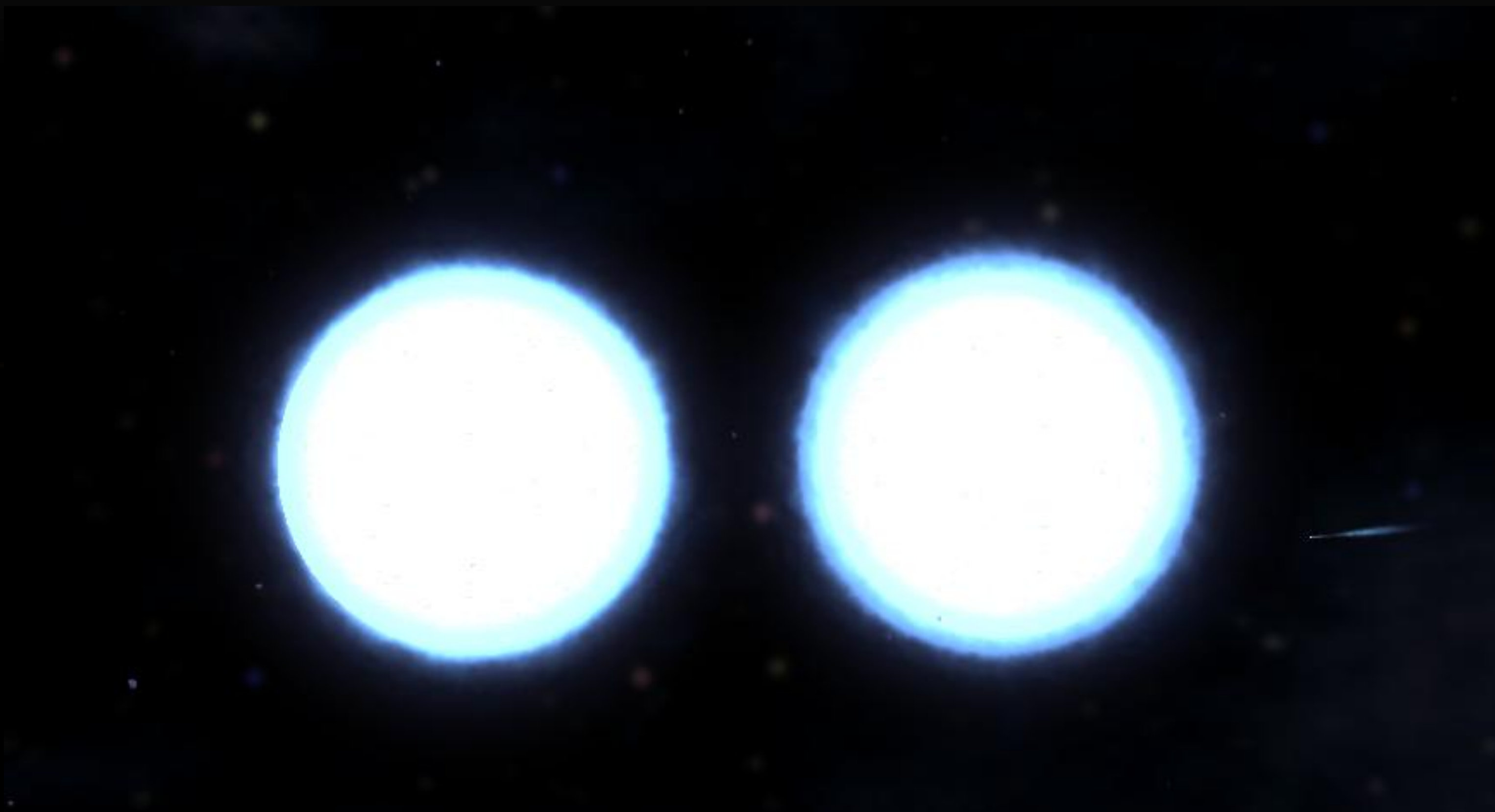
Tidally-Excited Stellar Pulsations in KEPLER KOI-54 and Resonance Locking

