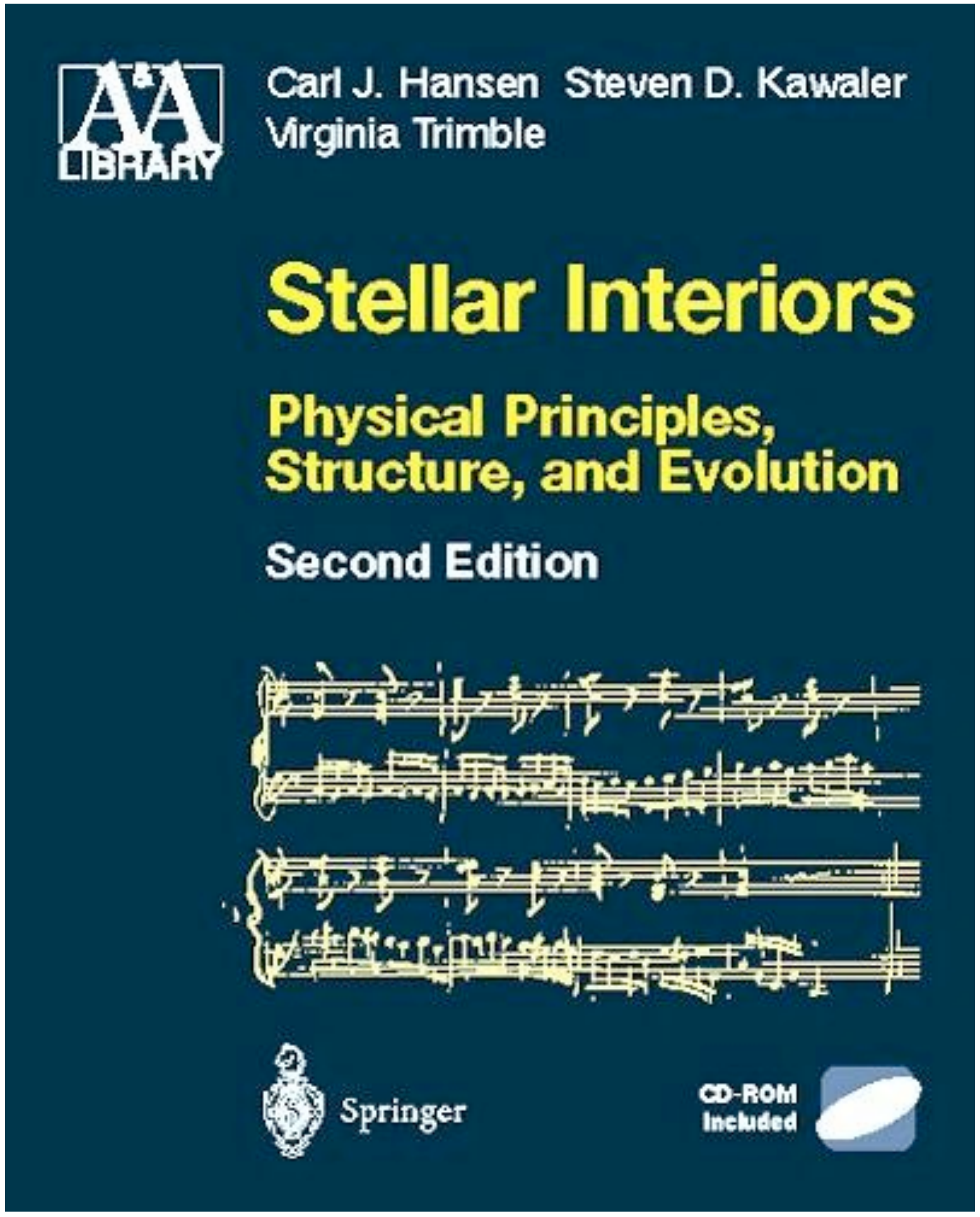
Pulsating Stars as Physics Probes: An Overview

Steve Kawaler
Iowa State University
(and KITP)

stellar structure in a nutshell

- a “reminder” of basic stellar structure and evolution

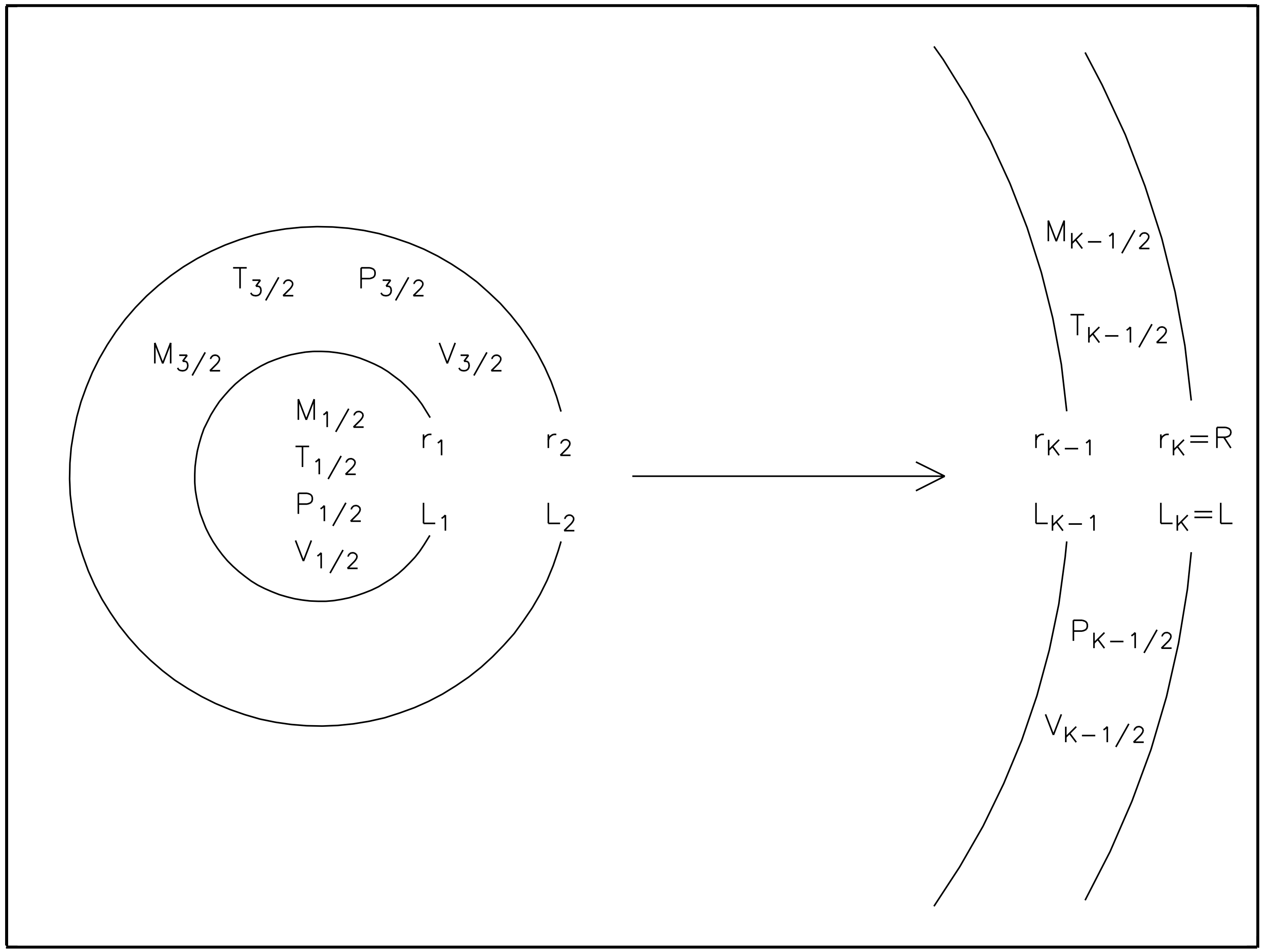


Dependent / Independent variables

- Independent variable - a measure of position
  - distance from center - r

-or-

  - mass fraction within - M_r



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  - **-or-**
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- Things that specify local conditions within a (hydrostatic) star:
  - ~~velocity~~
  - density: ρ or $n = N_A \rho/\mu$
  - pressure: P
  - temperature: T
  - chemical composition (fraction by mass): X_i
  - ion / charge balance: Y_i , n_e
  - internal energy (per unit mass): U
  - entropy (per unit mass): S
  - heat flow parameters/ x-sections (/mass): κ_{rad}, κ_{cond},
  - energy flow: L_r , F_{conv}
  - energy generation/loss (per unit mass): ε_{nuc} , ε_ν

the ‘core four’

- primary *mechanical* quantities
  - r (or M_r)
  - P
- primary *thermal* quantities
  - T
  - L_r

the ‘core four’... plus

- primary *mechanical* quantities
  - r (or M_r)
  - P
- primary *thermal* quantities
  - T
  - L_r
- necessary extra information
  - composition (element mass fraction X_i)
- necessary *derived* quantities
  - Equation of state: ρ, μ, U, S, ∇_{ad}, Y_i, etc.
  - Atomic physics: y_i, κ_{rad}, κ_{cond}
  - Nuclear physics: ε_{nuc}, ε_{ν}
  - confusing physics: F_{conv}
- NOTE - for pulsation, need partial derivatives of these

the basic equations

- Continuity

$$\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$$

- HSE

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$$

- Energy conservation

$$\frac{dL_r}{dM_r} = -T\frac{\partial S}{\partial t} + \epsilon$$

- Energy transport

$$\frac{dT}{dM_r} = -\nabla \frac{GM_r}{4\pi r^4}\frac{T}{P}$$

- Equation of state
  - $\rho\,(P, T, \mu)$
  - $\mu(P, T, X_i)$
  - $S\,(P, T, \mu)$
- Energy generation
  - $\varepsilon_{nuc}(\rho, T, X_i)$
  - $\varepsilon_{\nu}(\rho, T, X_i)$
- Energy transport
  - $\nabla_{rad} \longrightarrow \kappa_{rad}\,(\rho, T, X_i)$
  - $\nabla_{cond} \longrightarrow \kappa_{cond}\,(\rho, T, X_i)$
  - $\nabla_{convective} \longrightarrow ????$

input physics

- Equation of state
  - easy version(s) - perfect gas, mixed ionization state, ...
  - *complications - degeneracy, non-ideal effects, disequilibrium*
- Nuclear reaction rates
  - easy version(s) - S(0) energy approximation and expansions
  - *complications - hidden low-energy resonances, ...*
  - *complications - neutrino emission*
- Radiative transport - opacities
  - easy version(s) - Kramers, electron scattering
  - *complications - real atoms, molecules, coupling, mixtures*
  - *complications - conduction*
- Convective transport
  - easy version(s) - adiabatic, mixing-length theory
  - *complications - turbulence happens in 3-D*
  - *complications - interaction with rotation*
  - *complications - interaction with magnetic field*

input physics questions

(that asteroseismology can address)

- Equation of State
  - non-ideal effects
  - Coulomb crystallization - pulsating cool, massive WDs
- Nuclear processes
  - difficult cross-sections - chemical profiles in WD interiors
  - neutrino emission - evolution rates in hot WD pulsators
- Radiative transport - opacities
  - Cepheid masses, driving, and the iron bump
  - sdB driving (with diffusion thrown in)
  - B star pulsations
- the convective flux
  - white dwarf driving and harmonics
  - solar-like oscillations

other issues (non-coefficient)

- time evolution of abundance
  - composition changes via nuclear burning
    - direct impact through dS/dt term
  - composition changes via chemical diffusion
    - diffusion coefficients via atomic physics
  - composition changes via turbulence
    - instantaneous mixing via convection
    - convective overshoot
    - partial mixing via semiconvection, other processes
    - rotational mixing
- mass loss / accretion
- rotation
- magnetic fields
- tidal interaction and other effects of companions

EOS - trouble in an ideal world: particles *interact*

- proximity effects - Coulomb interactions between ions
  - Coulomb potential between two ions: Z^2e^2/a,
  - Coulomb effects are expected to become important when $Z^2e^2/a \sim kT$. Thus form the ratio

$$\Gamma_C \equiv \frac{Z^2e^2}{akT} = 2.27\ Z^2 \left(\frac{\rho}{10^6\,\mathrm{g\,cm^3}}\right)^{1/3} \left(\frac{T}{10^7 K}\right)^{-1} \left(\frac{A}{12}\right)^{-1/3}$$

  - when $\Gamma_C = 1$, effects begin to be felt
    - solar interior: Γ_C = 0.1
  - **if $\Gamma_C >> 1$, effects are strong (mutual ion repulsion)**
    - $\Gamma_C = 175$: ion-ion forces can cause crystallization (first proposed by Salpeter 1961)

$$T_{\mathrm{xtal}} = 3.4 \times 10^6\ Z^2 \left(\frac{A}{12}\right)^{-1/3} \left(\frac{\rho}{10^6\mathrm{g\,cm^{-3}}}\right)^{1/3} K$$

    - white dwarf interior Γ_C = 150 to 250 or more

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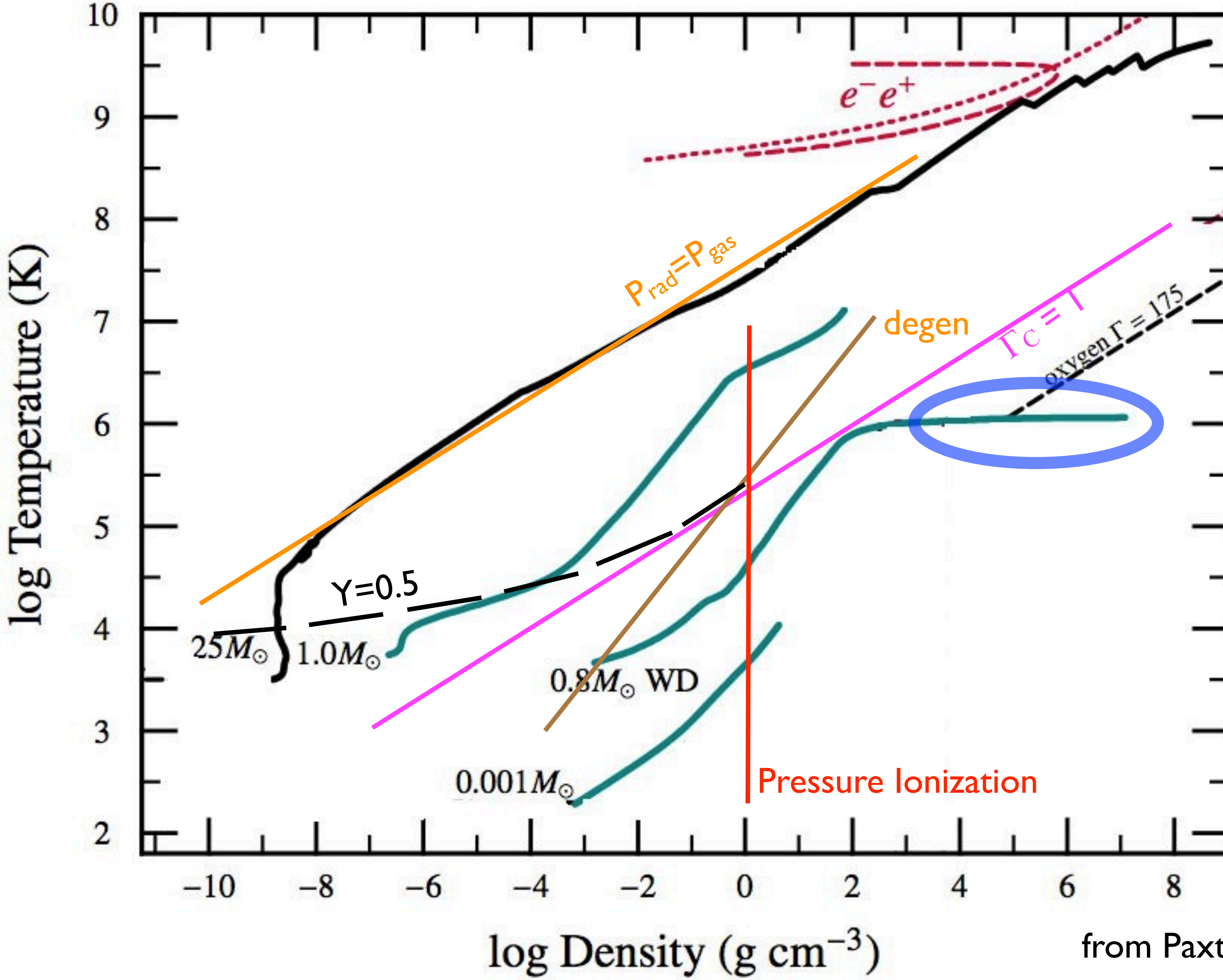
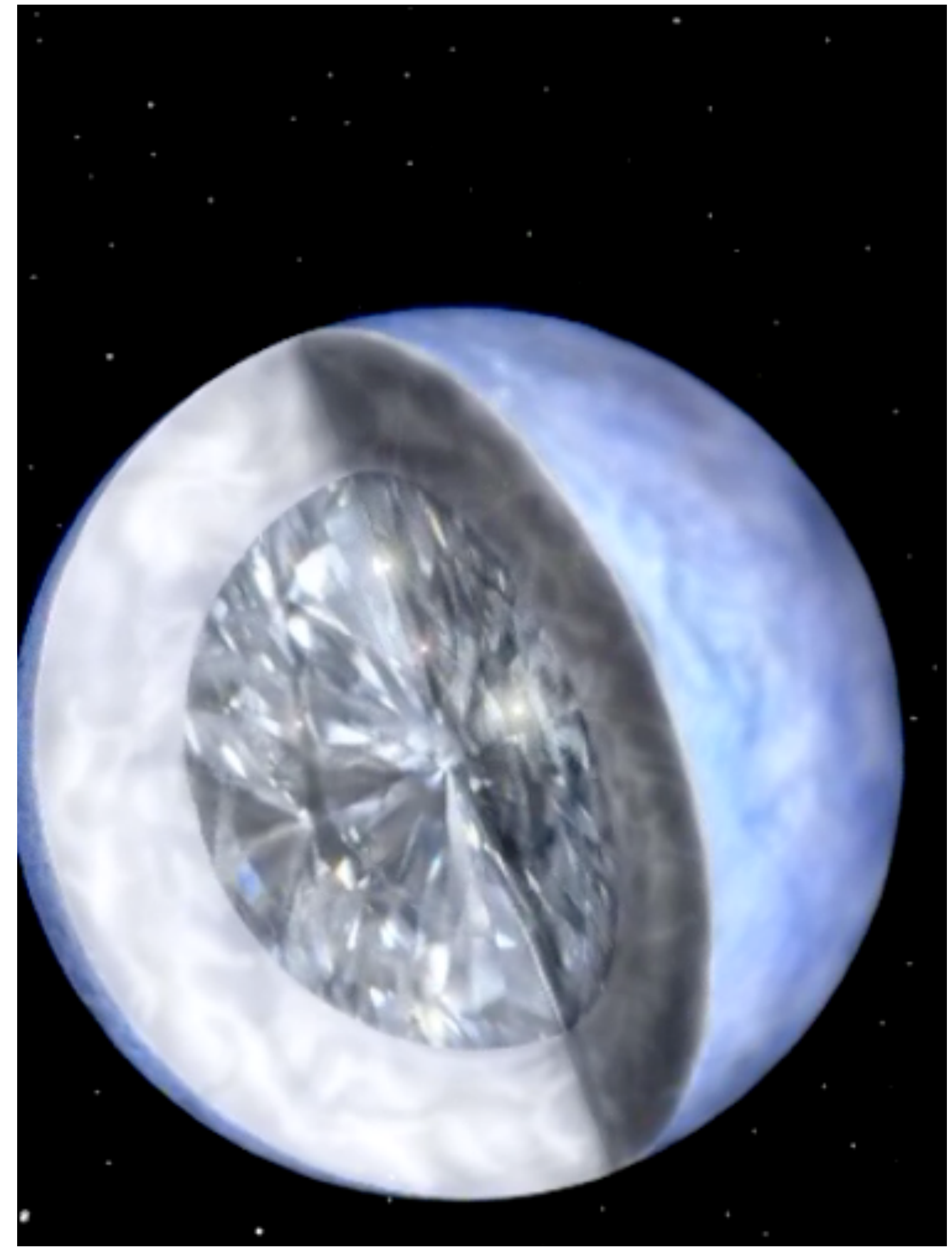


WD Xtalization

from Paxton et al. 2011

crystalline white dwarfs?

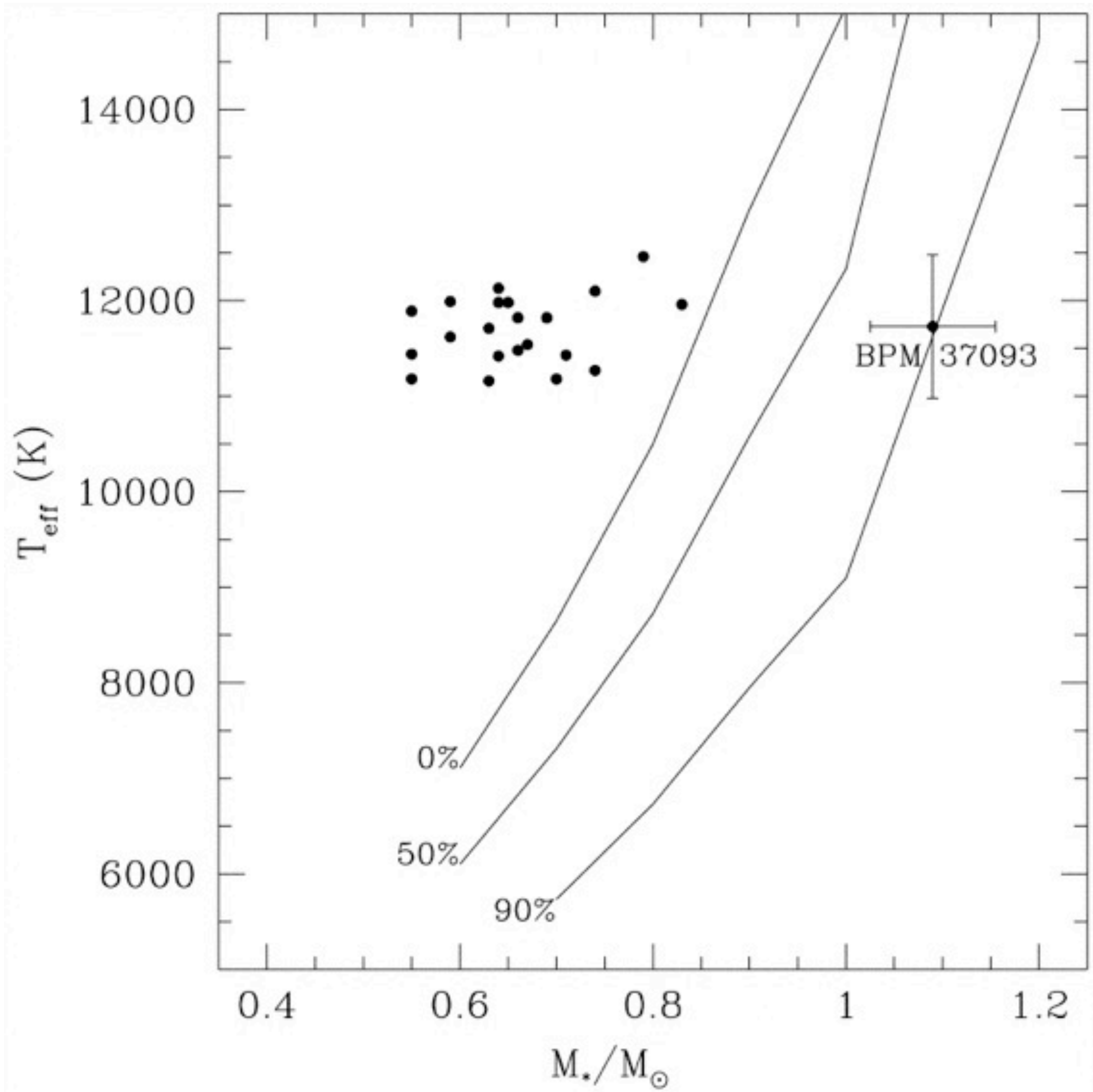
- at sufficient pressure, fully ionized metals can lock into a crystalline lattice
- conditions realized within cores of massive white dwarfs while surface still warm
- nonradial pulsations can reveal the crystallization boundary



"Twinkle, twinkle, little star,
How I wonder what you are!
Up above the world so high,
Like a *diamond* in the sky!"
Jane Taylor (1783-1824)

