Stellar pulsations as convection probes

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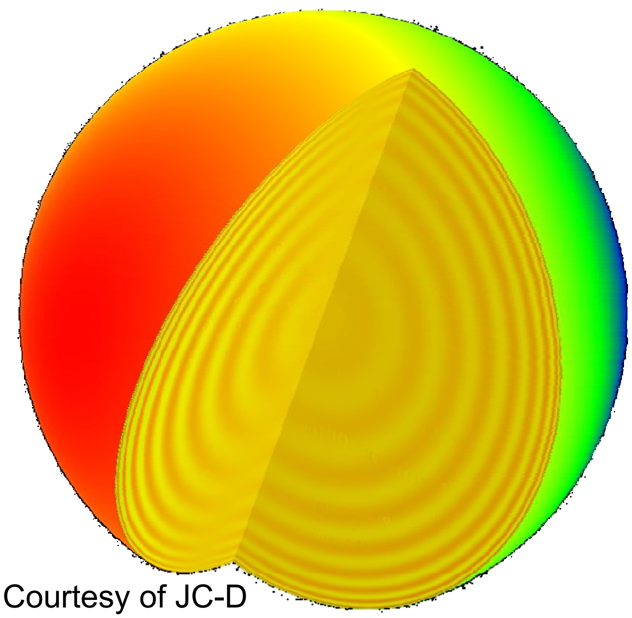
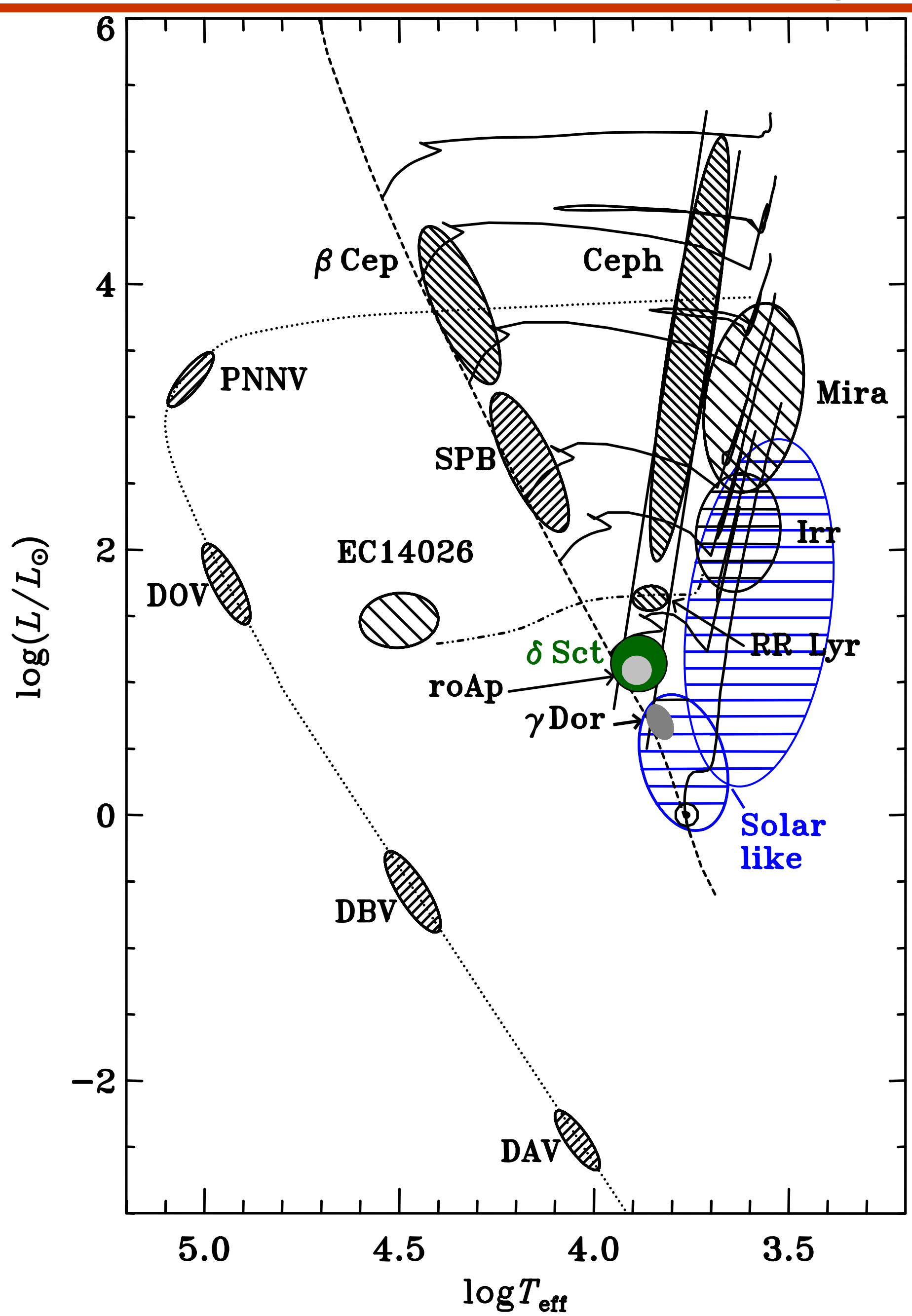


Courtesy of JC-D

The impact of asteroseismology across stellar astrophysics
Santa Barbara – October 2011

Pulsating H-R diagram



δ Scuti stars

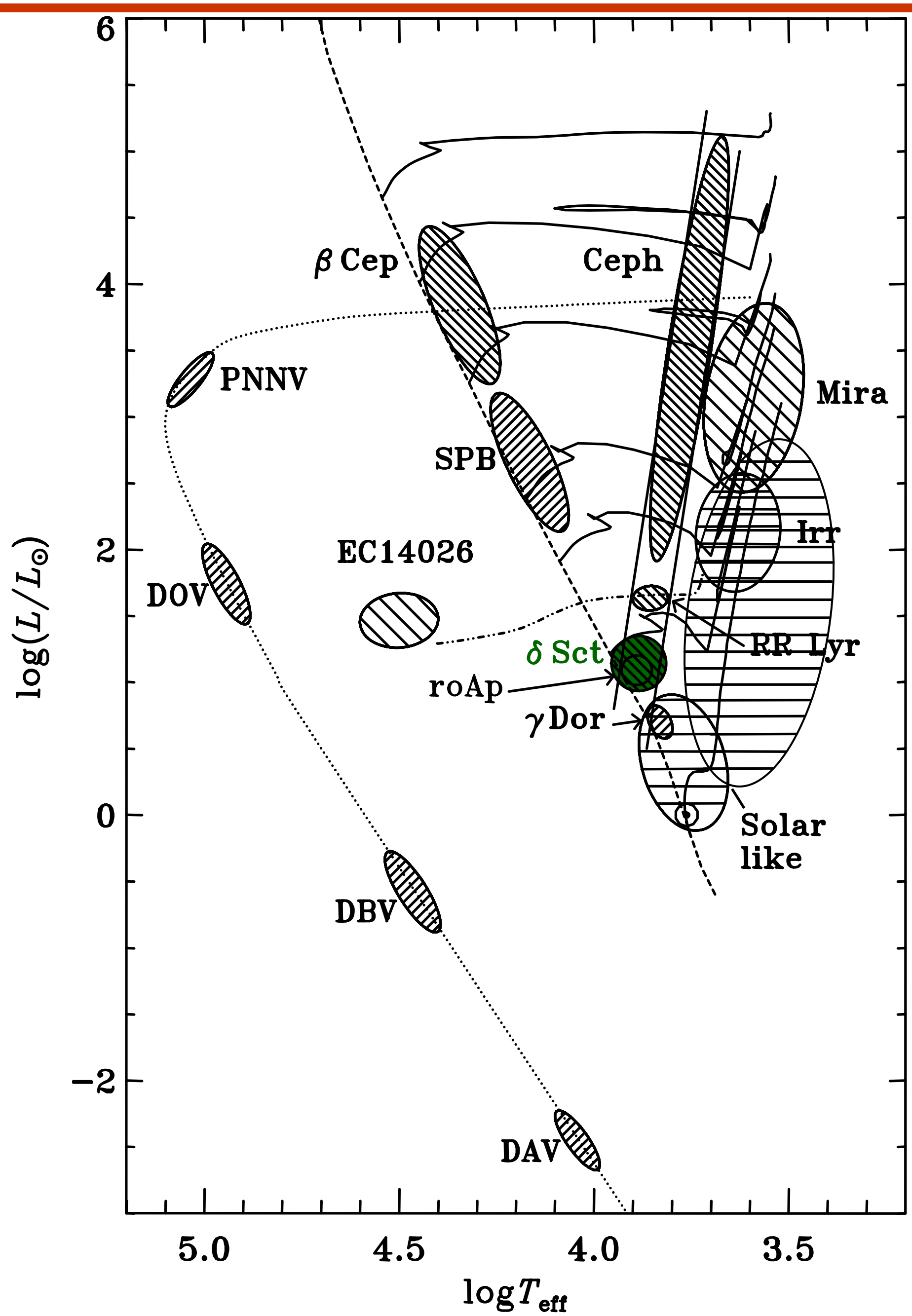
- location of cool edge of IS
- acoustic radiation

Solar-like pulsators

- surface effects
- amplitudes

Courtesy of Christensen-Dalsgaard

δ Scuti stars



- Central or shell hydrogen burning phase
- $1.5\,\mathrm{M}_\odot < M < 2.5\,\mathrm{M}_\odot$
- $0.6 < \log L/\mathrm{L}_\odot < 2.0$
- Multiperiodic behaviour (e.g. FG Vir: 79 modes)
- Amplitudes from 10^{-3} ⟶ 0.1 mag
- low-order p modes
- Periods between 18 min and 8 h
- Driven by the κ mechanism in the HeII ionization zone

δ Scuti stars

Overview of selected **time-dependent convection models** (applications by)

Unno (1967, 89) ➞ Gabriel (1998) ➞ Grigahcéne et al. (2004) : Dupret; Théado

Nonlinear mixing-length equations for a **Boussinesq fluid**
Infinite lifetime of fluid elements

Gough (1965, 77) ➞ Balmforth (1992) : Baker; Balmforth; Cunha; Gough; GH

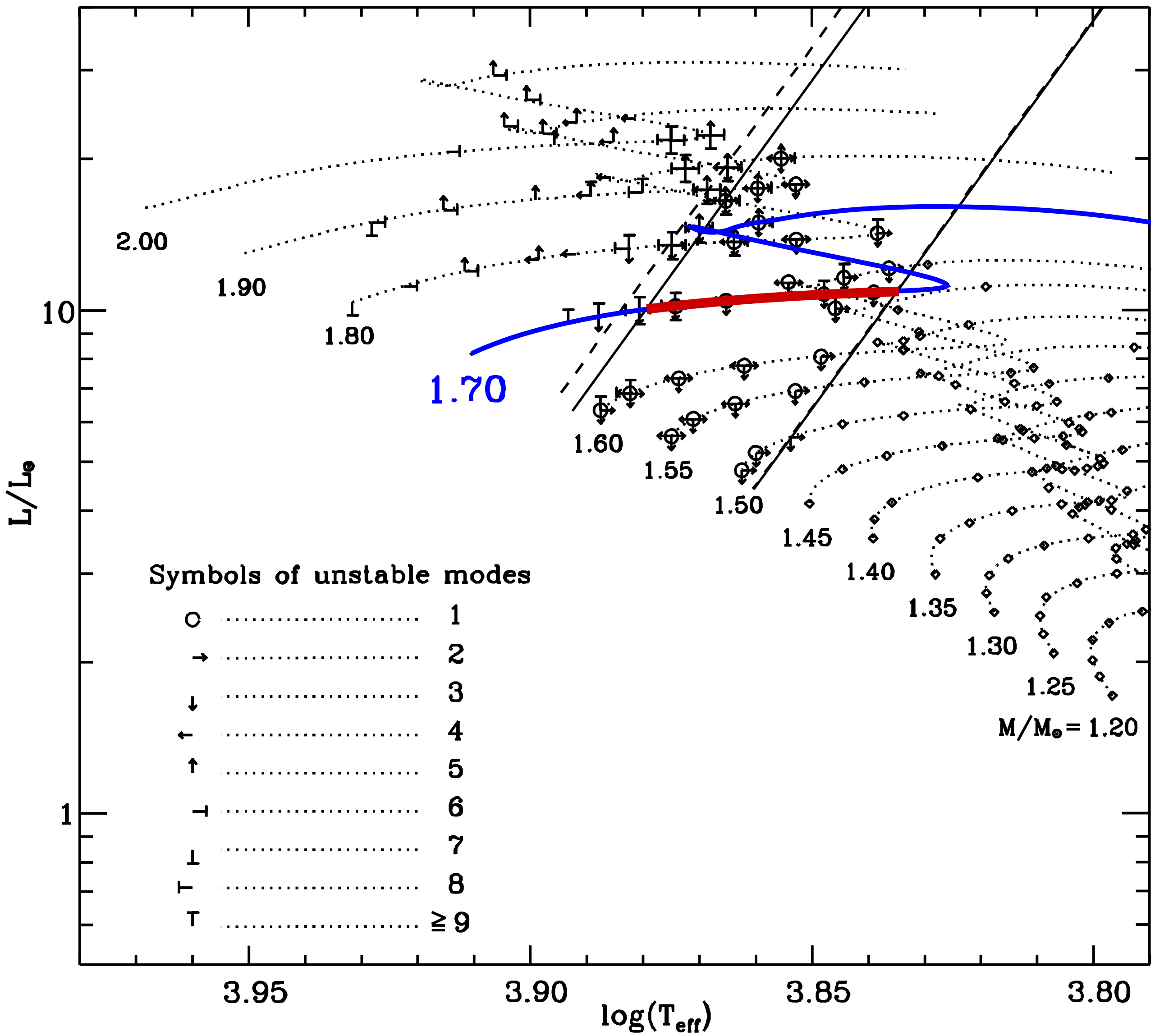
Linearized mixing-length equations for a **Boussinesq fluid**
Finite lifetime of fluid elements (linear growth rates)

