Estimating stellar mean density through seismic inversions

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Introduction

The importance of stellar mass

- dominant role in evolution and final fate of stars
- a key parameter when characterizing exoplanetary systems
- however, it can be difficult to obtain for single stars

Various approaches for determining stellar mass

- comparisons with evolutionary tracks in HR diagrams
  - large error bars & regions with overlapping tracks
- mean density from asteroseismology & radius from parallax
  - simple scaling laws
  - search for models in a grid
  - full density inversion & integration to get mean density

Theoretical aspects

- starting point: reference model which is not too far from the observed star
- this leads to frequency differences which can be related to differences on the structure:

$$\frac{\delta\nu_{n\ell}}{\nu_{n\ell}} = \int_0^1 K^{n\ell}_{\rho,\Gamma_1}(x)\frac{\delta\rho}{\rho}\mathrm{d}x + \int_0^1 K^{n\ell}_{\Gamma_1,\rho}(x)\frac{\delta\Gamma_1}{\Gamma_1}\mathrm{d}x + \frac{F_{\mathrm{surf}}(\nu_{n\ell})}{Q_{n\ell}},$$

- $\frac{\delta\nu_{n\ell}}{\nu_{n\ell}} = \frac{\nu_{\mathrm{obs}}-\nu_{\mathrm{ref}}}{\nu_{\mathrm{ref}}}$
- the kernels $K^{n\ell}_{\rho,\Gamma_1}$ and $K^{n\ell}_{\Gamma_1,\rho}$ are deduced via the variational principle

Mean density difference

- the difference in mass between the star and the reference model is:

$$\delta M = \int_0^R 4\pi\delta\rho r^2 \mathrm{d}r = \int_0^R 4\pi\rho r^2 \frac{\delta\rho}{\rho}\mathrm{d}r$$

- the difference in mean density is:

$$\frac{\delta\underline{\rho}}{\underline{\rho}} = \int_0^1 4\pi x^2 \frac{\rho R^3}{M}\frac{\delta\rho}{\rho}\mathrm{d}x,$$

- where $\underline{\rho} = 3M/(4\pi R^3)$
- this last equation still applies even if the star and the model don't have the same radii

- a linear combination of the $\delta\nu/\nu$ can then be re-expressed as:

$$\sum_i c_i \frac{\delta\nu_i}{\nu_i} = \int_0^1 \underbrace{\left\{\sum_i c_i K^i_{\rho,\Gamma_1}\right\}}_{K_{\mathrm{avg}}} \frac{\delta\rho}{\rho}\mathrm{d}x + \int_0^1 \underbrace{\left\{\sum_i c_i K^i_{\Gamma_1,\rho}\right\}}_{K_{\mathrm{cross}}} \frac{\delta\Gamma_1}{\Gamma_1}\mathrm{d}x + \underbrace{\sum_i c_i \frac{F_{surf}(\nu_i)}{Q_i}}_{\text{surface terms}}$$

- in order to obtain $\delta\underline{\rho}/\underline{\rho} \simeq \sum_i c_i \frac{\delta\nu_i}{\nu_i}$, one needs:
  - K_{avg} (="averaging kernel") goes to $4\pi\rho R^3 x^2/M$
  - K_{cross} (="cross-term kernel") and the surface terms go to 0

- the following condition ensures the correct inversion result for a homologous transformation:

$$\sum_i c_i = 2$$

- inversion procedures which satisfy this condition will be called "unbiased"

SOLA method

Minimization of the following function

$$J(c_i) = \underbrace{\int_0^1 \left\{4\pi\frac{\rho R^3}{M}x^2 - K_{\mathrm{avg}}(x)\right\}^2 \mathrm{d}x}_{\mathbf{I}} + \underbrace{\beta\int_0^1 \{K_{\mathrm{cross}}(x)\}^2\,\mathrm{d}x}_{\mathbf{II}}$$

$$+ \underbrace{\tan\theta \sum_i \frac{c_i^2\sigma_i^2}{\langle\sigma^2\rangle}}_{\mathbf{III}} + \underbrace{\lambda\left\{1 - \int_0^1 K_{\mathrm{avg}}\,dx\right\}}_{\mathbf{IV}} + \underbrace{\sum_{m=0}^{M_{\mathrm{surf}}} a_m \sum_i \frac{c_i\Psi_m(\nu_i)}{Q_i}}_{\mathbf{V}}$$

