Reflection-Driven MHD Turbulence in the Solar Atmosphere and Solar Wind

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Kavli Institute for Theoretical Physics, September, 2019

Collaborators

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- Francois Foucart, Joe Hollweg (U. New Hampshire)
- Alexander Tchekhovksoy (Northwestern U.)

Outline

I. Background: turbulence and the origin of the solar wind

II. Numerical simulations of reflection-driven MHD turbulence in the solar atmosphere and solar wind

III. The spectrum of imbalanced MHD turbulence

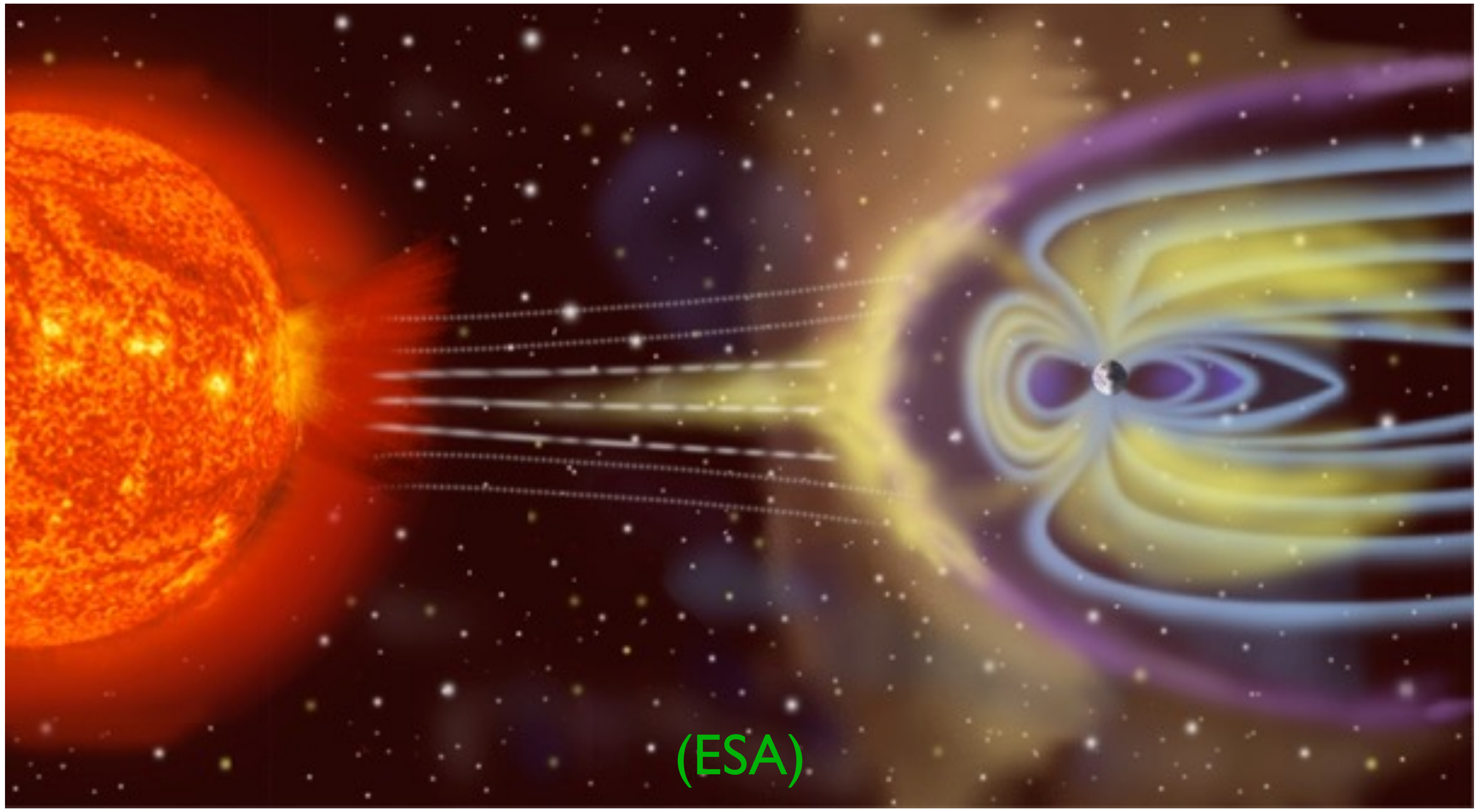
IV. Accounting for additional physics:

   A. compressibility (parametric decay)

   B. general relativity (relevance to disk winds?)

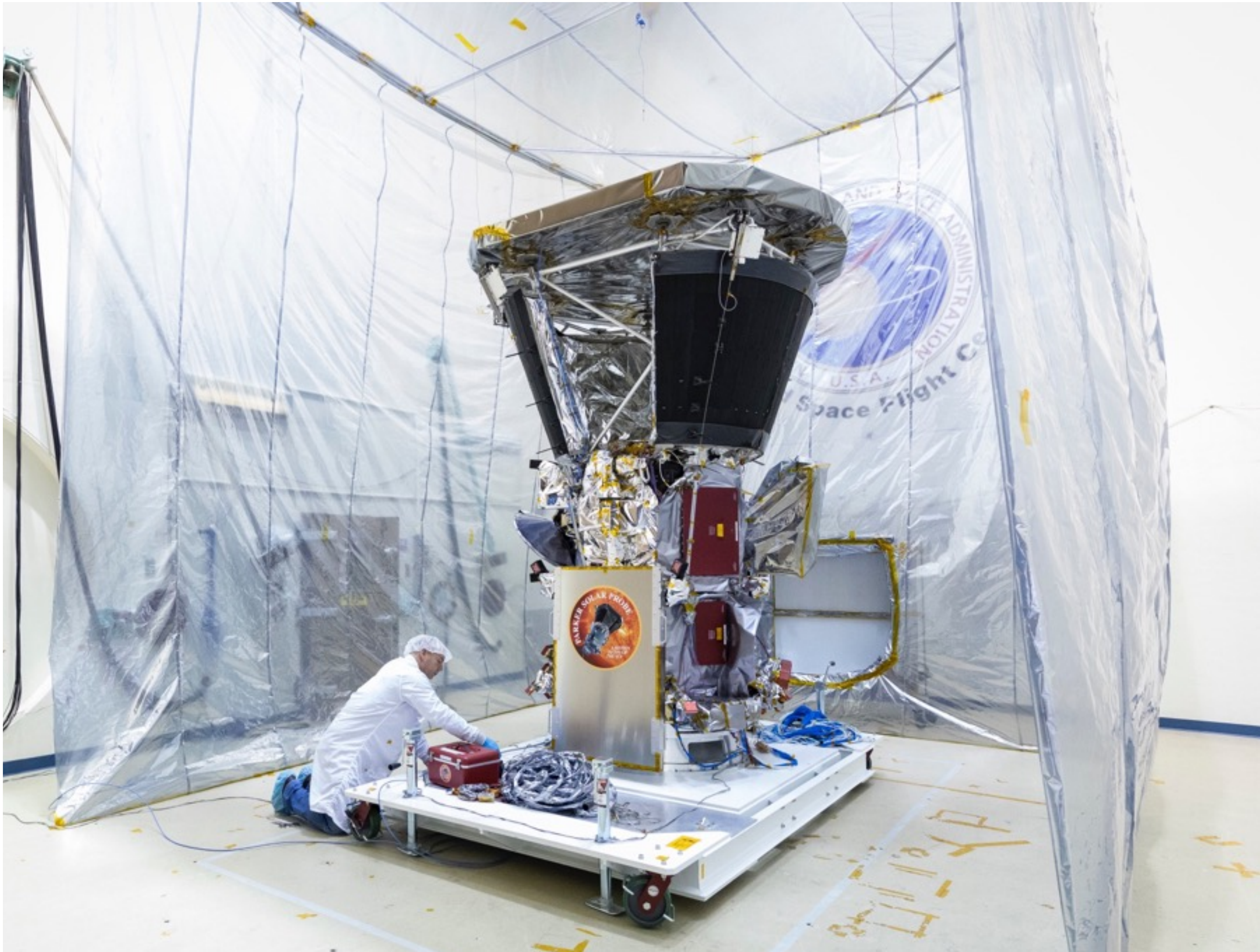
   C. spherical polarization (longitudinal fluctuations)

Solar Wind: a Quasi-Steady, Radial Outflow of Plasma



- At r = 1 AU: U ~ 300-800 km/s, T~10^5 K, n~5 cm^{-3}
- $\dot{M} \sim 10^{-14} M_\odot \text{ yr}^{-1} \qquad L_{\rm mech} \sim 10^{-6} L_\odot$
- Theatre for all of space physics and space weather and a laboratory for astrophysical plasma physics.

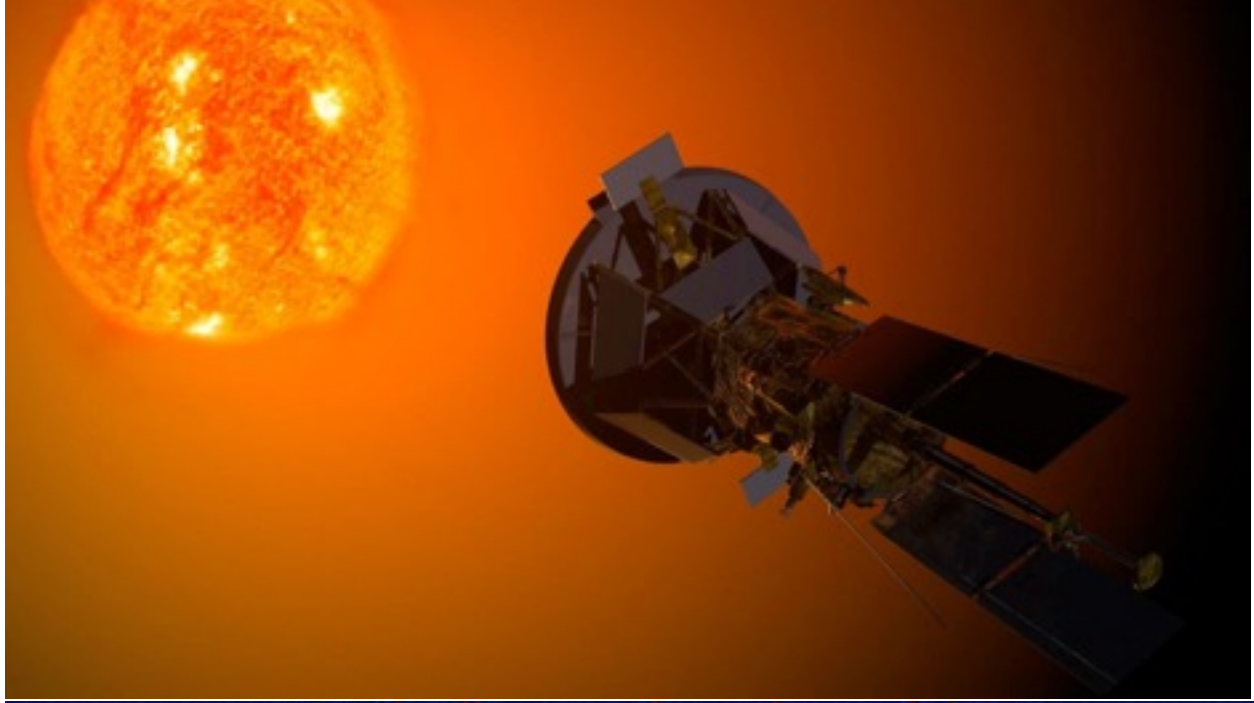
NASA’s Parker Solar Probe

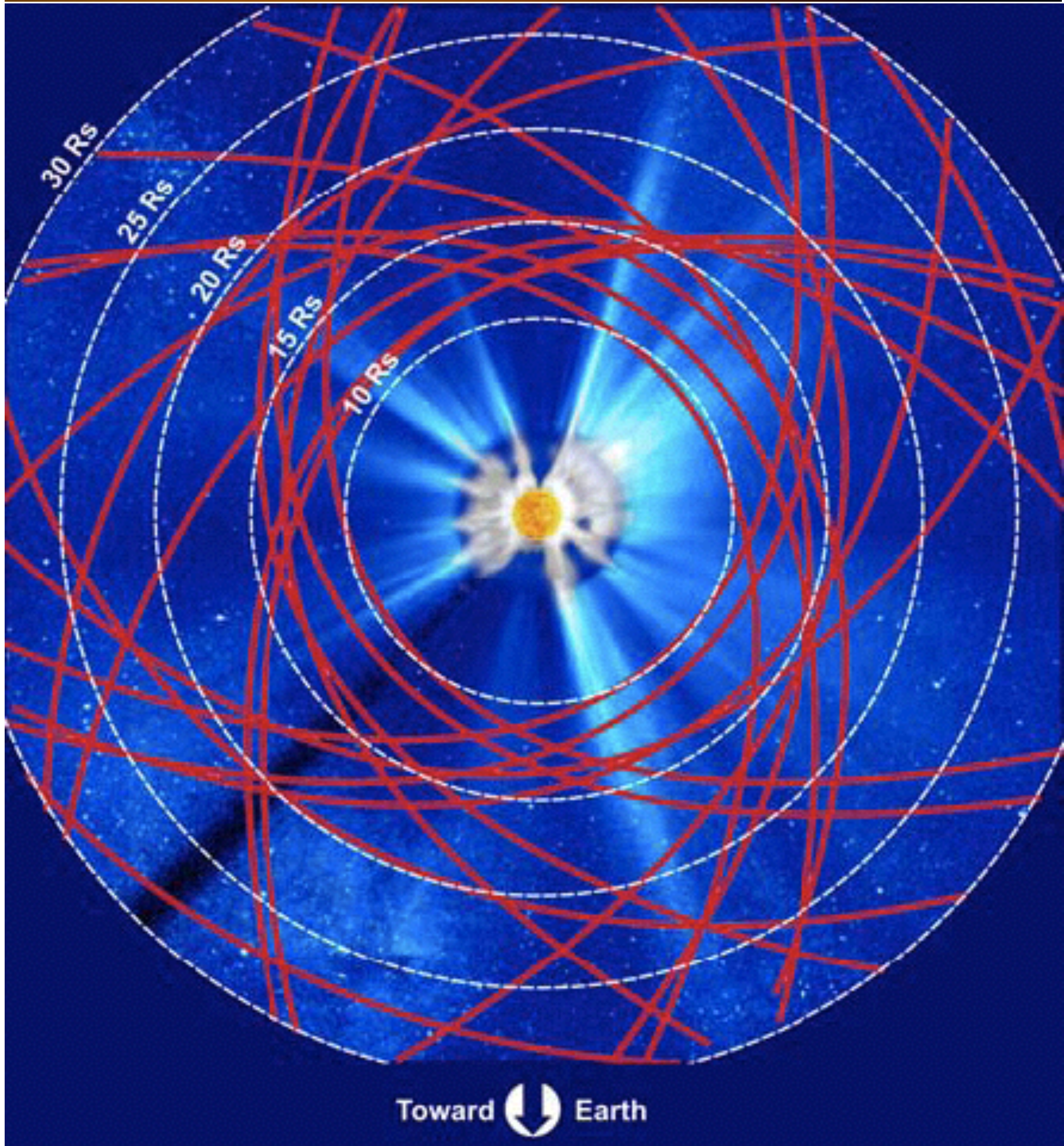


August 12, 2018

Parker Solar Probe

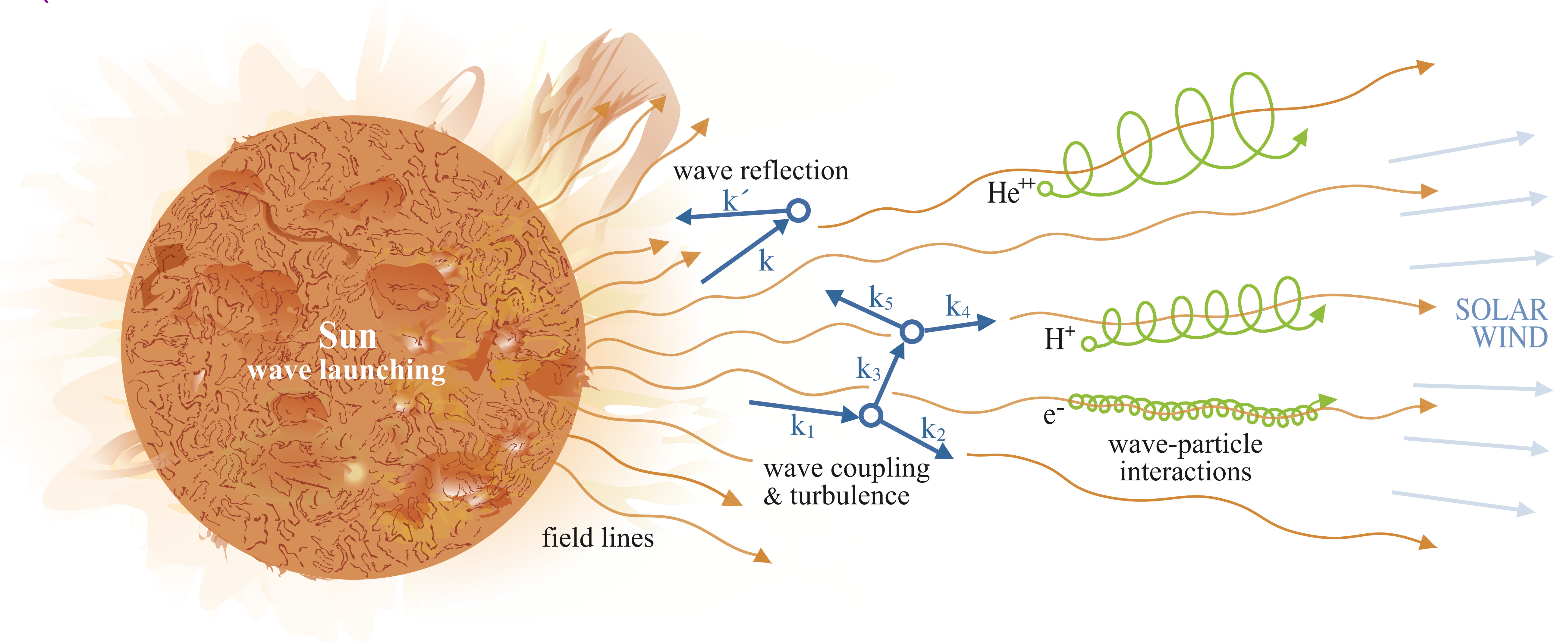
- Several passes to within 10 solar radii of Sun.
- Measures *E, B,* ***u****, T, f*(*v*), energetic particles.
- First in situ measurements ever of the solar-wind acceleration region.





