

Asteroseismic Diagnostics of Subdwarf B Stars

SdB pulsators as probes of stellar astrophysics

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Usual collaborators

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Outline

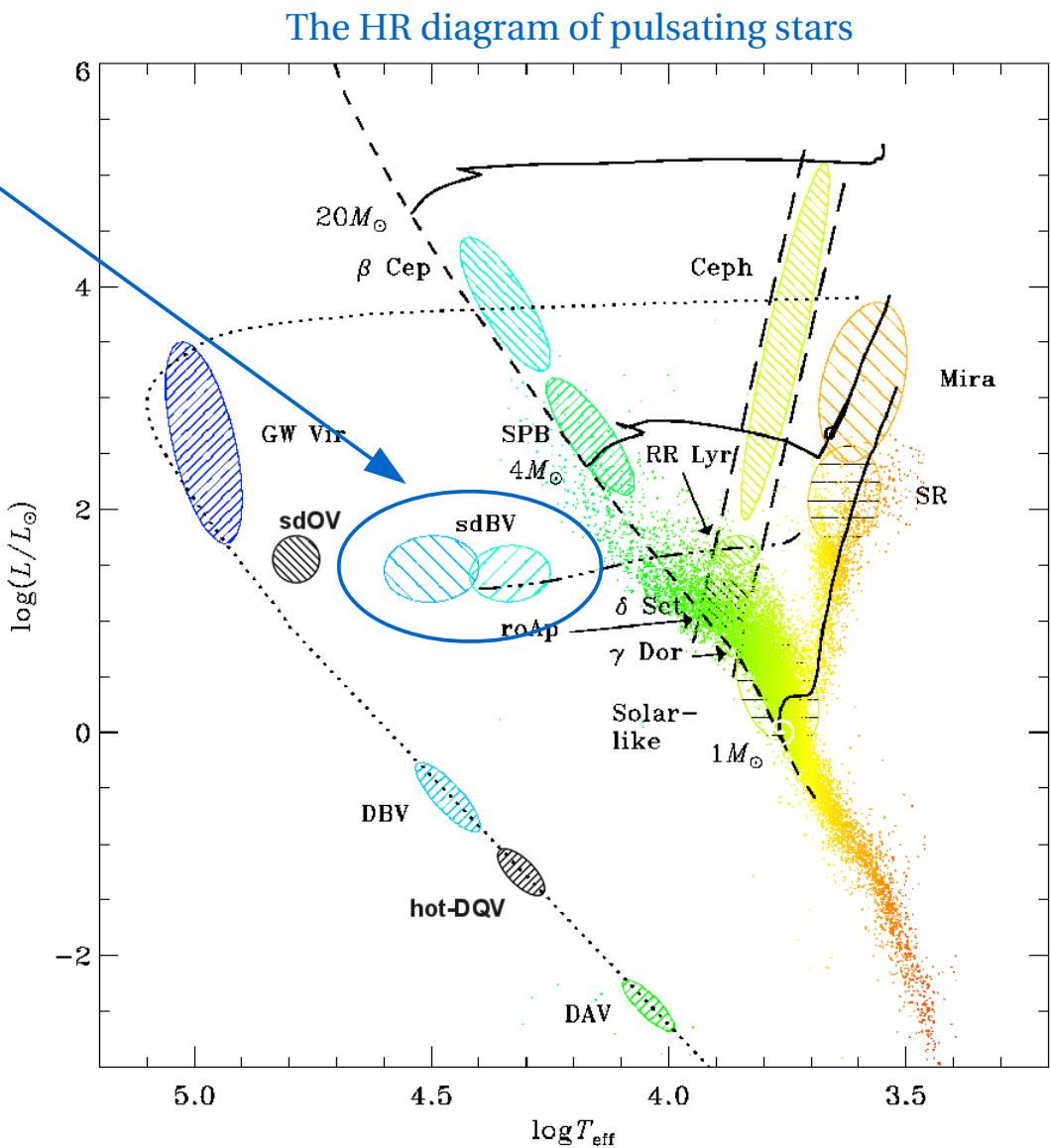
1. Introduction
2. Adiabatic asteroseismology of sdB stars
- (3. Nonadiabatic asteroseismology of sdB stars)
4. Summary and Prospects

Introduction

He core burning EHB (sdB) stars

Most are post-RGB stars having their H-rich envelope almost entirely removed
→ hot and compact (RG almost naked cores)
(see Van Grootel's talk on ways to produce sdB stars)

M peaking around 0.47 Msun
Teff ~ 23,000 – 40,000 K
 $\log g \sim 5.1 - 6.2$



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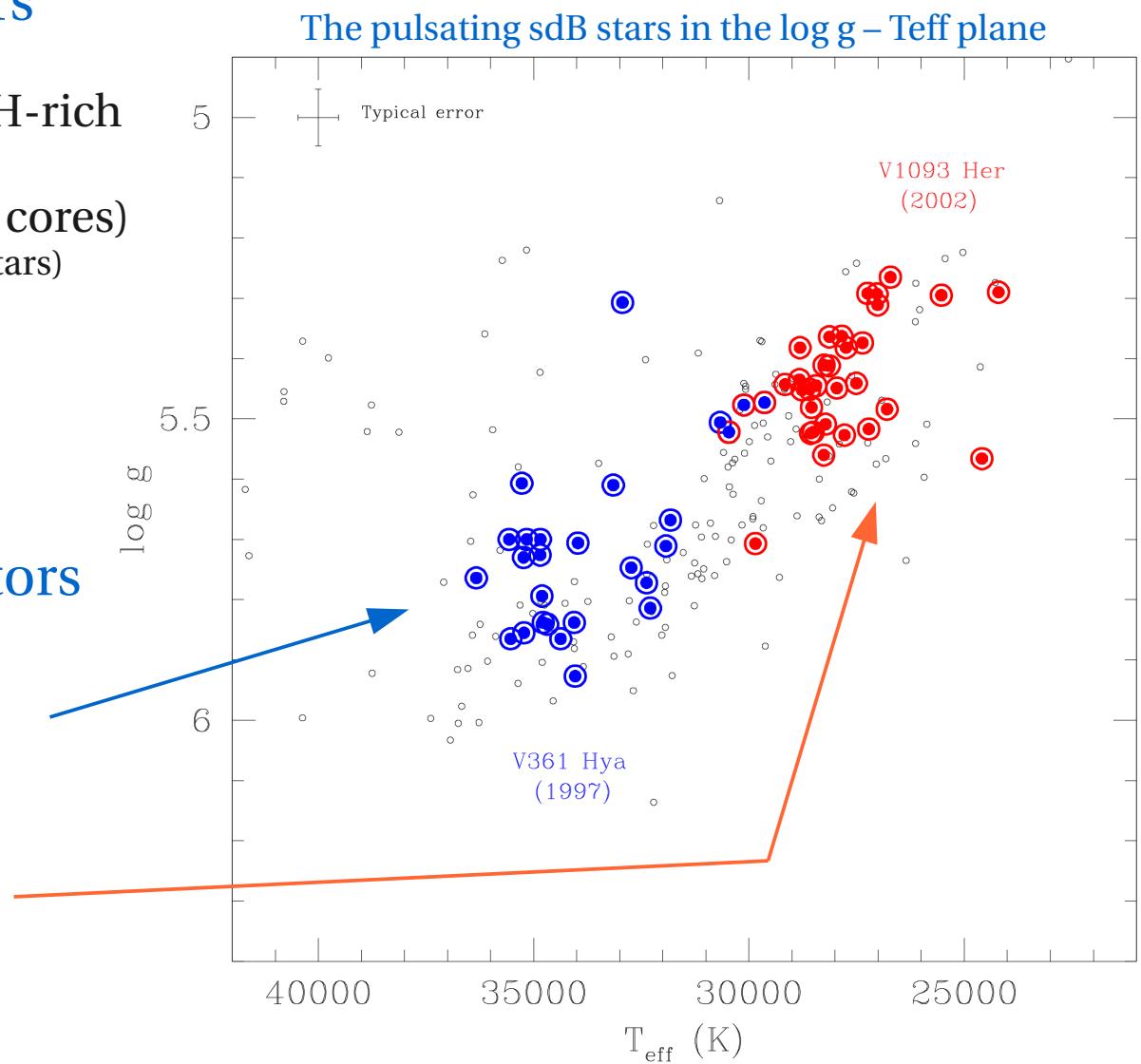
M peaking around 0.47 Msun
Teff \sim 23,000 – 40,000 K
 $\log g \sim 5.1 - 6.2$

Two groups of nonradial pulsators

P-modes: periods of $\sim 1 - 10$ minutes
(*V361 Hya* stars; Kilkenny et al. 1997)

G-modes: periods of $\sim 1 - 4$ hours
(*V1093 Her* stars; Green et al. 2003)

+ Hybrid pulsators



Adiabatic asteroseismology

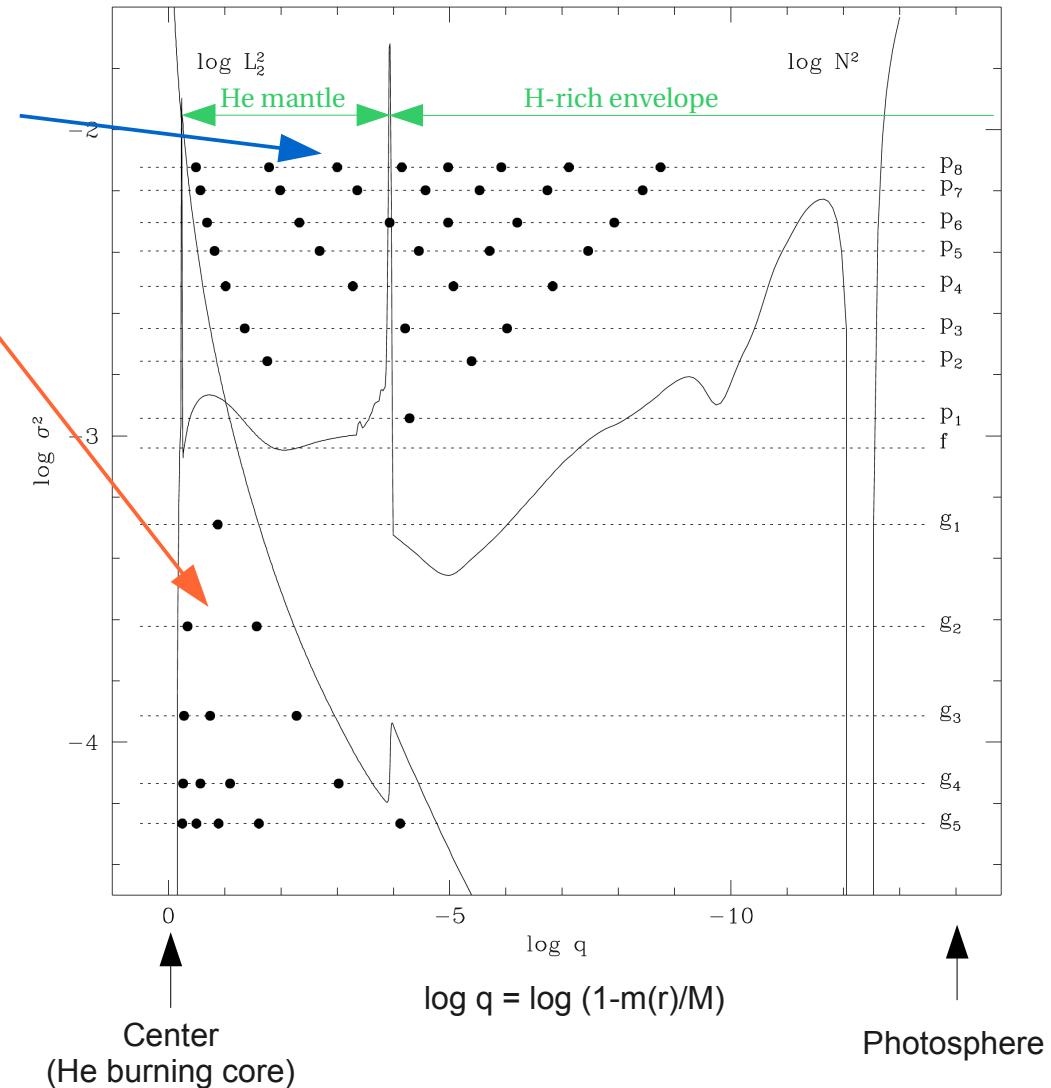
learning about the star interior from the frequencies

Mode properties in sdB stars

P-modes probe mostly the H-rich envelope and the upper He mantle (insensitive to the core)

G-modes probe much deeper regions (He mantle and C-O/He core boundary)

Propagation diagram (Charpinet et al. 2000)