Role of different terms

- **I** optimizes K_{avg}
- **II** minimizes K_{cross}
- **III** regularization
- **IV** normalizes K_{avg}
- **V** minimizes surface effects

Free parameters

β

θ

M_{surf}

Large frequency separation

- scaling law with large frequency separation:

$$\langle \Delta\nu \rangle \propto \sqrt{\underline{\rho}}$$

- in differential form, this law becomes:

$$2\frac{\delta\langle \Delta\nu \rangle}{\langle \Delta\nu \rangle} = \frac{\delta\underline{\rho}}{\underline{\rho}}$$

- the left hand = a linear combination of $\delta\nu_i/\nu_i$
  - this leads to linear inversion coefficients c_i
  - this allows the construction of K_{avg} and K_{cross}

The KBCD method

- Kjeldsen et al. (2008) proposed a method for correcting for surface effects
- as a by-product, this method also yields the mean density:

$$\frac{\underline{\rho}_{\mathrm{obs}}}{\underline{\rho}_{\mathrm{ref}}} \simeq \left\{ \frac{b-1}{b\frac{\langle\nu\rangle_{\mathrm{ref}}}{\langle\nu\rangle_{\mathrm{obs}}} - \frac{\langle\Delta\nu\rangle_{\mathrm{ref}}}{\langle\Delta\nu\rangle_{\mathrm{obs}}}} \right\}^2$$

- to first order, this becomes

$$\frac{\delta\underline{\rho}}{\underline{\rho}} \simeq 2\frac{b\frac{\delta\langle\nu\rangle}{\langle\nu\rangle} - \frac{\delta\langle\Delta\nu\rangle}{\langle\Delta\nu\rangle}}{b-1}$$

- in what follows, we will use $b = 4.9$, *i.e.* the solar value
- once more, the right hand = a linear combination of $\delta\nu_i/\nu_i$

Non-linear extension

- one can prescale the reference model by a scale factor, s, to try to bring the differences to the linear regime:

$$\underline{\rho} \longrightarrow s^2 \underline{\rho}_{\mathrm{ref}} \qquad \nu^{\mathrm{ref}} \longrightarrow s\nu_i^{\mathrm{ref}}$$

- the inverted mean density becomes:

$$\underline{\rho}_{\mathrm{inv}}(s) = \underline{\rho}_{\mathrm{ref}} s^2 \left\{ 1 + \sum_i c_i \left[\frac{1}{s} \left(\frac{\delta\nu_i}{\nu_i} + 1 \right) - 1 \right] \right\}$$

- if $\sum_i c_i = 2$ (the inversion procedure is unbiased), this simplifies to:

$$\underline{\rho}_{\mathrm{inv}}(s) = \underline{\rho}_{\mathrm{ref}} \left\{ -s^2 + s \left(2 + \sum_i c_i \frac{\delta\nu_i}{\nu_i} \right) \right\}$$

Non-linear extension

- the previous equation is a 2^{nd} order polynomial with the following maximum:

$$\underline{\rho}_{\text{max}} \equiv \underline{\rho}_{\text{inv}}\left(s_{\text{max}}\right) = \rho_{\text{ref}}\, s_{\text{max}}^2$$