DYNAMICAL TIDES IN KOI-54



Jim Fuller, Cornell

KITP Asteroseismo

Oct 11, 2011

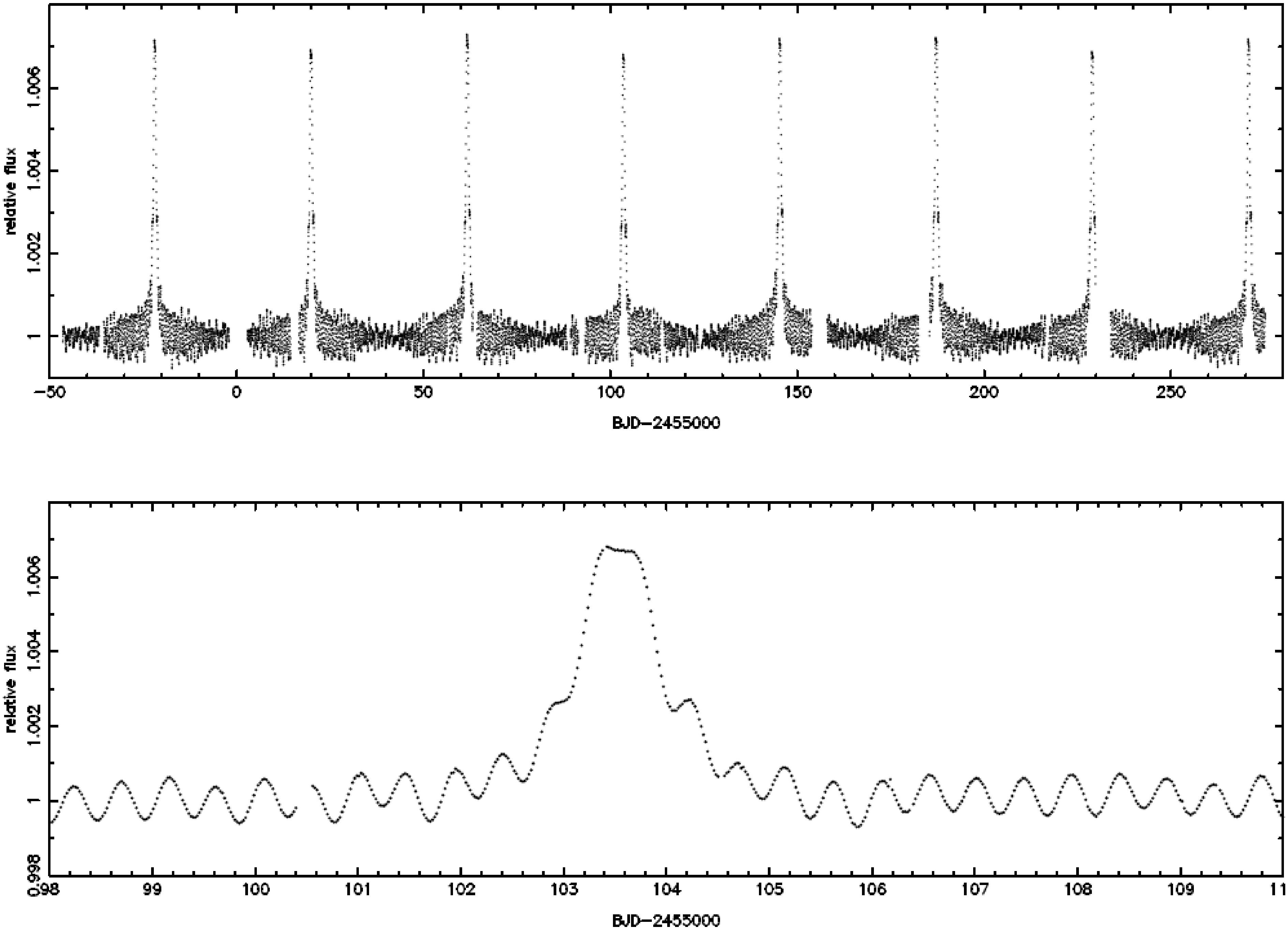


FIG. 1.— *upper:* The detrended and normalized *Kepler* light curve of KOI-54. *lower:* A detailed view of a brightening event.

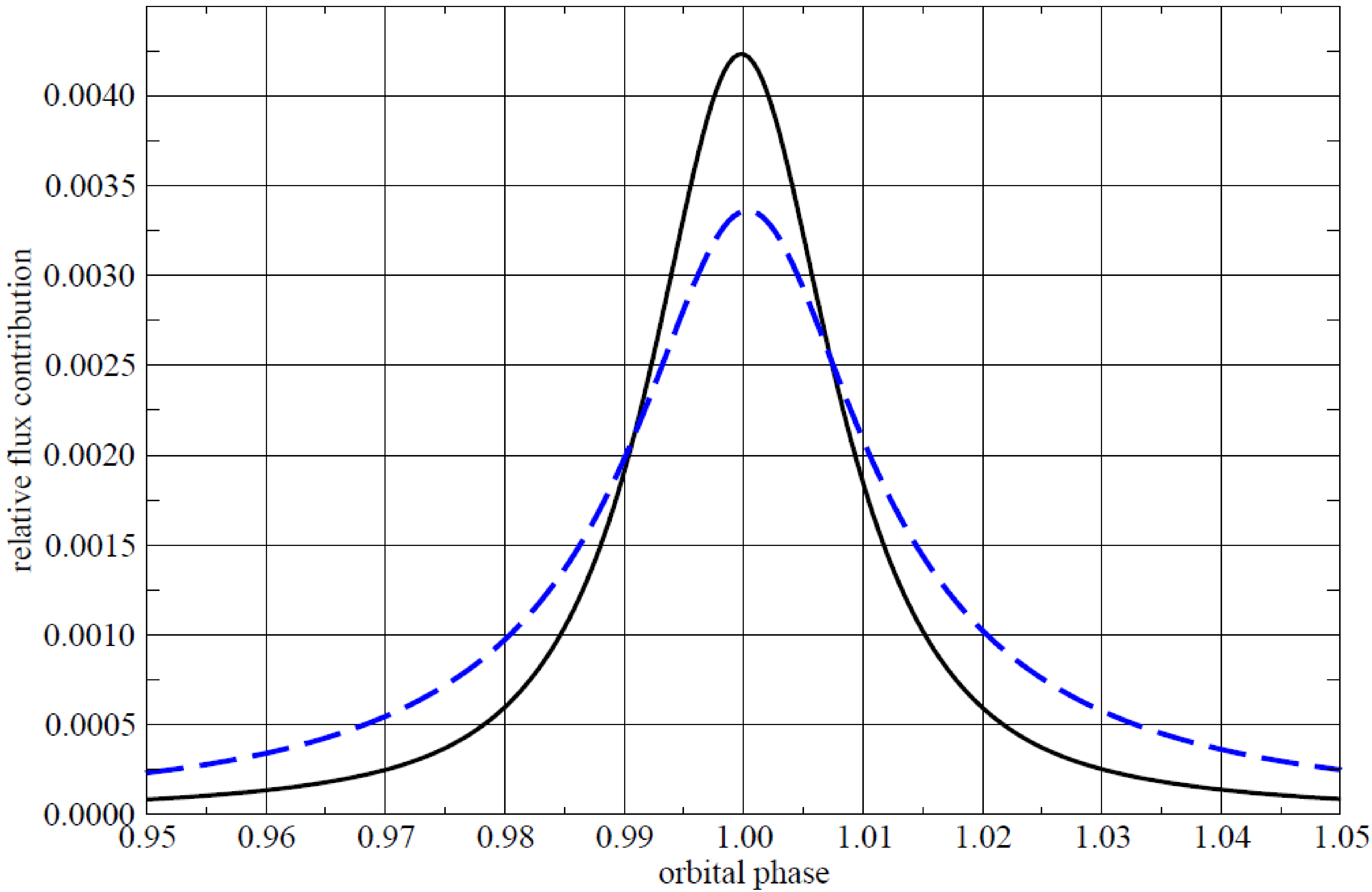
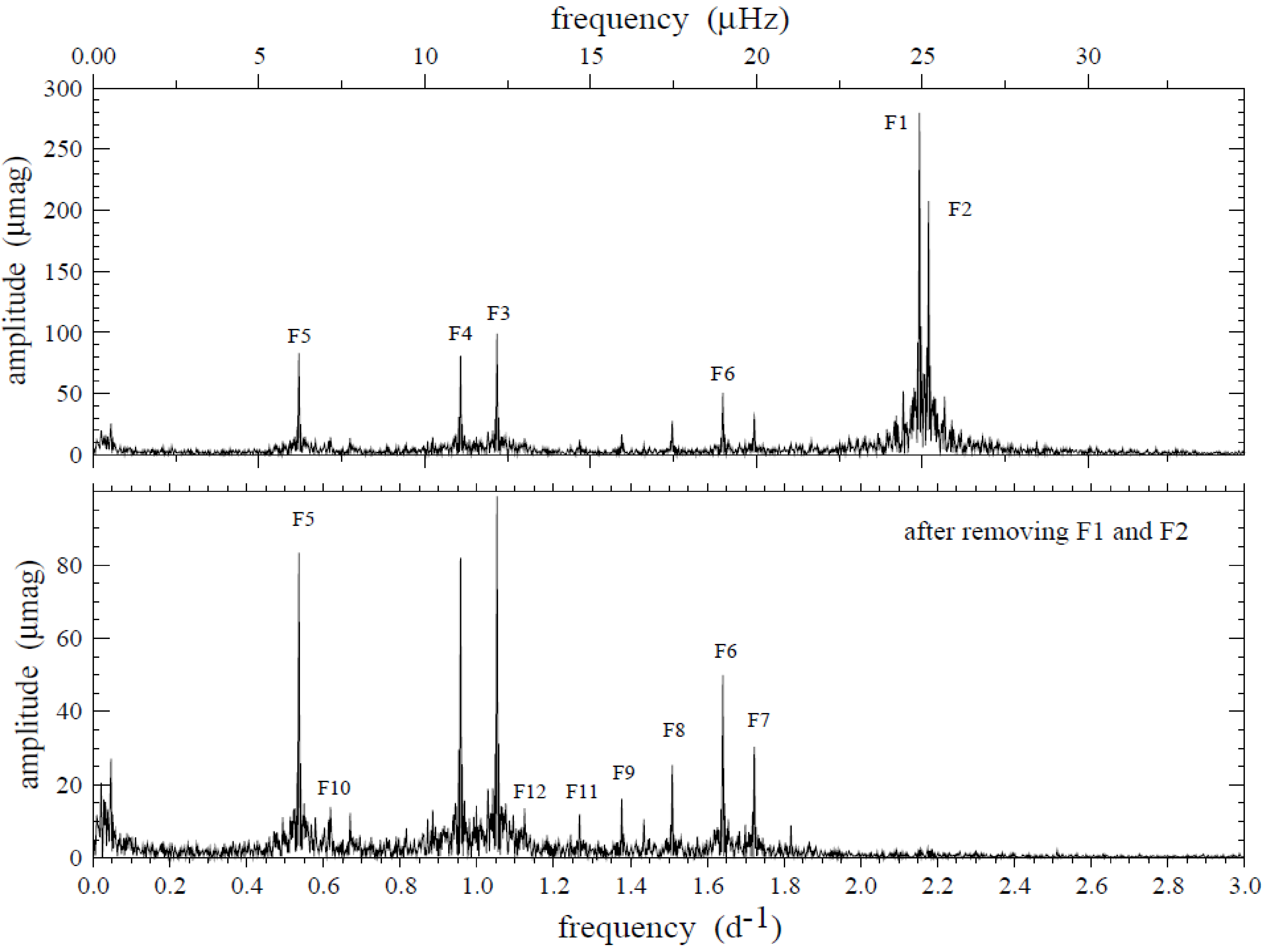


