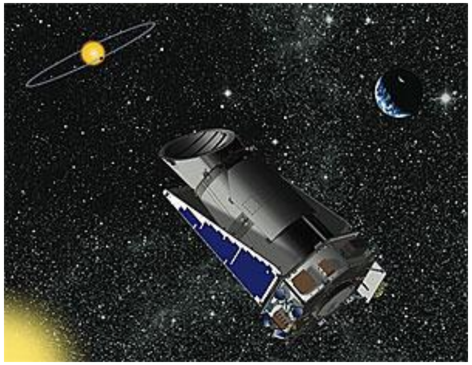
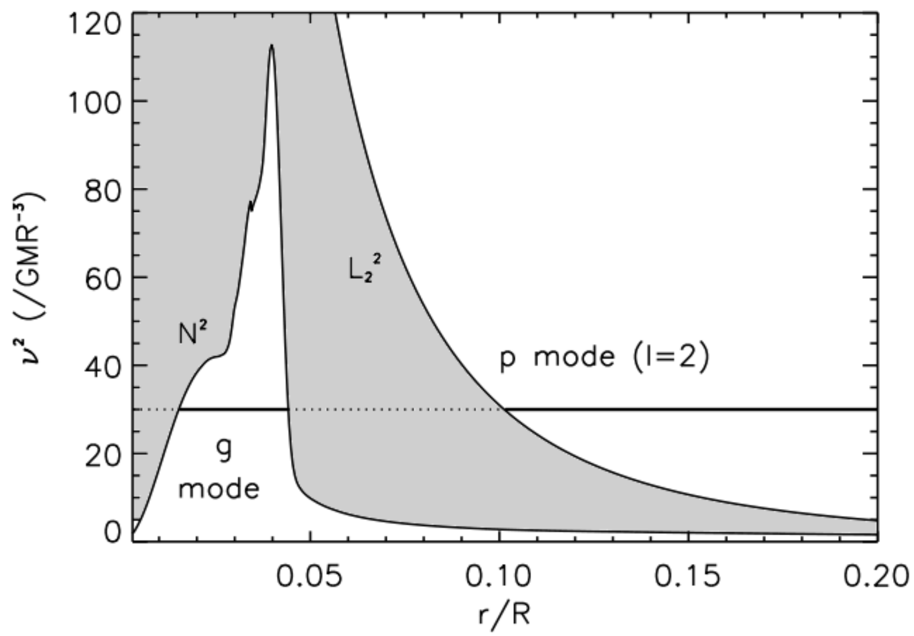
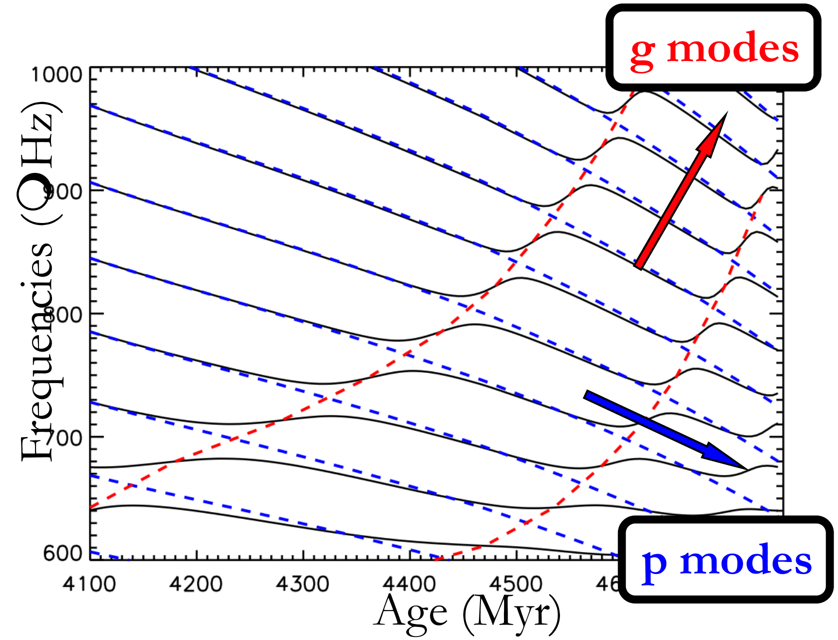
What can mixed modes tell us about the internal structure of subgiants?

S. Deheuvels



Mixed modes





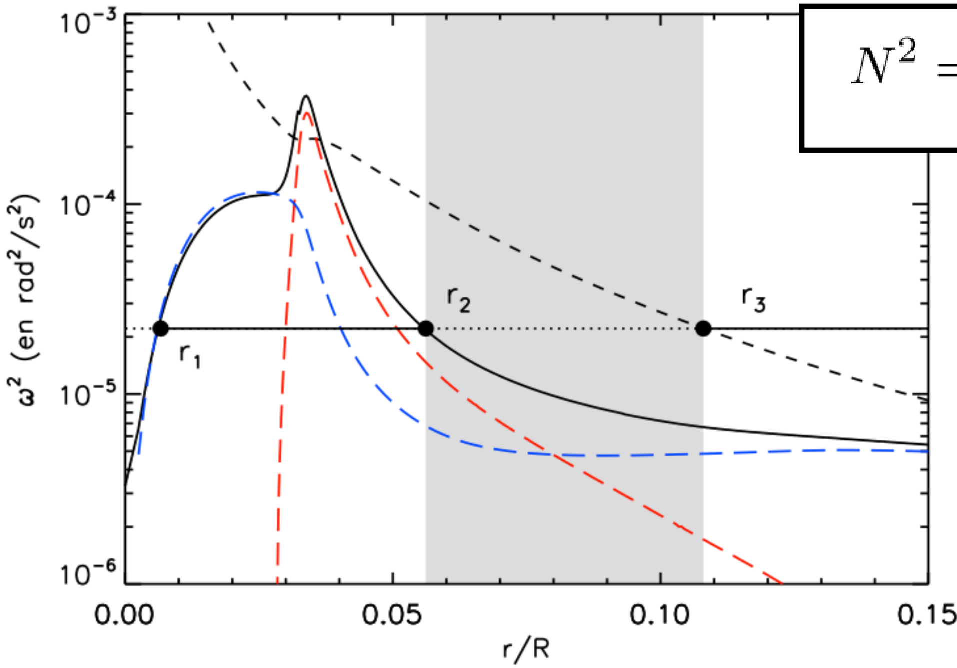
- Interest of mixed modes:
  1. They are **sensitive to the core**
  2. They have much **higher amplitudes** than pure g modes

Mixed modes

- Interest of having access to the frequency of a g mode

$$\nu_{\rm g} \propto \int_{r_1}^{r_2} \frac{N}{r}\,\mathrm{d}r$$

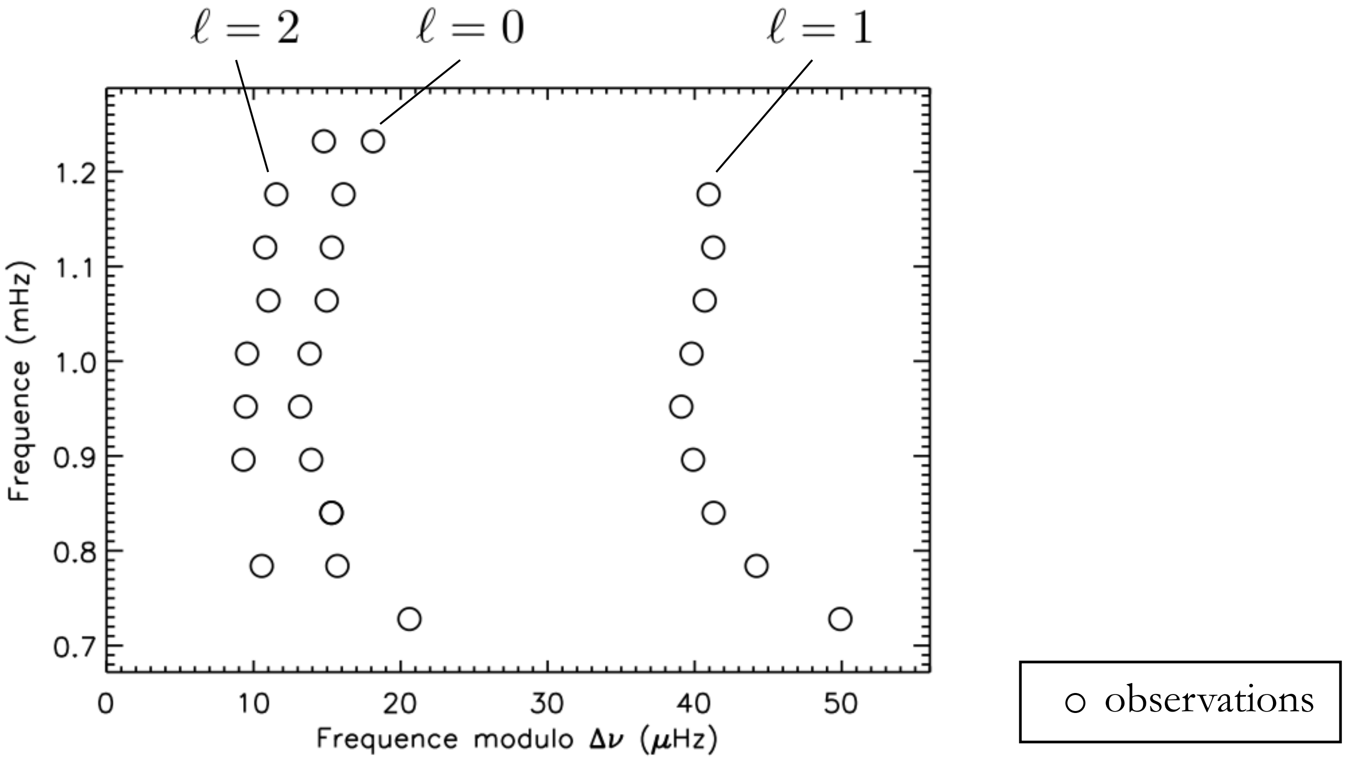
$$N^2 = \frac{g}{H_p}\left(\nabla - \nabla_{\rm ad}\right) + \frac{g}{H_p}\nabla\mu$$



⇒ possible constraints on the amount of core overshooting (Dziembowski & Pamyatnikh 1991)

The case of HD49385

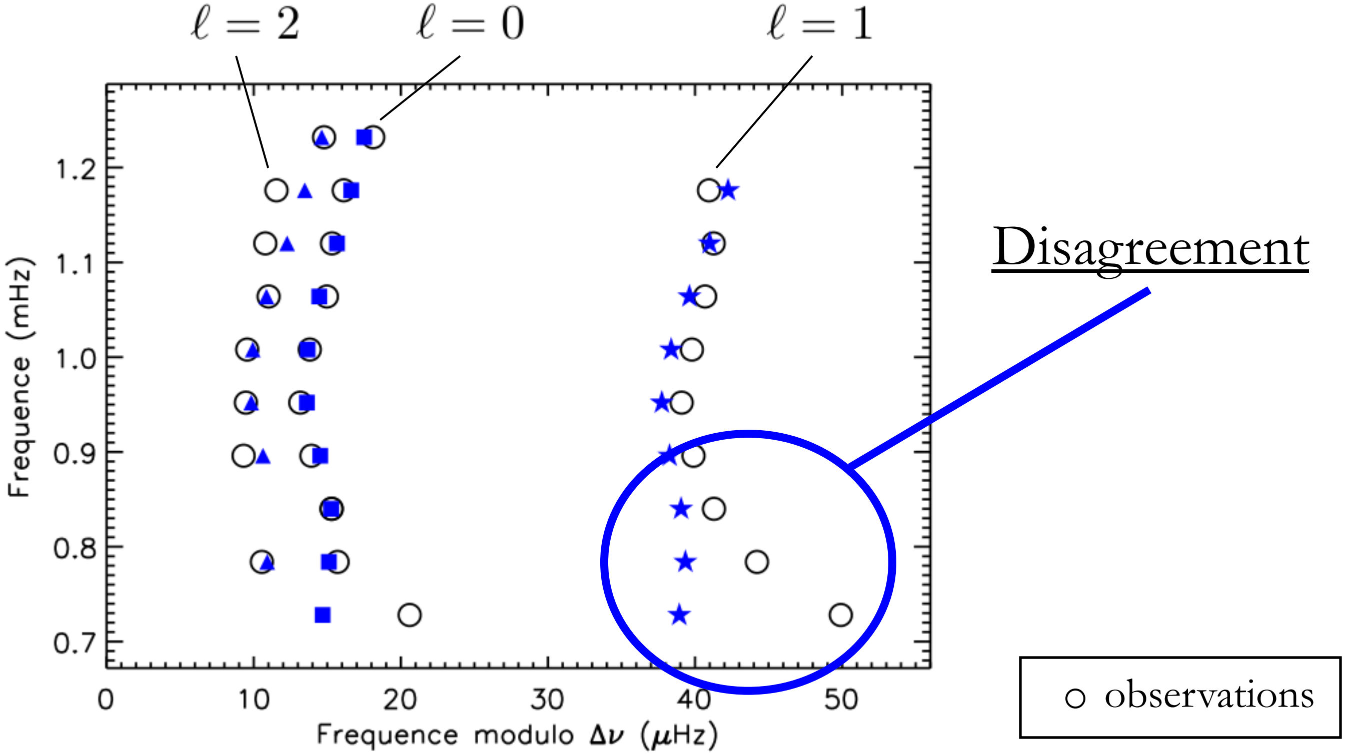
- HD49385: evolved solar-like pulsator
  - CoRoT target (137 days)
  - 27 p modes detected with ℓ=0, 1, and 2 (Deheuvels et al. 2010)