Leading Model for Origin of the Solar Wind

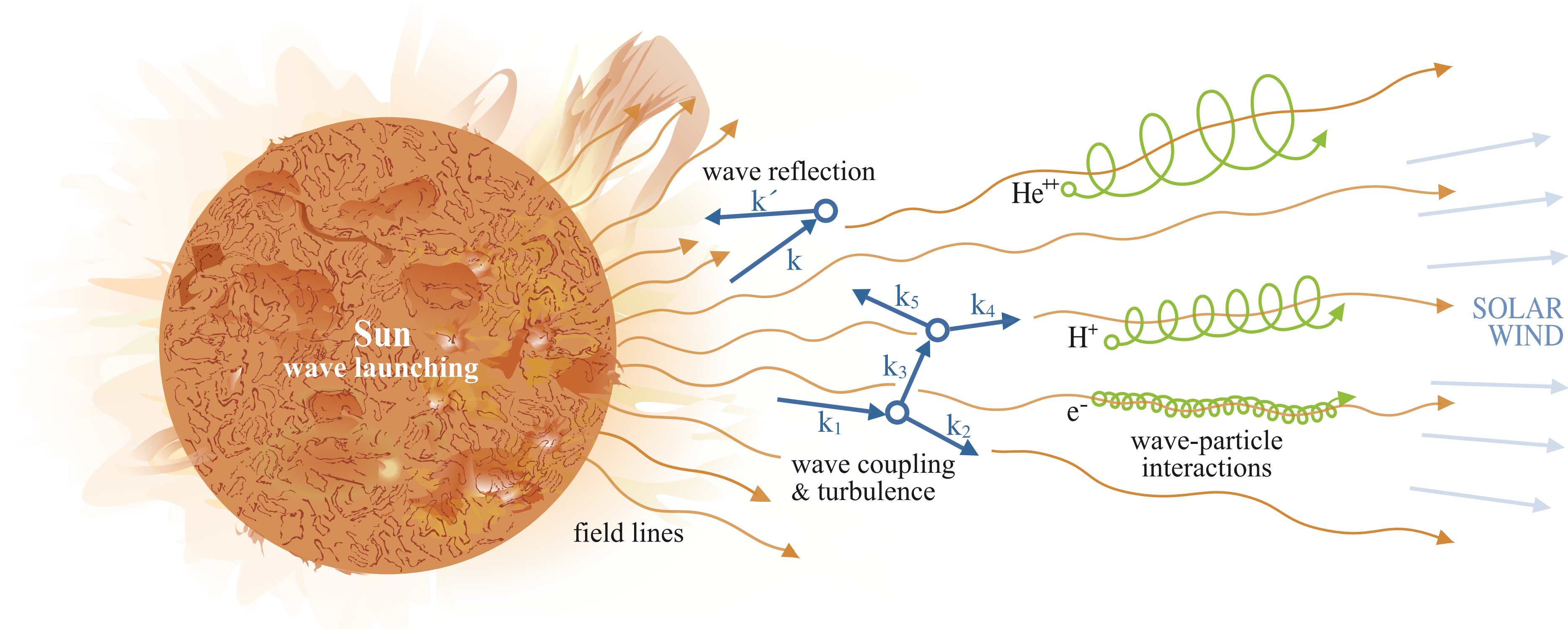
(Parker 1965, Coleman 1968, Velli et al 1989, Zhou & Matthaeus 1989, Cranmer et al 2007)



- The Sun launches Alfven waves, which transport energy outwards
- The waves become turbulent, which causes wave energy to ‘cascade’ from long wavelengths to short wavelengths
- Short-wavelength waves dissipate, heating the plasma. This increases the thermal pressure, which, with some contribution from wave pressure, accelerates the solar wind.

Why Is Turbulence Needed in This Model?

(Parker 1965, Coleman 1968, Velli et al 1989, Zhou & Matthaeus 1989, Cranmer et al 2007)



- Linear Alfvén wave damping is so weak that turbulence is needed to speed up the dissipation of the wave energy.
- If the mechanical luminosity $L_{\rm mech} \sim \mathcal{E} v \times {\rm area}$ stays in Alfvén waves, the energy density (pressure) $\mathcal{E}$ is small because the Alfvén speed is so large near the Sun. Low pressure means little acceleration.
- To accelerate the solar wind, wave energy must be transferred to the plasma. The propagation speed of the energy is then much smaller near the Sun, the energy density (pressure) $\mathcal{E}$ goes up, and you get much more acceleration. (Turbulence also needed for coronal heating.)

Ideal, Adiabatic MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \frac{p}{\rho^\gamma} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

- Magnetic forces: magnetic pressure and magnetic tension
- Frozen-in Law: magnetic field lines are like threads that are frozen to the plasma and advected by the plasma.
- Alfven waves are like waves on a string, where the magnetic field plays the role of the string.

Transverse, Non-Compressive Fluctuations

(Whang 1980, Zhou & Matthaeus 1989; Velli et al 1989)

- $\mathbf{v} = \mathbf{U} + \delta\mathbf{v} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$
- $\delta\mathbf{v} \perp \mathbf{B}_0 \qquad \delta\mathbf{B} \perp \mathbf{B}_0 \qquad \nabla \cdot \delta\mathbf{v} = 0$ (transverse, non-compressive approximation)
- $\mathbf{z}^{\pm} = \delta\mathbf{v} \mp \delta\mathbf{B}/\sqrt{4\pi\rho}$ (Elsasser variables)
- $\mathbf{v}_{\mathrm{A}} = \mathbf{B}_0/\sqrt{4\pi\rho} \qquad p_{\mathrm{tot}} = p + B^2/8\pi.$ (Alfven velocity and total pressure)

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\quad \pm \mathbf{v}_{\mathrm{A}}) \cdot \nabla \mathbf{z}^{\pm} \cdot$$

$$= -\left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} + \frac{\nabla p_{\mathrm{tot}}}{\rho} \right)$$

For a stationary (U=0) and homogeneous background.

- $\mathbf{z}^{+}$ represents Alfvén waves (AWs) propagating parallel to $\mathbf{B}_0$
- $\mathbf{z}^{-}$ represents AWs propagating anti-parallel to $\mathbf{B}_0$.

Transverse, Non-Compressive Fluctuations

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For a stationary (U=0) and homogeneous background.

The nonlinear term leads to energy cascade, but only in the presence of both z^+ and z^-. If all the waves propagate in same direction, they don't interact. **Only counter-propagating waves interact nonlinearly to produce turbulence. For turbulence to develop, we need some source of inward-propagating waves.**

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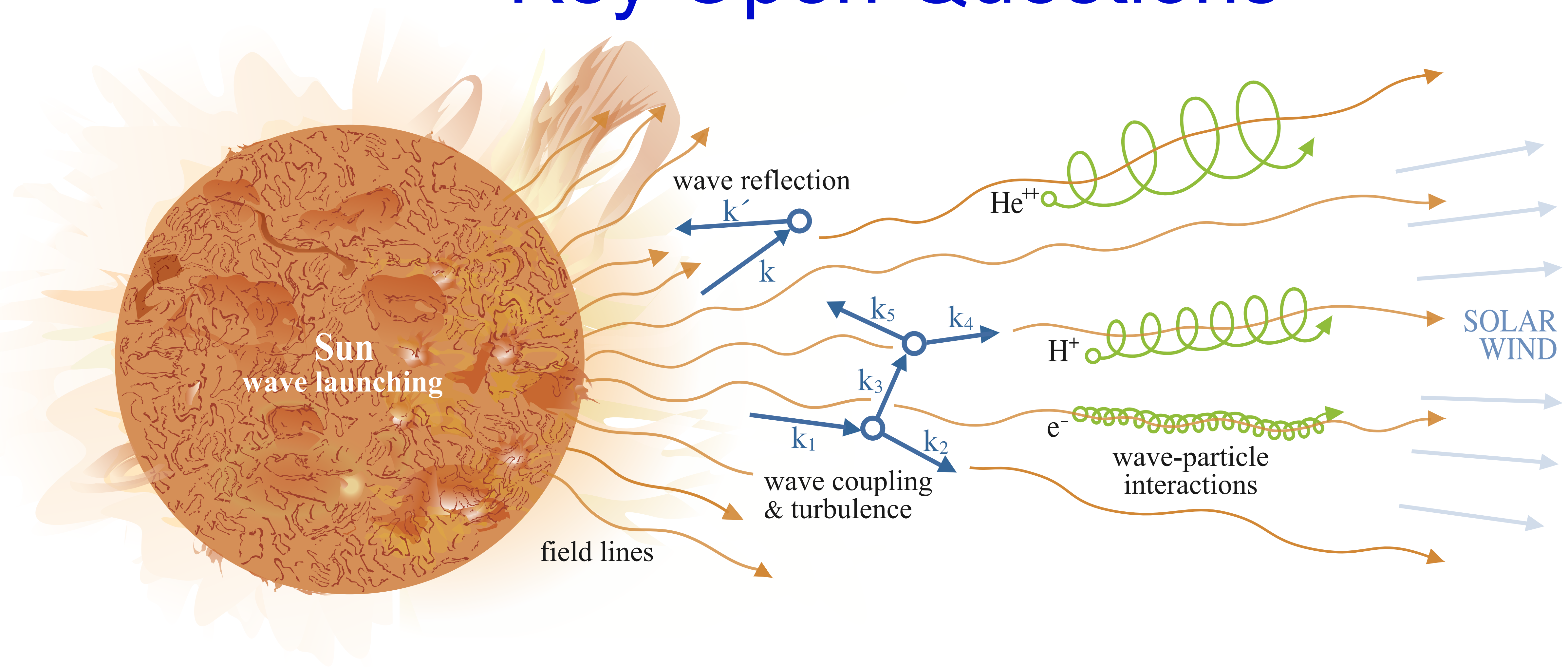
$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\mathbf{U} \pm \mathbf{v}_{\mathrm{A}}) \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla (\mathbf{U} \mp \mathbf{v}_A)$$
$$+ \frac{1}{2}(\mathbf{z}^{-} - \mathbf{z}^{+}) \left(\nabla \cdot \mathbf{v}_{\mathrm{A}} \mp \frac{1}{2} \nabla \cdot \mathbf{U} \right) = -\left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} + \frac{\nabla p_{\mathrm{tot}}}{\rho} \right)$$

Here we have allowed for background flow and inhomogeneity, and we have taken **U** to be parallel to $\mathbf{B}_0$.

Inward-propagating waves (z^-) and outward-propagating waves (z^+) are coupled via linear terms, which lead to (non-WKB) wave reflection. **Basic idea**: z^+ partially reflects, producing z^-, leading to turbulence.

Key Open Questions



1. Is the heating from reflection-driven MHD turbulence enough to generate the solar wind?

2. What is the spectrum of ‘imbalanced’ reflection-driven MHD turbulence?

Transverse, Non-Compressive Fluctuations

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$$+ \frac{1}{2}(\mathbf{z}^{-} - \mathbf{z}^{+}) \left(\nabla \cdot \mathbf{v}_{\mathrm{A}} \mp \frac{1}{2} \nabla \cdot \mathbf{U} \right) = - \left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} + \frac{\nabla p_{\mathrm{tot}}}{\rho} \right)$$

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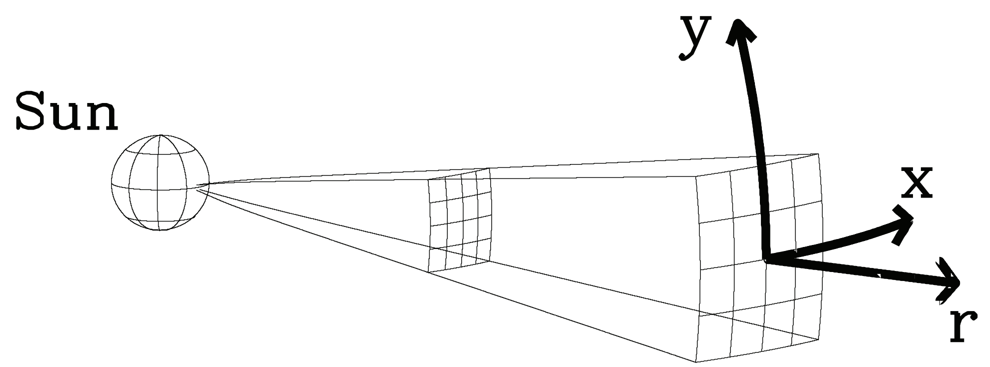
Three Direct Numerical Simulations

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)

- Simulation domain: narrow magnetic flux tube aligned with the radial direction. No solar rotation. 256^2x16387 grid points.



- Time-independent profiles for ρ, U, and B_0 that resemble fast solar wind.
- Inner boundary is the photosphere. The outer boundary is at r = 21 R_s.
- We use 8th order hyperdissipation. periodic boundary conditions and FFTs in the xy "plane", and a Chebyshev expansion in the radial direction.
- We impose a random, time-dependent, photospheric velocity field with rms amplitude 1.3 km/s, a perpendicular scale comparable to the box size, and a characteristic timescale of 5 to 20 minutes. The perpendicular box size is 4,000 to 16,000 km at the base of the corona.
- The simulations last 12 physical hours. Between 0 and 4 hours, $(z^+)^2$ gradually increases. We compute averages between 6 and 12 hours to focus on the statistically steady-state regime.

Parameters in Numerical Simulations

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)

Quantity		Run 1	Run 2	Run 3
$\delta v_{\rm ph,rms}$	....	1.3 km/s	1.3 km/s	1.3 km/s
$\tau_v^{\rm (ph)}$	....	3.3 min	9.6 min	9.3 min
$L_{\rm box}(1 R_\odot)$	....	4.1×10^2 km	4.1×10^2 km	1.6×10^3 km
$L_{\rm box}(1.0026 R_\odot)$	....	4.1×10^3 km	4.1×10^3 km	1.6×10^4 km
Number of grid points	....	$256^2 \times 16385$	$256^2 \times 16385$	$256^2 \times 16385$

Analytic Model of Reflection-Driven MHD Turbulence

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)

- Sun launches two populations of z^+ fluctuations: $z^+_{\rm LF}$ fluctuations with a longer radial correlation length, and $z^+_{\rm HF}$ fluctuations with a shorter radial correlation length.

- z^- fluctuations are produced by non-WKB reflection of $z^+_{\rm LF}$ fluctuations, and z^- and $z^+_{\rm LF}$ fluctuations become aligned, weakening their nonlinear interactions.

- $z^+_{\rm HF}$ fluctuations cascade more rapidly than $z^+_{\rm LF}$ fluctuations.

- change in sign of $dv_{\rm A}/dr$ at $r = r_{\rm m} = 1.7 R_\odot$ reduces z^- at $r < r_{\rm m}$.

- z^- fluctuations are anomalously coherent in a reference frame that moves with the z^+ fluctuations.

Using these conjectures, we approximate the nonlinear terms in the governing equations, obtaining ODEs, which we then solve.

Analytic Model of Reflection-Driven MHD Turbulence

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)

$$z^-_{\rm rms} = \frac{c^- L_\perp}{A}\left(\frac{U + v_{\rm A}}{2v_{\rm A}}\right)\left|\frac{{\rm d}v_{\rm A}}{{\rm d}r}\right|$$

$$\frac{(z^+_{\rm LF,rms})^2}{(z^+_{\rm LFb})^2} = \frac{Uv_{\rm A}(U_{\rm b}+v_{\rm Ab})^2}{U_{\rm b}v_{\rm Ab}(U+v_{\rm A})^2} \times \begin{cases} \left(\dfrac{v_{\rm Ab}}{v_{\rm A}}\right)^{c_{\rm I}} & \text{if } r_{\rm b} < r < r_{\rm m} \\ \left(\dfrac{v_{\rm Ab}}{v_{\rm Am}}\right)^{c_{\rm I}}\left(\dfrac{v_{\rm A}}{v_{\rm Am}}\right)^{c_{\rm O}} & \text{if } r > r_{\rm m} \end{cases}$$

$$\frac{(z^+_{\rm HF,rms})^2}{(z^+_{\rm HFb})^2} = \frac{Uv_{\rm A}(U_{\rm b}+v_{\rm Ab})^2}{U_{\rm b}v_{\rm Ab}(U+v_{\rm A})^2} \times \begin{cases} \left(\dfrac{v_{\rm Ab}}{v_{\rm A}}\right)^{c_{\rm I}/A_0} & \text{if } r < r_{\rm m} \\ \left(\dfrac{v_{\rm Ab}}{v_{\rm Am}}\right)^{c_{\rm I}/A_0} w^{c_{\rm O}/A_0}e^{-H} & \text{if } r > r_{\rm m} \end{cases}$$

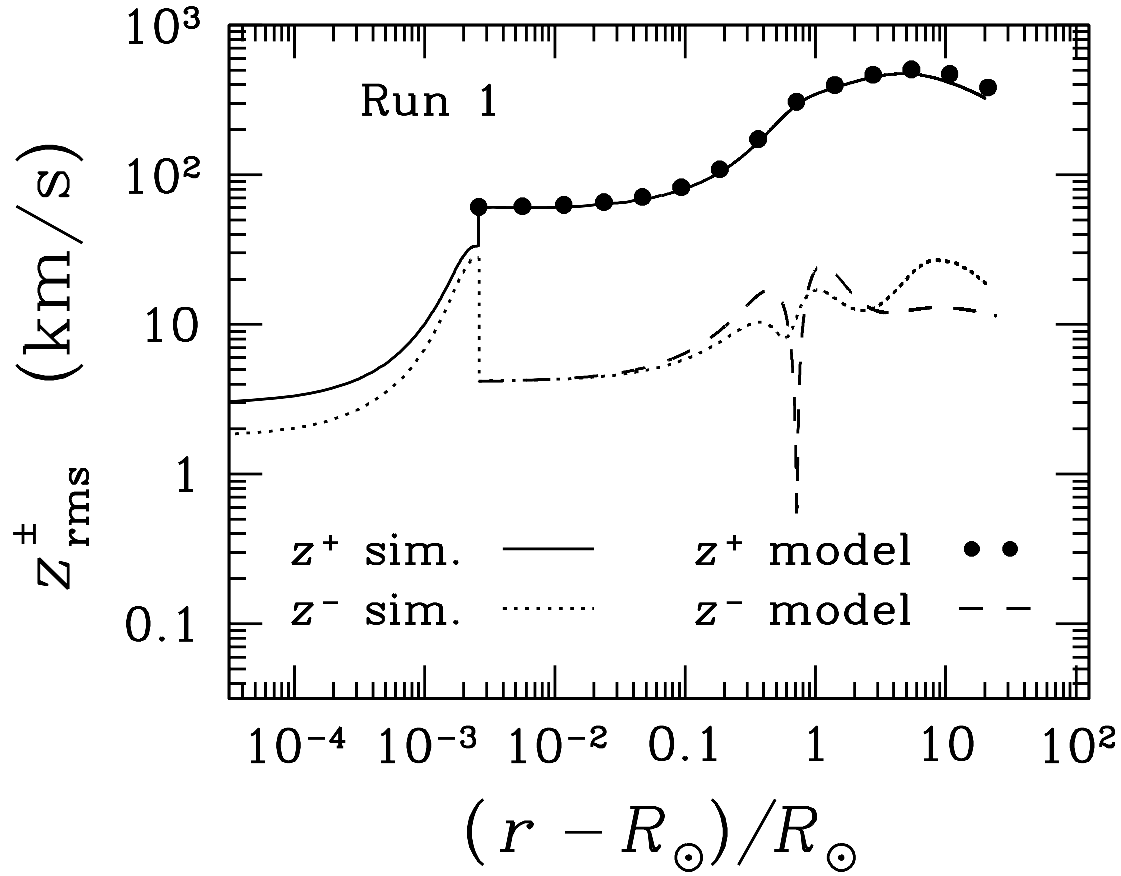
$$H = \frac{c_\theta c_{\rm O}^2 \Gamma_{\rm b}}{\sigma^2 A_0}\left(\frac{v_{\rm Am}}{v_{\rm Ab}}\right)^{(1-c_{\rm I})/2}(\sigma w^\sigma \ln w - w^\sigma + 1)$$