Xiong (1977, 1989) : Cheng; Deng; Xiong

Reynold's transport equations for a **Boussinesq fluid**
Third-order moments approximated with diffusion-like expressions
using parametrized length scales (closure coefficients)

$\longrightarrow$ **estimates of turbulent flux perturbations:** $\delta F_{\rm c}$; $\delta p_{\rm t}$

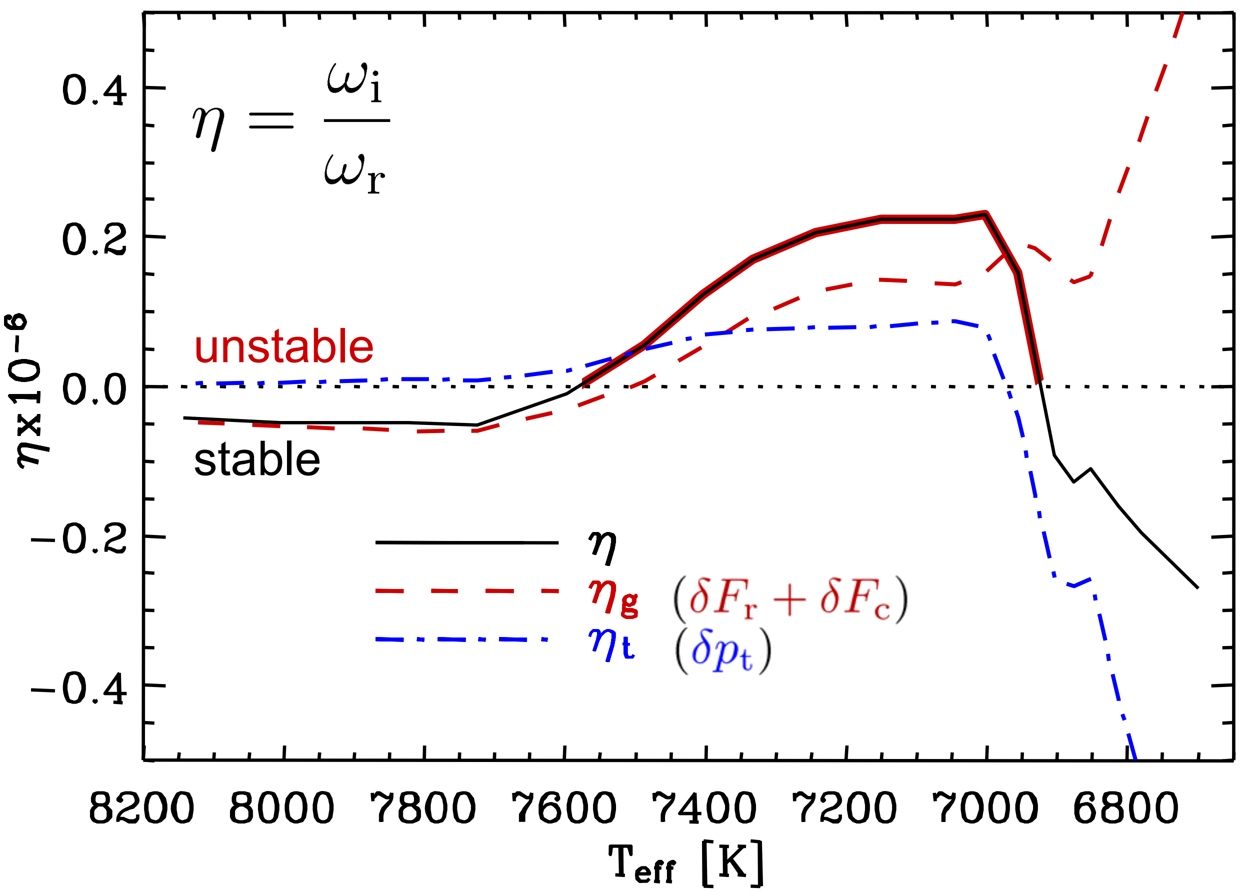
δ Scuti stars



GH et al. (1999)

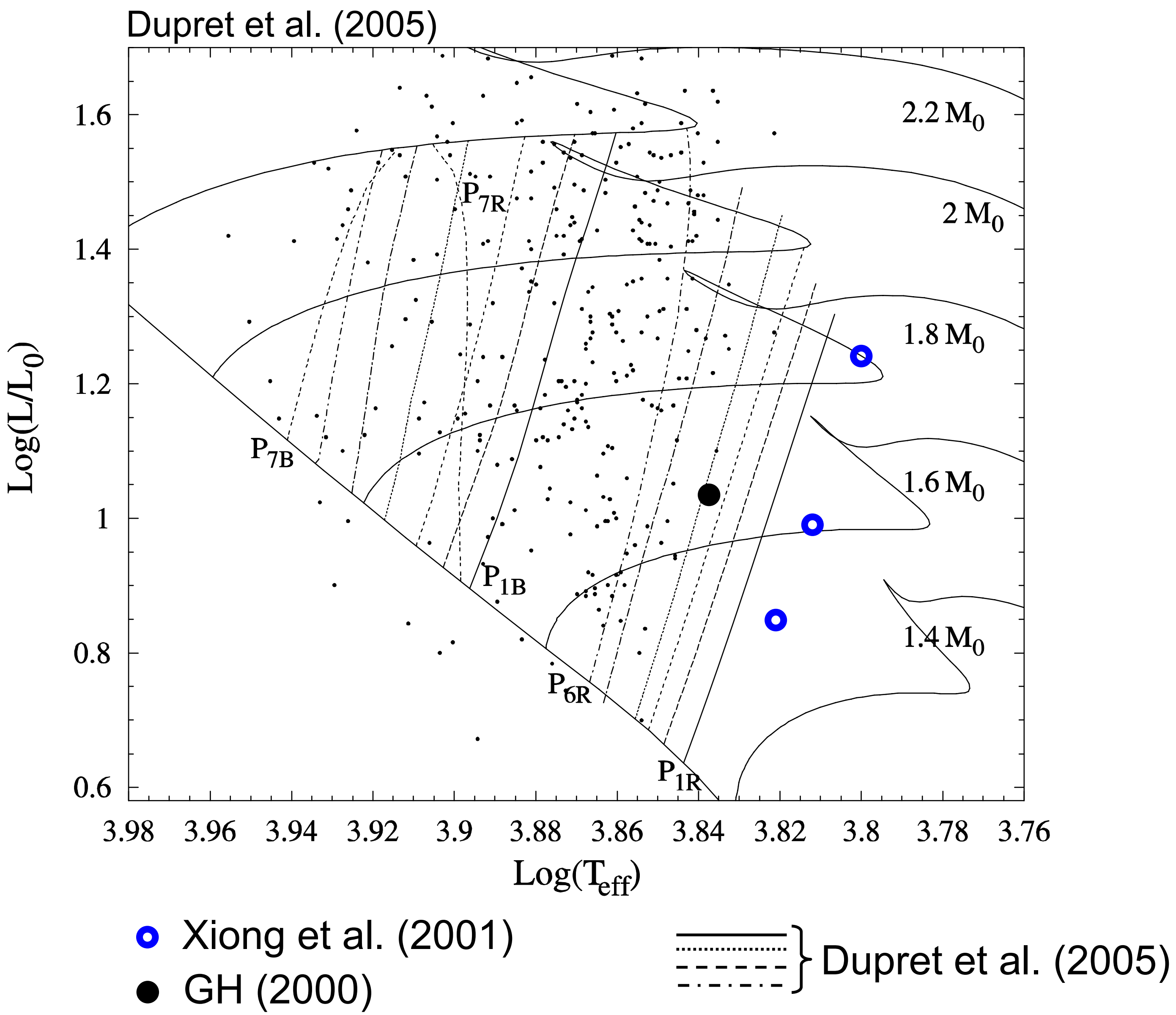
δ Scuti stars

1.7 M_ δ Scuti model evolving during H-core burning phase



GH (2000)

δ Scuti stars

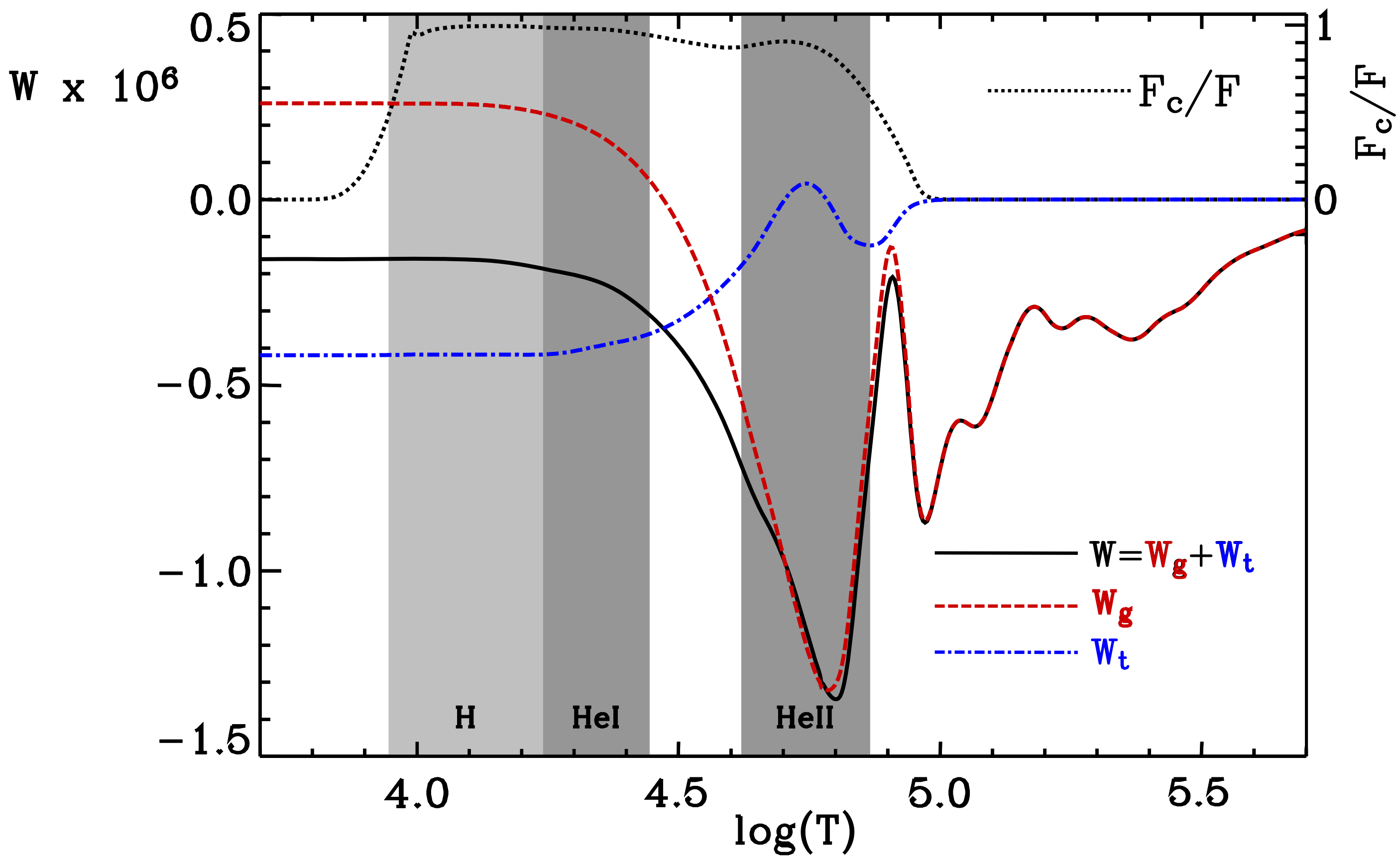


δ Scuti stars

Accumulated work integral

$$W_g = -\mathrm{Im}\left[\int_M \delta\rho^\star \delta p_g \, \rho^{-2} \, \mathrm{d}m\right]$$

$$W_t \simeq -\mathrm{Im}\left[\int_M \delta\rho^\star \delta p_t \, \rho^{-2} \, \mathrm{d}m\right]$$