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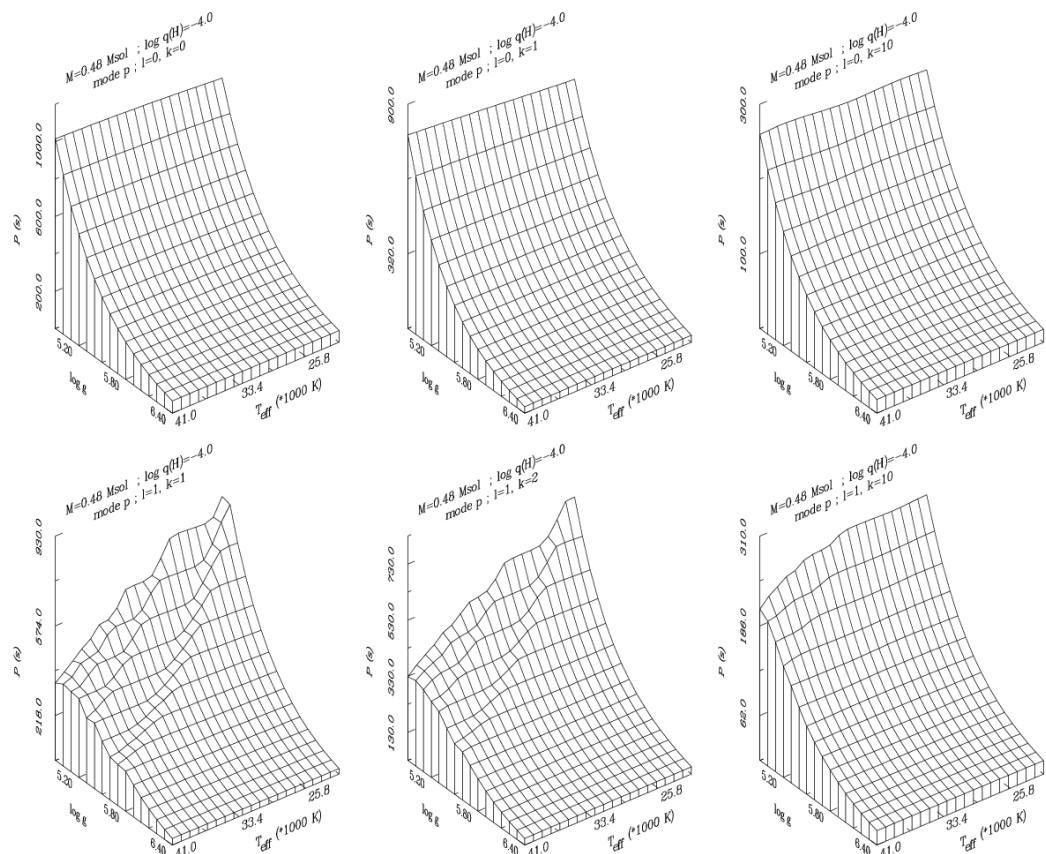
Sensitive to main stellar parameters

P-mode periods strongly sensitive to $\log g$ (or R_*).

Also sensitive to the stellar mass (not shown)

Avoided crossings at high-Teff, low $\log g$ (near TAEHB and post-EHB stars)

P-mode period sensitivity to Teff / $\log g$ (Charpinet et al. 2002)



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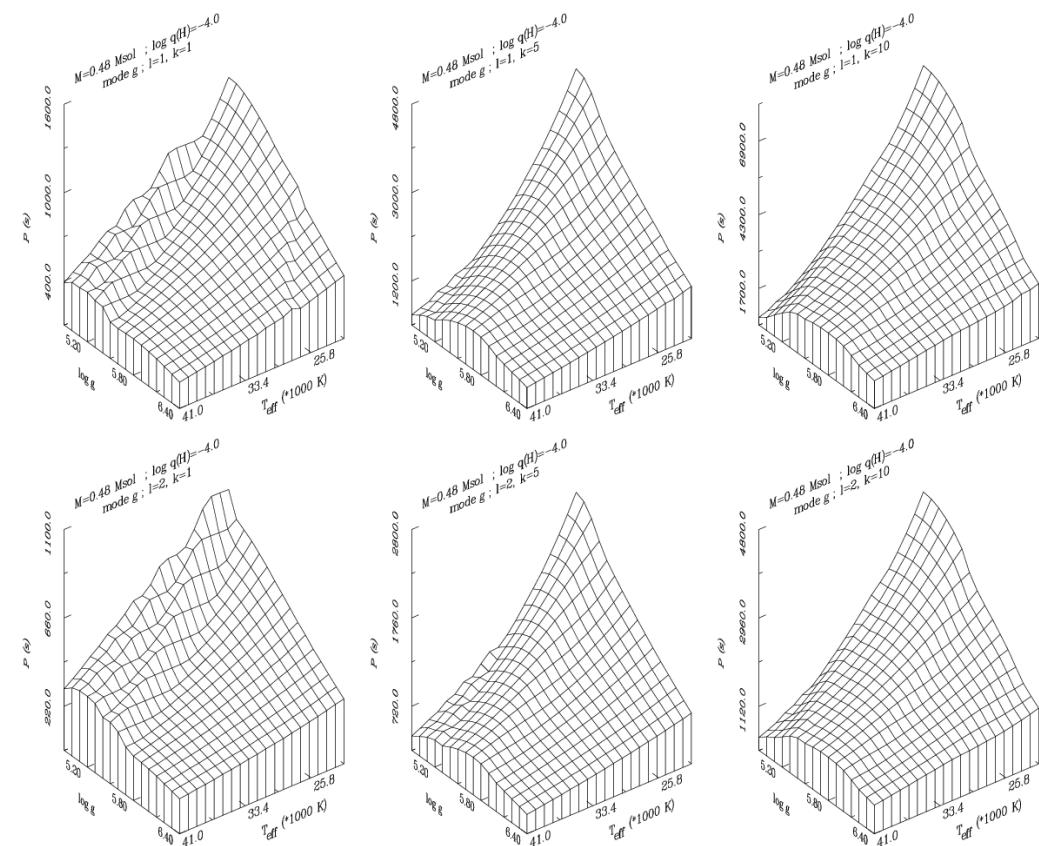
Sensitive to main stellar parameters

G-mode periods are also sensitive to the location of the star in the log g – Teff plane

... and to the stellar mass (not shown)

Avoided crossings at high-Teff, low log g (near TAEHB and post-EHB stars)

G-mode period sensitivity to Teff / log g (Charpinet et al. 2002)



Adiabatic asteroseismology

learning about the star interior from the frequencies

Mode properties in sdB stars

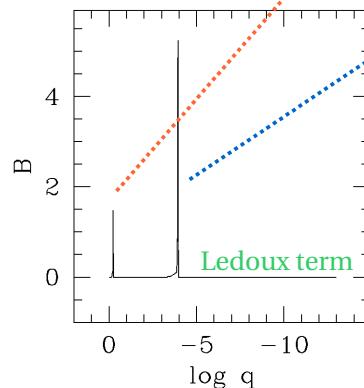
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Sensitive to chemical transitions

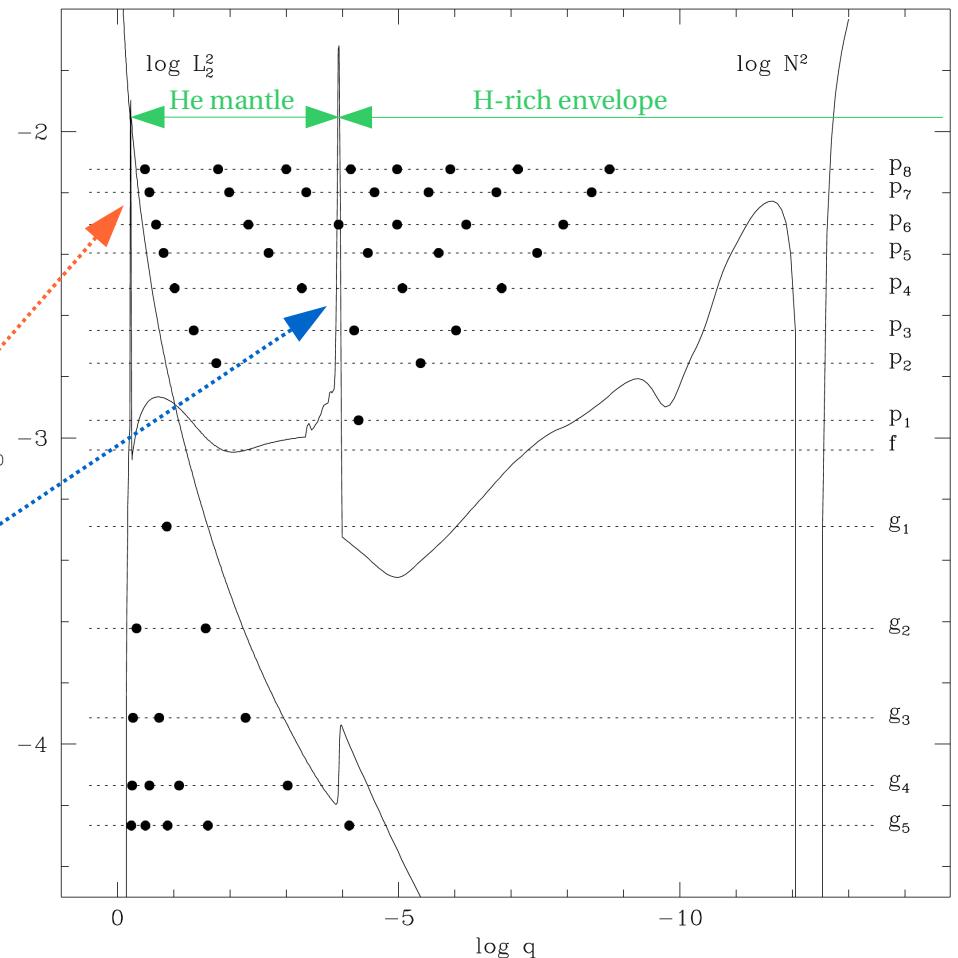
$$N^2 = \frac{g^2 \rho}{P} \frac{\chi_T}{\chi_\rho} (\nabla_{\text{ad}} - \nabla + B)$$

$$B \equiv -\frac{1}{\chi_T} \sum_{i=0}^{N_c-1} \chi_{x_i} \frac{d \ln X_i}{d \ln P}$$



Trapping effects are generated at the C-O/He core and He/H envelope boundaries by composition gradients

Propagation diagram (Charpinet et al. 2000)



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learning about the star interior from the frequencies

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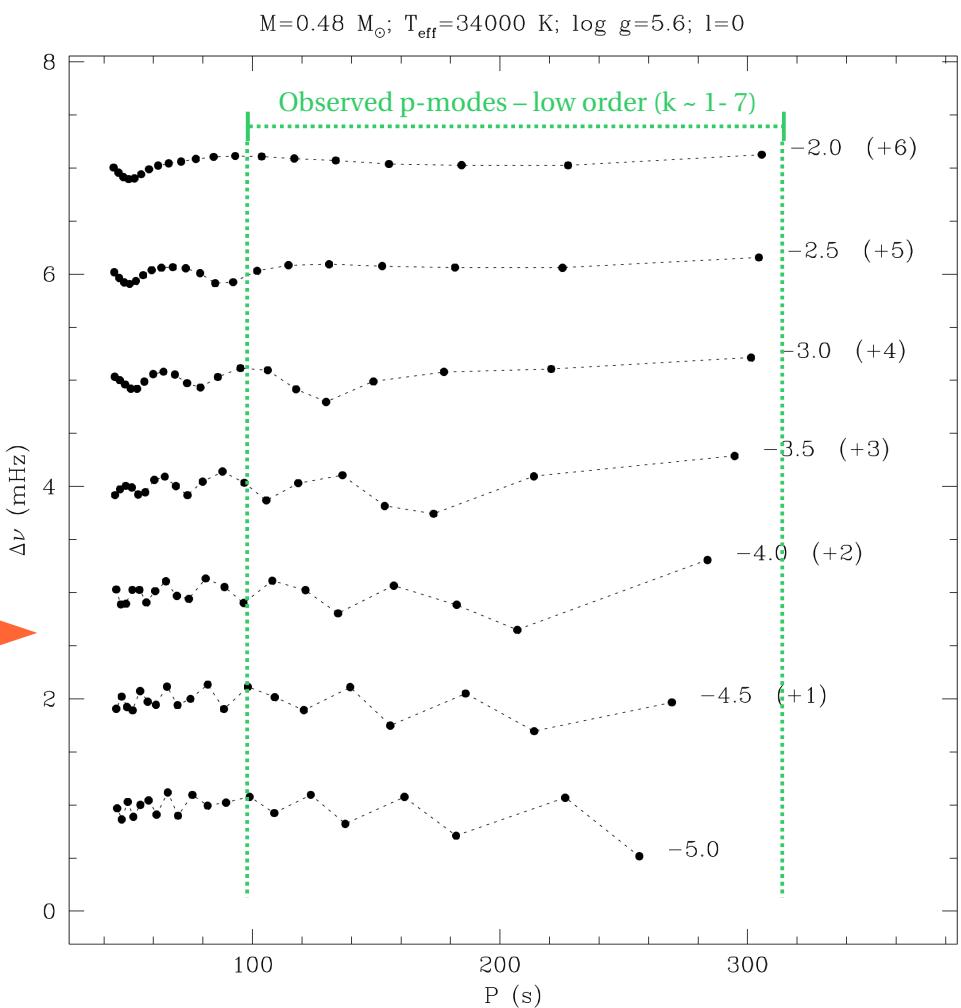
P-mode “micro-trapping”
(or “acoustic glitches”)

Acoustic mode distribution sensitive to the location of the He/H chemical transition
(sharp sound speed variation)

→ access to the mass of the H-rich envelope

But mostly insensitive to the core transition

Trapping of p-modes (Charpinet et al. 2000)