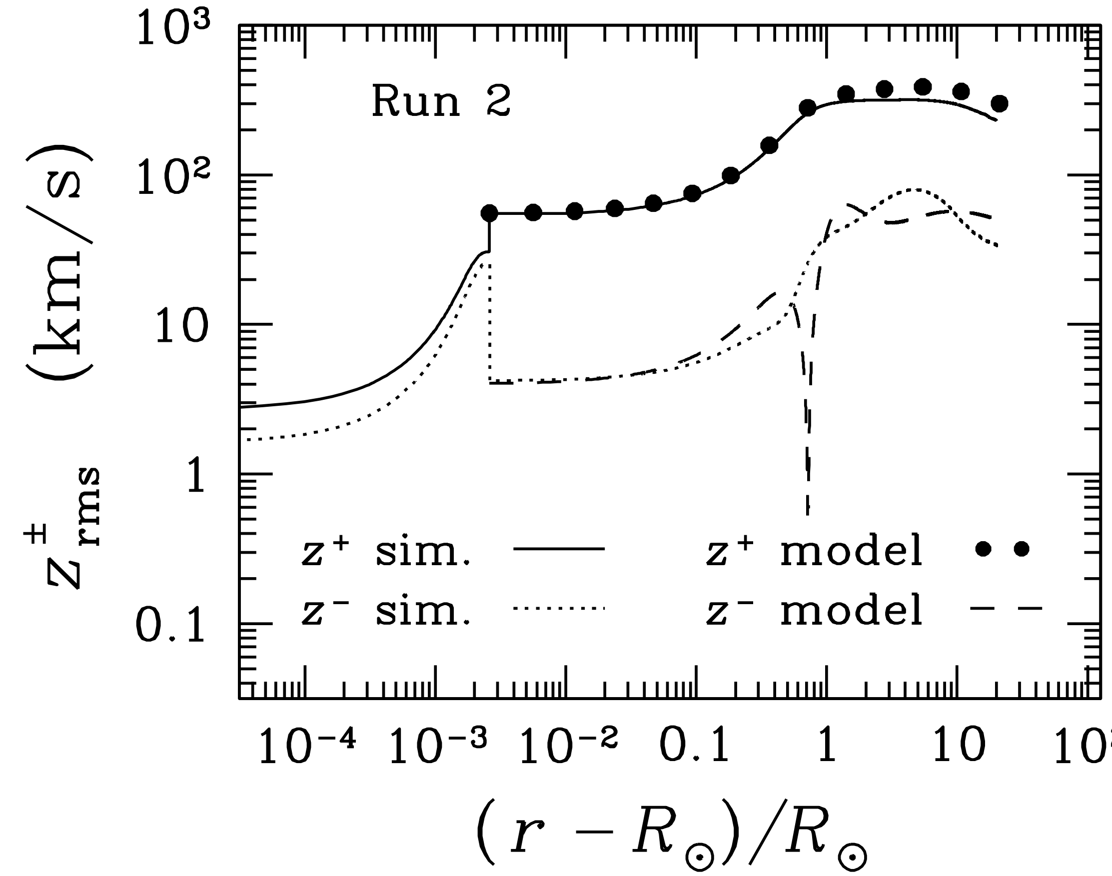
$$w = \frac{v_{\rm A}}{v_{\rm Am}} \qquad \sigma = \frac{1+c_{\rm O}}{2}$$

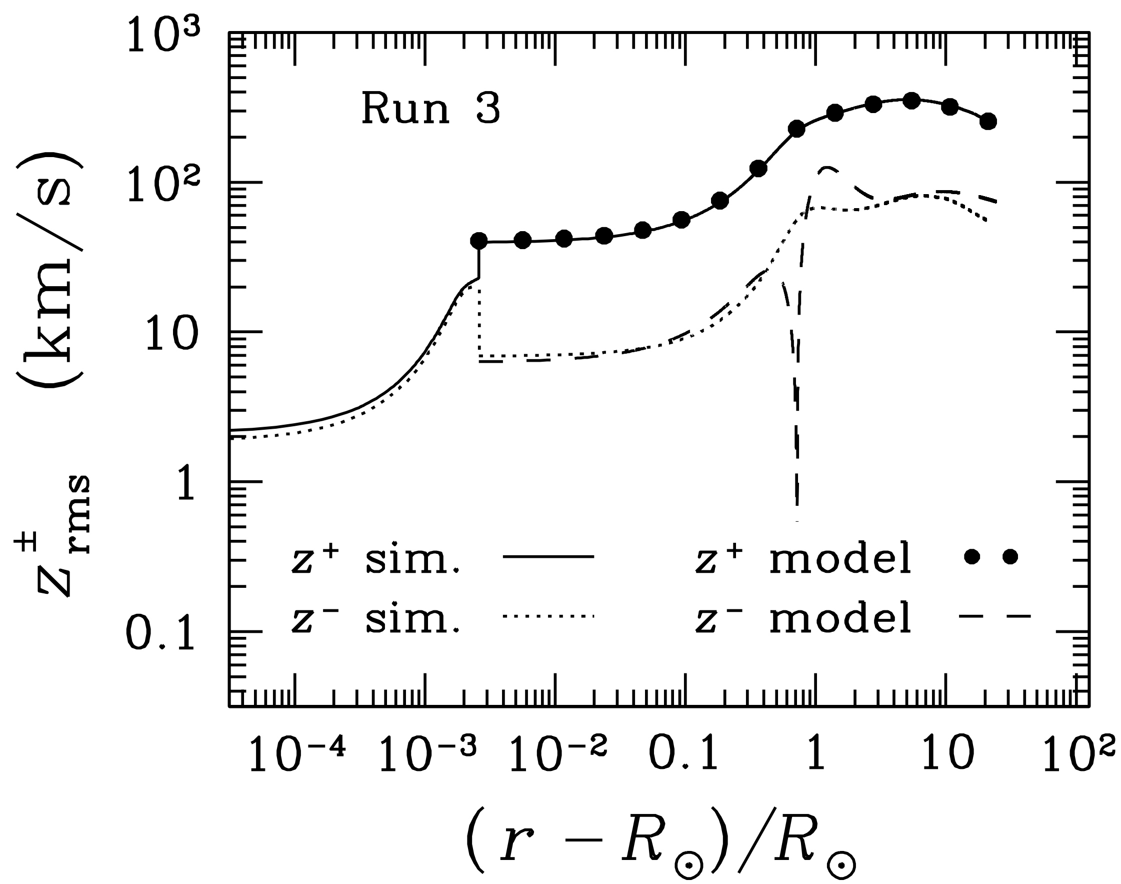
5 free parameters — same set is used for comparisons with all three runs.

Radial Profile of Fluctuation Amplitudes

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)



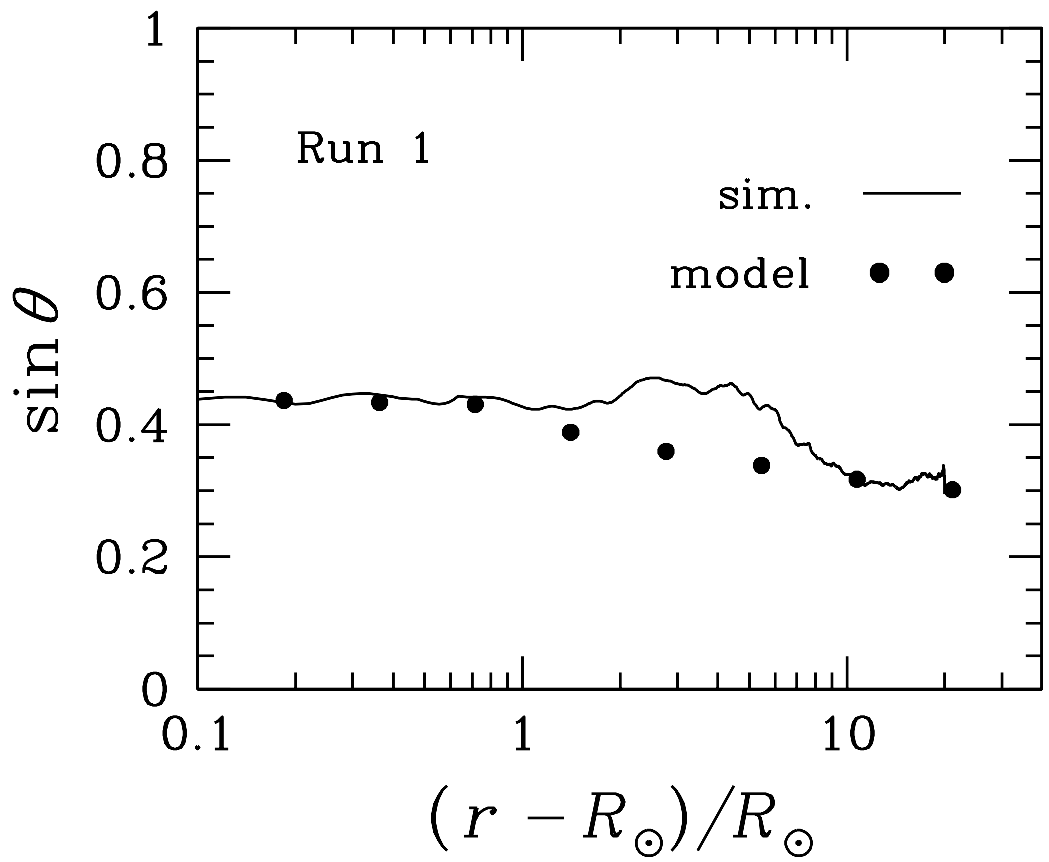


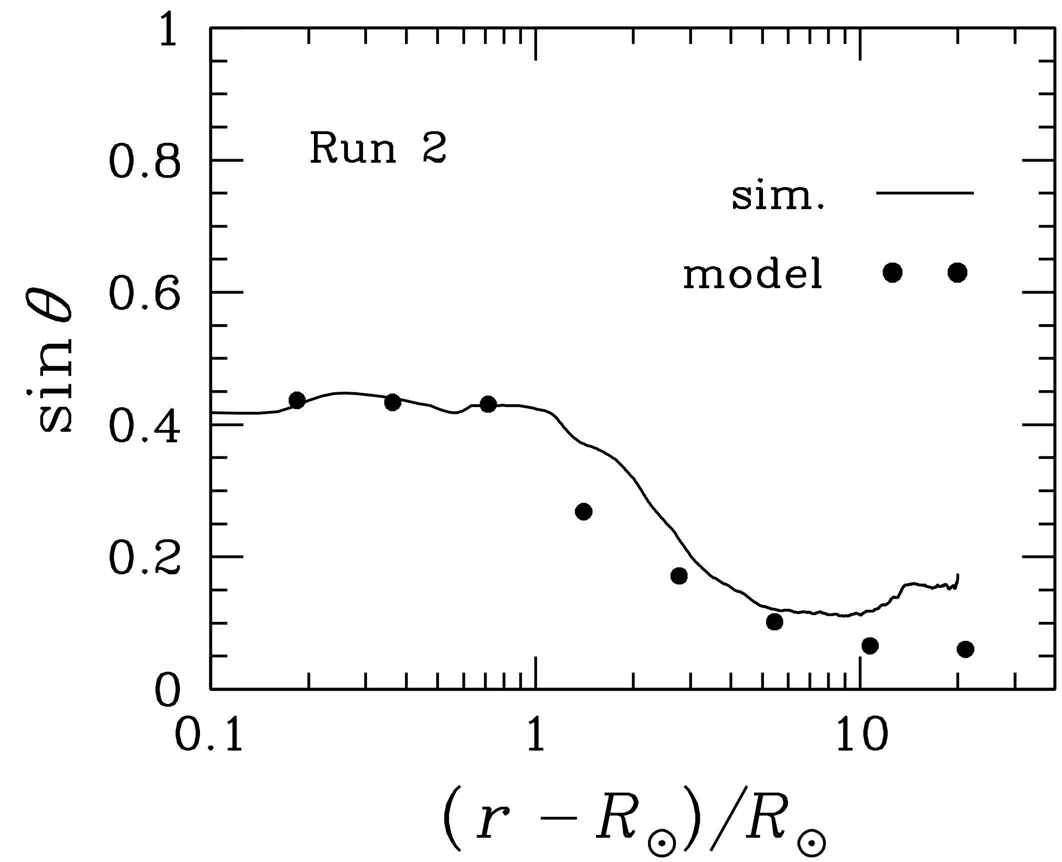


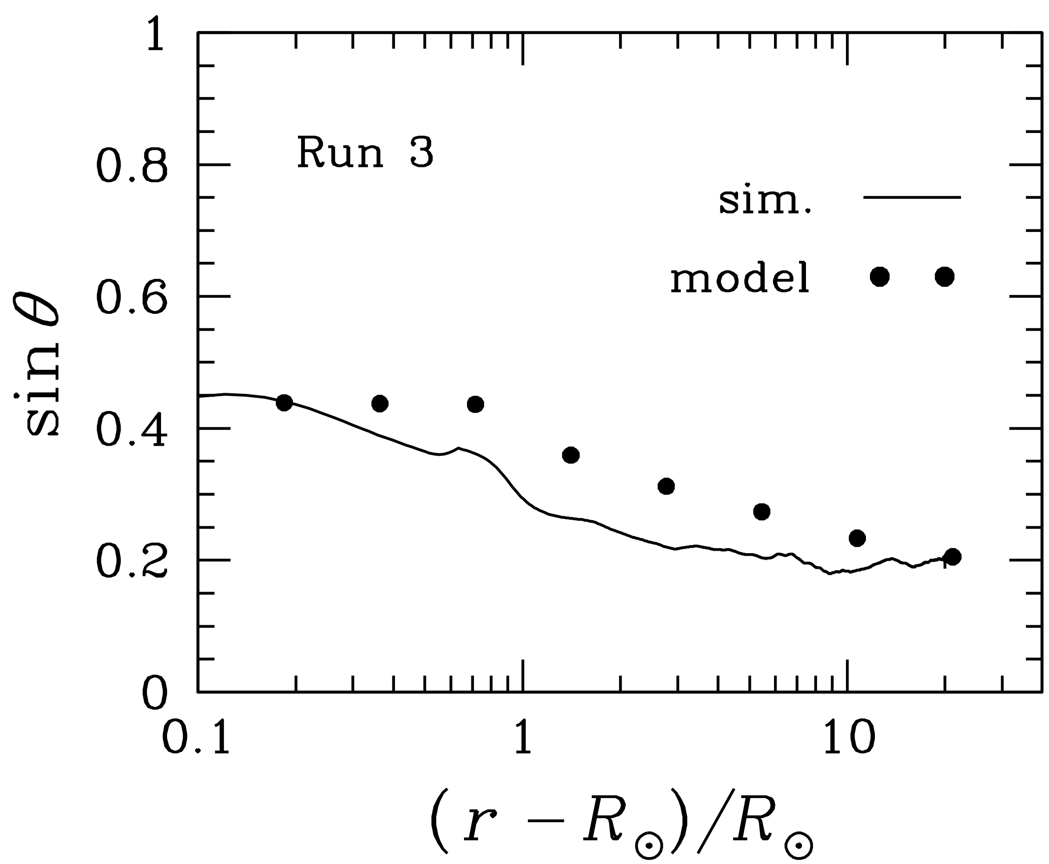
- Radial profiles of fluctuation amplitudes are similar to the profile in an analytic model (to be discussed below), and consistent with several constraints (photospheric motions, line widths in corona).

Alignment Angle Between z^+ And z^-

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)