- where

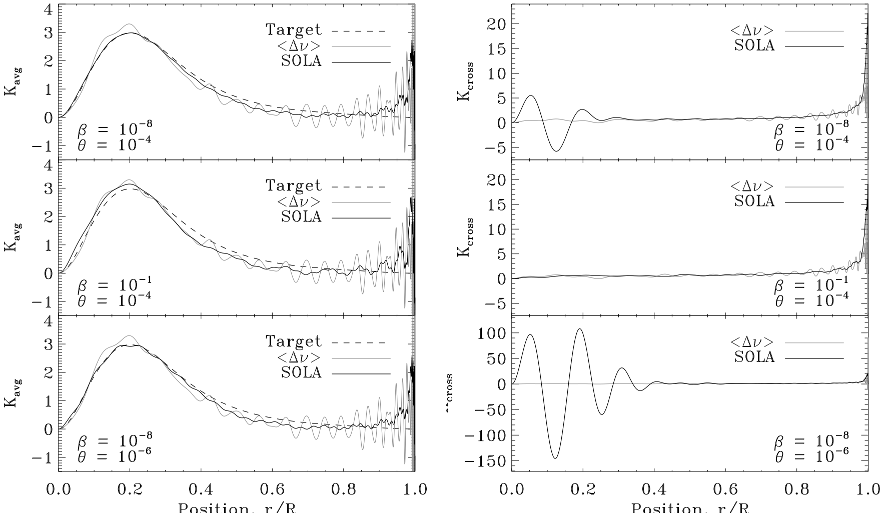
$$s_{\text{max}} = 1 + \frac{1}{2}\sum_i c_i \frac{\delta\nu_i}{\nu_i} = \frac{1}{2}\sum_i c_i \frac{\nu_i^{\text{obs}}}{\nu_i^{\text{ref}}}$$

- this maximum corresponds to the best mean density estimate:
    - linear inversions bring no further corrections
    - when used on linearized scaling and KBCD laws, the original non-linear laws are (nearly) retrieved

The sun

- use model S (Christensen-Dalsgaard et al. 1996) as a reference models
- use 104 GOLF frequencies (Lazrek 1997) as observed frequencies

The sun


Target
<Δν>
SOLA
K_{avg}
K_{cross}
$\beta = 10^{-8}$
$\theta = 10^{-4}$
$\beta = 10^{-1}$
$\theta = 10^{-4}$
$\beta = 10^{-8}$
$\theta = 10^{-6}$
Position, r/R

The sun

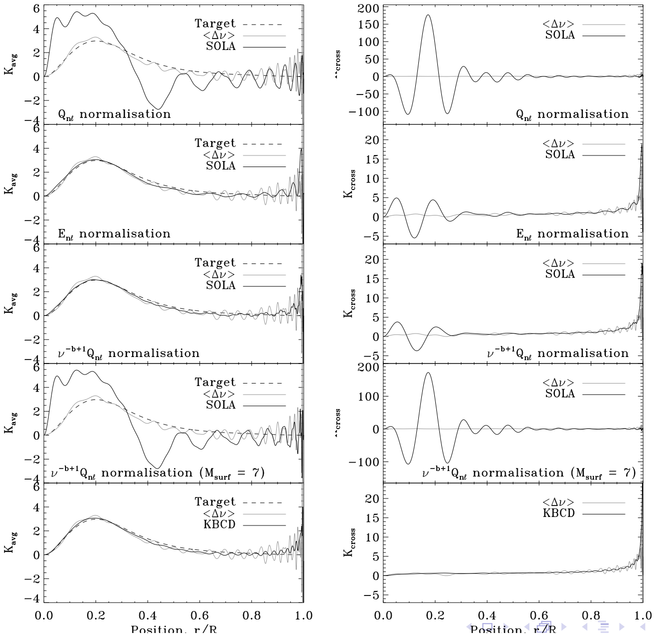
β	θ	$\delta\underline{\rho}/\underline{\rho}$	$\sigma_{\delta\underline{\rho}/\underline{\rho}}$	$\|\Delta K_{\rm avg}\|_2$	$\|K_{\rm cross}\|_2$
10^{-8}	10^{-4}	$-1.9e-3$	$5.3e-4$	0.32	2.54
10^{-1}	10^{-4}	$-5.1e-3$	$1.9e-3$	0.35	1.80
10^{-8}	10^{-6}	$-1.2e-3$	$1.2e-2$	0.31	40.6
$\langle\Delta\nu\rangle$	scaling	$-1.2e-2$	$4.1e-4$	1.36	2.77