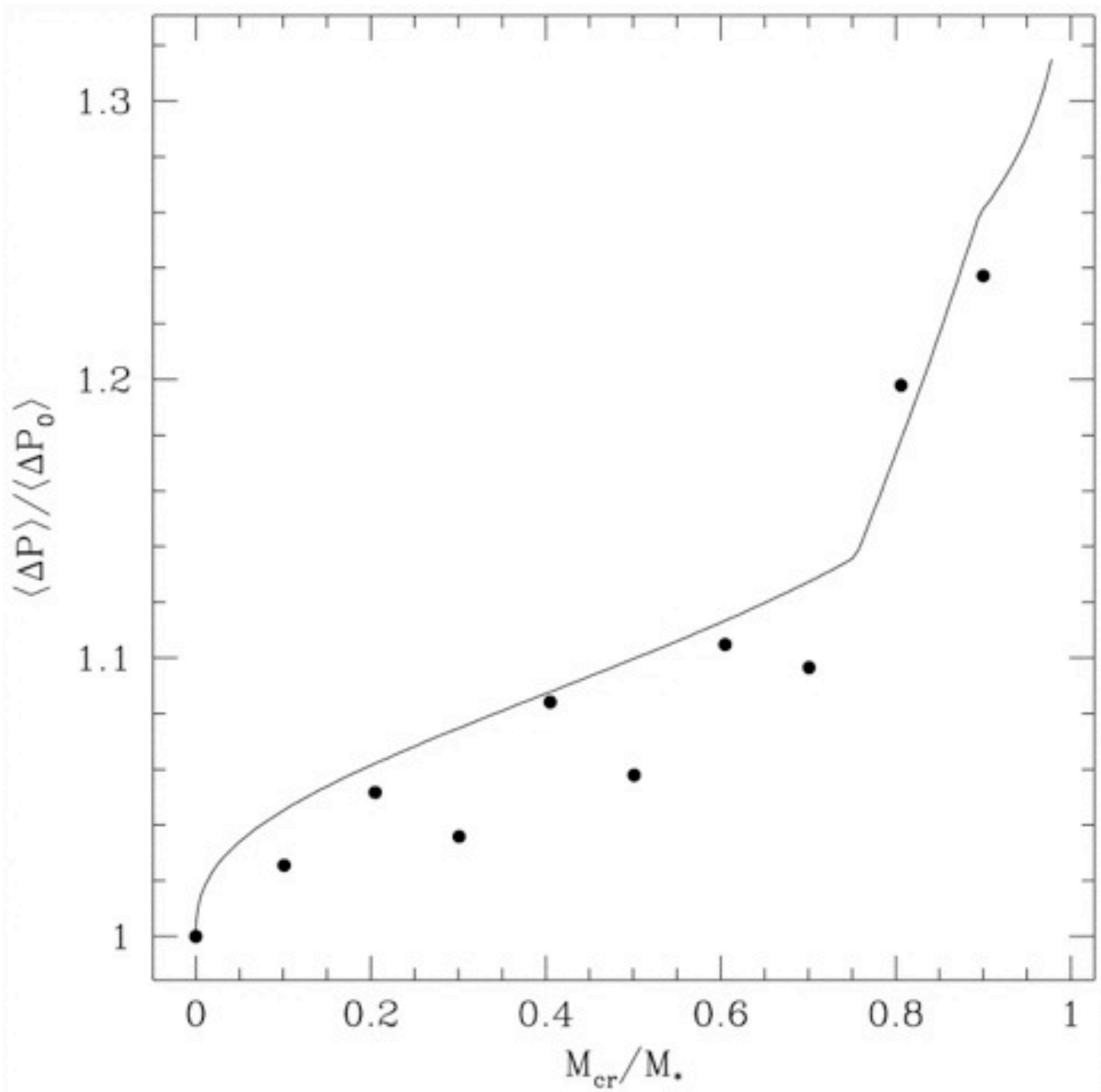
Credit: Travis Metcalfe and Ruth Bazinet
Harvard-Smithsonian Center for Astrophysics

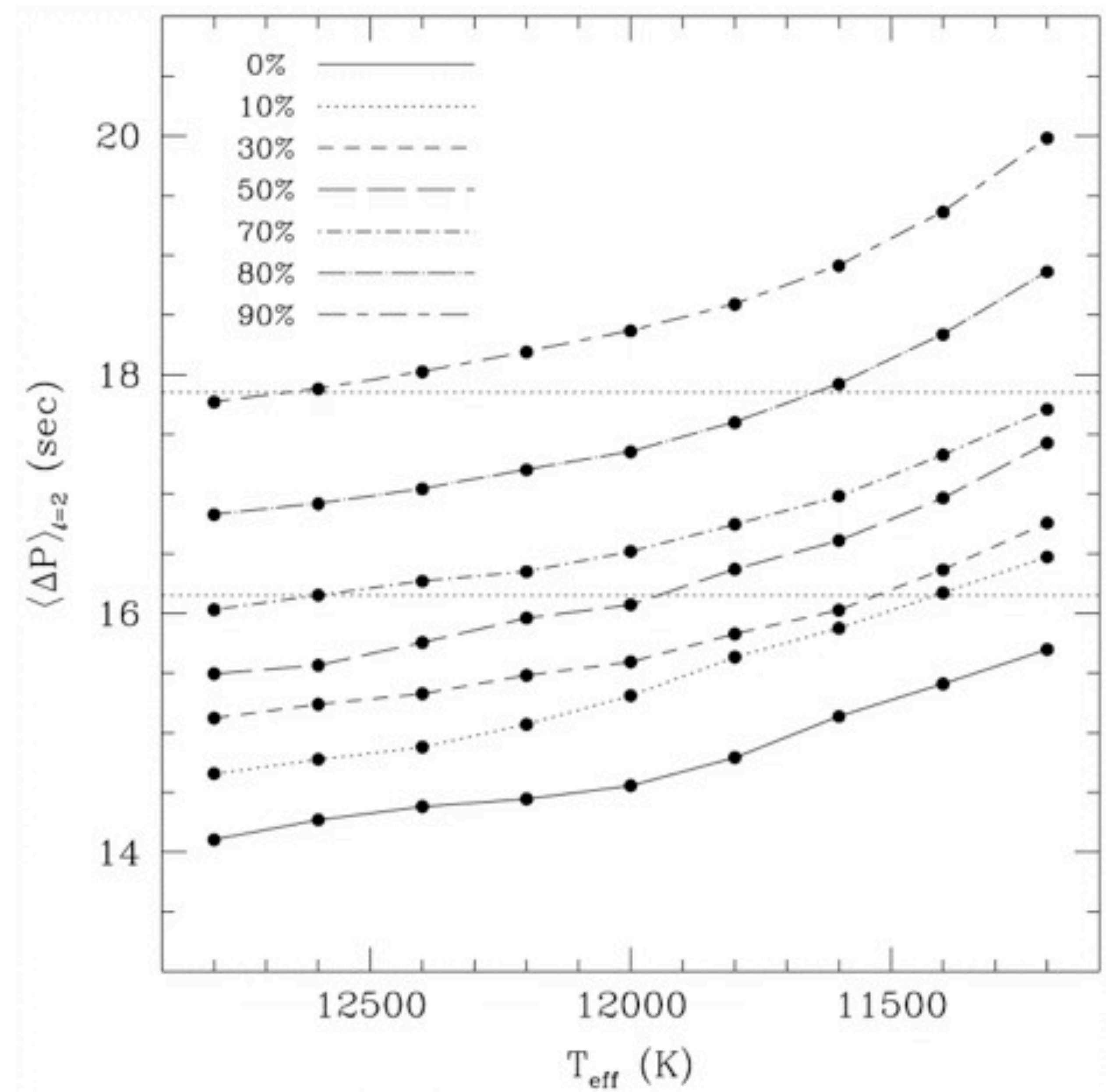
Pulsating DA white dwarfs



Montgomery & Winget 1999

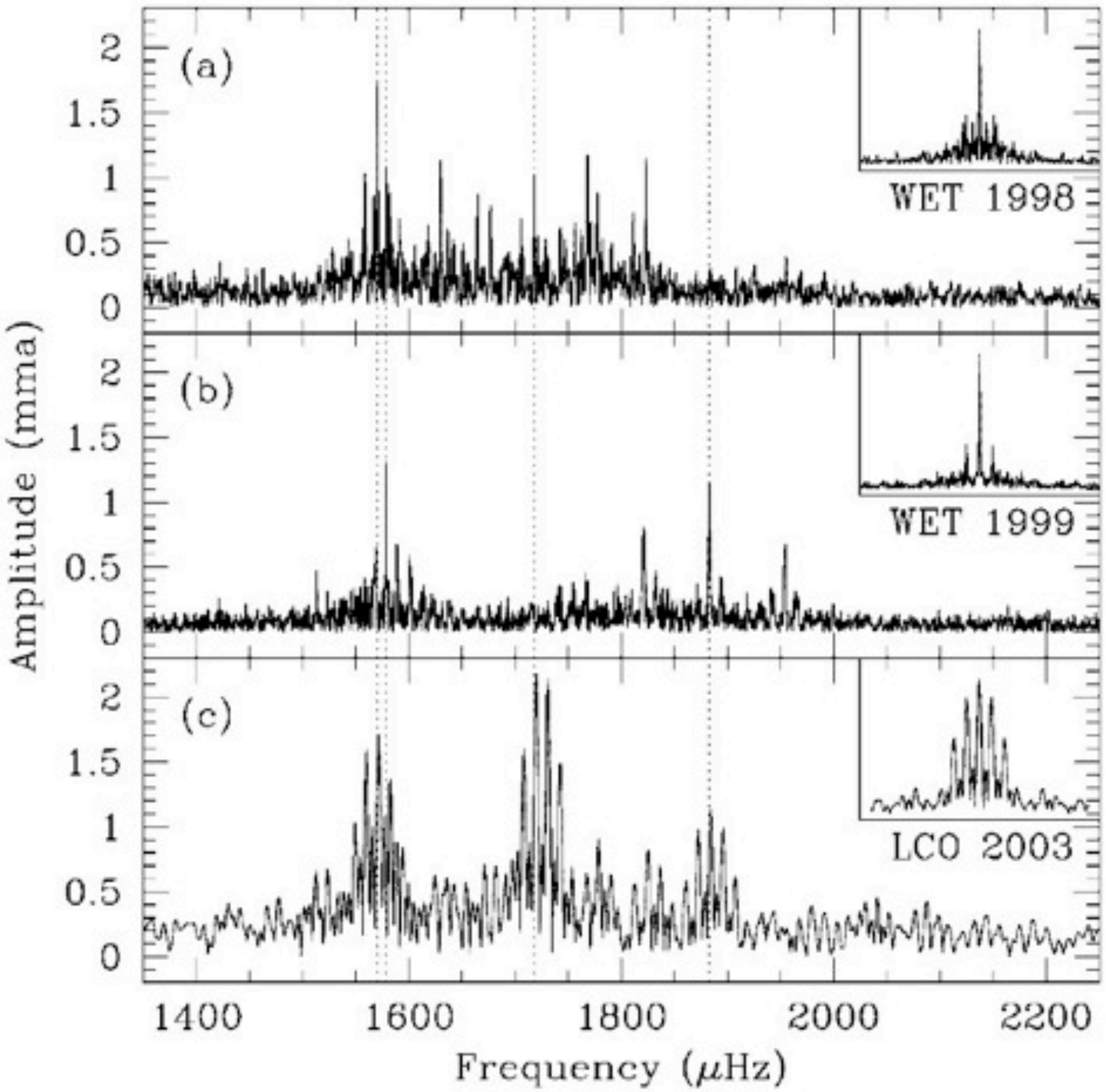
Montgomery & Winget 1999





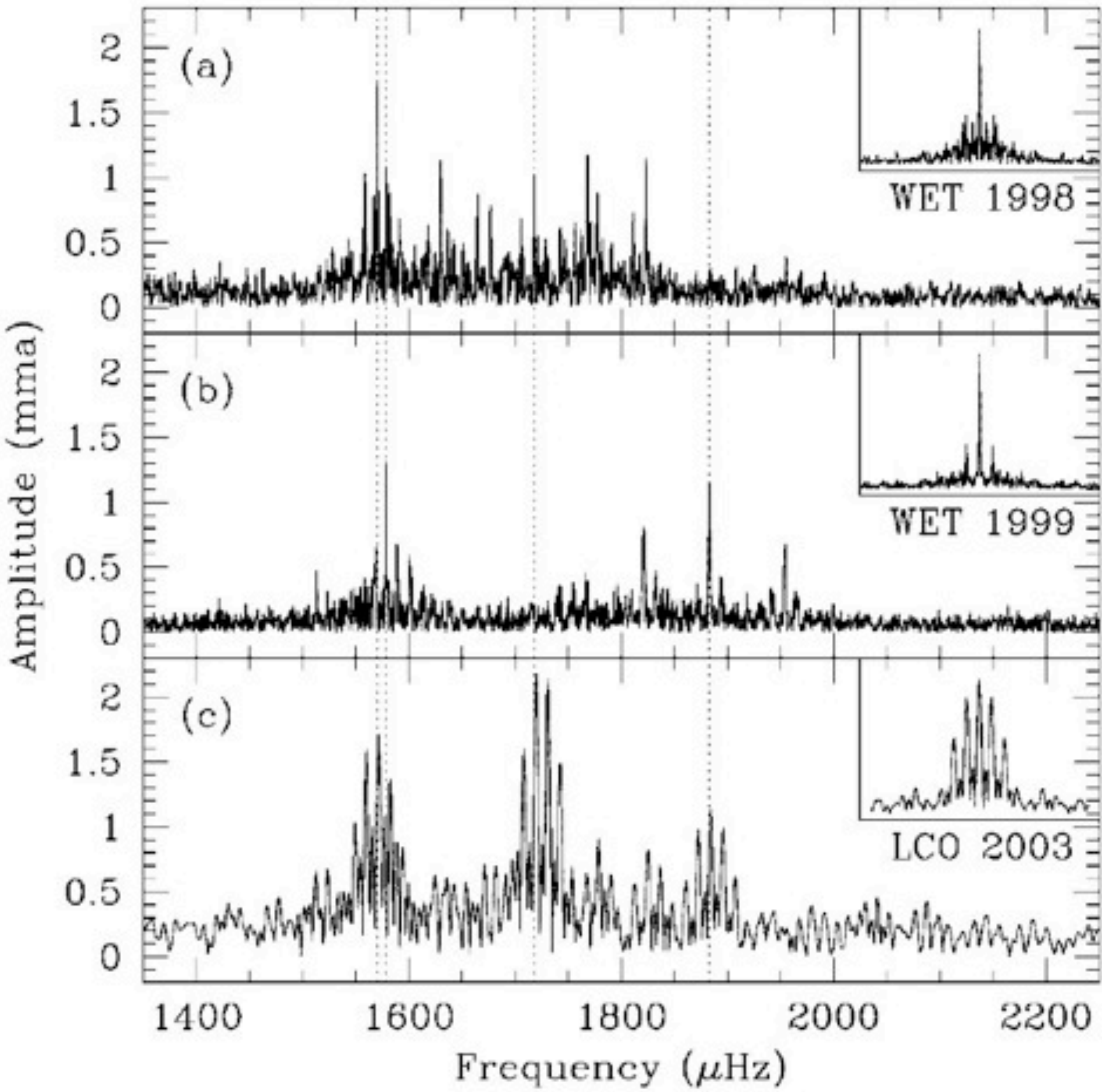
period spacing vs. crystallization fraction

Dependence on T_{eff}



BPM 37093: "the diamond star"

- 8 modes, unknown *l, n*
- period range 511 s to 636 s
- M, T spectroscopy constraints on models:
  - 12 available modes, *l*=1,2



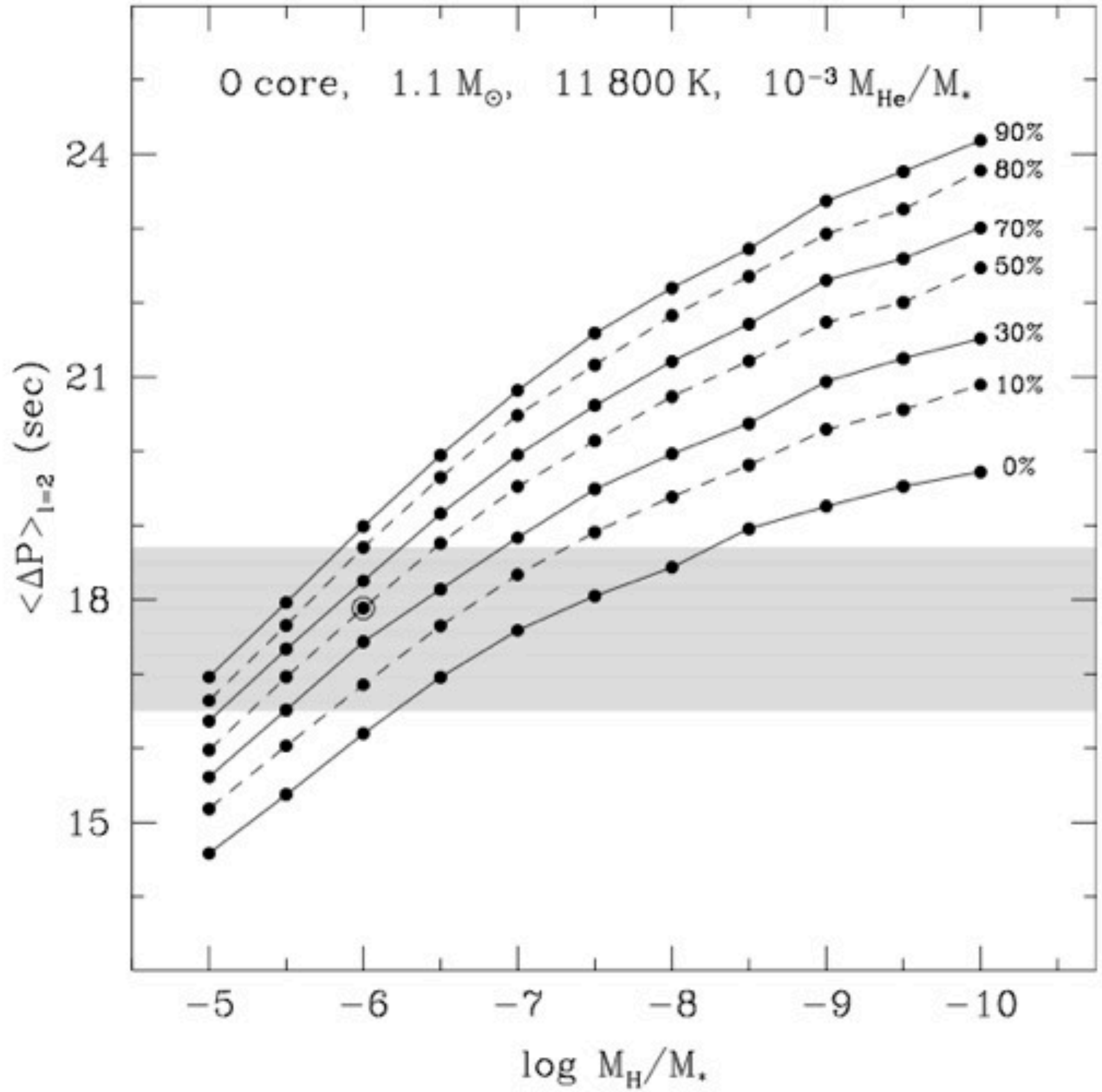


TABLE 2

FIXED-MASS OPTIMAL MODELS FOR BPM 3[illegible]

PARAMETER	1.00 $M_\odot$ Pure C	1.00 $M_\odot$ Pure O	1.03 $M_\odot$ Pure C	1.03 $M_\odot$ Pure O	1.10 $M_\odot$ Pure C	1.10 $M_\odot$ Pure O
$T_{\rm eff}$ (K)	13,700	14,500	11,100	11,300	10,500	11,500
log $(M_{\rm He}/M_*)$	−2.20	−2.00	−2.12	−2.26	−2.60	−2.22
log $(M_{\rm H}/M_*)$	−4.20	−5.00	−4.28	−4.36	−4.60	−5.64
$M_{\rm cr}/M_*$	0.90	0.90	0.90	0.90	0.90	0.90
σ_P (s)	1.24	1.04	1.08	1.14	0.95	0.86

Metcalfe et al. 2004,
Kanaan et al. 2005

BPM 37093: “the diamond star”

THE SUNDAY TIMES
ABSOLUTELY FRANK | E-COLI MENACE | £50,000 PLAY WORLD CUP FANTASY FOOTBALL | ARSENAL'S TWIN PEAKS
17 MAY 1998 · THE SUNDAY TIMES

Astronomers find a diamond in the sky

by Trushar Barot

ASTRONOMERS believe they have located the first star made of a solid diamond the size of Earth. Barely visible in the constellation Centaurus and at 17 light years from Earth — a stone's throw in galactic distances — it is one of the closest stars in the Milky Way.

It is known only by the unromantic designation of BPM37093, although its status is to be transformed by the scientists, who have amassed evidence of its gem-like qualities.

For 30 years astronomers have wondered whether diamonds could be present under the unusual conditions of a white dwarf, when a star's nuclear fuel has run out and its redundant ash of carbon and oxygen continue to smoulder at a modest 12,000C.

Now they think they are on the verge of a discovery that could make this star every girl's best friend. "We think BPM37093 is primarily made up of carbon and oxygen in a crystallised state," said Steve Kawaler of Iowa State University. "That would make it a diamond with a blue-green tint. This truly could be a diamond in the sky."

The intense gravity of a white dwarf creates the enormous pressures that enable carbon at such high temperatures to exist as a crystal, Kawaler said. "A white dwarf is a very, very dense star. One teaspoon of matter from these stars weighs a ton."

A diamond the size of Earth would give BPM37093 a carat weight of 10 billion trillion trillion, according to the Iowa scientists. The world's biggest diamond, owned by the diamond mining company De Beers, weighs 1,462 carats.

“A diamond the size of Earth would have a carat weight of 10 billion trillion trillion. The world's biggest diamond weighs 1,462 carats”

Scientists believe the diamond star will help them to estimate the age of the Milky Way and ultimately the universe. They theorise that BPM37093 could be 11-12 billion years old.

"This finding will extend the lower limit of the age of the Milky Way and, in turn, extend the estimated age of the universe," said Kawaler.

The Iowa researchers used the Hubble Space telescope to examine the star, notching up another remarkable success for astronomy. Earlier this year, scientists announced they had used the telescope to observe the first planet in orbit around a neighbouring star. It has always been assumed that around many of the stars in the night sky there are planets like our own, but until now none has been seen. The discovery of the planet around Proxima Centauri heightened the possibility that we may eventually find another Earth. Even more dramatic were the first photographs taken by the telescope of star birth at the centre of the galaxy, amid giant curls of gas and dust.

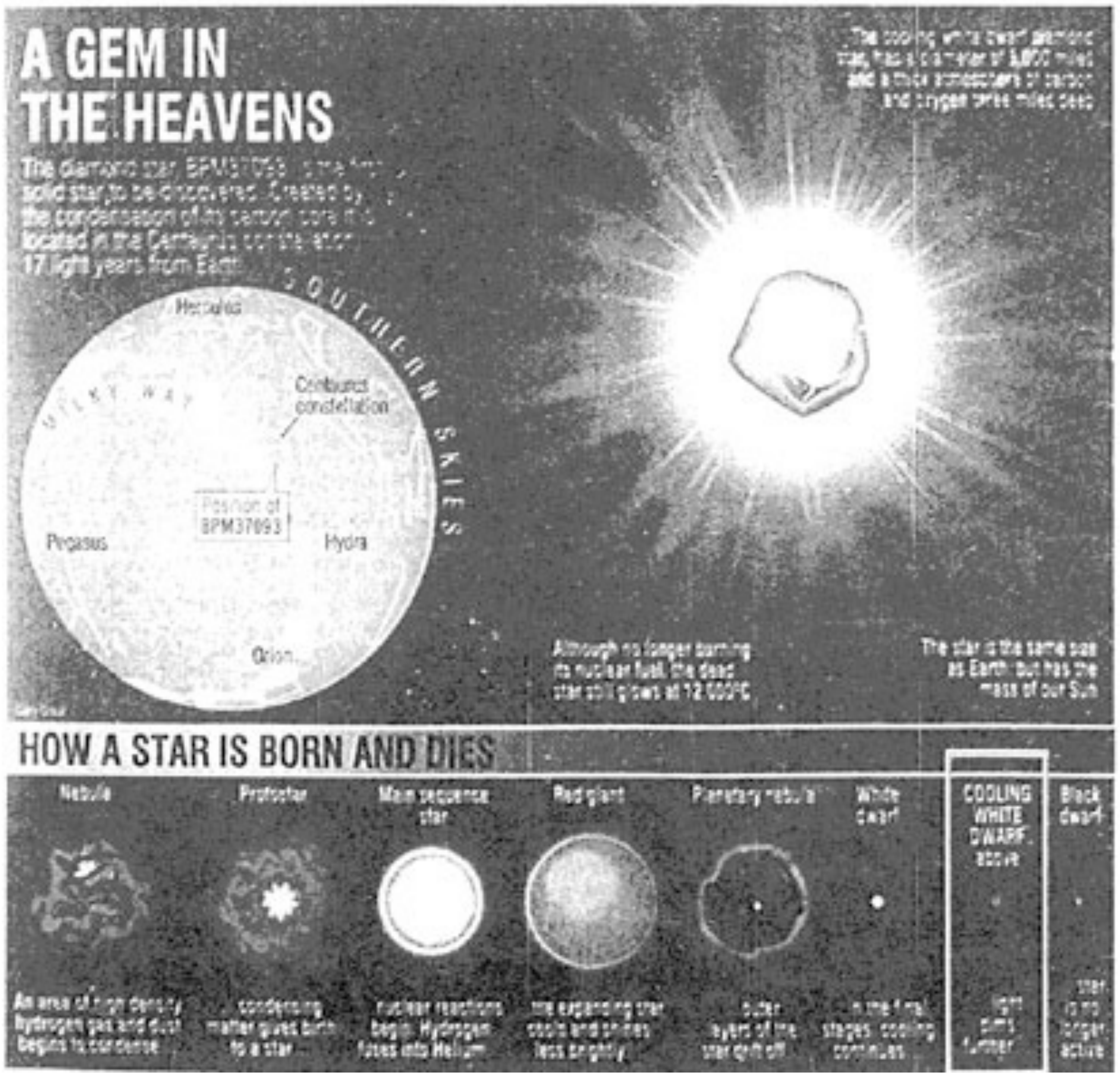
The most famous of astronomy's latest discoveries is of ice on the moon by Nasa's Lunar Prospector satellite, giving a boost to the long-term prospects of deep-space exploration. The presence of water could allow for the construction of a manned space station on the moon as a staging post for journeys into the darkest corners of the solar system.