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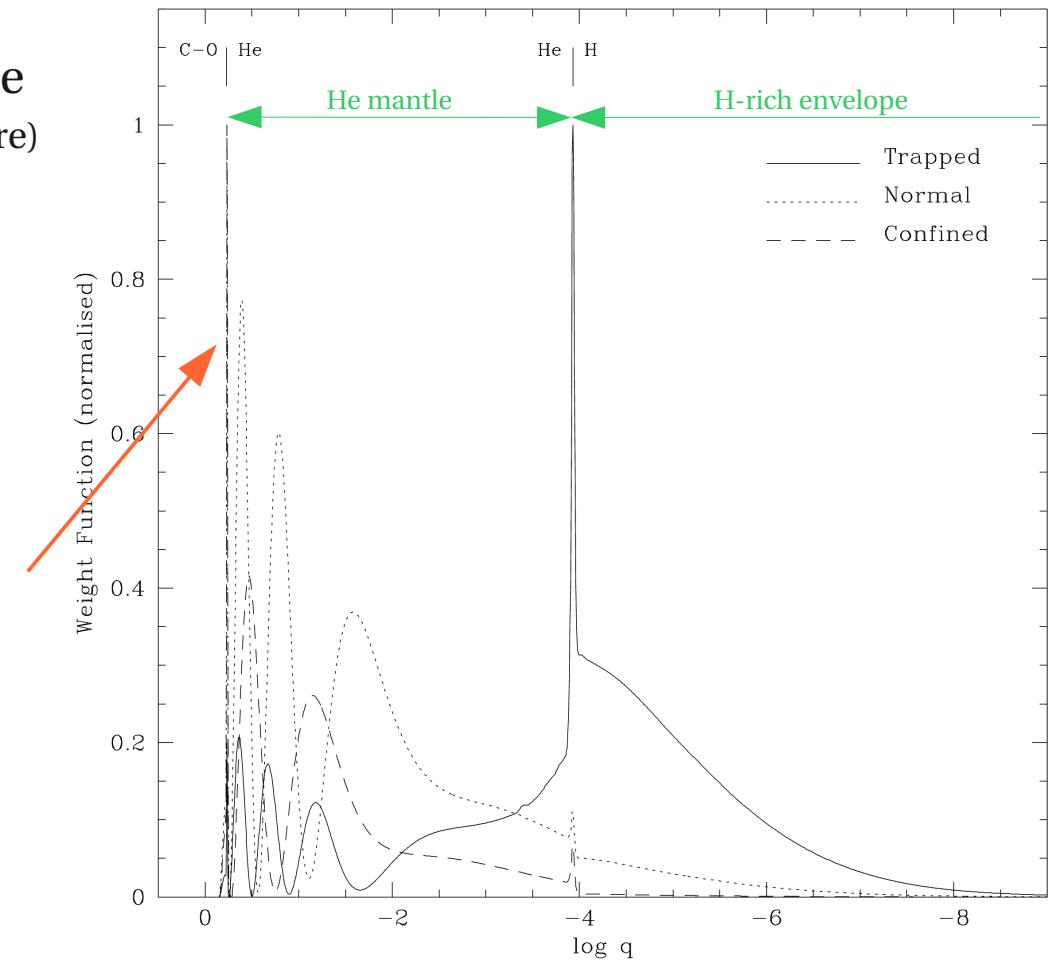
G-modes probe much deeper regions (He mantle and C-O/He core boundary)

G-modes provide deeper probes

Stronger influence of the He/H transition

Also sensitive to the core boundary

Typical g-mode weight functions (Charpinet et al. 2000)



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Mode properties in sdB stars

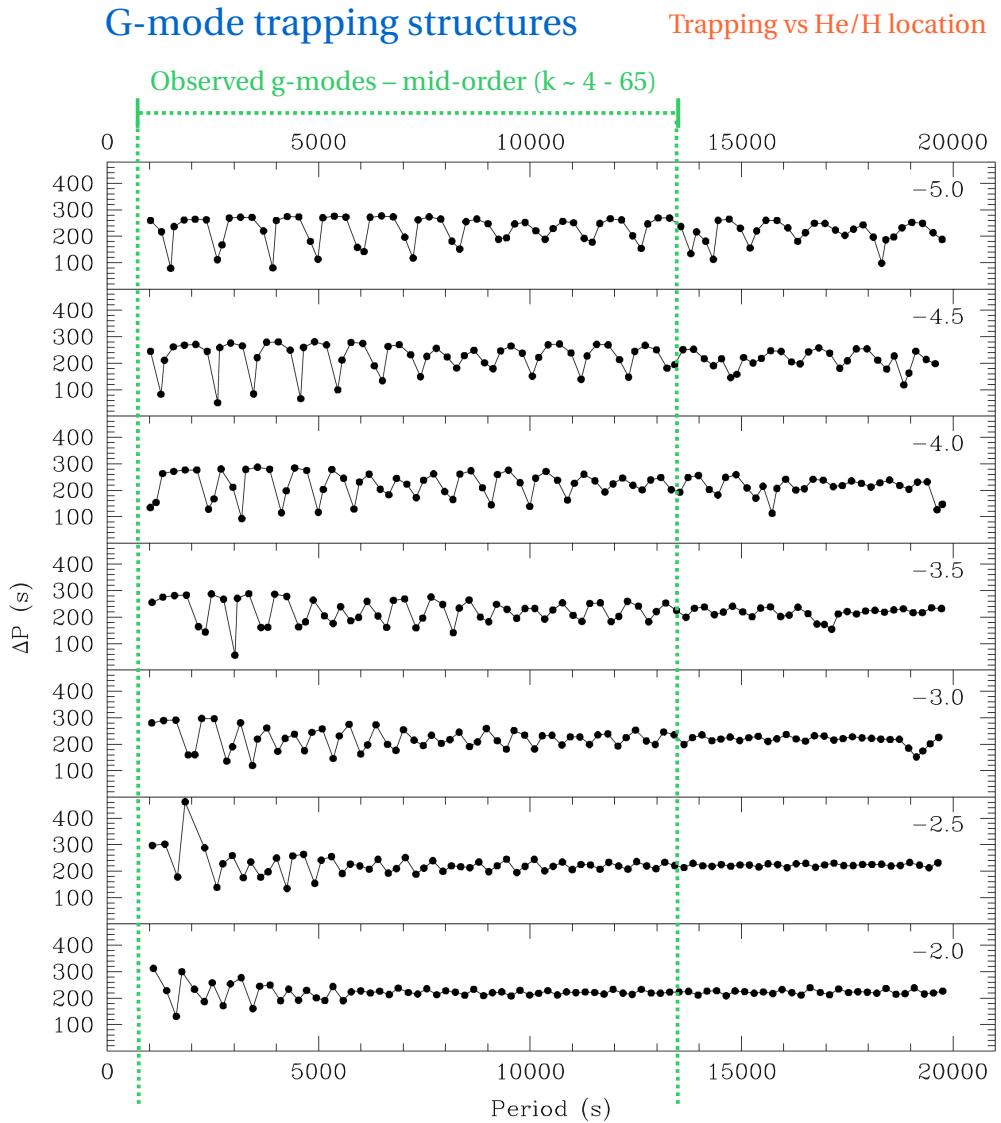
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G-mode envelope trapping

Cyclic perturbations of the g-mode periods depending on the He/H transition position

→ access to the mass of the H-rich envelope



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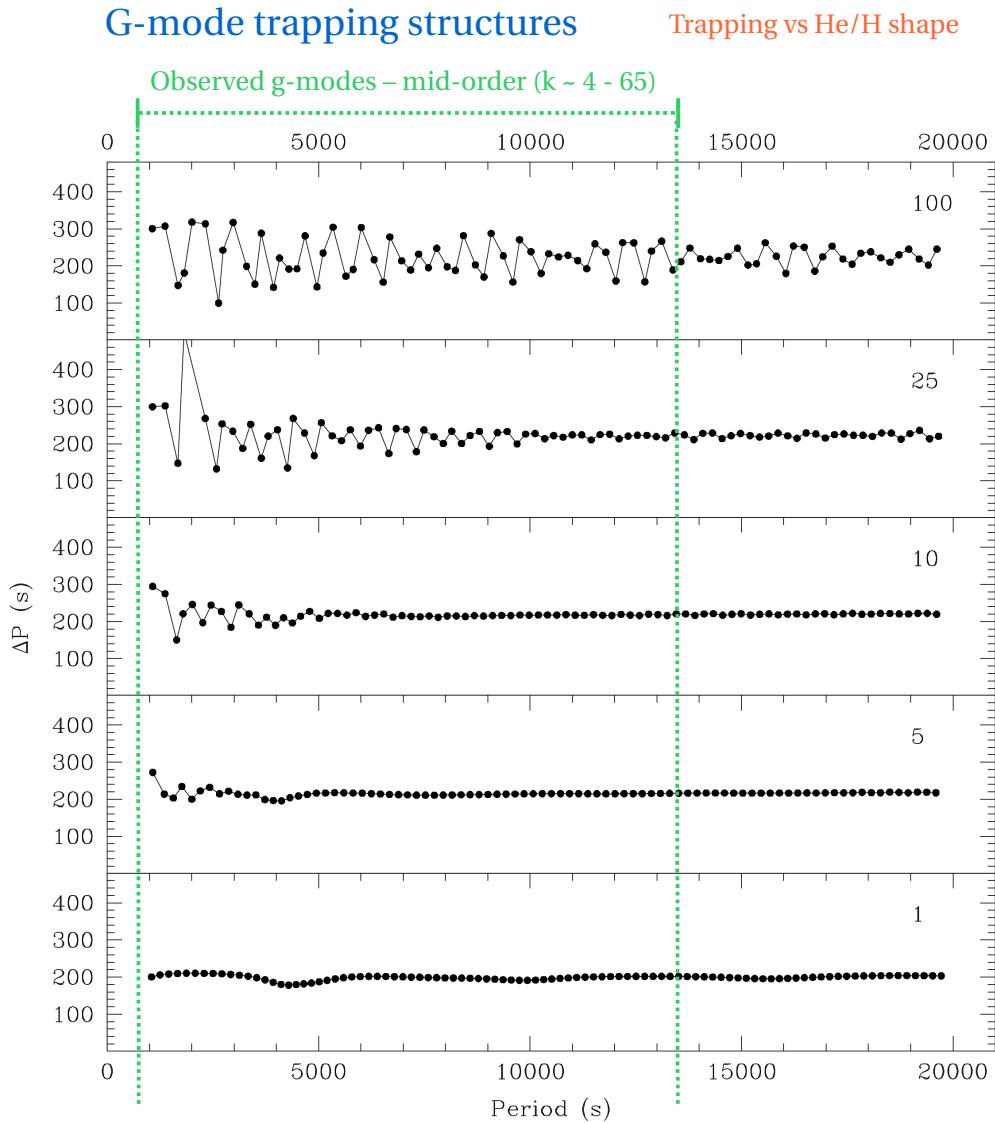
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Cyclic perturbations of the g-mode periods depending on the He/H transition position

→ access to the mass of the H-rich envelope

Sensitive also to the shape of this transition (smooth or sharp composition gradient)



Adiabatic asteroseismology

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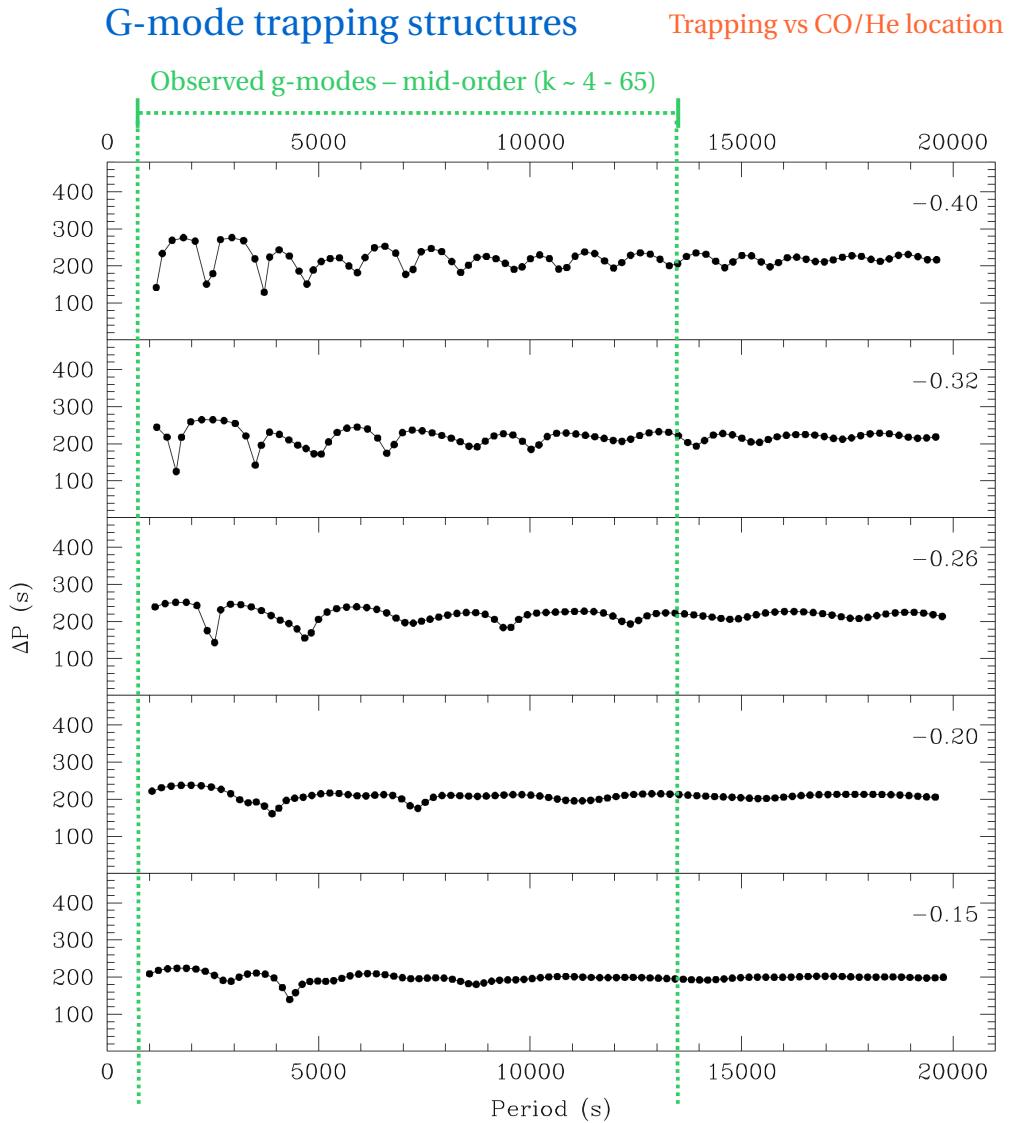
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G-mode connection with the core