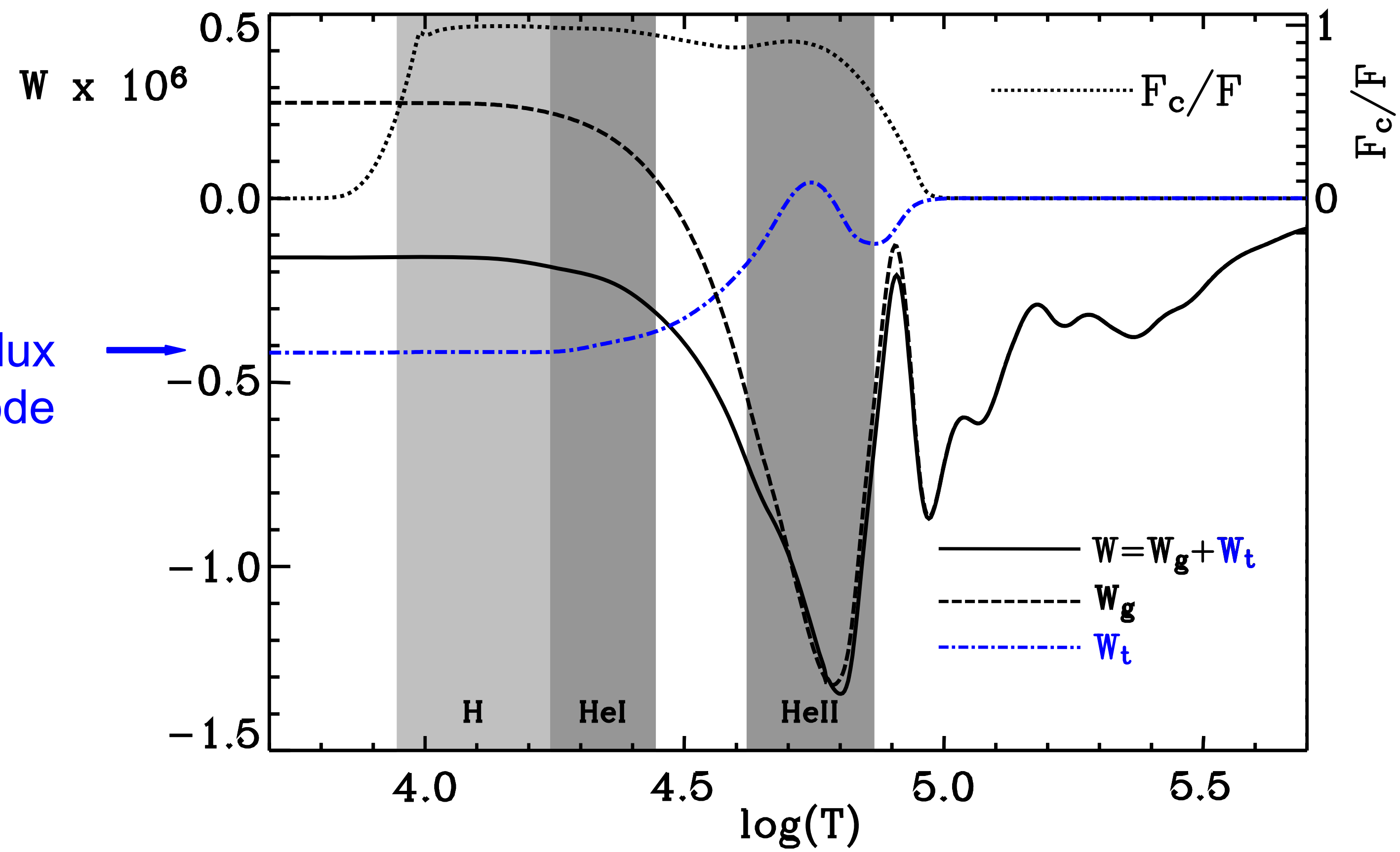


GH (2000)

δ Scuti stars

Accumulated work integral

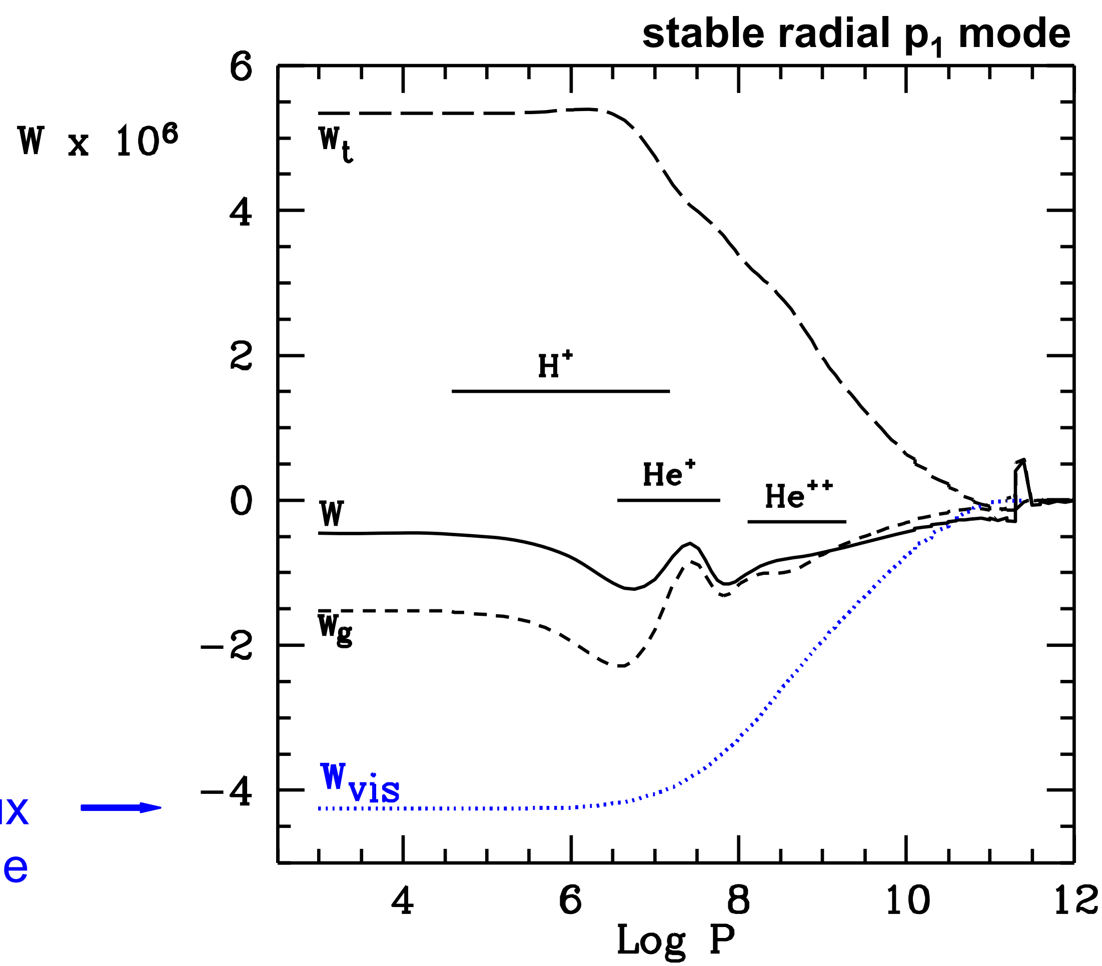
momentum flux
stabilizes mode



GH (2000)

δ Scuti stars

Accumulated work integral

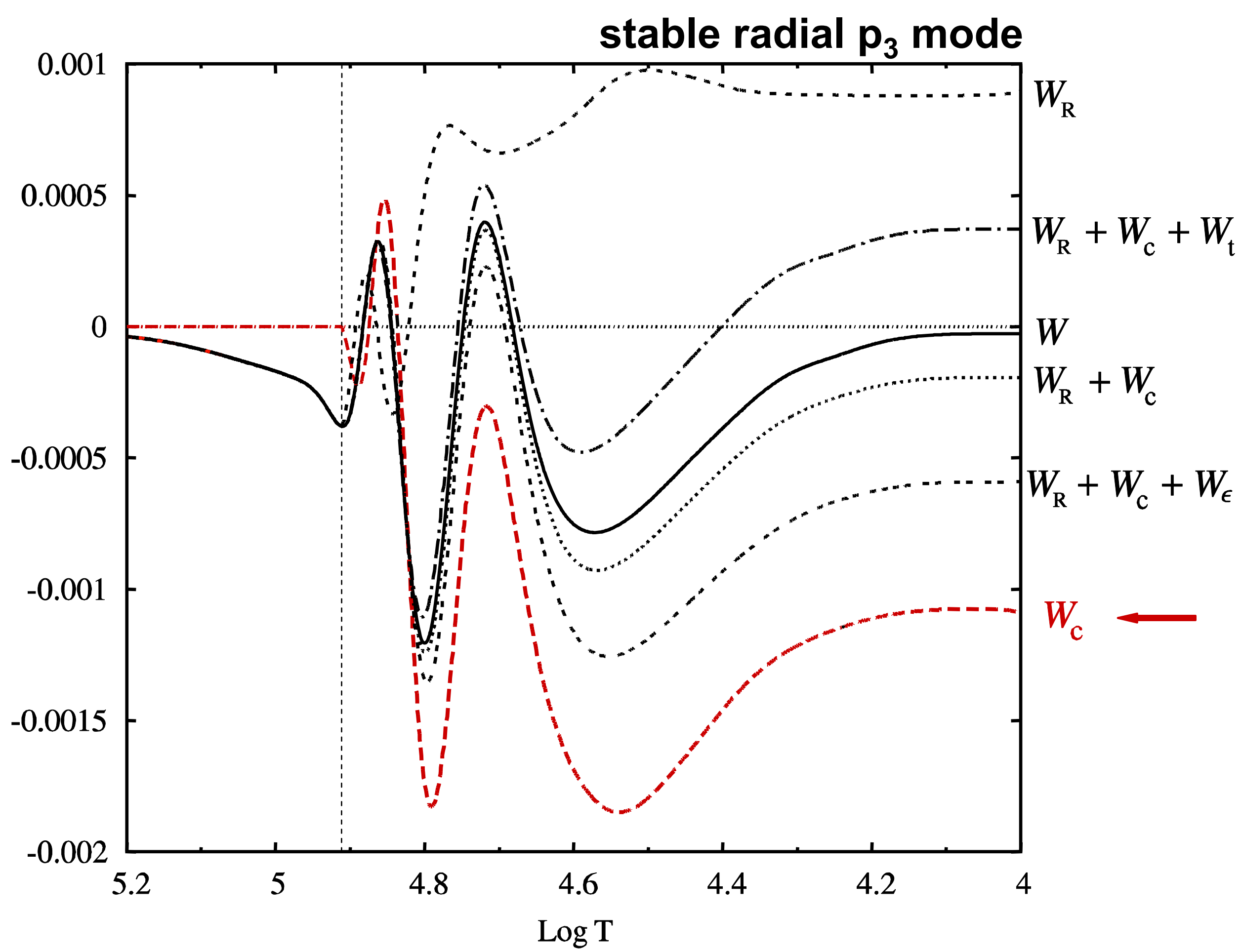


viscous flux stabilizes mode

Xiong & Deng (2001, 2007)

δ Scuti stars

Accumulated work integral



convective heat flux stabilizes mode

Dupret et al. (2005)

