Nonlinear pulsation

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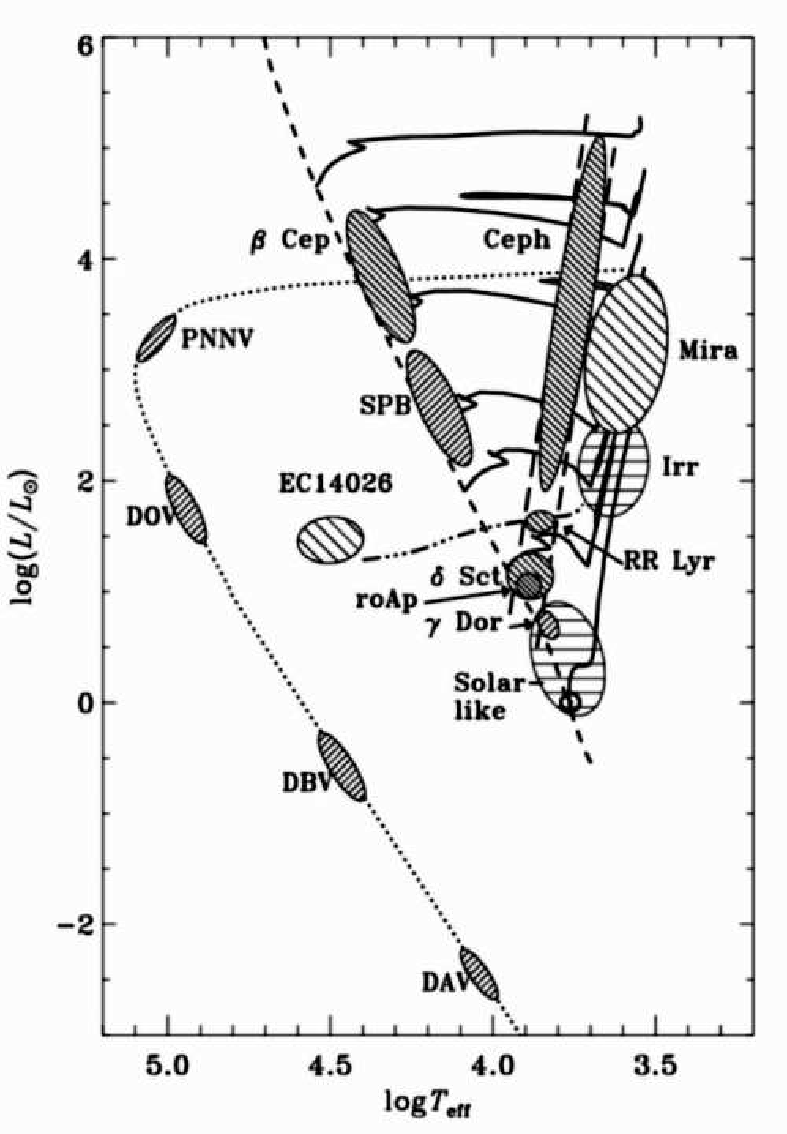
ÖFG

Nonlinear pulsation - Introduction



Christensen-Dalsgaard (1992)

Large amplitude pulsation in the classical instability strip

- Cepheids, RR Lyrae stars
- opacity driven pulsation
- radial pulsation

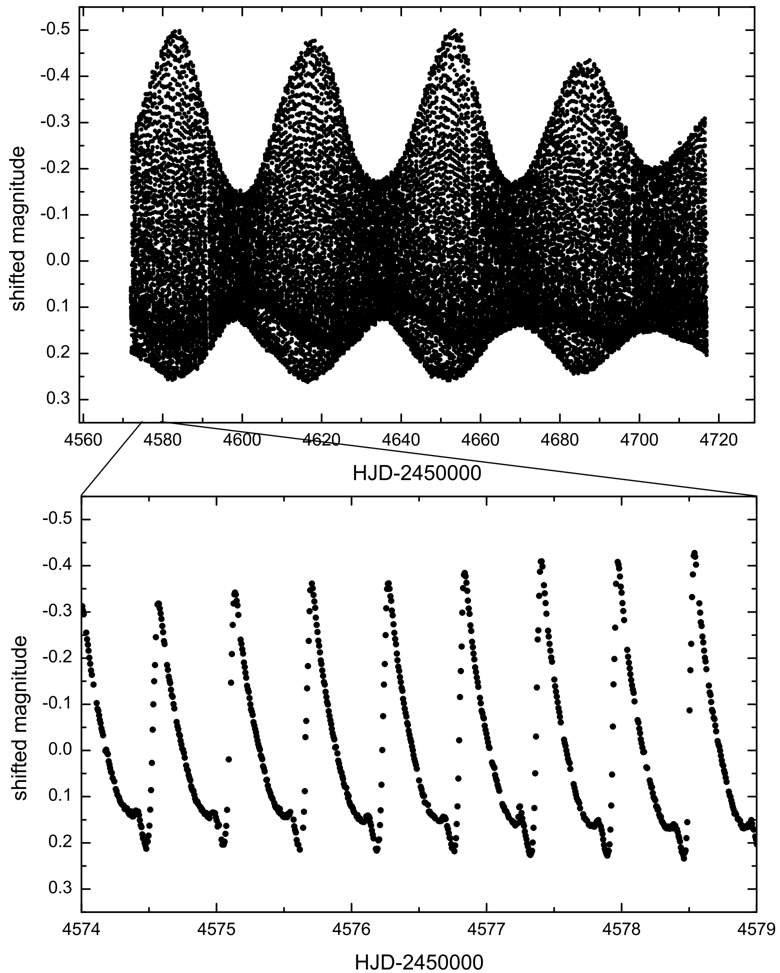
Focus on multimode pulsation

- ⋆ new discoveries (*Kepler*, *CoRoT*, ground)
- ⋆ what we see and
- ⋆ ...do we understand it

Multiperiodic pulsation: new discoveries

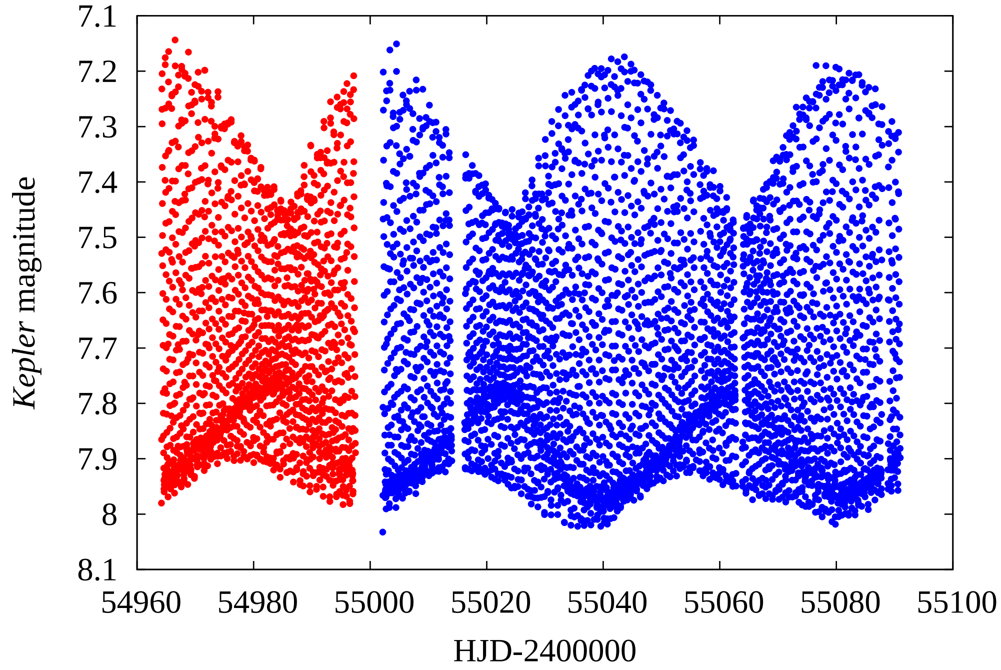
New discoveries: *Kepler*, *CoRoT*

Huge progress in understanding the Blazhko effect – Robert Szabó talk



CoRoT ID 105288363

Guggenberger et al. (2011)



RR Lyr

Kolenberg et al. (2011)

New discoveries: *Kepler*, *CoRoT*

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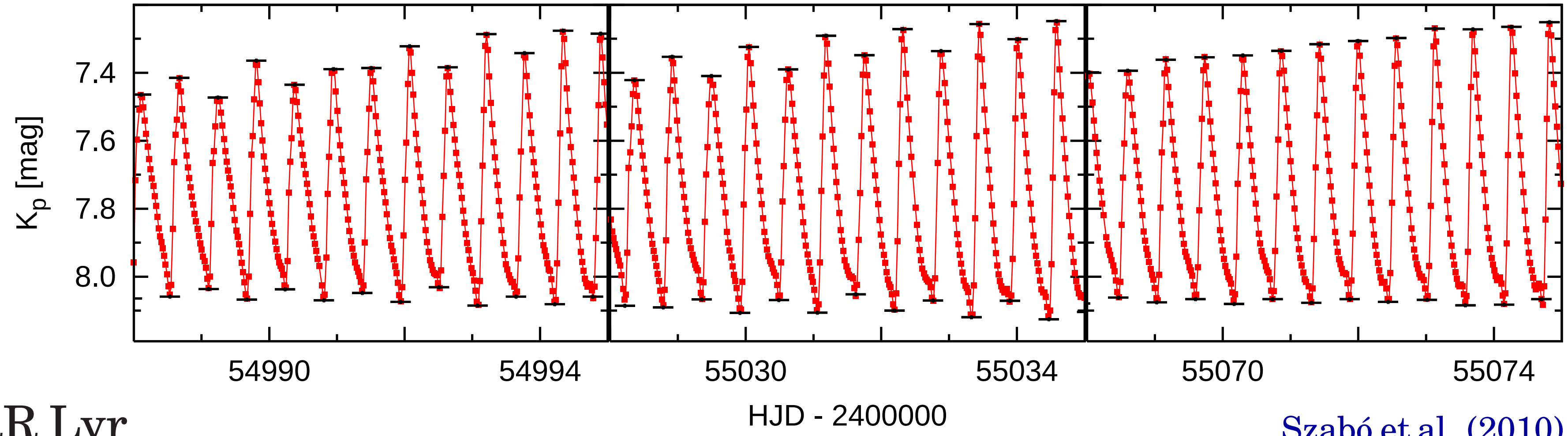


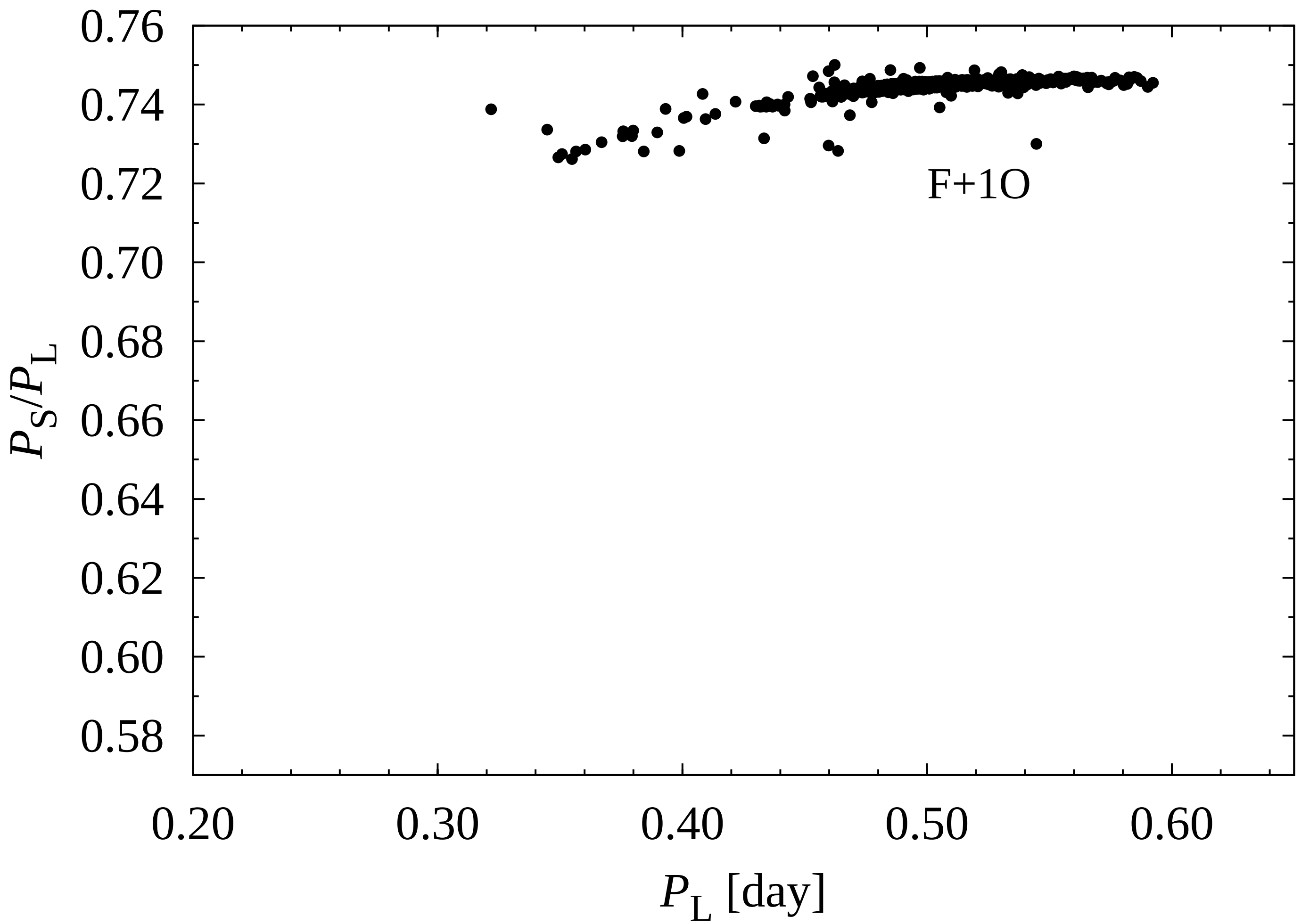
RR Lyr

Szabó et al. (2010)