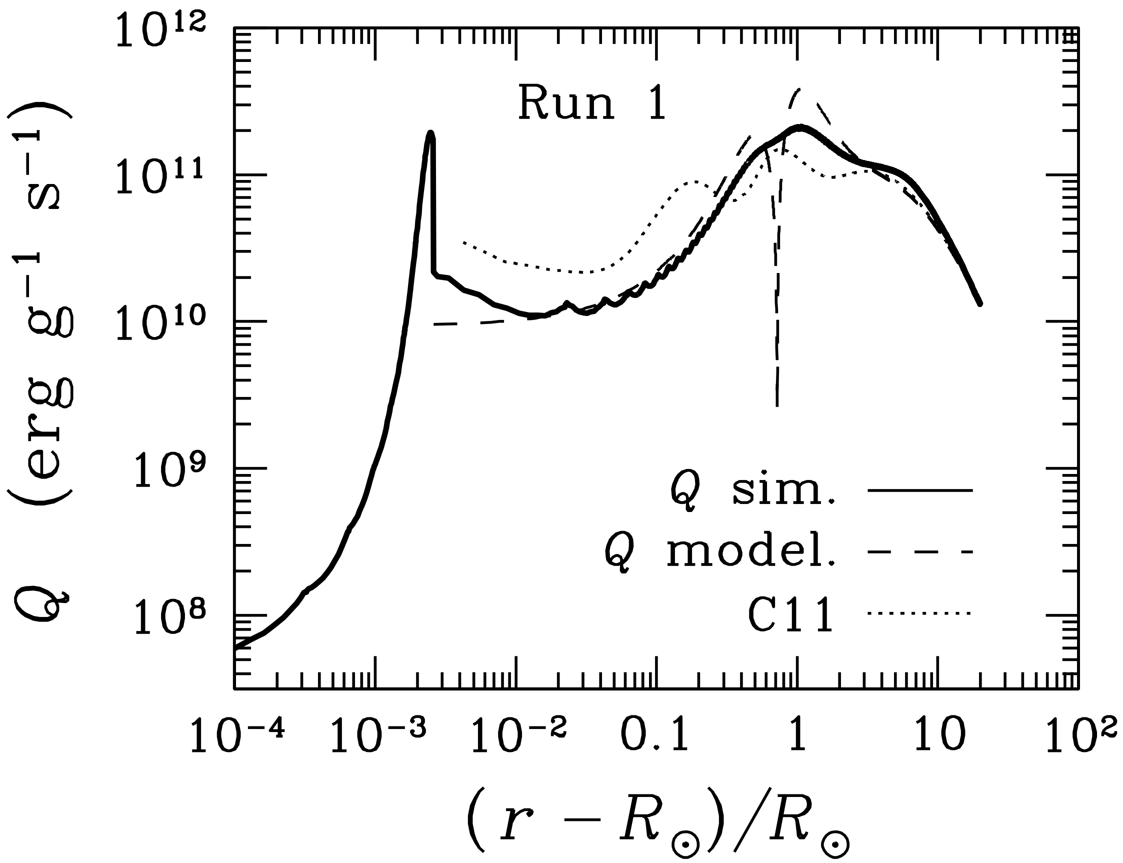


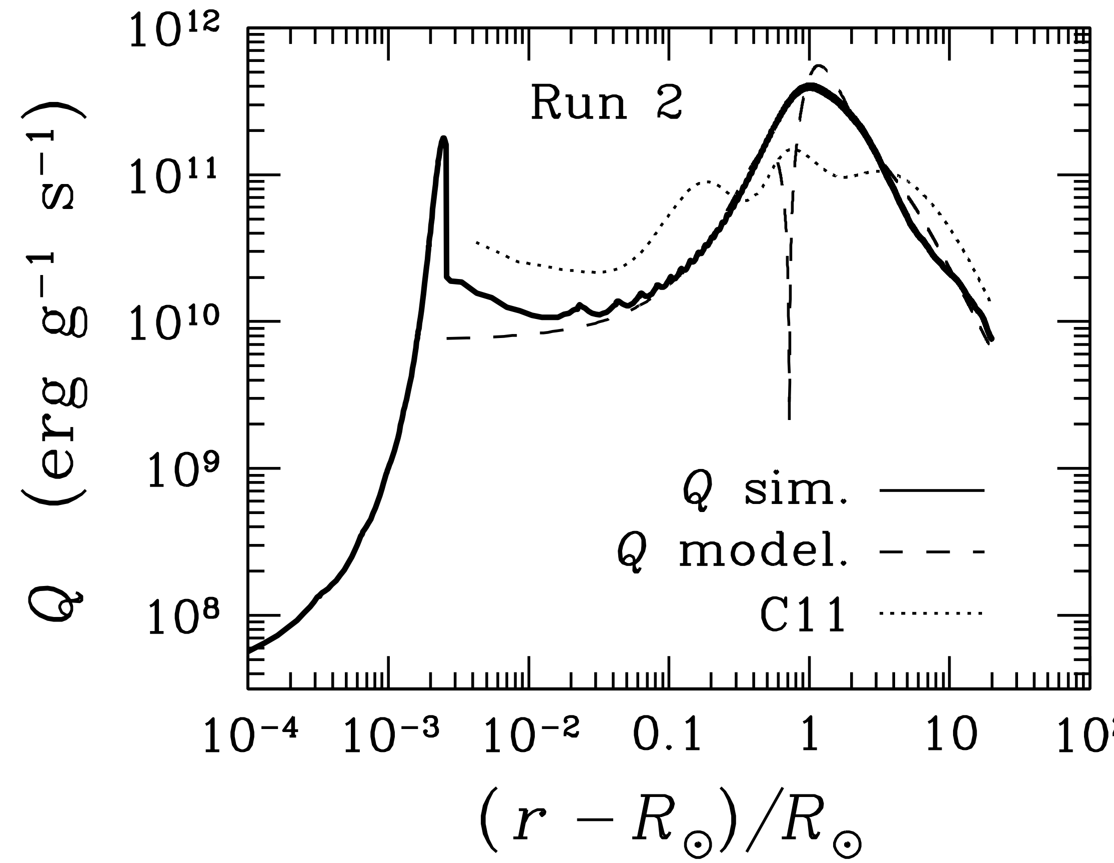


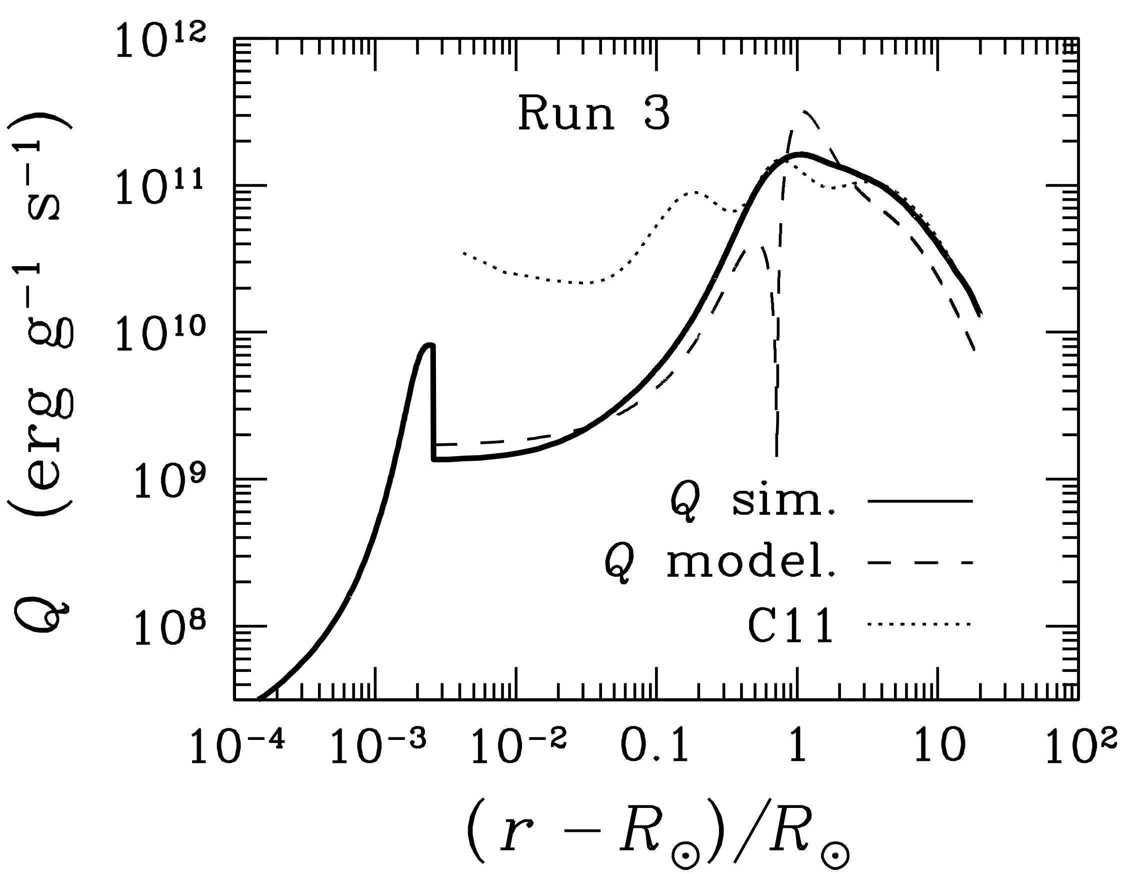
$$\sin\theta = \frac{\langle|\boldsymbol{z}^+ \times \boldsymbol{z}^-|\rangle}{\langle|\boldsymbol{z}^+|\rangle\langle|\boldsymbol{z}^-|\rangle}$$

Turbulent Heating Rate

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)

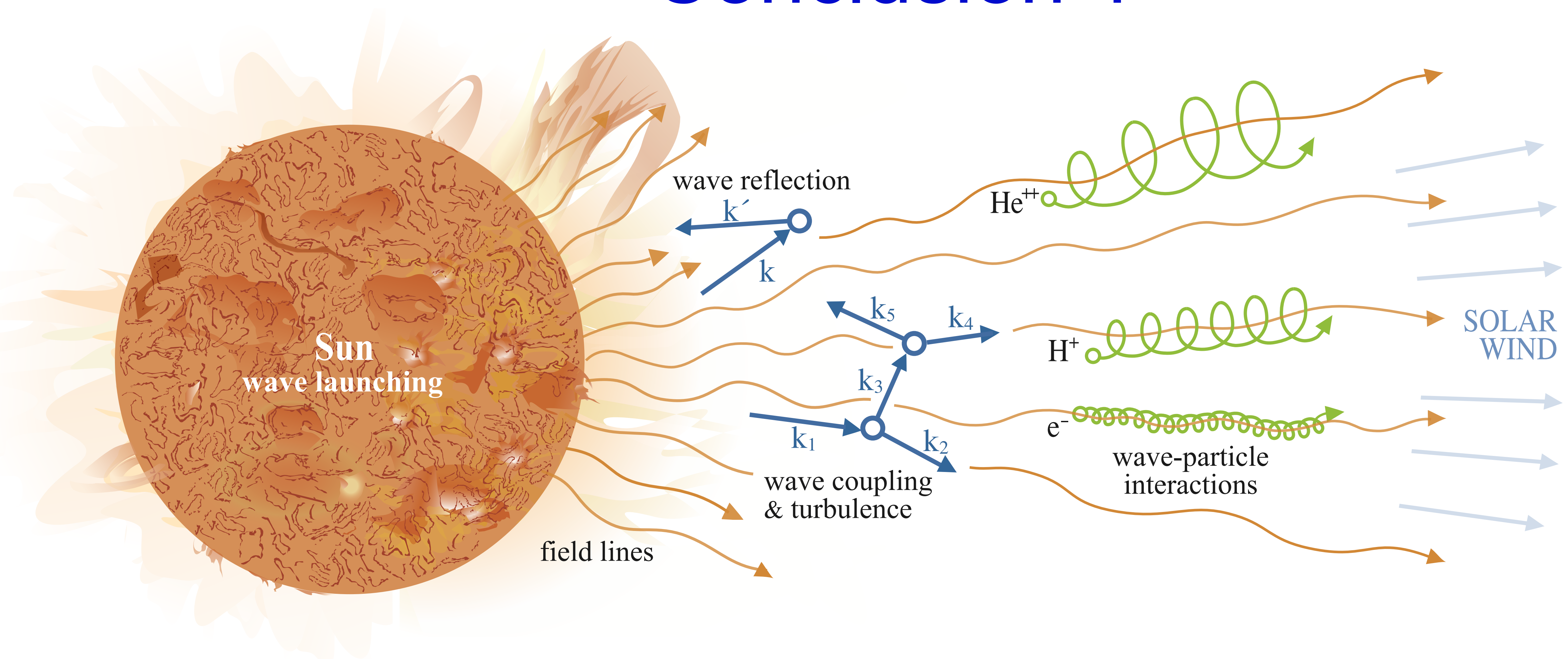






- The turbulent heating rate in the simulations is comparable to the heating rates in a fast-wind model (C11), and also the the heating rate in the analytic model (to be discussed momentarily).

Conclusion 1



1. Is the heating from reflection-driven MHD turbulence enough to generate the solar wind? YES

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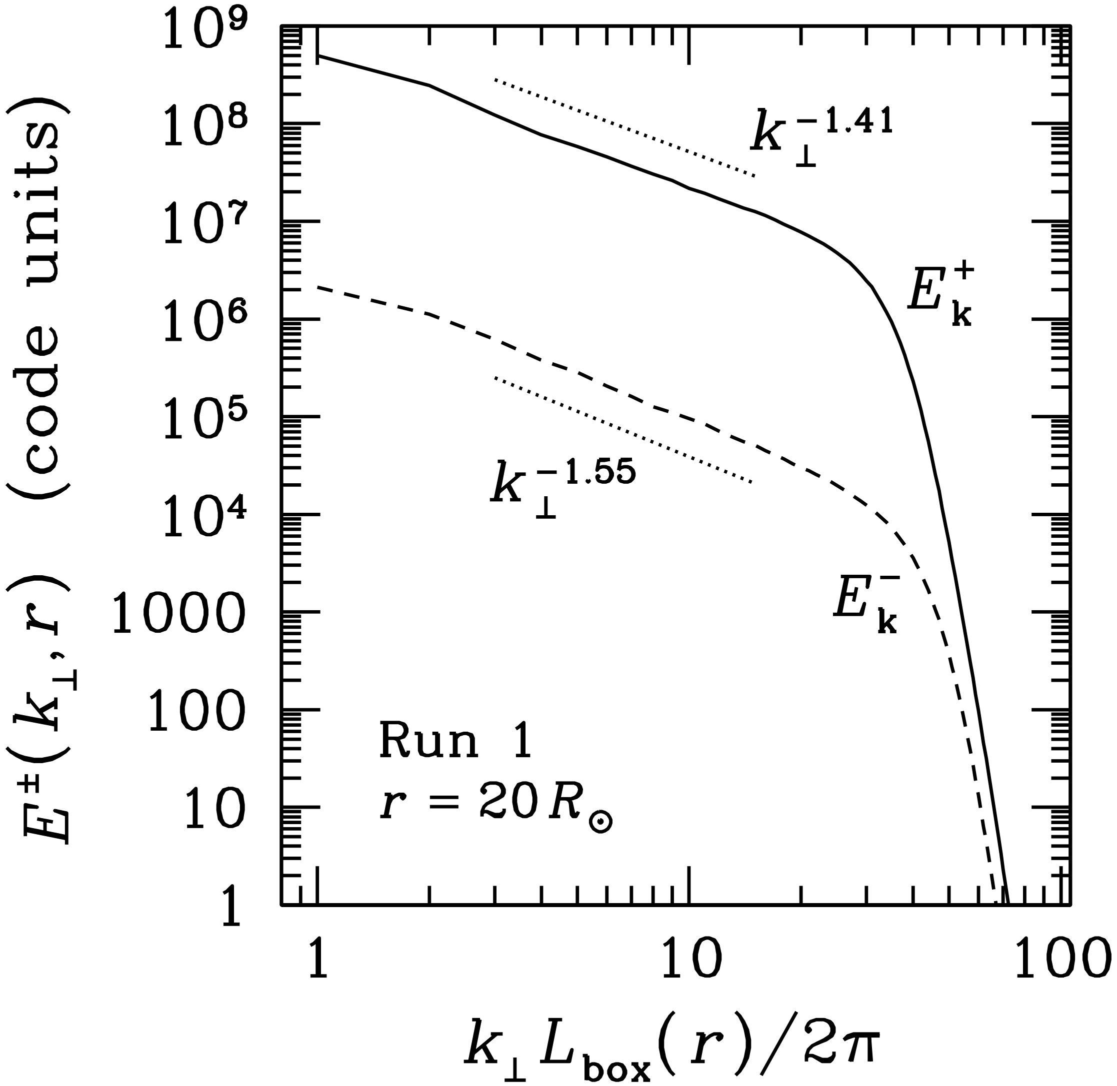
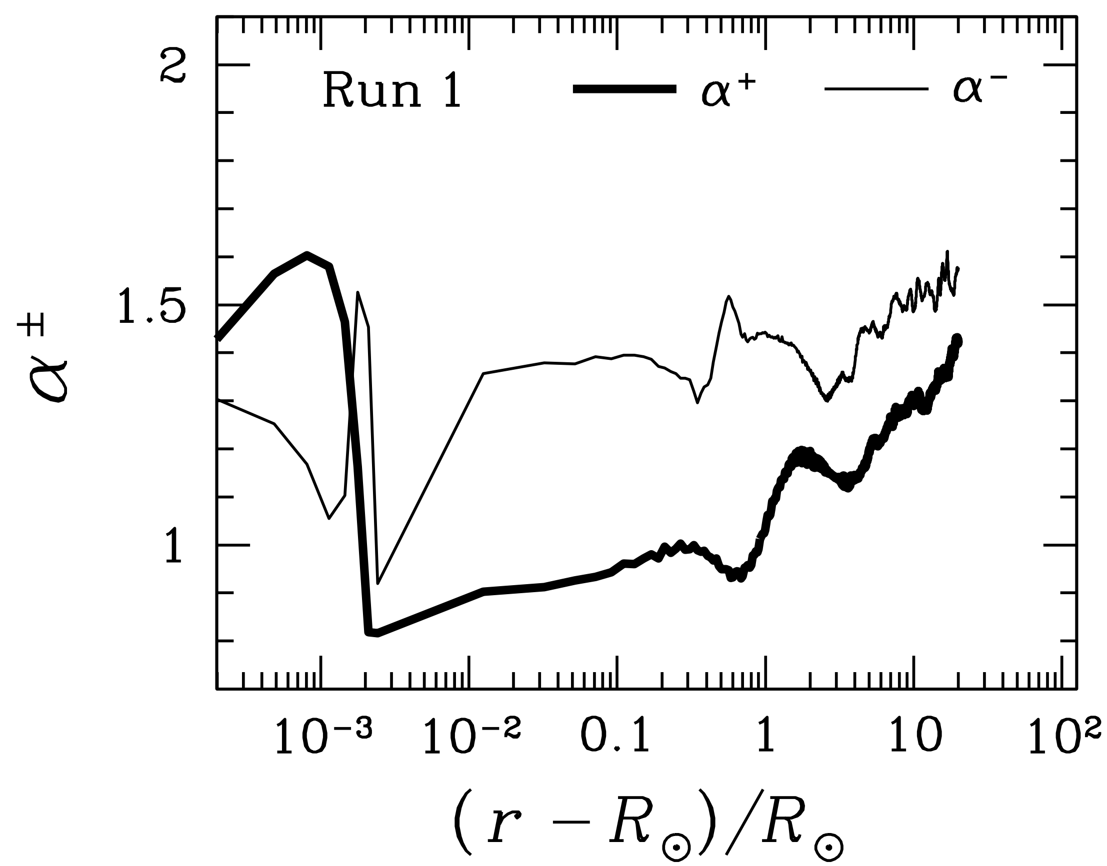
Power Spectra

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)





Elsasser power spectra: $E^{\pm}(k_\perp) \propto k_\perp^{-\alpha^{\pm}}$

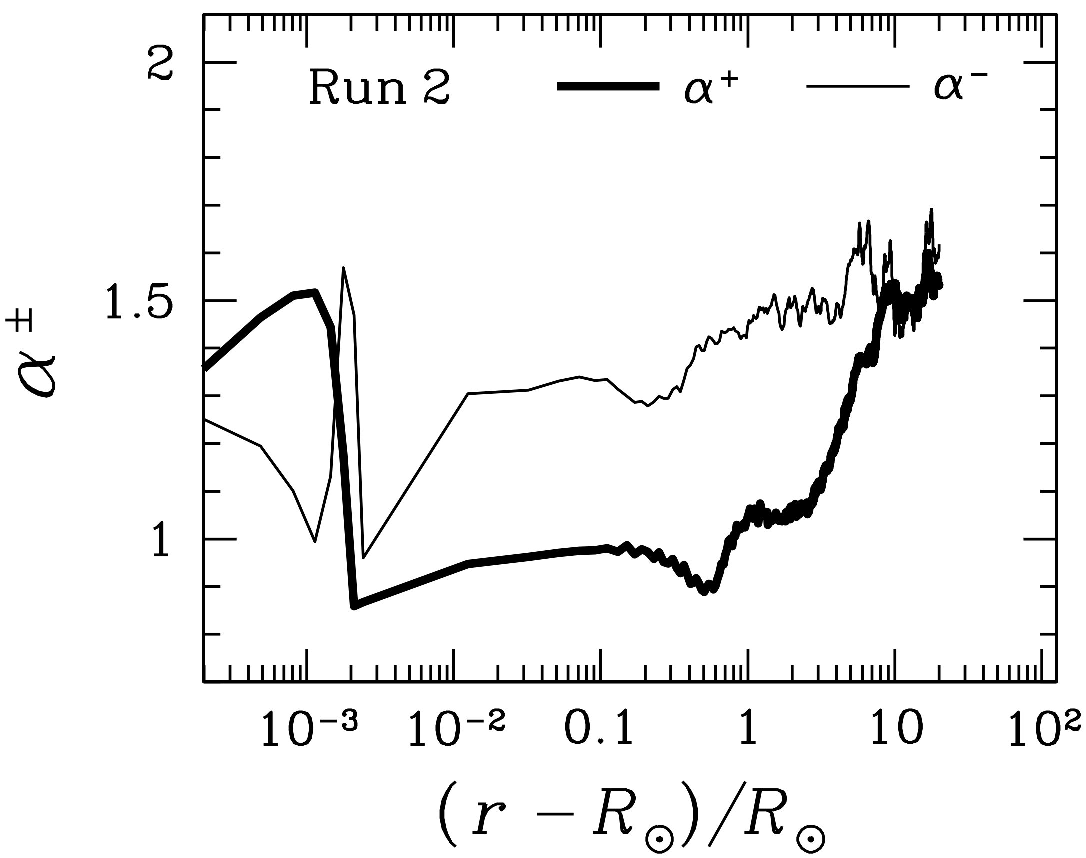
- Although we impose only a large-scale velocity at the photosphere, a broad power spectrum with spectral index of roughly -3/2 develops in the chromosphere. [First demonstrated by Van Ballegooijen et al (2011).]

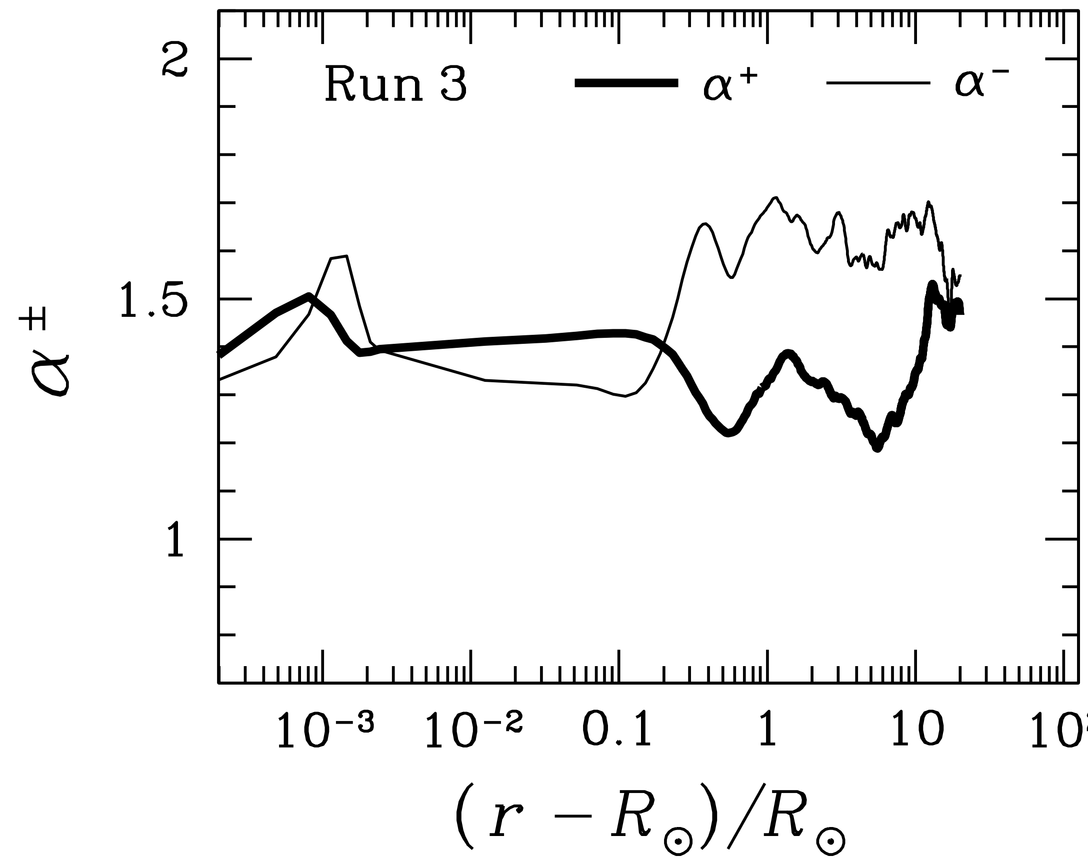
- **High-pass filtering in upper chromosphere.** (Velli 1993, Reville et al 2018)

- The z^+ spectrum flattens to ~ $k_\perp^{-3/2}$ at r exceeding ~ 10 solar radii.

