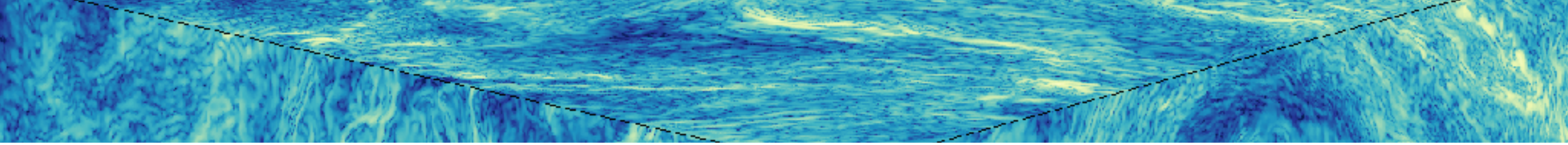
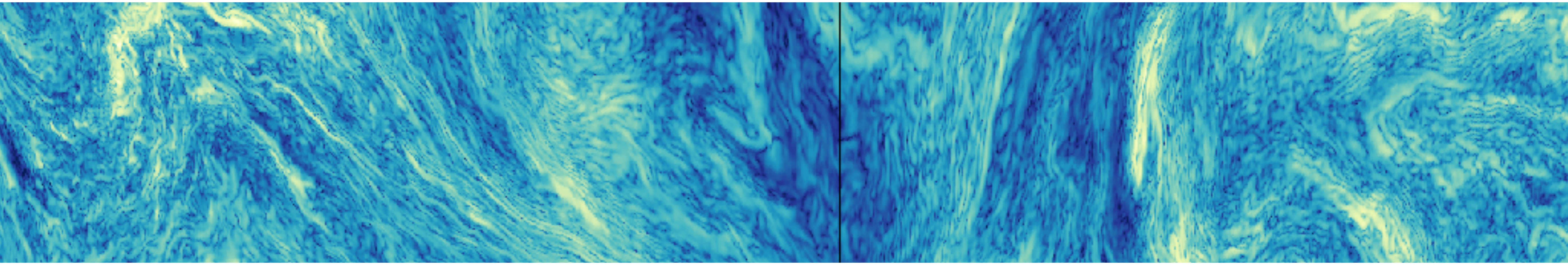
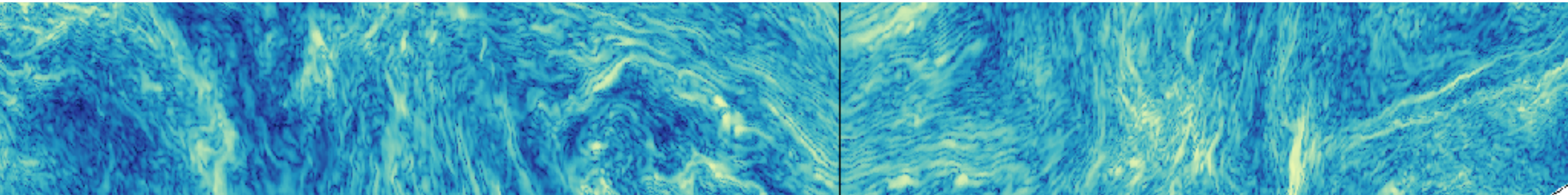