Period doubling effect

- detected in 9 stars all showing the Blazhko effect
- caused by the 9:2 resonance, $9\omega_0 = 2\omega_9$ (Kolláth et al 2011)
- solution to the Blazhko puzzle? (Buchler & Kollath 2011)

New discoveries: RR Lyrae stars – new pulsation modes



F+1O: 1335 stars in OIII-CVS

Soszyński et al. (2009, 2010, 2011)

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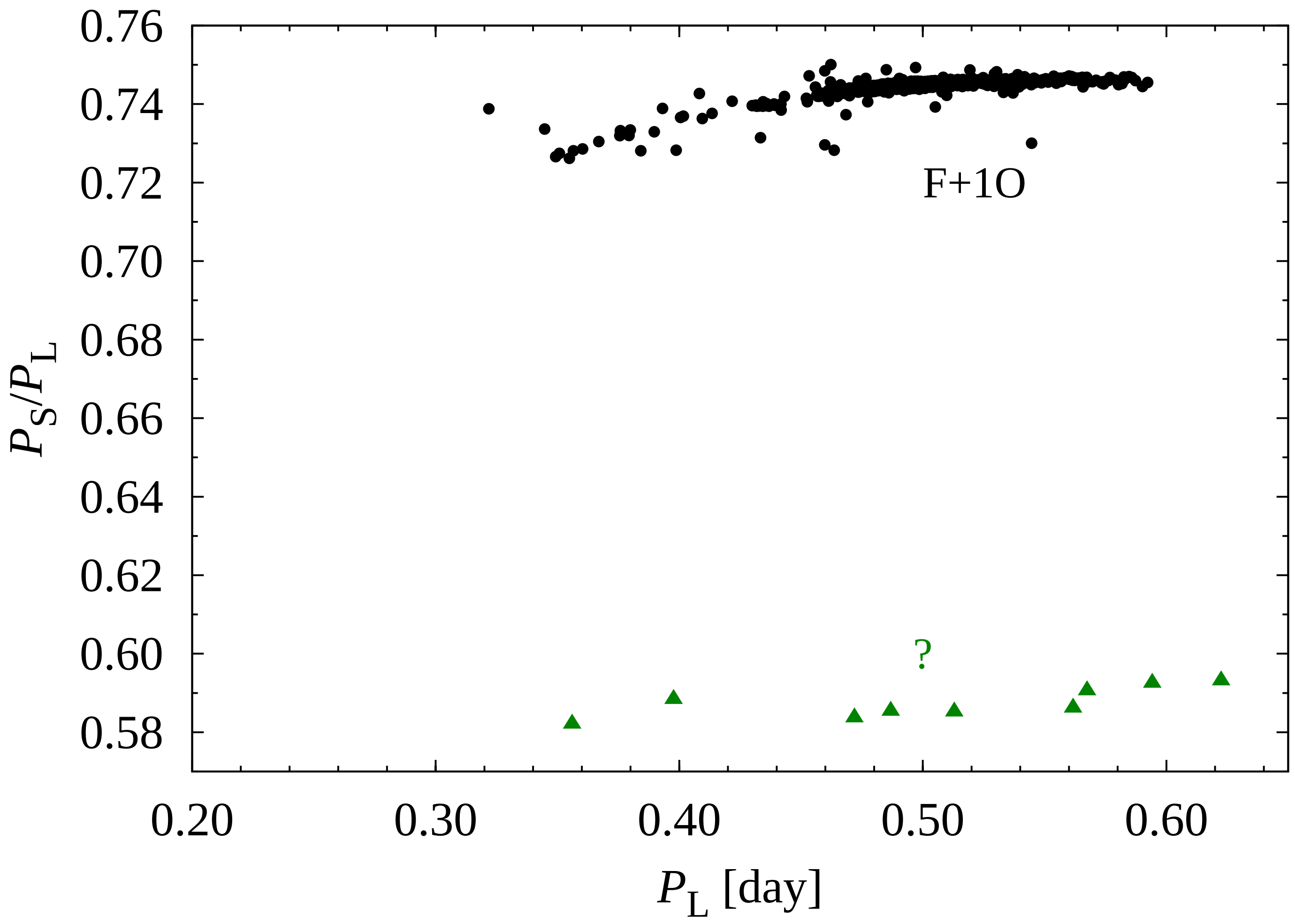


F+1O: 1335 stars in OIII-CVS

Soszyński et al. (2009, 2010, 2011)

?: 9 stars

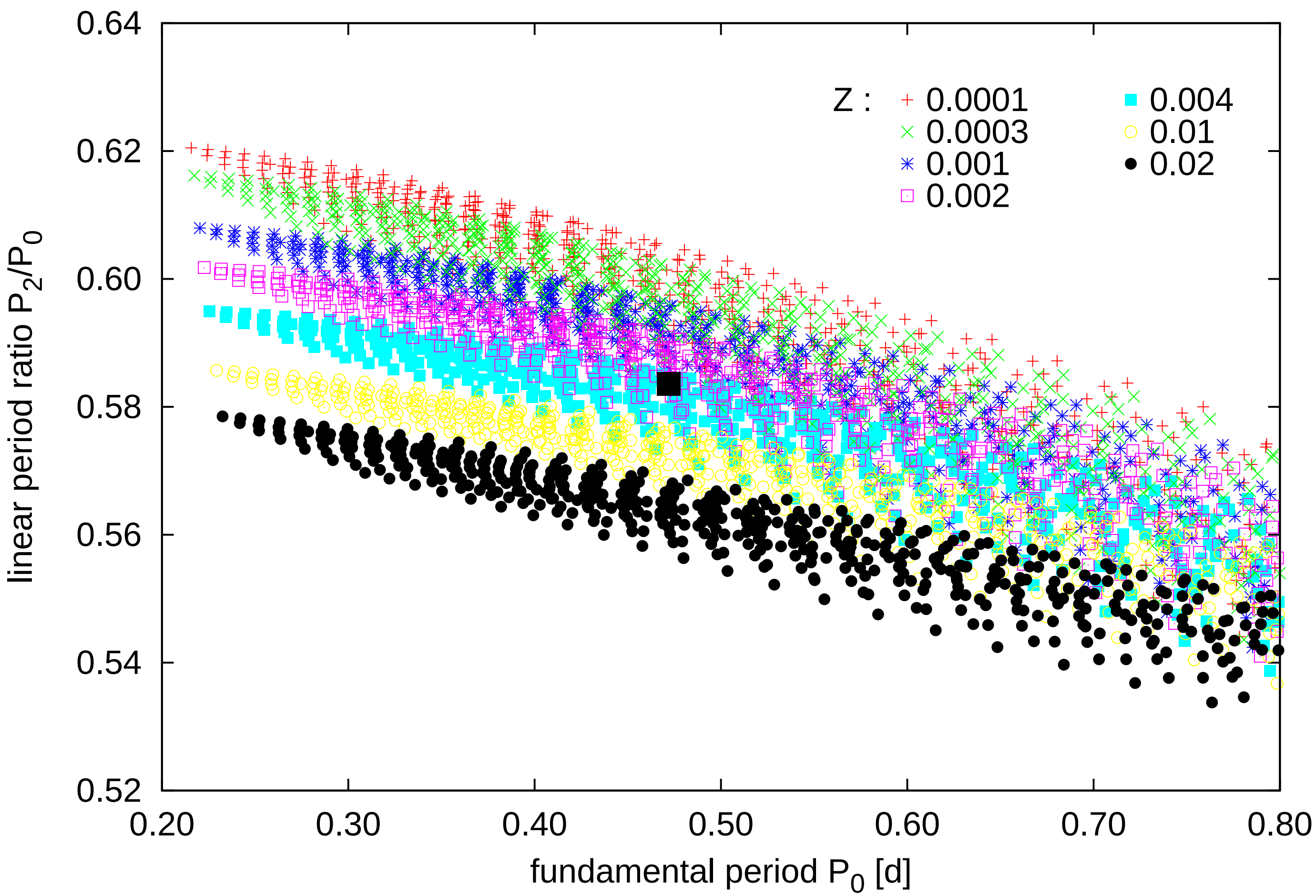
Kepler: 5

CoRoT: 3

MW Lyr

Chadid et al. (2009), Poretti et al. (2010), Benkö et al. (2010), Guggenberger et al. (2011)

New discoveries: RR Lyrae stars – new pulsation modes

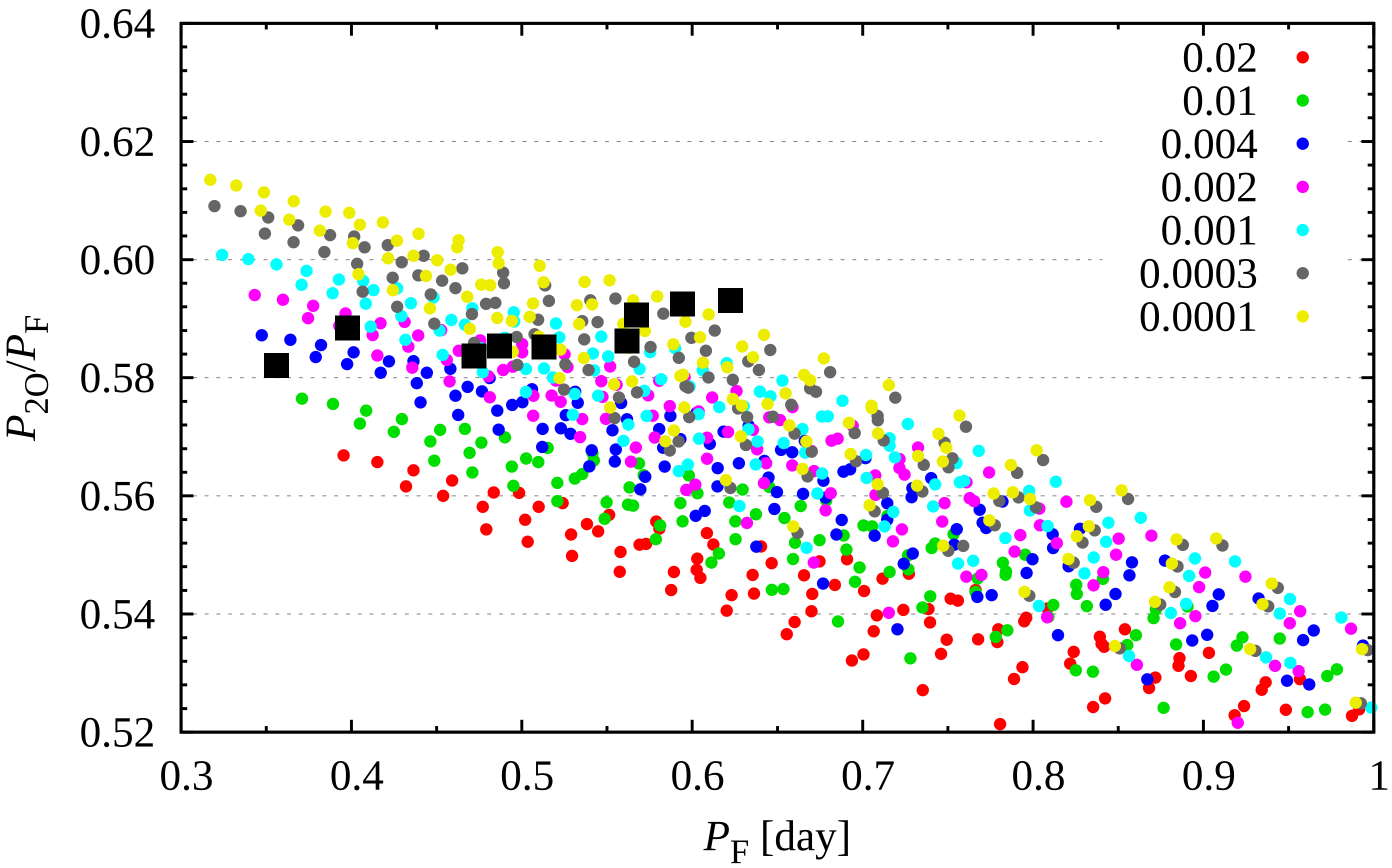


CoRoT 101128793

Poretti et al. (2010)