Confined modes sensitive to the C-O/He transition (amount of He left in the core)

Confined modes sensitive to the position of this transition (core boundary location)

→ access to the size of the mixed core



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learning about the star interior from the frequencies

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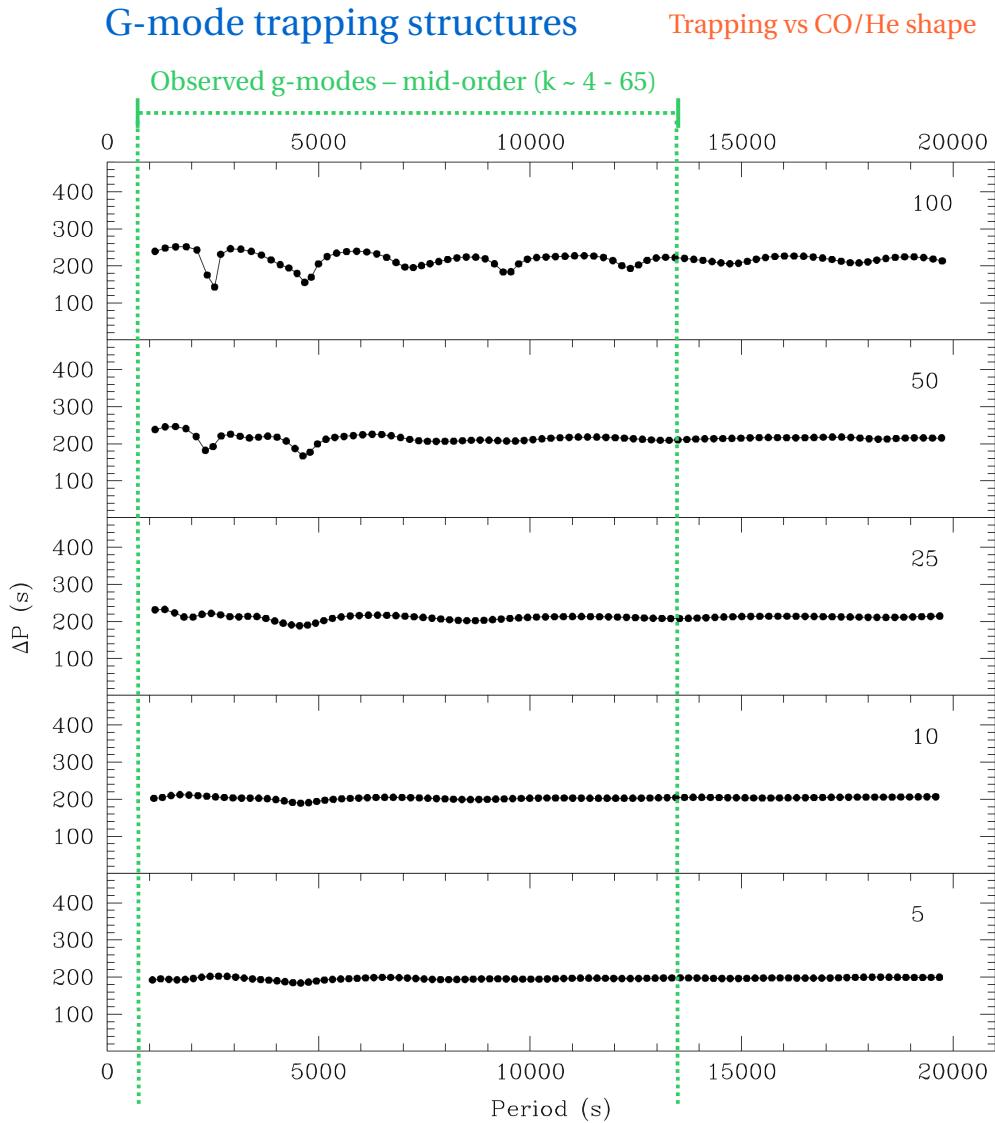
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Adiabatic asteroseismology

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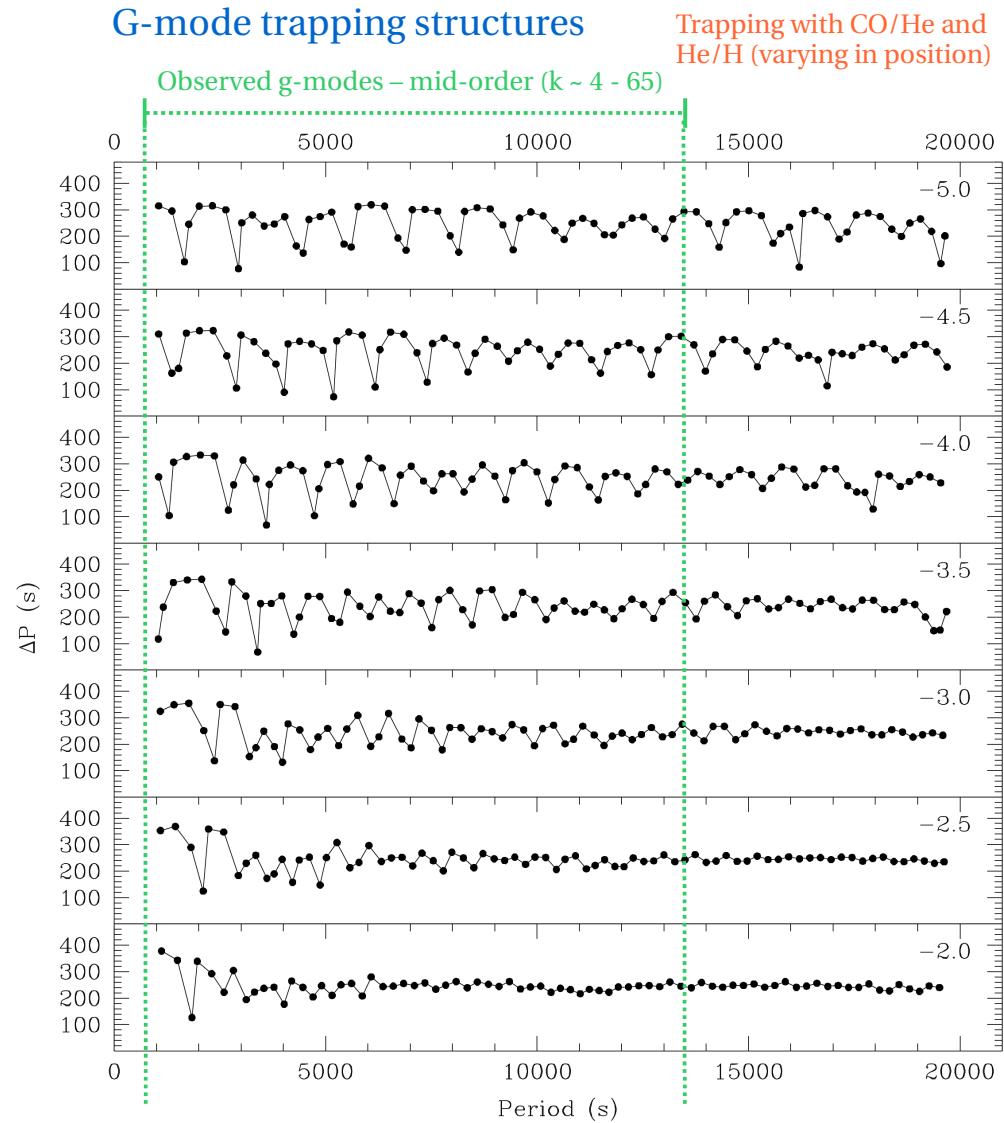
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G-mode trapping patterns

Various trapping effects interfere, leading to complex g-mode period perturbations



Adiabatic asteroseismology

learning about the star interior from the frequencies

Pulsation frequencies \leftrightarrow structural parameters : a complex problem

- Trapping structures interfere in a complicated way
- P-modes (and a few g-modes) out of asymptotic regime (sometimes mixed modes)
- Observed pulsation spectra are generally incomplete (modes are missing)
- Independent mode identification (of the degree ℓ) is usually not available

→ Simple seismic diagnostic tools are mostly ineffective

Although attempts to detect and interpret g-mode period spacings have been made
(Randall et al. 2006; Reed et al. 2011)

Adiabatic asteroseismology

learning about the star interior from the frequencies

Pulsation frequencies \leftrightarrow structural parameters : a complex problem

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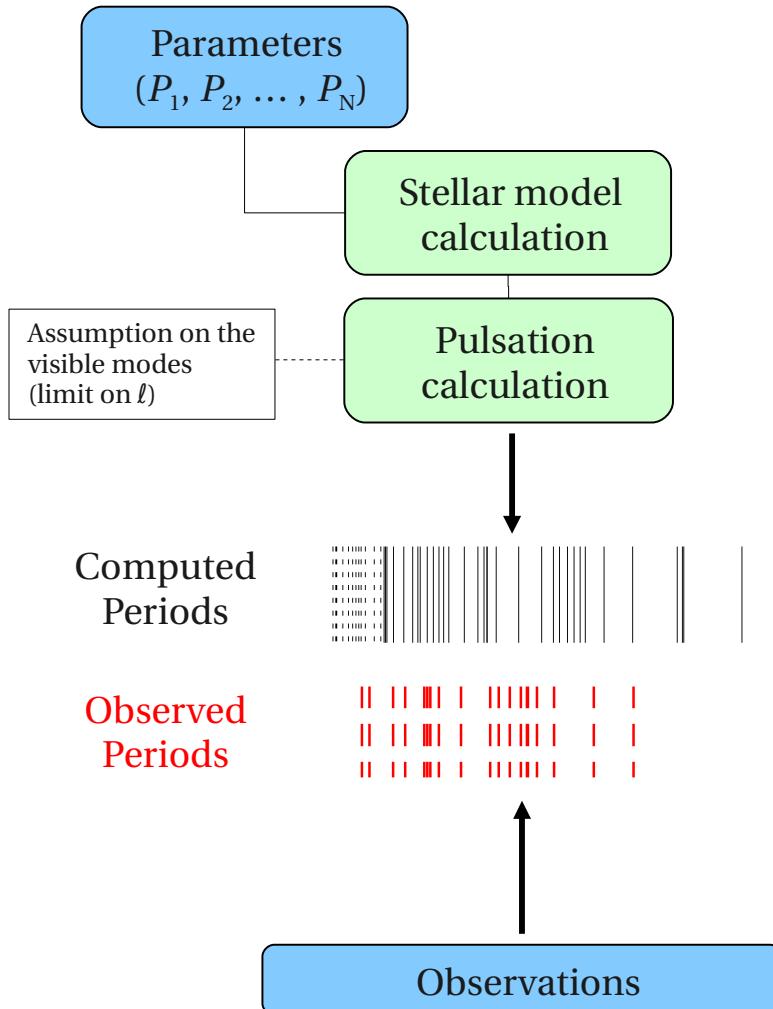
The problem has to be handled with the forward modeling approach

- Principle: searching for the optimal stellar model(s) that best match the observed frequencies (through a χ^2 -type merit function minimisation)
- Method: a global optimisation problem solved by using large grids (brute force method) or optimisation tools to explore a vast multi-dimensional parameter space.

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The forward modeling approach

Basic operation



The first optimization

Search for the best simultaneous fit of N observed periods among M ($M > N$) theoretical periods available from a given model.

→ find among all possible associations the period combination such that

$$\bar{\chi}^2 = \min \left\{ \chi^2 = \sum_{i=1}^N \left(\frac{P_{\text{theo},i} - P_{\text{obs},i}}{\sigma_i} \right)^2, \forall \text{ combinations} \right\}$$

A combinatorial optimization problem (we solve it with a GA-based period matching code).

Period matching
code

Use mode identification constraints here (if available)

$$\bar{\chi}^2(P_1, P_2, \dots, P_N)$$

+

Mode identification

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The forward modeling approach

The second optimization

Searching for the minima of $S^2(P_1, \dots, P_N)$ in the N -dimensional model parameter space with external constraints (e.g., Teff/log g from spectro.)