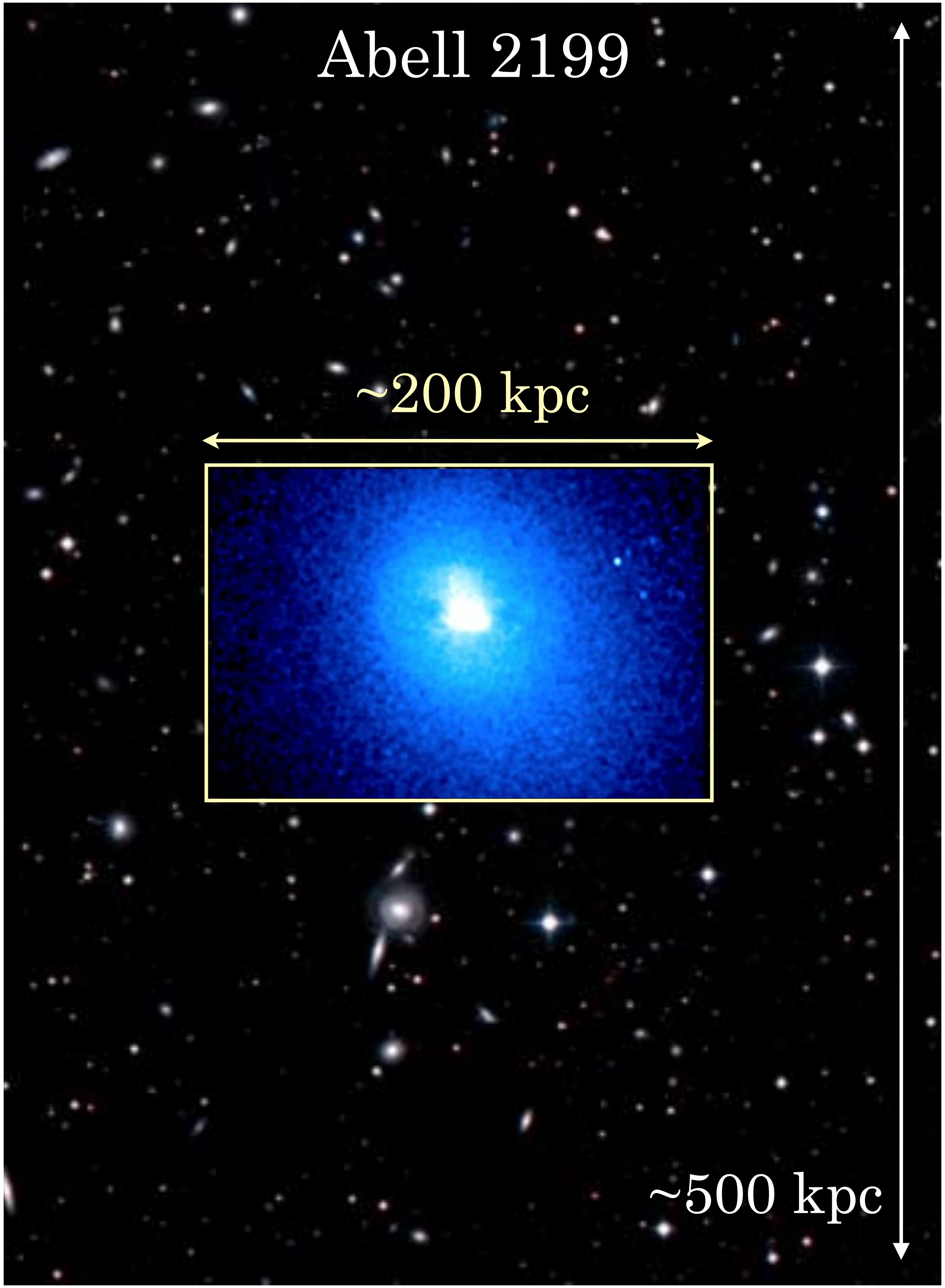
Fluctuation Dynamo in Weakly Collisional Plasmas

(or, I get by with help from my little friends)



Denis St-Onge
Matthew Kunz
PRINCETON UNIVERSITY
with Jono Squire and Alex Schekochihin