Near-surface effects

- the surface effects take on the following form:

$$\frac{F_{\text{surf}(\nu_i)}}{Q_i}$$

- F_{surf} = slowly varying ad-hoc function of frequency only
- Q_i: normalized mode inertia (typically used in structural inversions)
- the surface effects can also be normalized by:
  - E_i: unnormalized mode inertia
  - $\nu^{-b+1}Q_i$: normalization based on Kjeldsen et al. (2008)

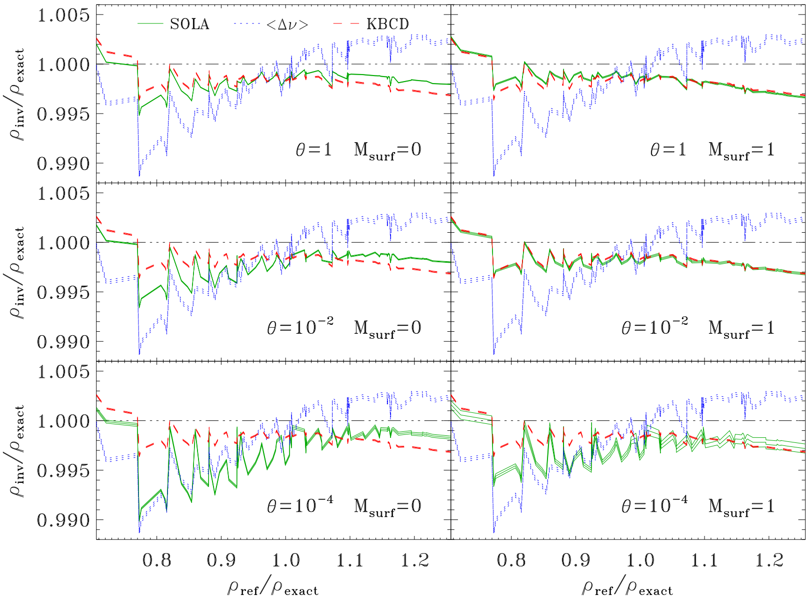

Target
<Δν>
SOLA
KBCD
K_{avg}
K_{cross}
$Q_{n\ell}$ normalisation
$E_{n\ell}$ normalisation
$\nu^{-b+1}Q_{n\ell}$ normalisation
$\nu^{-b+1}Q_{n\ell}$ normalisation (M_{surf} = 7)
Position, r/R

Description	M_{surf}	$\delta\underline{\rho}/\underline{\rho}$	$\sigma_{\delta\underline{\rho}/\underline{\rho}}$	$\|\Delta K_{\mathrm{avg}}\|_2$	$\|K_{\mathrm{cross}}\|_2$
$Q_{n\ell}$ normalization	1	$-5.6e-2$	$2.2e-2$	1.84	47.0
$E_{n\ell}$ normalization	1	$-1.0e-3$	$1.2e-3$	0.50	2.55
$\nu^{-b+1}Q_{n\ell}$ normalization	1	$-8.5e-4$	$6.2e-4$	0.40	2.25
$\nu^{-b+1}Q_{n\ell}$ normalization	7	$-5.8e-2$	$2.3e-2$	1.86	45.9
Kjeldsen et al. (2008)	–	$-1.9e-3$	$5.5e-5$	0.41	2.03

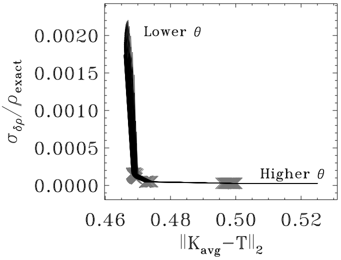
Grid of models

- systematic study of the different inversion procedures: grid of models
  - 93 pre-main and main sequence models
  - mass: 0.80 to 0.92 $M_{\odot}$
  - age: 28 Myrs to 17.6 Gyrs after birthline
  - source: http://www.astro.up.pt/corot/models/cesam/ (Marques et al. 2008)
- 3 “observed” stars:
  - Model A: same physics, different initial condition
  - Model A$'$: same as Model A but with altered surface
  - Model B: different physics (different composition, diffusion, mixing)