The case of HD49385

- HD49385: evolved solar-like pulsator
  - CoRoT target (137 days)
  - 27 p modes detected with ℓ=0, 1, and 2 (Deheuvels et al. 2010)

Main Sequence models



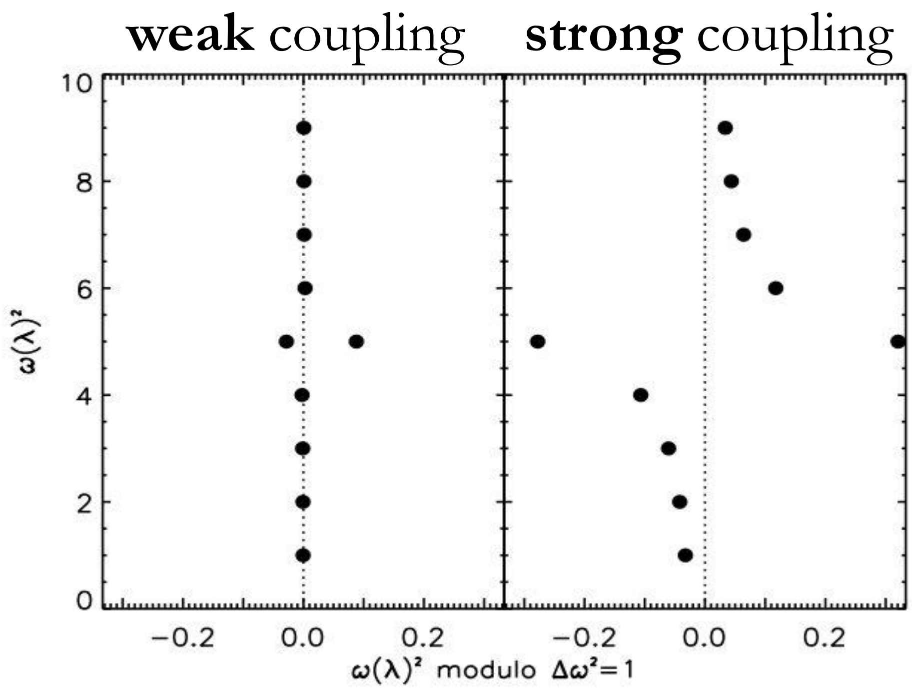
Avoided crossing with strong coupling

- Avoided crossings traditionally seen as a 2-mode interaction
- Analogy with harmonic oscillators (Christensen-Dalsgaard 2003)

$$\frac{\mathrm{d}^2 y_\mathrm{p}}{\mathrm{d}t^2} = -\omega_\mathrm{p}(\lambda)^2 y_\mathrm{p} \quad + \alpha y_\mathrm{g}$$

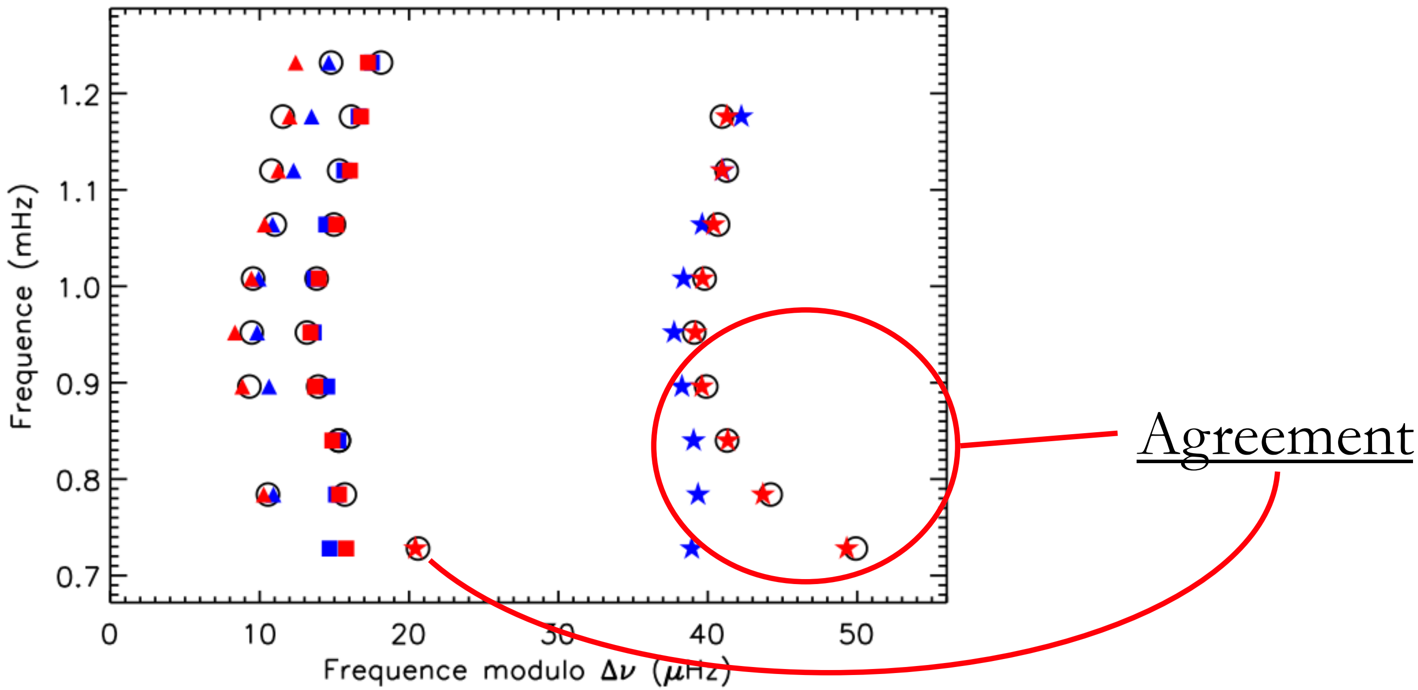
$$\frac{\mathrm{d}^2 y_\mathrm{g}}{\mathrm{d}t^2} = -\omega_\mathrm{g}(\lambda)^2 y_\mathrm{g} \quad + \alpha y_\mathrm{p}$$

- Extension of the analogy to the case of n modes (Deheuvels & Michel 2010)



Avoided crossing with strong coupling

- Post-main sequence models for HD49385

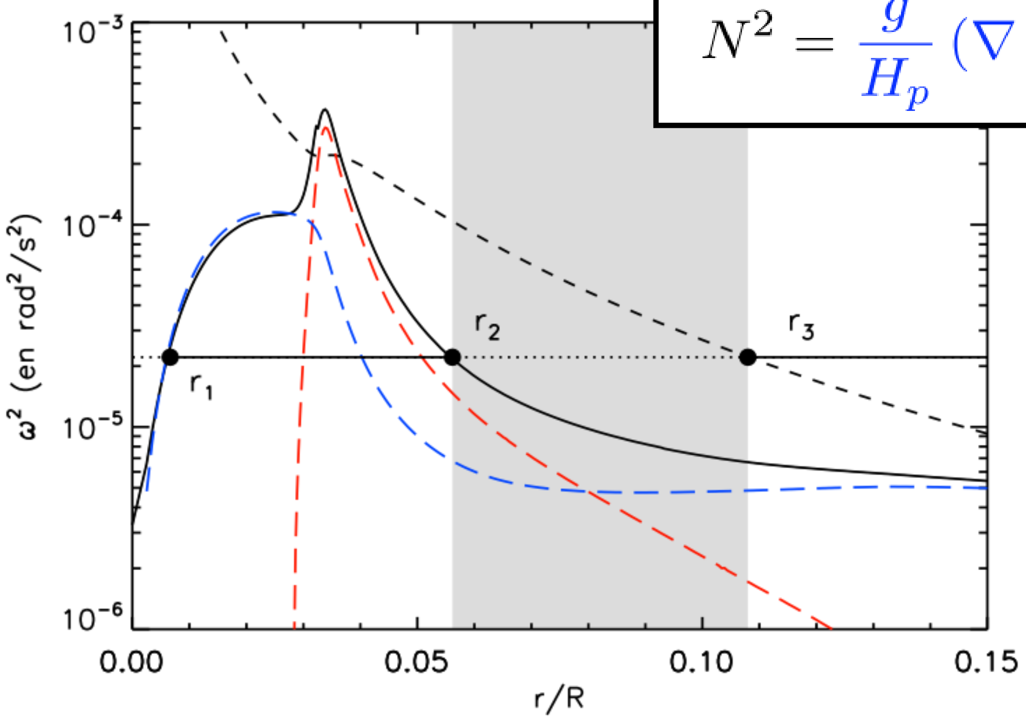


- HD 49385 is in the post main sequence stage
- **Firm detection** of mixed modes in avoided crossing

Interest of mixed modes

- Shape of the ℓ=1 ridge gives constraints on the strength of the coupling

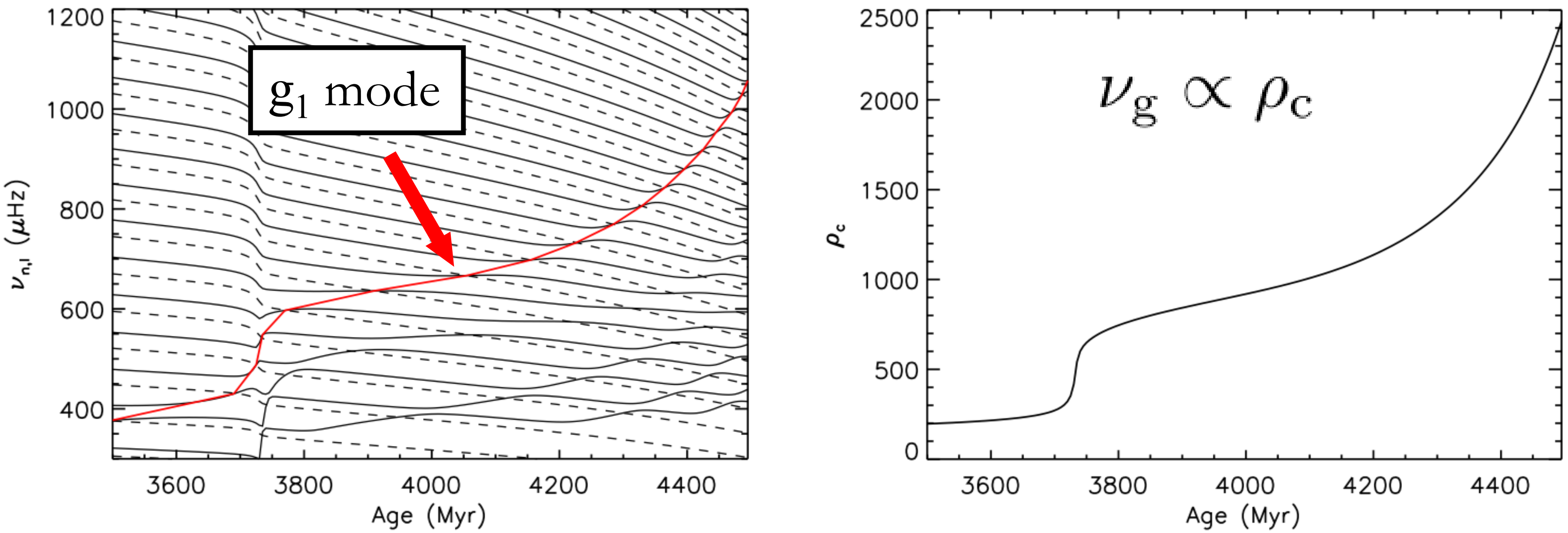
$$N^2 = \frac{g}{H_p}(\nabla - \nabla_{\rm ad}) + \frac{g}{H_p}\nabla\mu$$