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The forward modeling approach : optimization tools

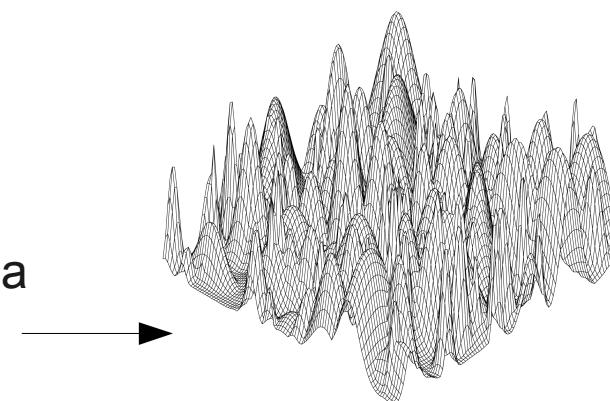
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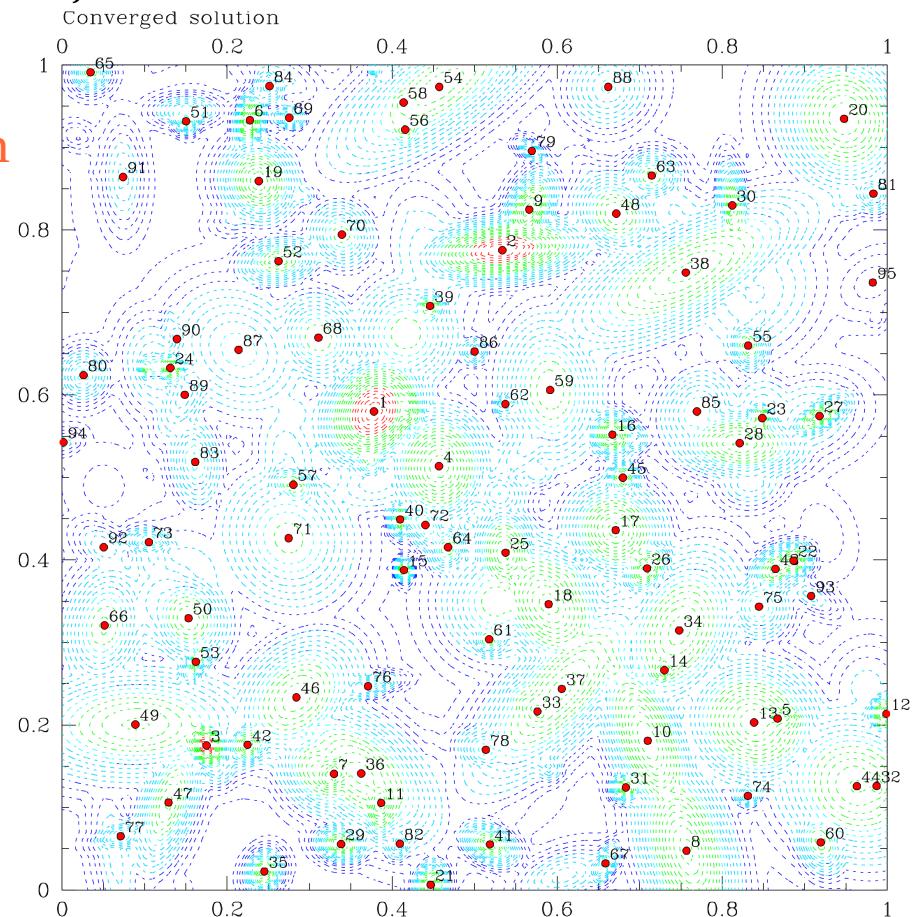
A massively parallel Genetic Algorithm for multi-modal, multi-dimension global optimization is used. It localizes all potential solutions (minima of S^2)

- Capture relevant solutions (wide or narrow loc.)
- Statement on uniqueness of the solution

Illustration with a test function



Maxima of a random Gaussian landscape ($M = 200$)



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Star structure modeling strategies

Models for asteroseismic probing of hot subdwarf pulsators

- Evolutionary models (see Haili Hu & Steven Bloemen Talks)

- suitable for p- and g-mode pulsators (EHB and post-EHB)
- pro: Stellar interior keeps the history of previous phases (chemical stratification)
but past history of sdB stars is not well known (various formation channels)
- cons: require large grid computations (and a grid resolution)

- Static envelope models (constant luminosity)

- suitable for pure p-mode pulsators only (EHB and post-EHB)
- pro: flexibility for asteroseismology with a higher level of parameterization of
inner structures that pulsation modes may be sensitive to
- pro: no grid, parameter space is explored in a continuous way

- Complete static models in thermal equilibrium (nuclear reactions included)

- suitable for p- and g-mode pulsators (EHB only)
- pro: flexibility for asteroseismology with a higher level of parameterization of
inner structures that pulsation modes may be sensitive to
- pro: no grid, parameter space is explored in a continuous way

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Historical record of sdB stars probed with asteroseismology since 2001

Name	$\log g$ (cm s ⁻²)	T_{eff} (K)	M (M_{\odot})	$\log M_{\text{env}}/M$	References
PG 0014+067	5.780±0.008	33550±380	0.490±0.019	-4.31±0.22	Brassard et al. (2001)
	5.775±0.009	34130±370	0.477±0.024	-4.32±0.23	Charpinet et al. (2005a)
	5.772	34130±370	0.478	-4.13	Brassard & Fontaine (2008)
PG 1047+003	5.800±0.006	33150±200	0.490±0.014	-3.72±0.11	Charpinet et al. (2003)
PG 1219+534	5.807±0.006	33600±370	0.457±0.012	-4.25±0.15	Charpinet et al. (2005b)
Feige 48	5.437±0.006	29580±370	0.460±0.008	-2.97±0.09	Charpinet et al. (2005c)
	5.462±0.006	29580±370	0.519±0.009	-2.52±0.06	Van Grootel et al. (2008a) → + rotation
EC 05217-3914	5.730	32000	0.490	-3.00	Billères & Fontaine (2005)
PG 1325+101	5.811±0.004	35050±220	0.499±0.011	-4.18±0.10	Charpinet et al. (2006a)
PG 0048+092	5.711±0.010	33300±1700	0.447±0.027	-4.92±0.20	Charpinet et al. (2006b)
EC 20117-4014	5.856±0.008	34800±2000	0.540±0.040	-4.17±0.08	Randall et al. (2006b)
PG 0911+456	5.777±0.002	31940±220	0.390±0.010	-4.69±0.07	Randall et al. (2007)
BAL 090100001	5.383±0.004	28000±1200	0.432±0.015	-4.89±0.14	Van Grootel et al. (2008b)
PG 1336-018	5.739±0.002	32780±200	0.459±0.005	-4.54±0.07	Charpinet et al. (2008) → + rotation
	5.248	32300±300	0.707	-5.78	van Spaandonk et al. (2008)
PG 1605+072	5.217	32300±300	0.561	-6.22	
	5.226±0.004	32300±300	0.528±0.002	-5.88±0.04	Van Grootel (2008)
	5.276	32630±600	0.731	-2.83	Van Grootel et al. (2010a)
	5.278	32630±600	0.769	-2.71	
EC 09582-1137	5.788±0.004	34805±230	0.485±0.011	-4.39±0.10	Randall et al. (2009)
KPD 1943+4058	5.520±0.030	27730±270	0.496±0.002	-2.55±0.07	Van Grootel et al. (2010b)
KPD 0629-0016	5.450±0.034	26485±195	0.471±0.002	-2.42±0.07	Van Grootel et al. (2010c)
KIC02697388	5.489±0.033	25395±225	0.463±0.009	-2.30±0.05	Charpinet et al. (2011)
	5.499±0.049	25395±225	0.452±0.012	-2.35±0.05	

13 p-mode pulsators

3 g-mode pulsators

Adiabatic asteroseismology

PG1336-018 : a Rosetta stone for the seismology of p-mode sdB pulsators

Properties :

sdB + dM eclipsing close binary (Porb = 2.4244 h)

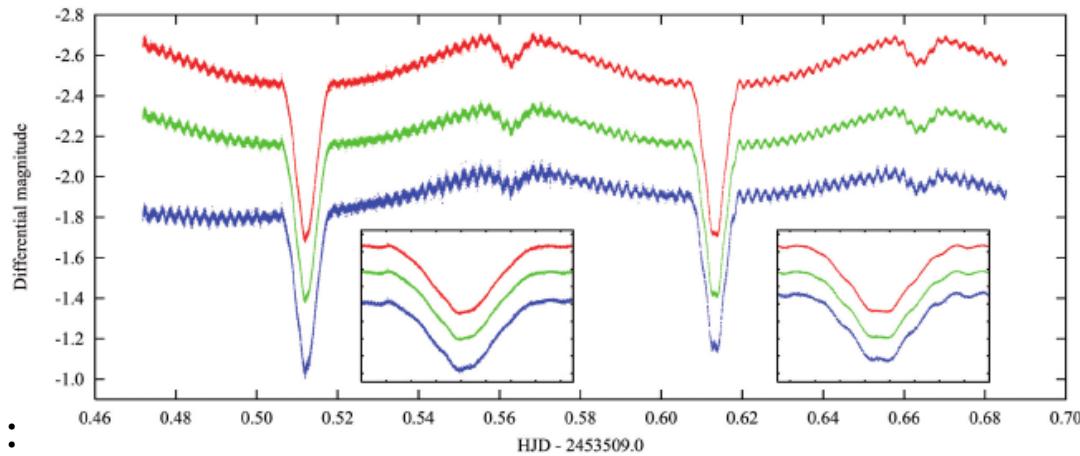
25 pulsation periods in the range 96 – 205 s
(Whole Earth Telescope campaign)

Complications :

Likely a fairly fast rotator phase-locked with
the orbital period of the close binary system :
Prot = Porb = 8727.78 seconds (2.4244 h)

Fairly large rotational splitting expected
(~ 114 μ Hz)

Effects of rotation had to be considered in the
optimisation procedure. First order solid-body
rotation assumed for the search



Ultracam/VLT data

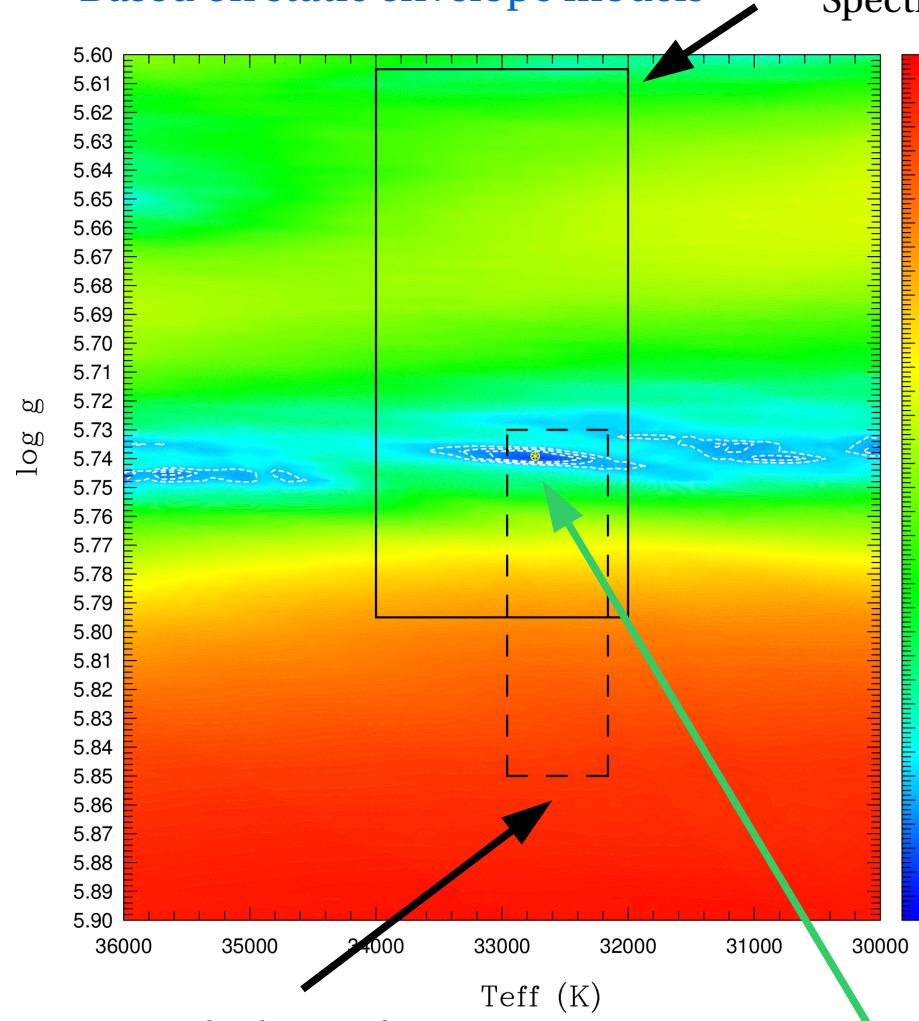
Figure from Vuckovic et al. 2007, A&A, 471, 605

$$\sigma_{klm} = \sigma_{kl0} - m \Omega (1 - C_{kl})$$

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PG1336-018 : a Rosetta stone for the seismology of p-mode sdB pulsators