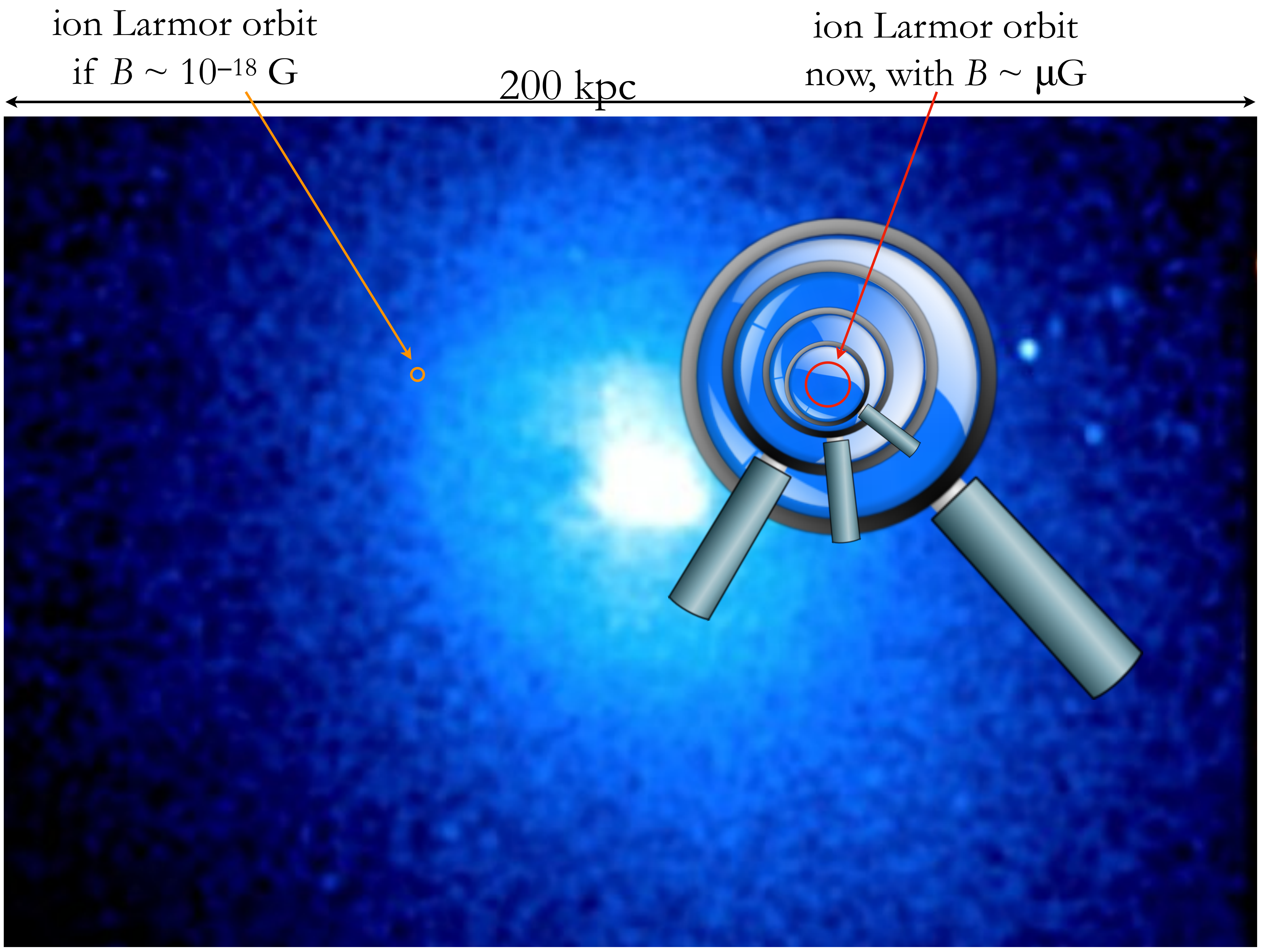




Galaxy Clusters

$M \sim 10^{14-15}\ \mathrm{M}_\odot$
in $\sim$1 Mpc

14% thermal plasma
$T \sim 1$–10 keV
$n \sim 10^{-4}$–$10^{-1}\ \mathrm{cm}^{-3}$


ion Larmor orbit
if $B \sim 10^{-18}$ G
200 kpc
ion Larmor orbit
now, with $B \sim \mu$G

$$\rho_i \sim \left(\frac{T}{1\ \mathrm{keV}}\right)^{1/2} \left(\frac{B}{10^{-18}\ \mathrm{G}}\right)^{-1}\ \mathrm{kpc}$$

$$\Omega_i \sim \left(\frac{B}{10^{-18}\ \mathrm{G}}\right)\ \mathrm{Myr}^{-1}$$

why? | how?