$\Rightarrow$ information on the **structure of the evanescent zone**

Modeling stars with mixed modes: *limitations of traditional techniques*

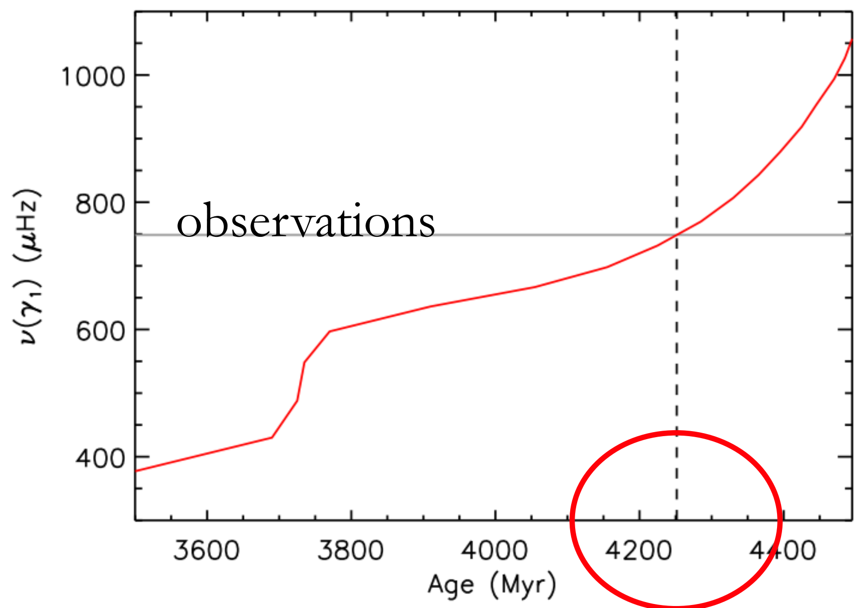
- Few studies were led so far to fit the properties of an avoided crossing: *Why?*
  1. Lack of precise enough observations
  2. $\tau_{\rm AC} \ll \tau_{\rm evol}$: avoided crossings occur on a **very short timescales** (problem for grids of models, automatic pipelines)



A possible solution

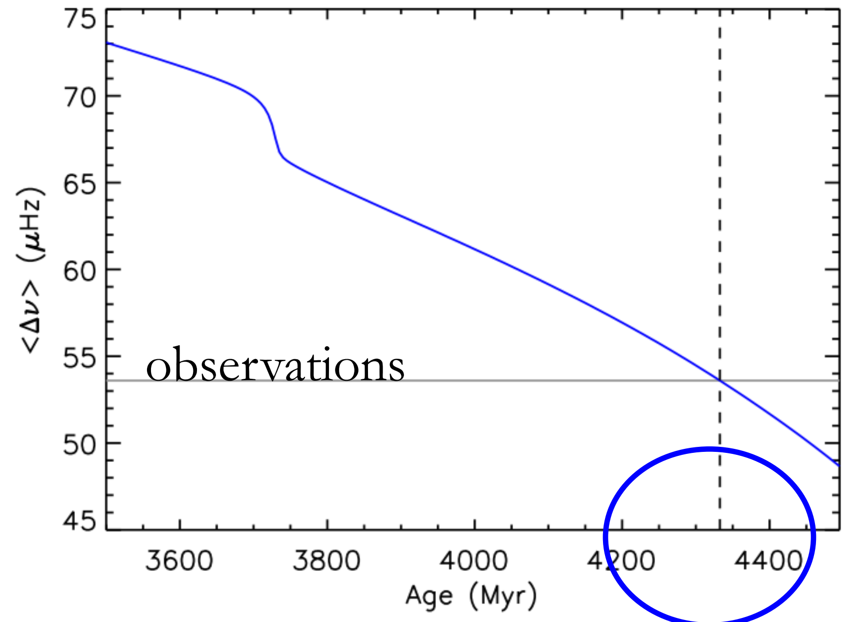
<u>g-mode frequencies</u>

ν_g



<u>large separation</u>

$\langle\Delta\nu\rangle \left(\propto \sqrt{\frac{GM}{R^3}}\right)$

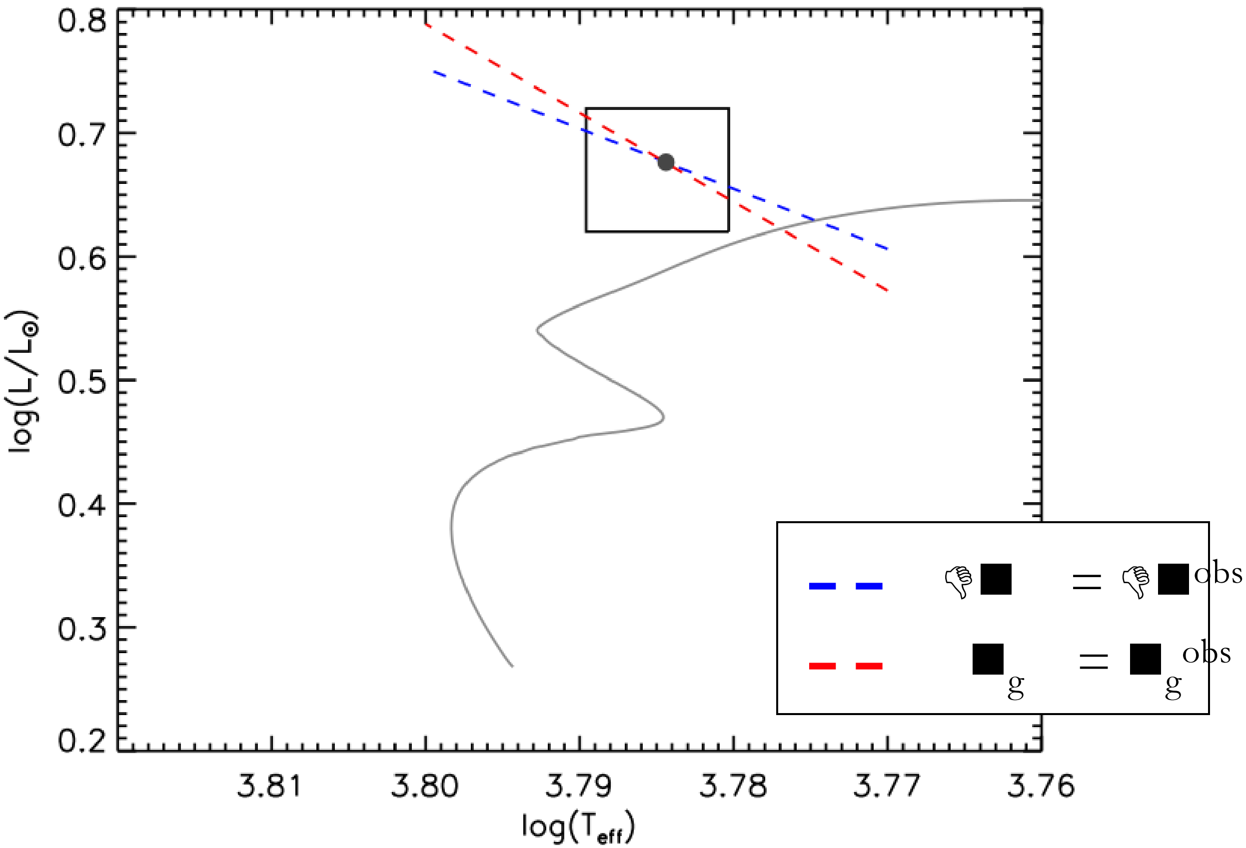


ages a priori different

Which are the models that fit both conditions simultaneously?

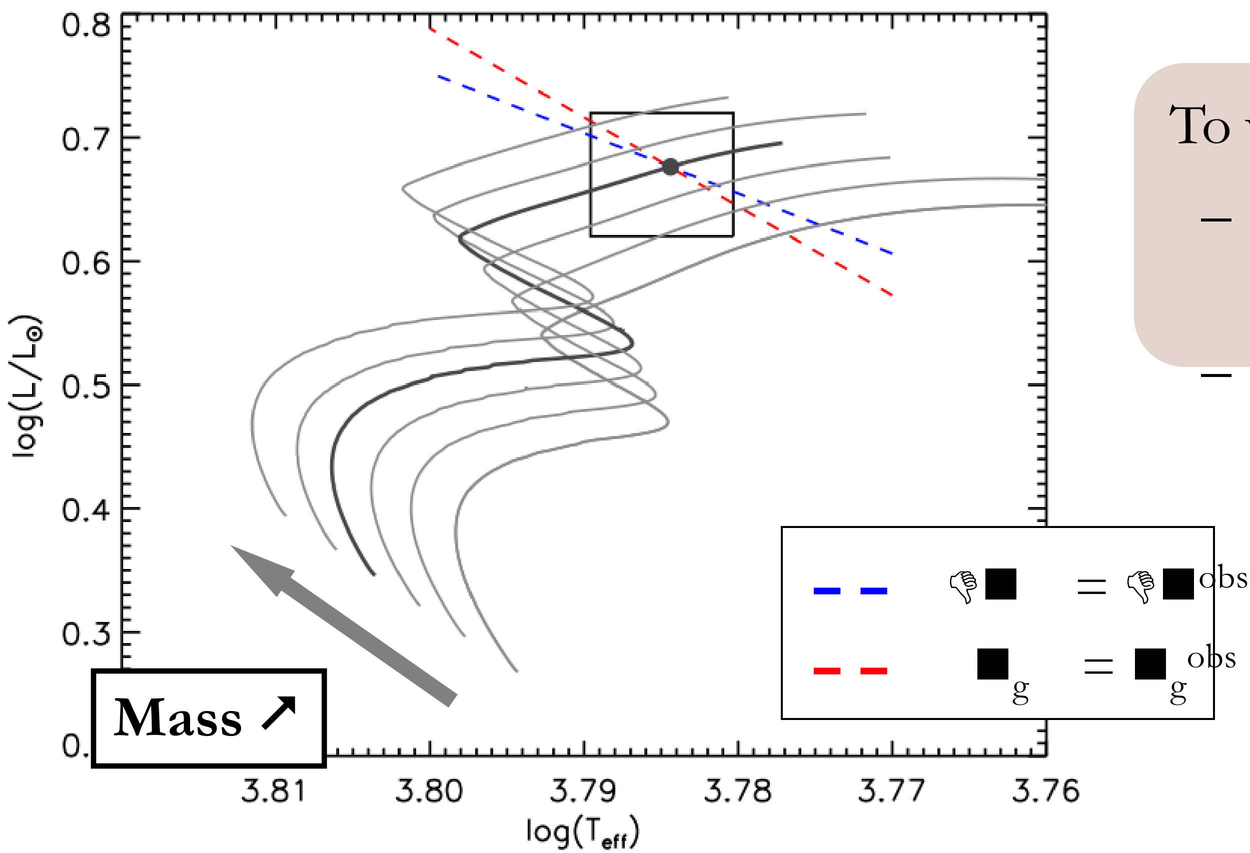
Effect of the mass

- We showed that for a given physics, the knowledge of 👎■ and $■_g$ imposes one and one only mass (Deheuvels & Michel 2011)



Effect of the mass

- We showed that for a given physics, the knowledge of 👎■ and $■_g$ imposes one and one only mass (Deheuvels & Michel 2011)

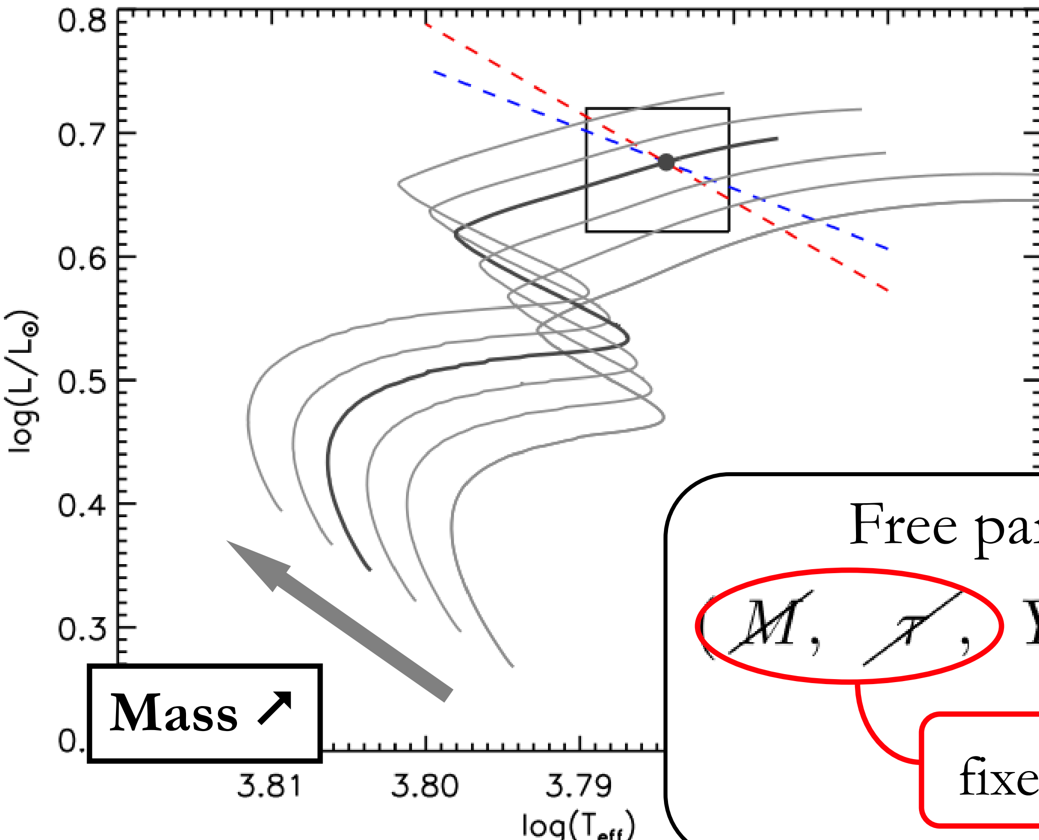


To verify both conditions

- **one mass only** ($\tilde{\mathbf{M}}$)
- **one age only** (♦)