Scientists have ruled out the possibility of life on planets around the diamond star. Ralph Wijers, an astronomer at the Institute of Astronomy in Cambridge, said: "The luminosity of an old white dwarf star gives out a lot less energy than our sun, so it would be difficult for life to exist. The more likely places to look for life are closer to home, such as Europa and Titan, the moons orbiting Jupiter and Saturn."

The diamond star's huge density and temperature mean human expeditions will not be possible, even if it could be reached. No human could survive the gravitational pressures, said Martin Barstow, an astronomer at Leicester University and co-ordinator of the Whole Earth Telescope project, a network of astronomers. "Everything would be flat. It simply would not be possible to have structures like houses or buildings. Everything would be compressed out of all recognition. The atmosphere would be five or six kilometres deep, like an incredibly thick blanket," he said.

Could there be planets of gold and silver out there somewhere? Barstow believes not. "It's only because carbon is the most abundant material that gets produced in these stellar reactions that the formation of a diamond structure is possible. But there are some exotic reactions between atoms and ultraviolet rays going in space, and there is no telling what formations could be out there," he said.

The world's diamond market is not in immediate danger of collapse. De Beers said: "As to whether this would pose a threat to our business, our geologists are on the case."



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non-resonant reaction rates

- simplest general form

$$\langle \sigma v \rangle_{ij} = \frac{K_0 S(0)}{Z_i Z_j} T^{-2/3} e^{-K_3 T^{-1/3}}$$

- S(0) is a quantity (related to the cross section at 0 energy) extrapolated from measurements at higher energy (why?)

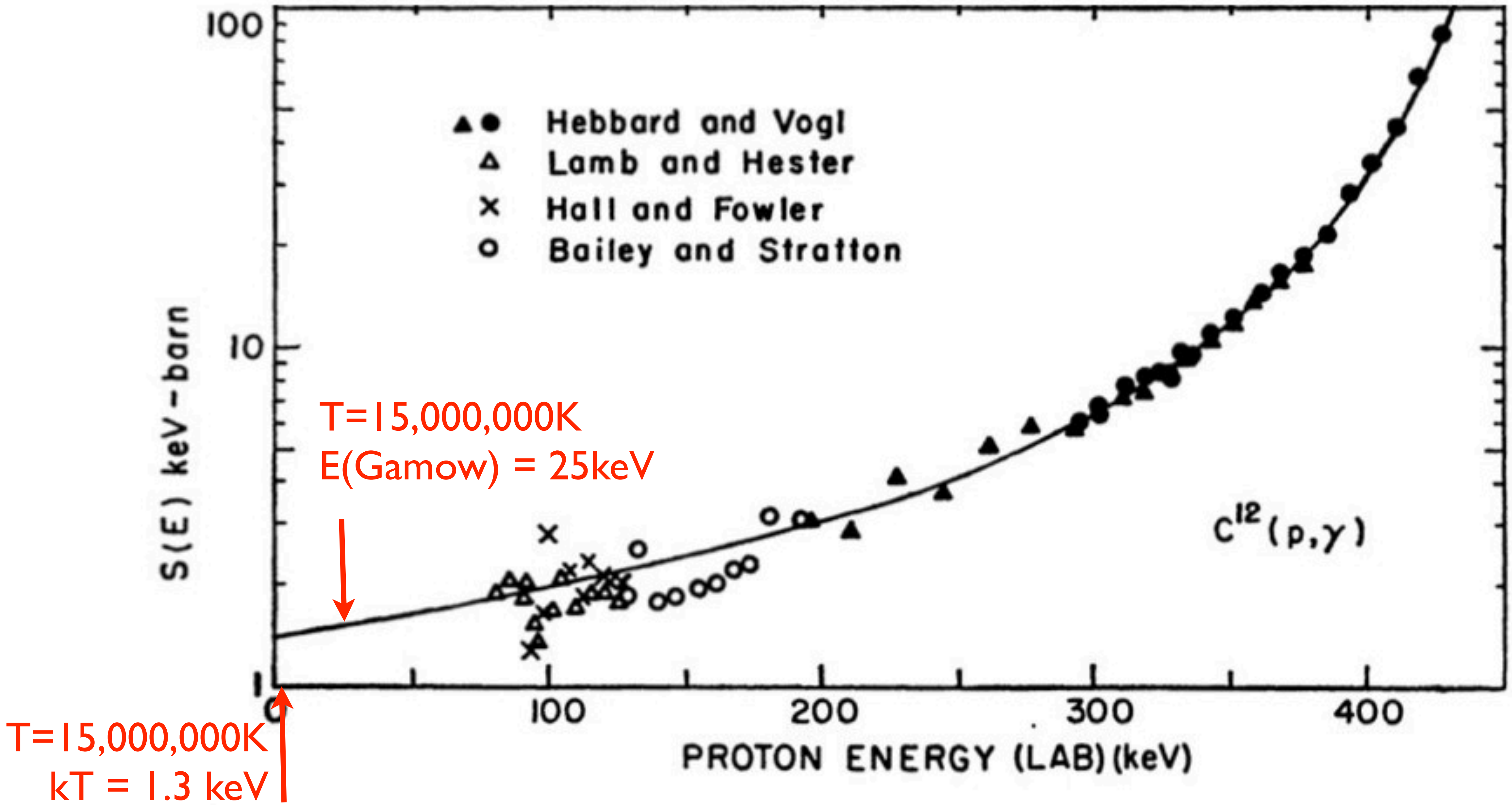


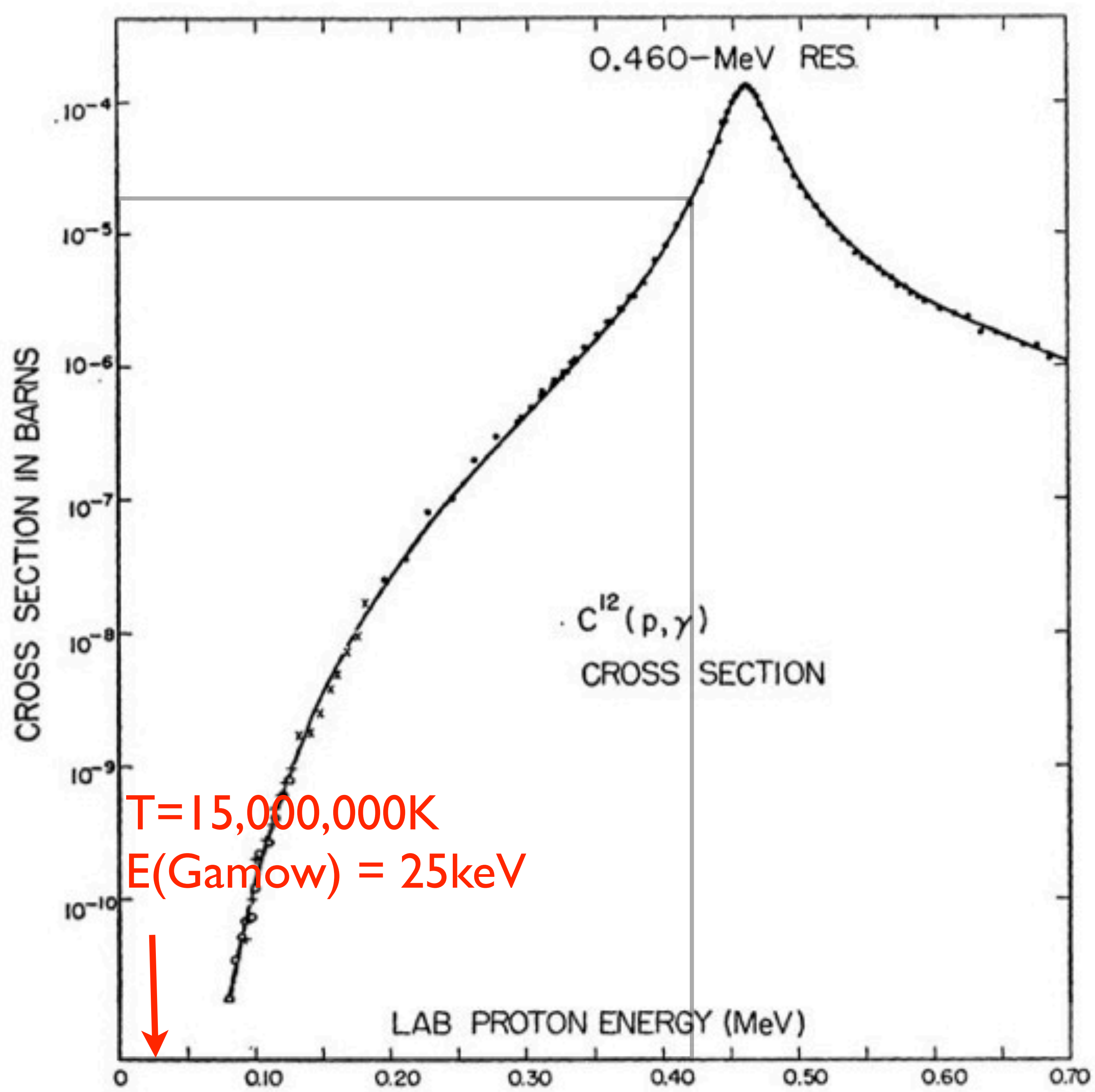
FIG. 3. The dependence on energy of the cross-section factors for $C^{12}(p,\gamma)N^{13}$

resonant reaction rates

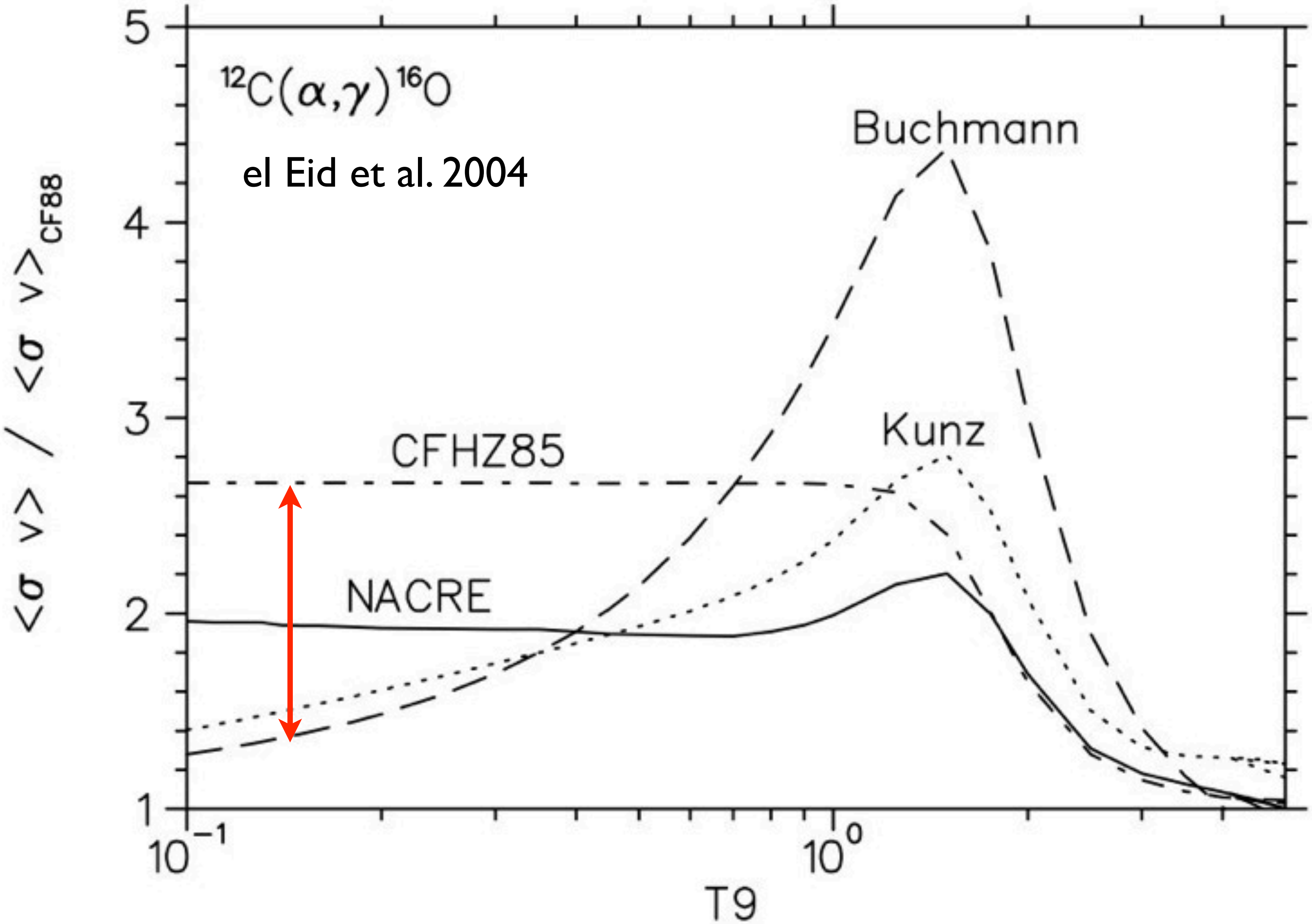
- Simplest general form

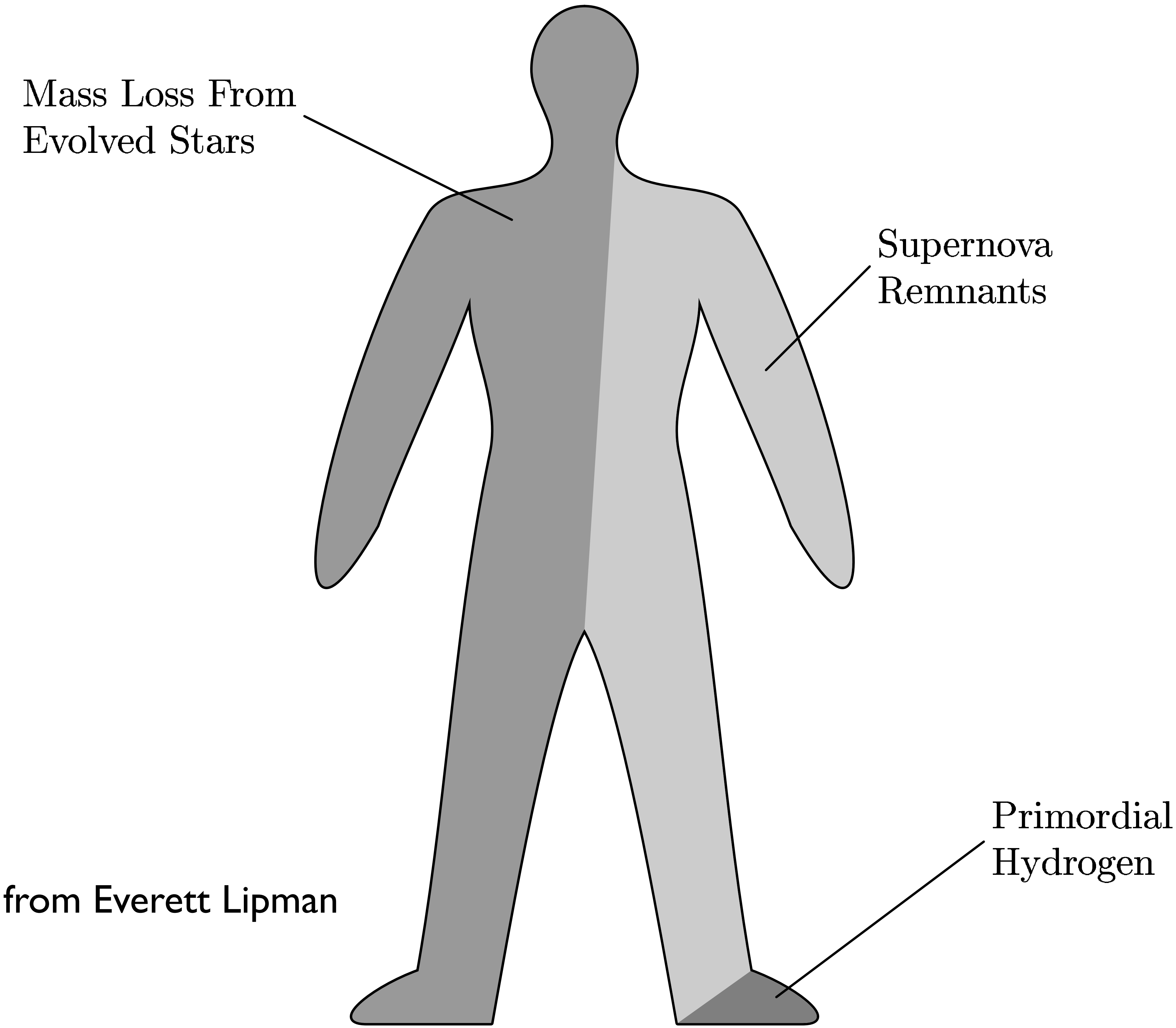
$$\langle \sigma v \rangle_{ij} = K_1\, g \frac{\Gamma_i \Gamma_j}{\Gamma} T^{-3/2} e^{-K_2/T}$$

- where the resonance at $E = E_{res}$ selects a single energy from the particle energy distribution



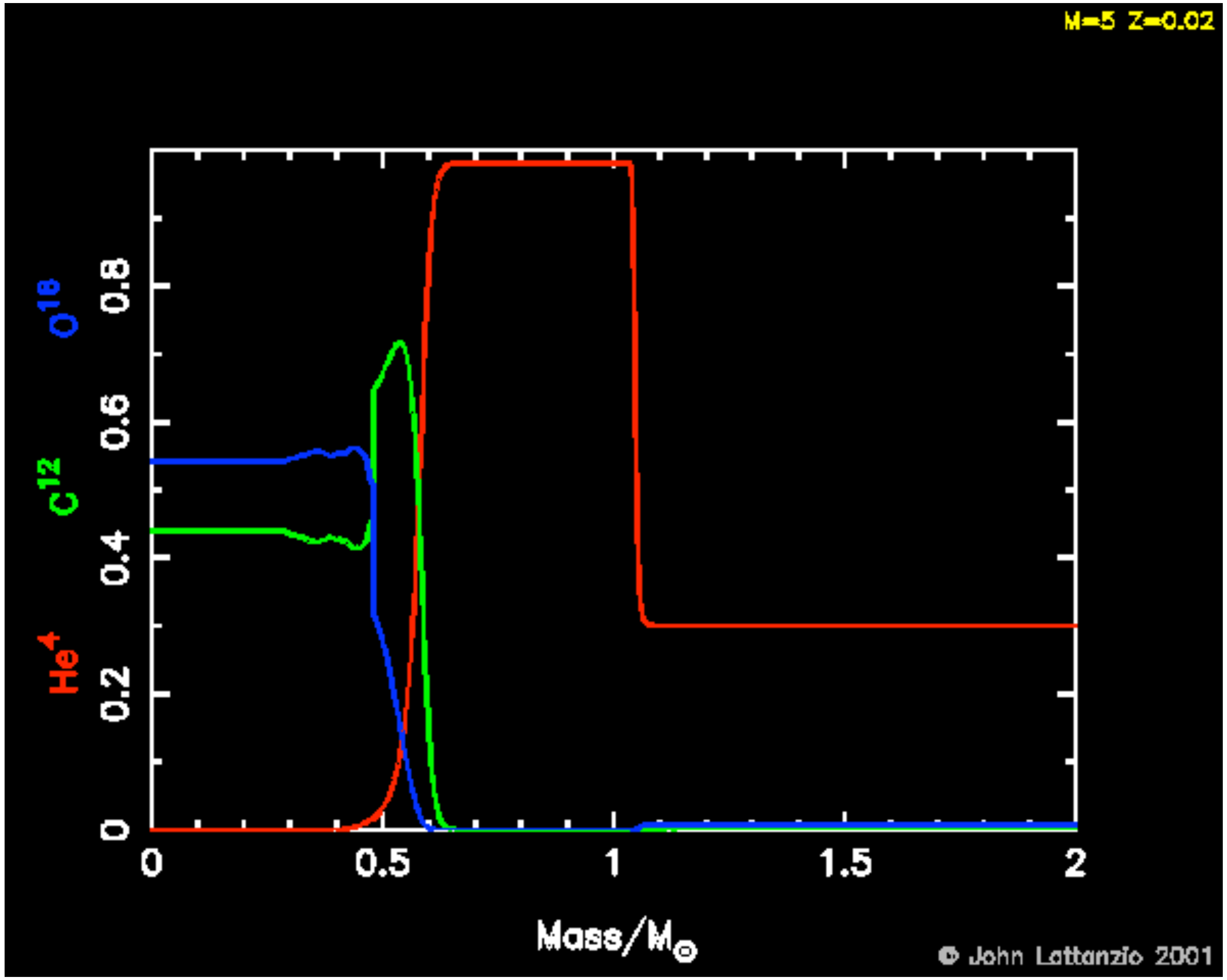
$^{12}C + {}^{4}He \rightarrow {}^{16}O$: rate poorly known





from Everett Lipman

core helium burning

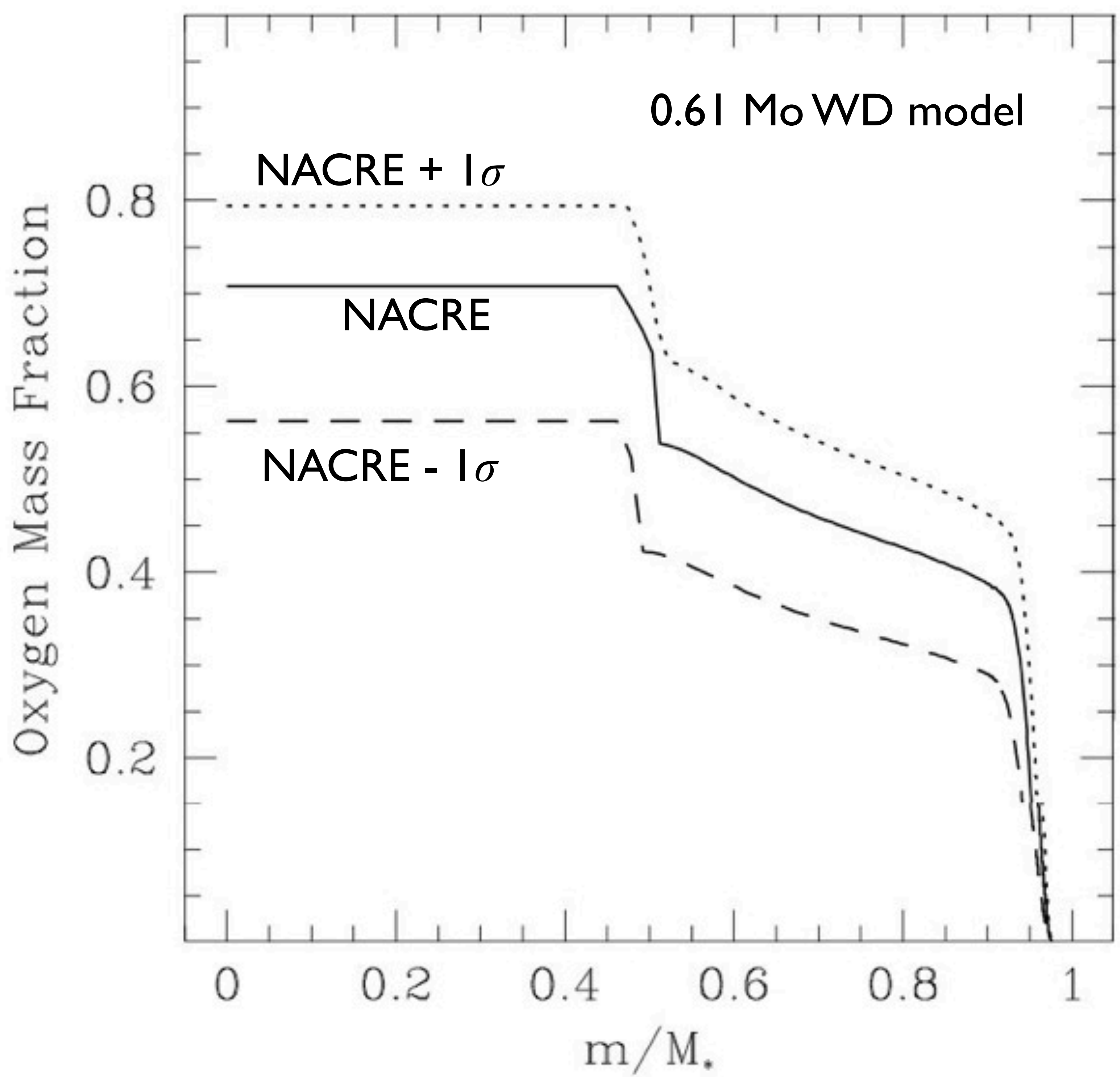


From John Lattanzio's online stellar evolution tutorial

- 3 ^{4}He ⇒ ^{12}C (triple alpha)
- as temperature increases, see more ^{12}C+^{4}He ⇒ ^{16}O
- final ^{16}O/^{12}C depends on the reaction rate of ↑