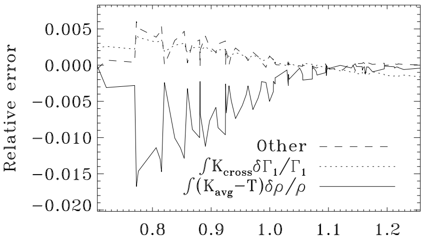
Model A


1.005
1.000
0.995
0.990
$\rho_{\rm inv}/\rho_{\rm exact}$
SOLA
$<\Delta\nu>$
KBCD
$\theta=1$ $M_{\rm surf}=0$
$\theta=1$ $M_{\rm surf}=1$
$\theta=10^{-2}$ $M_{\rm surf}=0$
$\theta=10^{-2}$ $M_{\rm surf}=1$
$\theta=10^{-4}$ $M_{\rm surf}=0$
$\theta=10^{-4}$ $M_{\rm surf}=1$
0.8
0.9
1.0
1.1
1.2
$\rho_{\rm ref}/\rho_{\rm exact}$

The errors

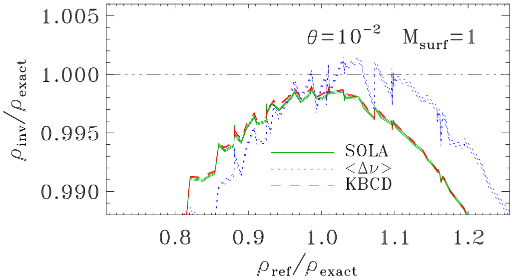


Trade-off curve

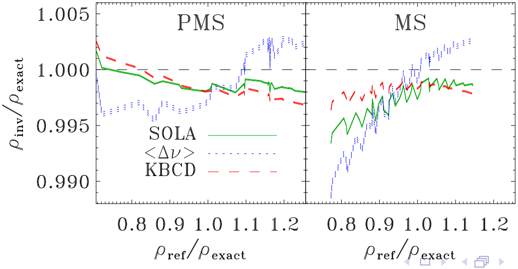


Different sources of error