Effect of the mass

- We showed that for a given physics, the knowledge of 👎■ and $■_g$ imposes one and one only mass (Deheuvels & Michel 2011)



To verify both conditions

- **one mass only** ($\tilde{\mathbf{M}}$)
- **one age only** (♦)

Free parameters of the fit

$$(\not{M},\ \not{\tau},\ Y_0,\ \alpha_{\rm ov},\ (Z/X)\ \ldots)$$

fixed to ($\tilde{M}$, ♦) by fitting (👎■, $■_g$)

Application to HD 49385

- Computation of a grid of models for HD 49385 with different input physics:

  ↄ3 ♋$_{\rm conv}$, Y_0, (Z/X), mixtures, ♋$_{\rm ov}$

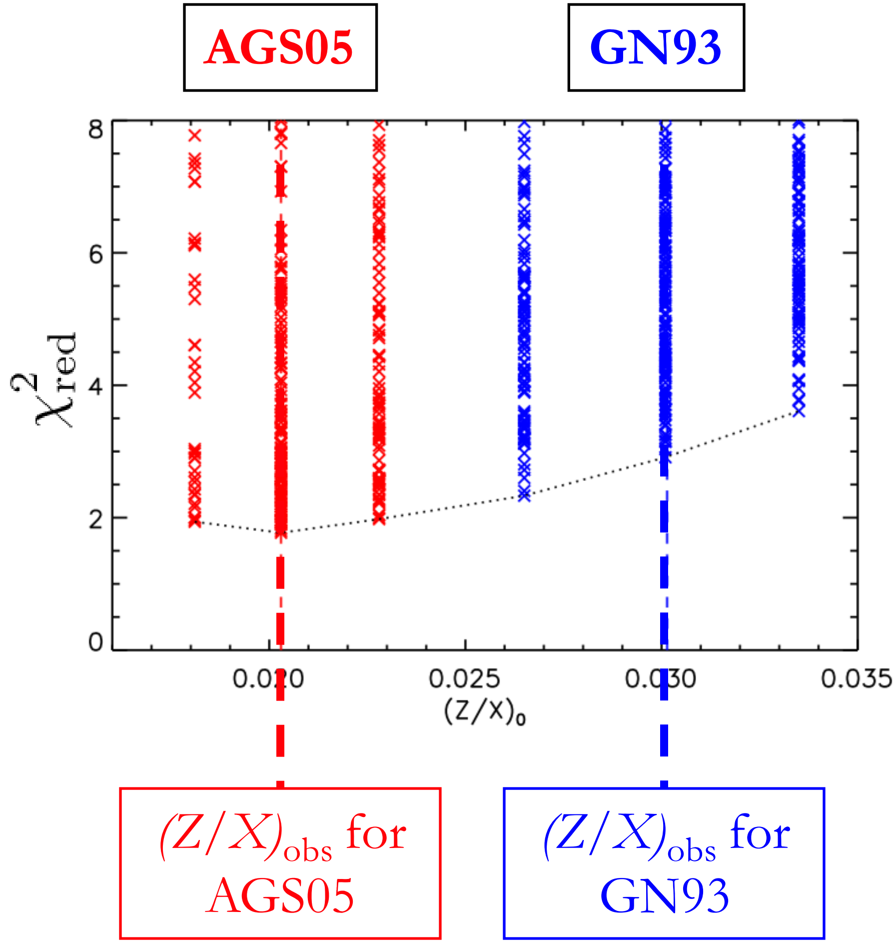
- Function of merit

$$\chi^2_{\rm red} \equiv \frac{1}{N-P} \sum_{i=1}^{N} \left(\frac{\mathcal{O}_i^{\rm obs} - \mathcal{O}_i^{\rm mod}}{\sigma_i^{\rm obs}} \right)^2$$

- Best model: $\chi^2_{\rm red} = 1.8$

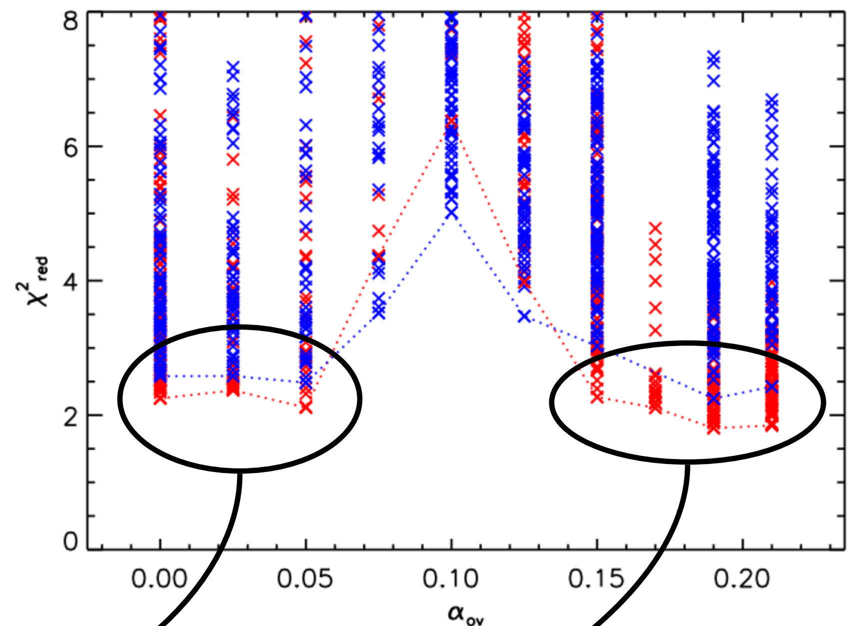
- Constraint on the metallicity results **in favor of the revised abundances of Asplund**

  (Z/X) = 0.0203 ± 0.0015



Results of the grid of models

- Constraint on the efficiency of the convection
  ♋$_{CGM}$ = 0.55 ± 0.04 (♋$_\odot$ = 0.64)
  - corroborated by *Piau et al. 2010*

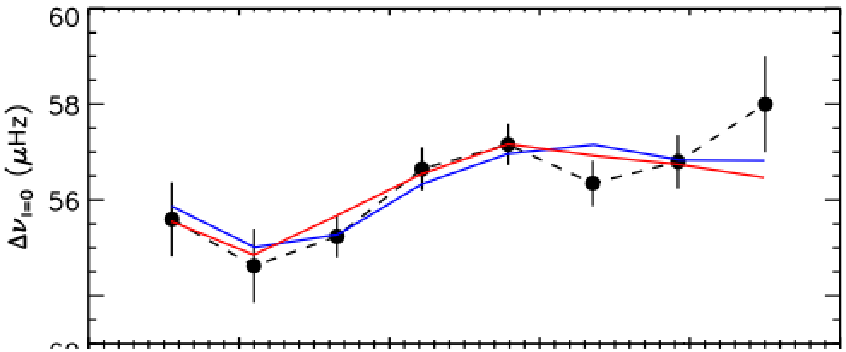


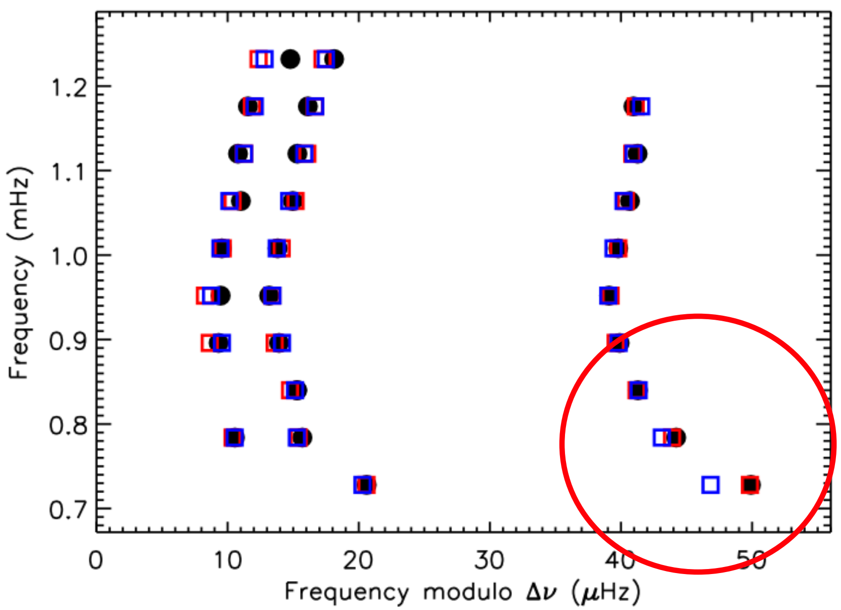
- Constraint on the amount of overshooting: two possible values
  - very low amount $\alpha_{\rm ov} < 0.05$
  - moderate amount $\alpha_{\rm ov} = 0.19 \pm 0.01$

AGS05
GN93

- Mass, radius and age efficiently constrained:
  M = 1.26 ± 0.04 M$_\odot$ (3%) **R = 1.94 ± 0.02 R$_\odot$** (1%) **♦ = 5.0 ± 0.3 Gyr**

Why are stellar parameters so well constrained in HD49385?

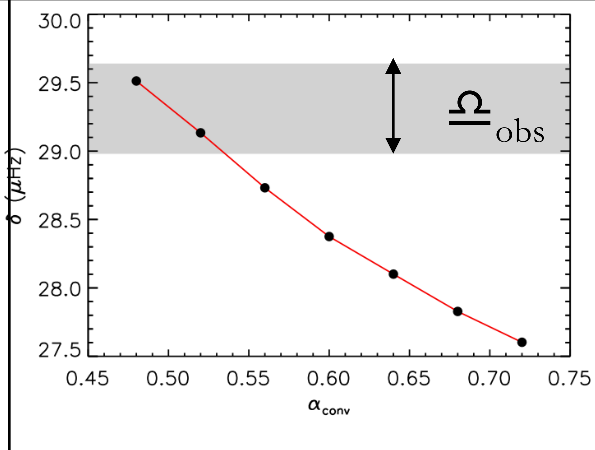
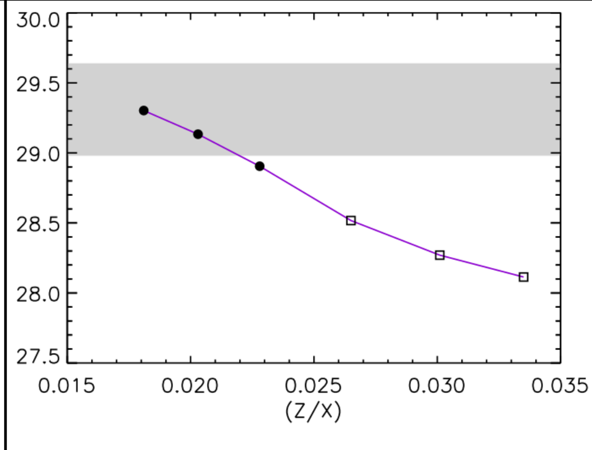
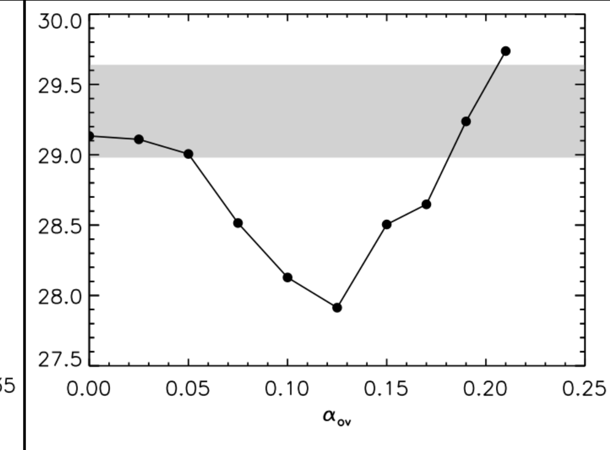