C/O WD core (Metcalfe et al. 2002)

post-helium burning core abundance profiles



DB pulsator fits - core C/O profile
(Metcalfe 2003)

- compositional stratification in two DB pulsators
- period spacings - composition transition zones
- periods - composition 'calibration' given external constraints on T_{eff} and log g

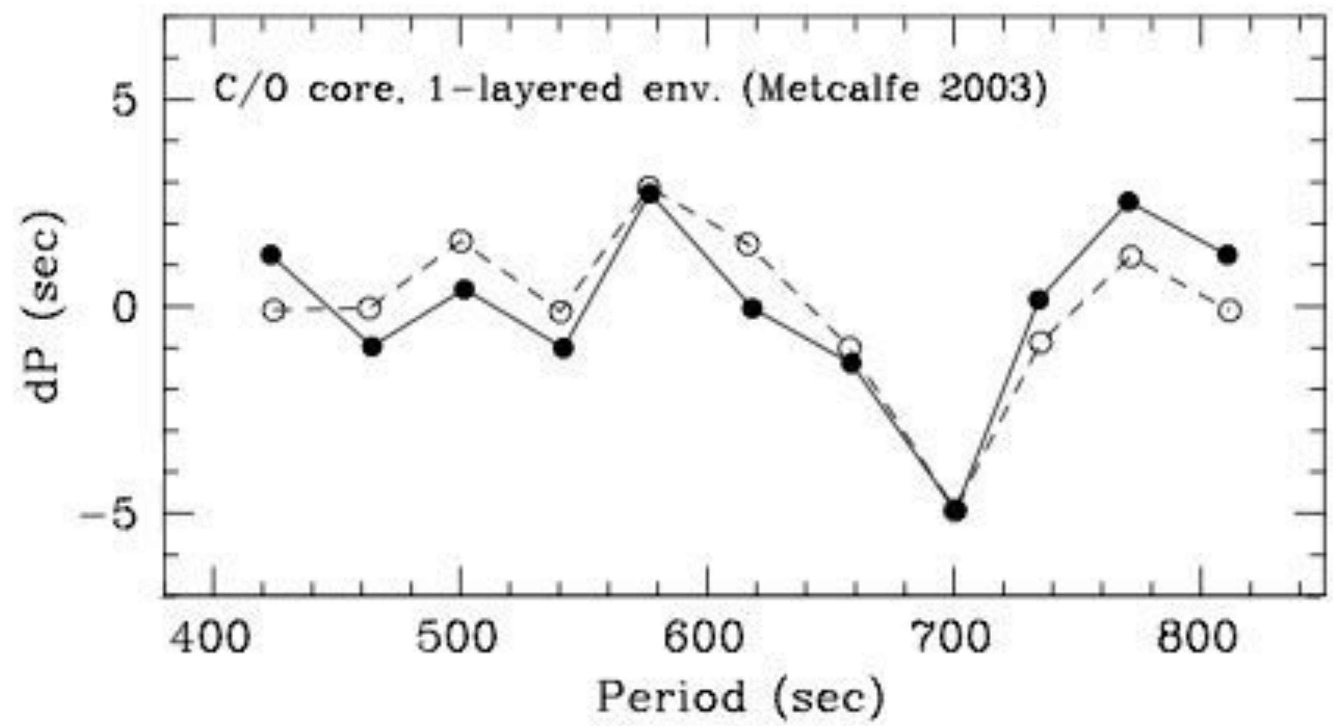
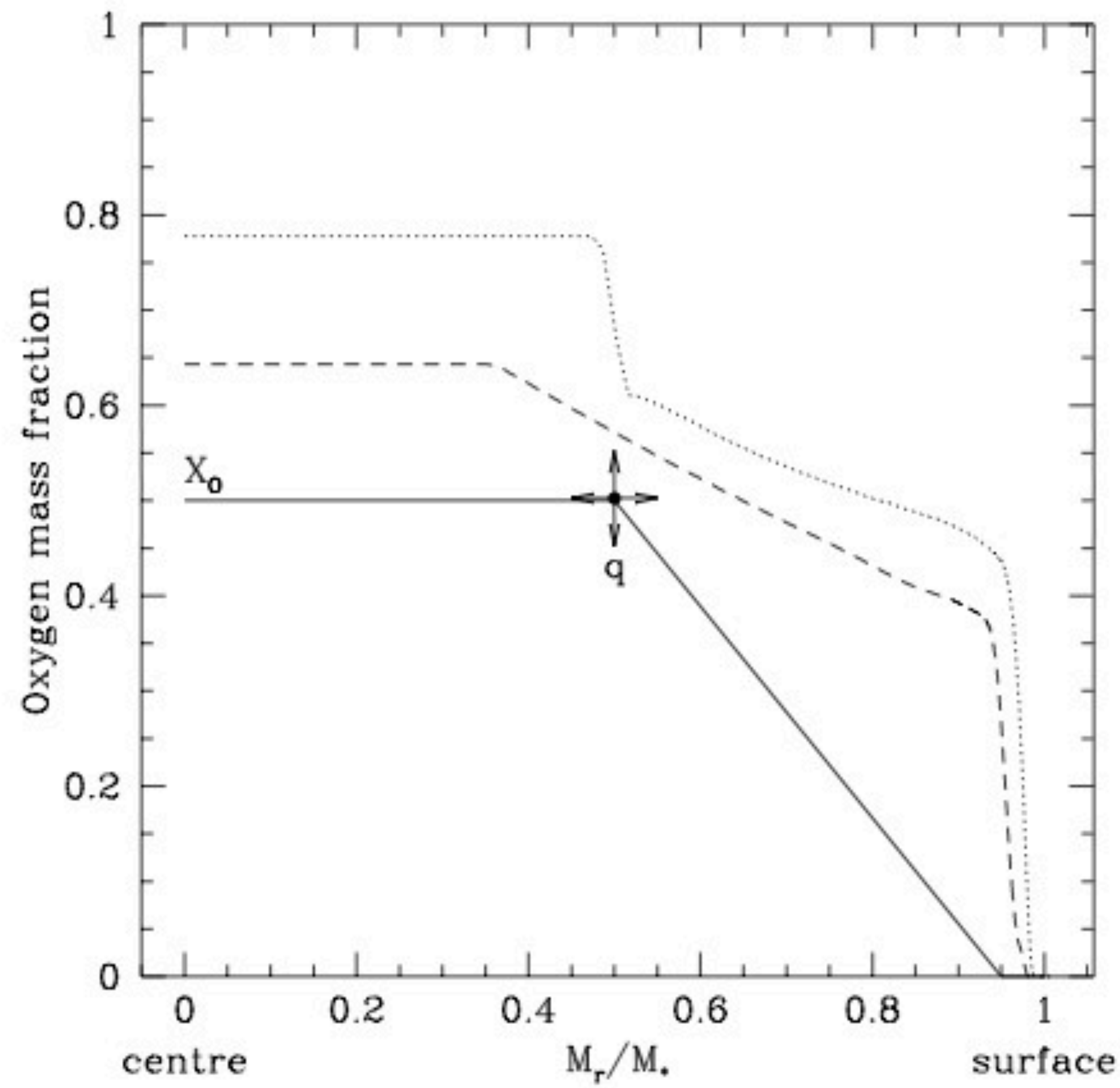


Figure 1. The pulsation periods observed in GD 358 (solid points) plotted against their deviations from the mean period spacing (dP), along with two physically distinct model fits (open points). The fit of Metcalfe (2003) has extra structure in the core



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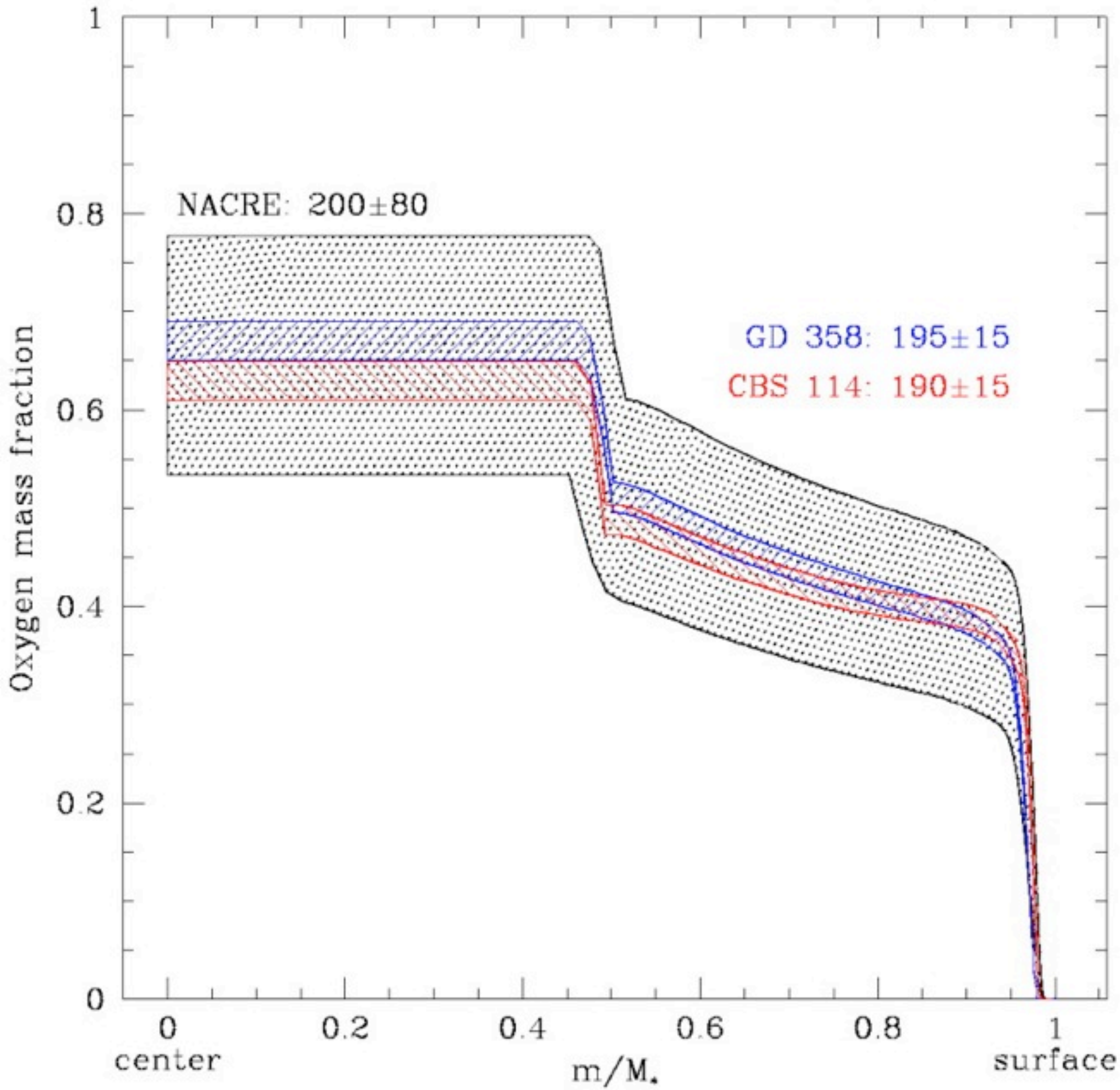


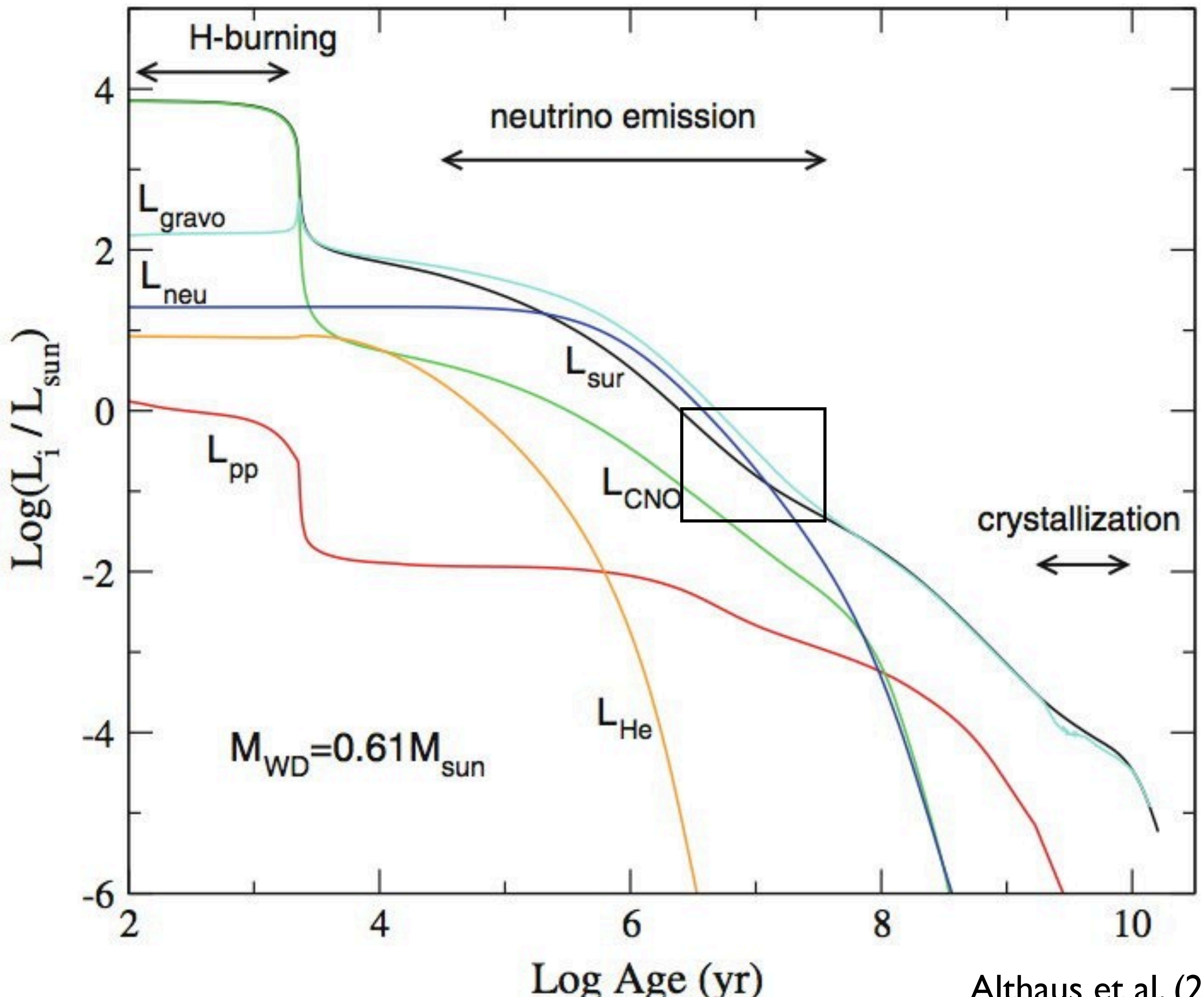
(thermal) neutrino emission

- "Coolant" in white dwarf evolution (ν produced, energy carried away, but not thermalized in stars → net energy loss ($\varepsilon < 0$)
  - 'Bremsstrahlung neutrinos':
    ν instead of γ in free-free scattering of ions, e^-
  - 'plasmon neutrinos': $\gamma^* \rightarrow \nu + \bar{\nu}$
    photon in plasma couples with the plasma, allowing it to decay and still conserve momentum and energy

- Energy loss mechanism in NS production (supernova):
  - 'pair neutrinos': $\gamma + \gamma \rightarrow e^+ + e^- \rightarrow \nu + \bar{\nu}$
  - 'photo neutrinos': $\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$

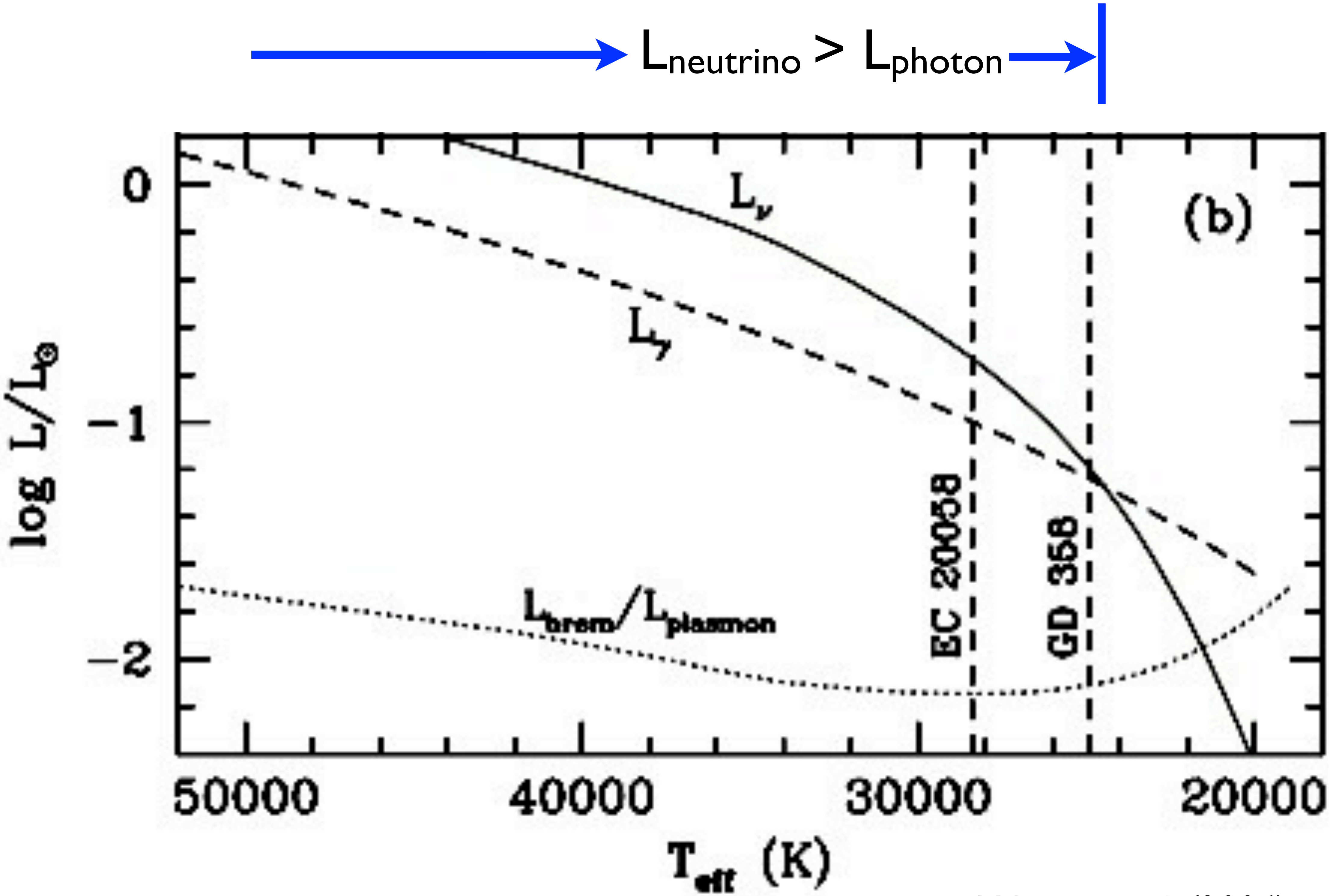
- **These production rates can be computed via QED, but have never been tested experimentally.**

neutrino emission from white dwarfs



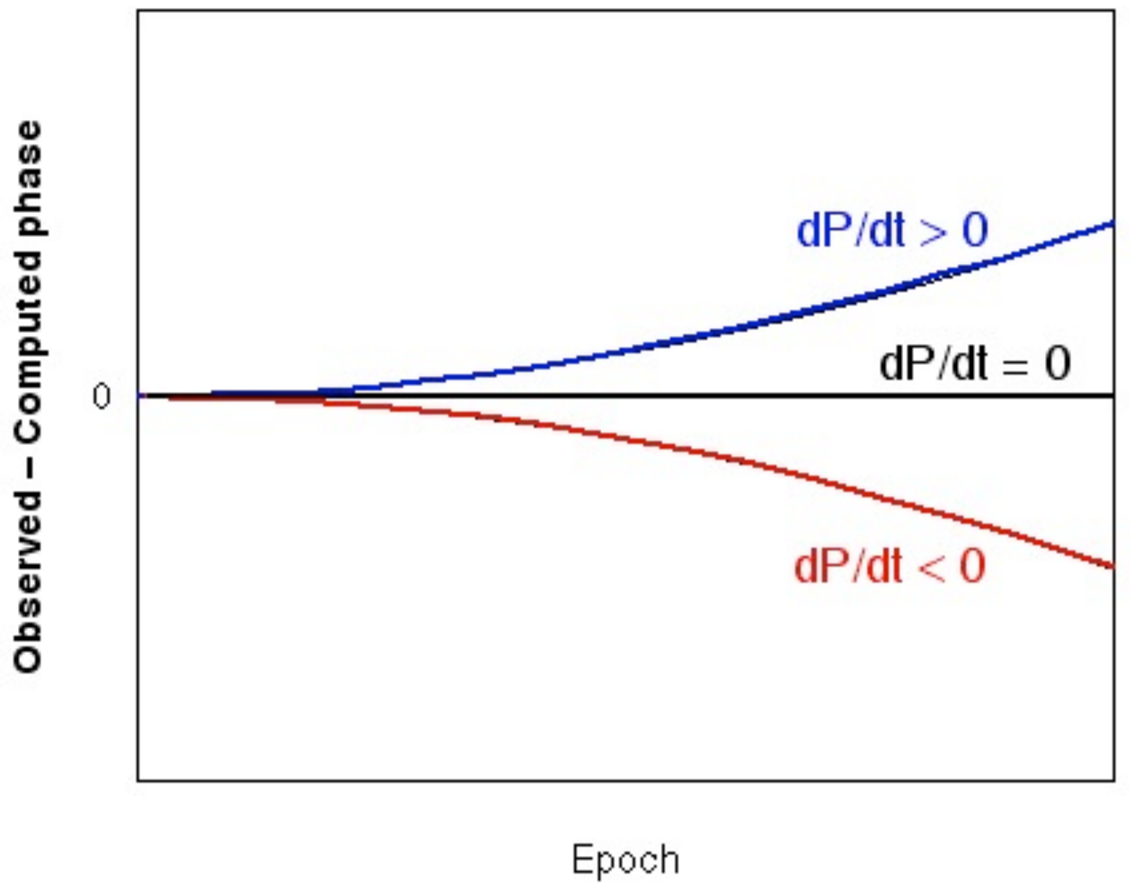
Althaus et al. (2010)

neutrino emission from white dwarfs



Winget et al. (2004)

deduce period changes via phase changes using (O-C) diagram



- secular change via stellar cooling - stellar evolution while you watch
- reflex orbital motion - low-mass companions

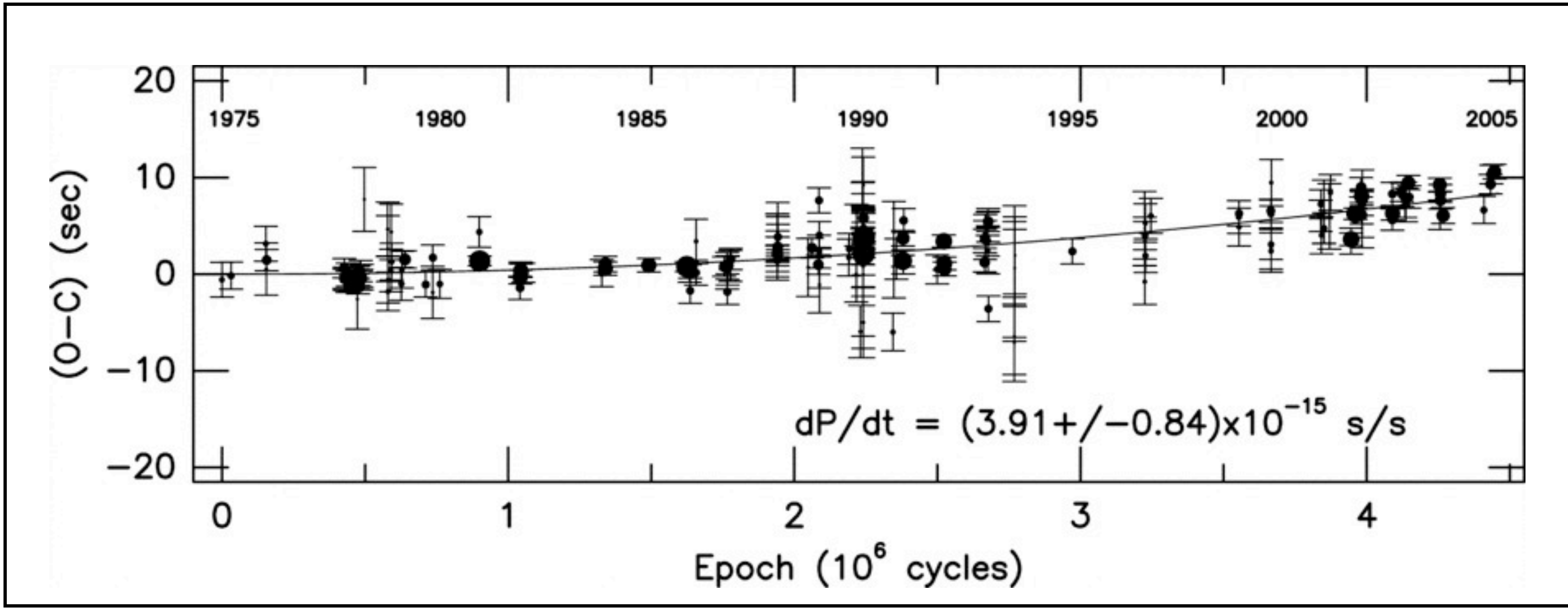
$$(O - C) \; = \; (\Delta t)^2 \frac{1}{P} \frac{dP}{dt}$$

for nonradial pulsations (g-modes)

$$\frac{1}{P}\frac{dP}{dt} = \frac{a}{R}\frac{dR}{dt} - \frac{b}{T}\frac{dT}{dt}$$