FIG. 7.— Relative contributions to the brightening from the tidal/ellipsoidal distortion only (solid curve) and irradiation/reflection only (dashed curve).

KOI-54 SYSTEM PARAMETERS

	parameter	value	error	unit
Observations	T_1	8500	200	K
	T_2	8800	200	K
	L_2/L_1	1.22	0.04	
	$v_{\mathrm{rot},1} \sin i_1$	7.5	4.5	km/s
	$v_{\mathrm{rot},2} \sin i_2$	7.5	4.5	km/s
	$[\mathrm{Fe/H}]_1$	0.4	0.2	
	$[\mathrm{Fe/H}]_2$	0.4	0.2	
Lightcurve/RV modeling	M_2/M_1	1.025	0.013	
	P_{orb}	41.8051	0.0003	days
	e	0.8342	0.0005	
	ω	36.22	0.90	degrees
	i	5.52	0.10	degrees
	a	0.395	0.008	AU
	M_1	2.32	0.10	$R_\odot$
	M_2	2.38	0.12	$R_\odot$
	R_1	2.19	0.03	$M_\odot$
	R_2	2.33	0.03	$M_\odot$



frequency (μHz)
0.00
5
10
15
20
25
30
300
250
200
150
100
50
0
amplitude (μmag)
F1
F2
F3
F4
F5
F6
F5
after removing F1 and F2
80
60
40
20
0
amplitude (μmag)
F10
F12
F11
F9
F8
F6
F7
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
frequency (d-1)

TABLE 3
THIRTY LARGEST KOI-54 PULSATIONS

ID	frequency (d^{-1})	frequency (μHz)	amplitude (μmag)	f/f_{orbit}	nearest harmonic
F1	2.1529	24.917	297.7	90.00	90
F2	2.1768	25.195	229.4	91.00	91
F3	1.0525	12.182	97.2	44.00	44
F4	0.9568	11.074	82.9	40.00	40
F5	0.5363	6.207	82.9	22.42	—
F6	1.6405	18.988	49.3	68.58	—
F7	1.7222	19.933	30.2	72.00	72
F8	1.5087	17.462	17.3	63.07	63
F9	1.3773	15.941	15.9	57.58	—
F10	0.6697	7.751	14.6	28.00	28
F11	1.2678	14.673	13.6	53.00	53
F12	1.1241	13.011	13.4	46.99	47
F13	0.9329	10.798	12.5	39.00	39
F14	1.4349	16.608	11.6	59.99	60
F15	0.8851	10.244	11.5	37.00	37
F16	1.6983	19.656	11.4	71.00	71
F17	0.6183	7.156	11.1	25.85	—
F18	1.8178	21.039	9.8	75.99	76
F19	0.8574	9.924	9.3	35.84	—
F20	0.6458	7.475	9.1	27.00	27
F21	1.0284	11.903	8.4	42.99	43
F22	1.0765	12.460	8.3	45.01	45
F23	1.5092	17.467	8.1	63.09	63
F24	0.8610	9.965	6.9	35.99	36
F25	1.4452	16.726	6.8	60.42	—
F26	1.2439	14.397	6.4	52.00	52
F27	1.0078	11.664	6.3	42.13	—
F28	0.7894	9.137	5.9	33.00	33
F29	0.6937	8.028	5.8	29.00	29
F30	1.1483	13.290	5.7	48.00	48