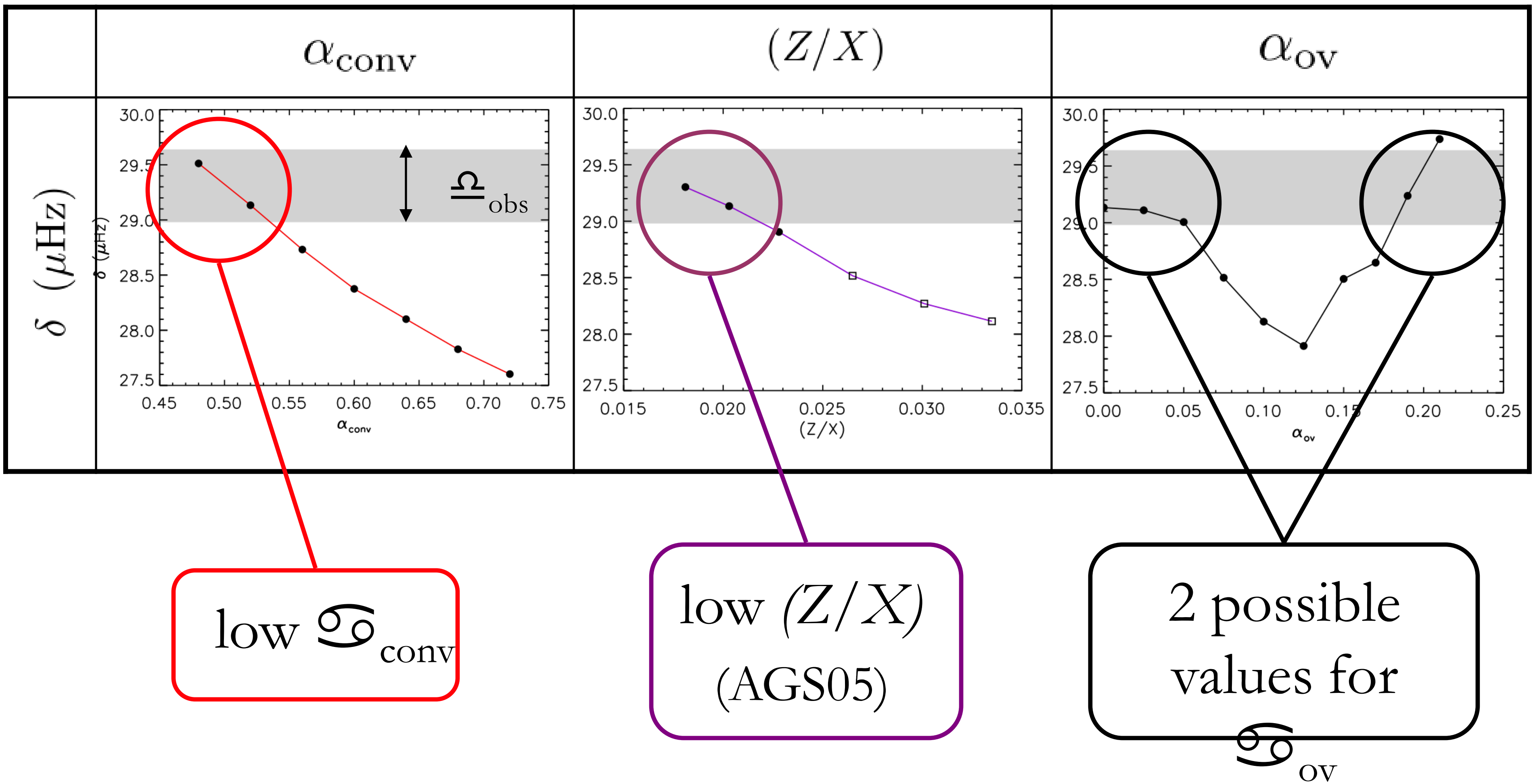
- Contributions to the value of the ♍^{2} function:
  - (i) position in the HR diagram
  - (ii) oscillation of the eigenfrequencies (acoustic depth of the **helium second ionization zone**)
  - (iii) curvature of the ℓ=1 ridge (**coupling** between the cavities)

♋$_{ov}$ = 0.10 : the curvature of the ridge contributes to about **80%** of the ♍^{2} value!

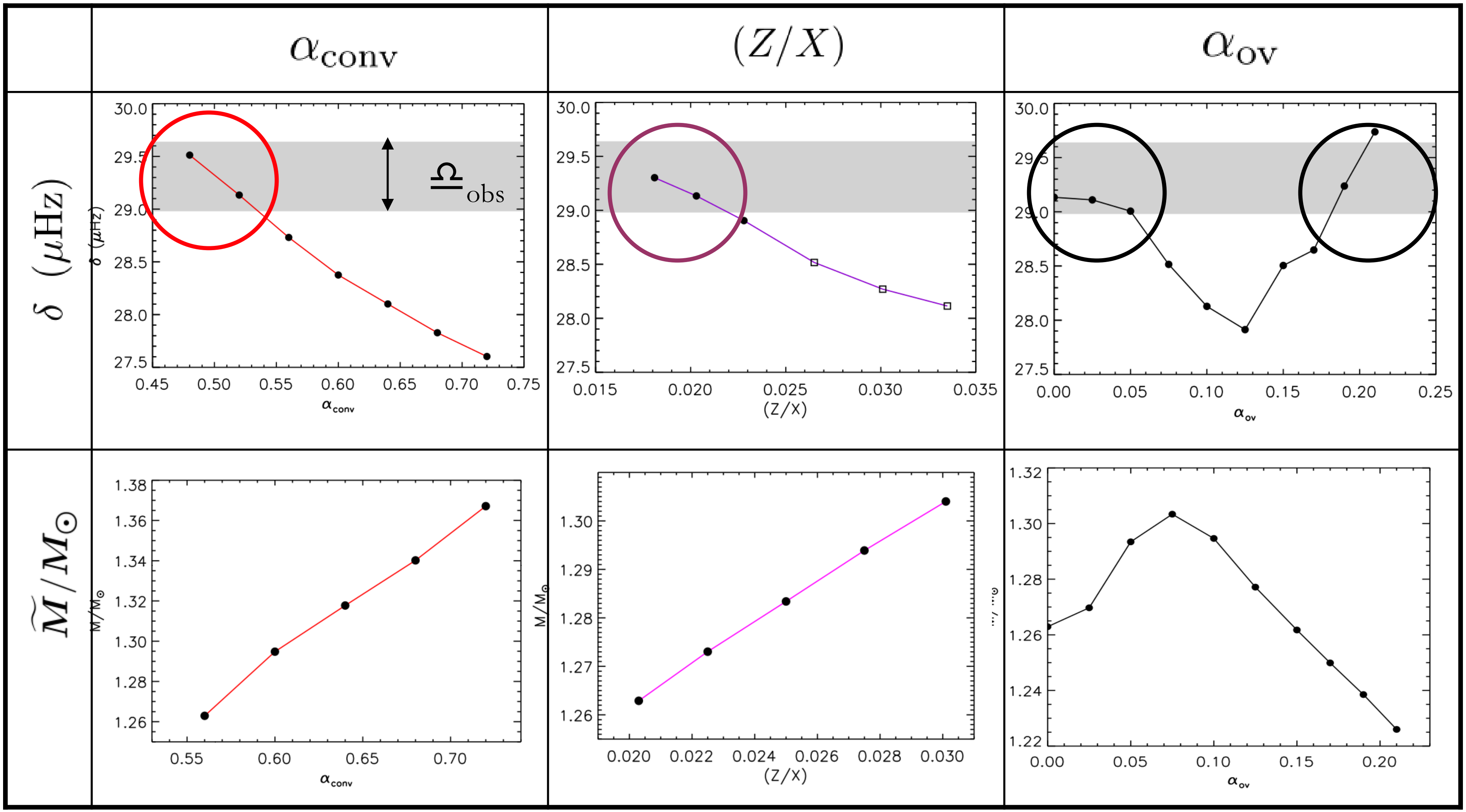
Dependence of □□□□□□□□□□□□ on the stellar parameters

	$\alpha_{\rm conv}$	(Z/X)	$\alpha_{\rm OV}$
δ (μHz)			

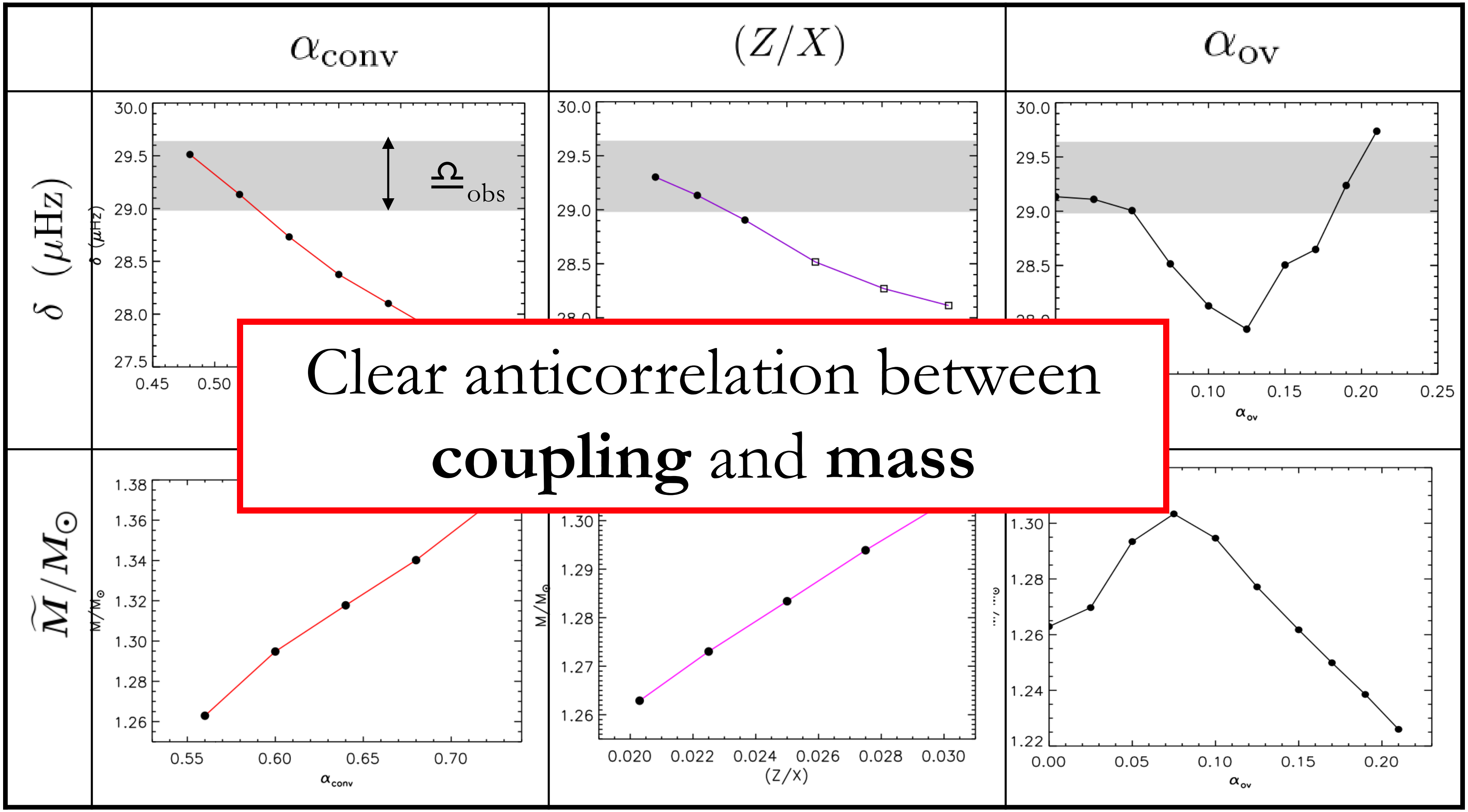
Coupling between the cavities in the models



Coupling between the cavities in the models



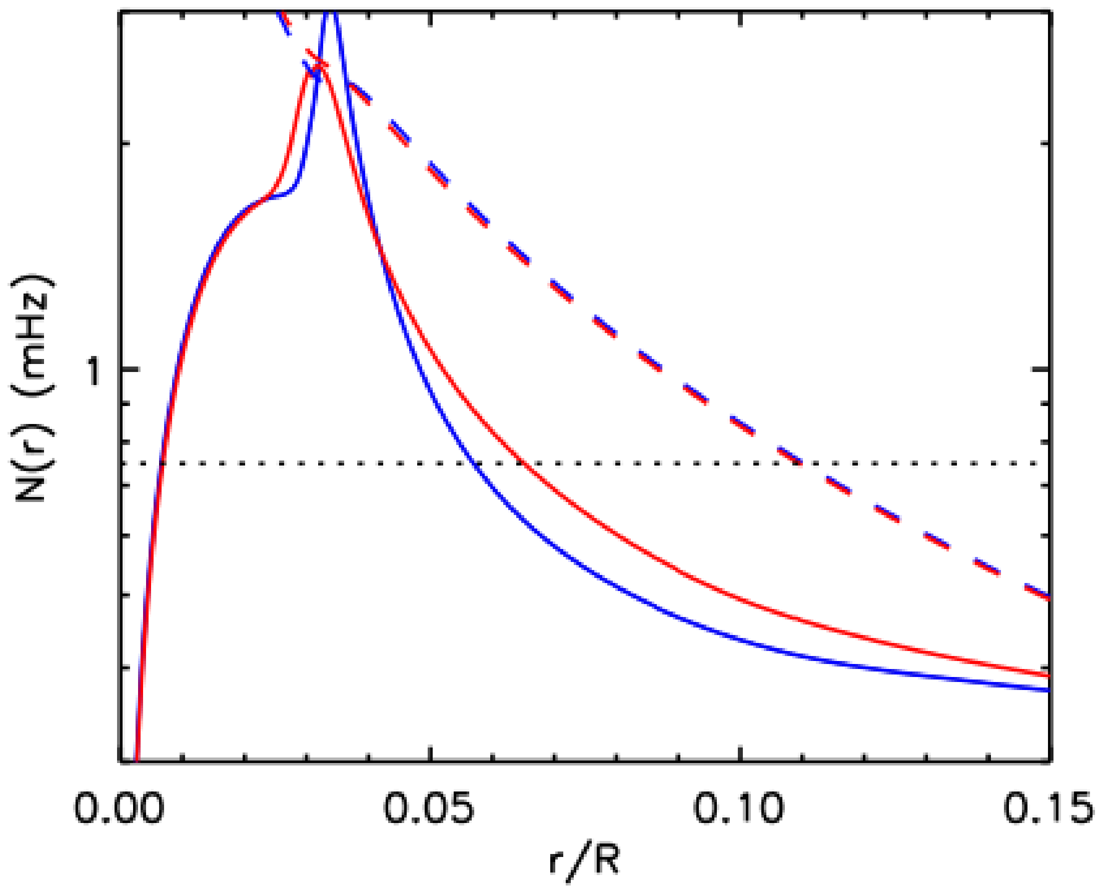
Coupling between the cavities in the models