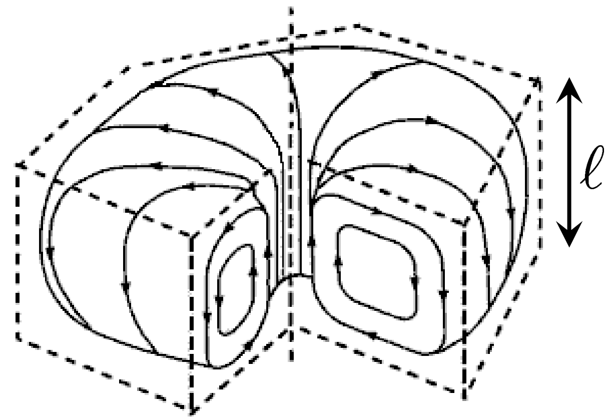
δ Scuti stars

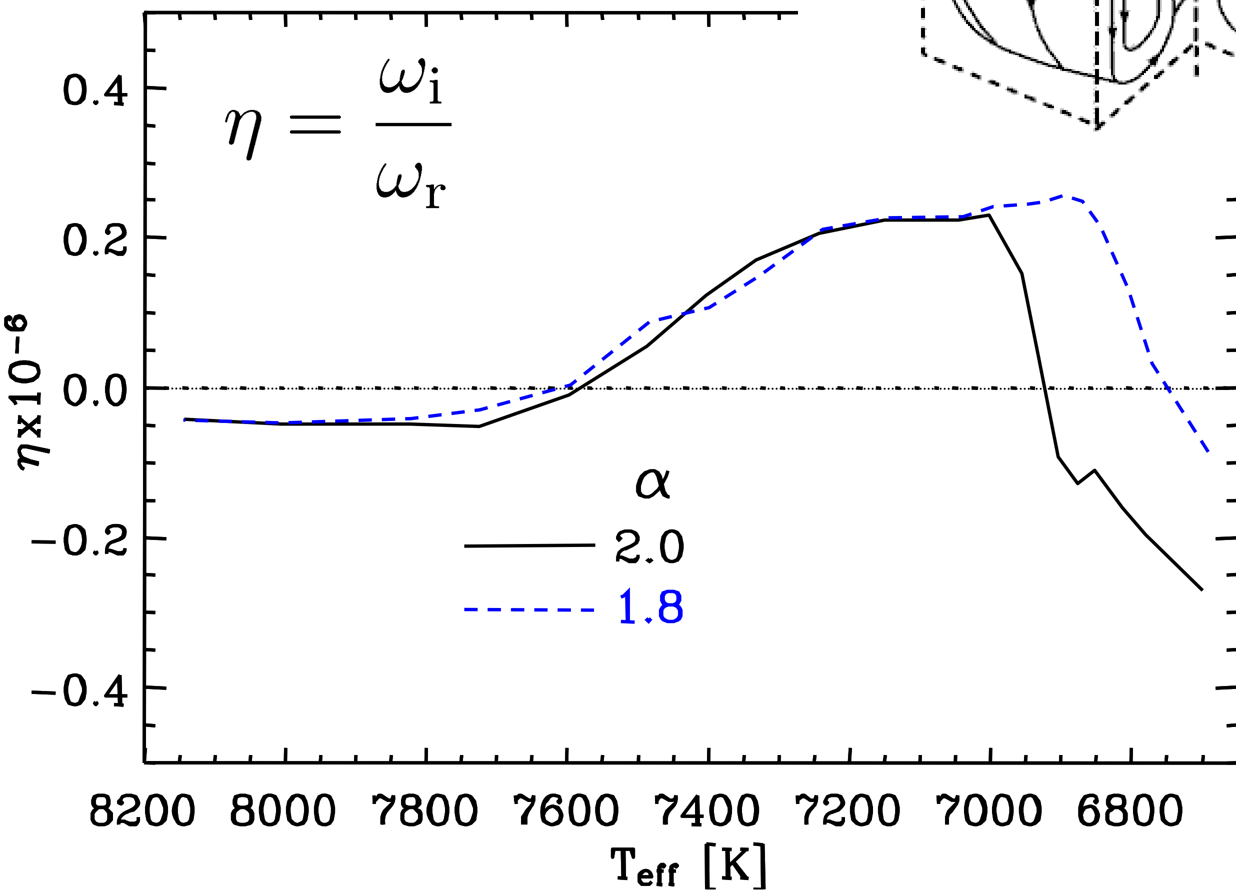
Acoustic radiation in the equilibrium model

(Houdek & Gough 1998; Houdek 2000; Samadi, Goupil, Houdek 2002)

overturning eddy: $$\frac{2w^2}{\ell} = g\frac{\tilde{\delta}}{T}T' - \frac{P_{\rm ac}}{\rho w}$$

acoustic drag





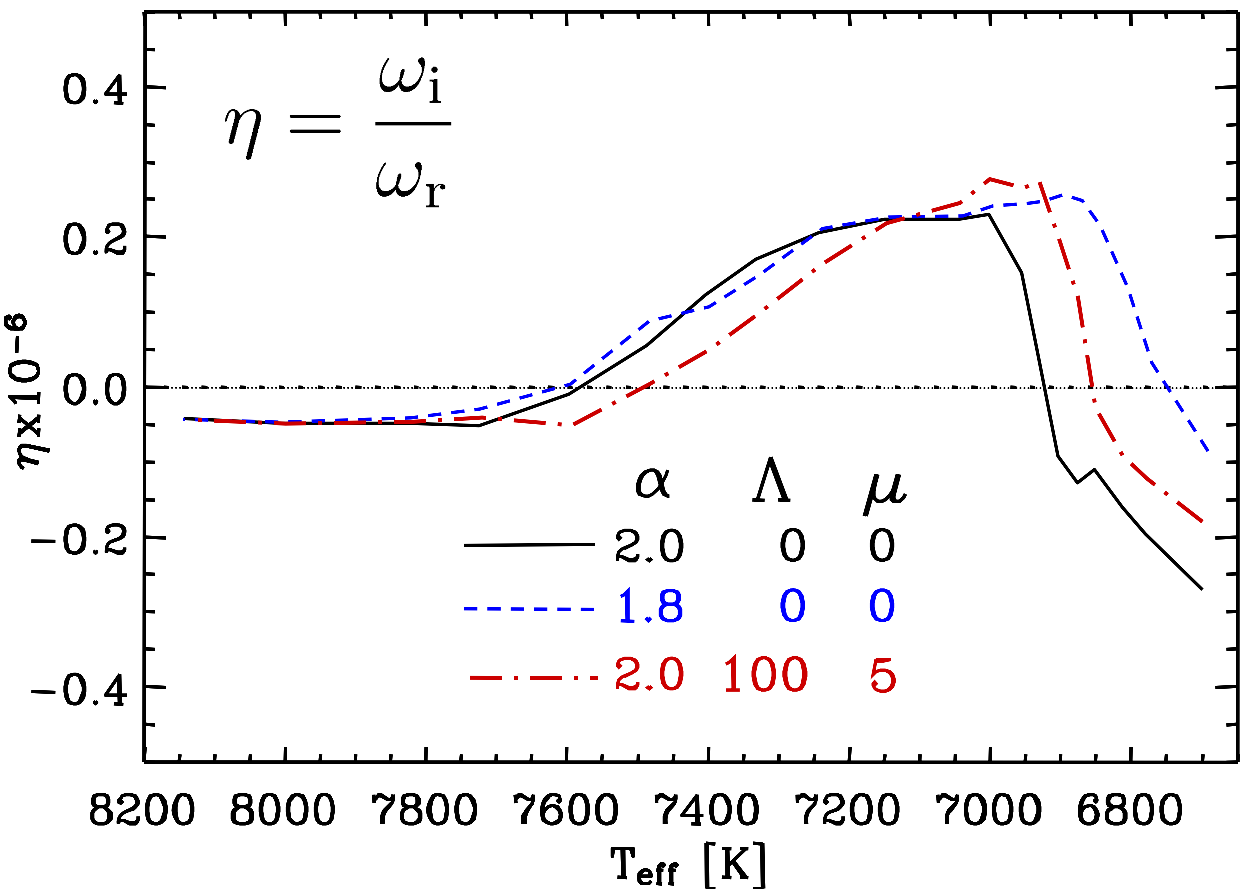
δ Scuti stars

Acoustic radiation in the equilibrium model

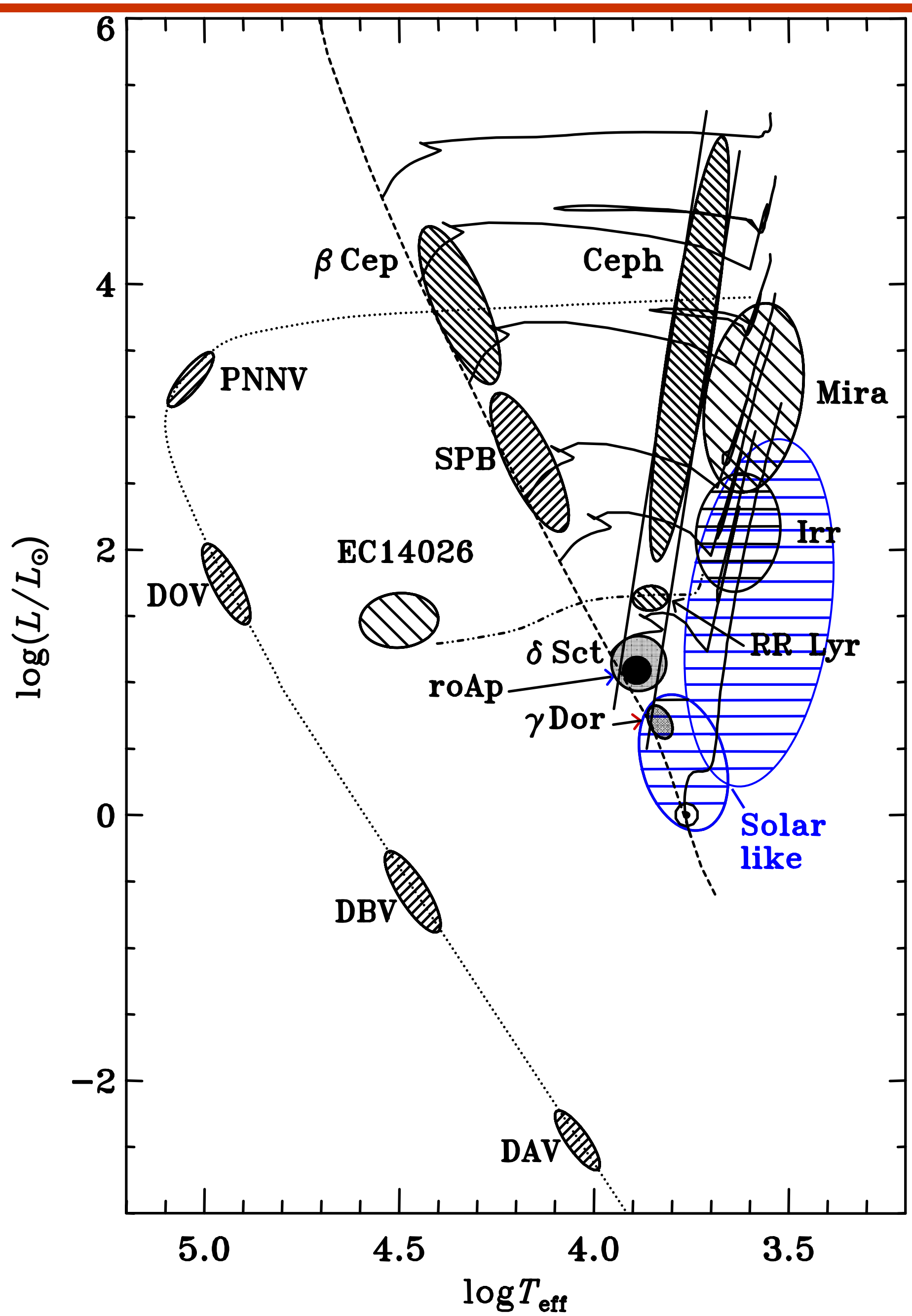
(Houdek & Gough 1998; Houdek 2000; Samadi, Goupil, Houdek 2002)

overturning eddy: $$\frac{2w^2}{\ell} = g\frac{\tilde{\delta}}{T}T' - \frac{P_{\rm ac}}{\rho w}$$

Lighthill-Proudman: $$P_{\rm ac} = \Lambda \frac{\rho w^3}{\ell}\left(\frac{w}{c}\right)^{\mu}$$