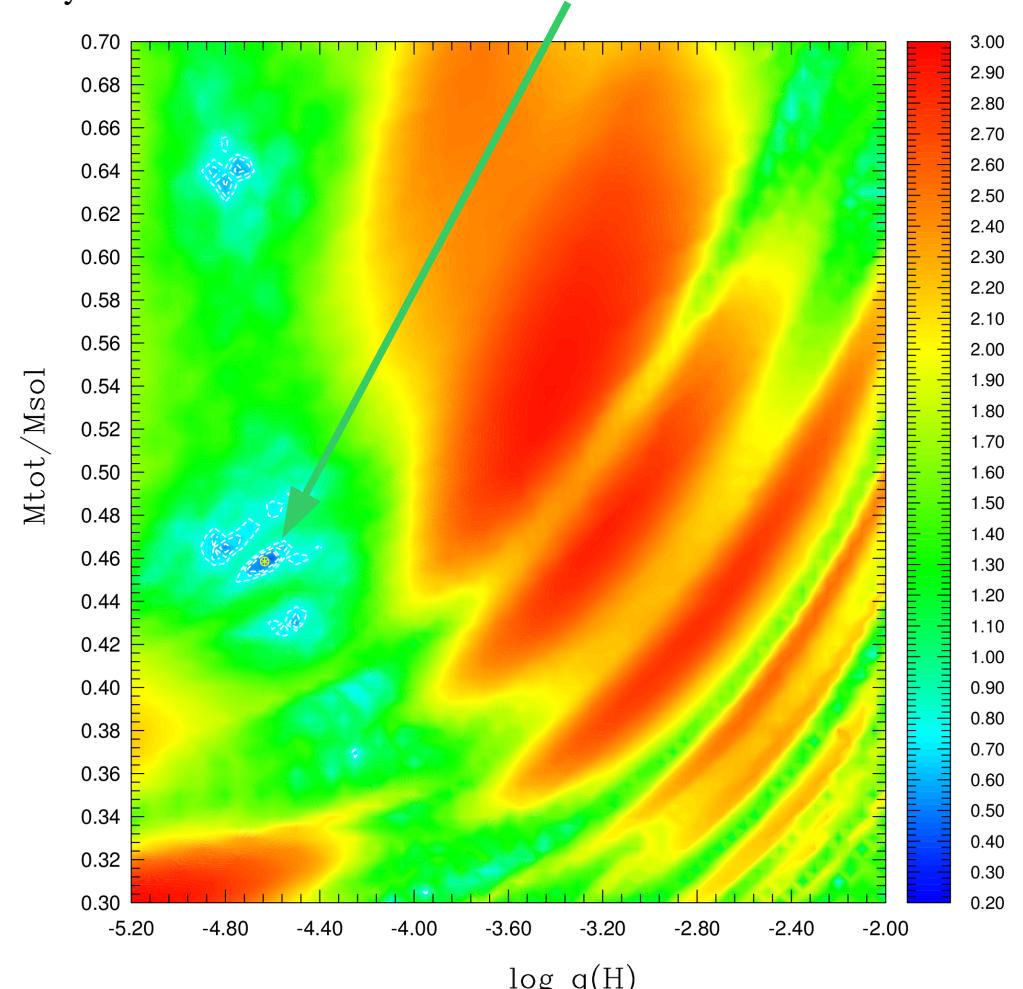
Based on static envelope models



Spectro : high-S/N, low resolution, NLTE

Spectro : Kilkenny et al. 2003

Optimal solution



Charpinet et al. 2008, A&A, 489, 377

Optimal solution

Adiabatic asteroseismology

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Charpinet et al. 2008, A&A, 489, 377

Table 1. Structural parameters of PG 1336-018 ($y = 13.450 \pm 0.093$; Wesemael et al. 1992) derived from asteroseismology and compared with the independent analysis of the orbital light curve by Vuckovic et al. 2007.

Quantity	Astroseismic analysis		Spectroscopy
	Estimated Value	BG9	Kilkenny et al. (1998)
T_{eff} (K)	$32,740 \pm 400$	$32,560 \pm 400$	$33,000 \pm 1000$
$\log g$	5.739 ± 0.002	5.79 ± 0.06	5.7 ± 0.1
M_*/M_{\odot}	0.459 ± 0.005		
$\log(M_{\text{env}}/M_*)$	-4.54 ± 0.07		
R/R_{\odot} (M_* , g)	0.151 ± 0.001		
L/L_{\odot} (T_{eff} , R)	23.3 ± 1.5		
M_V (g , T_{eff} , M_*)	4.49 ± 0.04		
$d(V, M_V)$ (pc)	619 ± 38		
P_{rot} (h) [†]	2.42438		
V_{eq} (P_{rot} , R) (km/s)	75.9 ± 0.6		

[†] assumed identical to the orbital period (value taken from the ephemeris of K

[‡] deemed unlikely and thus rejected by Vuckovic et al. (2007) due to the high

Adiabatic asteroseismology

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M_*/M_{\odot}	0.459 ± 0.005			0.389 ± 0.005	0.466 ± 0.006	0.530 ± 0.007
$\log(M_{\text{env}}/M_*)$	-4.54 ± 0.07					
$R/R_{\odot} (M_*, g)$	0.151 ± 0.001			0.14 ± 0.01	0.15 ± 0.01	0.15 ± 0.01
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Independent values from eclipsing binary
lightcurve modeling

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Adiabatic asteroseismology

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Convergence of 3 techniques on the derived values of $\log g$, M , and R

Adiabatic asteroseismology

PG1336-018 : a Rosetta stone for the seismology of p-mode sdB pulsators

Internal rotation

Checking solid-body rotation with a two-zone model :

$P_{\text{surf}} = P_{\text{orb}} = 8727.78 \text{ s}$ (sync at the surface)

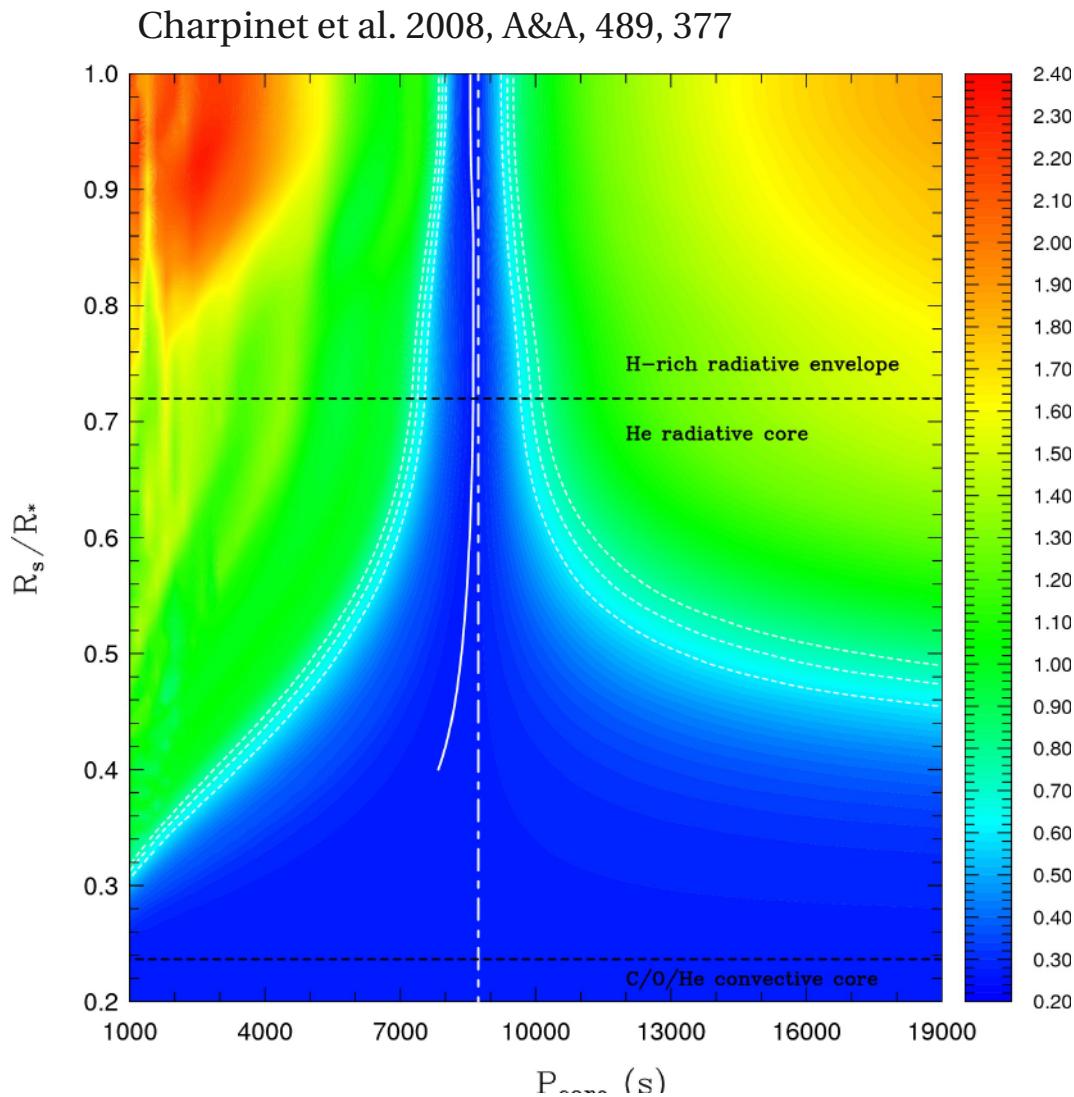
P_{core} varied between 1000 and 19000 s

R_s/R^* , the transition between the two zones, is varied between 0.2 and 1.0

A vertical valley of best fit solutions indicating solid internal rotation

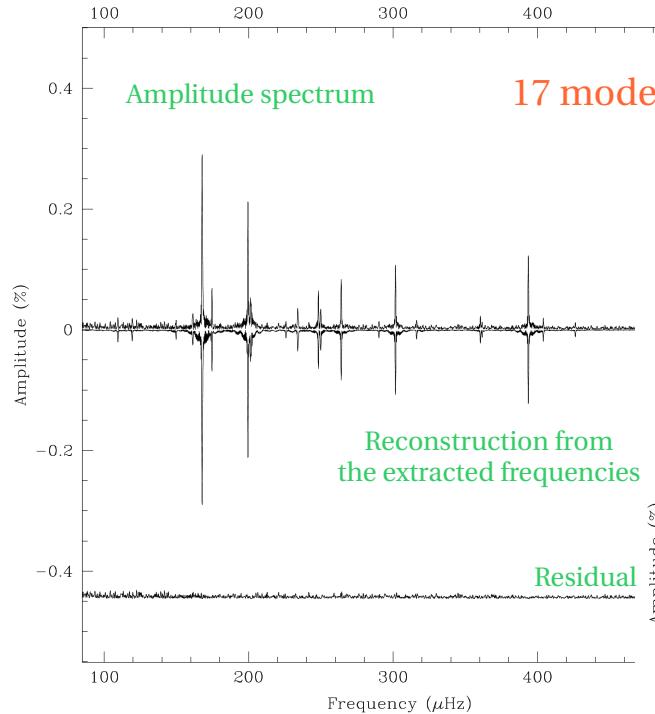
~ 50 % of the star (in radius) is probed

Similar results for Feige 48
(Van Grootel et al. 2008)

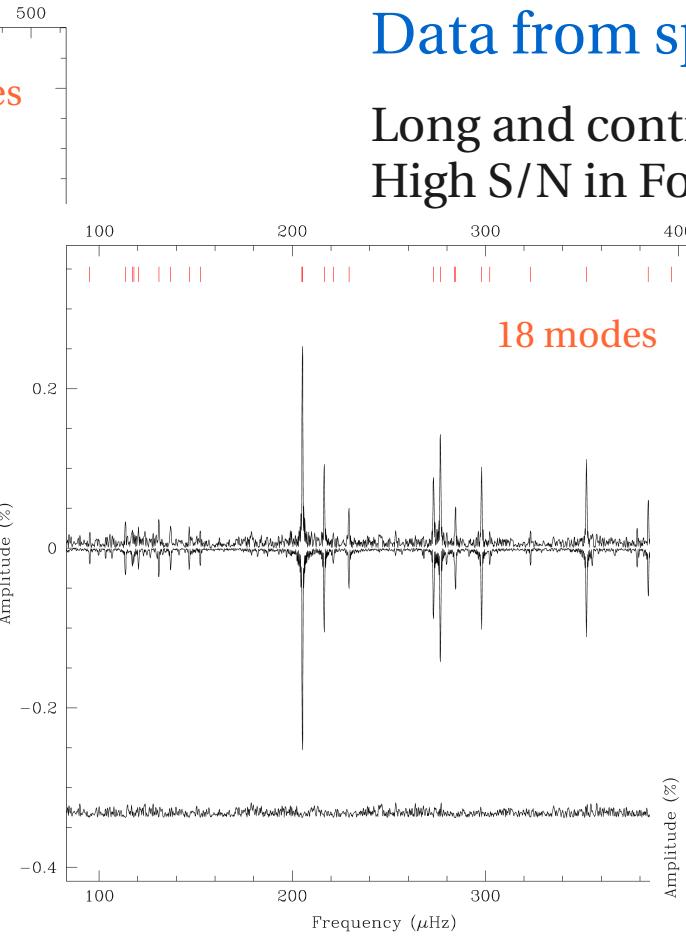


Adiabatic asteroseismology

Space observations: a decisive step for g-mode seismology of sdB stars

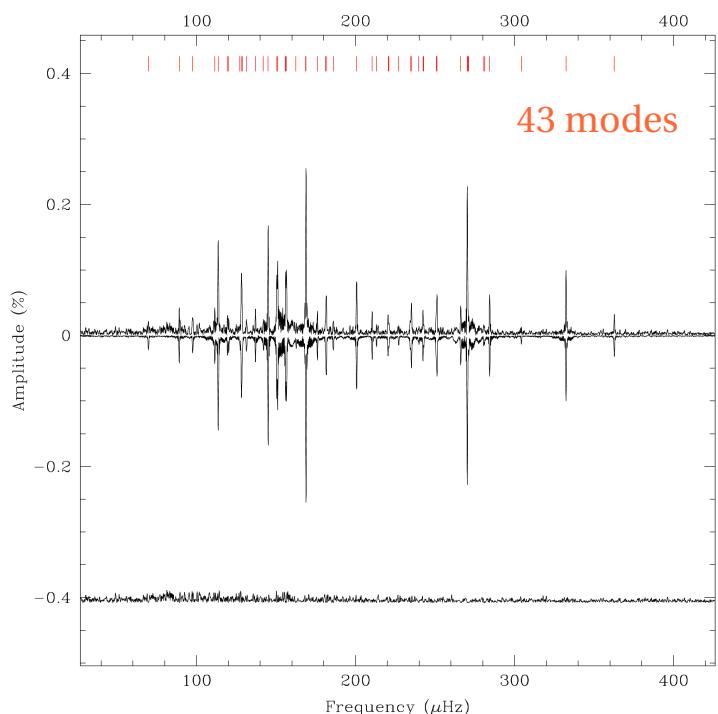


KPD0629-0016 (CoRoT – 24d)
(Charpinet et al. 2010)