Clear anticorrelation between **coupling** and **mass**

Relation between coupling and mass

- Two models in the grid with $\widetilde{M}_1 > \widetilde{M}_2$



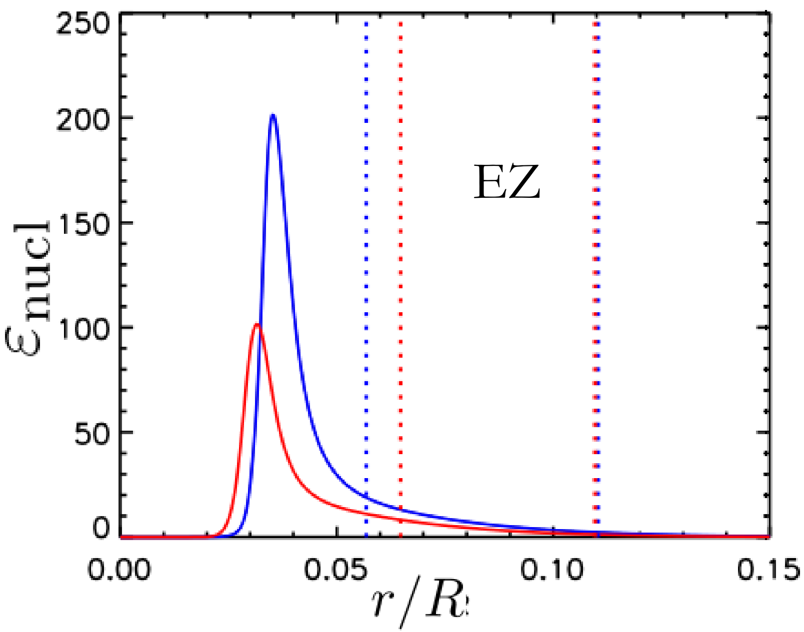
Relation between coupling and mass

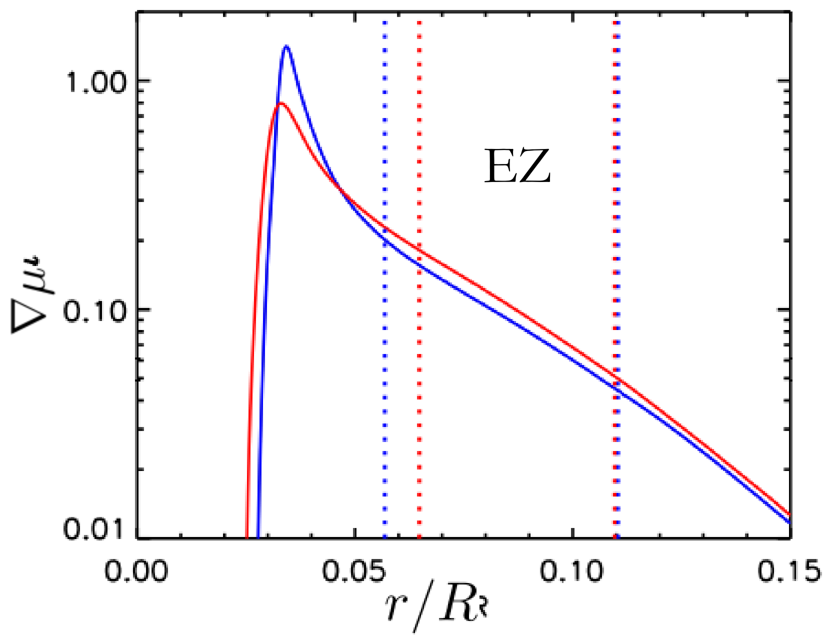
- Coupling depends on the Brunt-Väisälä in the EZ

$$N^2 = \frac{g}{H_p}\left(\nabla_{\rm ad} - \nabla + \nabla\mu\right)$$

- <u>When mass ↗</u>
  - $\varepsilon_{\rm nucl}$ ↗ in the H-burning layer
  - $\nabla\mu$ ↘ in the evanescent zone
  - N(r) ↘ in the EZ
  - <u>coupling ↘</u>

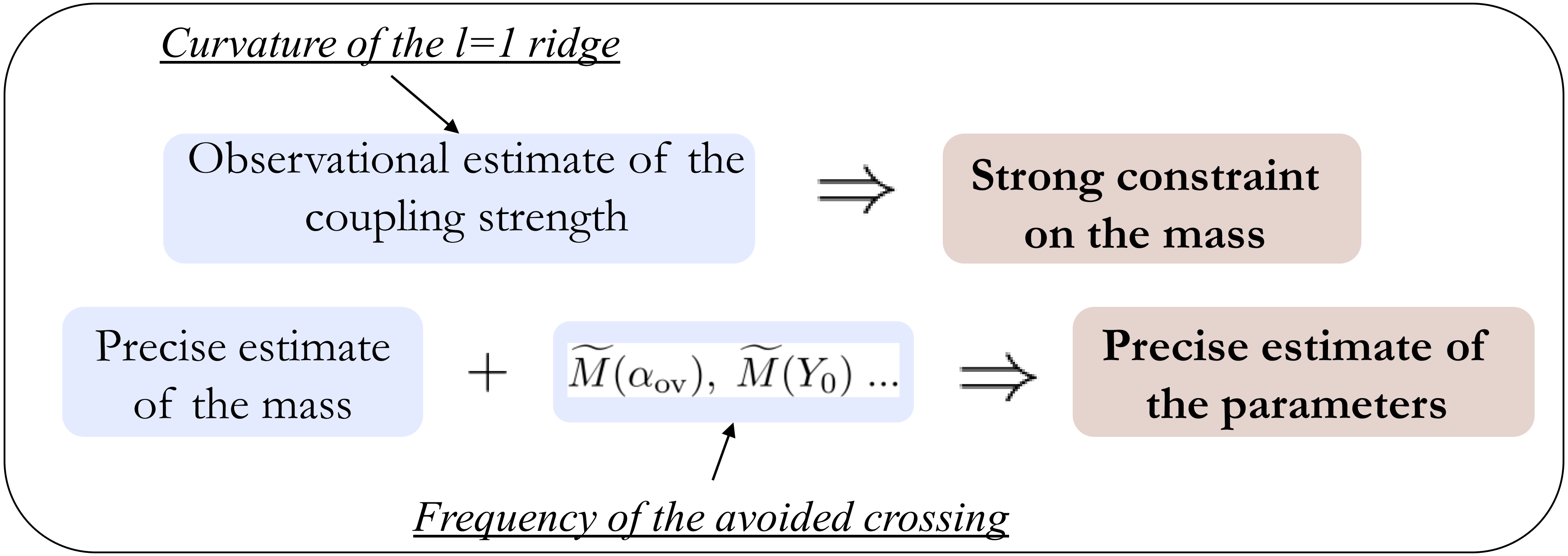
$\widetilde{M}_1 > \widetilde{M}_2$





Summary

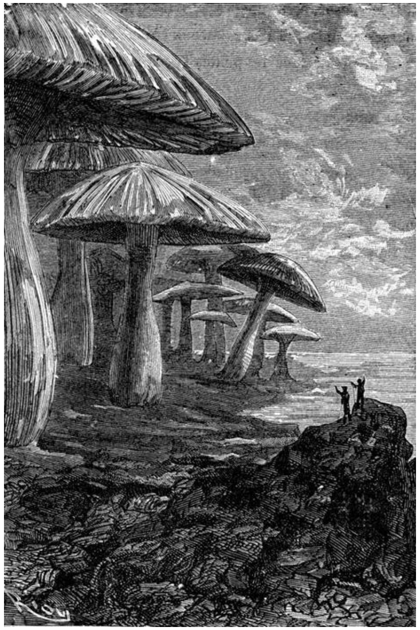
- Schematic summary:



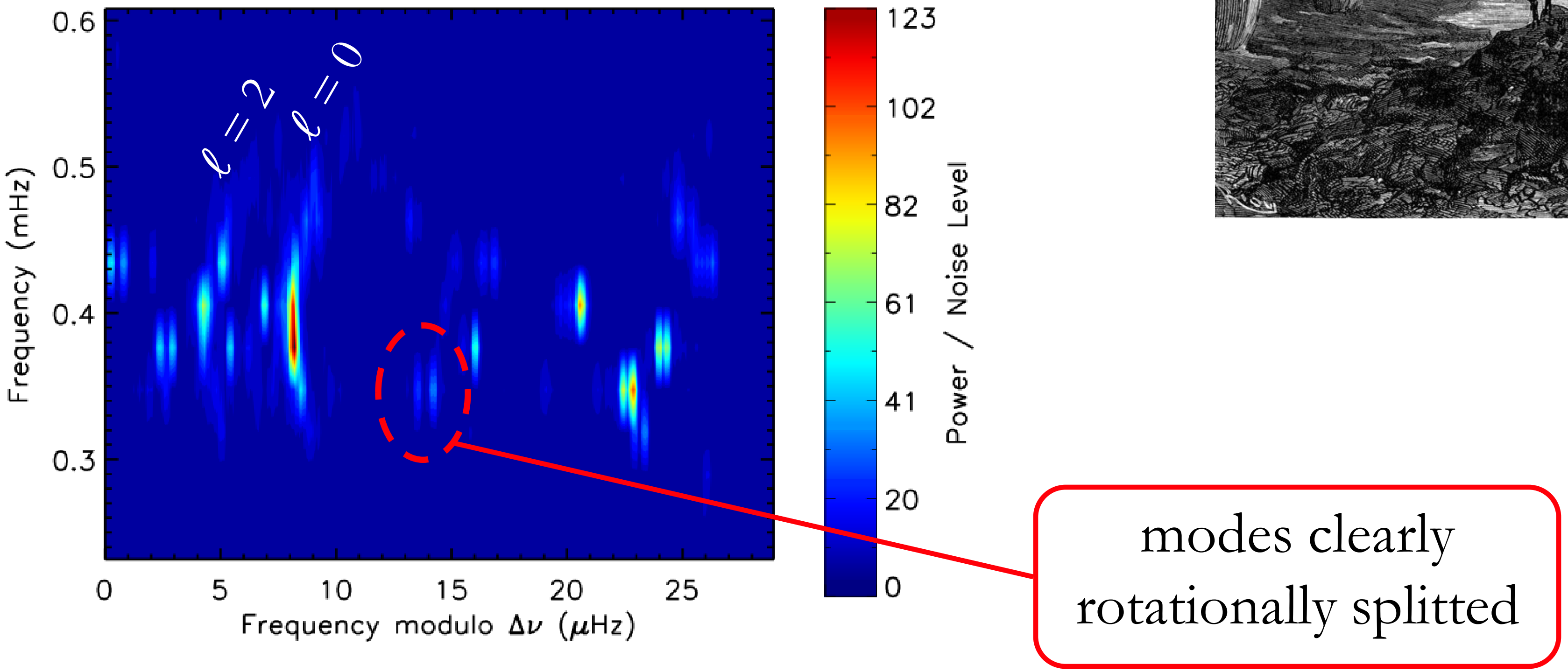
- Method is currently being applied to Kepler targets (a dozen with detected avoided crossings)

The exciting case of the Kepler subgiant Otto

- **Otto**: first star for which we can probe the rotation in the deepest layers (named after Otto Lidenbrock in *Voyage au centre de la Terre*)

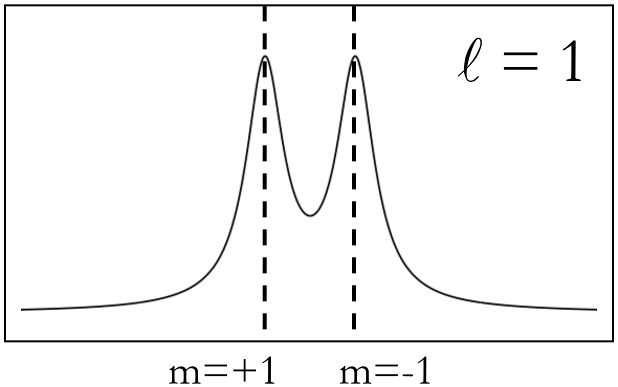


(Deheuvels et al. in prep)