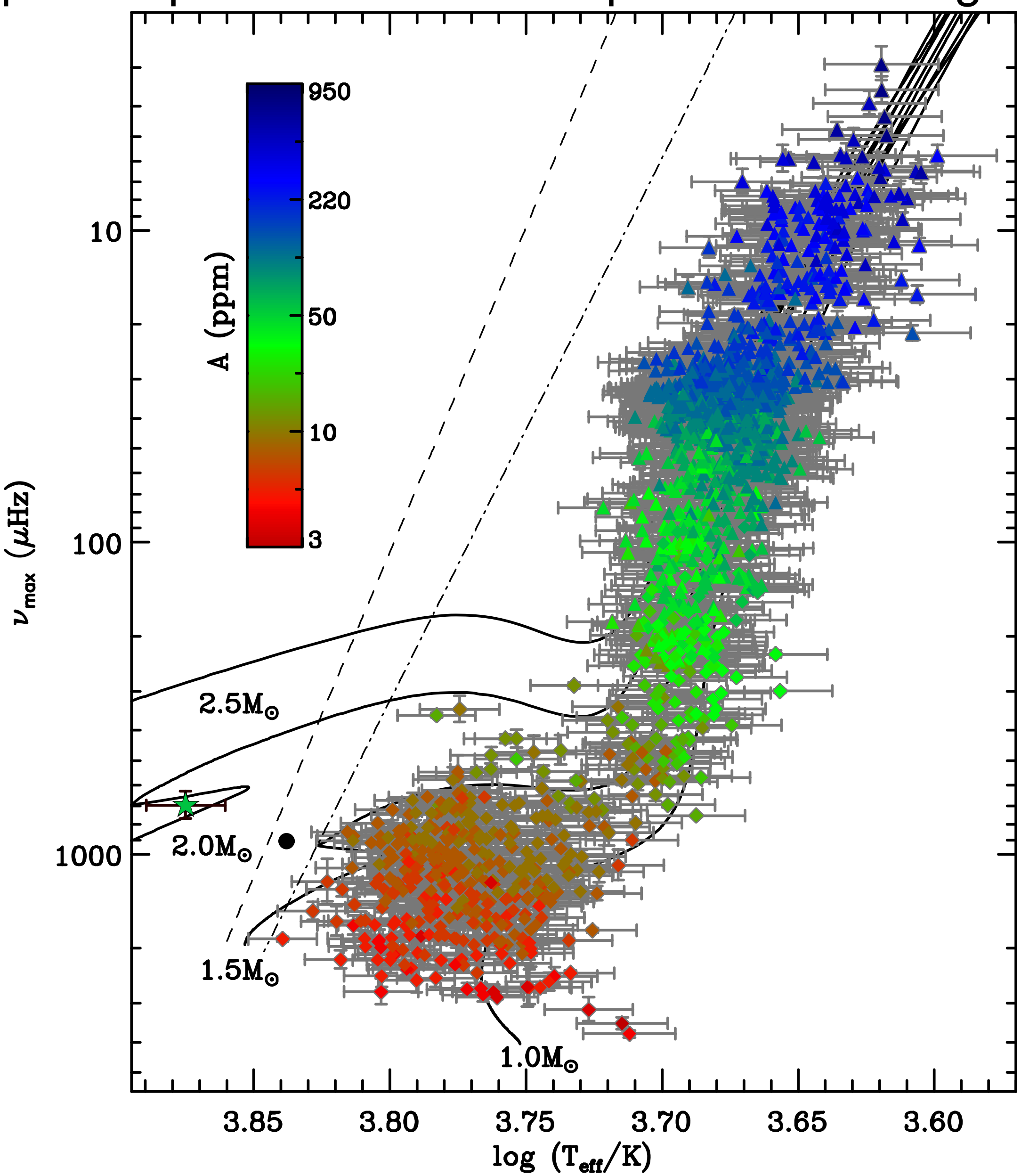
Solar-like pulsator



- Main-sequence/post main-sequence phase
- In the Sun up to 10 million p modes
- Amplitudes: $cms^{-1} \longrightarrow ms^{-1}$
  few ppm $\longrightarrow$ hundreds of ppm
- Driven stochastically by the turbulence in the outer stellar layers

Solar-type pulsator

Kepler amplitudes of main-sequence and red-giant stars

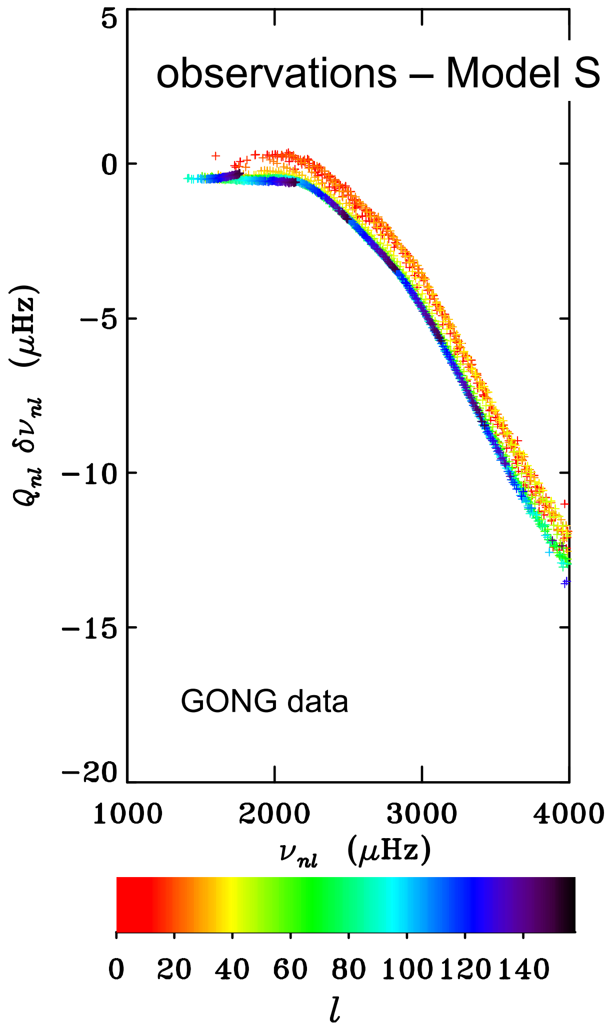


Huber et al. (2011)

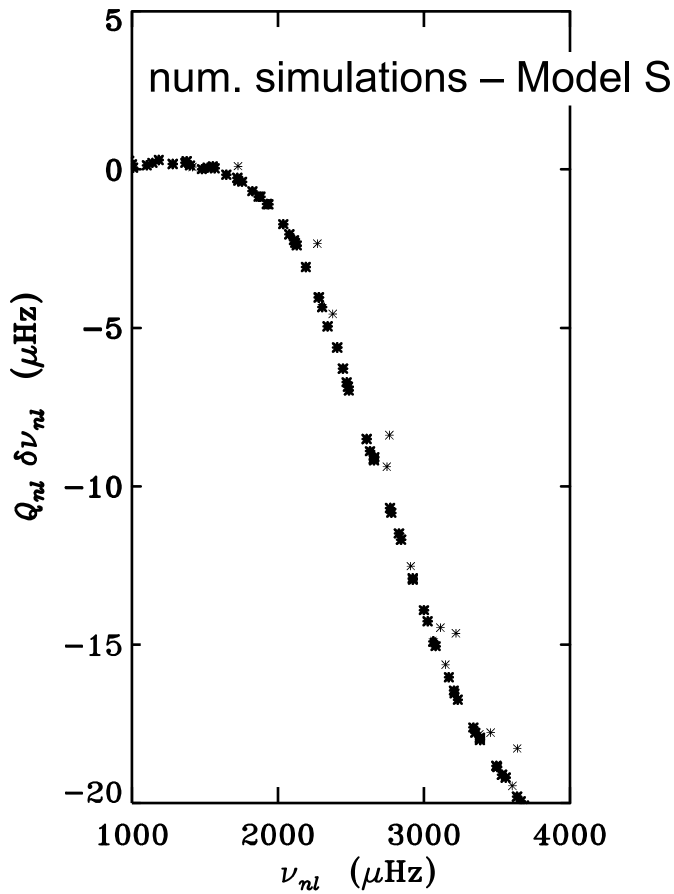
Solar-like pulsator: surface effects

Solar observations – **adiabatic** calculations

Christensen-Dalsgaard et al. (1996)



Rosenthal et al. (1995)

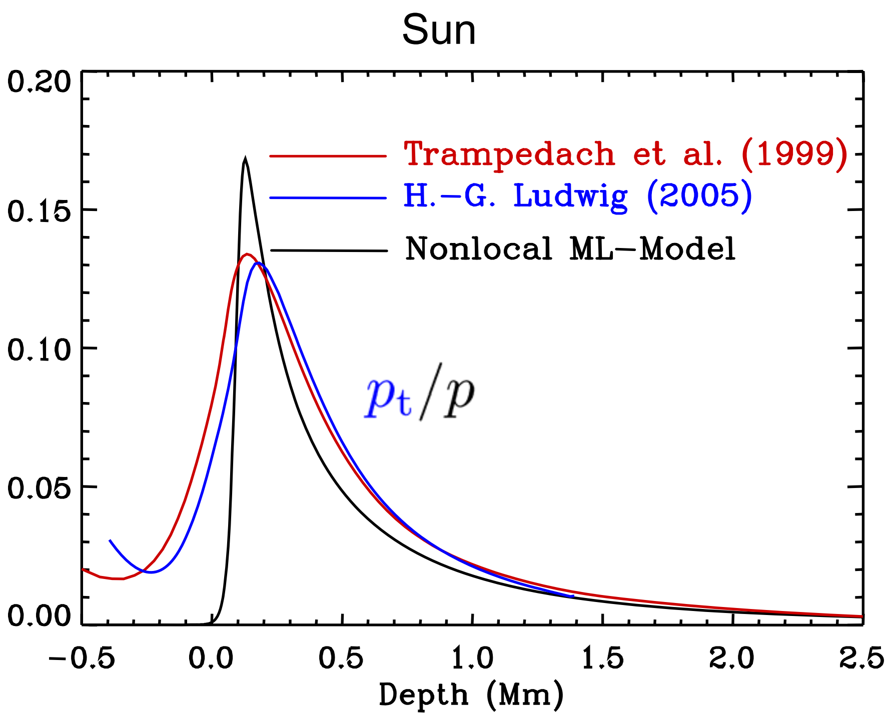


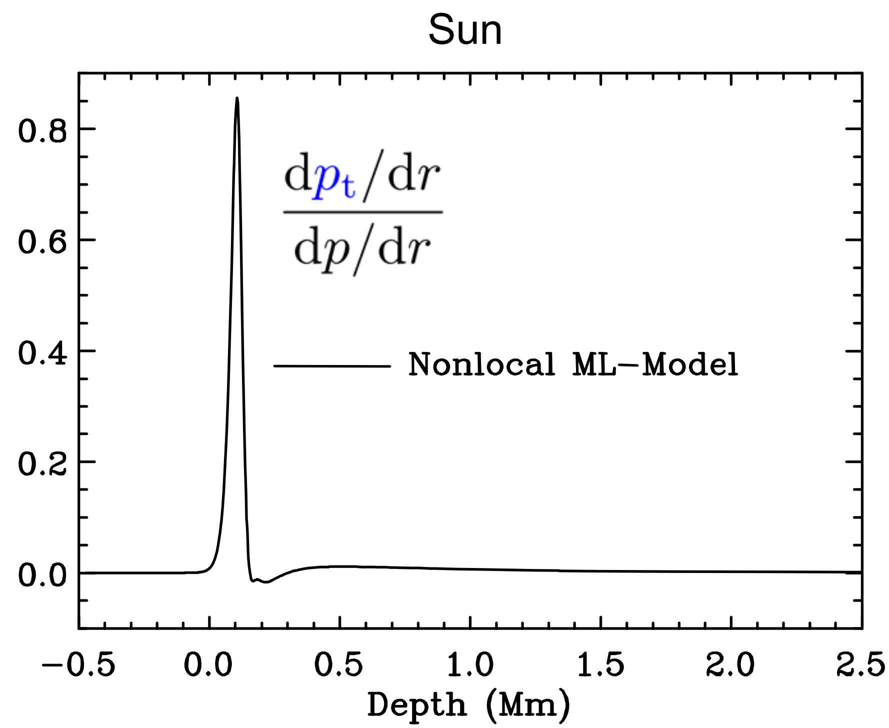
Solar-like pulsator: surface effects

Momentum equation of stellar (envelope) structure:

$$\frac{\partial}{\partial m}\underbrace{(p_{\rm g} + p_{\rm t})}_{p} + (3 - \Phi)\frac{p_{\rm t}}{4\pi r^3 \rho} = -\frac{1}{4\pi r^2}\left(\frac{Gm}{r^2} + \frac{\partial^2 r}{\partial t^2}\right)$$

(mean) turbulent momentum flux (turbulent pressure): $p_{\rm t} = \overline{\rho w^2}$





GH (2006)