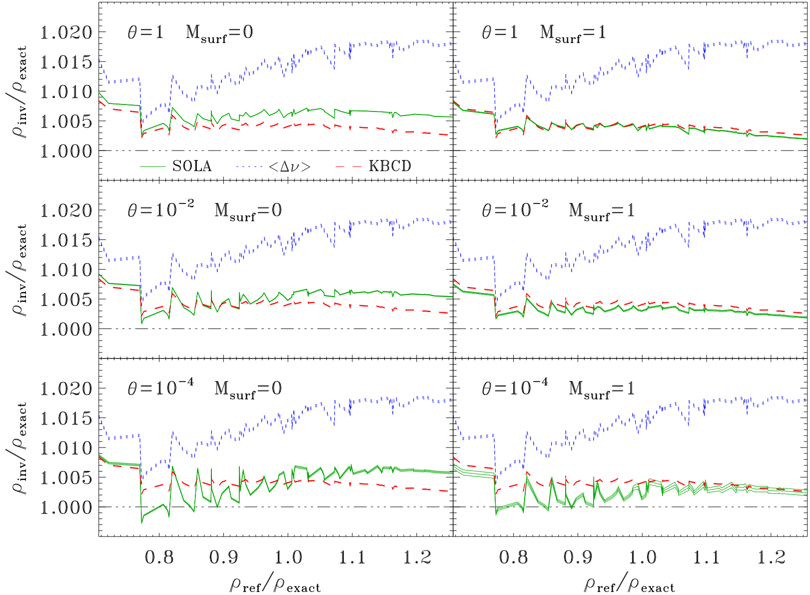
Without prescaling



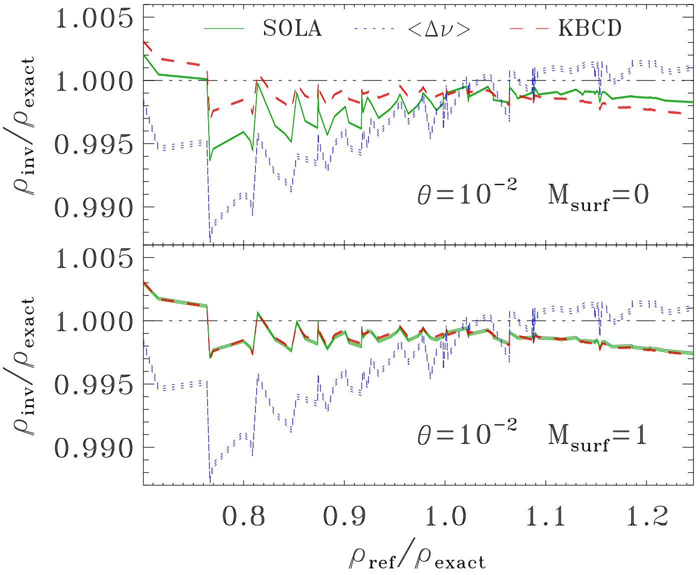
PMS vs. MS



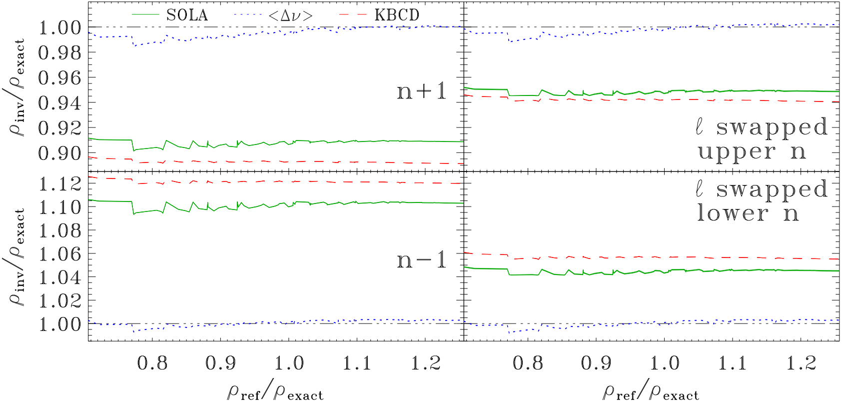
Model A′


θ=1 M_surf=0
θ=1 M_surf=1
θ=10^-2 M_surf=0
θ=10^-2 M_surf=1
θ=10^-4 M_surf=0
θ=10^-4 M_surf=1
SOLA
<Δν>
KBCD
ρ_inv/ρ_exact
ρ_ref/ρ_exact
1.000
1.005
1.010
1.015
1.020
0.8
0.9
1.0
1.1
1.2

Model B

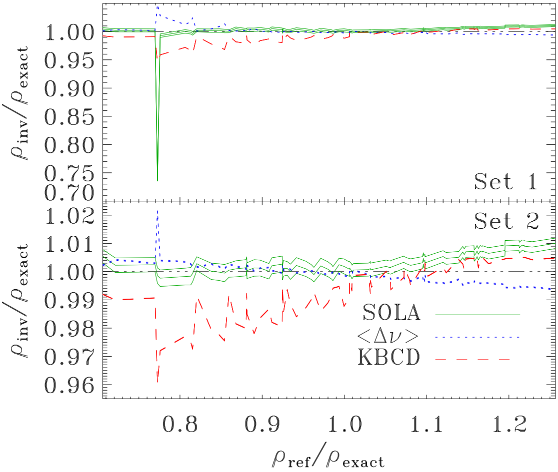

1.005
1.000
0.995
0.990
SOLA
<Δν>
KBCD
θ=10^-2 M_surf=0
θ=10^-2 M_surf=1
ρ_inv/ρ_exact
0.8
0.9
1.0
1.1
1.2
ρ_ref/ρ_exact