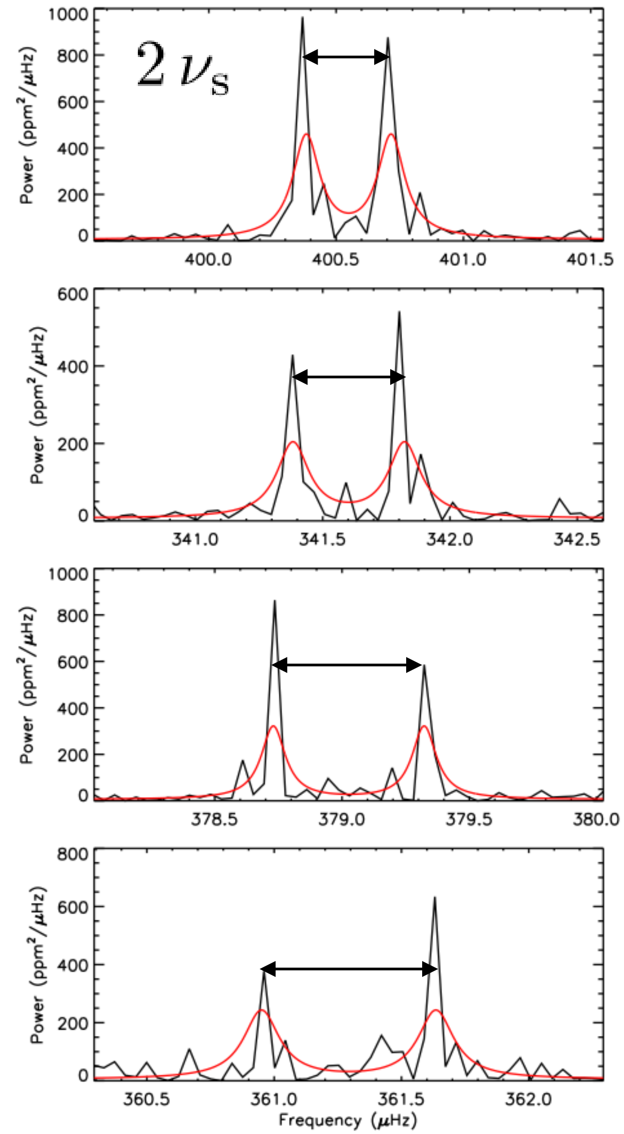
A varying splitting for Otto

- First fit of the modes



$$i = 90^\circ$$
$$\nu_s = 0.2\,\mu\text{Hz}$$

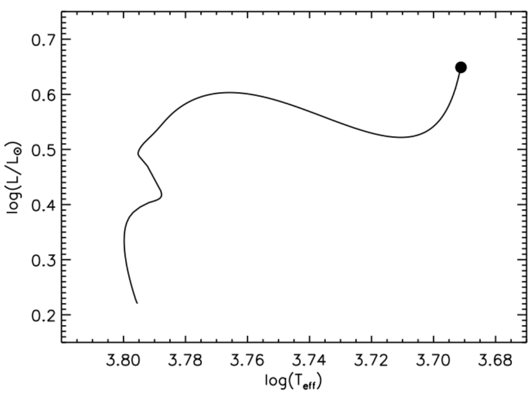
Surface rotation period of $\sim$ 60 days

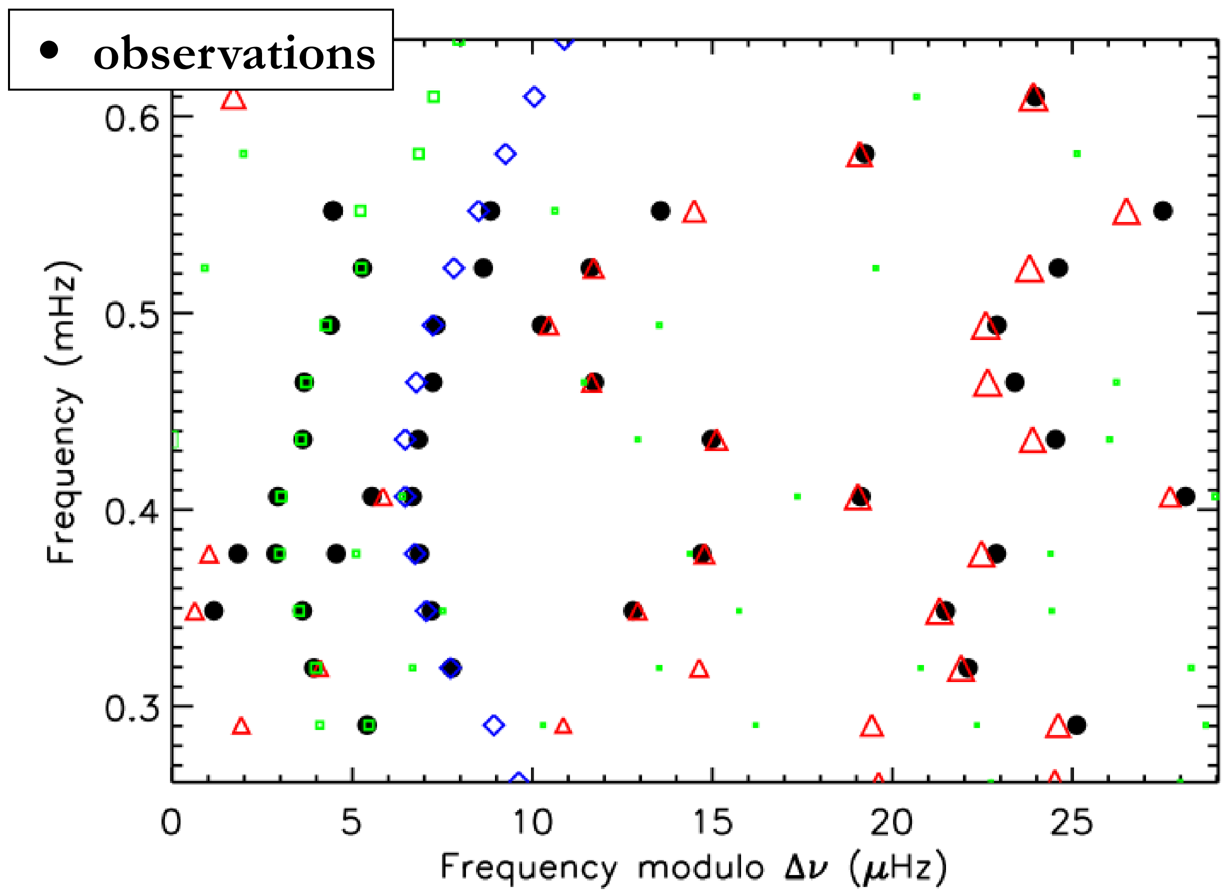


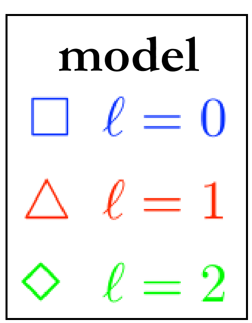
- Profiles of different ℓ=1 modes
  - ■$_s$ varies from one mode to another
- Second fit of the modes performed with a varying splitting

Finding a model for Otto

- Necessary to have access to the **rotational kernels** $K_{n,\ell}(r)$
- Modeling using the method of Deheuvels et al. 2011 (adapted)



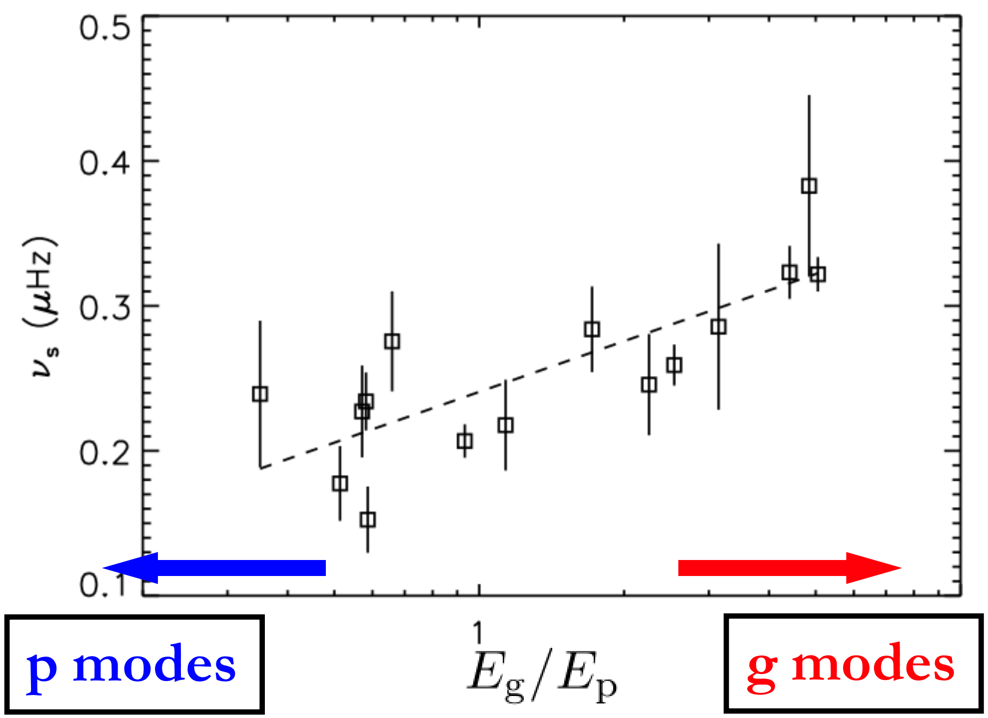




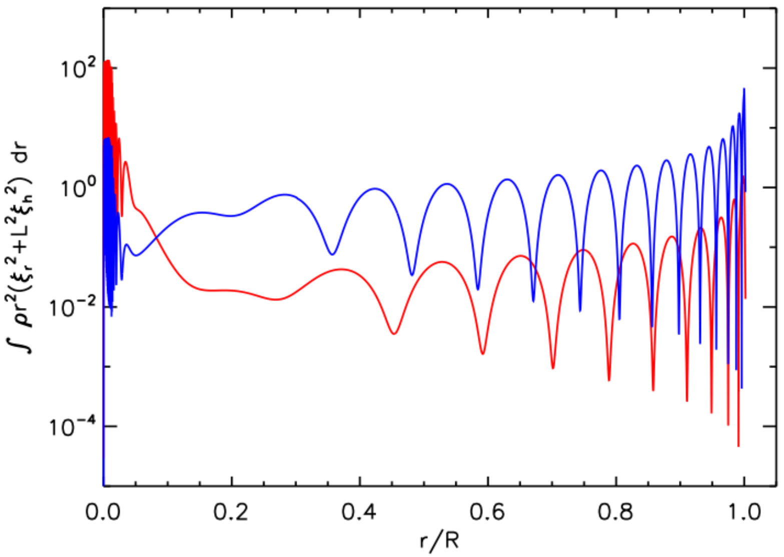
A fast rotating core?

- Trapping of the modes

$$\frac{E_\mathrm{g}}{E_\mathrm{p}} = \frac{\int_\mathrm{g} \rho r^2 (\xi_\mathrm{r}^2 + L^2 \xi_\mathrm{h}^2) \mathrm{d}r}{\int_\mathrm{p} \rho r^2 (\xi_\mathrm{r}^2 + L^2 \xi_\mathrm{h}^2) \mathrm{d}r}$$



- Kinetic energy of a p mode and a g mode of Otto

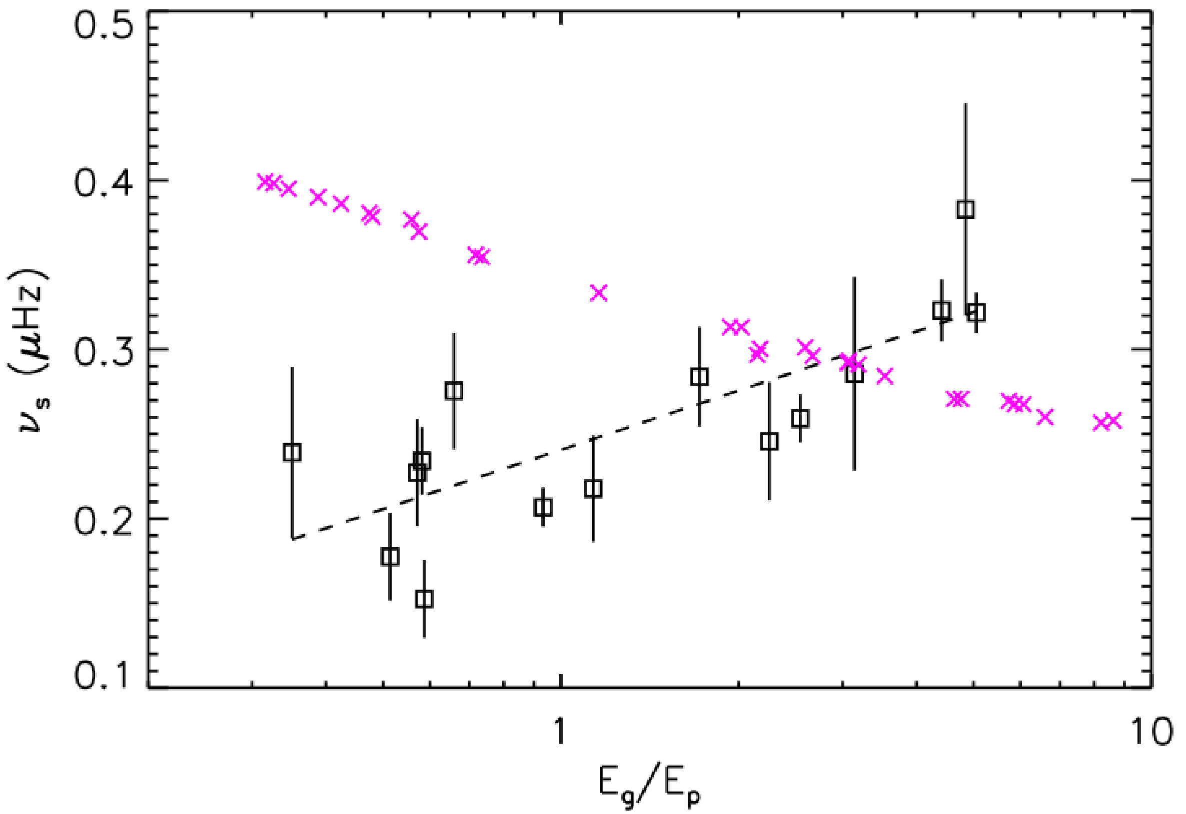


suggests that

$\Omega_{core} > \Omega_{surface}$!