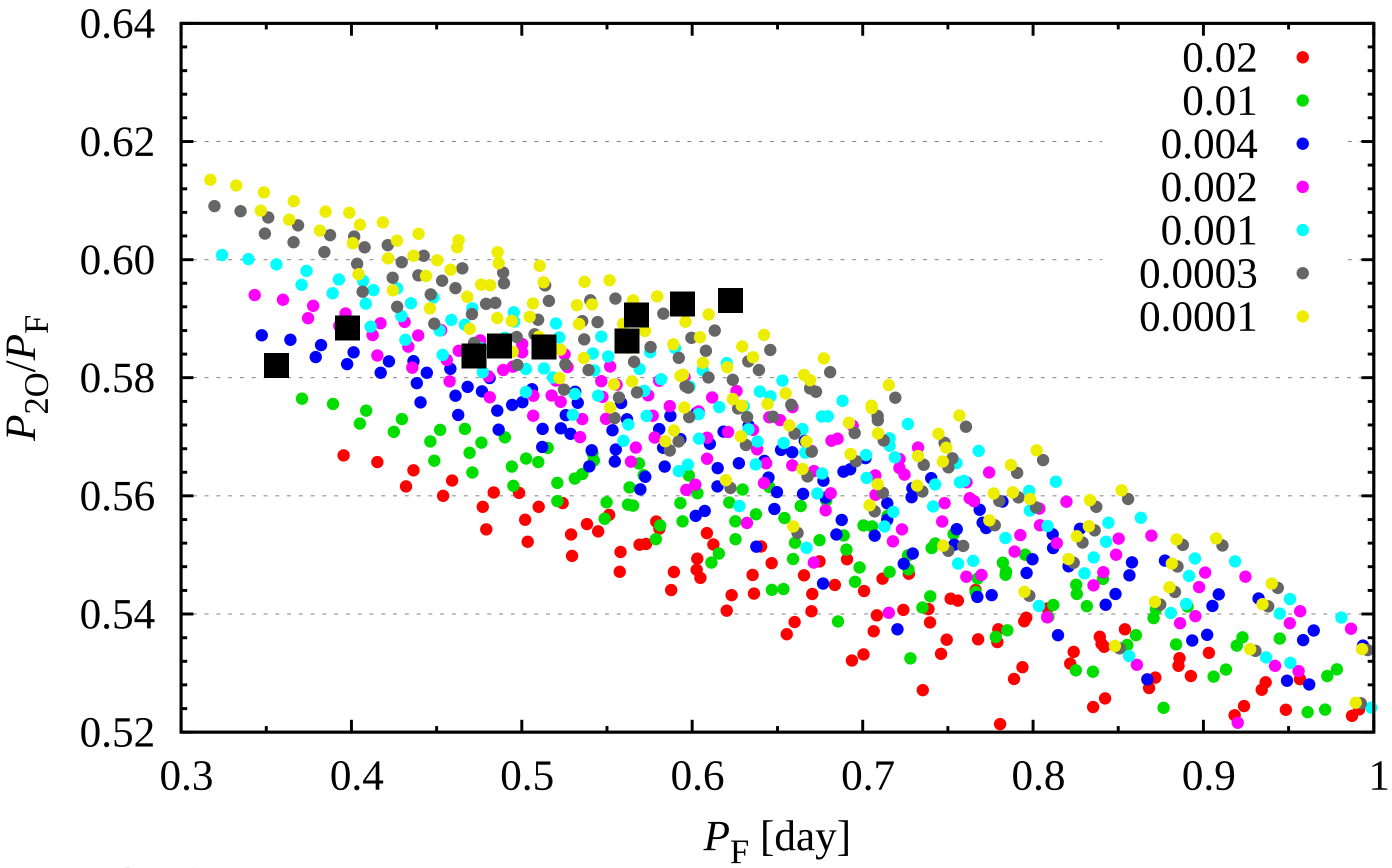
also Benkő et al. (2010)

New discoveries: RR Lyrae stars – new pulsation modes



Smolec, unpubl.

New discoveries: RR Lyrae stars – new pulsation modes

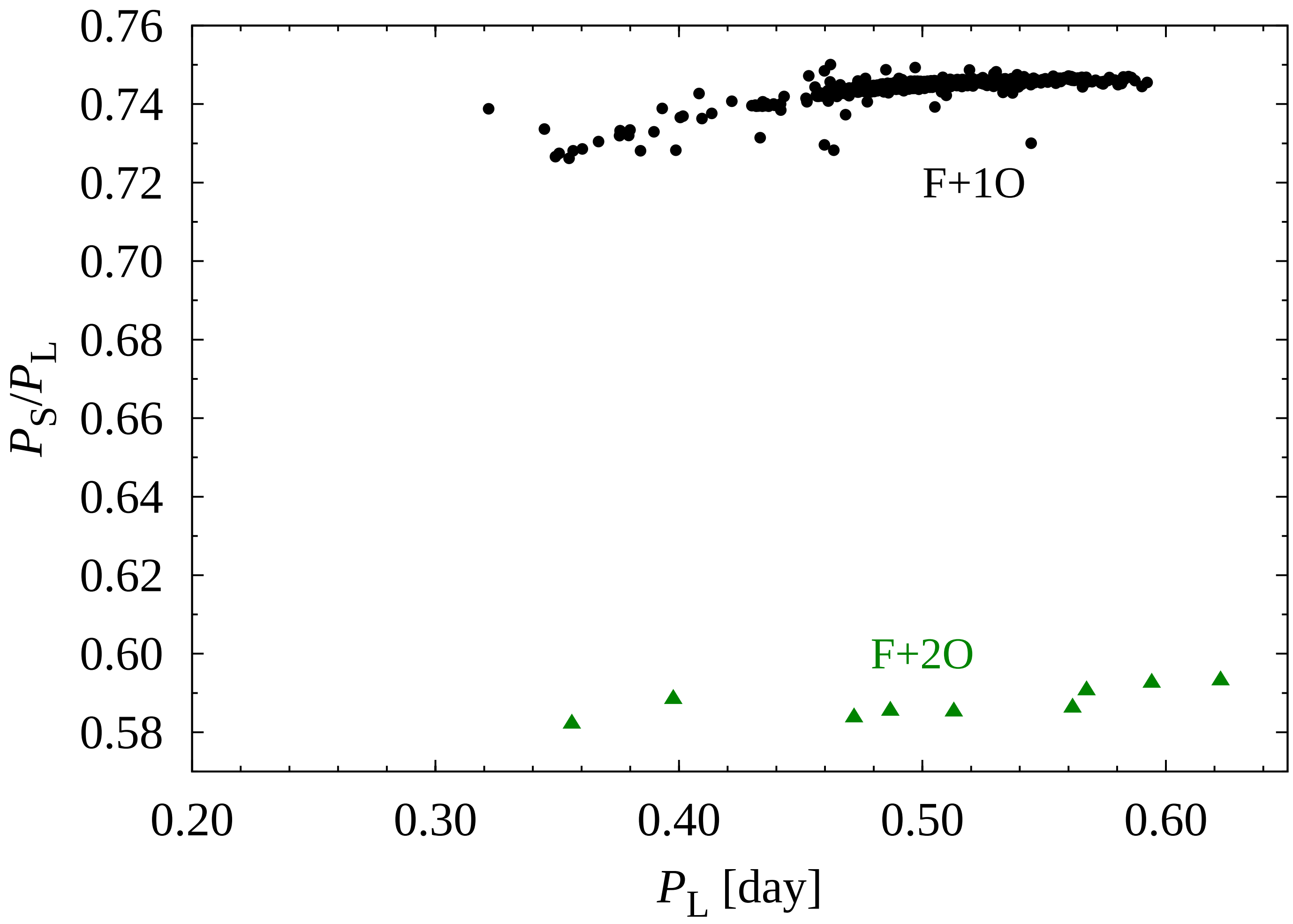


F+2O, but:

Smolec, unpubl.

- ⋆ **what about 1O?**
- ⋆ **pulsational stability of 2O?**
- ⋆ **resonances?**

New discoveries: RR Lyrae stars – new pulsation modes



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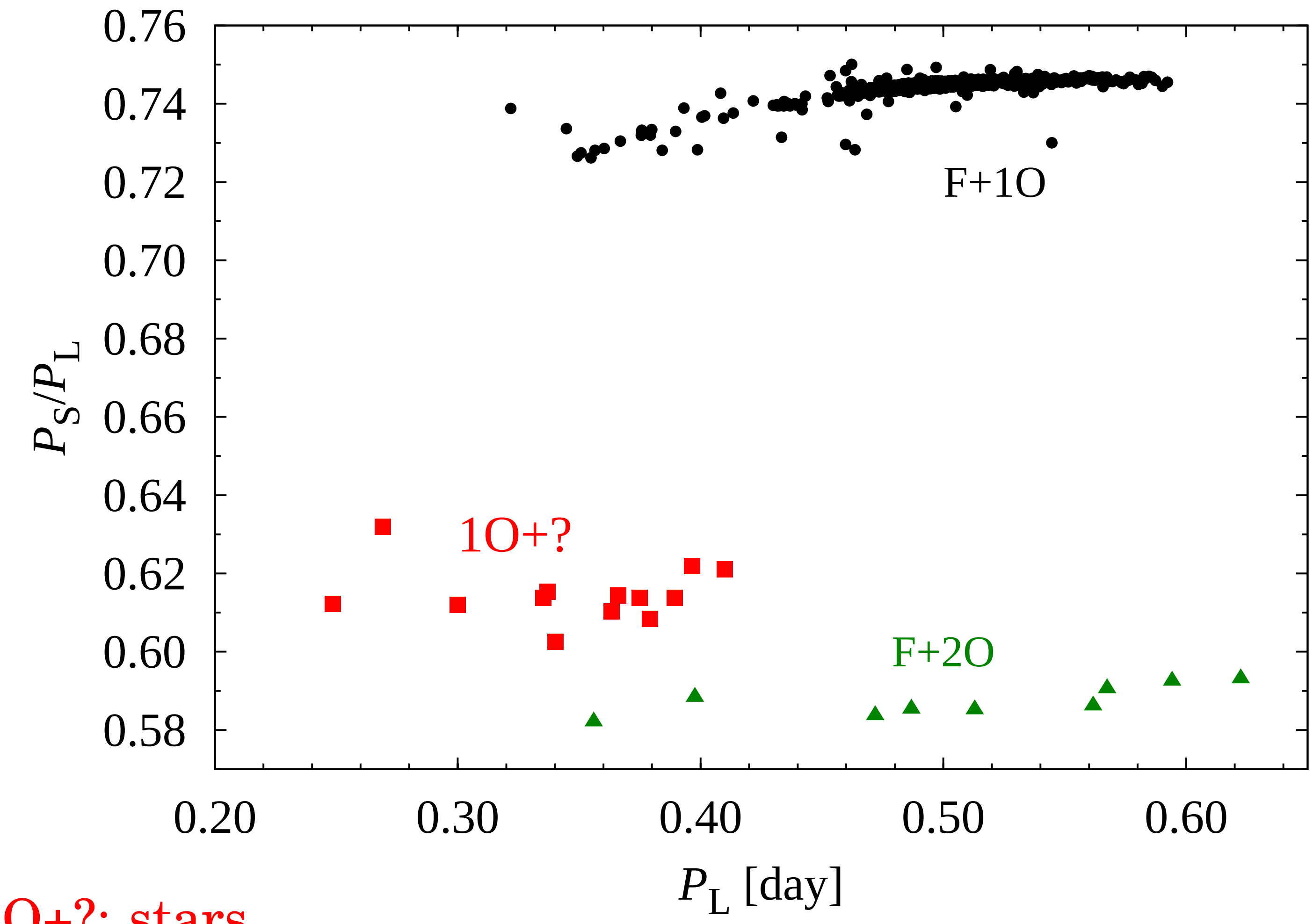

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MW Lyr

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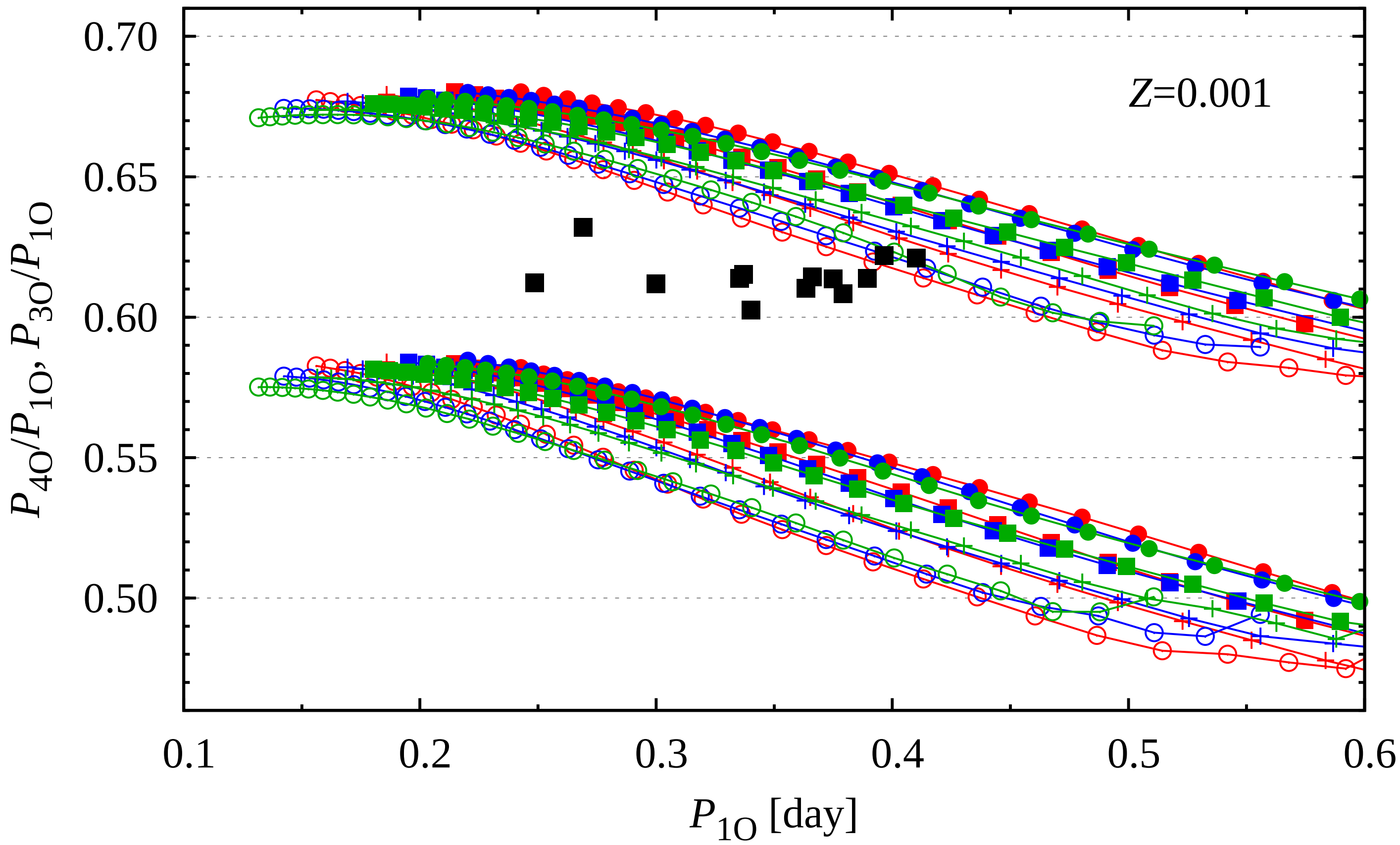
1O+?: stars