Spectral Indices in Runs 2 and 3

(Chandran & Perez, J. Plasm. Phys., 85, 905850409, 2019)





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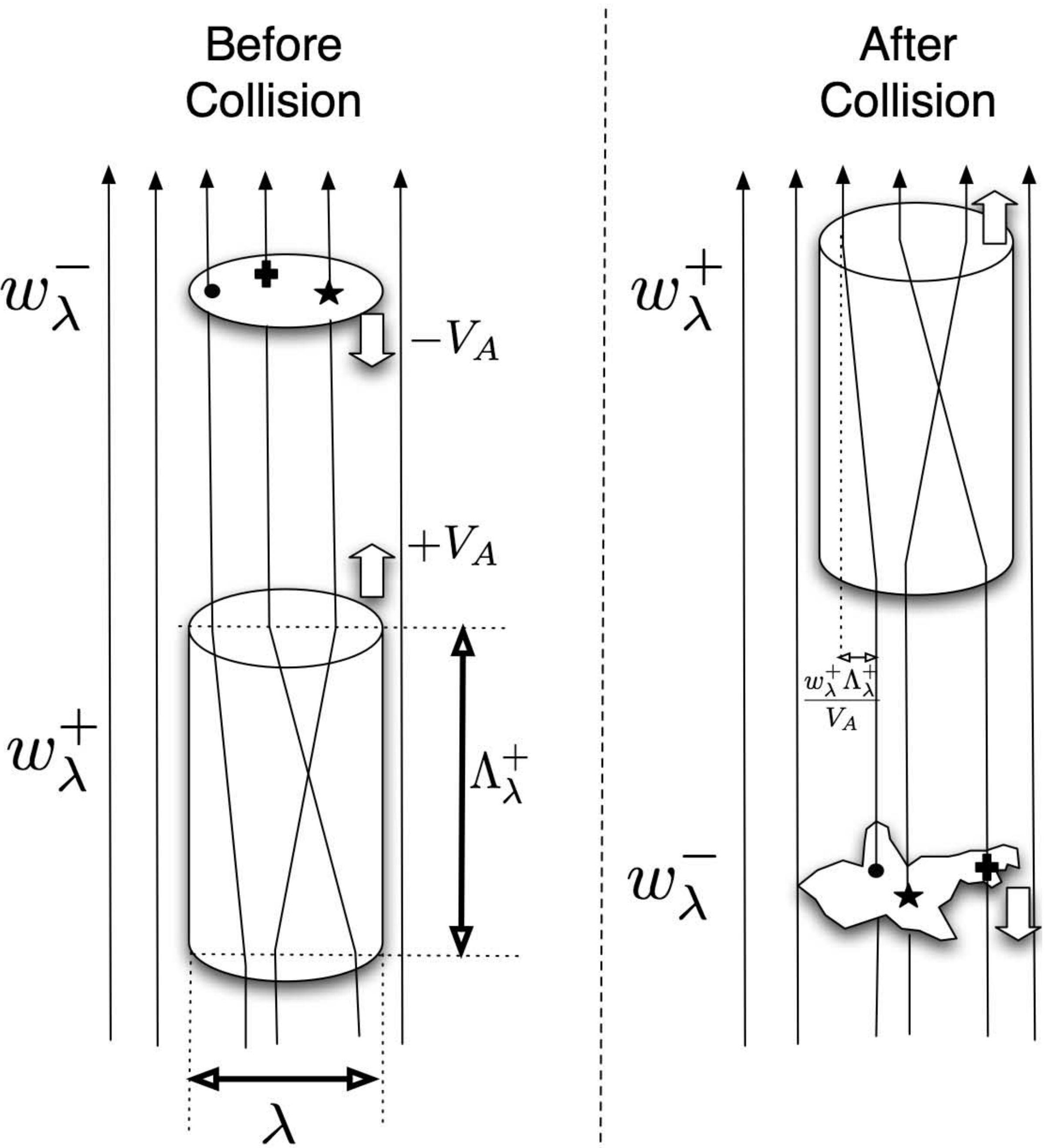
Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



- their $\boldsymbol{w}^{\pm}$ is my $\boldsymbol{z}^{\pm}$. $w_\lambda^+ \gg w_\lambda^-$.
- duration of collision: $\Lambda_\lambda^+ / v_{\mathrm{A}}$

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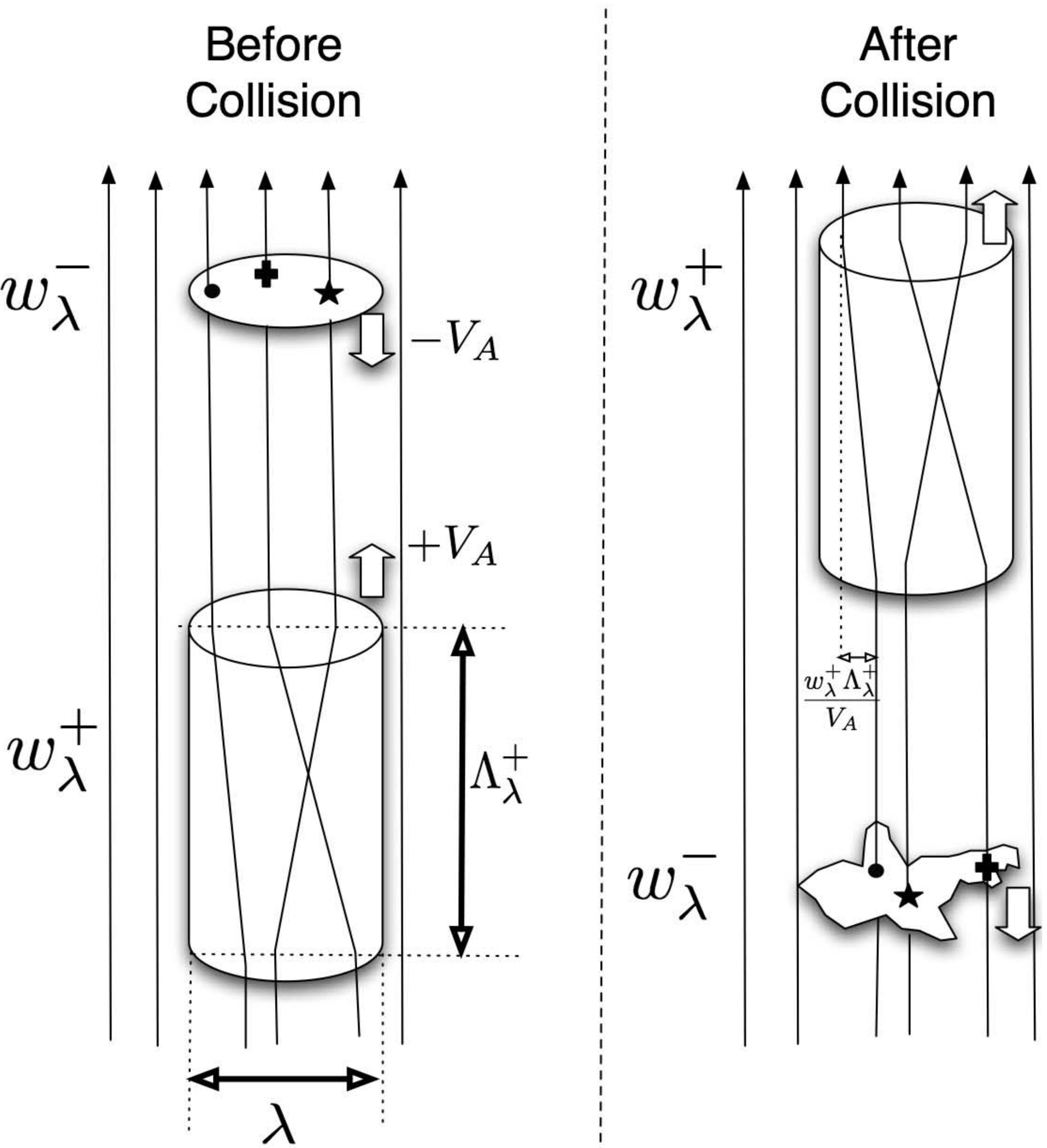

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$$\chi_\lambda^+ \sim \frac{w_\lambda^+\Lambda_\lambda^+}{\lambda v_A}$$

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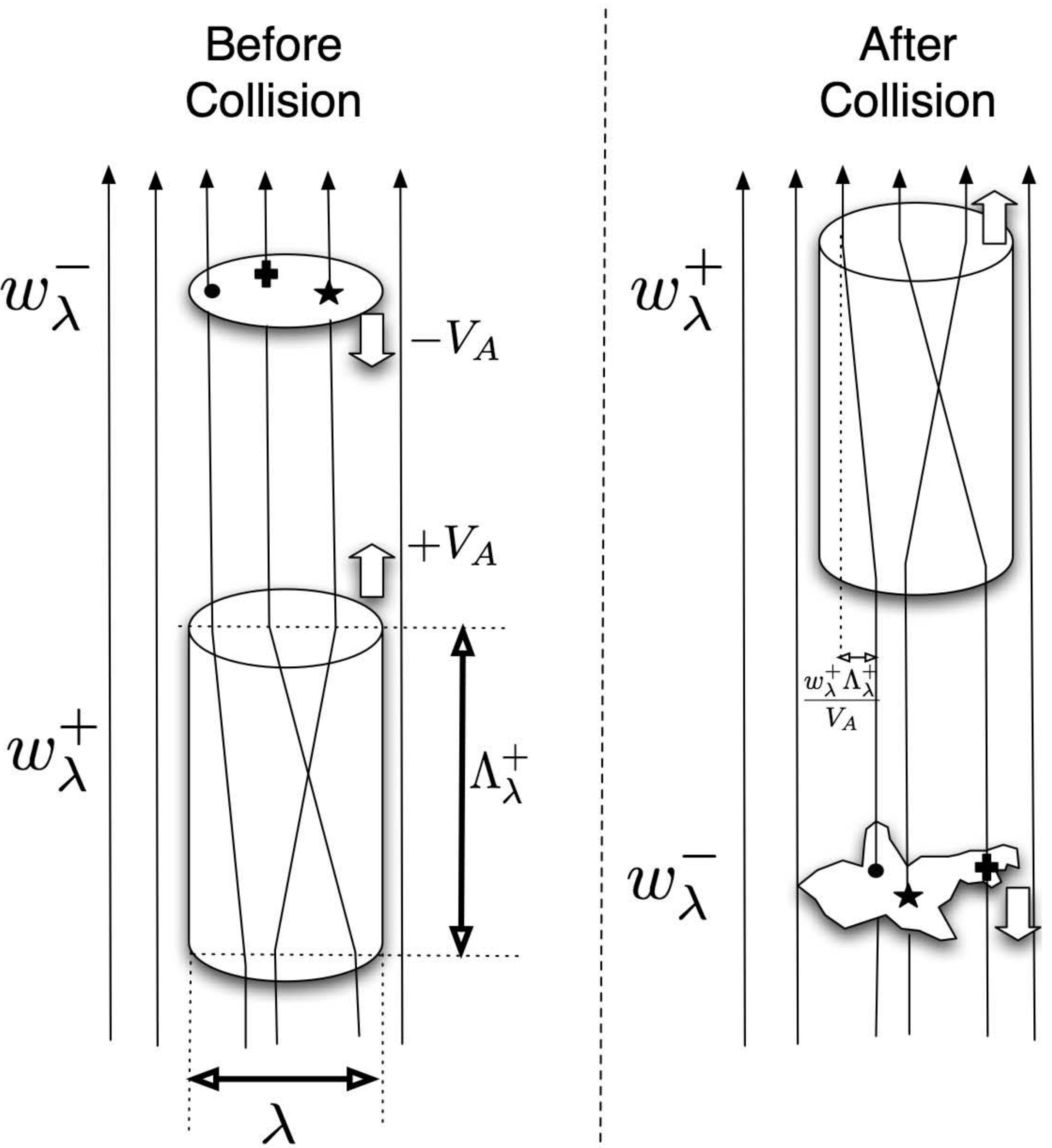

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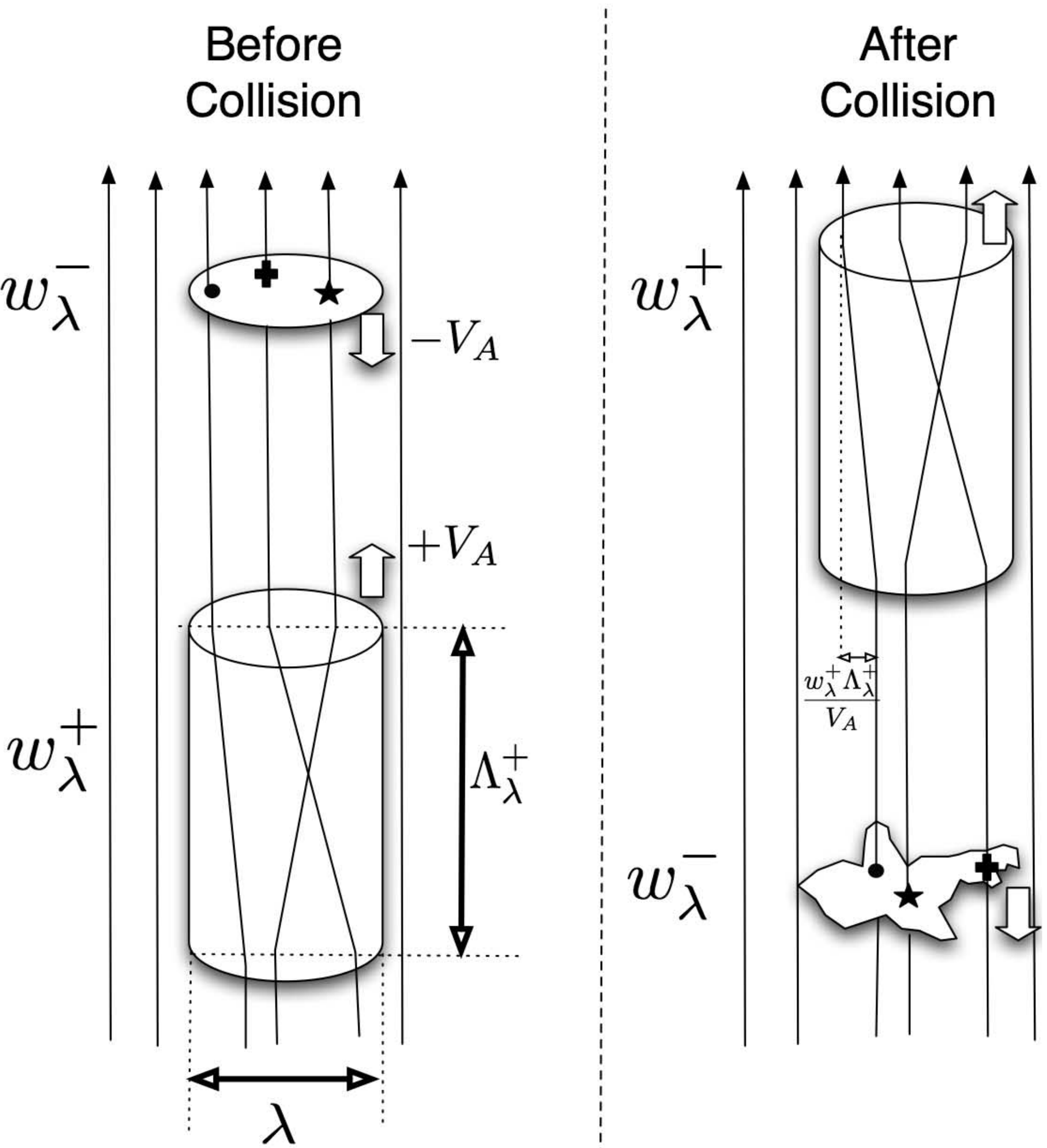

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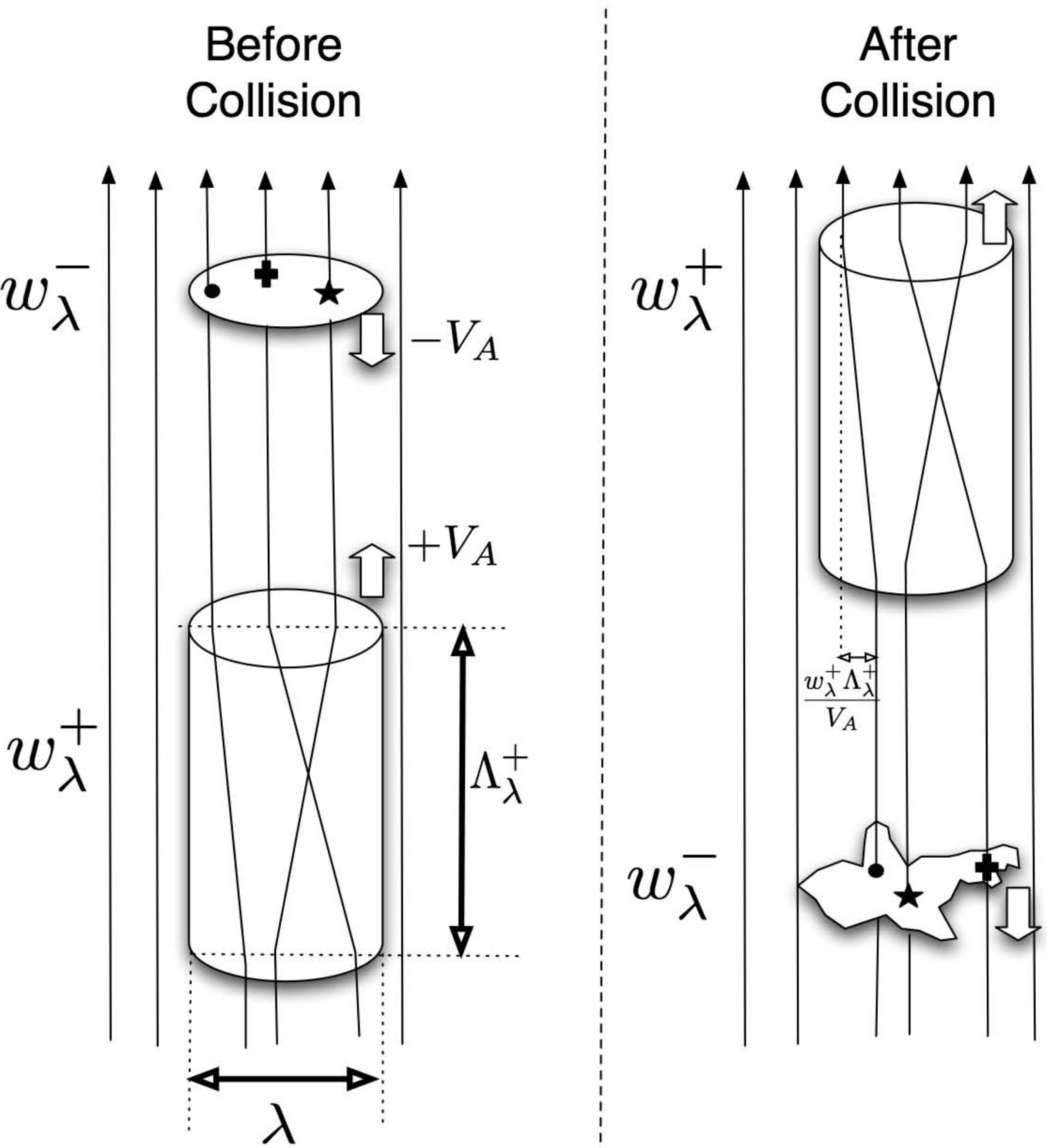