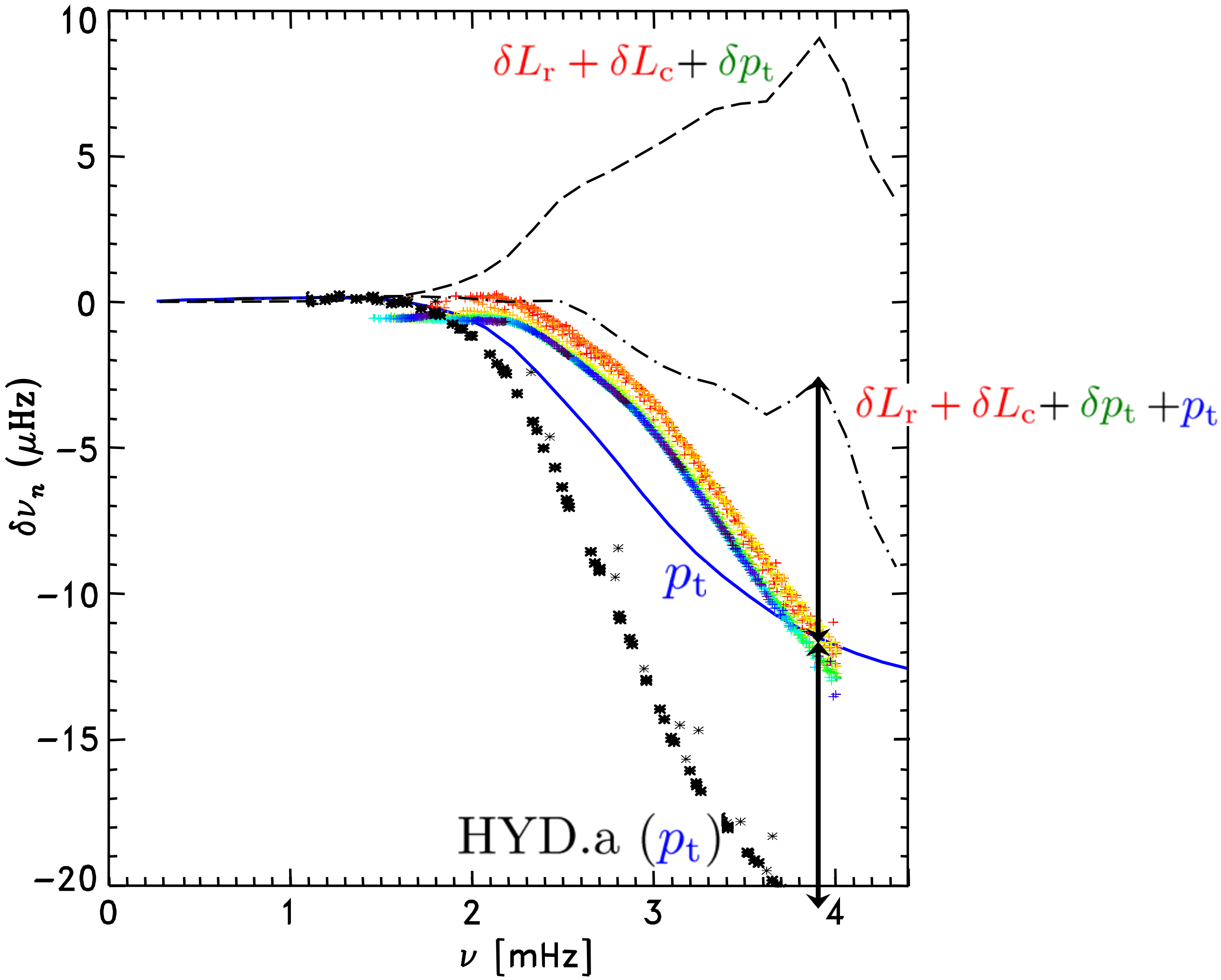
Solar-like pulsator: surface effects

Nonadiabaticity:

$$\frac{\partial}{\partial m}\left(\frac{\delta(L_{\rm r}+L_{\rm c})}{L}\right) = -{\rm i}\omega\frac{c_pT}{L}\left(\frac{\delta T}{T}-\nabla_{\rm ad}\frac{\delta p}{p}\right)\ ;$$

Convection dynamics:

$$\frac{\partial}{\partial m}\left(\frac{\delta p}{p}\right) = f\left(\frac{\delta r}{r},\frac{\delta T}{T},\frac{\delta p}{p},\frac{\delta p_{\rm t}}{p},\frac{\delta \Phi}{\Phi}\right)$$



Solar-like pulsator: surface effects

Nonadiabaticity:

$$\frac{\partial}{\partial m}\left(\frac{\delta(L_\mathrm{r}+L_\mathrm{c})}{L}\right) = -\mathrm{i}\omega\frac{c_pT}{L}\left(\frac{\delta T}{T}-\nabla_\mathrm{ad}\frac{\delta p}{p}\right)\ ;$$

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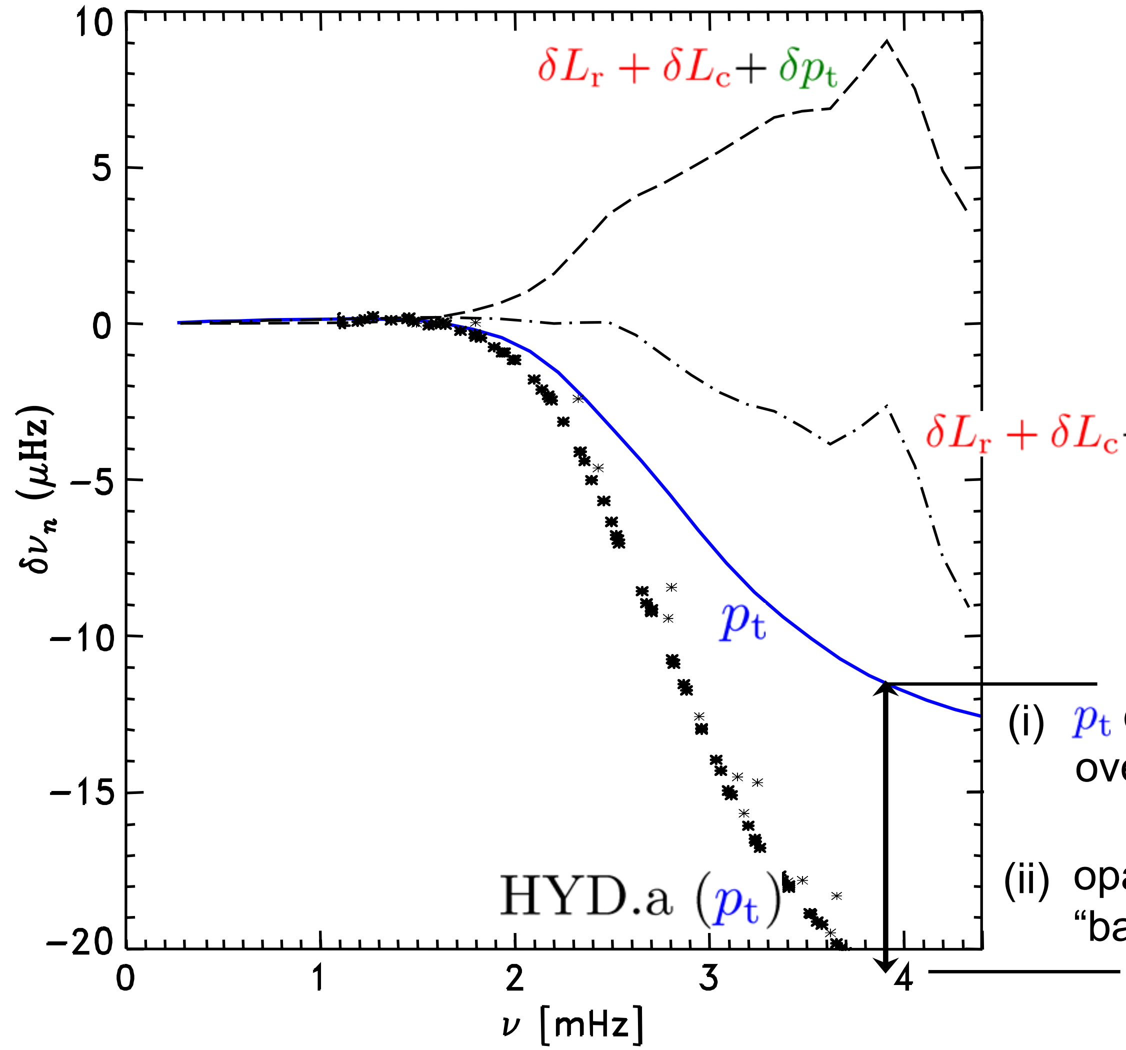


(i) p_t contribution in the overshoot region

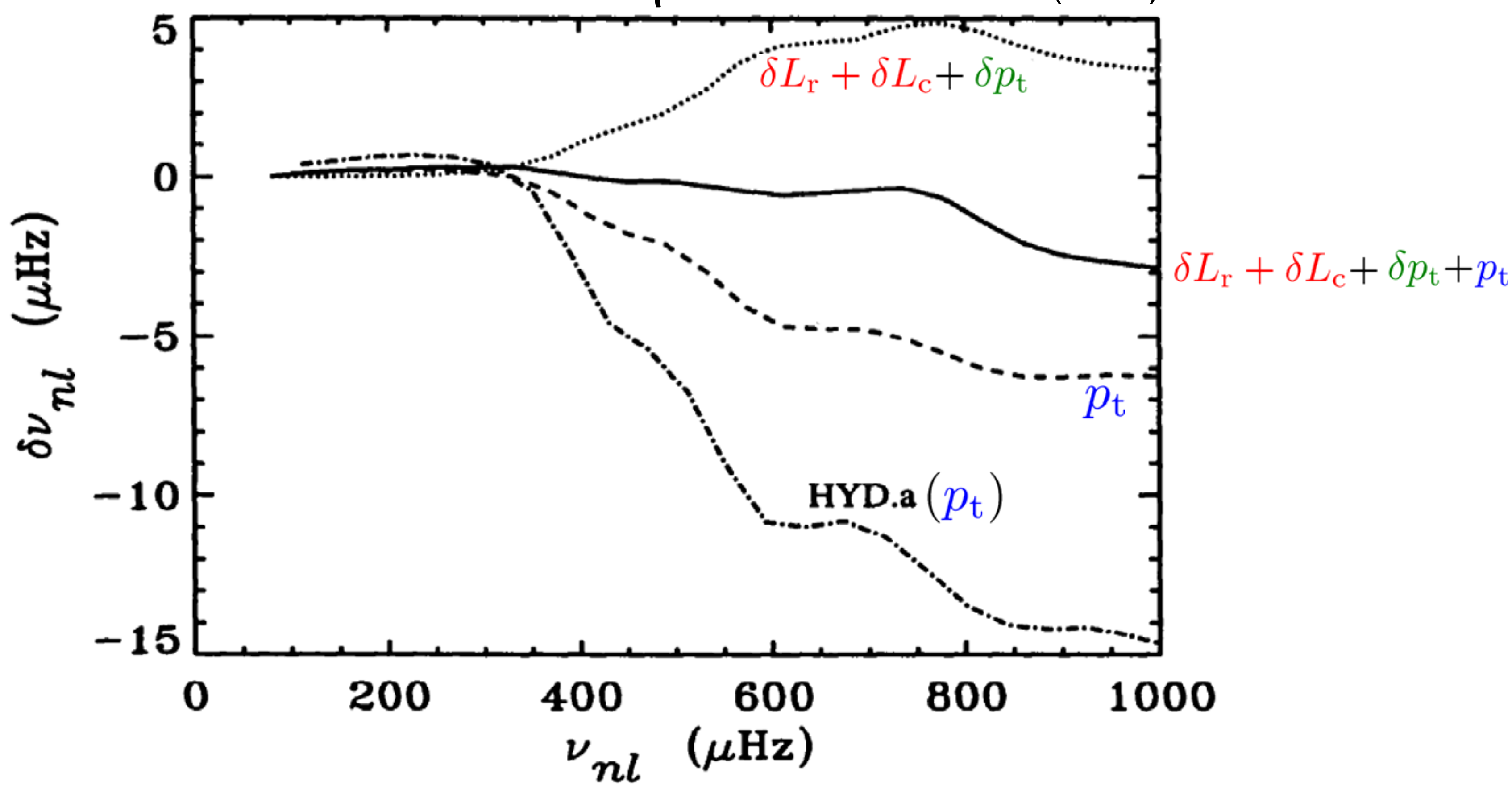
(ii) opacity effect: “back warming”

Solar-like pulsator: surface effects

perturbed energy equation: $$\frac{\partial}{\partial m}\left(\frac{\delta(L_{\rm r}+L_{\rm c})}{L}\right) = -{\rm i}\omega\frac{c_pT}{L}\left(\frac{\delta T}{T} - \nabla_{\rm ad}\frac{\delta p}{p}\right)$$

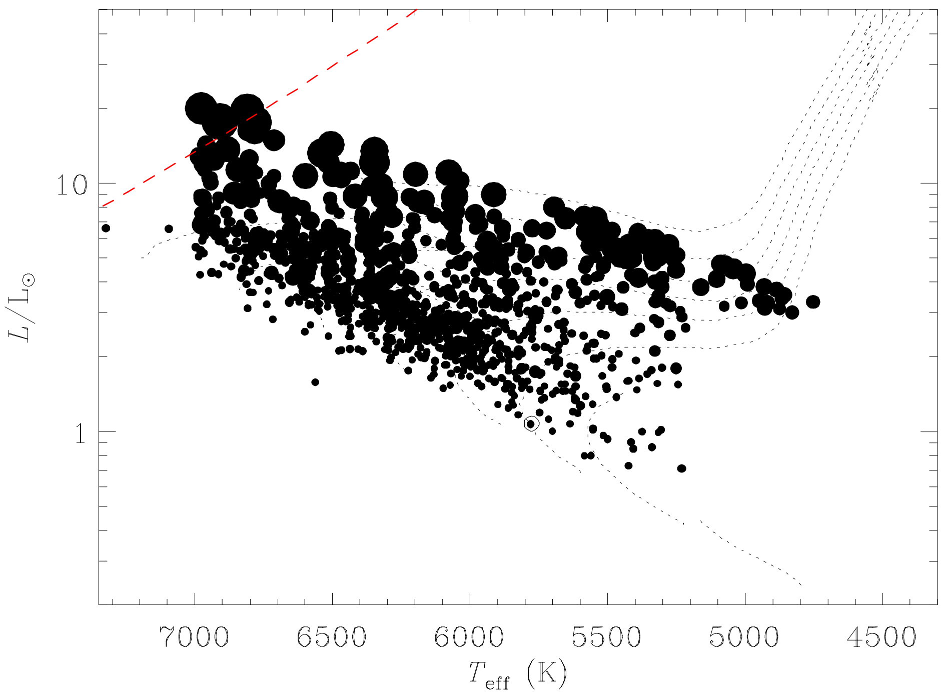
perturbed momentum equation: $$\frac{\partial}{\partial m}\left(\frac{\delta p}{p}\right) = f\left(\frac{\delta r}{r}, \frac{\delta T}{T}, \frac{\delta p}{p}, \frac{\delta p_{\rm t}}{p}, \frac{\delta \Phi}{\Phi}\right)$$