13 stars including 4 in the *Kepler* field

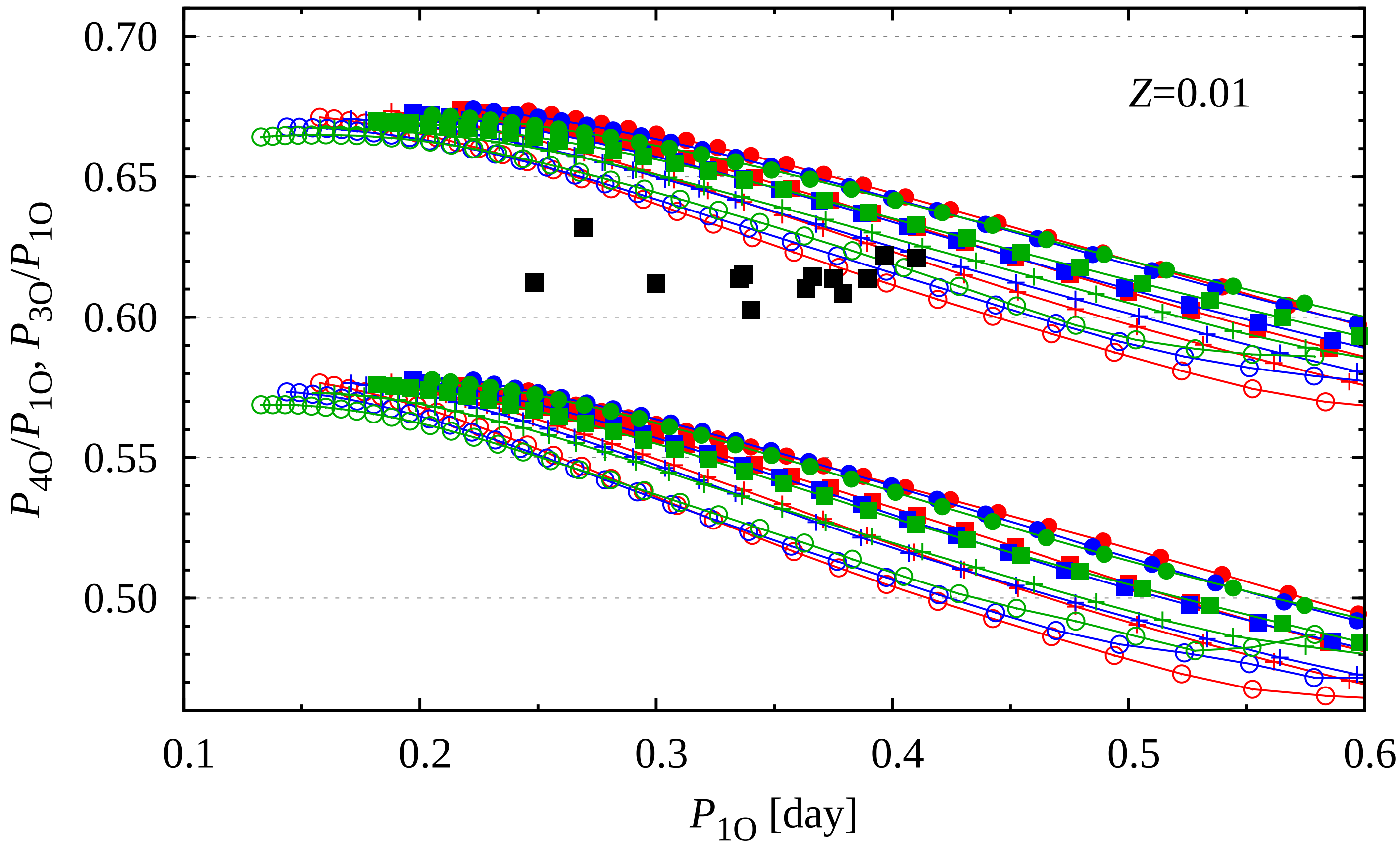
Gruberbauer et al. (2007), Olech & Moskalik (2009), Soszynski et al. (2009), Moskalik et al. (2011, unp.)

New discoveries: RR Lyrae stars with period ratio around 0.6



$Z = 0.001$ Smolec, unpubl.

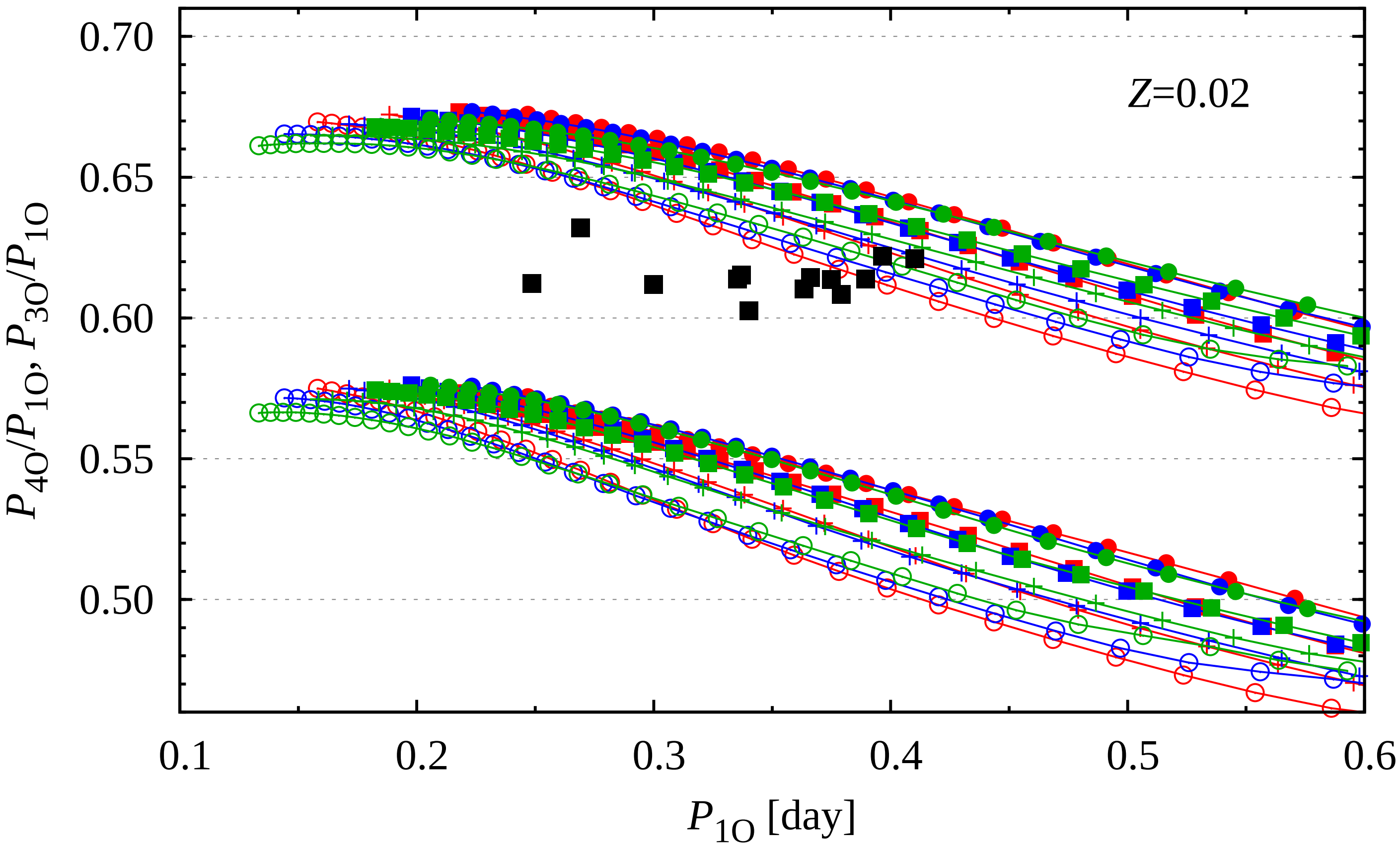
New discoveries: RR Lyrae stars with period ratio around 0.6



$Z = 0.01$

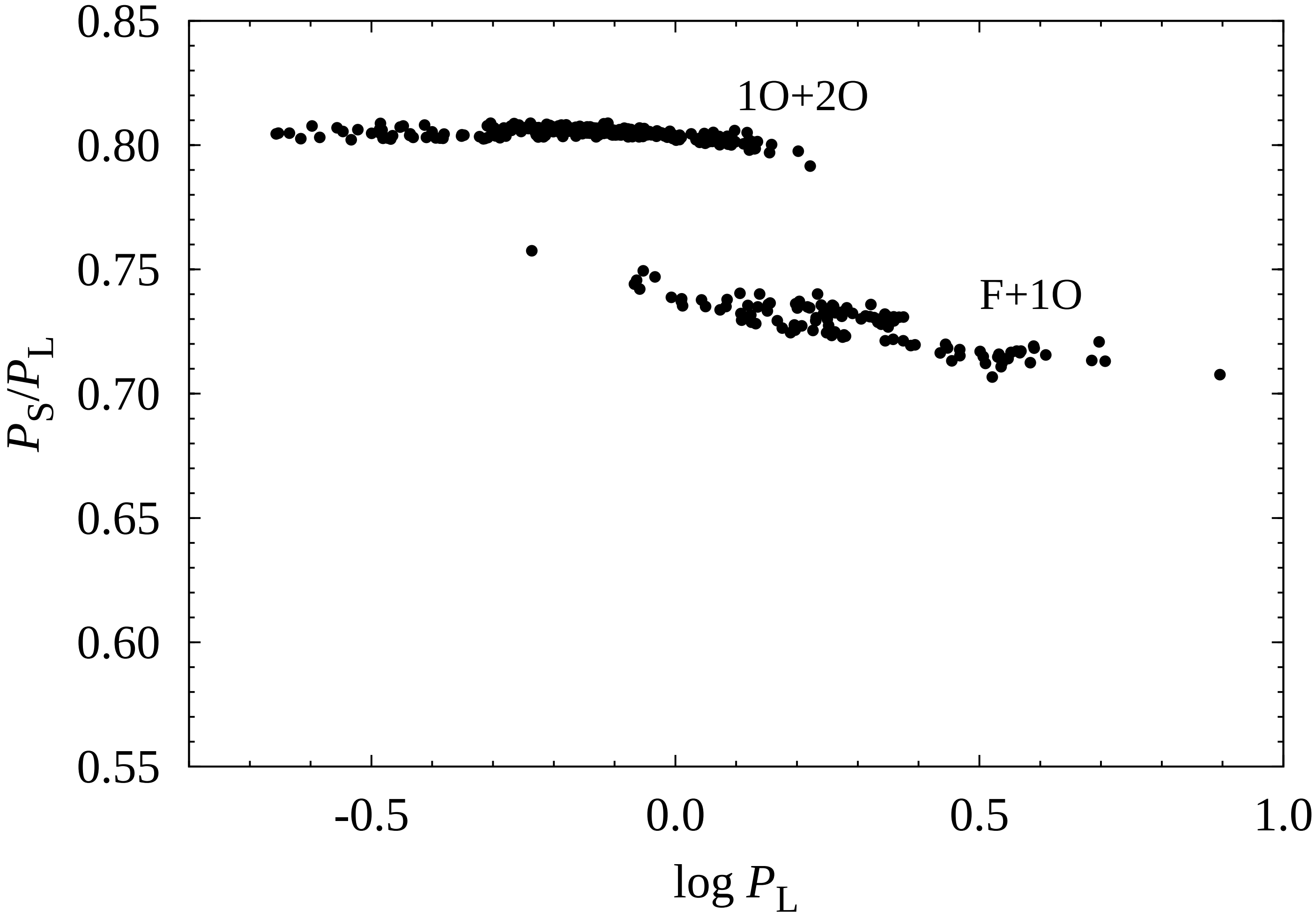
Smolec, unpubl.

New discoveries: RR Lyrae stars with period ratio around 0.6



$Z = 0.02$ Smolec, unpubl.