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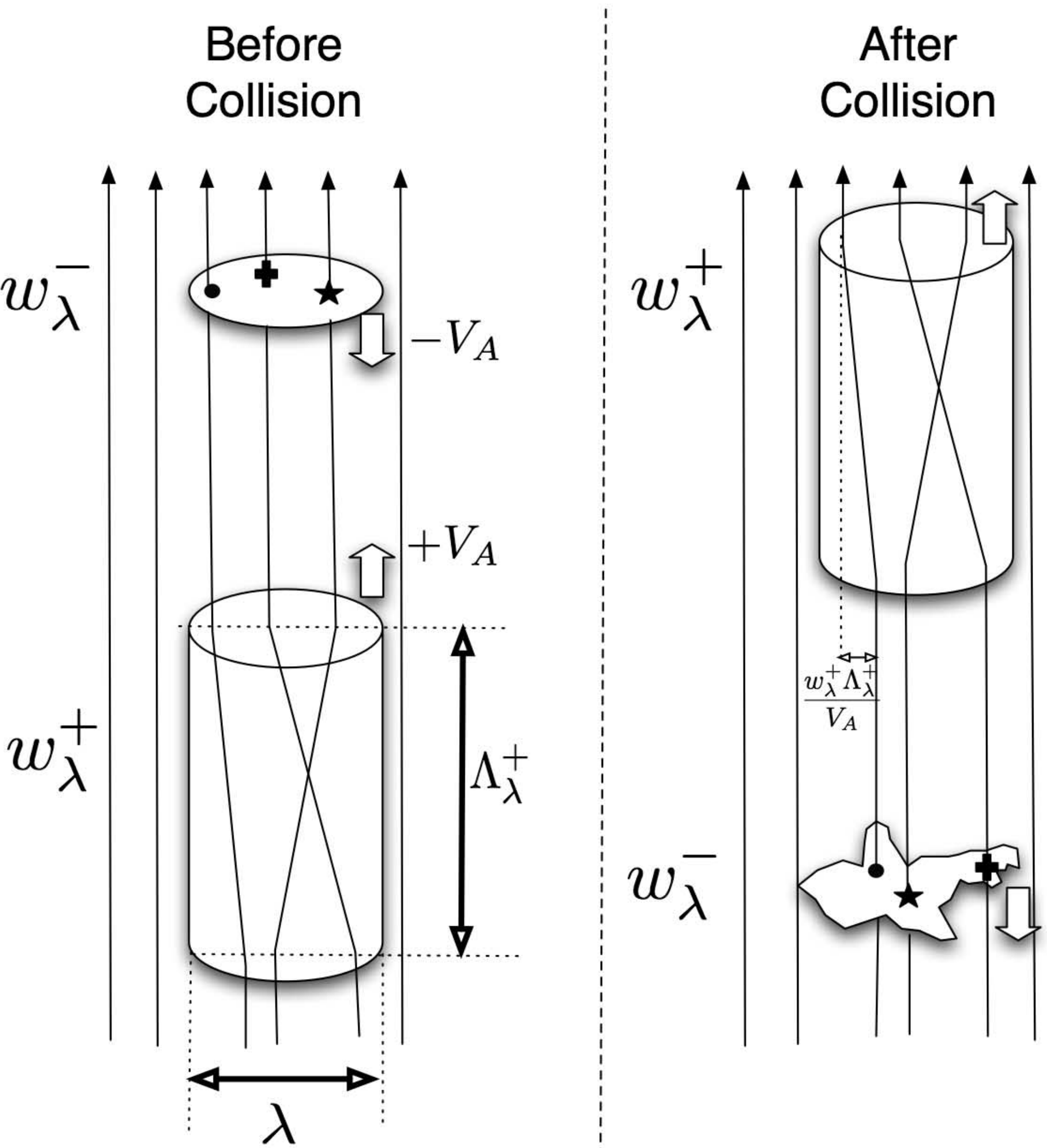
- w_λ^- parallel correlation length: $\Lambda_\lambda^- \sim v_A \lambda/w_\lambda^+$.

Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



- their $\boldsymbol{w}^\pm$ is my $\boldsymbol{z}^\pm$. $w_\lambda^+ \gg w_\lambda^-$.
- duration of collision: $\Lambda_\lambda^+/v_{\rm A}$
- fractional change of w_λ^- "slice" during collision:

$$\chi_\lambda^+ \sim \frac{w_\lambda^+ \Lambda_\lambda^+}{\lambda v_{\rm A}}$$

- strong turbulence *assumption*: $\chi_\lambda^+ \gtrsim 1$
- w_λ^- cascade time scale: λ/w_λ^+
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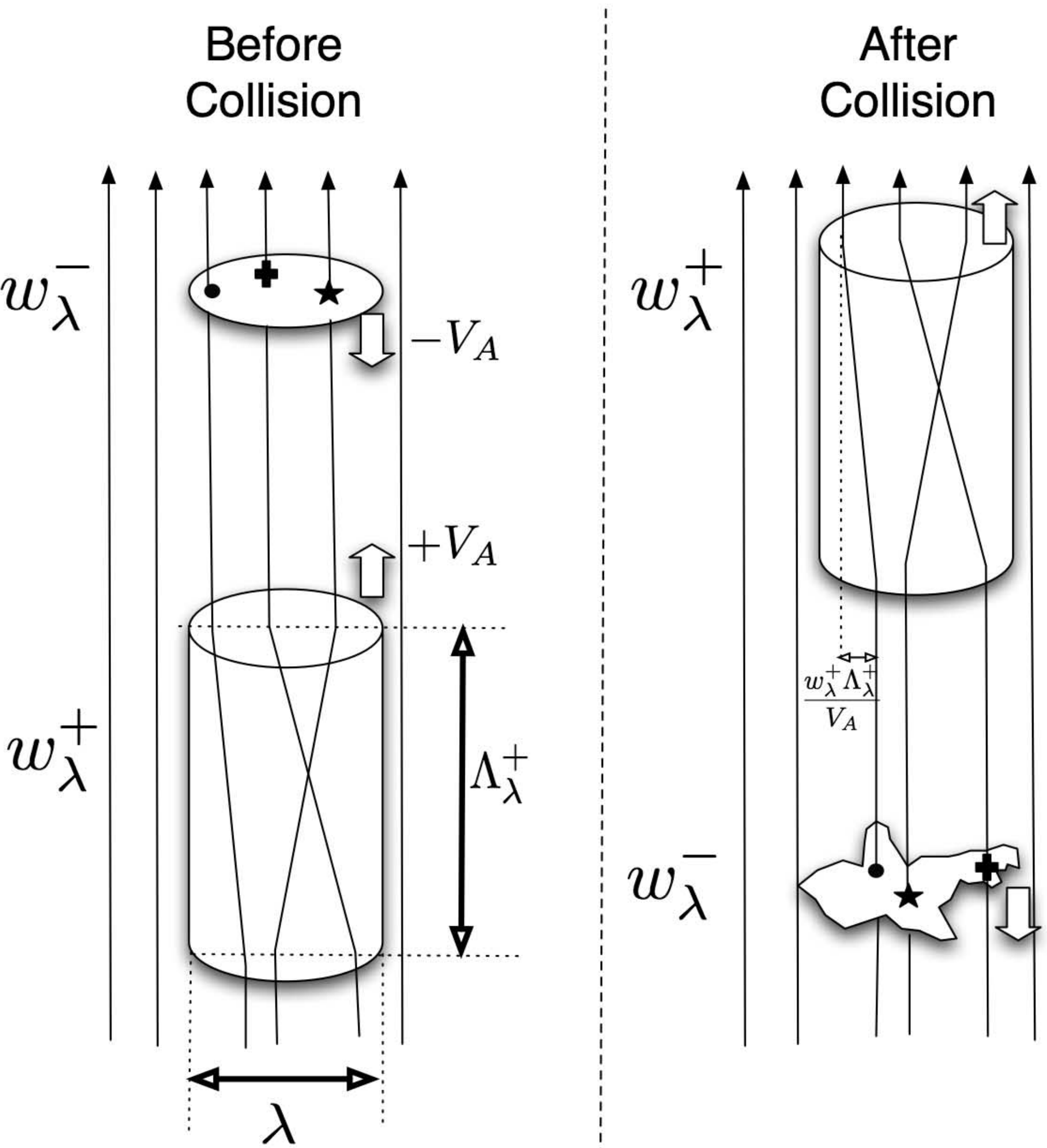

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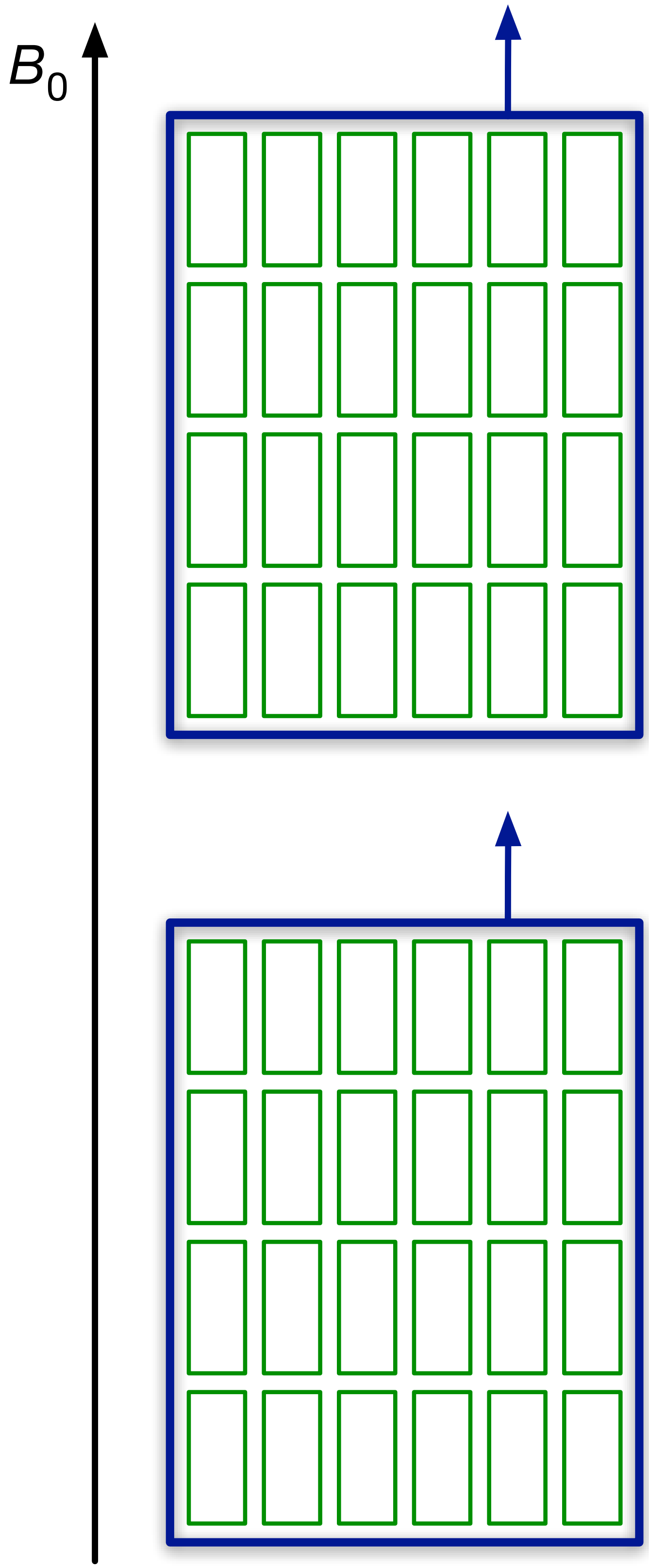
- $\longrightarrow \quad \chi_\lambda^+ \sim \dfrac{w_\lambda^+ \Lambda_\lambda^+}{\lambda v_\mathrm{A}} \sim \dfrac{w_\lambda^+}{\lambda v_\mathrm{A}} \times \dfrac{v_\mathrm{A}\lambda}{w_\lambda^+} \sim 1$

Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



- suppose you start with a full inertial range of nested, w_λ^+ eddies.
- suppose w_λ^- is infinitesimal
- then $\boldsymbol{w}^+$ is static in the 'w^+ frame'.

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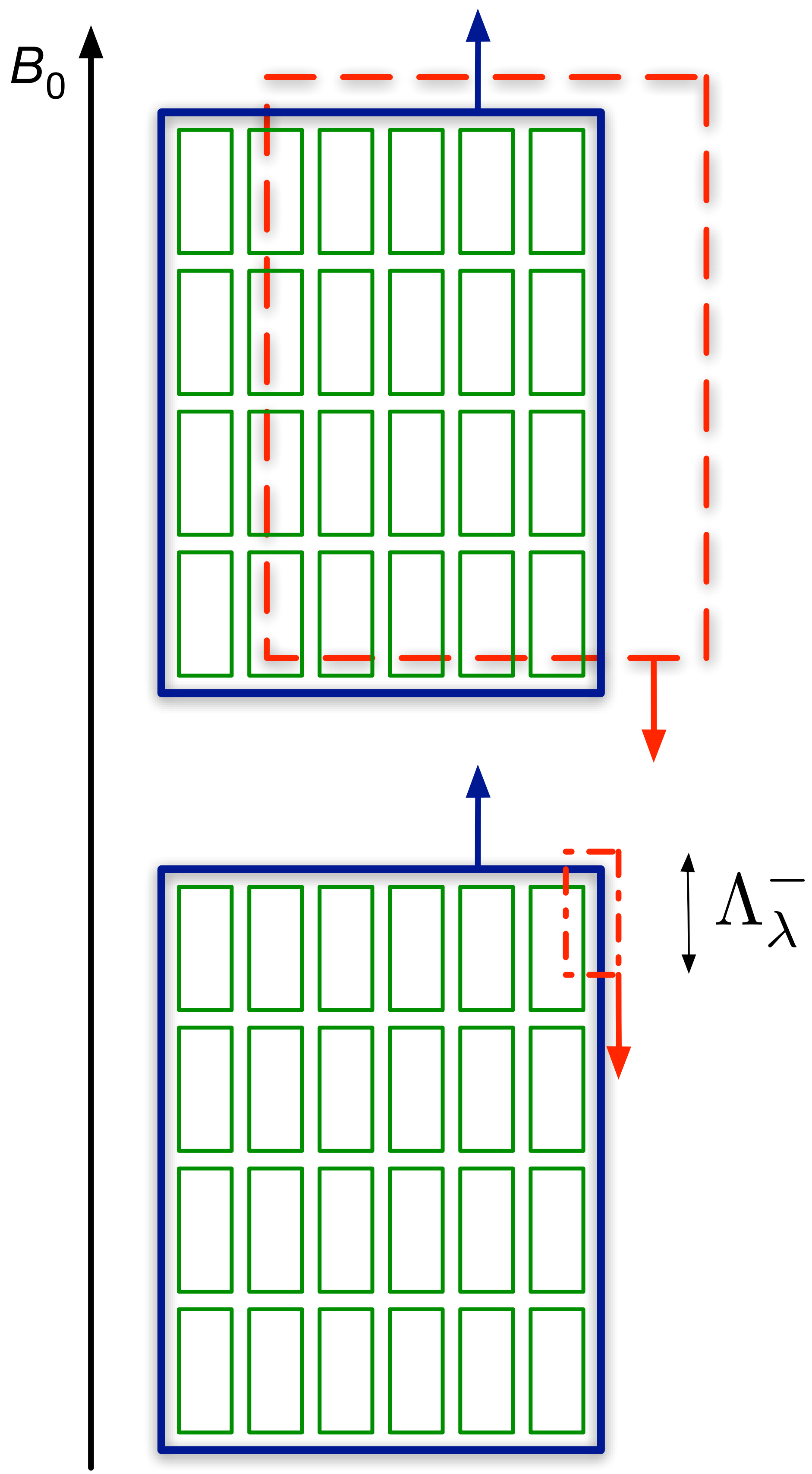
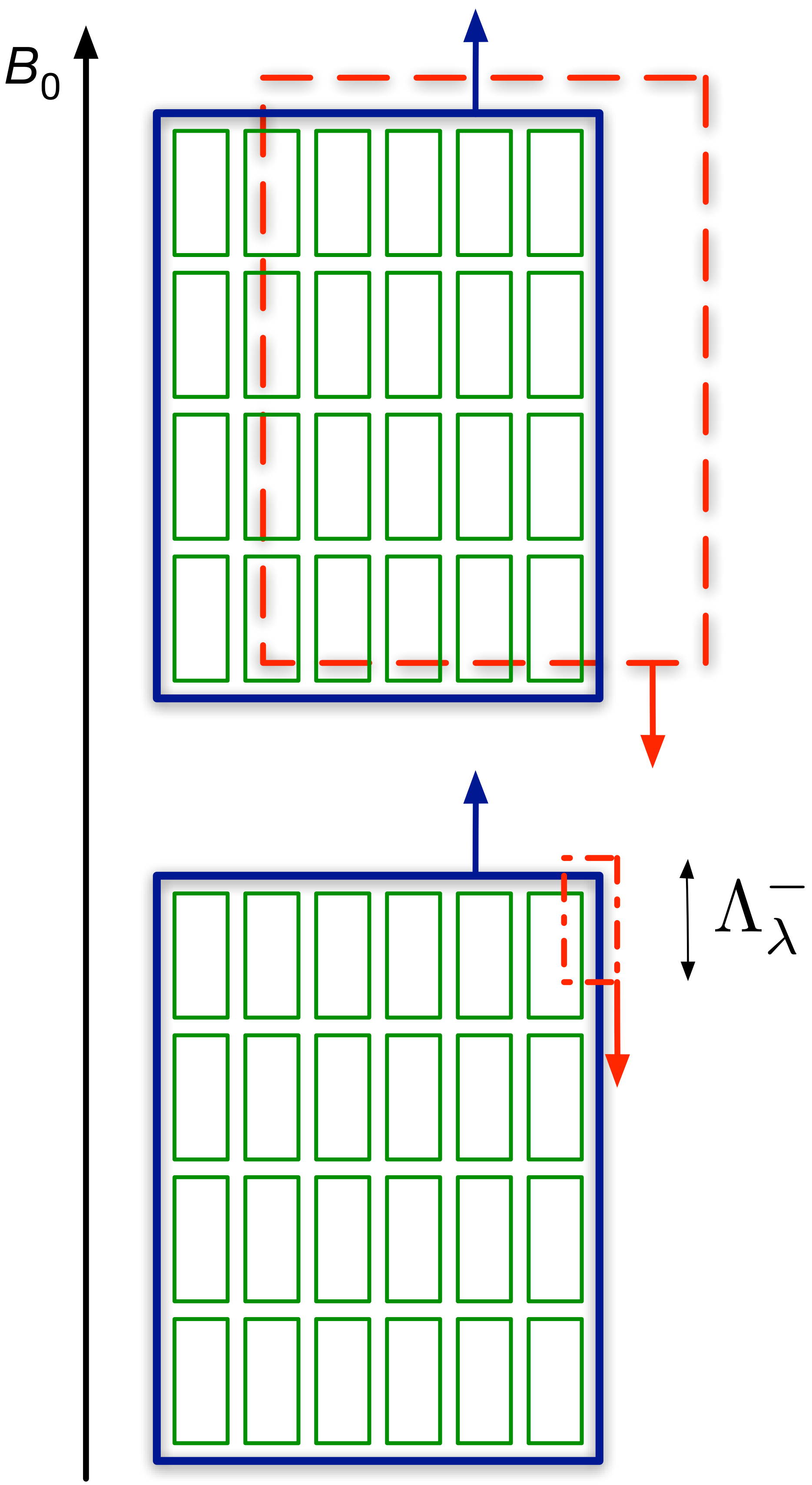


- "Coherence assumption": suppose $\boldsymbol{w}^-$ in injected at the outer scale with a coherence time *in the* w^+ *frame* that is at least as long as the outer-scale w_λ^+ cascade time scale.
- As w_λ^- is reduced towards 0, what is the coherence time $t_{\mathrm{coh},\lambda}^+$ of the shearing experienced by w_λ^+?
- $\Lambda_\lambda^-/v_{\mathrm{A}}$?

Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



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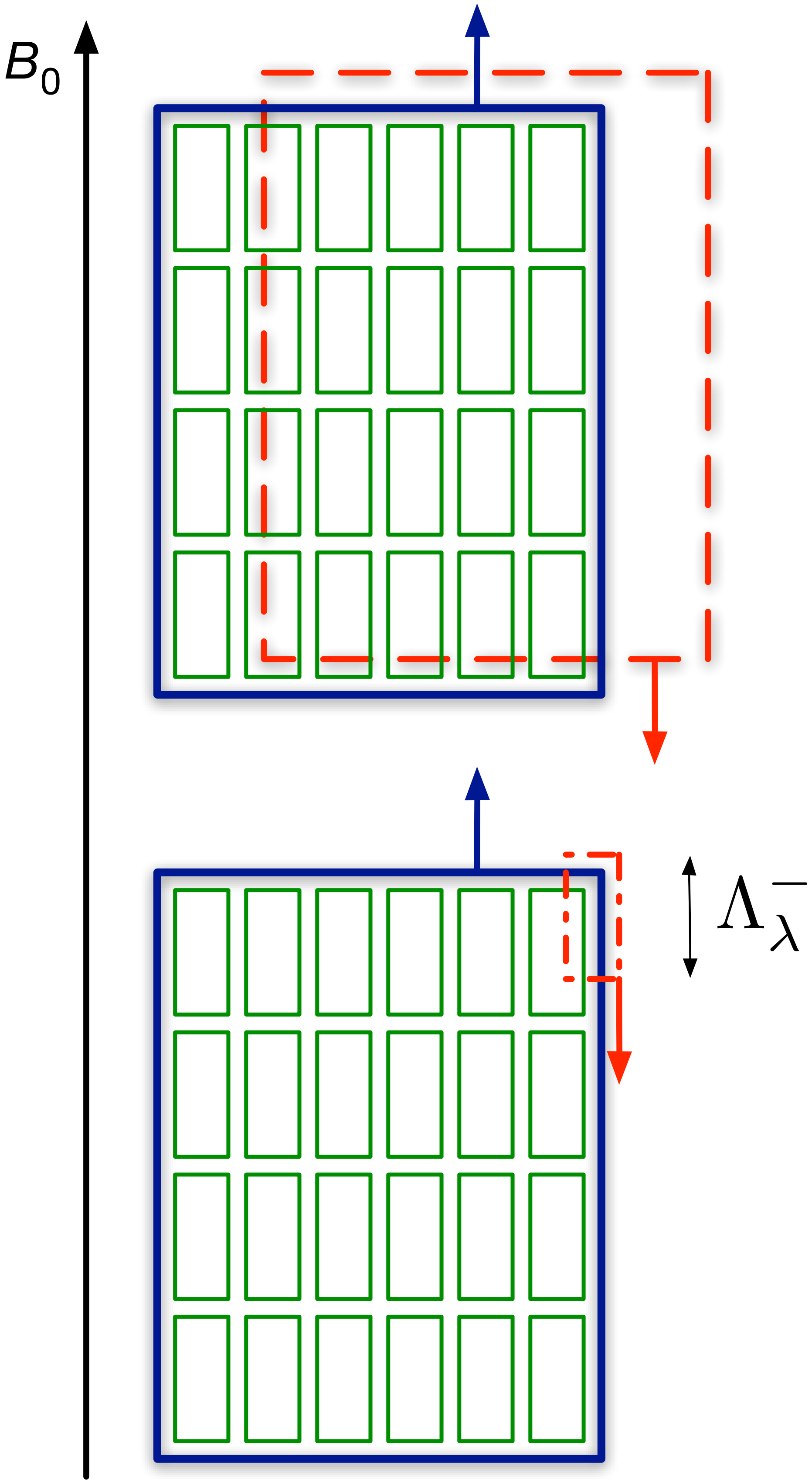

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- As w_λ^- is increased, $t_{\mathrm{coh},\lambda}^+$ is approximately the cascade time scale of w_λ^+.

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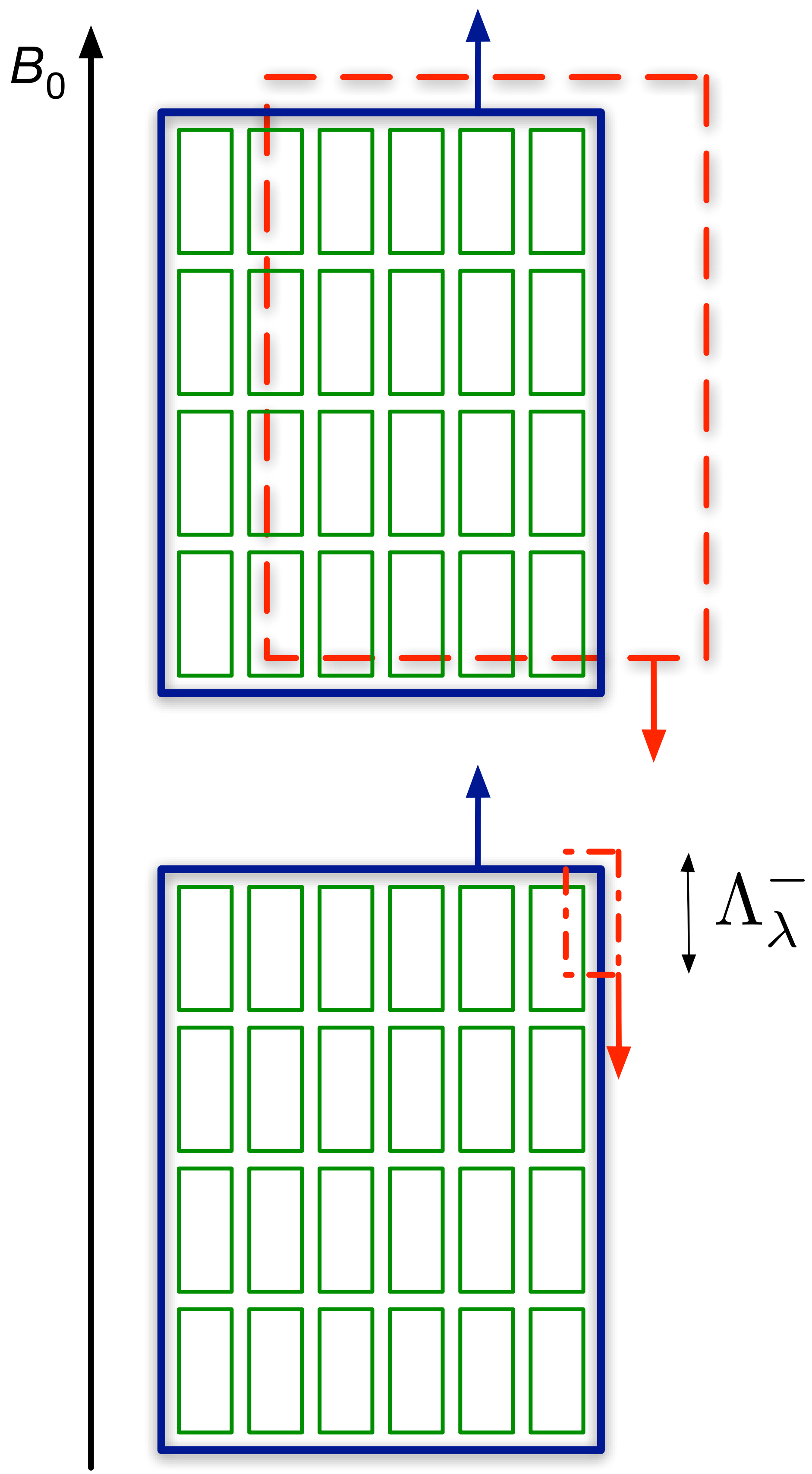

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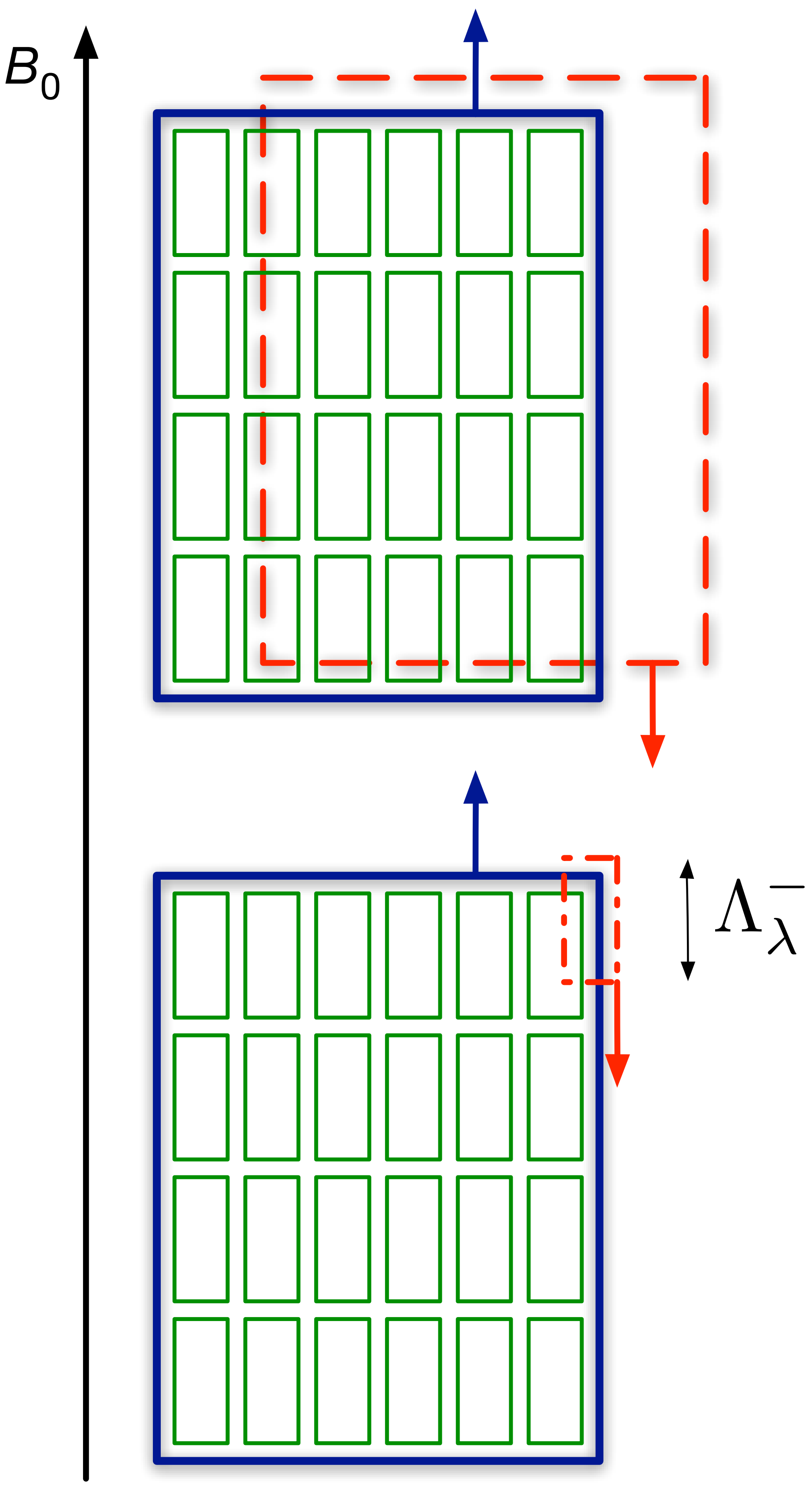
- $\epsilon^+ \sim \dfrac{(w_\lambda^+)^2 w_\lambda^-}{\lambda} \qquad \epsilon^- \sim \dfrac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}$

Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



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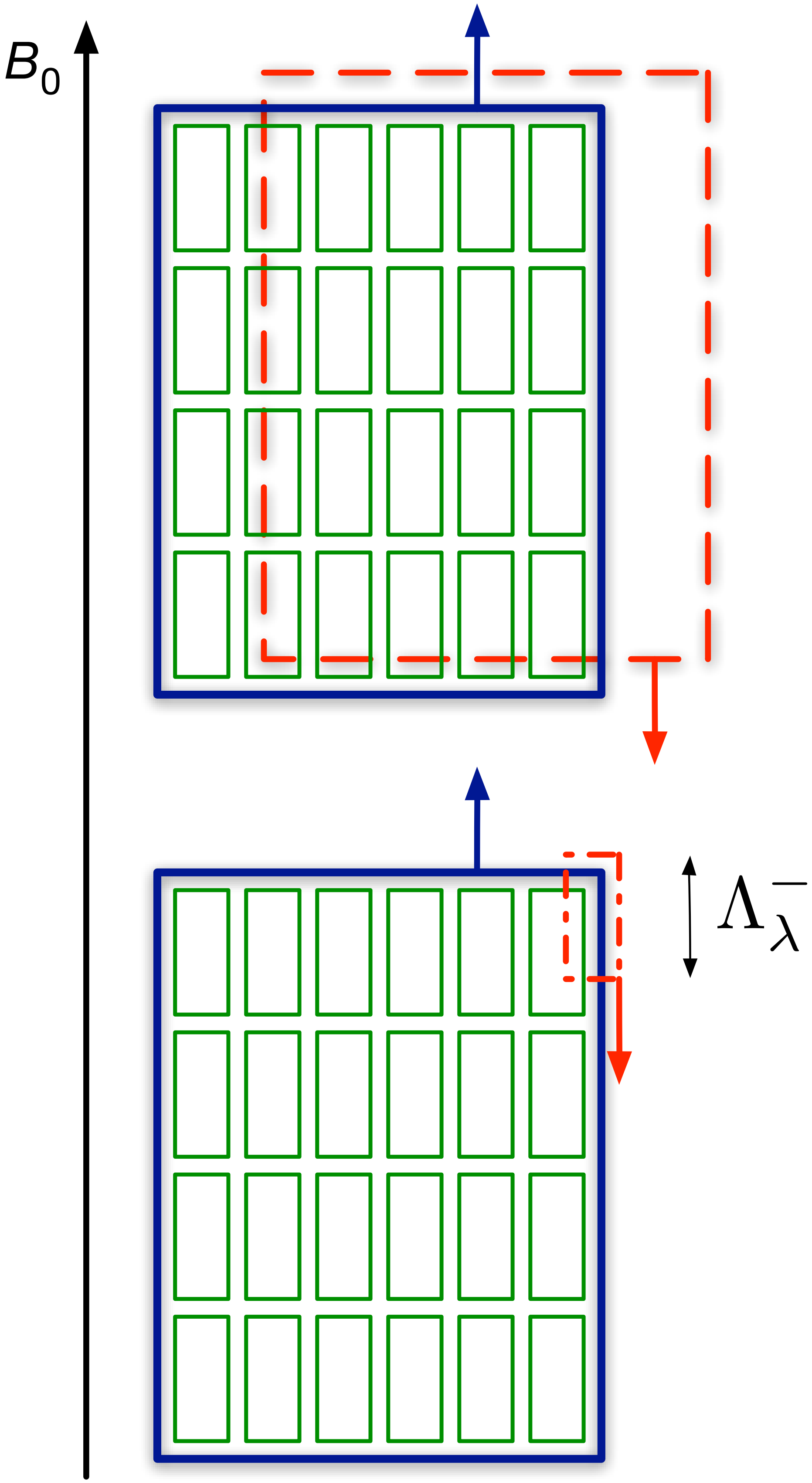
- $\epsilon^+ \sim \dfrac{(w_\lambda^+)^2 w_\lambda^-}{\lambda}$ $\quad\epsilon^- \sim \dfrac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}$
- $(\epsilon^+)^2/\epsilon^- \propto \dfrac{(w_\lambda^+)^3}{\lambda} \propto \lambda^0$
- $w_\lambda^+ \propto \lambda^{1/3}$ $\quad w_\lambda^- \propto \lambda^{1/3}$

i.e., $E^\pm(k_\perp) \propto k_\perp^{-5/3}$

Lithwick, Goldreich, & Sridhar (2007) Model (`LGS' Model)



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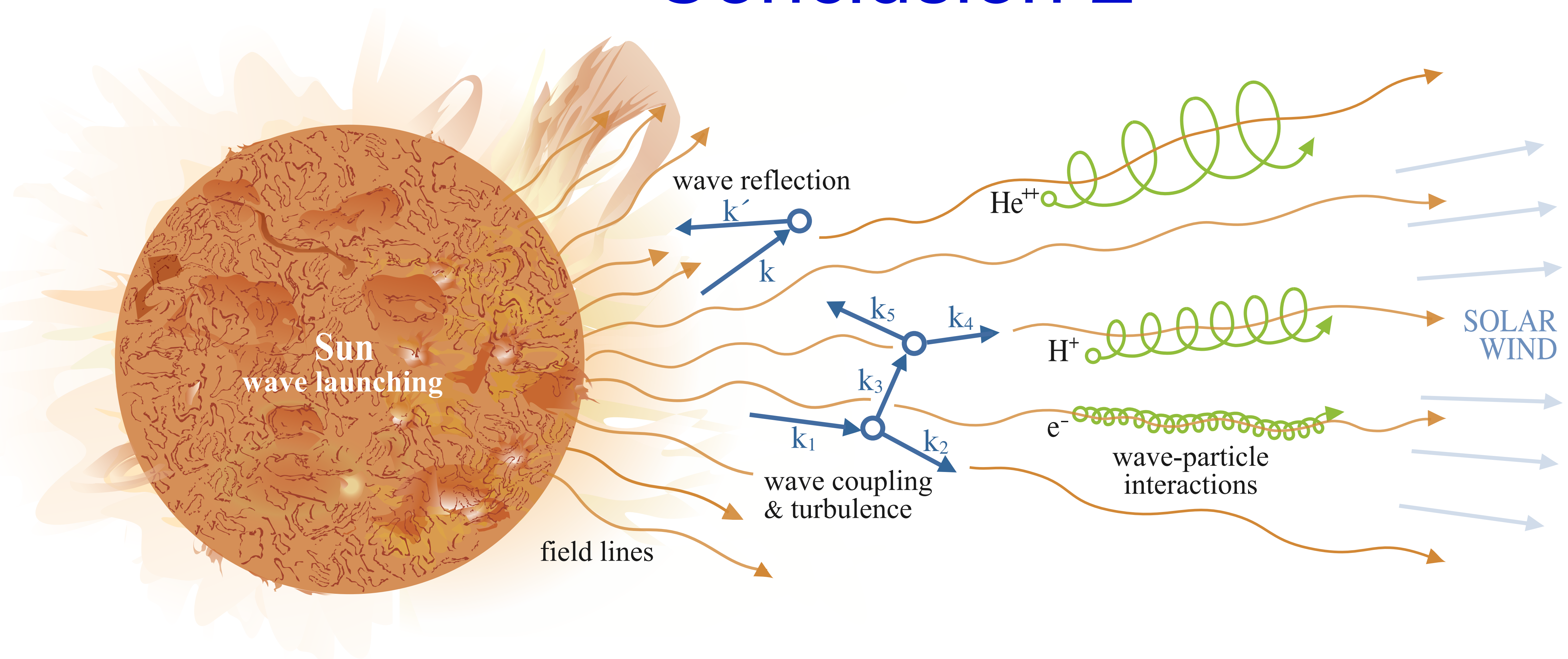
Coherence assumption is reasonable for reflection-driven turbulence!