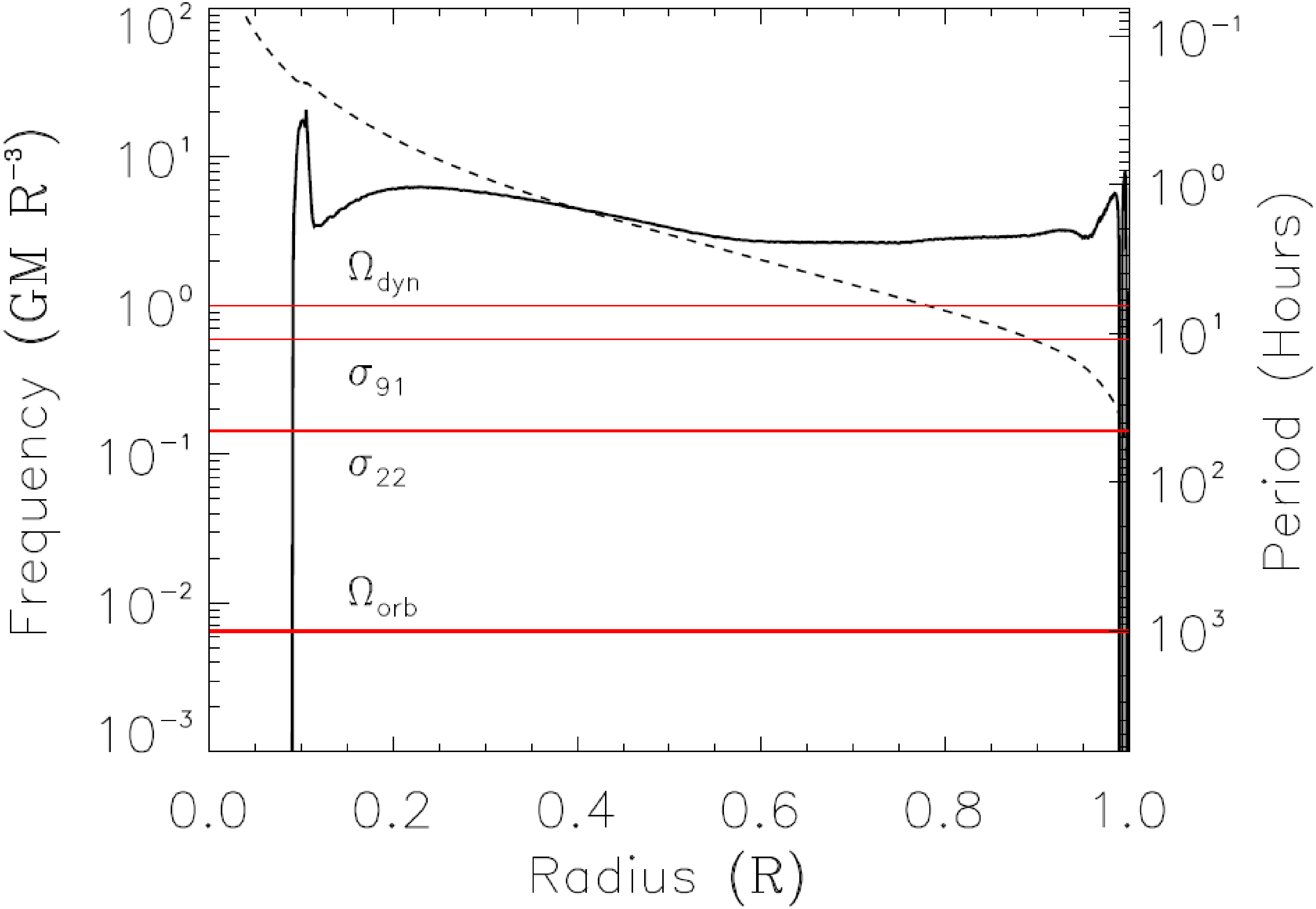
NOTE. — Formal uncertainty in amplitudes is 0.3 μmag. Orbital frequency f_{orbit} was found via least-squares fit to best match the harmonics: $f_{orbit} = 0.0239205\ d^{-1} = 0.276857\ \mu$Hz.

NOMENCLATURE

- Orbital Frequency Ω
- Spin Frequency Ω_s
- Inertial Mode Frequency

$$\sigma_\alpha = \varepsilon_\alpha + m(1 - C_\alpha)\Omega_s$$
$$= \omega_\alpha + m\Omega_s$$

PROPAGATION DIAGRAM



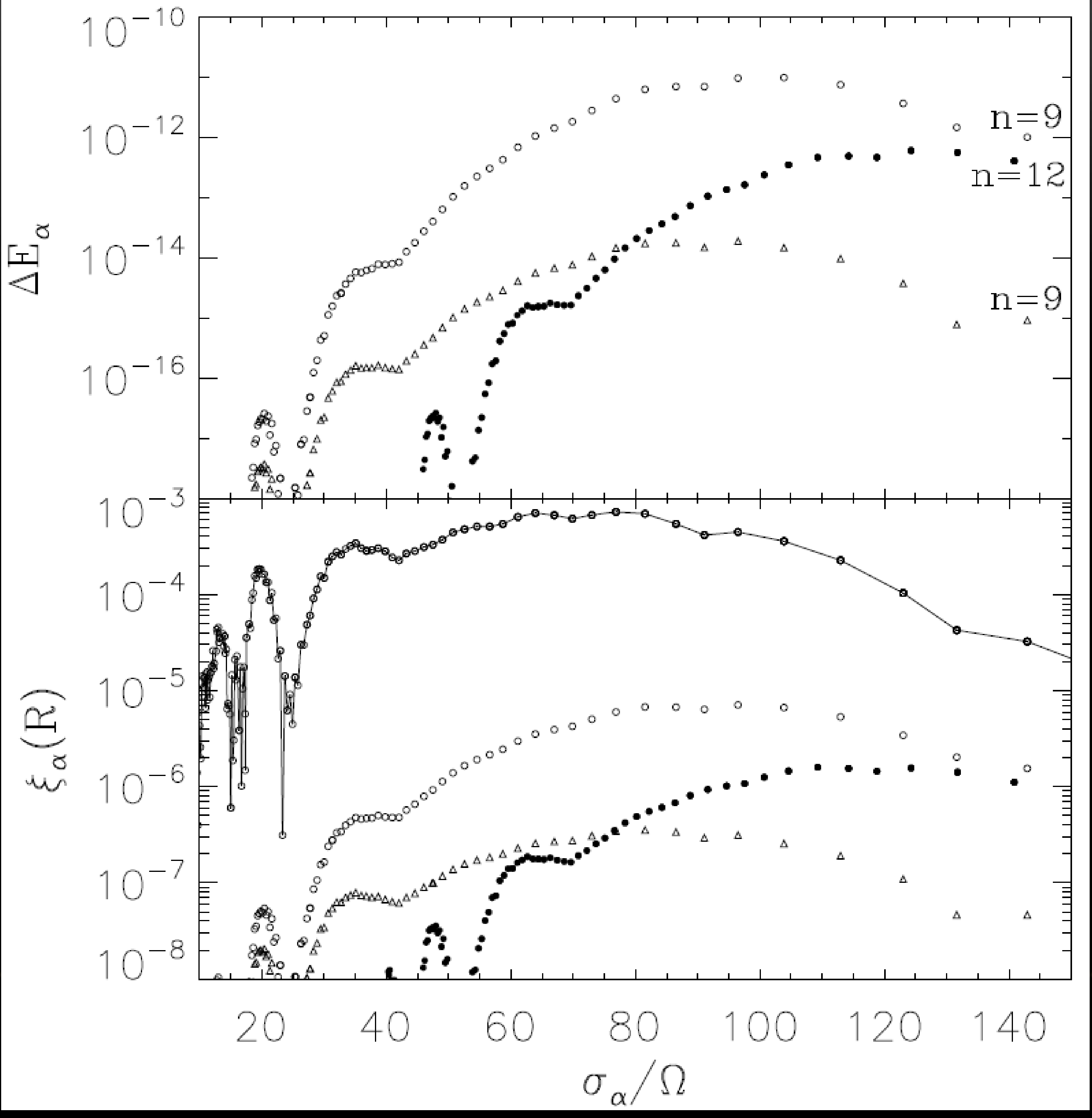
NON-RESONANT MODE ENERGIES

- Forced, adiabatic modes, ignoring Coriolis force

$$\Delta E_\alpha = \frac{GM'^2}{R}\left(\frac{R}{D_p}\right)^{2(l+1)} \frac{2\pi^2\sigma_\alpha}{\varepsilon_\alpha} |Q_\alpha K_{lm}(\sigma_\alpha)|^2$$