New discoveries: Cepheids – new pulsation modes



F+1O: 120 stars

1O+2O: 421 stars

Soszyński et al. (2008, 2010)

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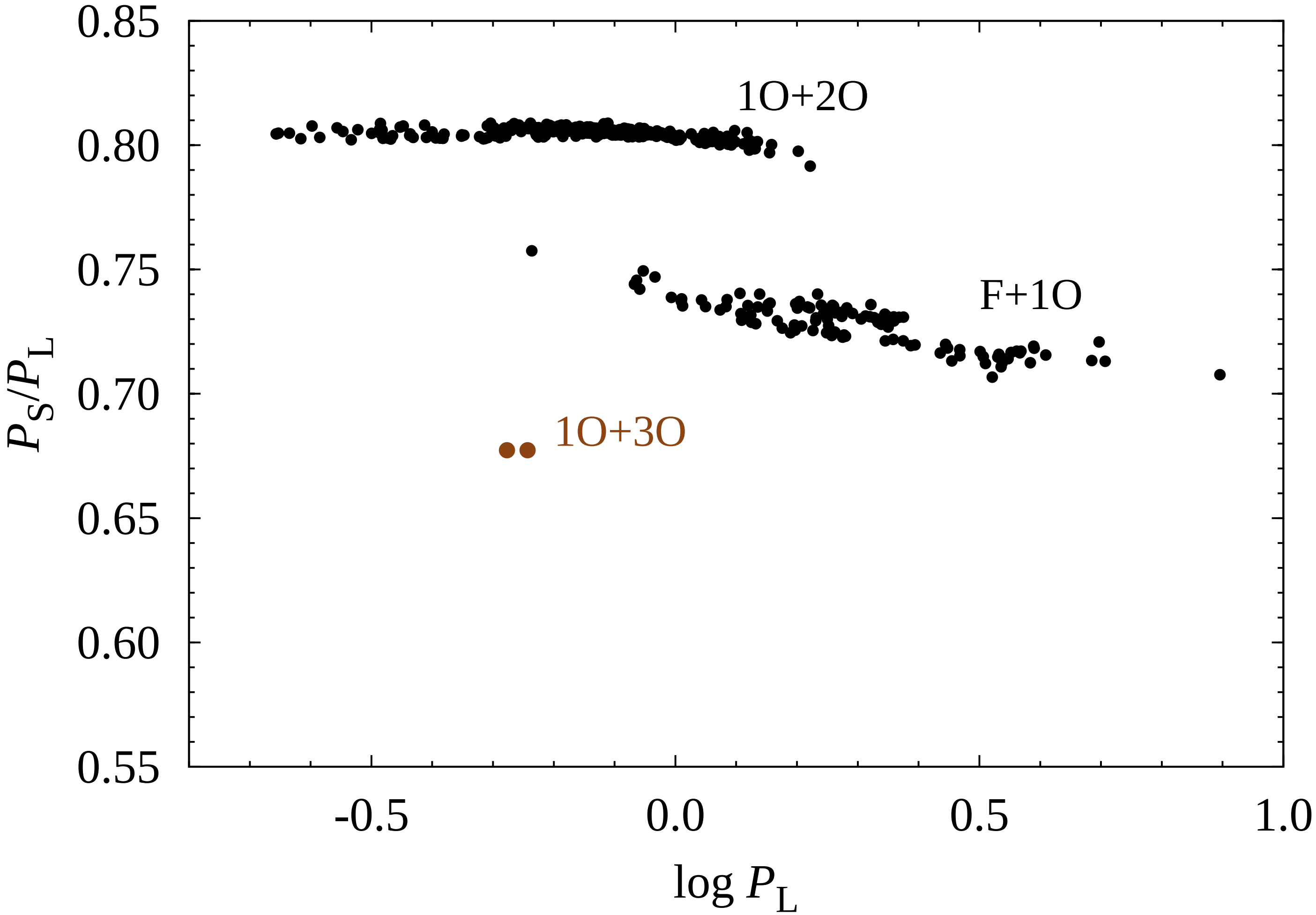

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1O+3O: 2 stars

Soszyński et al. (2008)

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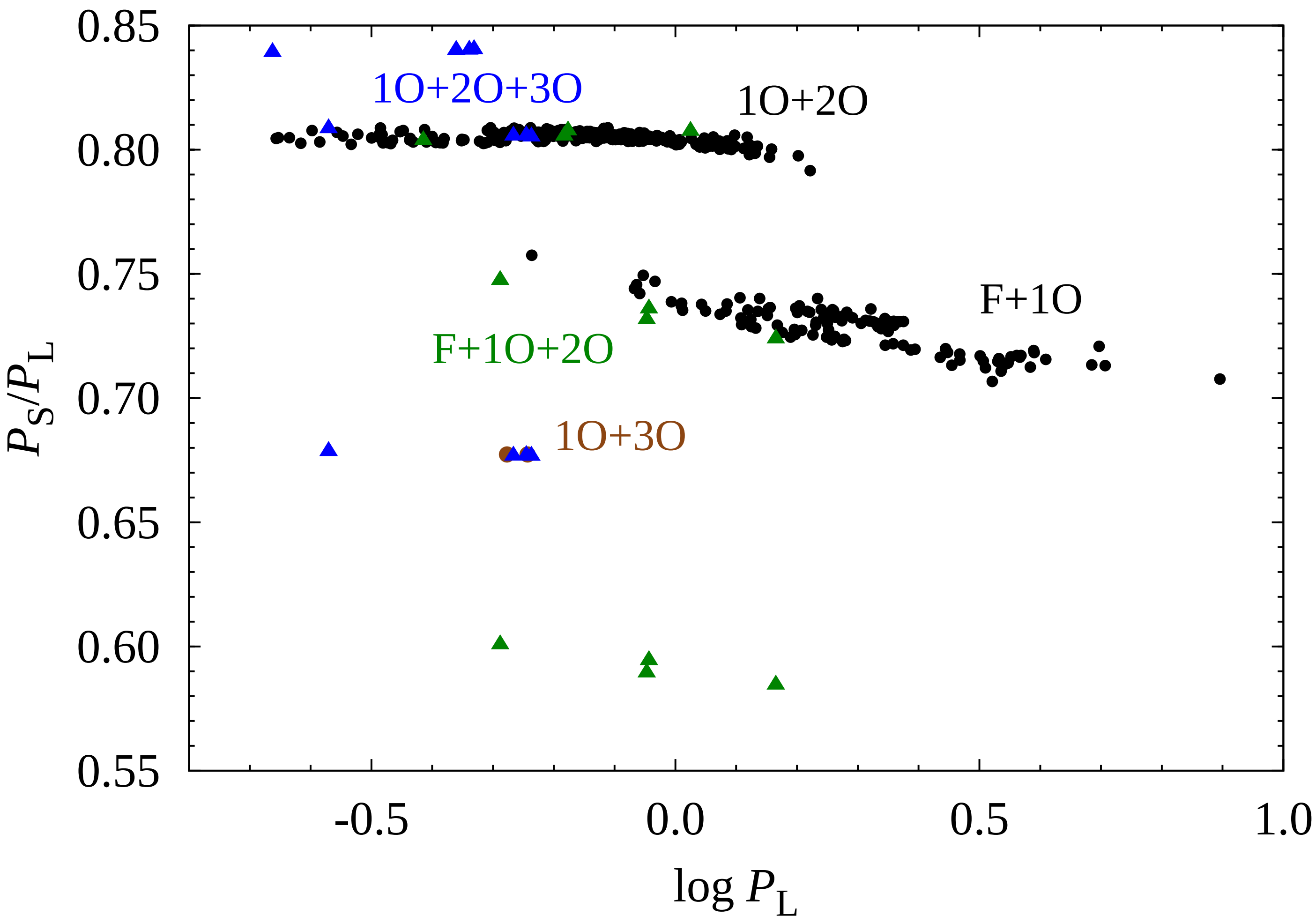

F+1O: 120 stars

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Soszyński et al. (2008, 2010)

1O+3O: 2 stars

Soszyński et al. (2008)

F+1O+2O: 4 stars

1O+2O+3O: 4 stars

Moskalik et al. (2004)

Soszyński et al. (2008, 2010)

modelling:

Moskalik & Dziembowski (2005)

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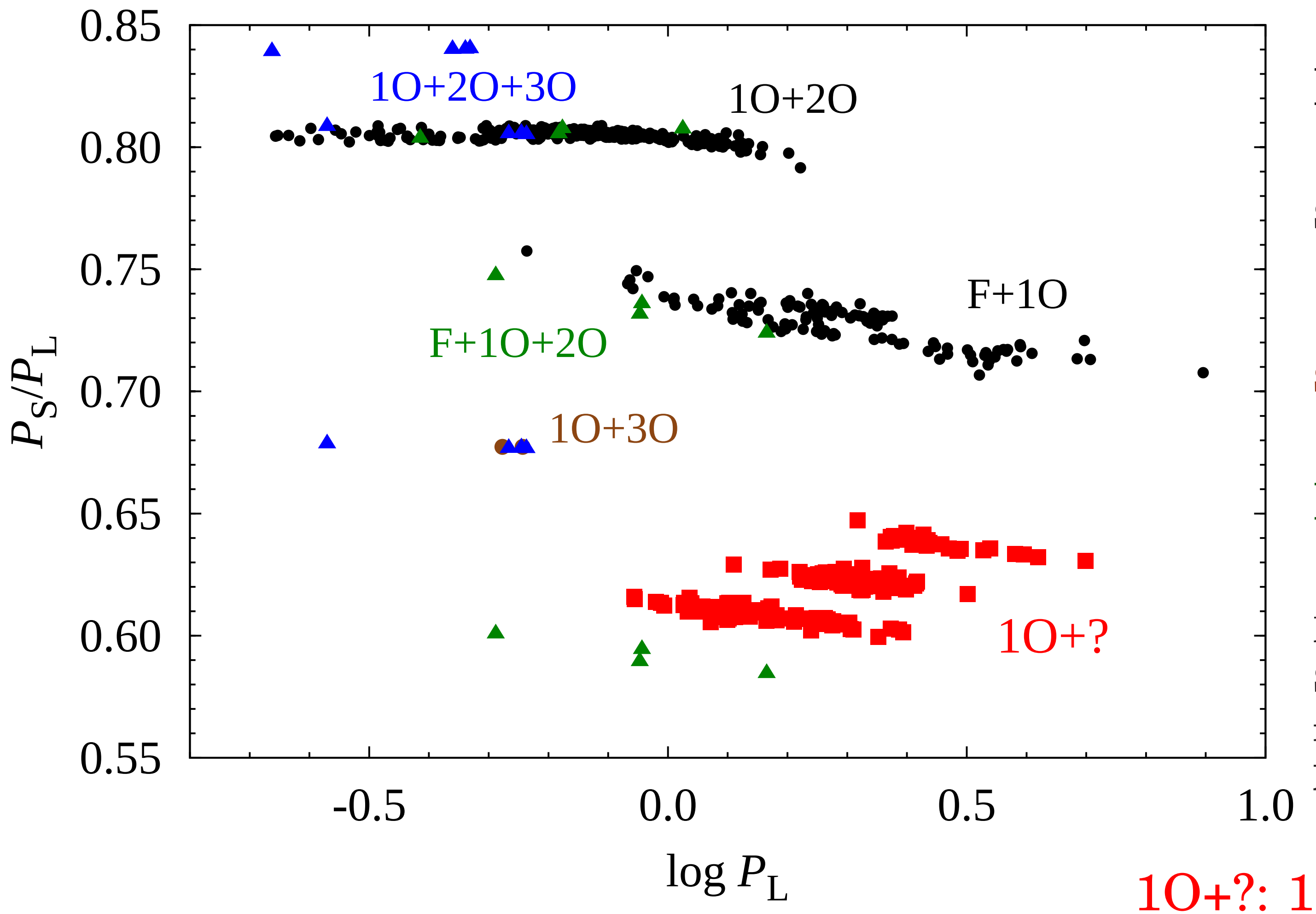

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Soszyński et al. (2008, 2010)

modelling:

Moskalik & Dziembowski (2005)