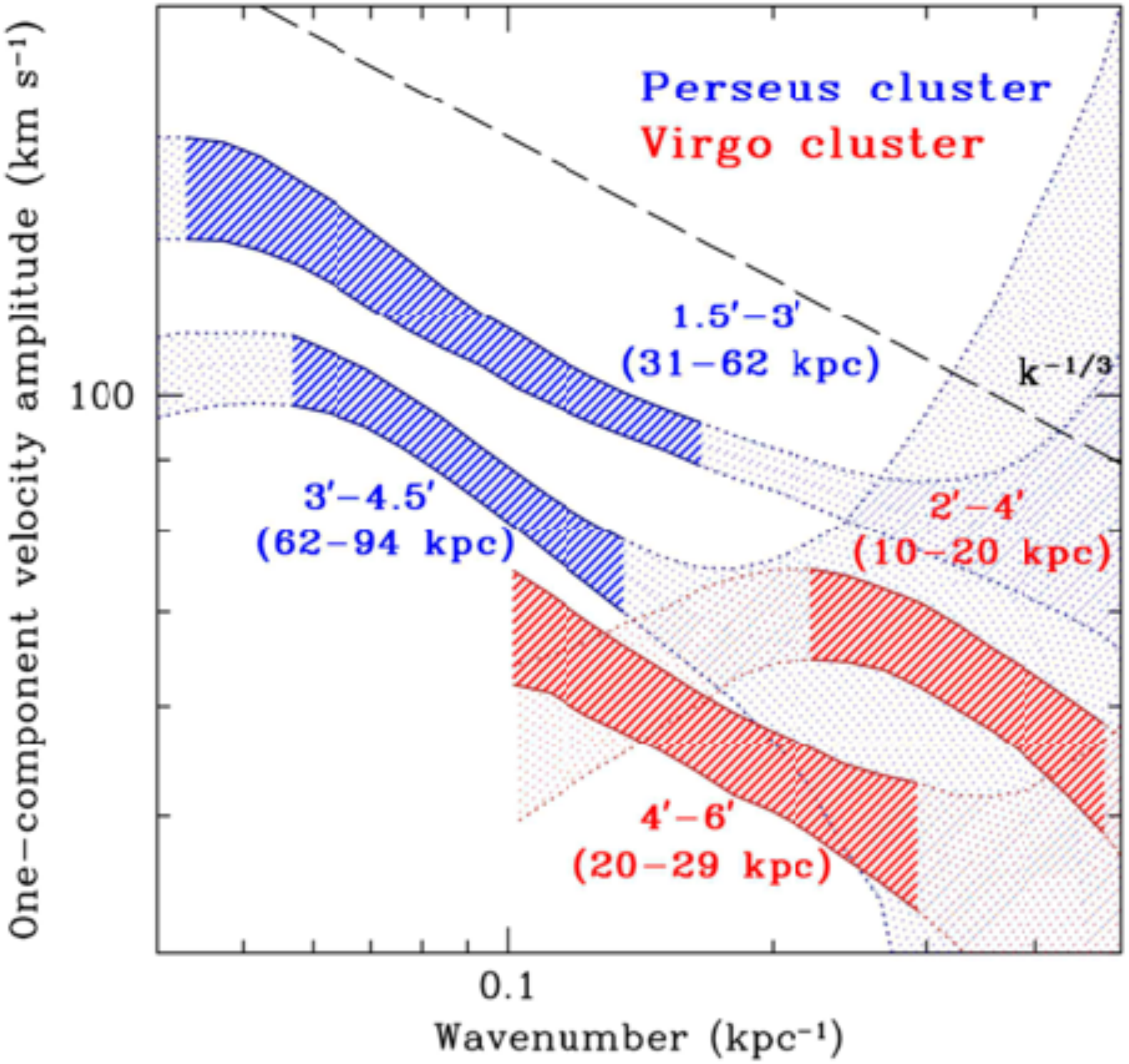
dynamo

$$\rho_i \sim \left(\frac{T}{1\ \mathrm{keV}}\right)^{1/2} \left(\frac{B}{10^{-6}\ \mathrm{G}}\right)^{-1}\ \mathrm{npc}$$

$$\Omega_i \sim \left(\frac{B}{10^{-6}\ \mathrm{G}}\right)\ \mathrm{min}^{-1}$$

(ion Larmor orbit ~ size of Jupiter) $(\beta \equiv 8\pi nT/B^2 \sim 10^{2-4})$

ICM is turbulent



Zhuravleva *et al.* 2014, Nature

Hitomi, before its death:
$u = 164 \pm 10$ km/s
in Perseus at ~50 kpc

Note:

$$v_\mathrm{A} = 154 \left(\frac{B}{10\ \mu\mathrm{G}}\right) \left(\frac{n}{0.02\ \mathrm{cm}^{-3}}\right)^{-1/2}\ \mathrm{kpc}$$

likely not a coincidence!!!