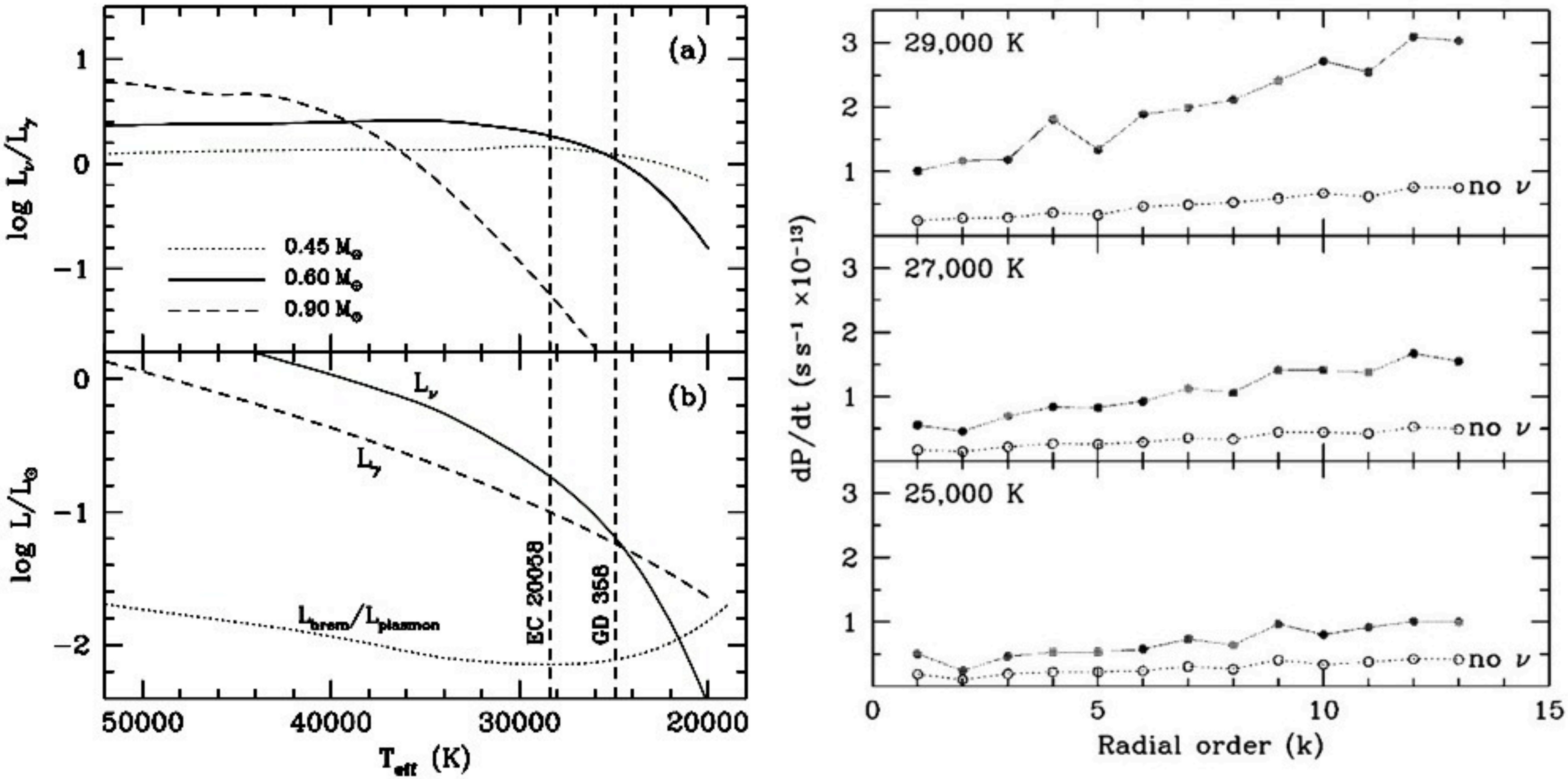
- contraction / heating
  - period decrease with time
- expansion / cooling
  - period increase with time

G117-B15a: a very stable optical clock (Kepler et al.)



consistent with WD cooling models as long as core composition lighter than neon - soon constrain C/O ratio!

neutrino signal in DB white dwarfs



DB white dwarfs (Winget et al. 2004)

how long to measure dP/dt?

$$(O - C) \ = \ (\Delta t)^2 \frac{1}{P} \frac{dP}{dt}$$

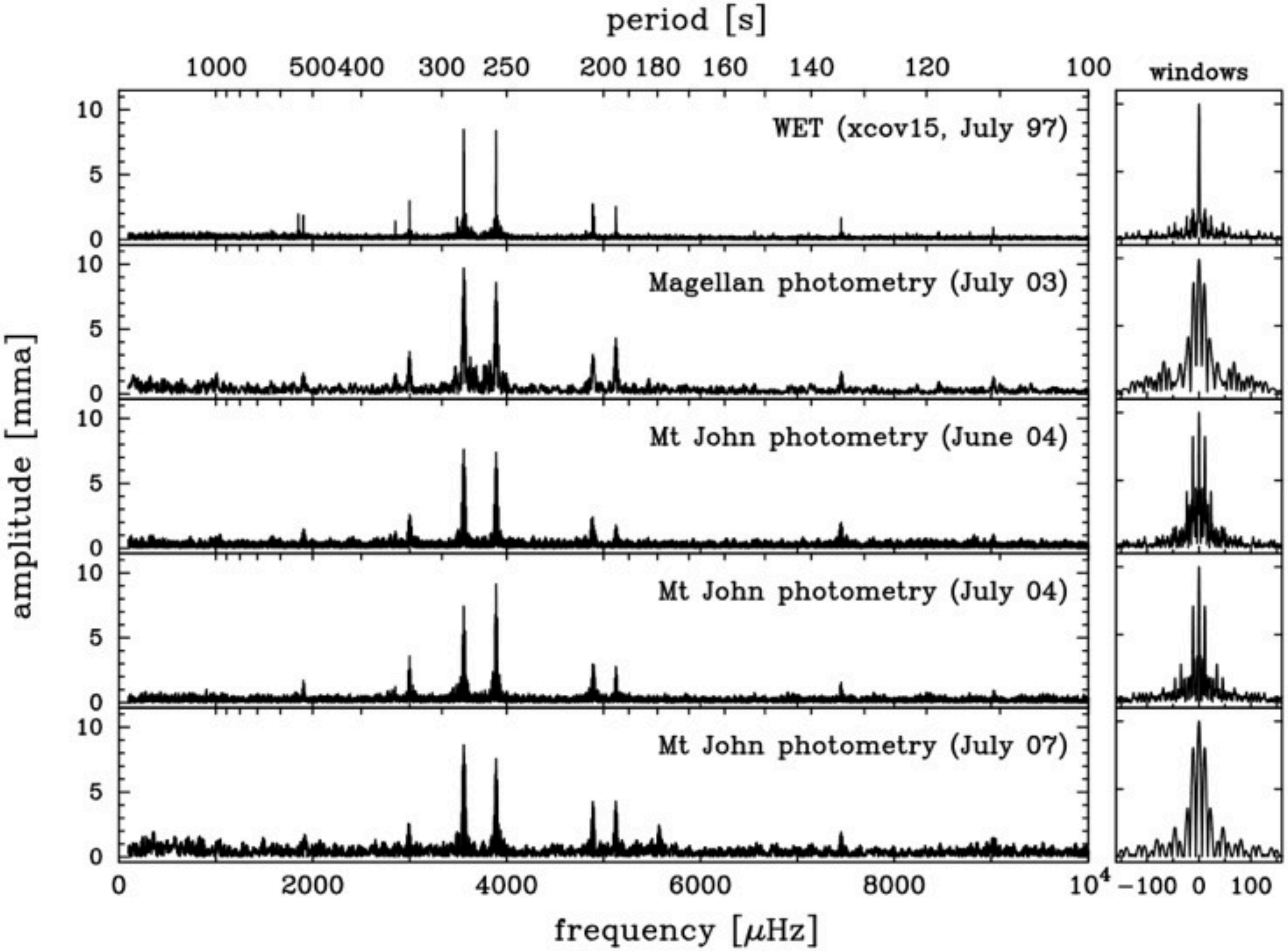
- P ~ 300 s ; dP/dt ~ 10^{-13}
- 1/P(dP/dt) ~ 3×10^{-16} s^{-1} ($\tau \sim 10^8$ years)
- phase uncertainty in a ‘good’ run ~ 10 s
- 10s = (O-C) = $(\Delta t)^2 \times 3\times10^{-16}$

  $(\Delta t)^2 = 10$ s / 3×10^{-16} s^{-1}

  $\Delta t \sim 3\times10^{16}$ s ~ 6 years for a measurable (O-C)

best candidate: EC20058

Denis Sullivan - 1997 - 2011+



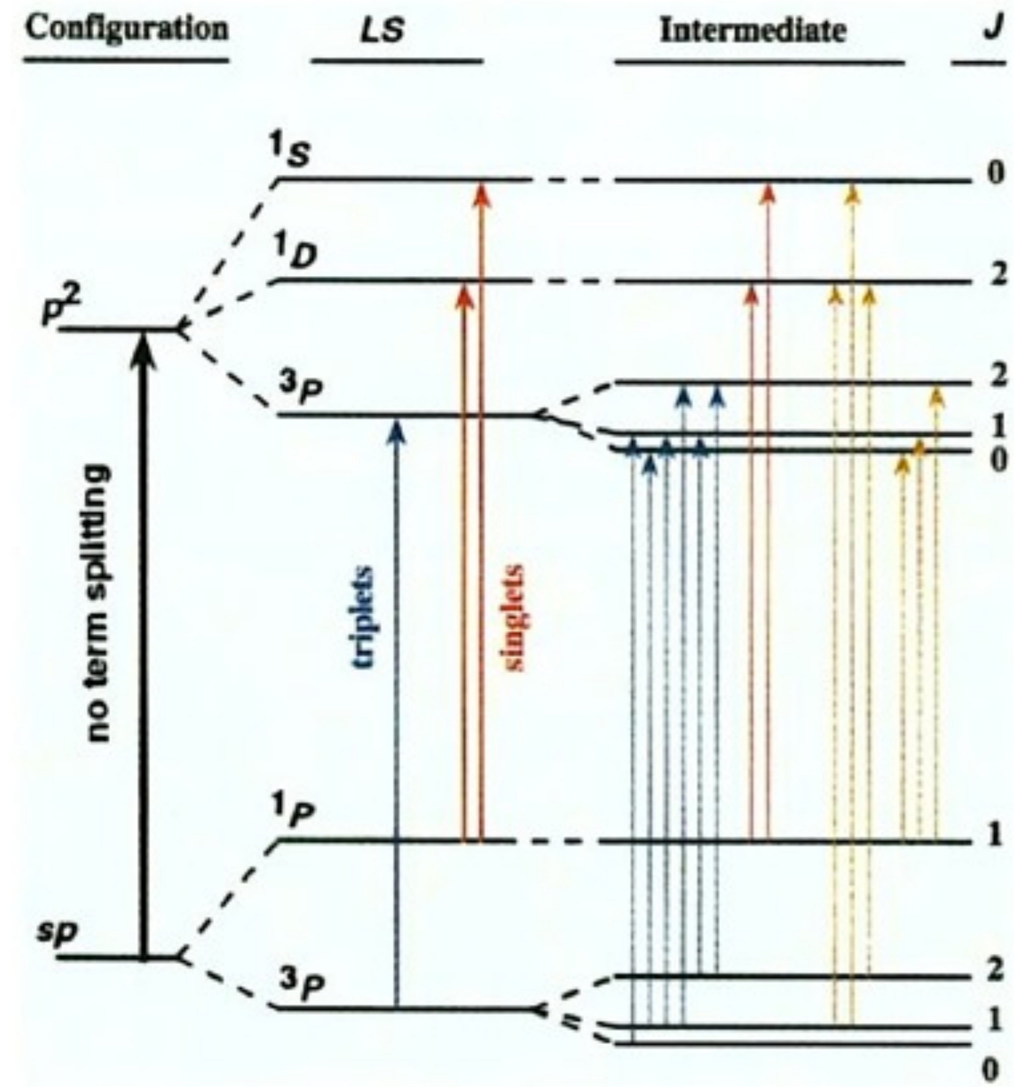
we're getting there...

Energy Transport - radiative

- As posed, it is contained in ∇: $$\frac{dT}{dM_r} = -\nabla \frac{GM_r}{4\pi r^4}\frac{T}{P}$$
- photon diffusion ('radiative' heat transport): $\nabla = \nabla_{\rm rad}$
$$\nabla_{\rm rad} \equiv \frac{3\kappa_r}{16\pi ac}\frac{L_r}{T^4}\frac{P}{GM_r}$$
- κ_r - the radiative opacity:
  - flux: $F_\nu = -\frac{4\pi}{3}\frac{1}{\rho\kappa_\nu}\frac{\partial B_\nu}{\partial T}\frac{dT}{dr}$
  - integrate: $F_{\rm rad} = \int_0^\infty F_\nu d\nu = -\frac{4\pi}{3}\frac{1}{\rho\bar{\kappa}_r}\frac{dT}{dr}\int_0^\infty \frac{\partial B_\nu}{\partial T}d\nu$
  - where $$\frac{1}{\bar{\kappa}_r} \equiv \frac{\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B_\nu}{\partial T}d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T}d\nu}$$ $=acT^3/\pi$
  - so $$F_r = -\frac{4}{3}\frac{ac}{\kappa_r\rho}T^3\frac{dT}{dr}$$

about that κ_ν

Rogers & Iglesias (1994)

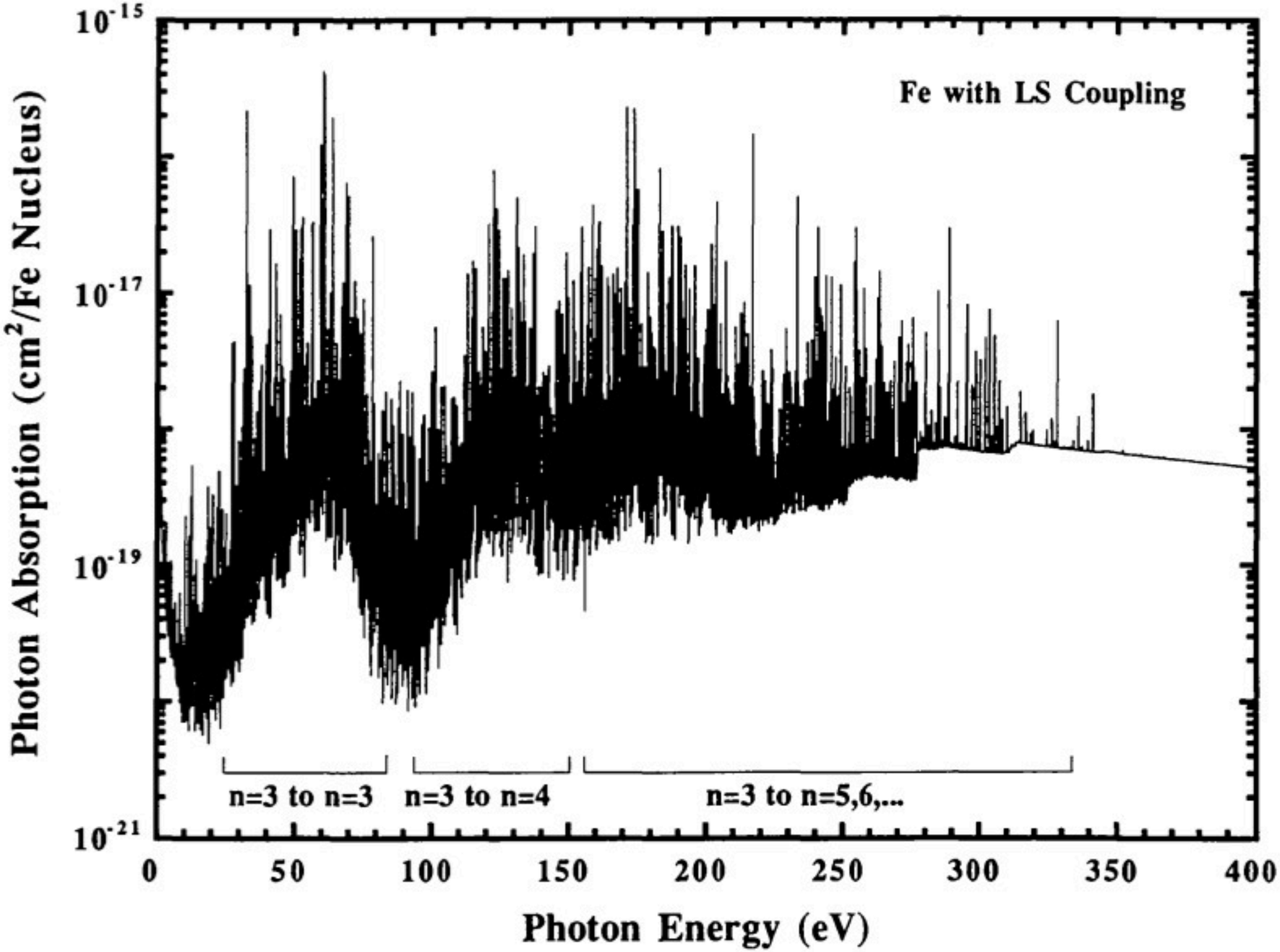


- atomic processes
  - electron scattering (easy...)
  - free-free scattering
  - bound-free absorption
  - bound-bound absorption - the messiest of all

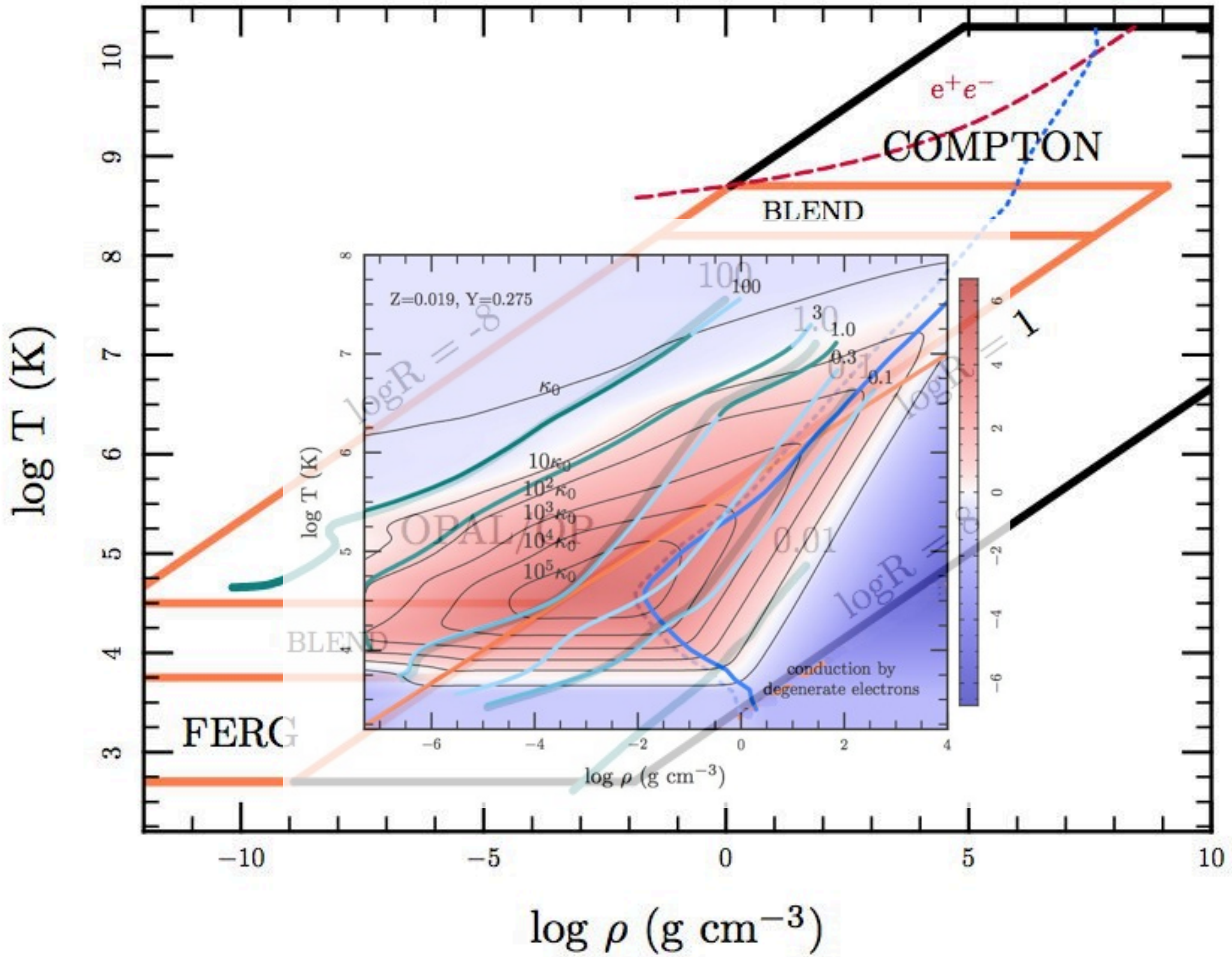
- molecular absorption - also messy!
  - H^-
  - CO, OH, H_2O, CH_4, ...

some simple examples of atomic κ_ν

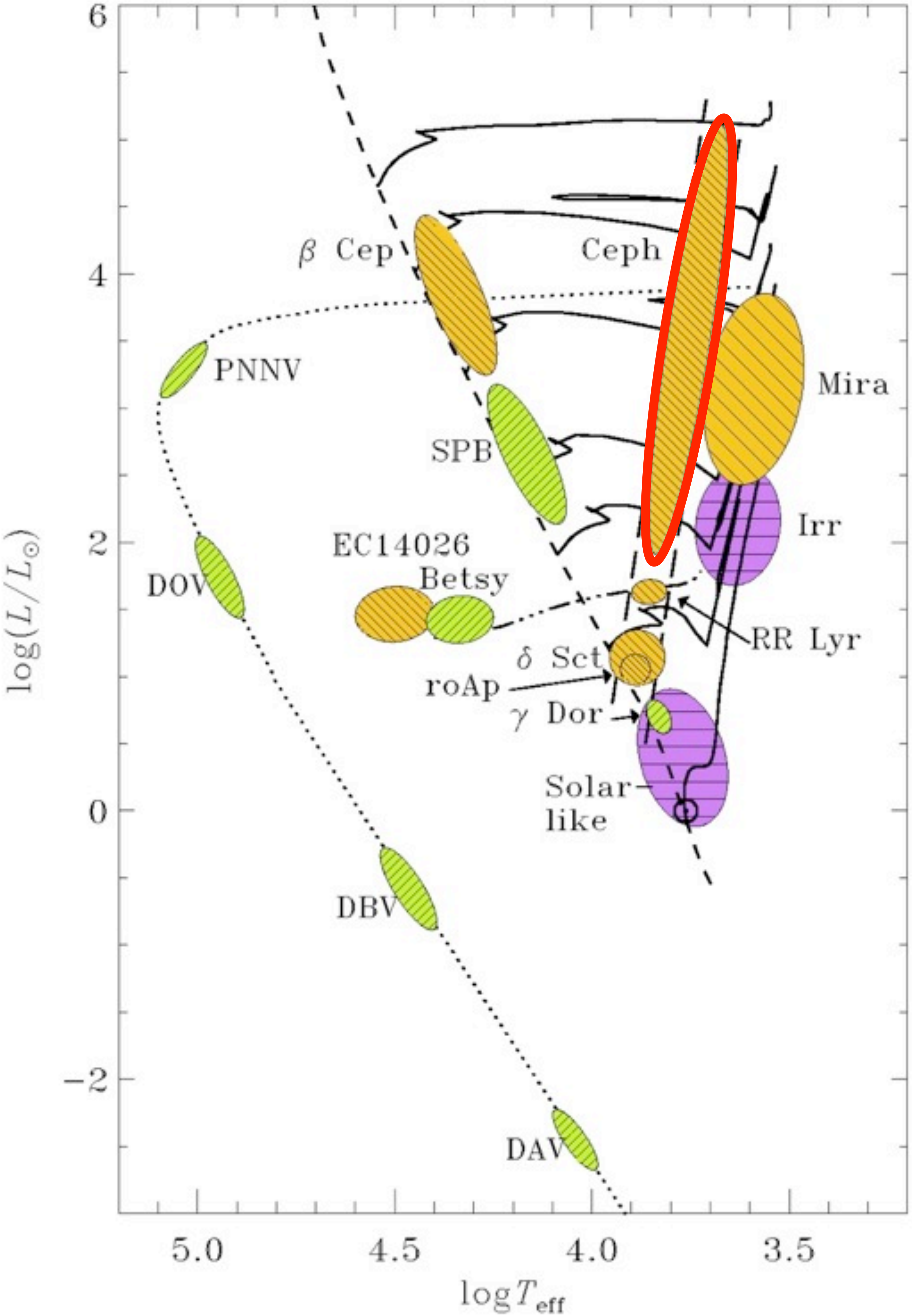
OPAL: Rogers & Iglesias (1992): log T=5.4, log ρ=-5.3