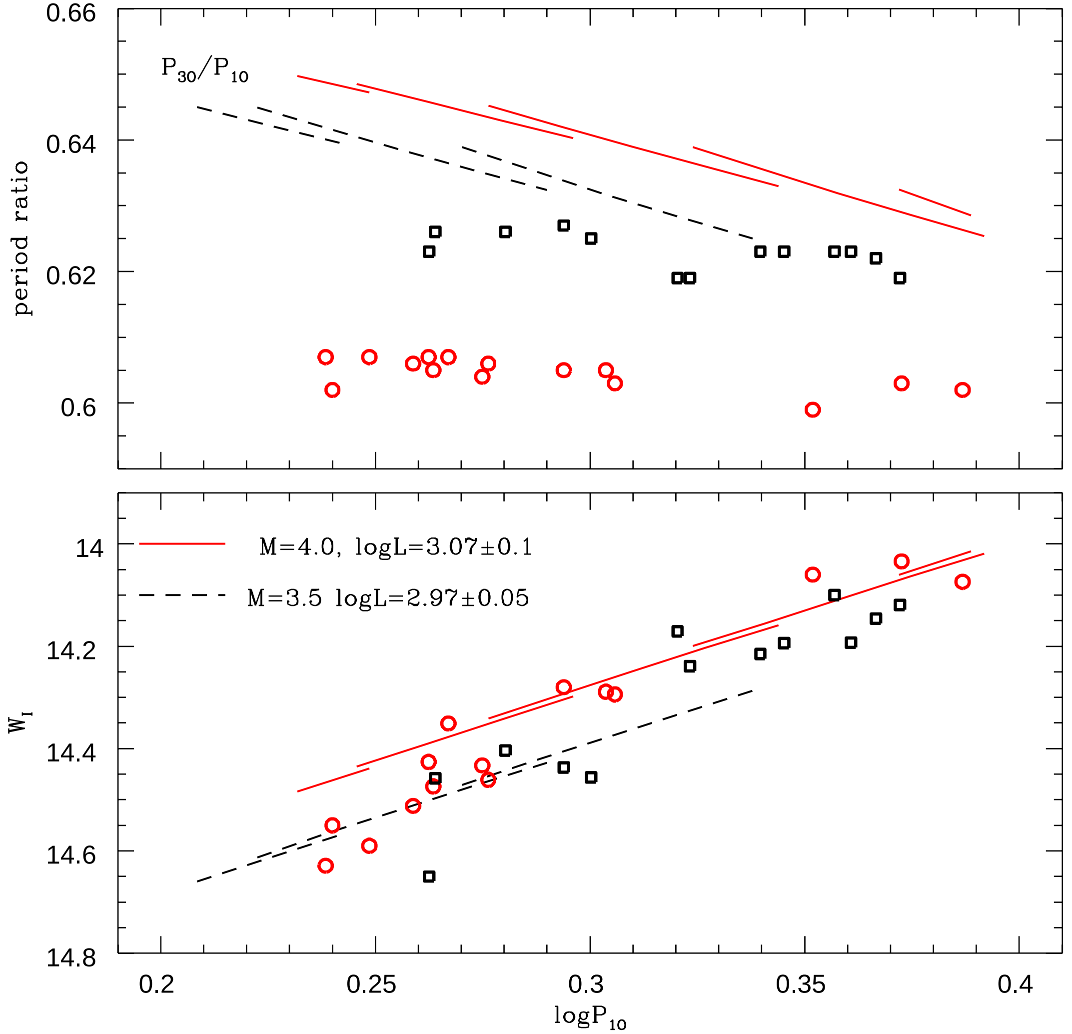
1O+?: 175 stars

in three groups (27+67+81)

Moskalik et al. (2004)

Soszyński et al. (2008, 2010)

New discoveries: Cepheids stars with period ratio around 0.6



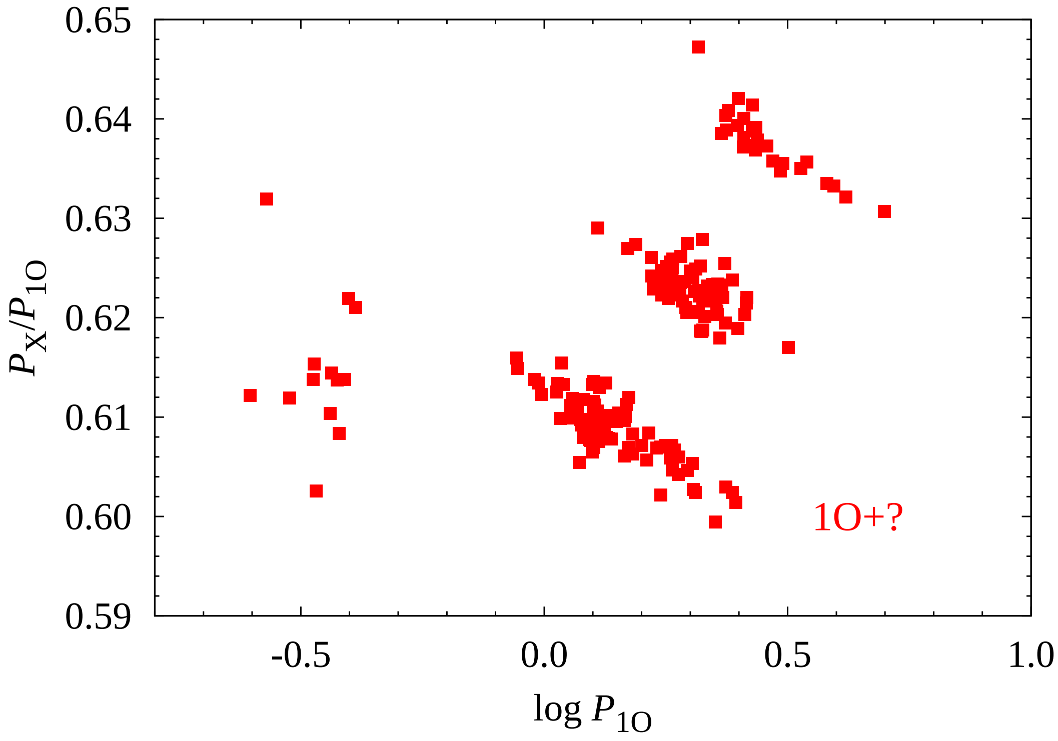
Dziembowski & Smolec (2009)

Strange period ratio ~ 0.6

- ⋆ Cepheids and RR Lyrs: 1O+?
- ⋆ radial modes
  - ▸ ruled out
- ⋆ non-radial modes

  Dziembowski (1977), Osaki (1977)

  Mulet-Marquis et al. (2007)
  - ▸ difficult to model
  - ▸ unstable modes $l > 5$



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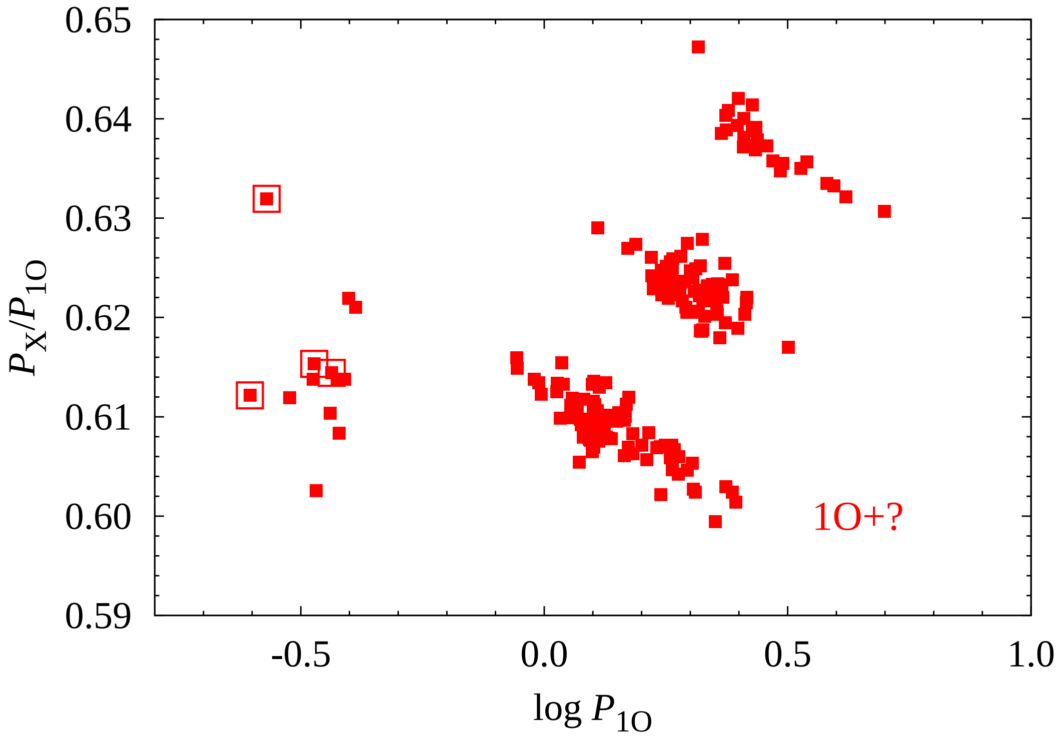
⋆ hint(?) from *Kepler*

▸ period doubling of a secondary mode

▸ strong variability of a secondary mode



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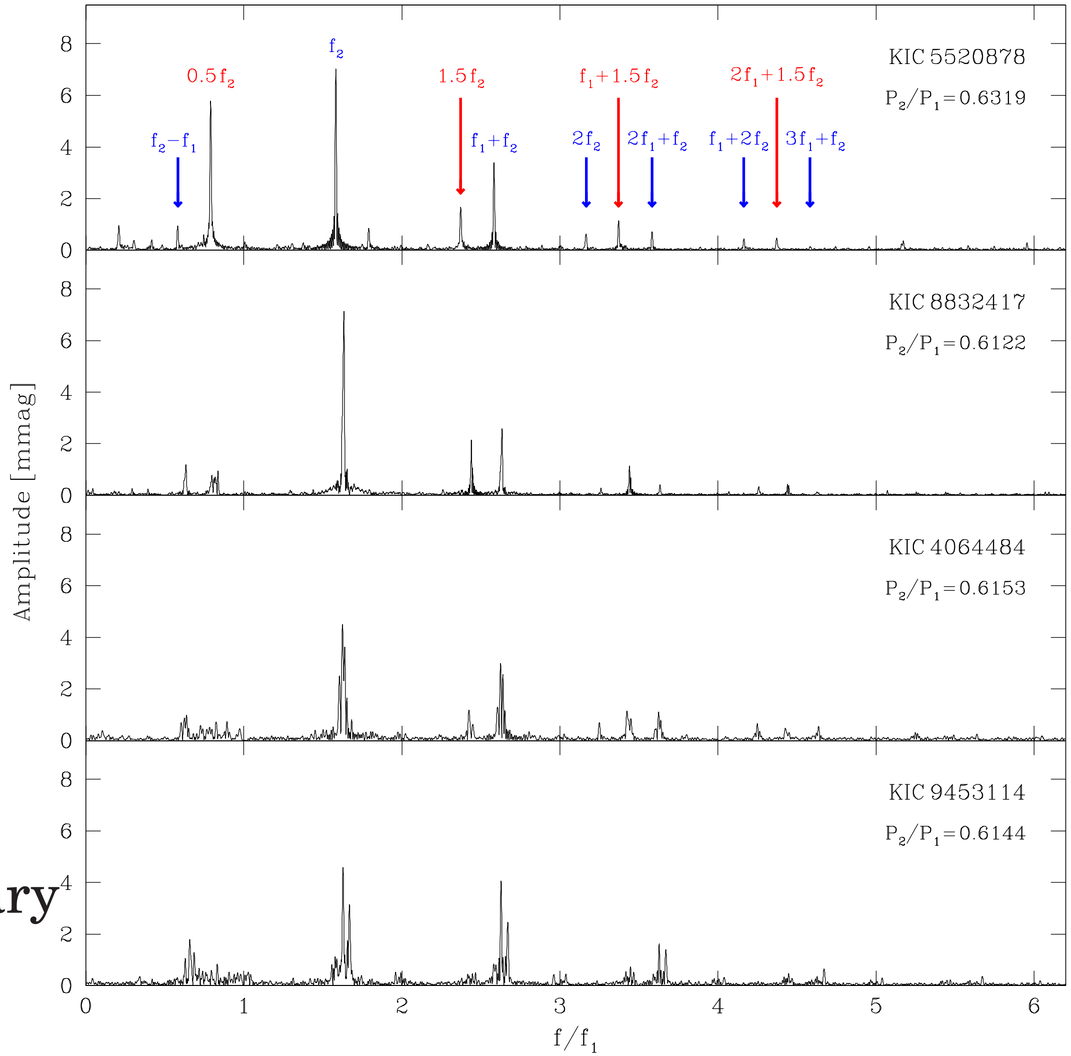

Moskalik et al. (2011)

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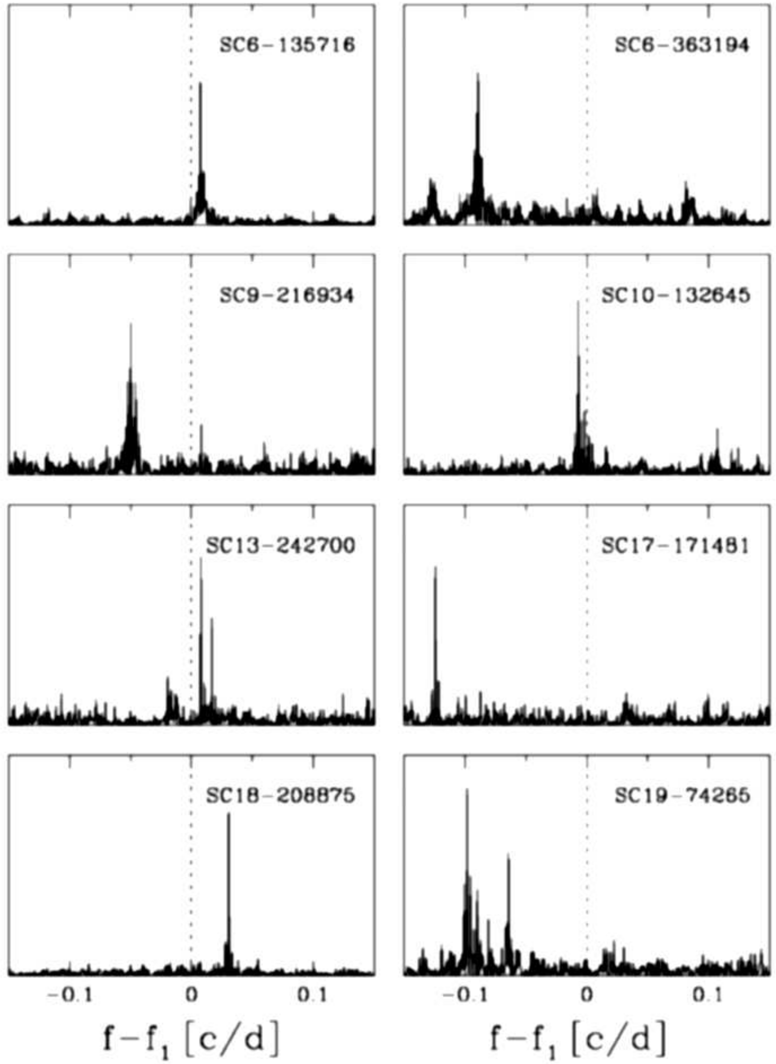

