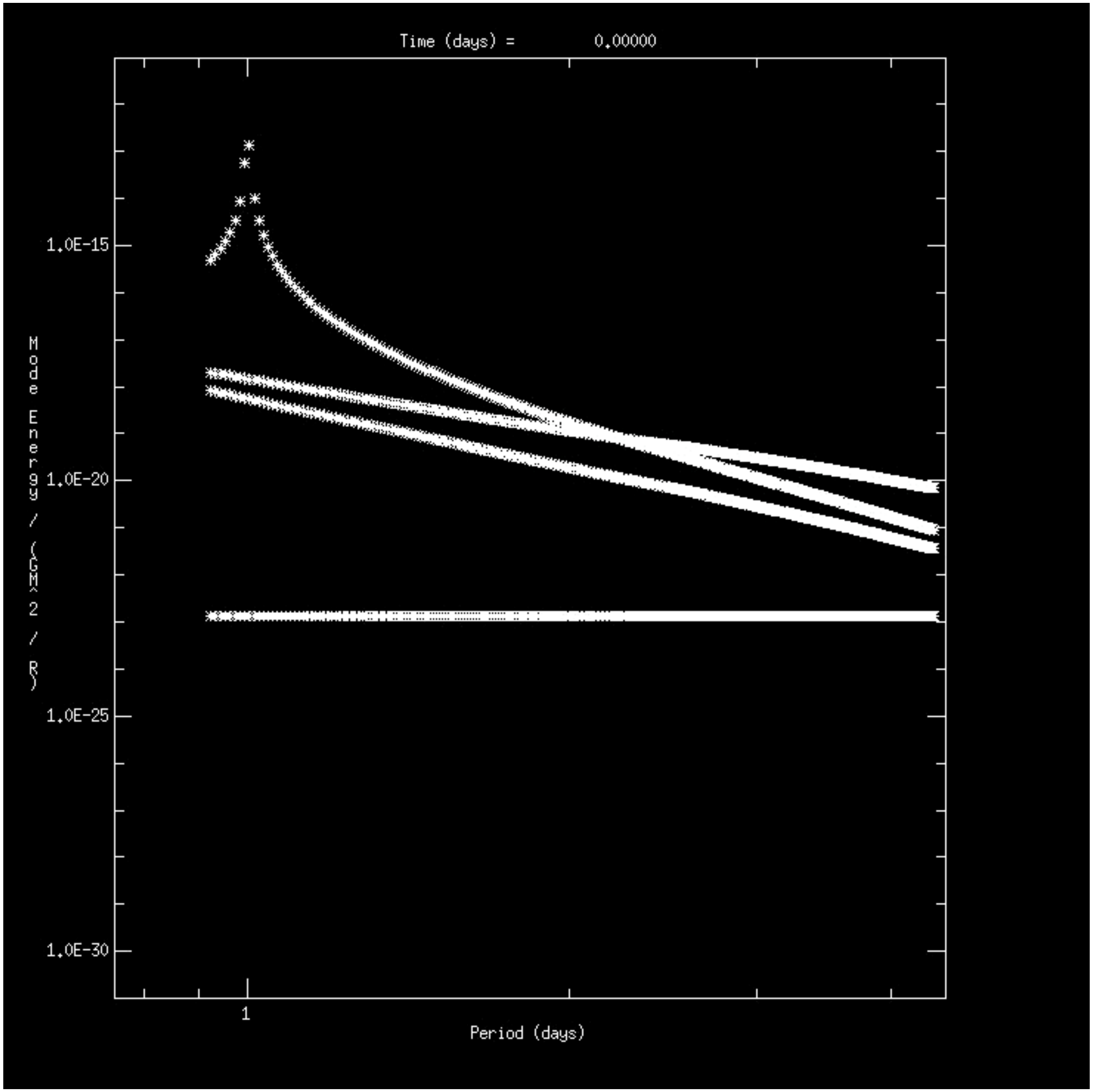
Parametric instability
causes unstable
daughters at
2 day periods.

All modes coupled.

Damping X 1000



“g-mode turbulent cascade”



Damping X 100

Summary

- Tidal evolution observed for binaries containing solar-type stars.

- "Dynamical tide" excitation/damping of inertial-gravity waves important. Inertial waves in convection zones a promising avenue toward larger larger dissipation rates needed to explain observations.

- Two important nonlinear effects: wave breaking
  + parametric instability.

A simple model for nonlinear tidal dissipation: Press, Wiita & Smarr (1975)

- Re >> 1 + instability gives turbulence.
- For forcing frequency σ, saturation with velocity and length scales

$$v_\ell \sim \epsilon \sigma R$$

$$\ell \sim R$$

- Turbulent viscosity

$$\nu_{\rm turb} \sim \ell v_\ell \sim \epsilon \sigma R^2$$

- Yields an <u>amplitude dependent</u> tidal Q

$$Q^{-1} \sim \sigma^{-1} \frac{\nu_{\rm turb}}{\ell^2} \sim \epsilon$$

<u>Possible improvements:</u>
Better tidal flow
waves not eddies
realistic dissipation
Identify instability