Conclusion 2



1. Is the heating from reflection-driven MHD turbulence enough to generate the solar wind? YES

2. What is the spectrum of ‘imbalanced’ reflection-driven MHD turbulence? evolves towards the LGS (2007) scalings, flattened from -5/3 towards -3/2. (but not for all simulation parameters…)

Outline

I. Background: turbulence and the origin of the solar wind

II. Numerical simulations of reflection-driven MHD turbulence in the solar atmosphere and solar wind

III. The spectrum of imbalanced MHD turbulence

IV. Accounting for additional physics:

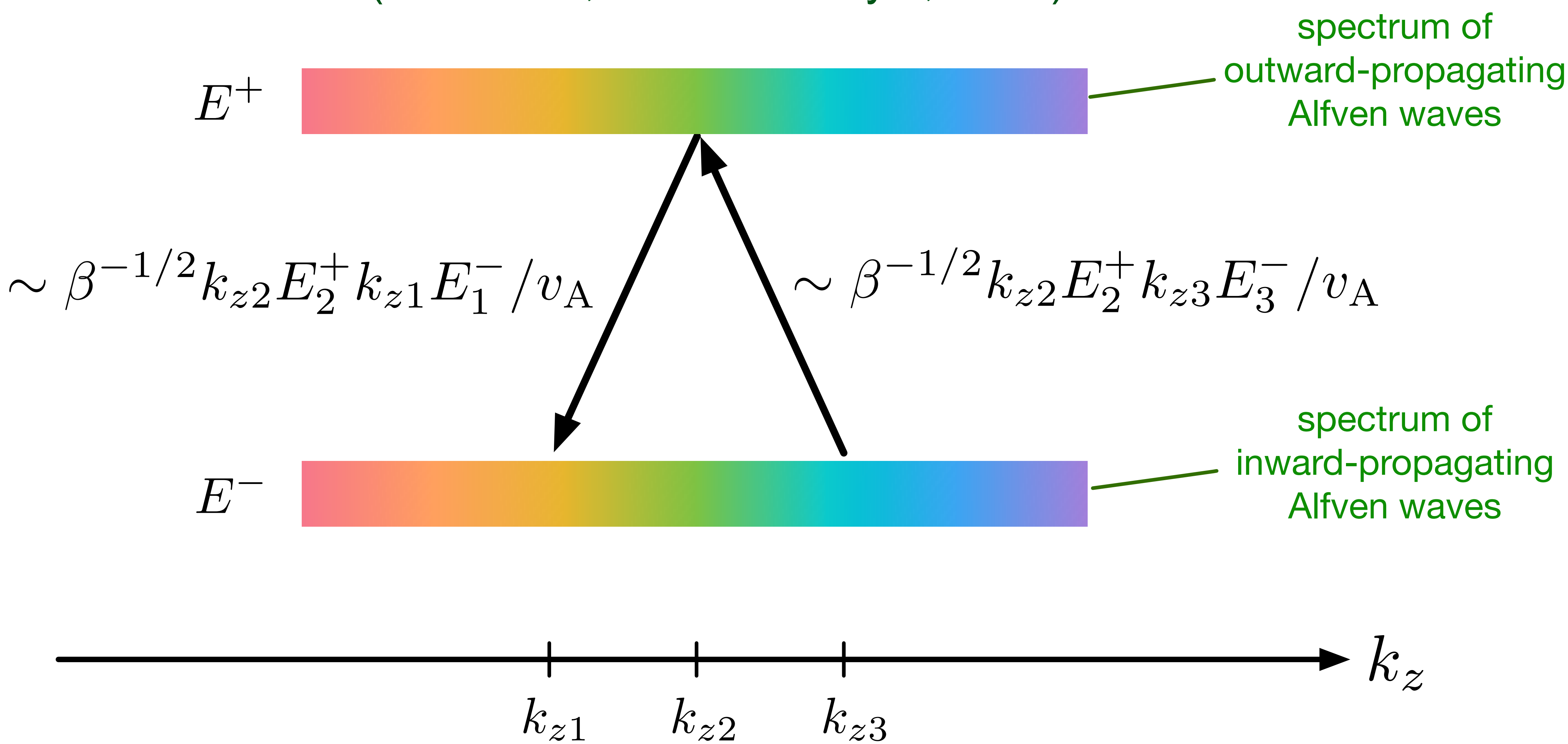
→ A. compressibility (parametric decay)

B. general relativity (relevance to disk winds?)

C. spherical polarization (longitudinal fluctuations)

Wave Kinetic Equations for Parametric Decay at Low β

(Chandran, J. Plasm. Phys., 2018)



$$\frac{\partial E_2^+}{\partial t} \sim \frac{\beta^{-1/2} k_{z2} E_2^+}{v_{\rm A}} \left(k_{z3} E_3^- - k_{z1} E_1^- \right)$$

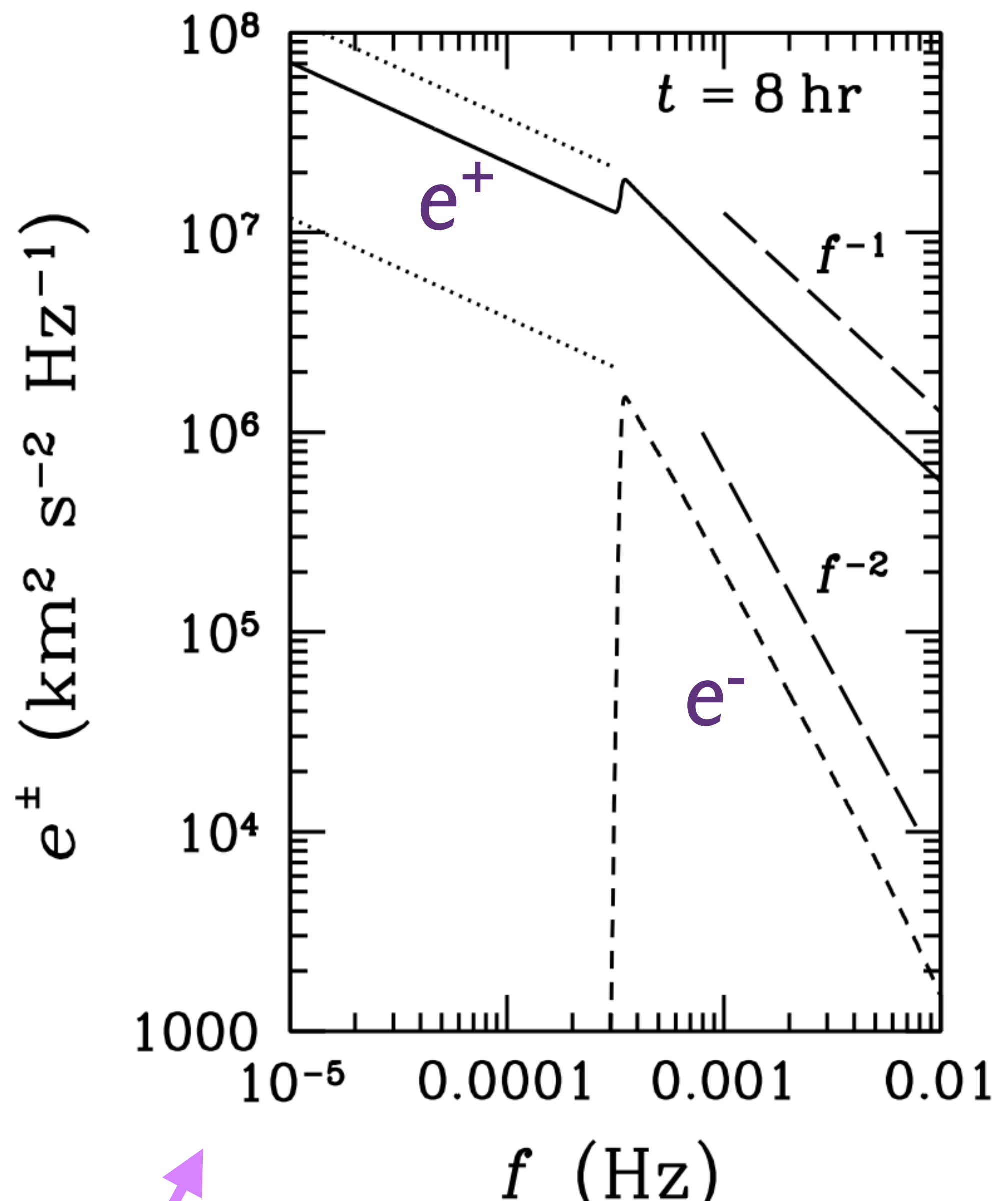
$$k_{z3} - k_{z1} \sim \beta^{1/2} k_{z2}$$

$$\frac{\partial E_2^+}{\partial t} \sim \frac{\beta^{-1/2} k_{z2} E_2^+}{v_{\rm A}} \times \beta^{1/2} k_{z2} \frac{\partial}{\partial k_z} \left(k_z E^- \right) \sim \frac{k_{z2}^2 E_2^+}{v_{\rm A}} \frac{\partial}{\partial k_z} \left(k_z E^- \right)$$

Nonlinear Evolution of Parametric Decay Instability in Solar Wind

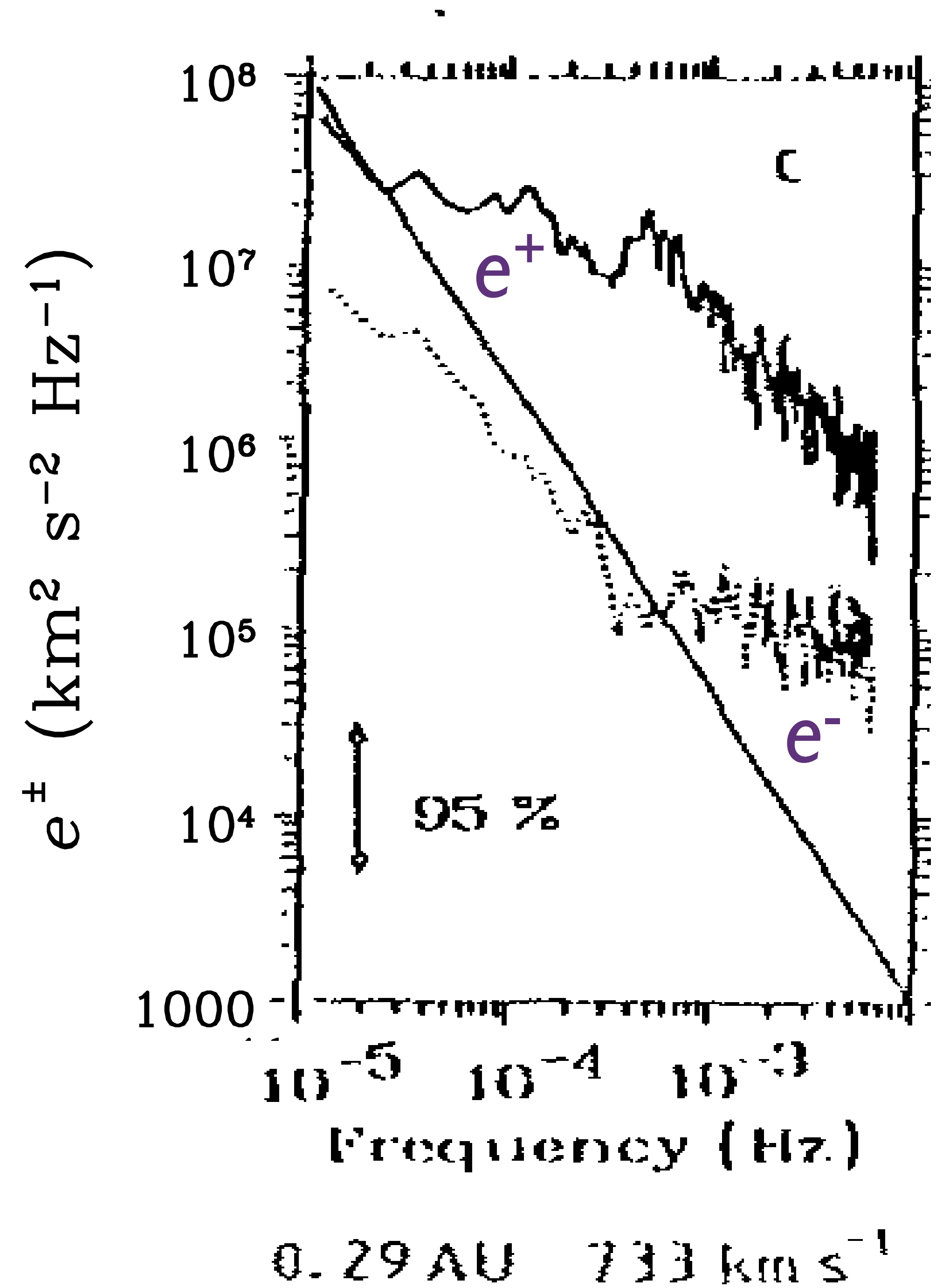
(Chandran, J. Plasm. Phys., 2018)

Weak Turbulence Calculation



Alfven speed = 150 km/s. Initial dominant frequency (maximum of f x E_f) is 0.01 Hz. Alfven travel time to 0.29 AU is 12 hours.

Helios Measurements



Tu & Marsch (1995)

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Reflection-Driven General Relativistic MHD Turbulence

(Chandran, Foucart, Tchekhovskoy, J. Plasm. Phys., 2018)

$$\overline{z_-^\nu}\partial_\nu M_+ + M_+\left(2\overline{z_-^\nu}_{;\nu} + \frac{3\overline{z_-^\nu}\partial_\nu\overline{\mathcal{E}}}{2\overline{\mathcal{E}}}\right) = -2\gamma_+ M_+$$

$$\gamma_+ = \left|\frac{\overline{z_-^\nu}\partial_\nu v_{\rm A}}{2v_{\rm A}}\right|$$

In the axisymmetric, steady-state case, can solve analytically for arbitrary profiles of the background flow and magnetic field:

- Between the base of the disk's corona and Alfven-speed maximum:

$$M_+ = M_{+\rm b}\left(\frac{\chi_{\rm b}}{\chi}\right)\left(\frac{v_{\rm Ab}}{v_{\rm A}}\right) \qquad \text{where} \qquad \chi = \frac{\overline{\mathcal{E}}^{3/2}(u_{\rm p}+v_{\rm Ap})^2}{\overline{\rho}^2 u_{\rm p}^2}$$

$u_{\rm p}$ and $v_{\rm Ap}$ are the poloidal components of the fluid velocity and Alfvén velocity, and $\mathcal{E} = \rho + u + p + b^2$

Reflection-Driven GRMHD Turbulence

(Chandran, Foucart, Tchekhovskoy, J. Plasm. Phys., 2018)

$$\overline{z_-^\nu} \partial_\nu M_+ + M_+ \left(2\overline{z_-^\nu}_{;\nu} + \frac{3\overline{z_-^\nu}\partial_\nu \overline{\mathcal{E}}}{2\overline{\mathcal{E}}} \right) = -2\gamma_+ M_+$$

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- Beyond Alfvén-speed maximum:

$$M_+ = M_{+\rm b} \left(\frac{\chi_{\rm b}}{\chi} \right) \left(\frac{v_{\rm Ab} v_{\rm A}}{v_{\rm Am}^2} \right)$$

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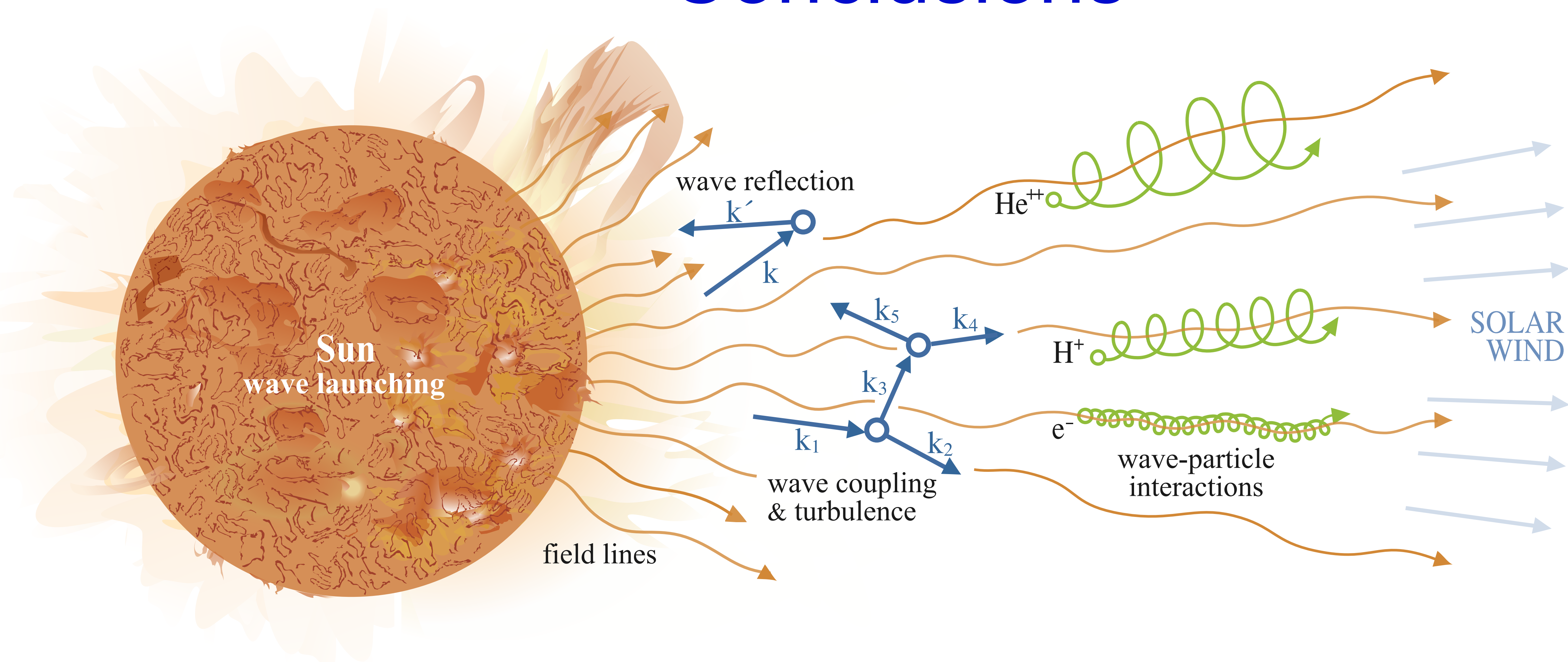
Important direction for future research.

Conclusions



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