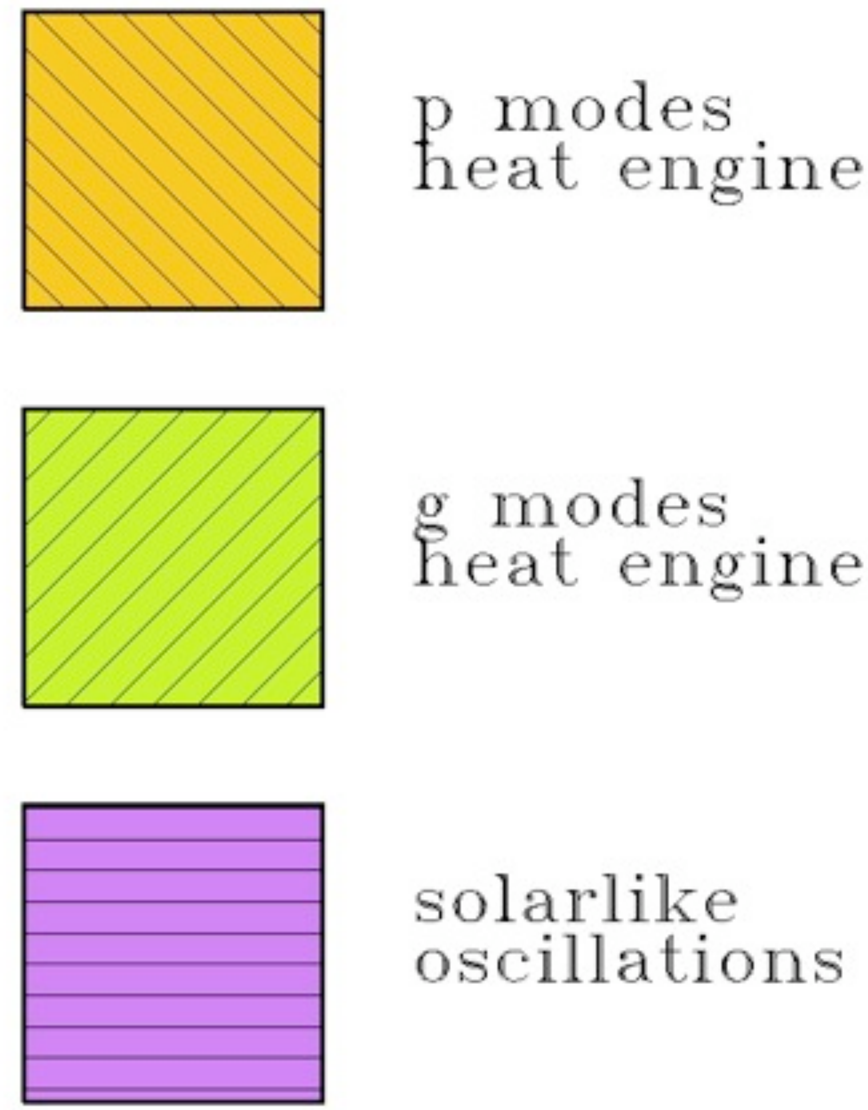


opacity sources in models
(MESA - Paxton et al. 2011)



Pulsating stars in the HR diagram





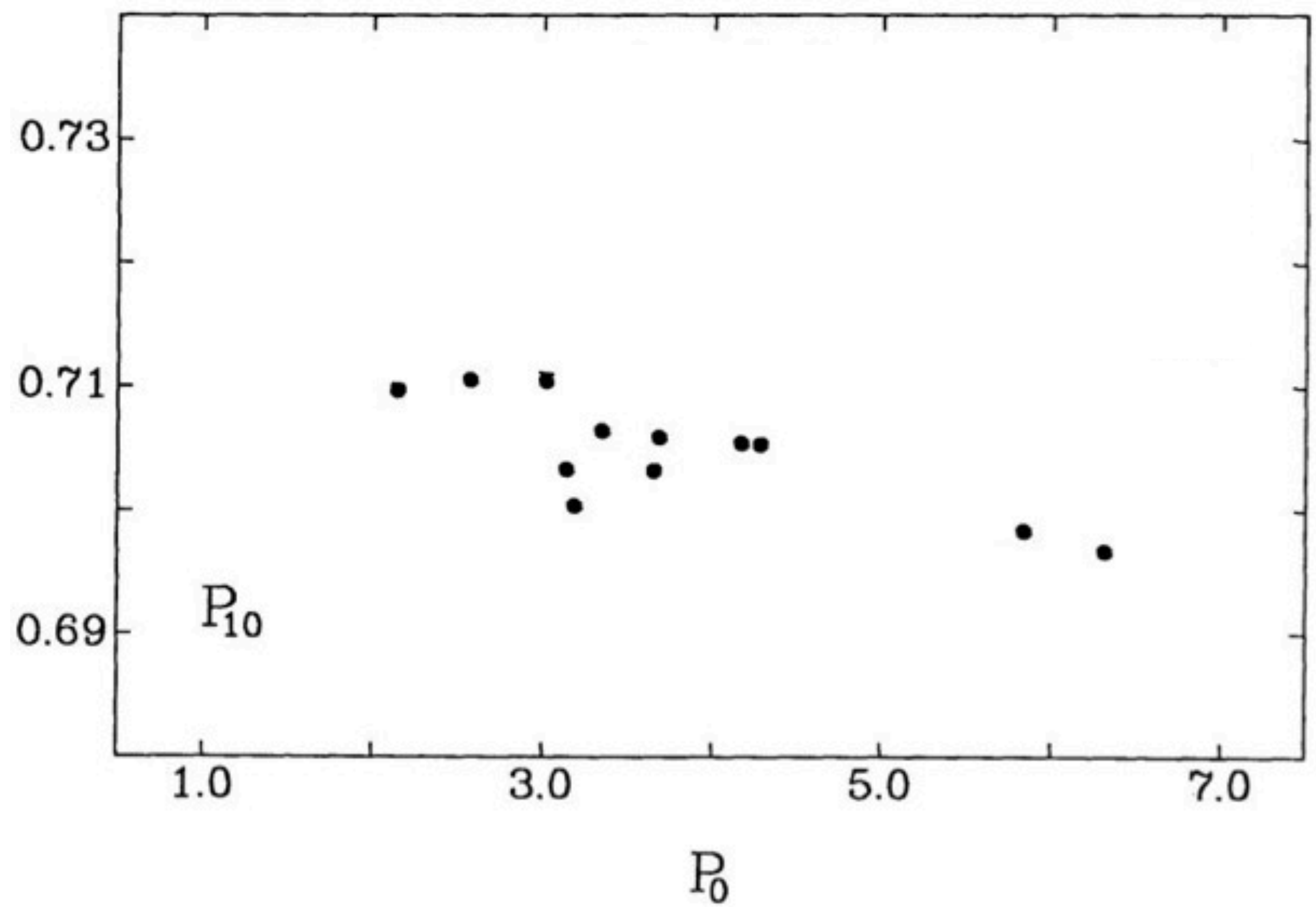
from J. Christensen-Dalsgaard

Cepheids: core helium burning Pop I stars

- lower mass limit - large enough to avoid core He flash ~ 2-3 $M_{\odot}$
- cross instability strip during "blue loop" of core He burning (and post-core exhaustion)
- most likely to find them at extreme blue end of the loop (slowest evolution)

Cepheid Masses circa 1985

- "Beat" Cepheids
  - period ratio of first overtone to fundamental: P_1/P_0
  - observed values: $0.70 < P_1/P_0 < 0.71$



from Moskalik et al. 1992

  - but... model periods were far from this value

but... a solution was already foreseen: Norman Simon: 1981

159TH AAS MEETING, BOULDER, COLORADO 871

olution Stages of Globular
heids. A. N. COX, and S. P.
al Laboratory - Population II
stage on the asymptotic giant
Herculis variables, and then
e, at periods longer than 10
bles. For some of these popu-
n periods and at least approxi-
sses can be obtained. Homogen-
els with 0.4, 0.5, 0.55, 0.65,
structed using the King Ia (Y =
e outer few percent of the mass
95% of the radius. Theoretical
e edge and lines of constant
ted for these masses and this
e of the mean periods of the
variables as a luminosity indi-
$_\odot$ values can be obtained for
in ω Cen (5), and M15 (1), in
nd M14 (2) of Oosterhoff group
n be found for the W Virginis
12 (3), and M15 (1) (Group II)
114 (3) (Group I). Using only
ividual luminosities, these
he theoretical H-R diagrams.
fit best on the 0.65 $M_\odot$ H-R
ty of about 0.1 $M_\odot$. Pulsation

35.04 Experimental Envelope Models for Cepheids. N.R. SIMON, U. Neb.-Lincoln. Numerical experiments are conducted with a view toward constructing Cepheid models which satisfy observational and theoretical constraints. Pulsation analysis is performed in the linear theory. Radiative models are studied, as well as those in which the H-zone is spread in a manner that mimics the effect of mixing-length convection. When the influence of convection on pulsation is examined in two artificial limits, adiabatic and isothermal, the former is found to be unsatisfactory, the latter tentatively acceptable. Following the results of earlier investigations, we test the effect of opacity on pulsational period ratios. It is found that an approximate doubling of the envelope opacity for temperatures $\gtrsim 10^5$ K seems sufficient (this work is still in a preliminary stage) to satisfy observational constraints with otherwise normal evolutionary models in both the double mode and bump Cepheid domains. This work is supported by the National Science Foundation under Grant # AST 8105064.

but... a solution was already foreseen: Norman Simon: 1982

TROPHYSICAL JOURNAL, 260:L87–L90, 1982 September 15

A PLEA FOR REEXAMINING HEAVY ELEMENT OPACITIES IN STARS

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Received 1982 April 5; accepted 1982 May 28

ABSTRACT

It is shown that increasing the opacity due to heavy elements by a factor of 2–3 leads to classical Cepheid models which reproduce observed period ratios at evolutionary masses and luminosities. Thus the mass anomalies are removed in both the double-mode and bump Cepheid regimes. The proposed increases may also serve to energize β Cephei variables, thus solving yet another important problem in the theory of pulsating stars. We argue that opacity changes of this order are not implausible and urge further work in this important area.

Subject headings: opacities — stars: Cepheids — stars: pulsation

but... a solution was already foreseen: Norman Simon: 1982

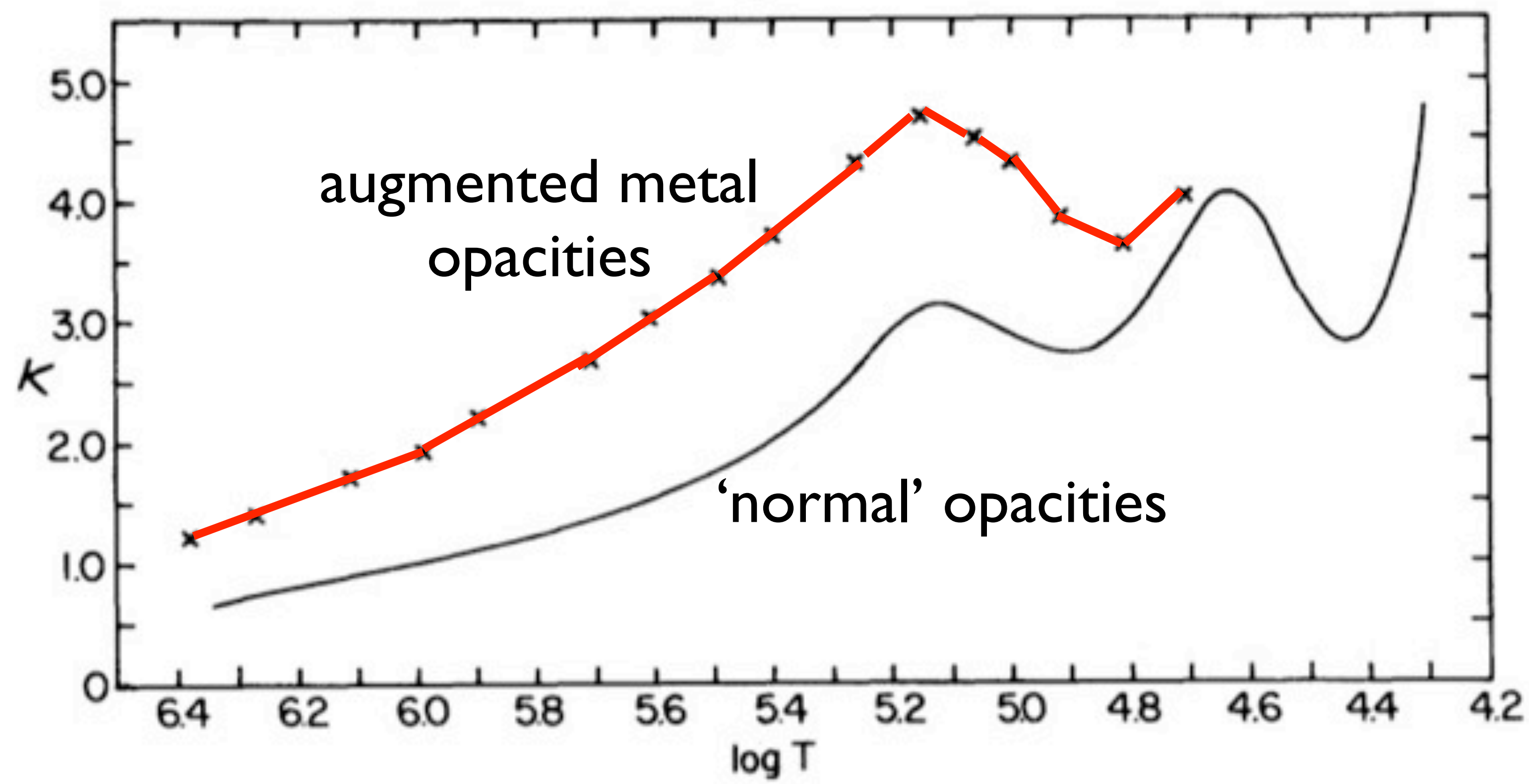


TABLE 1

LNA Results for Double-Mode Cepheid Model[a]

Model	P_0	$(-\sigma_i/\sigma_r)_0$	$(\Delta\phi)_0$	P_1/P_0
NO	3.126	8.92(−4)	122	0.742
AMO	3.306	6.47(−4)	117	0.713

[a] $M = 5\ M_\odot$; $L = 1100\ L_\odot$; $T_e = 5800$ K; $X = 0.70$; $Z = 0.02$.

TABLE 2

LNA Results for Bump Cepheid Model

Model	P_0	$(-\sigma_i/\sigma_r)_0$	$(\Delta\phi)_0$	P_2/P_0
NO	9.731	4.10(−3)	113	0.539
AMO	10.58	2.14(−3)	106	0.498

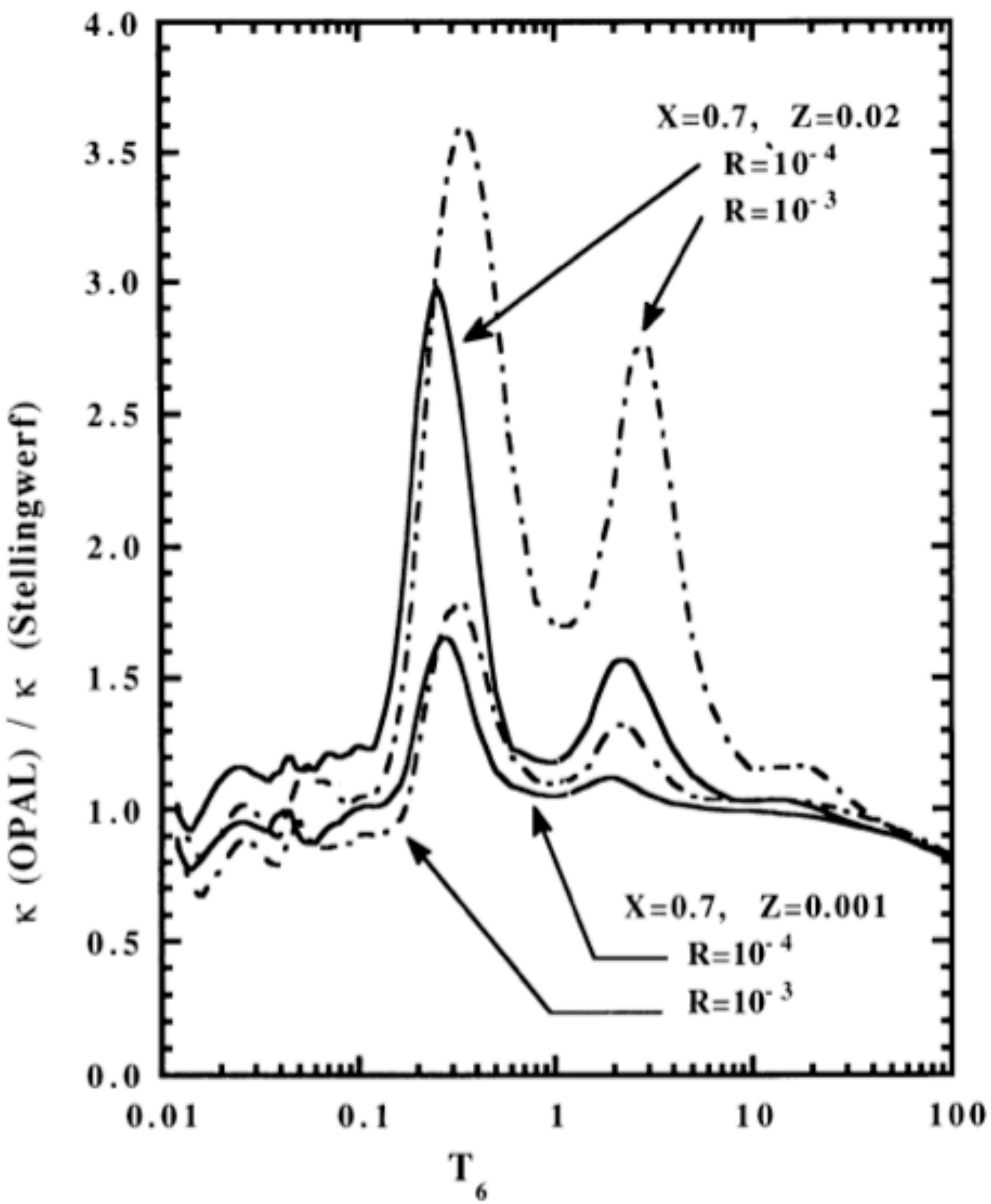
[a] $M = 7\ M_\odot$; $L = 4742\ L_\odot$; $T_e = 5623$ K; $X = 0.70$; $Z = 0.03$.

but... a solution was already foreseen: Norman Simon: 1982

played in Figure 5 of Stellingwerf (1978). Although the present classical Cepheid models differ in density from models in the β Cephei regime, it is nonetheless expected that AMO models of the latter objects would also show the enhanced opacity feature. It is thus quite possible that, by the single stroke of augmenting the heavy element opacities by factors of 2–3, we can bring into line with the theory of stellar structure and evolution not only the double-mode and bump Cepheids, but the β Cephei pulsators as well.

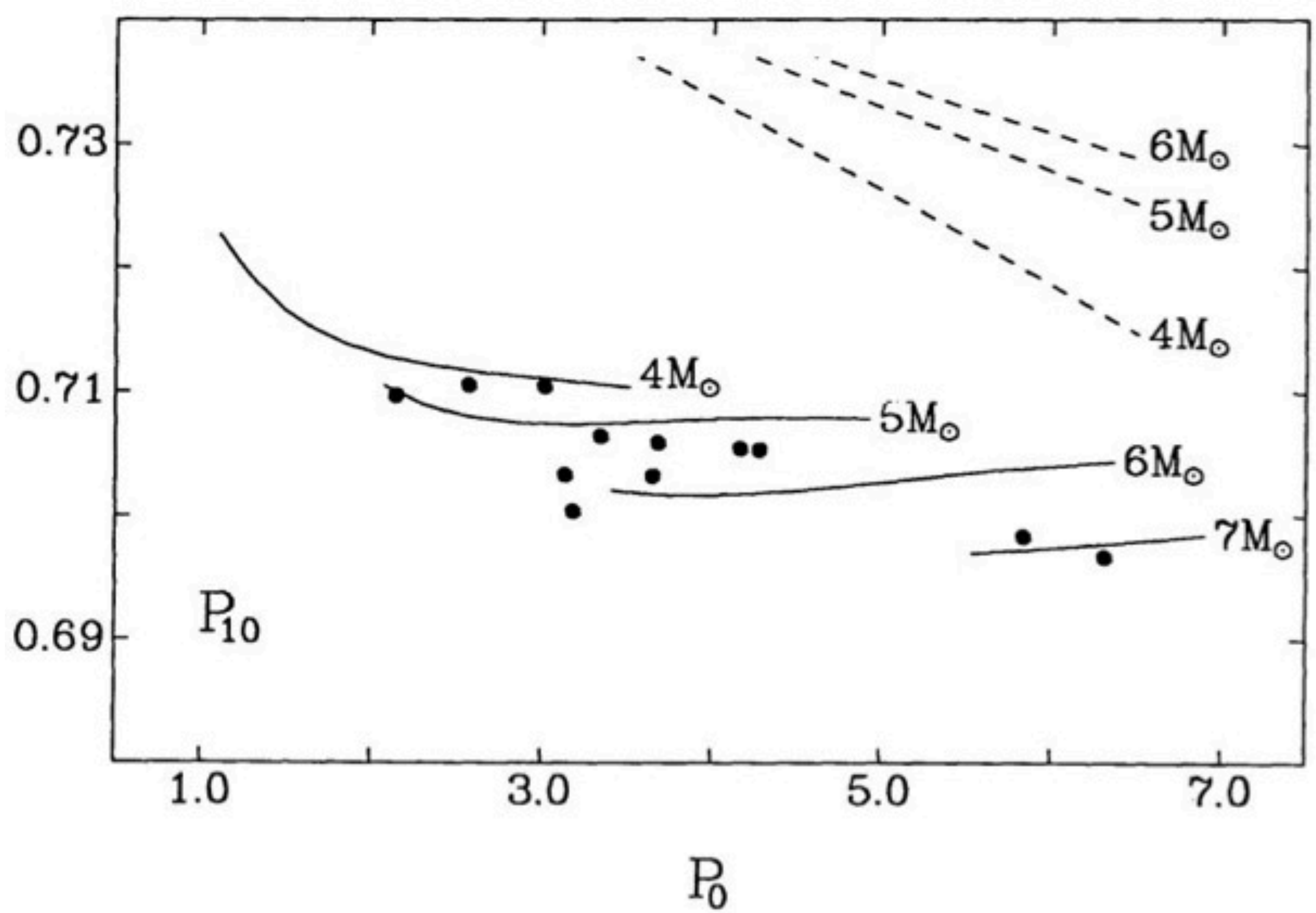
OPAL to the rescue

- early version of OPAL opacities found much higher opacities precisely where Simon said: i.e.
  - Iglesias & Rogers 1991



Cepheid Masses circa 1992

- "Beat" Cepheids
  - period ratio of first overtone to fundamental: P_1/P_0
  - observed values: $0.70 < P_1/P_0 < 0.71$



from Moskalik et al. 1992

  - new models with new opacities are just fine

non-adiabatic pulsation:
driving, damping, and the convective flux

- Radiative energy transport - opacities and driving
  - Cepheid masses, driving, and the iron bump
  - (massive pulsators and the iron bump)
  - sdB driving (with diffusion thrown in)
- the convective flux
  - white dwarf driving and harmonics

the kappa mechanism

$$W_{\rm rad} = \int_R dr \left(\frac{\delta T}{T}\right)^2 \frac{d}{dr}\left\{\left(\underbrace{\left[\frac{\partial \ln \kappa}{\partial \ln T}\right]_\rho}_{\kappa_T} + \frac{1}{\Gamma_3 - 1}\underbrace{\left[\frac{\partial \ln \kappa}{\partial \ln \rho}\right]_T}_{\kappa_\rho}\right) L_R\right\}$$

- if the integrand is > 0, then that region contributes to a positive value of $W_{\rm rad}$ and therefore pulsation driving
- typical values for power law opacity:

  $\kappa_T \sim -3.5$; $\kappa_\rho \sim 1.0$; $\Gamma_3 \sim 5/3$ so ( ) < 0

- ** Damping or driving when thermal response time of the layer is comparable to the pulsation period:

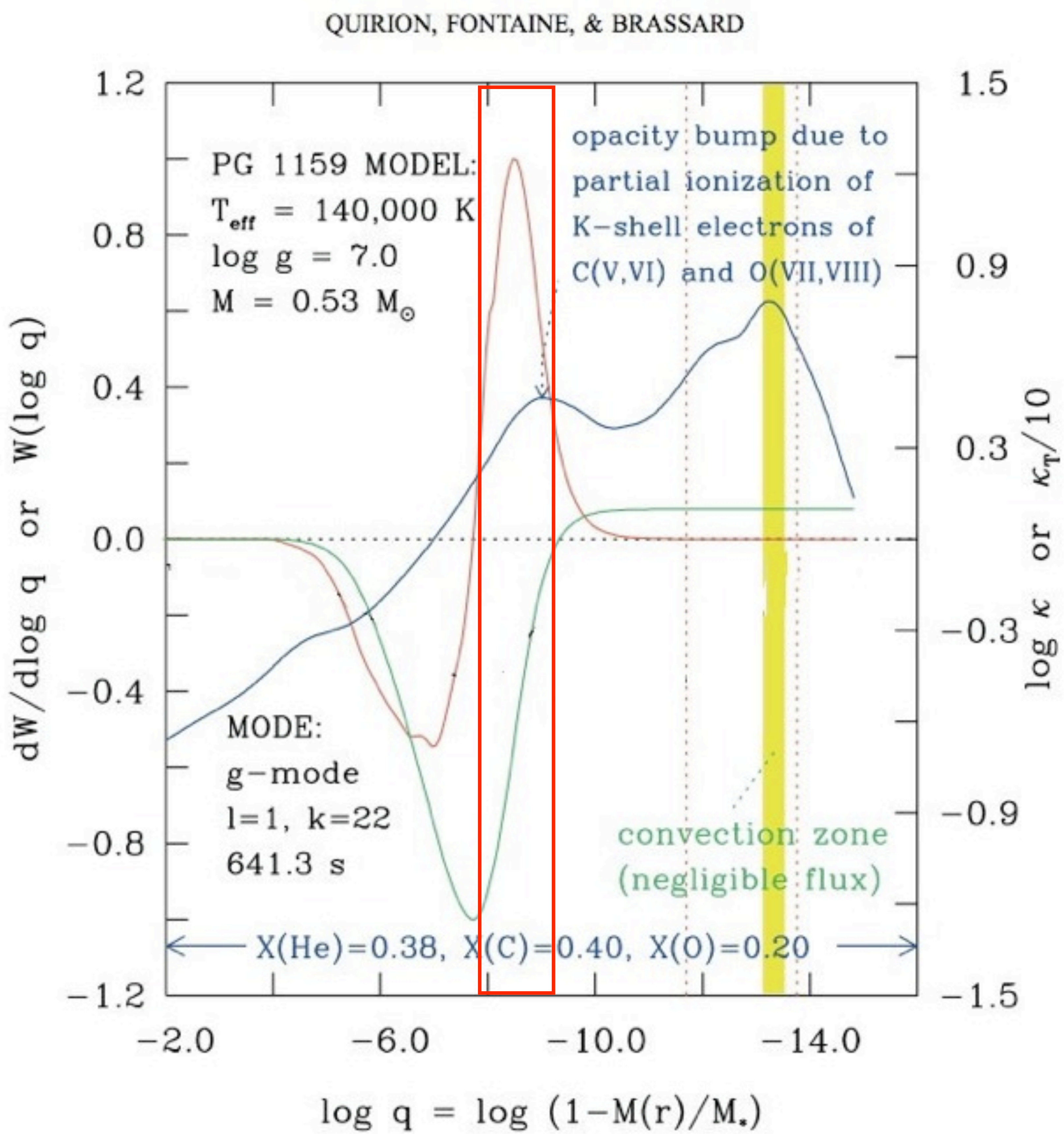
$$\tau_{\rm therm} \equiv \frac{\int_{M_r}^{M} c_V T dm}{L} \approx P_{\rm puls}$$

the kappa mechanism

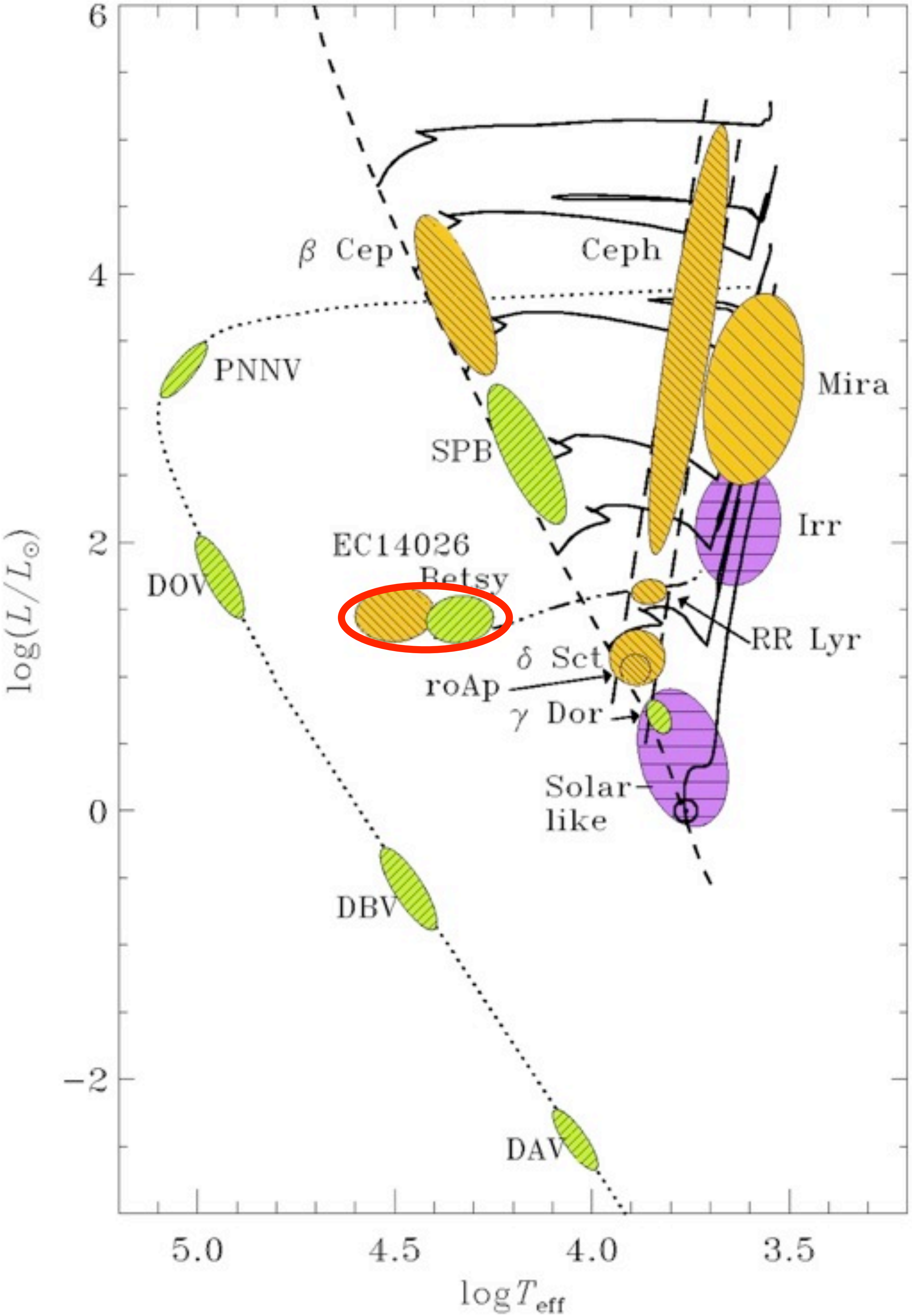
- ‘normal’ situation
  - compression cycle:
    - T, ρ increase, κ decreases:
    - region becomes ‘leakier’ to radiation
    - Flux can increase, so energy is not ‘bottled up’
    - $W_{tot} < 0$
- ‘unstable’ situation (partial ionization)
  - compression cycle:
    - T, ρ increase
    - κ does not decrease
    - (energy goes into ionization)
    - Flux ‘bottled up’
    - can only release energy on ‘downstroke’ when T falls

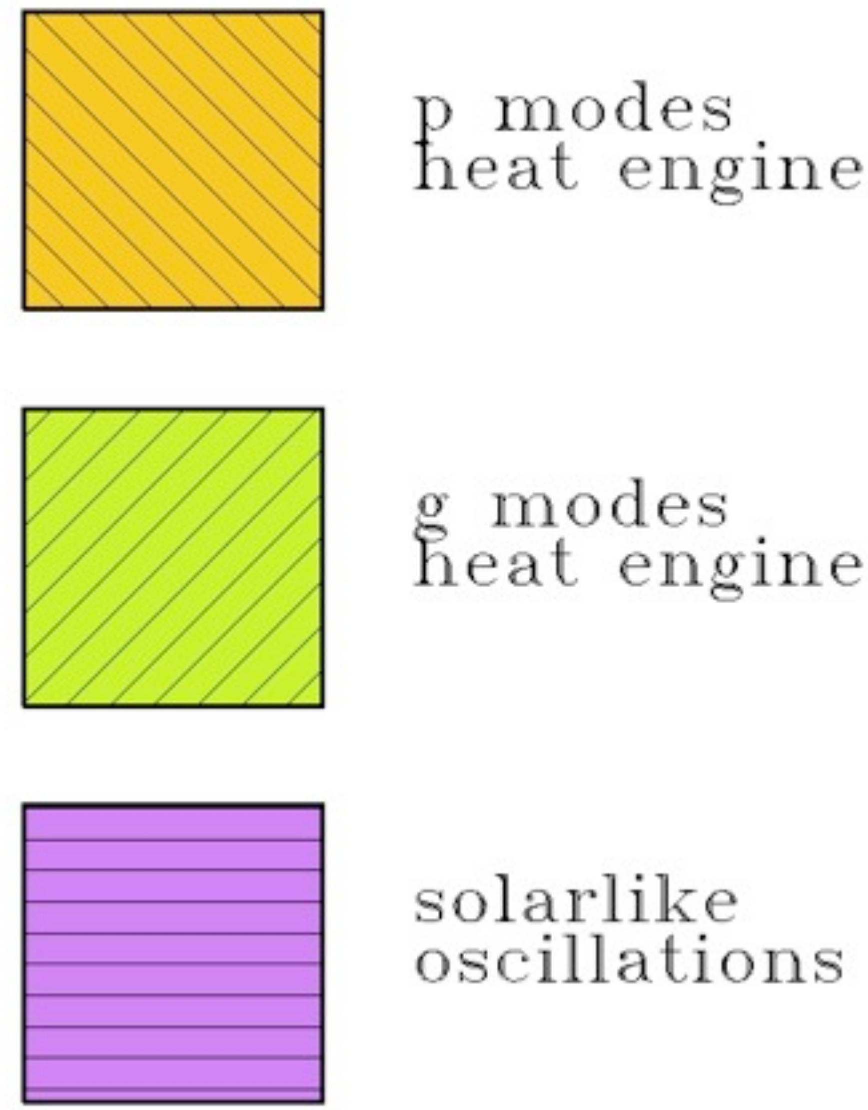
- **GENERAL CONDITION - partial ionization**

an example of pure kappa



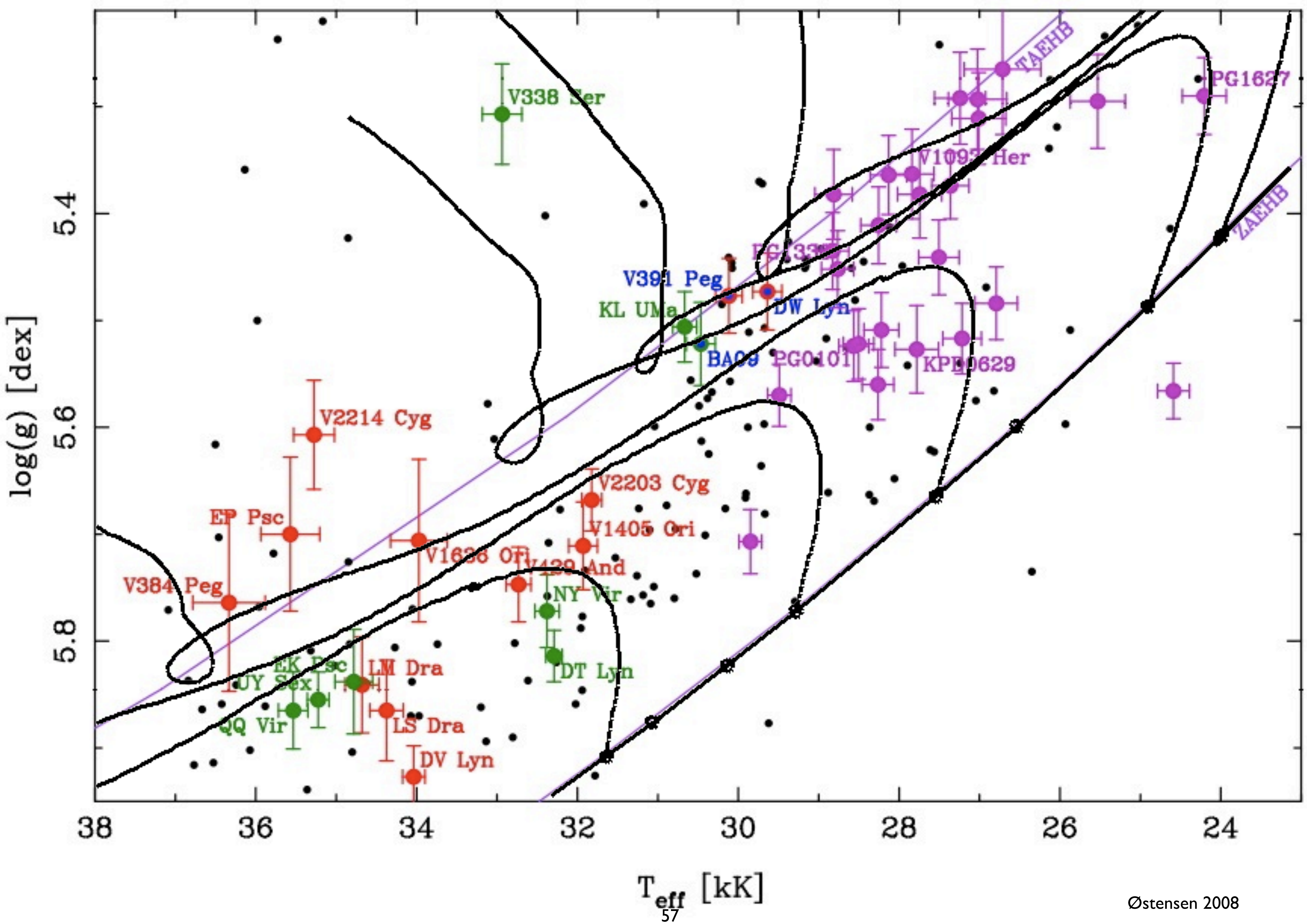
Pulsating stars in the HR diagram





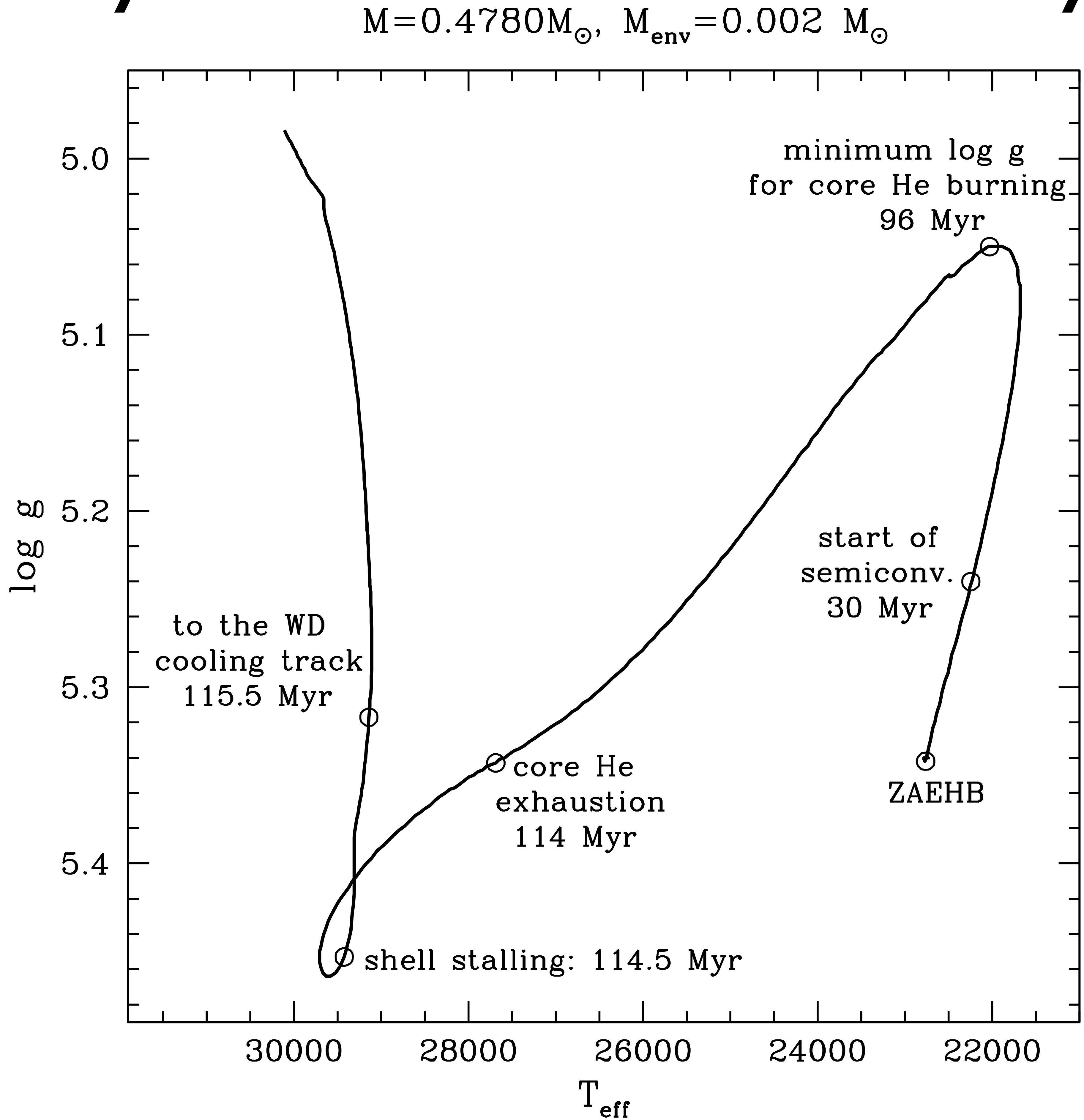
from J. Christensen-Dalsgaard

sdB stars and standard models



Østensen 2008

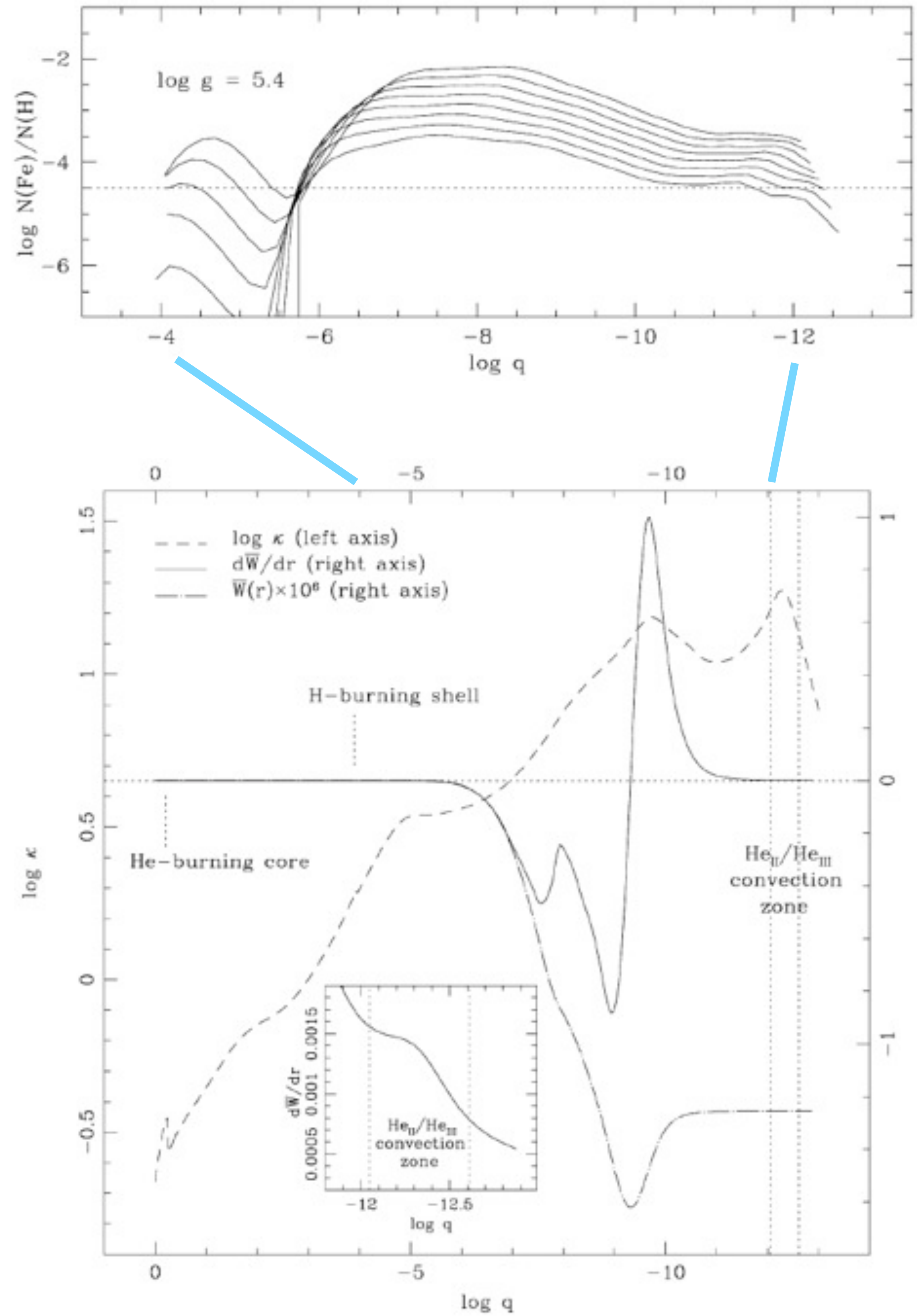
anatomy of an sdB evolutionary track



Driving Mechanism:

opacity effect with ***levitated Iron***

(Charpinet et al. 1996-2002)

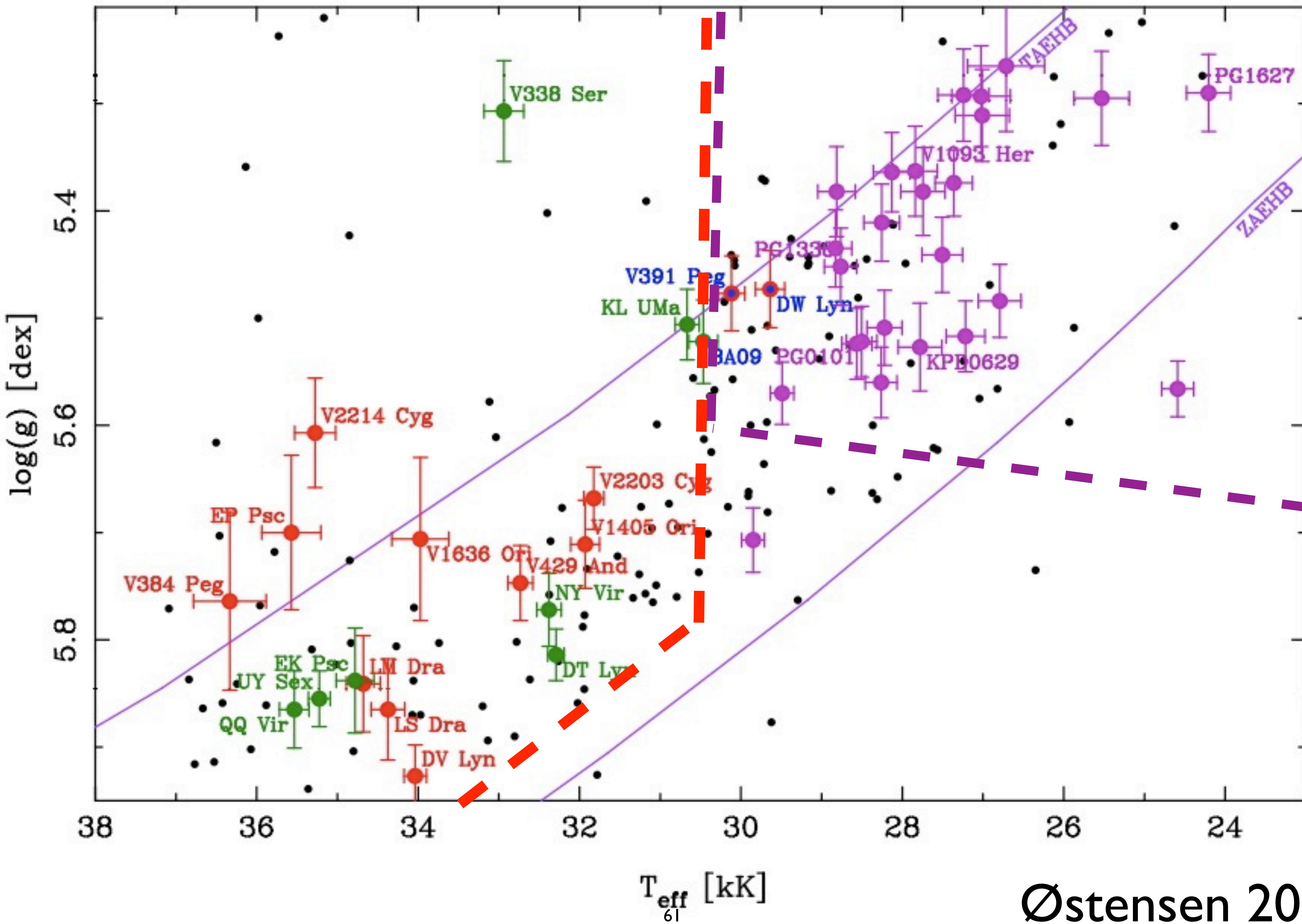


Charpinet, Fontaine, & Brassard 2009: "nonadiabatic asteroseismology" of sdB stars