Moskalik & Kołaczkowski (2009)

Multiperiodic pulsation: understanding

1D nonlinear pulsation codes:

Radiative models

- e.g., Stellingwerf (1975)

$$\frac{dU}{dt} = -\frac{1}{\rho}\nabla(p) - \nabla\phi$$

$$\frac{de}{dt} + p\frac{dV}{dt} = -\frac{1}{\rho}\nabla(F_{\mathrm{r}})$$

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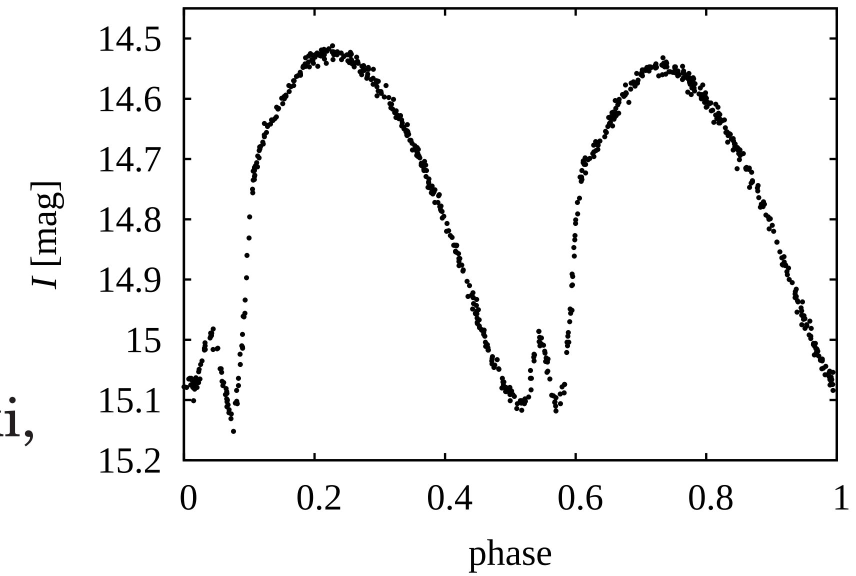
$$\frac{de}{dt} + p\frac{dV}{dt} = -\frac{1}{\rho}\nabla(F_{\rm r})$$

- **Period doubling in BL Herculis stars**
- predicted 1992 (Buchler & Moskalik), radiative 1D code
- first detection 2011 (Smolec, Soszyński, Moskalik et al.)



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Convective models

- Stellingwerf (1982)
  Bono & Stellingwerf (1992)
- Kuhfuß (1986)
  Feuchtinger (1999)
  Kolláth et al. (1998)
  Smolec & Moskalik (2008)

$$\frac{dU}{dt} = -\frac{1}{\rho}\nabla(p + p_{\mathrm{t}}) + U_q - \nabla\phi$$

$$\frac{de}{dt}+p\frac{dV}{dt} = -\frac{1}{\rho}\nabla(F_{\mathrm{r}}+F_{\mathrm{c}})-(S - D - D_{\mathrm{r}})$$

$$\frac{de_{\mathrm{t}}}{dt} + p_{\mathrm{t}}\frac{dV}{dt} = -\frac{1}{\rho}\nabla F_{\mathrm{t}} + E_q + (S - D - D_{\mathrm{r}})$$

Modelling of double-periodic Cepheid pulsation: controversy

- ⋆ inclusion of turbulent convection into the models led to success (Kolláth et al. (1998) – Cepheid models, Feuchtinger (1998) – one RR Lyrae model)
- ⋆ systematic double-periodic model surveys – only Florida-Budapest group (Kolláth et al. 2002, Szabó et al. 2004)
- ▸ despite computation of many model sequences, we haven't found any double-periodic Cepheid model with Warsaw pulsation hydrocodes (Smolec & Moskalik 2008)

  → our codes also use Kuhfuß model!

small difference in the model equations ⇒ huge difference for the mode selection problem

Reliability of 1D convective models

Origin: Reynolds averaging

- $x = \langle x \rangle + x'$
- continuity, momentum and thermal energy equations decomposed into mean and fluctuating part
- equation for turbulent energy $e_\mathrm{t} = \langle w'^2/2 \rangle$

Approximations and closure relations

- e.g. down gradient approximations, $F_\mathrm{c} \propto \nabla s$, $F_\mathrm{t} \propto \nabla e_\mathrm{t}$

Reliability of 1D convective models

$$\frac{dU}{dt} = -\frac{1}{\rho}\nabla(p + p_{\rm t}) + U_q - \nabla\phi$$

$$\frac{de}{dt} + p\frac{dV}{dt} = -\frac{1}{\rho}\nabla(F_{\rm r} + F_{\rm c}) - (S - D - D_{\rm r})$$

$$\frac{de_{\rm t}}{dt} + p_{\rm t}\frac{dV}{dt} = -\frac{1}{\rho}\nabla F_{\rm t} + E_q + (S - D - D_{\rm r})$$

$e_{\rm t}$	turbulent energy	
$\boldsymbol{S}$	source function	$\alpha\alpha_{\rm s}TpQe_{\rm t}^{1/2}\mathcal{Y}/H_p$
D	turbulent dissipation	$(\alpha_{\rm d}/\alpha)(e_{\rm t}^{3/2}/H_p)$
$D_{\rm r}$	radiative cooling	$D_{\rm r} = 4\sigma{\gamma_{\rm r}}^2/\alpha^2(T^3V^2e_{\rm t})/(c_p\kappa H_p^2)$
$p_{\rm t}$	turbulent pressure	$\alpha_{\rm p}\rho e_{\rm t}$
E_q, U_q	eddy-viscous terms	$-(4/3)\alpha\alpha_\nu H_p e_{\rm t}^{1/2}R\frac{\partial(U/R)}{\partial R}$
$\boldsymbol{F}_{\rm c}$	convective flux	$\alpha\alpha_{\rm c}\rho Tc_pe_{\rm t}^{1/2}\mathcal{Y}$
$F_{\rm t}$	turbulent flux	$-\alpha_{\rm t}\alpha\rho H_pe_{\rm t}^{1/2}\frac{\partial e_{\rm t}}{\partial R}$

8 free parameters!

$\boldsymbol{F}_{\rm c} \propto \mathcal{Y}$, $\boldsymbol{S} \propto \mathcal{Y}$, $\mathcal{Y} = \nabla - \nabla_{\rm a}$

Reliability of 1D convective models

Ambiguities: source term (buoyant driving/damping)

$S \propto e_{\mathrm{t}}^{1/2}\mathcal{Y}$

Kuhfuß (1986)

or

$S \propto e_{\mathrm{t}}\mathrm{sgn}(\mathcal{Y})\sqrt{|\mathcal{Y}|}$

Stellingwerf (1982)

no DM solutions

Smolec & Moskalik (2008,2010)

no DM solutions

G. Bono, private comm.

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$S \propto e_{\mathrm{t}}^{1/2}\mathcal{Y}_{+}$: negative buoyancy neglected

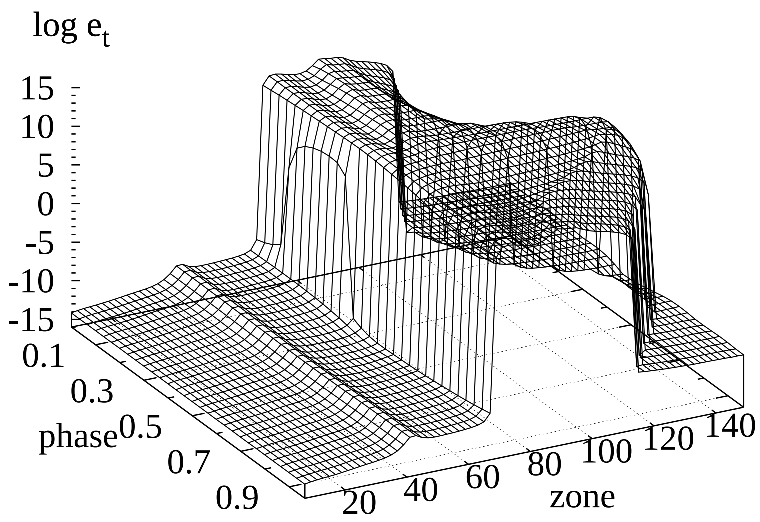
Kolláth et al. (1998)

DM pulsation

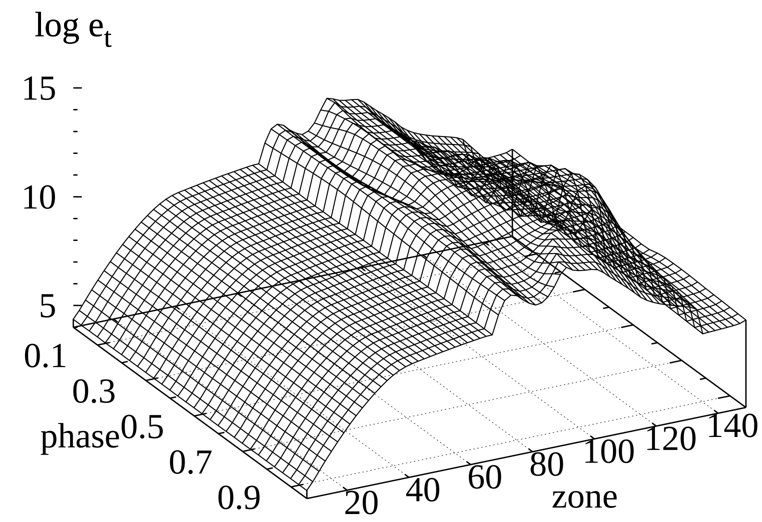
e.g., Kolláth et al. (1998,2002)

Double-periodic Cepheid pulsation: treatment of negative buoyancy

Negative buoyancy included:



Negative buoyancy neglected:



$$\mathcal{Y} < 0$$

$$\frac{de_{\rm t}}{dt} = \boldsymbol{S} - \boldsymbol{D} + \boldsymbol{E_q}$$

$$\frac{de_{\rm t}}{dt} = - \boldsymbol{D} + \boldsymbol{E_q}$$

Smolec & Moskalik (2008)

Double-periodic Cepheid pulsation: treatment of negative buoyancy

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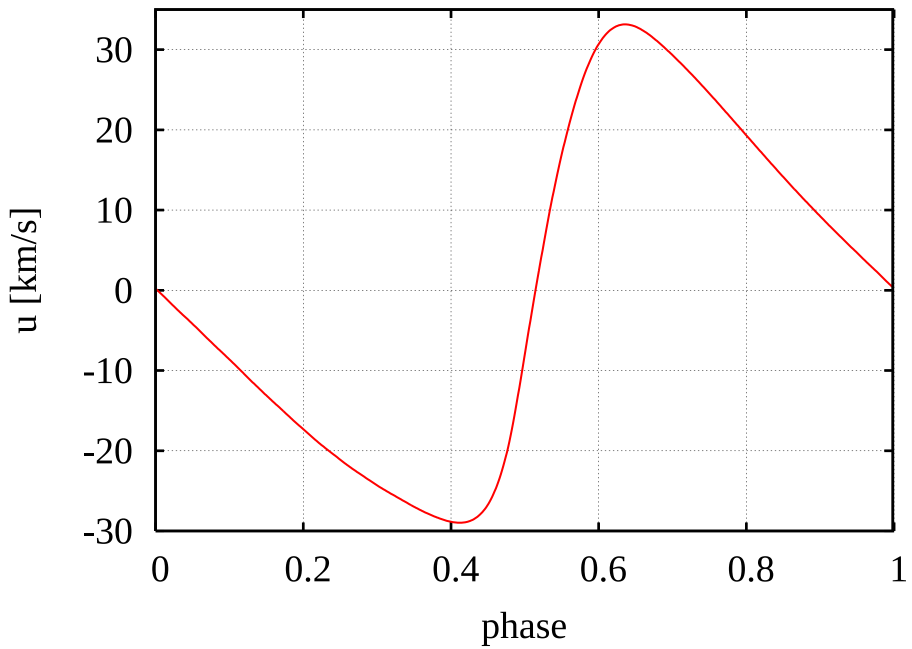

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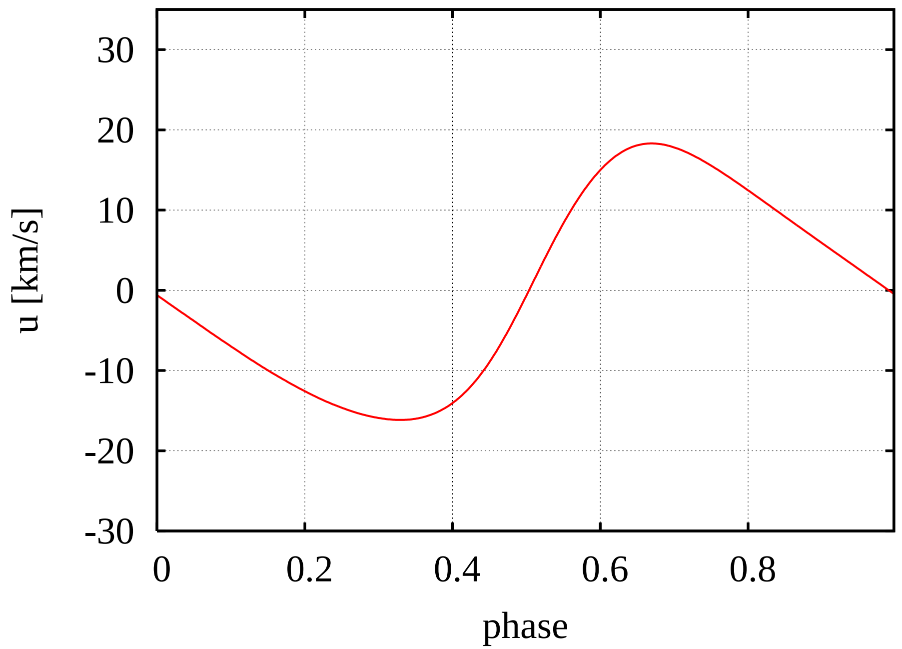

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Smolec & Moskalik (2008)

Unsolved problem: Modelling of double-periodic Cepheid pulsation

- Amplitude reduction is differential: amplitude of the F-mode is reduced stronger
- Result: self saturation exceeds the cross saturation → double-mode pulsation emerge
- but is a result of unpysical assumption

Double periodic pulsators exist – can we improve the 1D formulae to model them?