What information do we get about Ω(r)?

- Forward approach $$\delta\omega_{n,\ell} = \int_0^R K_{n,\ell}(r)\Omega(r)\,\mathrm{d}r$$
- Solid body rotation rejected



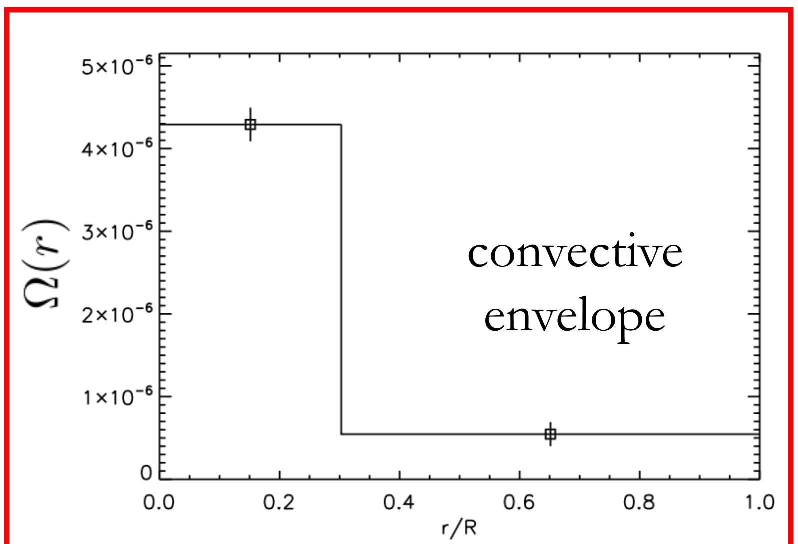
An attempt to invert Ω(r)

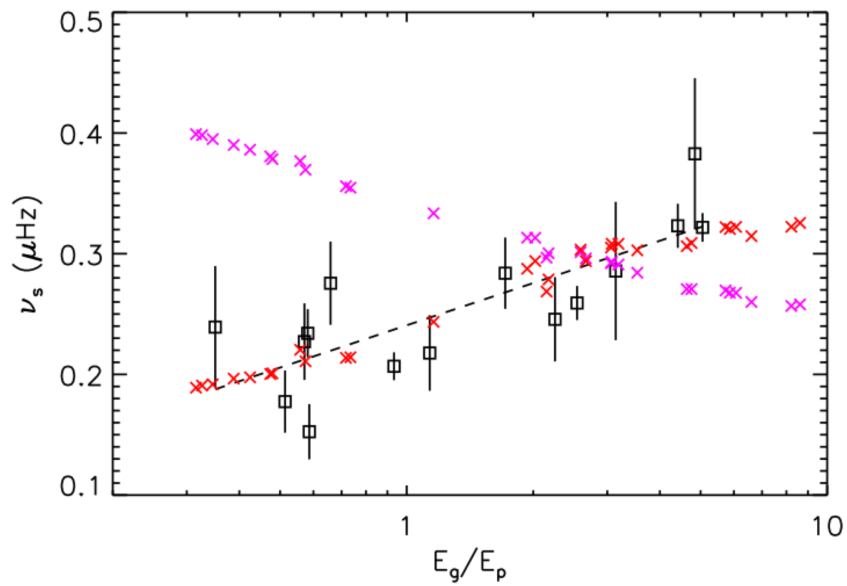
- RLS method: find the function Ω(r) which minimizes

$$S \equiv \sum_{k=1}^{M} \left[\frac{\int_0^R K_k(r)\Omega(r)\mathrm{d}r - \delta\omega_k}{\sigma_k} \right]^2 + \gamma \left\| \Omega''(r) \right\|^2$$

regularization

- Simple profiles:
  - Solid body rotation Ω(r) = Ω_0 : $\chi^2 = 10.1$
  - Different rotation core/envelope: $\chi^2 = 1.2$

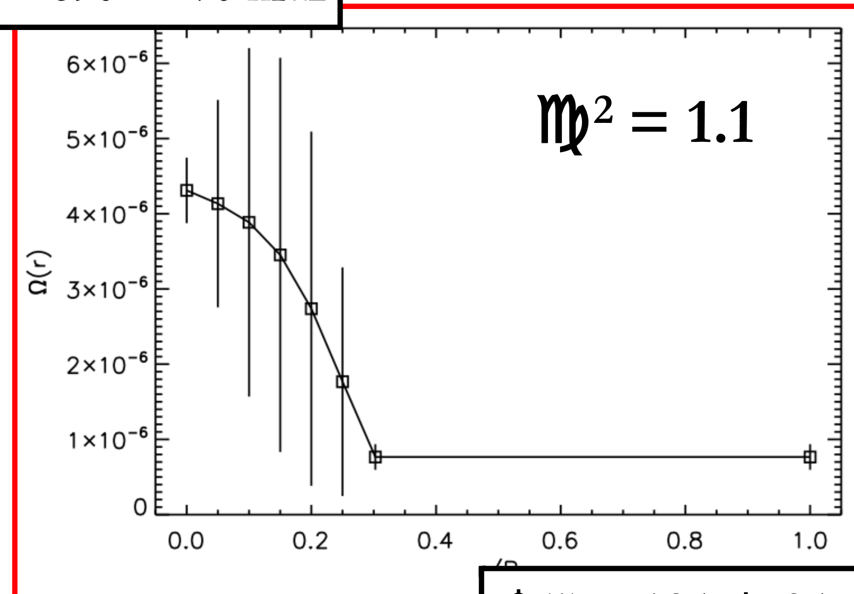




An attempt to invert ♁(r)

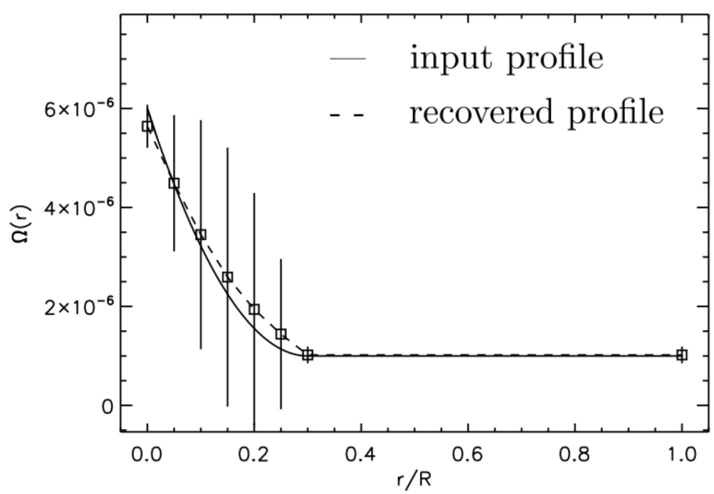
- RLS with more complex profiles assuming solid body rotation in the envelope
  regularization factor ♑ = 0.02

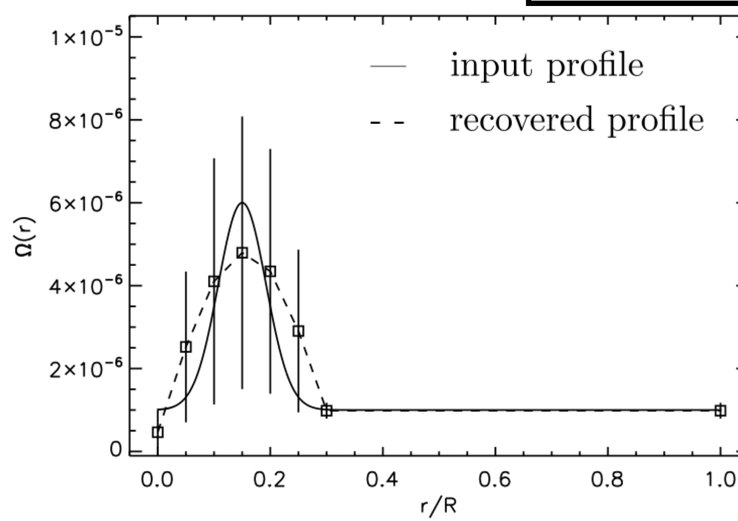
♁(0) = 690 ± 70 nHz



♁(1) = 125 ± 25 nHz

- Simulations with the same modes:

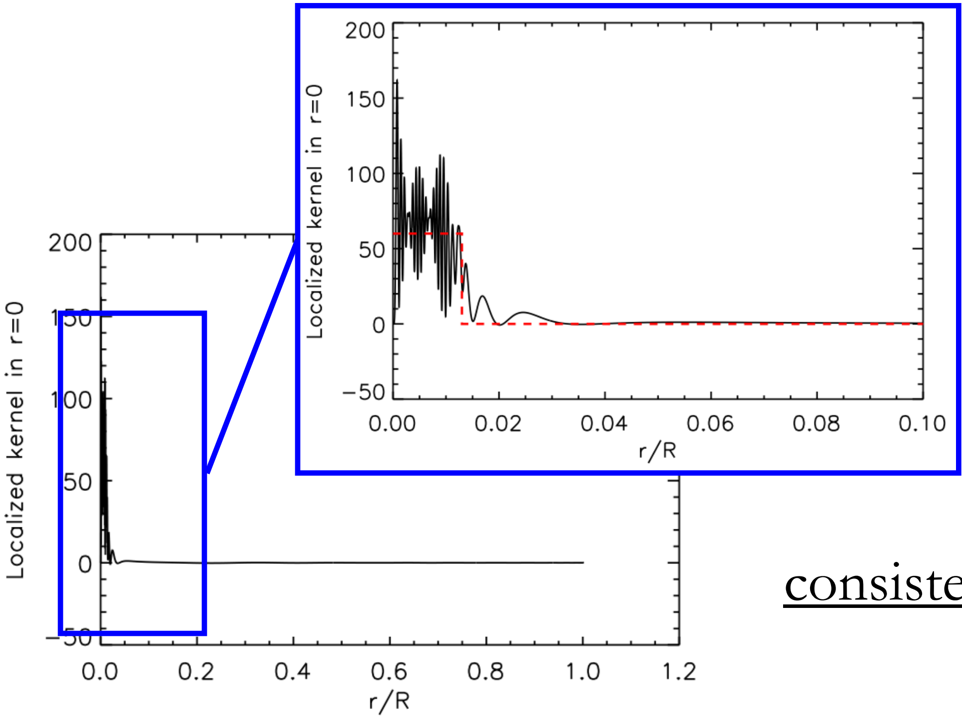




An attempt to invert $\Omega(r)$

- OLA inversion method (Optimally Localized Averages)
  - *principle*: create a linear combination of the rotational kernels which is as localized as possible

localized kernel in r=0.01



$\Rightarrow$ Precise estimate of the **most central rotation**
$\Omega(0.01) = 710 \pm 50$ nHz

consistent with RLS: $\Omega(0) = 690 \pm 70$ nHz

Possible follow-ups

➓ ⌽$_{core}$ = 6 ⌽$_{surf}$ for Otto!

- The case of ℓ=2 modes in the spectrum of Otto

  possible **asymmetry of multiplets** during the avoided crossings?

- Search for other Kepler subgiants like Otto

- Indication about the rotation in the Sun?

- Fast rotating core also in red giants *(Beck et al. 2011)*

  pre-white dwarfs *(Corsico et al. 2011)*