$$K_{lm}(\sigma_\alpha) = \frac{W_{lm}}{2\pi}\int_{-P/2}^{P/2} dt \left(\frac{R}{D}\right)^{l+1} e^{i\sigma_\alpha t - im\Phi}$$

$$\begin{aligned} Q_\alpha &= \langle \xi_\alpha | \nabla(r^l Y_{lm}) \rangle \\ &= \int d^3x\, \rho \xi_\alpha^* \cdot \nabla(r^l Y_{lm}) \\ &= \int d^3x\, \delta\rho_\alpha^* r^l Y_{lm}. \end{aligned}$$



10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-3}
ΔE_α
n=9
n=12
n=9
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\xi_\alpha(R)$
20
40
60
80
100
120
140
σ_α/Ω

FORCED OSCILLATOR EQUATIONS

- Mode amplitude satisfies

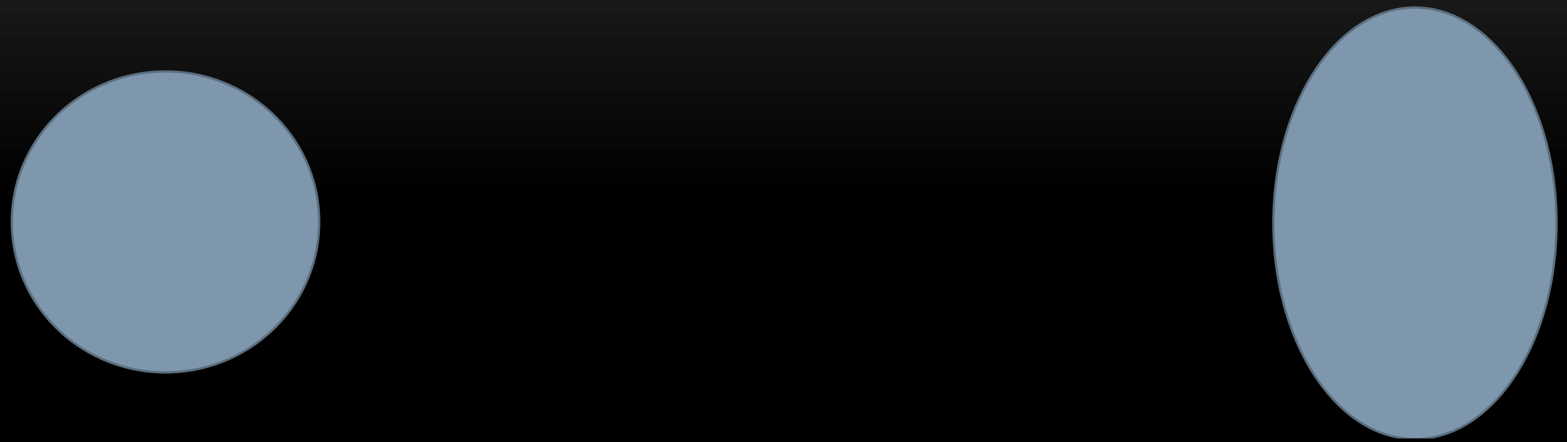
$$\dot{c}_\alpha + (i\omega_\alpha + \gamma_\alpha)c_\alpha = \frac{iGM'W_{lm}Q_\alpha}{2\varepsilon_\alpha a^{l+1}} \times \sum_{N=-\infty}^{\infty} F_{Nm} e^{i(m\Omega_s - N\Omega)t}$$

- Nonhomogeneous solution:

$$c_\alpha(t) = \frac{GM'W_{lm}Q_\alpha}{2\varepsilon_\alpha a^{l+1}} \sum_{N=-\infty}^{\infty} \frac{F_{Nm} e^{-i(N\Omega - m\Omega_s)t}}{(\sigma_\alpha - N\Omega) - i\gamma_\alpha}$$

MODE VISIBILITY

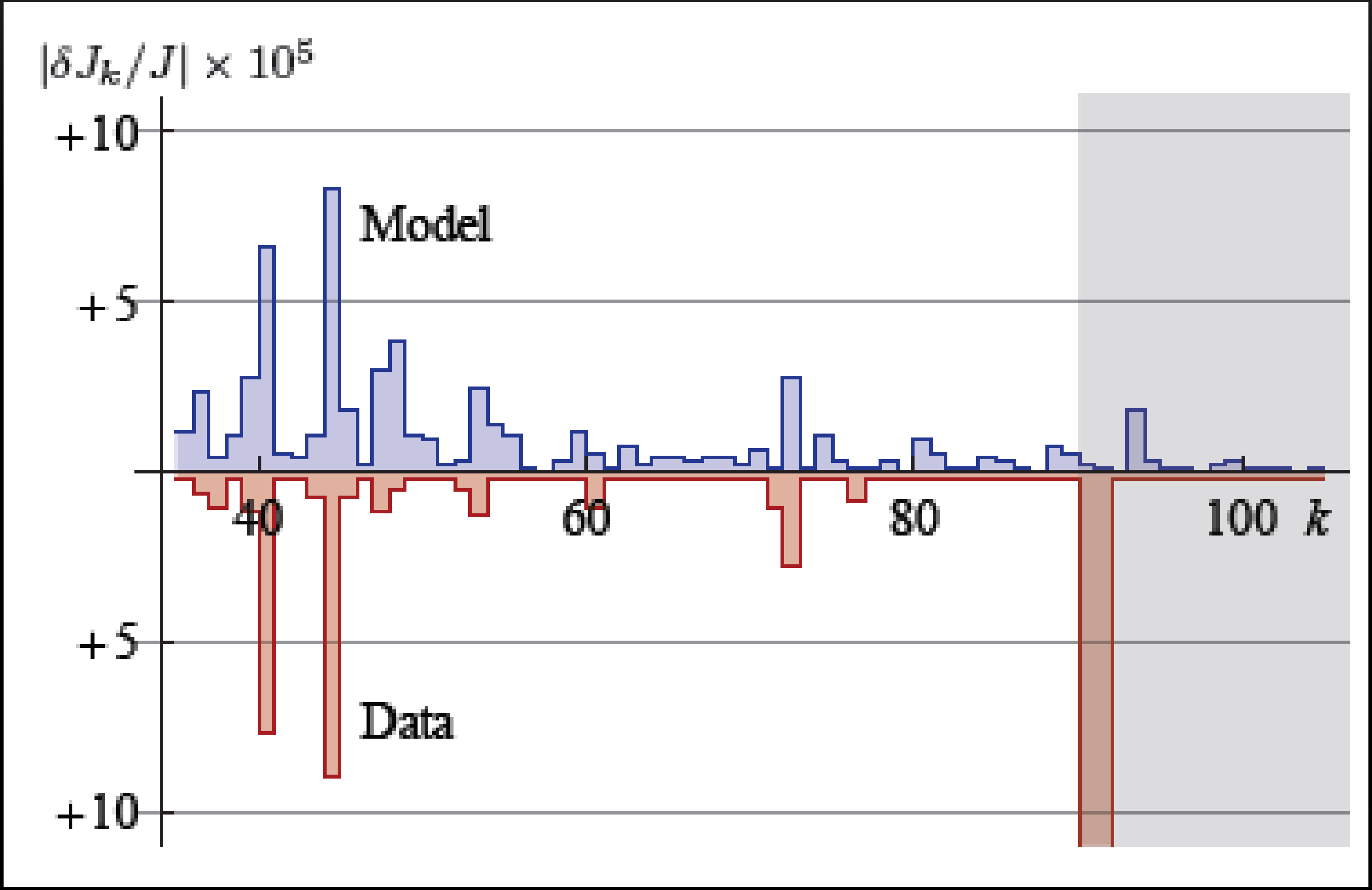
$$|\Delta \mathrm{mag}|_\alpha \propto \left[3\cos^2(\theta) - 1\right]\xi_r(R) \qquad \left(\text{for } m = 0\right)$$