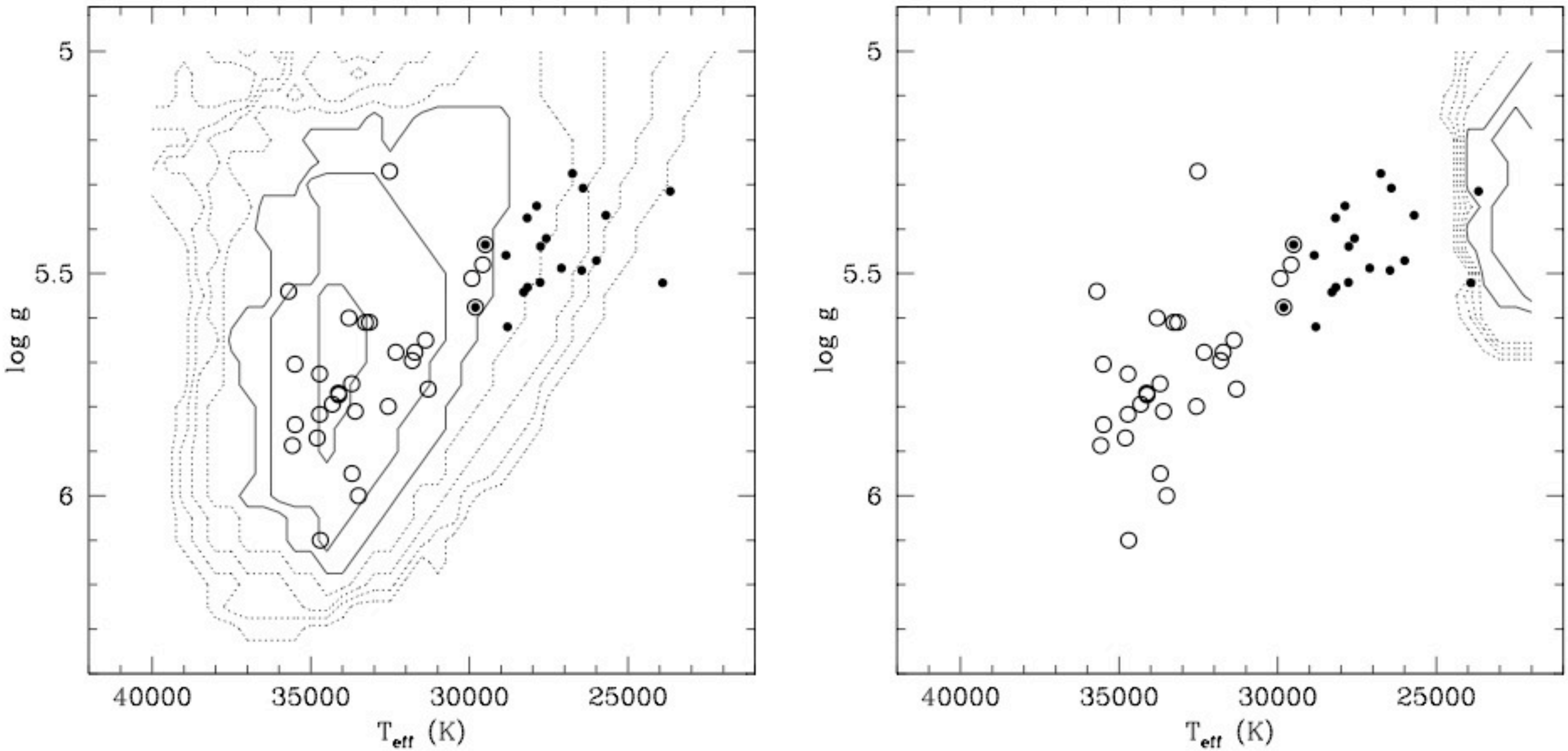
- kappa mechanism is robust in sdB stars
- predictive
- observed pulsations (given Teff, log g) **require levitated iron**
- use observed period ranges to place strong constraints on subsurface Fe (and on processes that would dilute the driving)

instability regions

red/green = short period purple = long-period

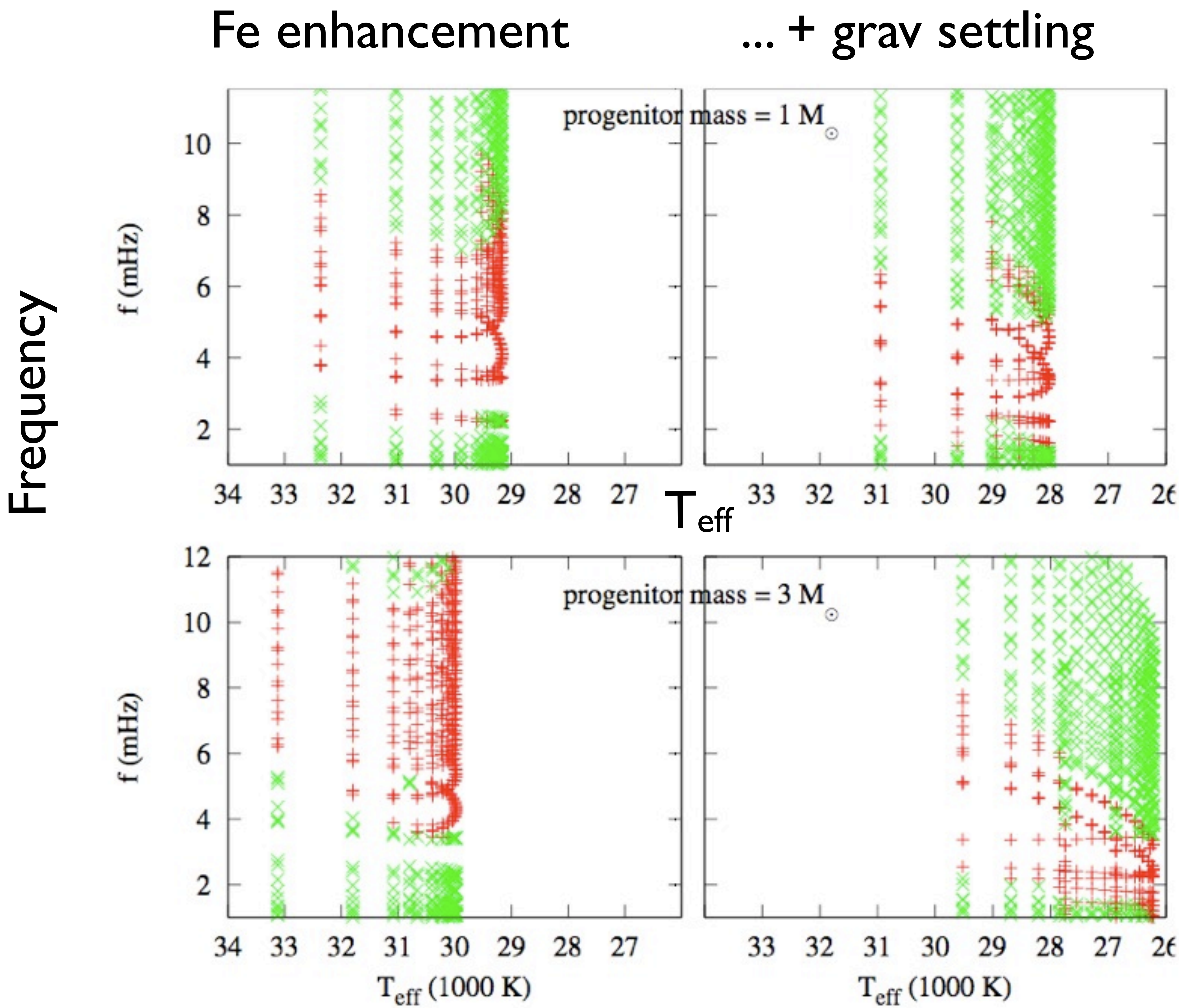


Østensen 2008

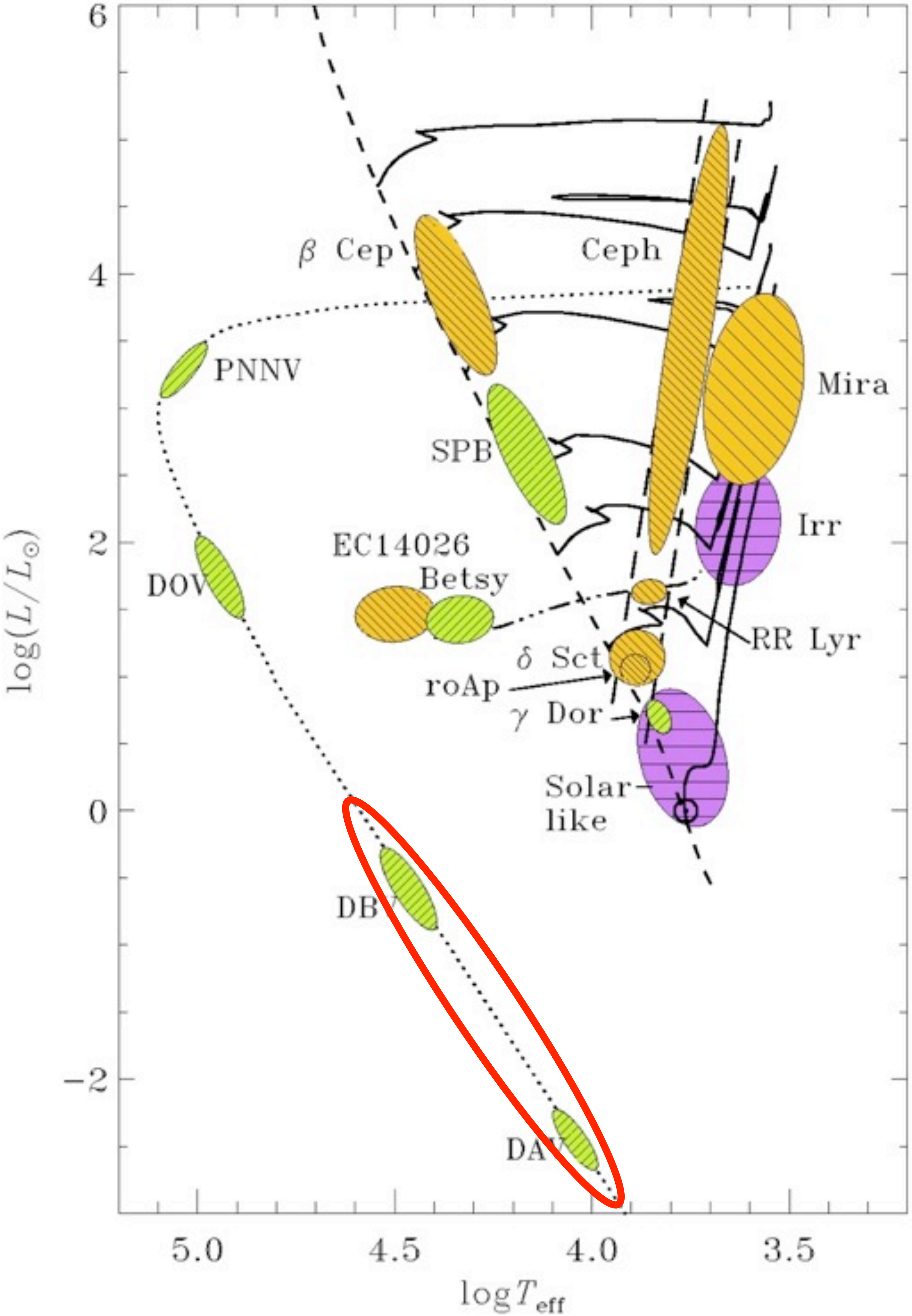


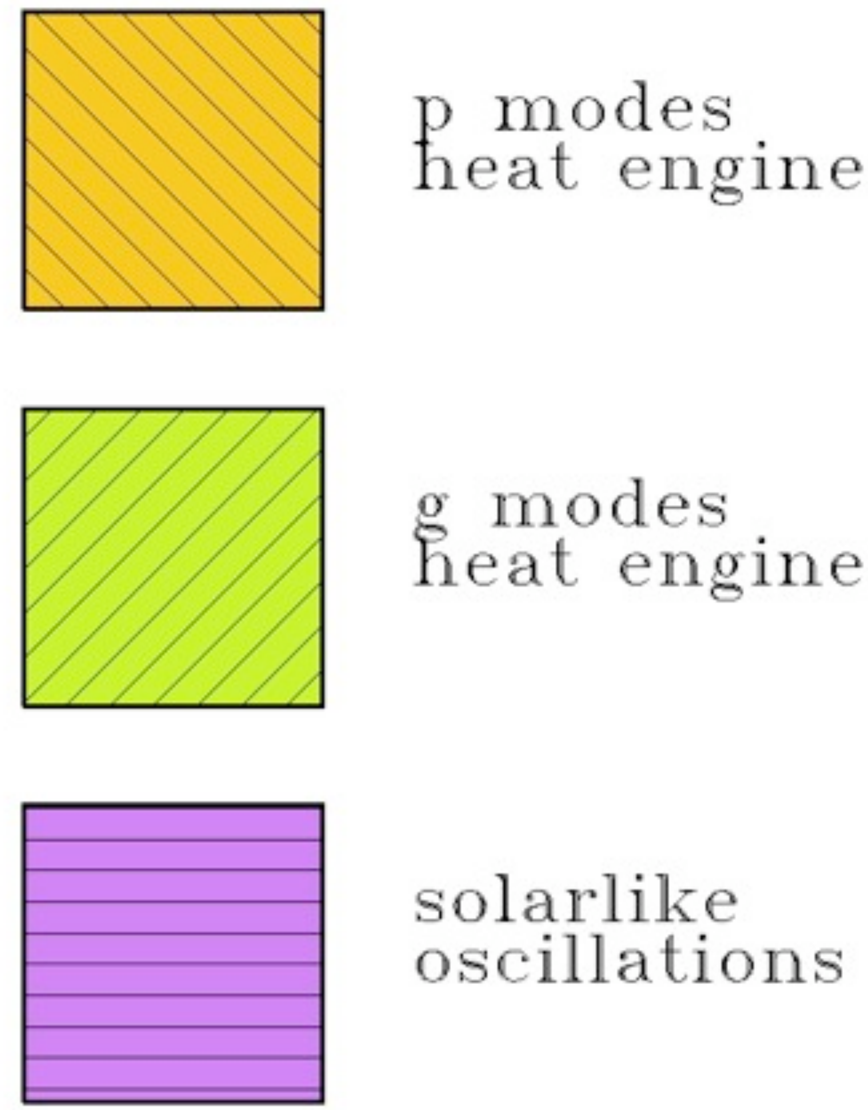
Charpinet et al. (2007)

Hu et al. (2010)


Fe enhancement
... + grav settling
Frequency
T_{eff}
progenitor mass = 1 $M_\odot$
progenitor mass = 3 $M_\odot$
f (mHz)
T_{eff} (1000 K)
34 33 32 31 30 29 28 27
33 32 31 30 29 28 27 26
2 4 6 8 10 12

Pulsating stars in the HR diagram





from J. Christensen-Dalsgaard

convection efficiency constraints from pulsating white dwarfs

- nonlinear pulse shape for high-amplitude pulsating white dwarfs depend on convective time scales near the surface

- Blue edge for DB white dwarfs is sensitive to the parameters of mixing length theory (and therefore efficiency of (near) surface convection)

S. O. Kepler and M. H. Montgomery in Mount Cuba Observatory.

pulse shapes

Mike Montgomery (2005, 2006, 2009)

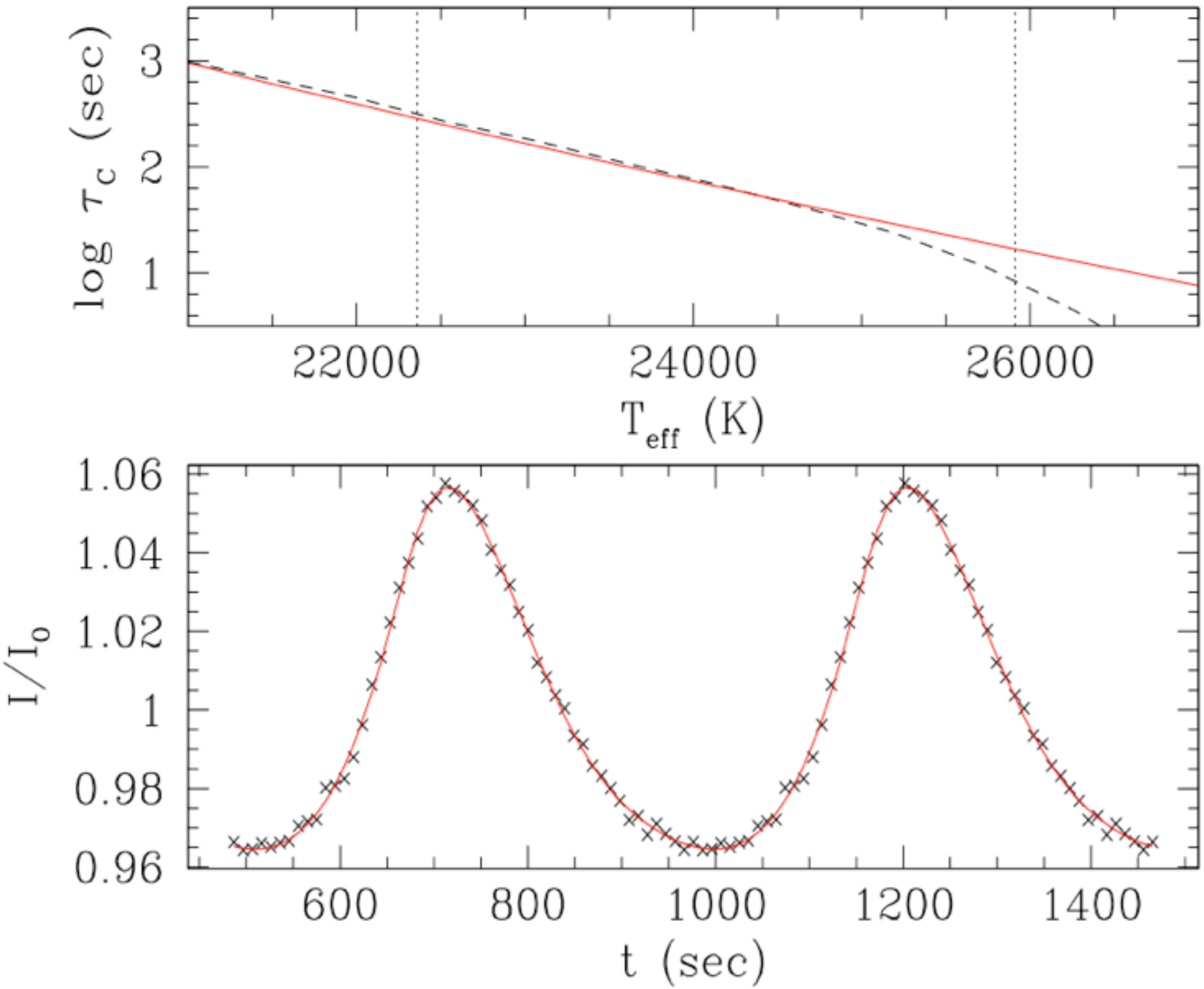
- subsurface flux variations are sinusoidal
- convective turnover time << pulsation periods
  - instantaneous response of convection zone to changes through a pulsation cycle (via MLT) yields

$$\tau_c \approx \tau_0 \left(\frac{T_{\rm eff}}{T_0} \right)^{-N}$$ where N ~ 25

- look at flux (and temperature) variations and model the light curve accordingly to yield a 'pulse shape'

- adjustable parameters: τ_o, N, and inclination θ

pulse shapes
Mike Montgomery (2006 - 2010)



pulse shapes
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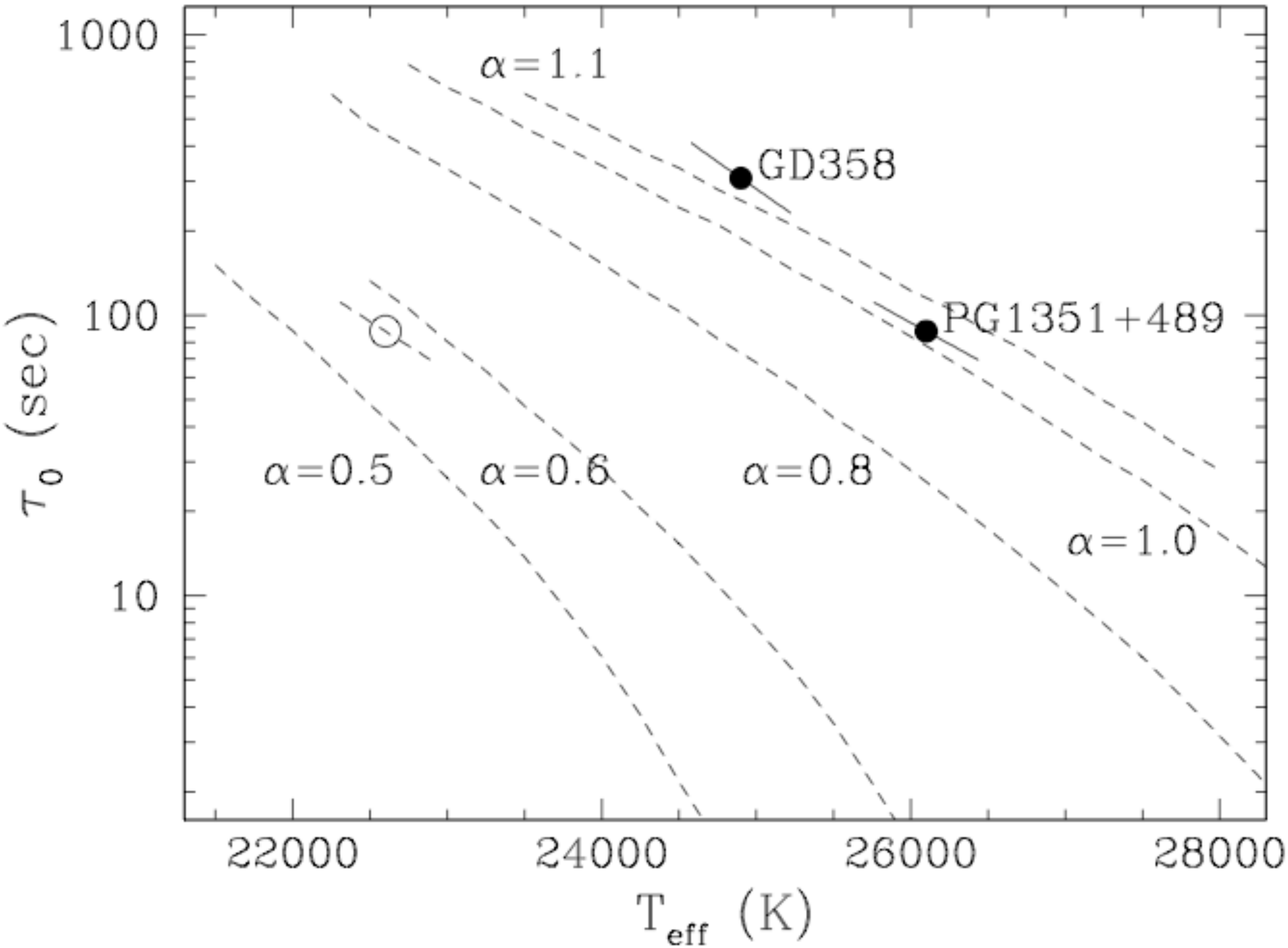

input physics questions

(that asteroseismology can address)

- Equation of State
  - non-ideal effects
  - Coulomb crystallization - pulsating cool, massive WDs
- Nuclear processes
  - difficult cross-sections - chemical profiles in WD interiors
  - neutrino emission - evolution rates in hot WD pulsators
- Radiative energy transport - opacities and driving
  - Cepheid masses, driving, and the iron bump
  - sdB driving (with diffusion thrown in)
  - B star pulsations
- the convective flux
  - white dwarf driving and harmonics
  - solar-like oscillations

other issues (non-coefficient)

- time evolution of abundance
  - composition changes via nuclear burning
    - direct impact through dS/dt term
  - composition changes via chemical diffusion
    - diffusion coefficients via atomic physics
  - composition changes via turbulence
    - instantaneous mixing via convection
    - convective overshoot
    - partial mixing via semiconvection, other processes
    - rotational mixing
- mass loss / accretion
- rotation
- magnetic fields
- **tidal interaction and other effects of companions**

Tidal (non) synchronization in a close binary (Pablo et al. 2011)

- sdB (pulsator) and low-mass (~0.2 $M_\odot$) M dwarf in a binary
- binary period = 9.56 hours ; orbital separation about 5 R_*
- Tidal synchronization time scale (a la Zahn 1975): ~ 10^9 yr
- so we expect that the sdB should *not* be in synchronous rotation

- *g*-mode pulsations in sdB show splitting corresponding to a rotation period of 9.63 *days*

- other sdB pulsators in binaries *do* appear to be synchronous
  - PG 1336 - shorter period; $\tau_{sync} < 10^6$ yr
  - Feige 48 - more massive companion; but $\tau_{sync} \sim 10^8$ yr

B4 triplets - P_{rot} = 9.63 d