KPD1943+4058 (Kepler – 27d)
(Van Grootel et al. 2010)

KIC02697388 (Kepler – 27d)
(Charpinet et al. 2011)



→ Clear and unambiguous detection of many g-modes
with reliable period (frequency) determinations

Adiabatic asteroseismology

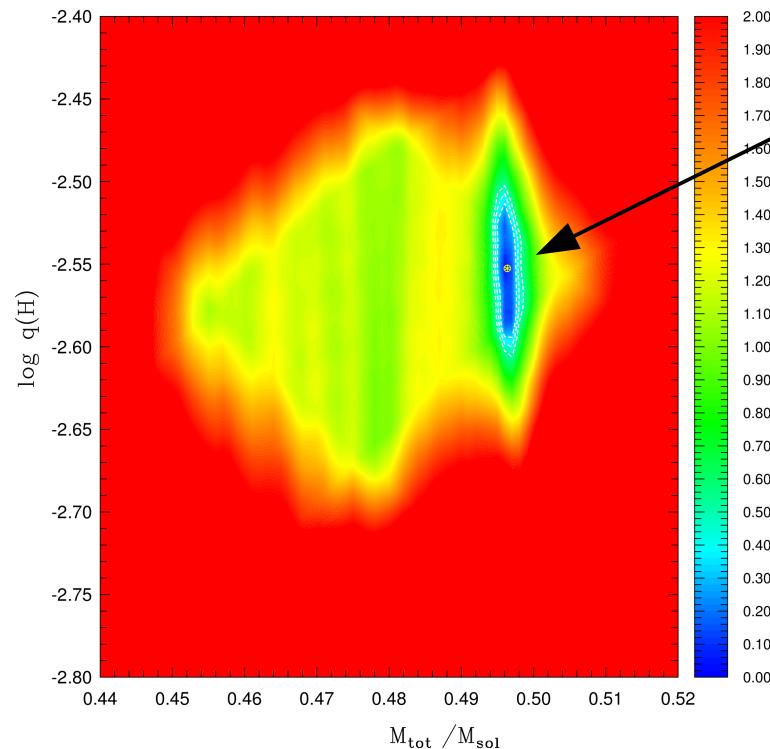
Probing the core structure with g-modes

KPD 1943+4058 (*Kepler*)

Van Grootel et al. 2010, ApJ, 718, L97

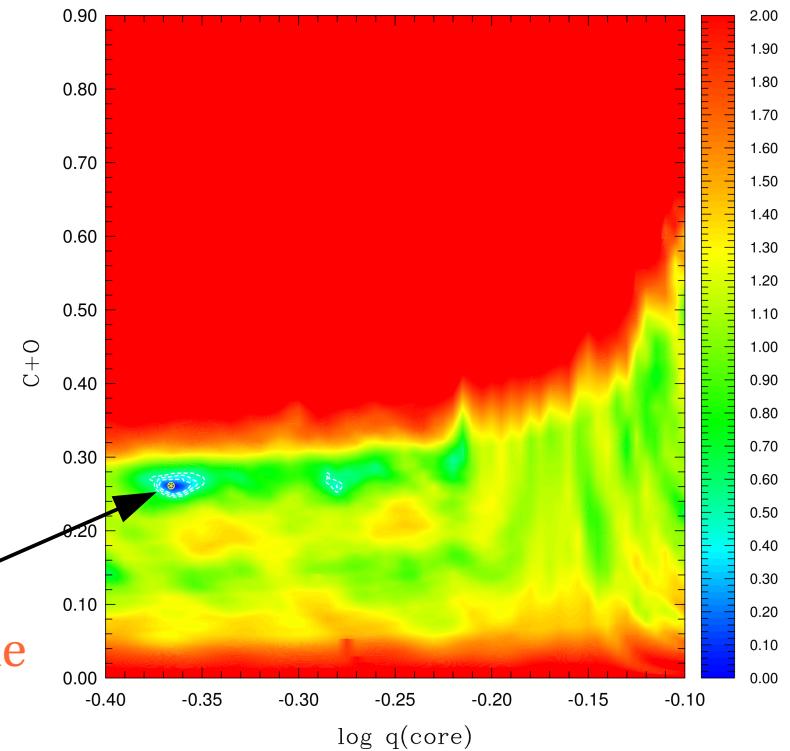
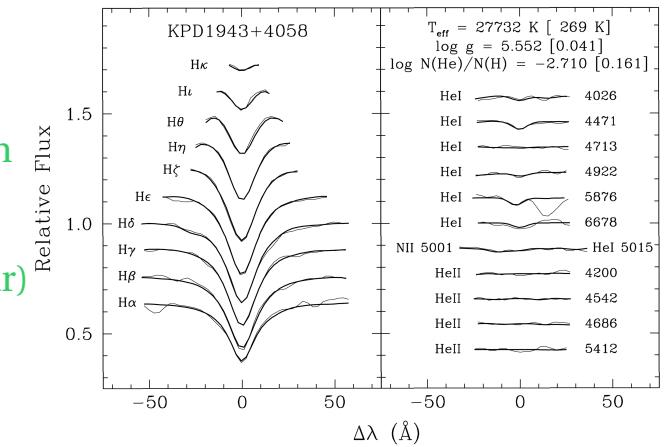
18 frequencies simultaneously fitted to
 $\ell = 1, 2$ (4) g-modes + 4D χ^2 -minimisation
 using complete static models:

M_* , M_{env} , M_{core} , $X_{\text{core}}(\text{C+O})$



Optimal solution:
 $M_{\text{env}} - M_*$ plane

$\log g - T_{\text{eff}}$ constraints
 from spectroscopy &
 NLTE atmospheres with
 Blanchette et al. (2008)
 mixture : C, O, Si (1/10
 solar) and N, S, Fe (solar)



Optimal solution:
 $M_{\text{core}} - X_{\text{core}}(\text{C+O})$ plane

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Probing the core structure with g-modes

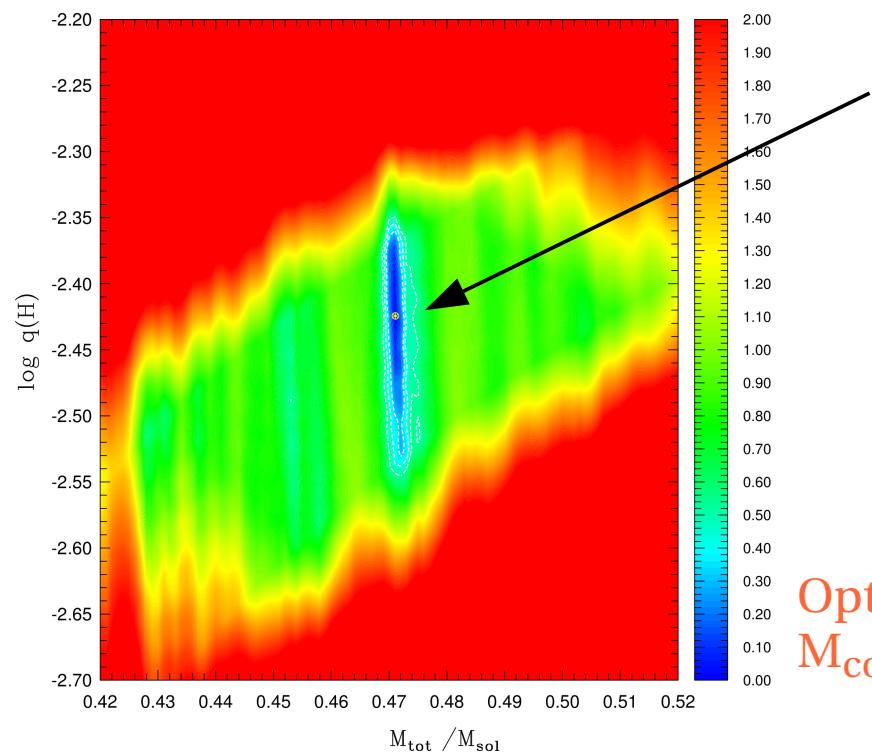
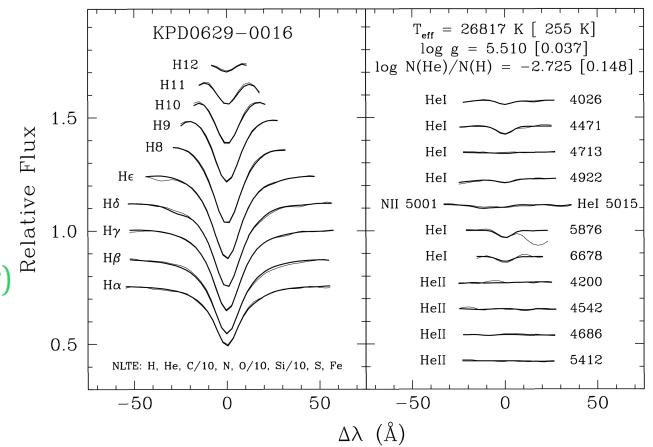
KPD 0629-0016 (CoRoT)

Van Grootel et al. 2010, A&A, 524, 63

17 frequencies simultaneously fitted to
 $\ell = 1, 2$ (4) g-modes + 4D χ^2 -minimisation:

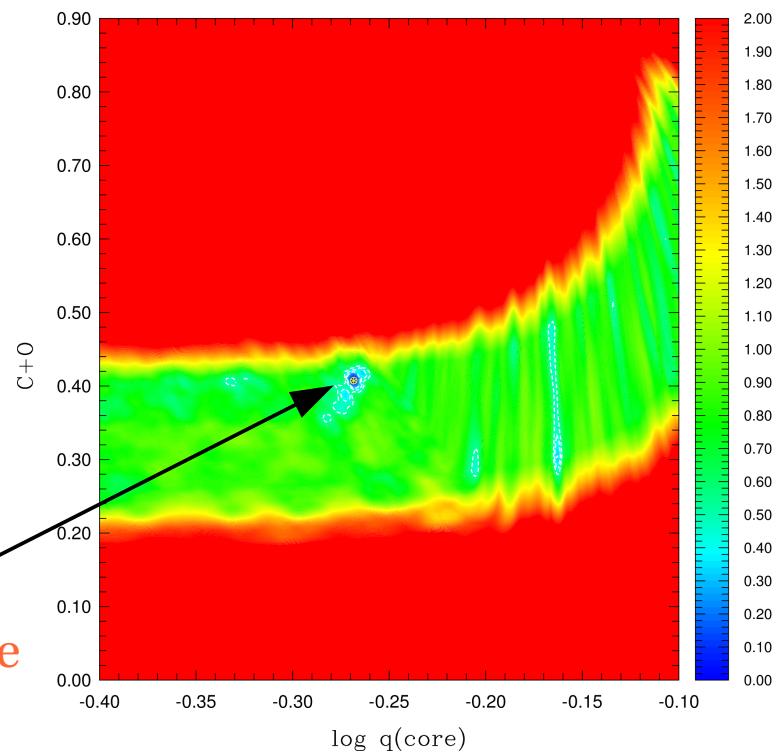
M_* , M_{env} , M_{core} , $X_{\text{core}}(\text{C+O})$

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Optimal solution:
 $M_{\text{env}} - M_*$ plane

Optimal solution:
 $M_{\text{core}} - X_{\text{core}}(\text{C+O})$ plane



Adiabatic asteroseismology

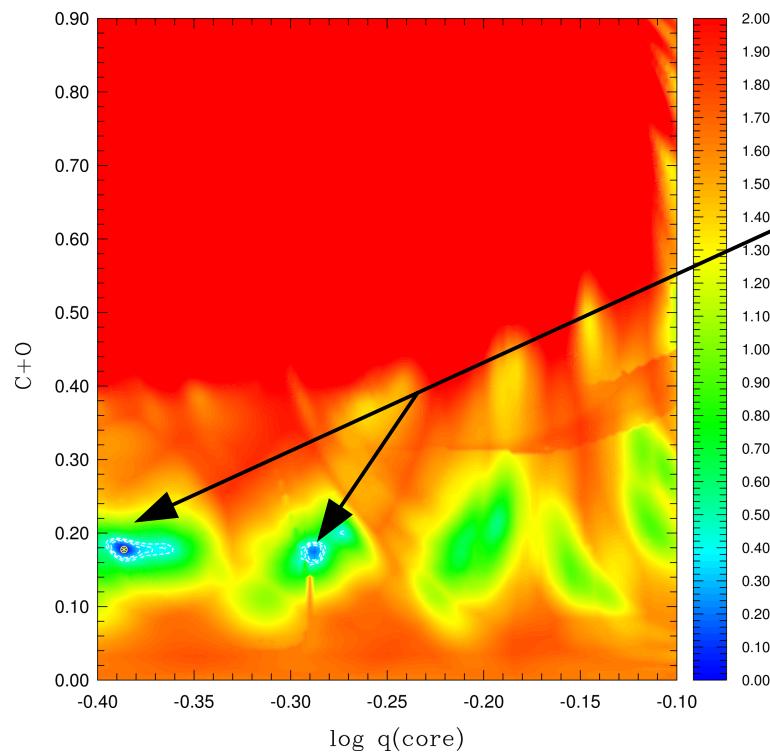
Probing the core structure with g-modes

KIC 02697388 (*Kepler*)

Charpinet et al. 2011, A&A, 530, 3

43 frequencies simultaneously fitted to
 $\ell = 1, 2$ (4) g-modes + 4D χ^2 -minimisation:

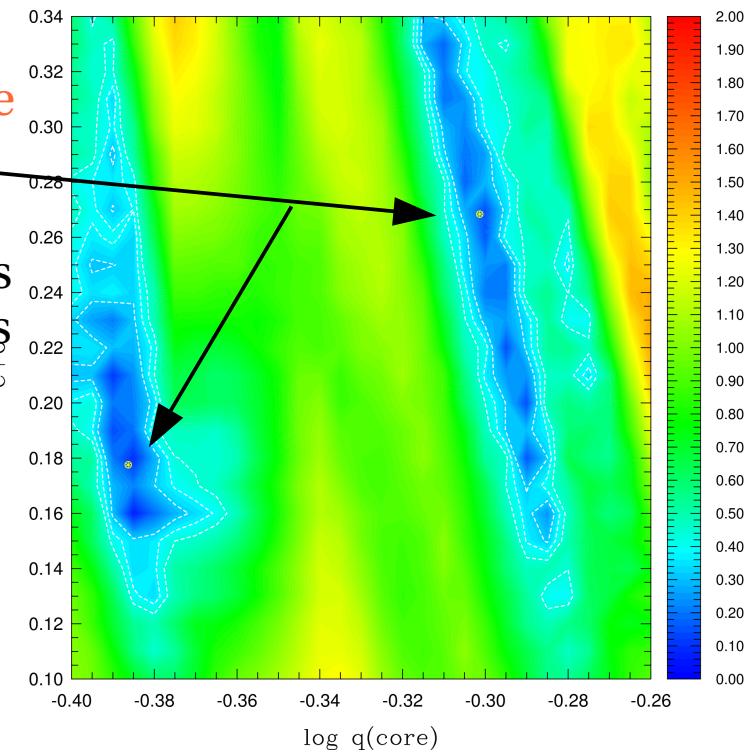
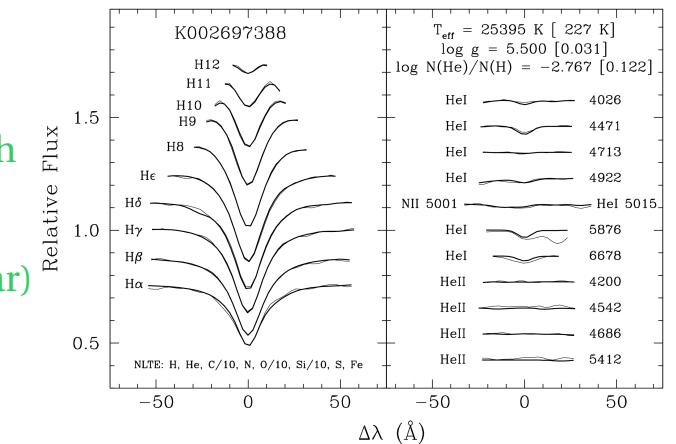
M_* , M_{env} , M_{core} , $X_{\text{core}}(\text{C+O})$



Optimal solutions:
 $M_{\text{core}} - X_{\text{core}}(\text{C+O})$ plane

Two families of solutions
undistinguishable at this
stage

4D merit function
projected on the
 $M_{\text{core}} - X_{\text{core}}(\text{C+O})$
plane



Adiabatic asteroseismology

Probing the core structure with g-modes

Main outcome : structural and core parameters

Table 3

Structural and Core Parameters Inferred for KPD 1943+4058 (All the Quoted Uncertainties are the Formal Fitting Ones)

Quantity	Estimated Value
T_{eff} (K)	$27730 \pm 270^{\text{a}}$
	$28050 \pm 470^{\text{b}}$
$\log g$	$5.552 \pm 0.041^{\text{a}}$
	$5.52 \pm 0.03^{\text{b}}$
M_*/M_{\odot}	0.496 ± 0.002
$\log(M_{\text{env}}/M_*)$	-2.55 ± 0.07
$\log(1 - M_{\text{cc}}/M_*)$	-0.37 ± 0.01
M_{cc}/M_{\odot}	0.28 ± 0.01
$X_{\text{core}}(\text{C+O})$	0.261 ± 0.008
Age (Myr)	$18.4 \pm 1.0^{\text{c}}$
R/R_{\odot} (M_*, g)	0.203 ± 0.007
L/L_{\odot} (T_{eff}, R)	22.9 ± 3.1
M_V (g, T_{eff}, M_*)	4.21 ± 0.11
$E(B - V)$	0.094 ± 0.017
$d(V, M_V)$ (pc)	1180 ± 95

Notes.

^a From spectroscopy.

^b From asteroseismology.

^c Since zero-age EHB.

Table 2. Structural and core parameters inferred for KPD 0629–0016

Quantity	Estimated Value
T_{eff} (K)	$26484 \pm 196^{\text{(1)}}$
	$26290 \pm 530^{\text{(2)}}$
$\log g$	$5.473 \pm 0.027^{\text{(1)}}$
	$5.450 \pm 0.034^{\text{(2)}}$
M_*/M_{\odot}	0.471 ± 0.002
$\log(M_{\text{env}}/M_*)$	-2.42 ± 0.07
$\log(1 - M_{\text{core}}/M_*)$	-0.27 ± 0.01
$M_{\text{core}}/M_{\odot}$	0.22 ± 0.01
$X_{\text{core}}(\text{C+O})$	0.41 ± 0.01
Age (Myr)	$42.6 \pm 1.0^{\text{(3)}}$
R/R_{\odot} (M_*, g)	0.214 ± 0.009
L/L_{\odot} (T_{eff}, R)	19.7 ± 3.2
M_V (g, T_{eff}, M_*)	4.23 ± 0.13
$E(B - V)$	0.128 ± 0.023
$d(V, M_V)$ (pc)	1190 ± 115

⁽¹⁾From spectroscopy ; ⁽²⁾From asteroseismology

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Table 6. Structural and core parameters inferred for KIC02697388 for the two solutions

Quantity	Estimated values	
	solution 1	solution 2
T_{eff} (K)	$25395 \pm 227^{\dagger}$	$\ddagger 25622^{+490}_{-350} \quad \ddagger 25555^{+480}_{-560}$
$\log g$	$5.500 \pm 0.031^{\dagger}$	$\ddagger 5.489^{+0.029}_{-0.036} \quad \ddagger 5.499^{+0.047}_{-0.051}$
M_*/M_{\odot}	$0.463^{+0.010}_{-0.008}$	$0.452^{+0.018}_{-0.005}$
$\log(M_{\text{env}}/M_*)$	$-2.30^{+0.03}_{-0.06}$	$-2.35^{+0.07}_{-0.02}$
$\log(1 - M_{\text{core}}/M_*)$	$-0.39^{+0.01}_{-0.02}$	$-0.30^{+0.03}_{-0.01}$
$M_{\text{core}}/M_{\odot}$	$0.274^{+0.008}_{-0.010}$	$0.225^{+0.011}_{-0.016}$
$X_{\text{core}}(\text{C+O})$	$0.18^{+0.06}_{-0.03}$	$0.27^{+0.07}_{-0.12}$
Age (Myr)*	40.8 ± 1.0	53.9 ± 1.0
R/R_{\odot} (M_*, g)	$0.203^{+0.009}_{-0.007}$	$0.198^{+0.013}_{-0.011}$
L/L_{\odot} (T_{eff}, R)	$16.0^{+1.9}_{-1.4}$	$15.1^{+2.3}_{-2.0}$
M_V (g, T_{eff}, M_*)	4.394 ± 0.133	4.450 ± 0.187
V		15.234 ± 0.021
$B - V$		-0.164 ± 0.030
$E(B - V)$	0.057 ± 0.030	0.056 ± 0.030
A_V	0.182 ± 0.096	0.179 ± 0.096
$d(V, M_V, A_V)$ (pc)	1355 ± 144	1321 ± 173

[‡] from asteroseismology

[†] from spectroscopy

* from the zero age extreme horizontal branch (ZAEHB)

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Core parameters: Mixed core more extended (by ~ 50% in mass) than expected from standard sdB evolutionary models ($M_{\text{c}} \sim 0.15 \text{ M}_{\odot}$) → additional sources of mixing

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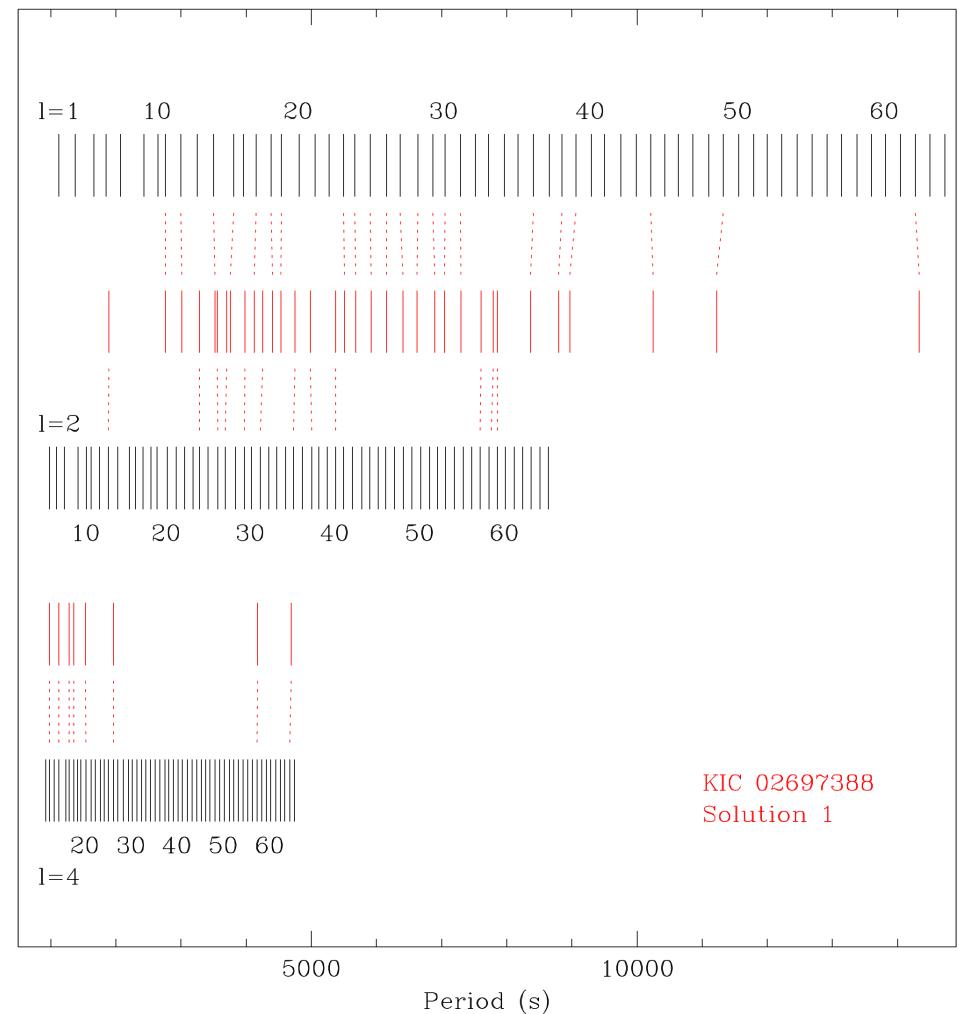
Mode identification

Identification of the observed modes

Periods associated to $\ell = 1, 2, and 4 g-modes$

Radial order $k \sim 10 - 65$, up to the g-mode cutoff period (which is ℓ -dependent)

Seismic solution for KIC02697388 (Charpinet et al. 2011)



Adiabatic asteroseismology

How well do the best-fit models fit?

KPD1943+4058

(Van Grootel et al. 2010a)

18 modes

$$\overline{|\Delta P/P|} \sim 0.22 \%$$

$$\overline{|\Delta P|} \sim 7.8 \text{ s}$$

$$\overline{|\Delta v|} \sim 0.697 \mu\text{Hz}$$

KPD0629-0016

(Van Grootel et al. 2010b)

17 modes

$$\overline{|\Delta P/P|} \sim 0.23 \%$$

$$\overline{|\Delta P|} \sim 11.8 \text{ s}$$

$$\overline{|\Delta v|} \sim 0.530 \mu\text{Hz}$$

KIC02697388

(Charpinet et al. 2011)

43 modes

$$\overline{|\Delta P/P|} \sim 0.35 \%$$

$$\overline{|\Delta P|} \sim 20.9 \text{ s}$$

$$\overline{|\Delta v|} \sim 0.811 \mu\text{Hz}$$

Formal resolution : $0.43 \mu\text{Hz}$ (27 day runs) and $0.042 \mu\text{Hz}$ (after 9 month)

Accuracy on measured frequencies : $\sim 1/10$ of the formal resolution (resolved modes)

- Models provide « close » fits to the observations but are not yet able to match the frequencies simultaneously at the precision of the observations.
- The next challenge is to identify the origin(s) of these remaining discrepancies and correct the model's structure accordingly

Summary & Prospects

Precise determination of the stellar main parameters

- An essential step for detailed investigations of individual objects
- Important for understanding the evolution history of sdB stars

Probing the internal stratification

- Mass of the H-rich envelope (remnant of the former red giant envelope)
- Extent of the internal mixed core (convection + extra mixing)
- Detecting He-flash signatures?

Internal rotation of sdB stars (not really discussed here)

- In binaries, constraints on tidal synchronization
- Core rotation from g-modes (Kepler data should reveal more about this soon)
They seem to be very slow rotating compact stars!

Nonadiabatic asteroseismology (not really discussed here):

- Radiative levitation + competing mixing (e.g., thermohaline conv.)
- Atomic physics (opacities): Fe, Ni