$$|\Delta \mathrm{mag}|_\alpha \propto \sin^2(\theta)\xi_r(R) \qquad \left(\text{for } m = 2\right)$$

Face-on (θ=0) Edge-on (θ =90)

TUNED MODEL FLUX VARIATIONS



RESONANCE LOCKING

- Modes absorb orbital energy, thus increasing the orbital frequency
- As modes are excited and damp, they spin up star, changing the mode frequency in the inertial frame

$$\sigma_\alpha = \varepsilon_\alpha + m(1 - C_\alpha)\Omega_s$$

- At resonance, $\sigma_\alpha = N\Omega$
- Mode can stay in resonance if $\dot{\sigma}_\alpha = N\dot{\Omega}$

RESONANCE LOCKING CONTINUED

- Modes also absorb orbital angular momentum

$$\dot{E}_{\rm orb} = -\sum_\alpha 2\gamma_\alpha E_\alpha, \quad \dot{J}_{\rm orb} = -\sum_\alpha 2\gamma_\alpha E_\alpha \frac{m}{\sigma_\alpha}$$

- Solve to find

$$N_c = m\left(\frac{B_\alpha \mu a^2}{3I}\right)^{1/2}$$

EVEN MORE RESONANCE LOCKING

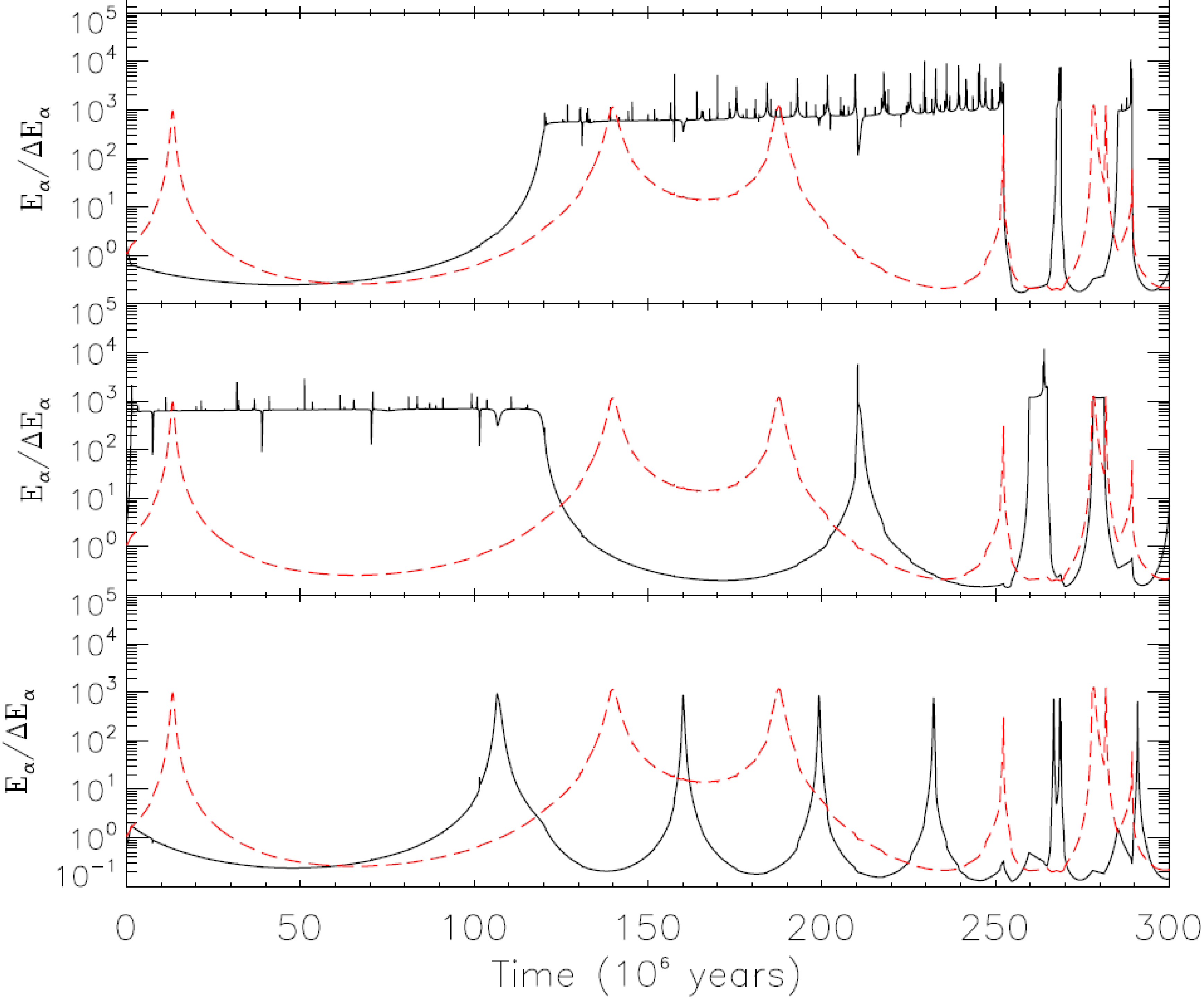
- Introduce spin down term to find modes can lock if