η Boo JCD et al. (1995)

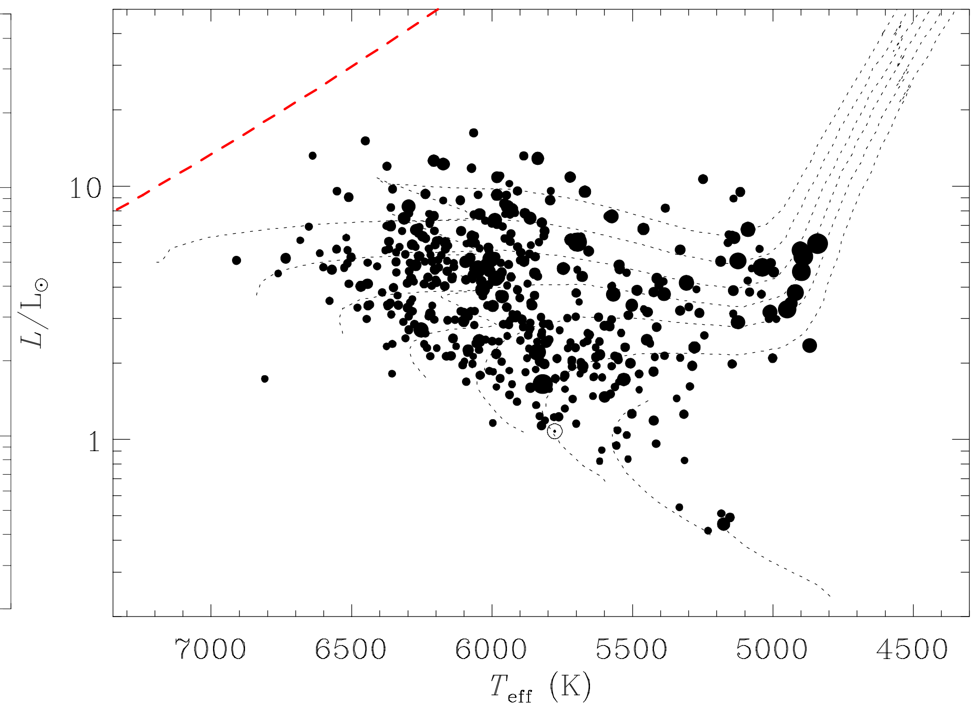


Pulsation amplitudes

Theroretical scaling: $V/V\_ \sim (L/M)^s$

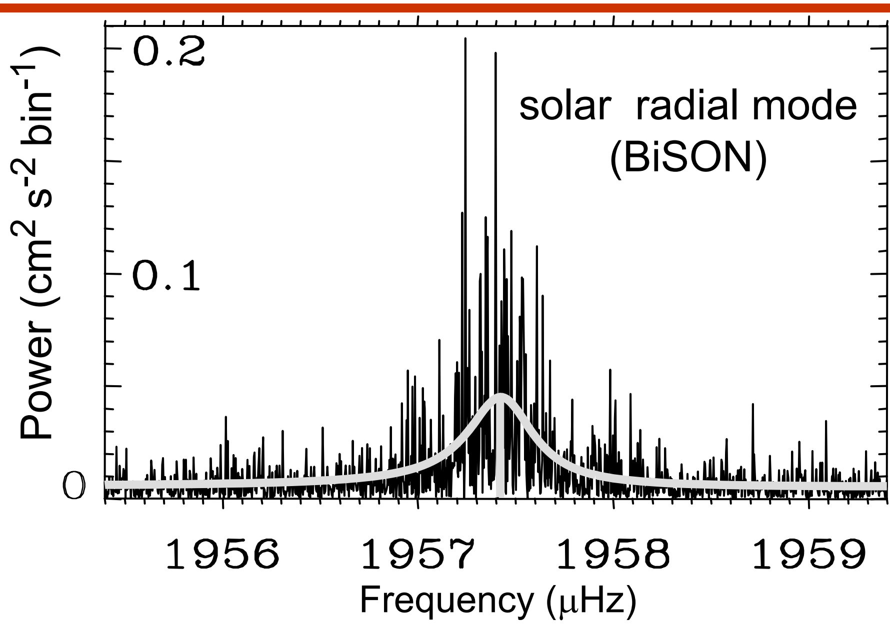


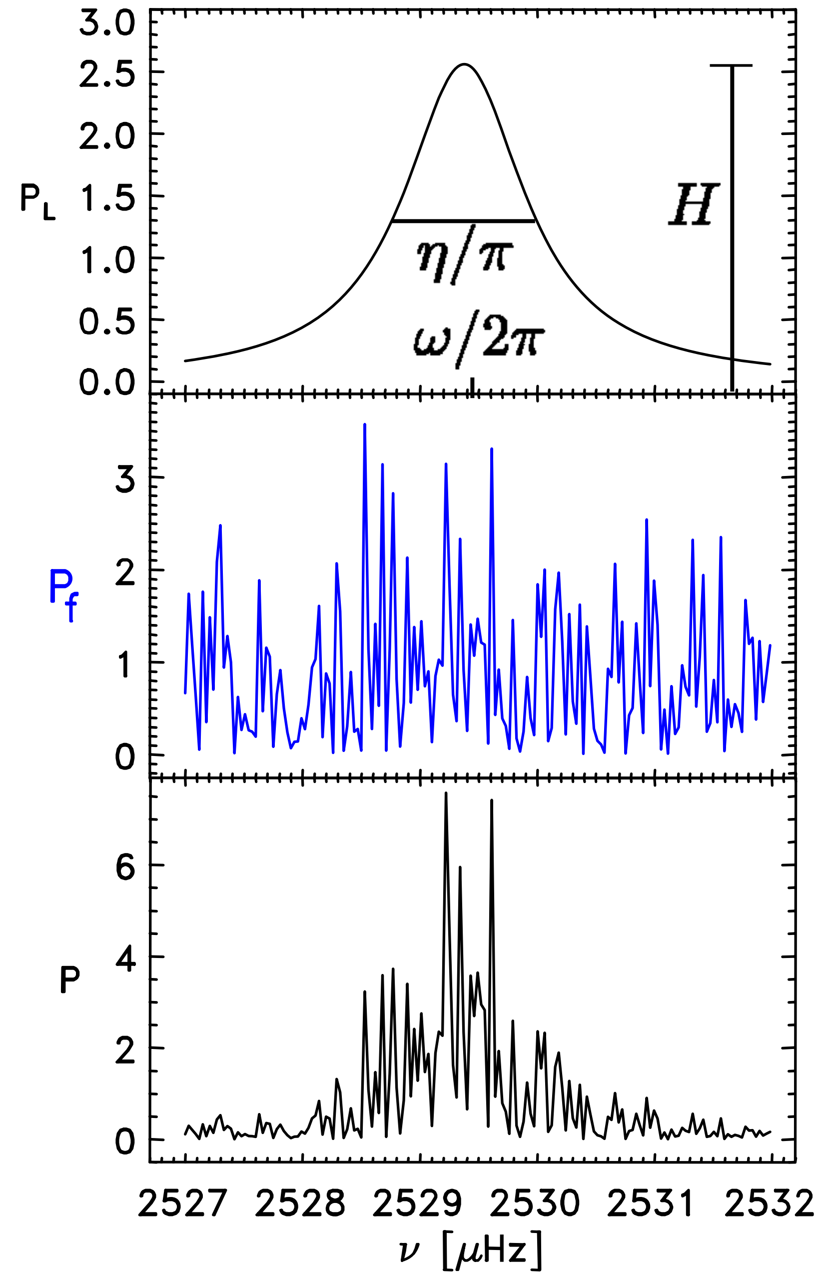
Kepler observations: SNR



(Chaplin et al. 2011)

Solar-type pulsator: excitation model





- Inhomogeneous wave equation:

$$\rho\left(\ddot{\xi} + 2\eta\dot{\xi} + \mathcal{L}\xi\right) = \boldsymbol{\mathcal{F}}(\boldsymbol{u}, t)$$

- Power (spectral density): $P \propto P_{\rm L}\, P_{\rm f}$

- Total integrated power (mean energy *E*):

$$E = IV^2 \propto \frac{P_{\rm f}(\omega)}{\eta} \propto I\eta H$$

Solar-type pulsator: excitation model

(Goldreich & Keeley 1977, Balmforth 1992, Samadi et al. 2001, Chaplin et al. 2005)

Reynolds stress contribution

$$P \propto I^{-1} \int_0^R \ell^3 \left(r \frac{\partial \xi_{i r}}{\partial r} p_{\mathrm{t}} \right)^2 \mathcal{S} \, \mathrm{d}r$$

mean turbulent pressure

$$H := P / \eta^2 I = 2 V^2 / \eta$$

Solar-type pulsator: excitation model

(Goldreich & Keeley 1977, Balmforth 1992, Samadi et al. 2001, Chaplin et al. 2005)

Reynolds stress contribution

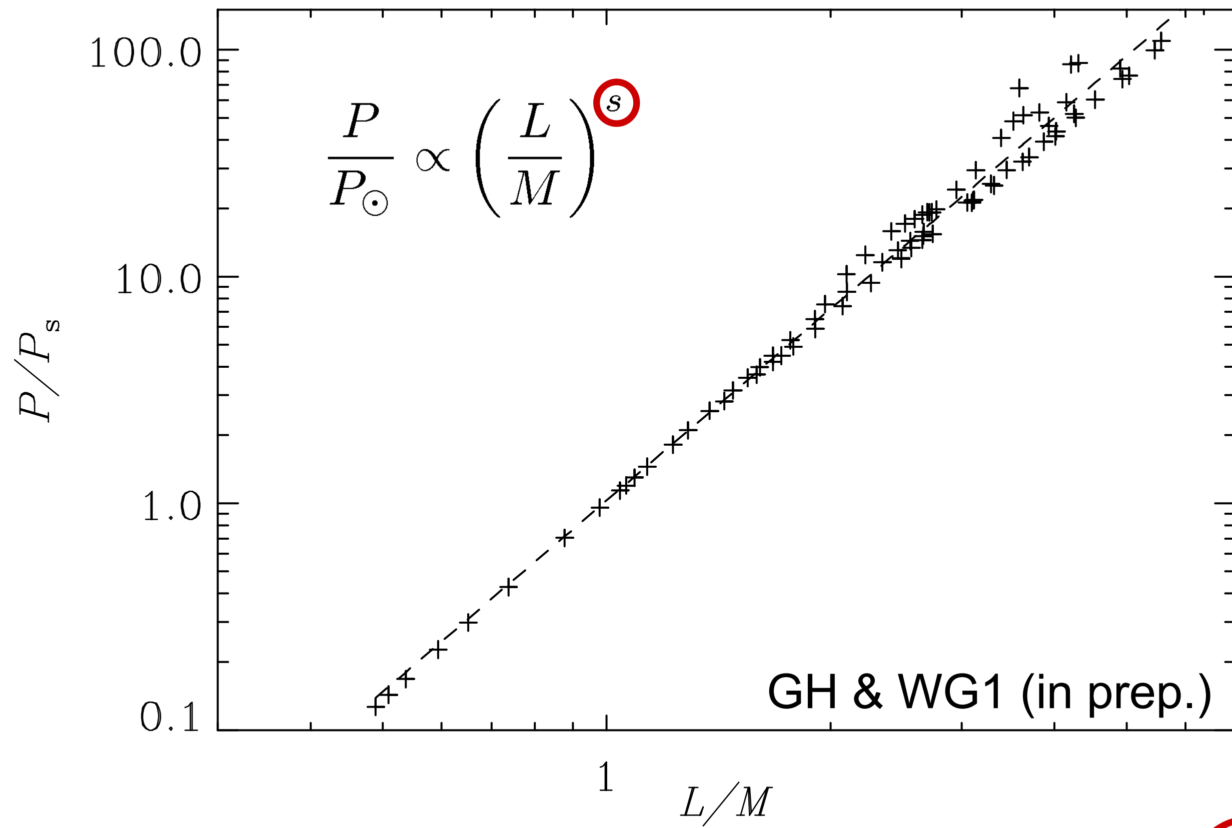
$$P \propto I^{-1} \int_0^R \ell^3 \left(r \frac{\partial \xi_{i\,r}}{\partial r} p_{\mathrm{t}} \right)^2 \mathcal{S} \, \mathrm{d}r$$

small-scale turbulence

$$H := P/\eta^2 I = 2V^2/\eta$$

Solar-type pulsator: excitation model

Energy supply rate ***P*** for 83 **models** with $0.9 \le M/\mathrm{M}_\odot \le 1.5$