Mode misidentification

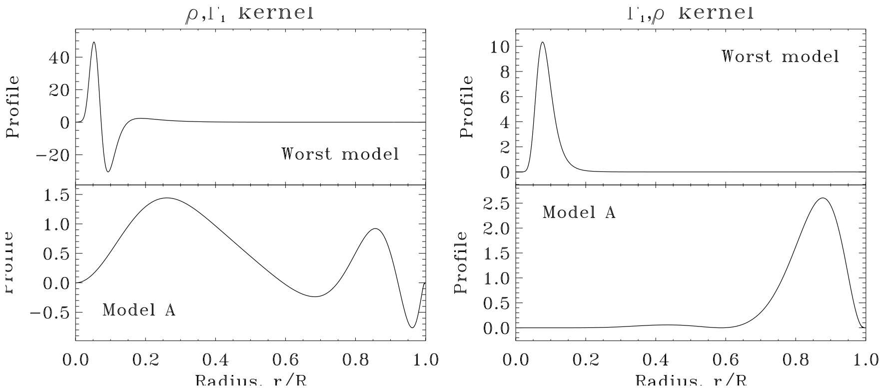


- divergent results can be used to eliminate erroneous mode identifications (also see Bedding et al. 2010, White et al. 2011)

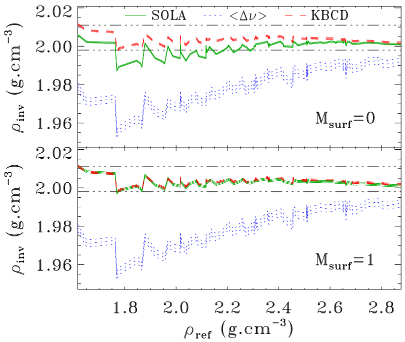
Kernel mismatch



Kernel mismatch

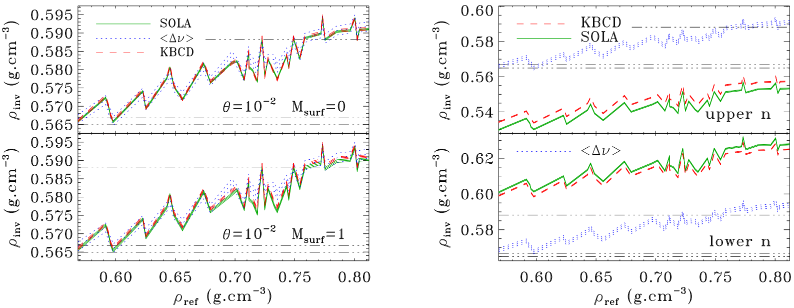


Observed stars – α Cen B



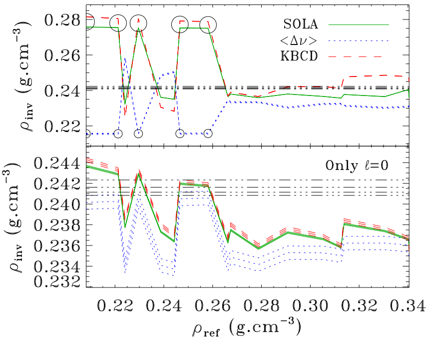
- inversion results:
  - lower than results from orbital parameters (Pourbaix et al. 2002) + parallax (Kervella et al. 2003)
  - agree with other seismic studies (Eggenberger et al. 2004, Kjeldsen et al. 2008)
- strong surface effects

HD 49933



- larger spread in results in this part of the HR diagram
- lack of surface effects
- inversions clearly favor the currently accepted identification (Benomar et al. 2009)

HD 49385



- mixed modes:
  - poor results for models which are far away – better results with $\ell = 0$ only
  - for very close models from Deheuvels et al. (2011), results are similar with and without mixed modes
  - kernels from mixed modes are poorly adapted

Conclusion

- large frequency scaling produces sub-optimal results
- KBCD and SOLA approach are similar:
  - similar mean density estimates
  - similar averaging and cross-term kernels
  - more robust to surface effects
- accuracy goes from $\pm 0.5\%$ to $\pm 2.5\%$
- using $\langle \Delta\nu \rangle$ scaling law + KBCD or SOLA method can identify modes
- importance of using models which are close to the observed star rather than scaling everything on solar values
- mixed modes do not improve results very much