$$N < N_c$$

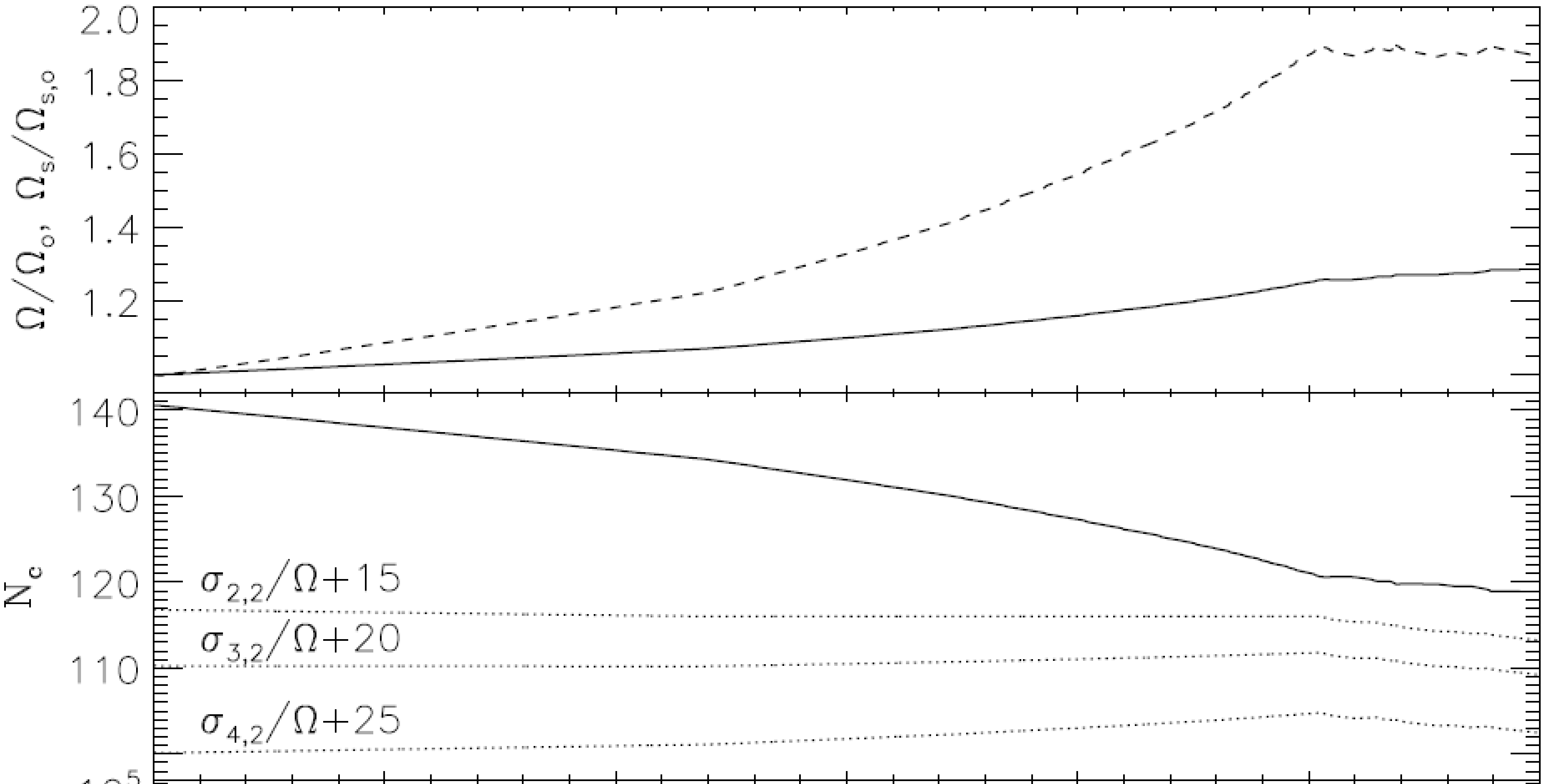
- Mode reaches equilibrium when

$$\epsilon_{\mathrm{eq}} \simeq \frac{1}{2\pi}\left[\frac{t_D \delta_N}{t_{d\alpha}} - (\gamma_\alpha P)^2\right]^{1/2} \ll 1.$$

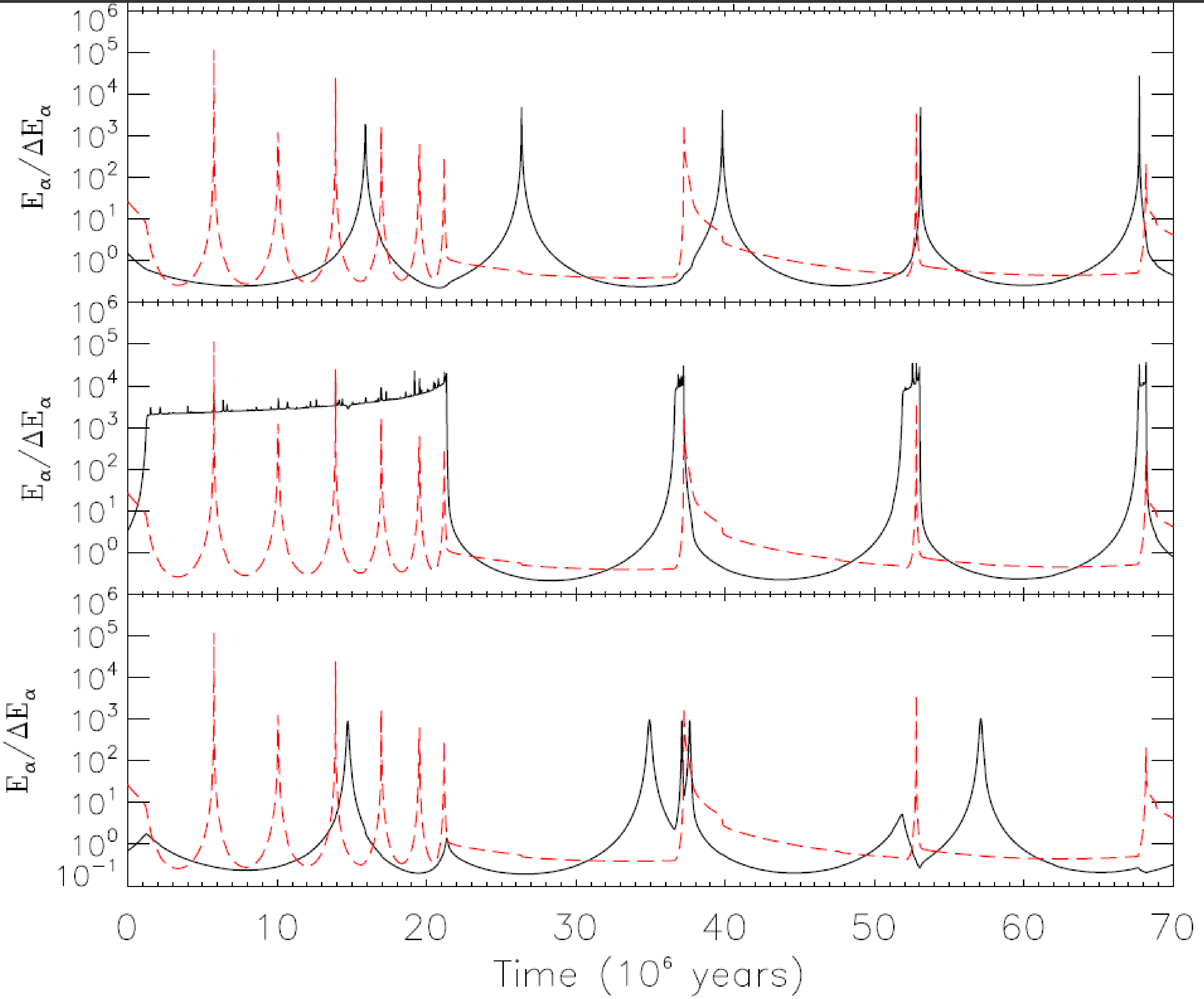
- Mode may reach 10^4 times normal energies