- use 3D models to improve 1D models (e.g., description of overshooting)

2D/3D pulsation hydrocodes:

⋆ several simulations for main-sequence/giant stars, e.g.,

Stein & Nordlund (1989, 1998), Nordlund et al. (2009)

Meakin & Arnett (2007), Arnett, Meakin & Young (2009)

Trampedach et al. (2007, 2010)

▸ 3D models used to improve the MLT (e.g., Arnett, Meakin & Young 2010)

⋆ 2D/3D modelling of large amplitude pulsations – a challenging problem

Early work: Deupree (1977)

Gastine & Dintrans (2011)

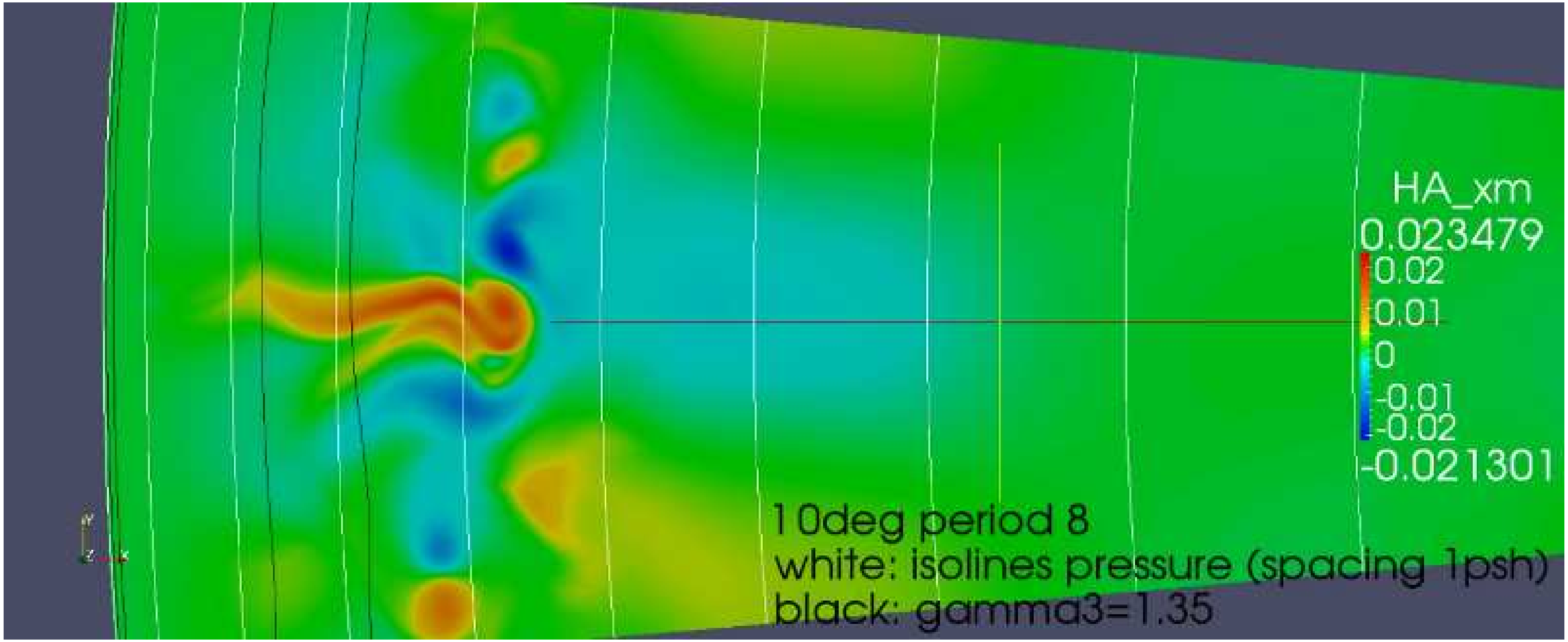
ANTARES (Muthsam et al. 2010)

SPHERLS (Geroux & Deupree 2011)

ANTARES (Muthsam, et al. 2010):

- ⋆ **A N**umerical **T**ool for **A**strophysical **RES**earch
- ⋆ time-dependent compressible hydrodynamics, RHD, 1D–3D
- ⋆ realistic microphysics (OPAL EOS, opacities)
- ⋆ high-resolution simulation of solar granulation (Muthsam et al. 2007)
- ⋆ 2D Cepheid models (5125K, 5500K)

ANTARES (Muthsam, et al. 2010):



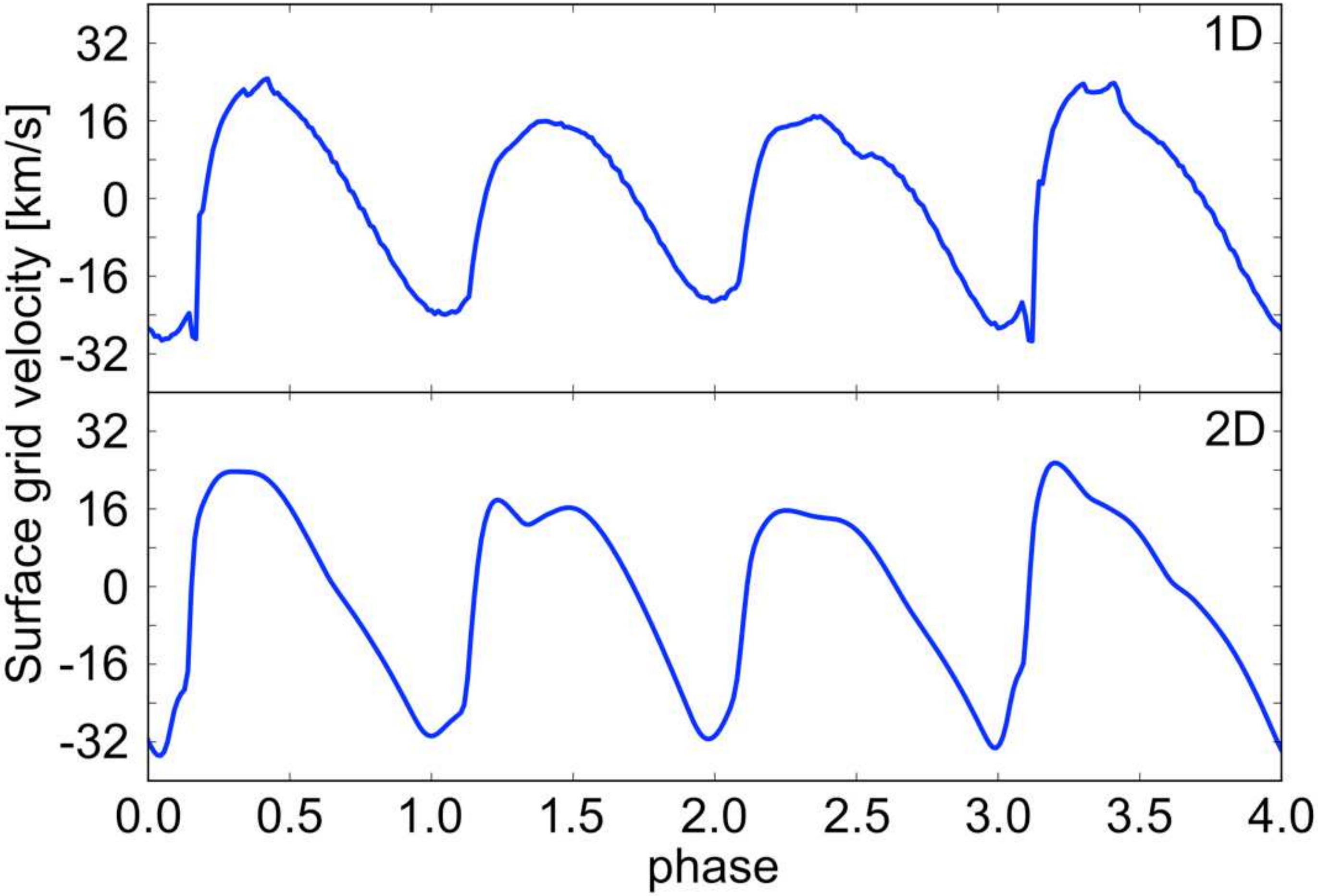
(Muthsam, Mundprecht)

- overshooting estimate – up to 1 H_p

SPHERLS (Geroux & Deupree 2011):

- **S**tellar **P**ulsation with a **H**orizontal **E**ulerian, **R**adial **L**agrangian **S**cheme
- 1D, 2D (working) and 3D code
- realistic EOS and opacities
- radiation in the diffusion approximation
- full amplitude pulsation for few models

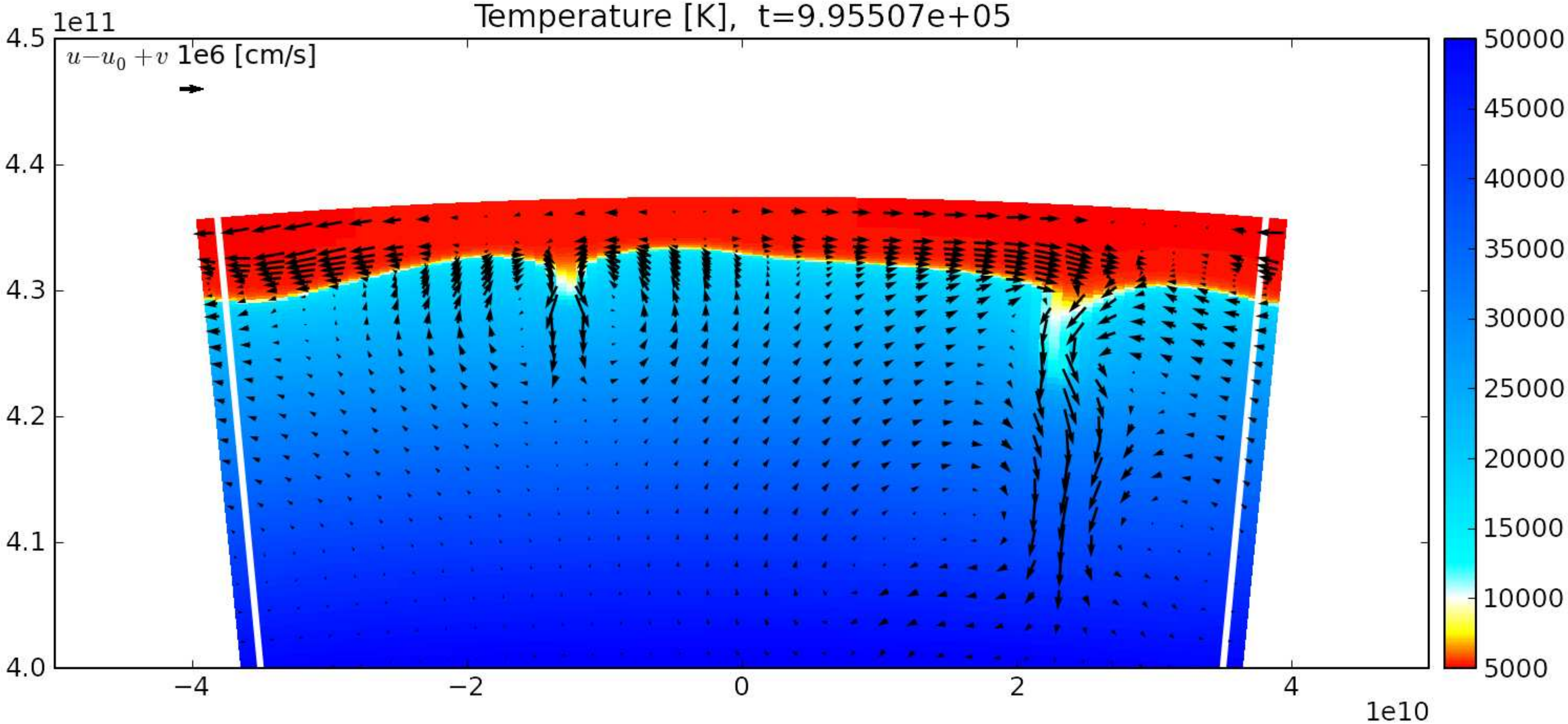
SPHERLS (Geroux & Deupree 2011):



6500K

Geroux (2011)

SPHERLS (Geroux & Deupree 2011):



Geroux (2011)

- phase dependent overshooting; estimates 0.25–0.5 H_p

Conclusions:

- modelling of non-radial pulsation is needed
- 3D hydrodynamic simulations

Many thanks to Herbert Muthsam, Eva Mundprecht and Chris Geroux for helpful discussions and figures!