Excitation model	Ω	*s*
Chaplin et al. (2005)	Gaussian: λ = 1.0	2.80
Samadi et al. (2003)	mod. Lorentzian: λ = 1.5; β = 3.2	2.87
Samadi et al. (2003)	mod. Lorentzian: λ = 1.0; β = 3.2 (Kaneda 1993)	2.72

Solar-type pulsator: excitation model

(Goldreich & Keeley 1977, Balmforth 1992, Samadi et al. 2001, Chaplin et al. 2005)

Reynolds stress contribution

eigenfunction ξ

$$P \propto I^{-1} \int_0^R \ell^3 \left(r \frac{\partial \xi_{i\,r}}{\partial r} p_\mathrm{t} \right)^2 \mathcal{S} \, \mathrm{d}r$$

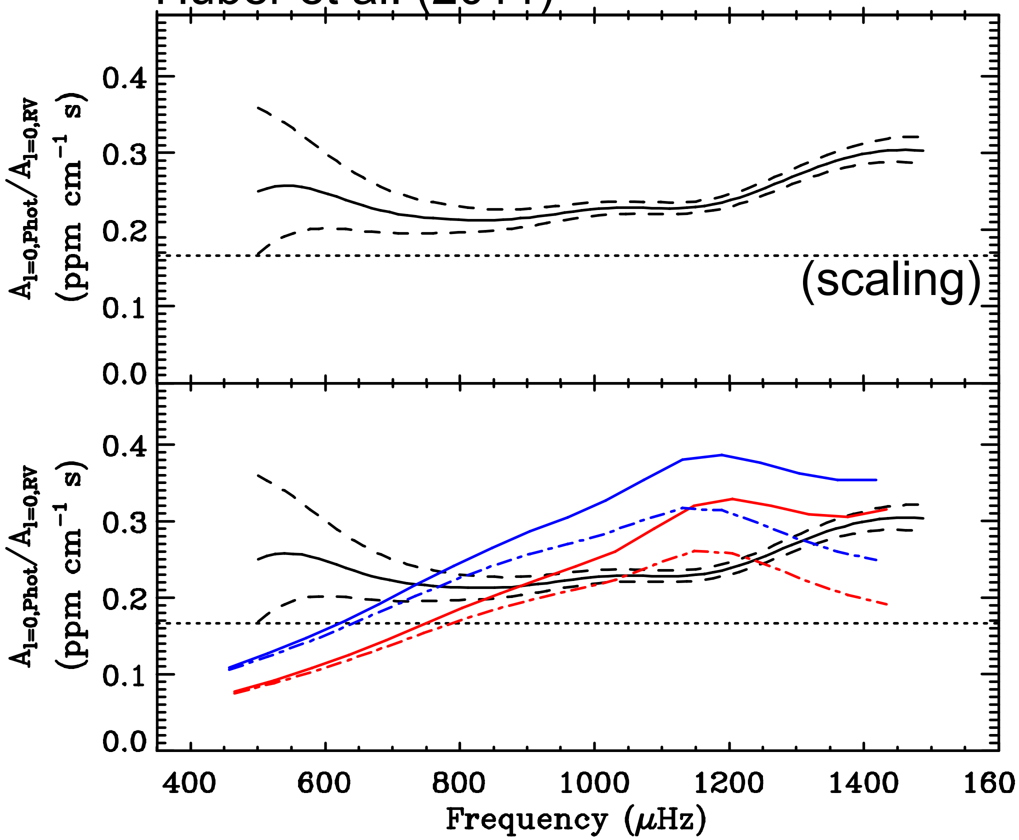
$$\boldsymbol{H \;:=\; P/\eta^2 I = 2V^2/\eta}$$

Solar-type pulsator: eigenfunctions ξ

Amplitude ratios in a model for Procyon A

modelled amplitude ratio = $\dfrac{\delta L/L}{\omega_{\mathrm{r}} r\ \delta r/r}$

Huber et al. (2011)



Data:
I : MOST
V: Arentoft et al. (2008)

Model: (GH 2006)
scaled VAL-C } atmos-
Eddington } phere

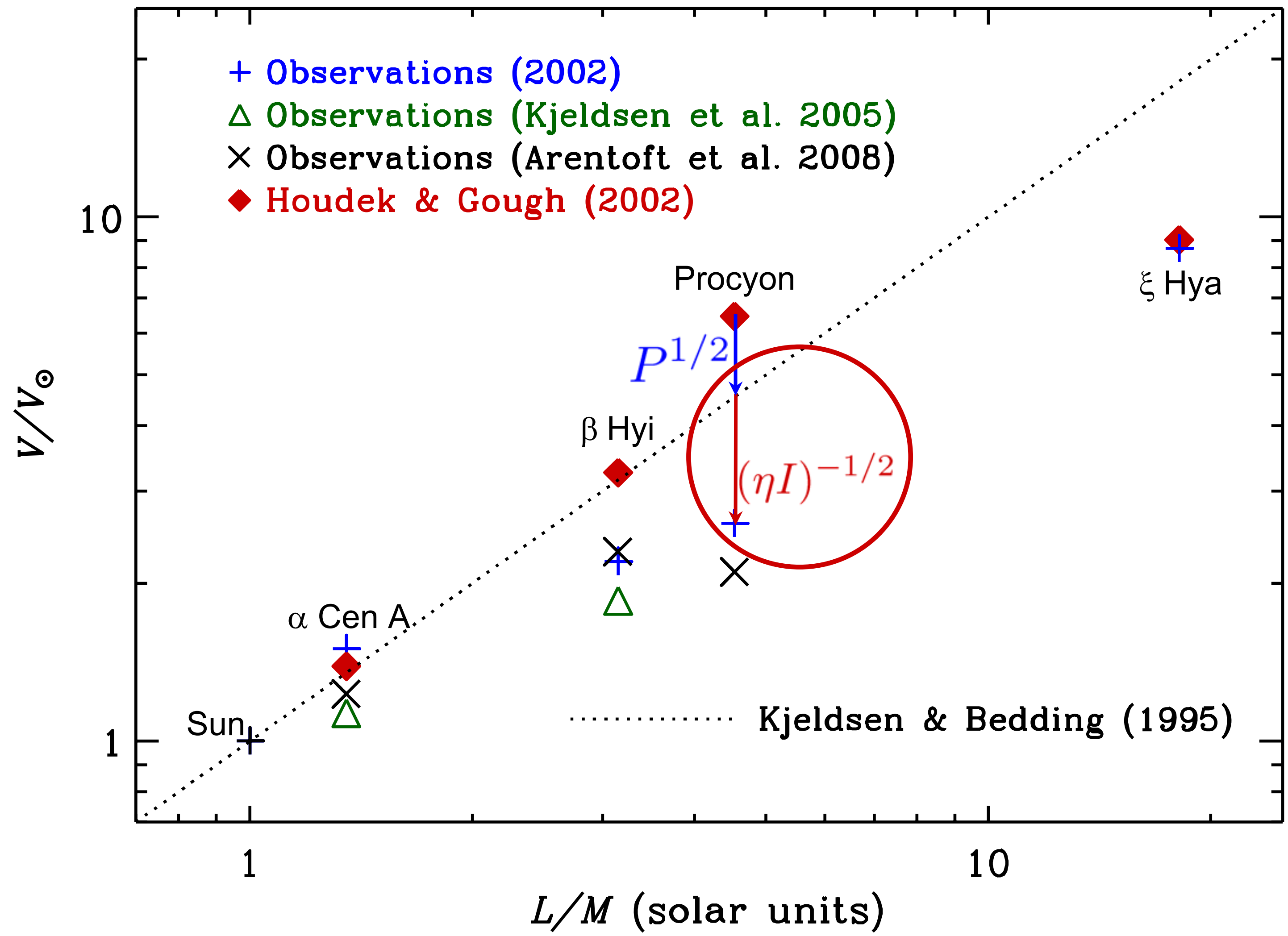
height: 300 km

height: 600 km

Note: amplitude ratios do not depend on a stochastic excitation model !!

Solar-type pulsator: mode lifetimes τ

Velocity amplitudes



$$H := P/\eta^2 I$$

GH (2006)

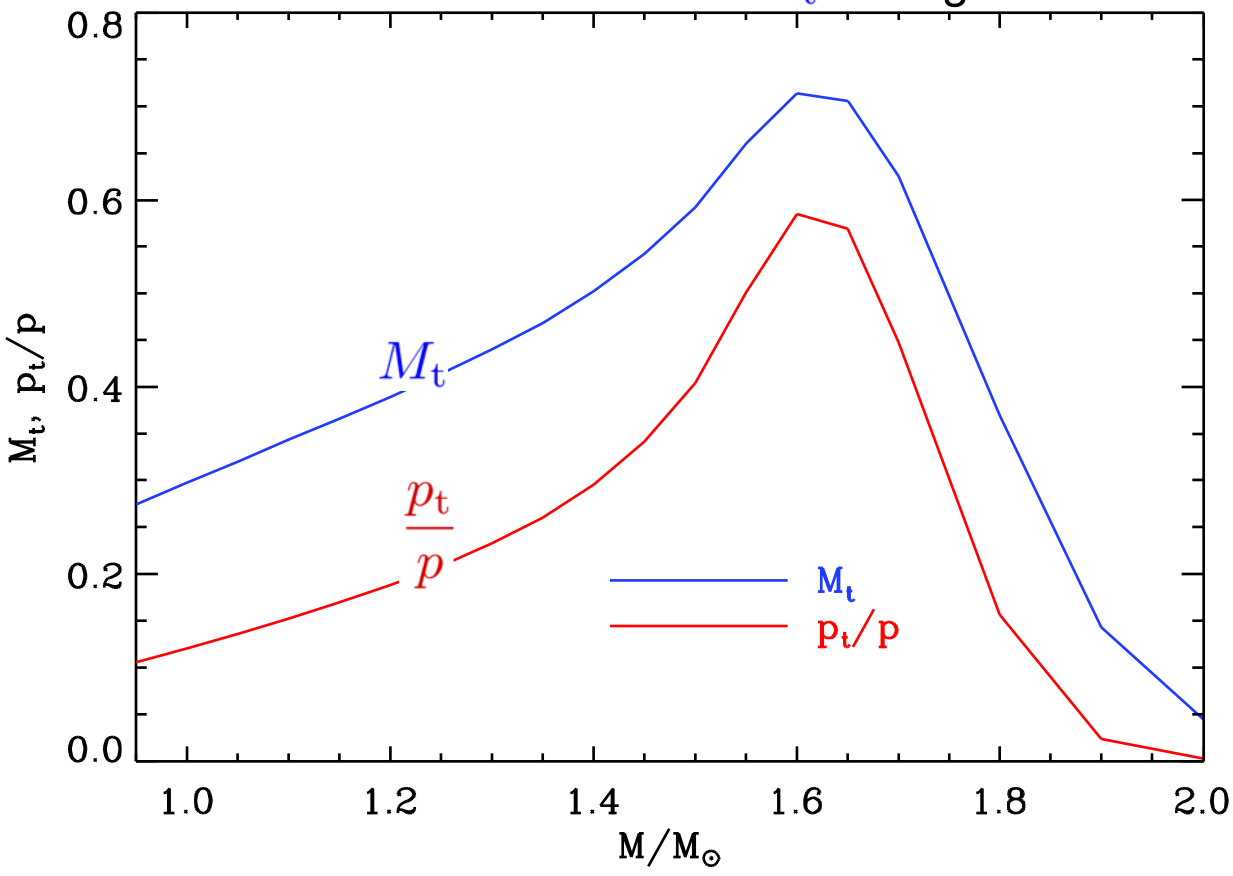
Solar-type pulsator: mode lifetimes τ

Scattering contribution (Goldreich, Murray 1994):

$$\eta_{\mathrm{s}} \sim \frac{2\omega}{\pi(n+1)} {M_{\mathrm{t}}}^2 \qquad\qquad M_{\mathrm{t}} := \frac{u}{c}$$

pulsation energy is scattered from low-l $\longrightarrow$ high-l modes
ultimate recipients: f and g modes

Turbulent mach number M_{t} along ZAMS



GH et al. (1999)

Summary/Conclusion

- Turbulent convection controls location of cool edge of IS in δ Scuti stars; the detailed mechanism (i.e. convective heat flux, momentum flux of turbulent viscosity) for return to stability, however, is still uncertain.

- Acoustic radiation in the equilibrium model affects both the hot & cool edge of IS in δ Scuti stars.

- Mean turbulent pressure appears to be dominating surface effect in explaining diffrences between observed and modelled frequencies of solar-like oscillations; nonadiabaticity and convection dynamics must also not be neglected.

- Poor modelling of pulsation eigenfunctions and mode lifetimes together with partial cancellation of acoustic excitation sources may explain disagreement between modelled and observed pulsation amplitudes of solar-like oscillations in hotter stars.