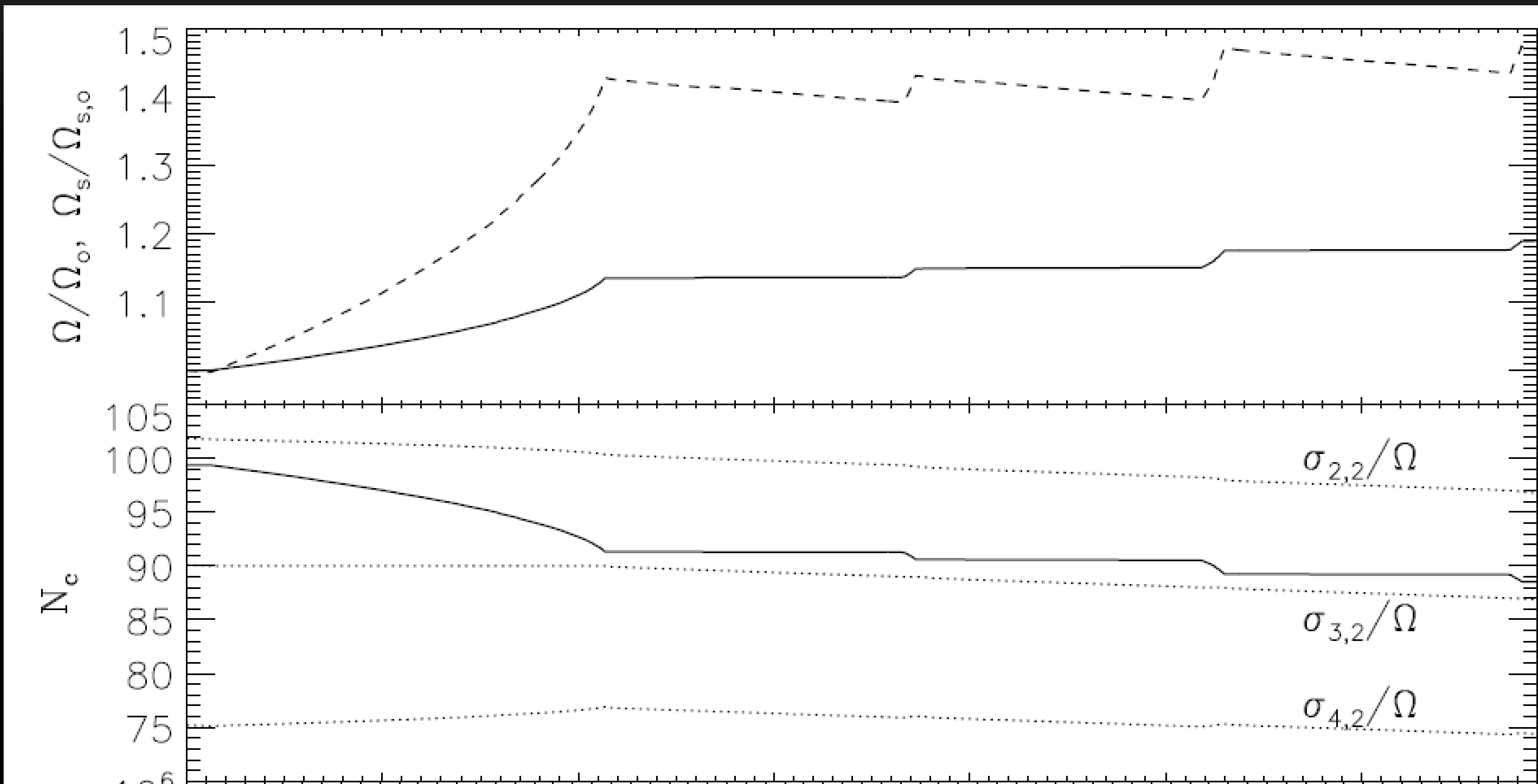
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
0
50
100
150
200
250
300
Time (10^6 years)



2.0
1.8
1.6
1.4
1.2
$\Omega/\Omega_o,\ \Omega_s/\Omega_{s,o}$
140
130
120
110
N_c
$\sigma_{2,2}/\Omega+15$
$\sigma_{3,2}/\Omega+20$
$\sigma_{4,2}/\Omega+25$



10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$E_\alpha/\Delta E_\alpha$
0
10
20
30
40
50
60
70
Time (10^6 years)



1.5
1.4
1.3
1.2
1.1
Ω/Ω_o, $\Omega_s/\Omega_{s,o}$
105
100
95
90
85
80
75
N_c
$\sigma_{2,2}/\Omega$
$\sigma_{3,2}/\Omega$
$\sigma_{4,2}/\Omega$

NON-LINEAR MODE COUPLING

- Several modes detected at non-integer multiples of orbital frequency
  - Likely the result of non-linear coupling to daughter modes
- Occurs when

$$\sigma_p \simeq \sigma_{d1} + \sigma_{d2}$$

- In KOI-54,

$$\sigma_2 \simeq \sigma_5 + \sigma_6$$

- Other non-integer modes likely due to non-linear coupling in which one of the daughter modes is invisible (e.g., the visible daughter mode has m=0, the invisible mode has m=2)

FUTURE PROSPECTS

- Identify modes using spectral techniques
- Measure orbital decay?
- Tidal asteroseismology