(~10 μG is typical B measurement from RM in core;
$B \propto n^{1/2}$ inferred in Coma: Bonafede *et al.* 2010)

it is then natural to attribute intracluster magnetic field to the **fluctuation ("turbulent") dynamo**

(Batchelor 1950; Kazantsev 1967; Zel'dovich et al. 1984; Childress & Gilbert 1995),

whereby a succession of random velocity shears stretches the field and leads on the average to its growth to dynamical strengths.

$$\frac{\mathrm{d}\ln B}{\mathrm{d}t} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}$$

magnetic energy grows in a 3D, smooth, chaotic velocity field

$$u_\ell \sim \ell^{1/3} \quad \Longrightarrow \quad \frac{\mathrm{d}\ln B}{\mathrm{d}t} \sim \frac{U}{L}\left(\frac{\ell_\nu}{L}\right)^{-2/3} \sim \frac{U}{L}\,\mathrm{Re}^{1/2}$$

depends on the material properties of the host plasma

Small-scale MHD dynamo evolution at Pm ≫ 1

1) kinematic

no feedback from B on u; exponential growth

Kazantsev $k^{3/2}$ spectrum, peaking at k_η

development of folded structure

2) nonlinear

tension affects viscous-scale eddies:

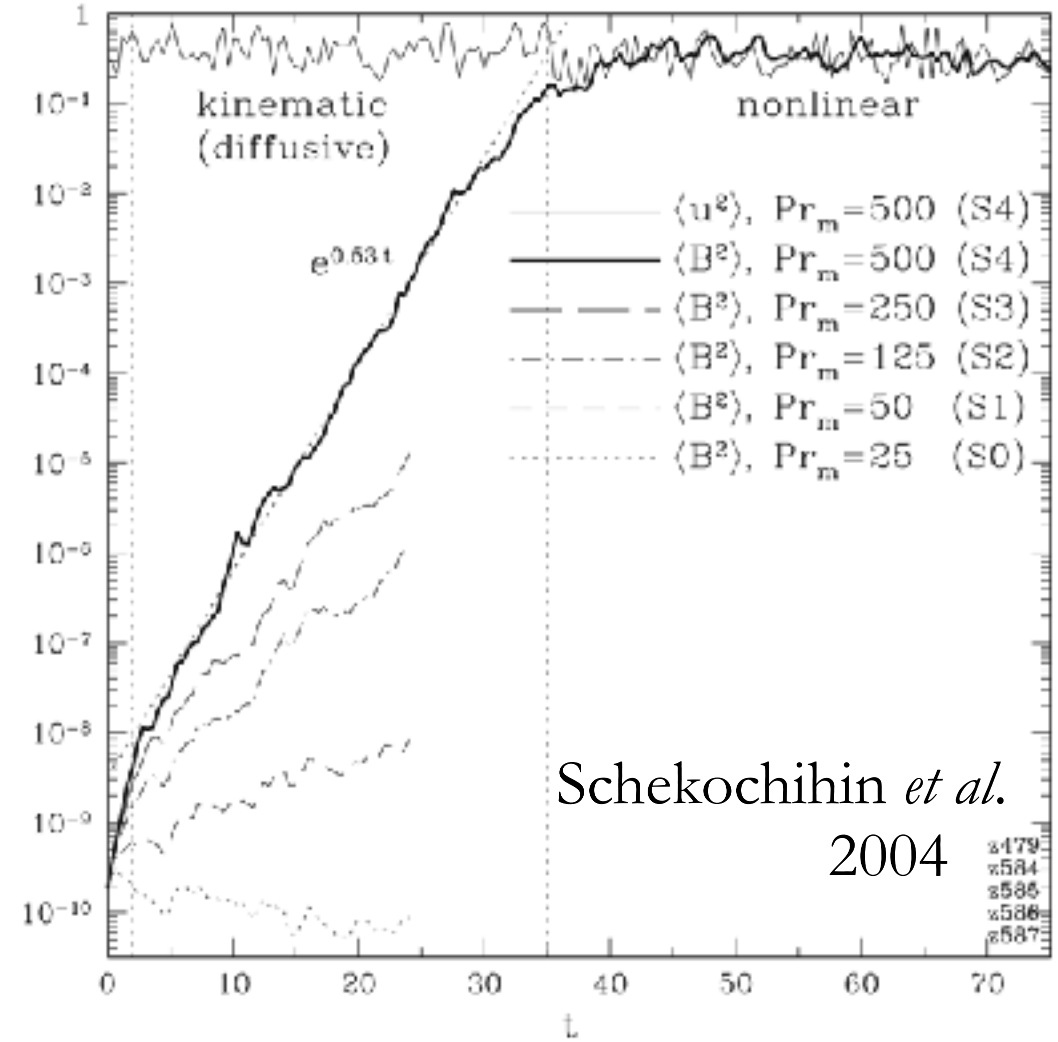
$$\boldsymbol{B}\cdot\nabla\boldsymbol{B} \sim \boldsymbol{u}\cdot\nabla\boldsymbol{u} \sim u_\nu^2/\ell_\nu$$

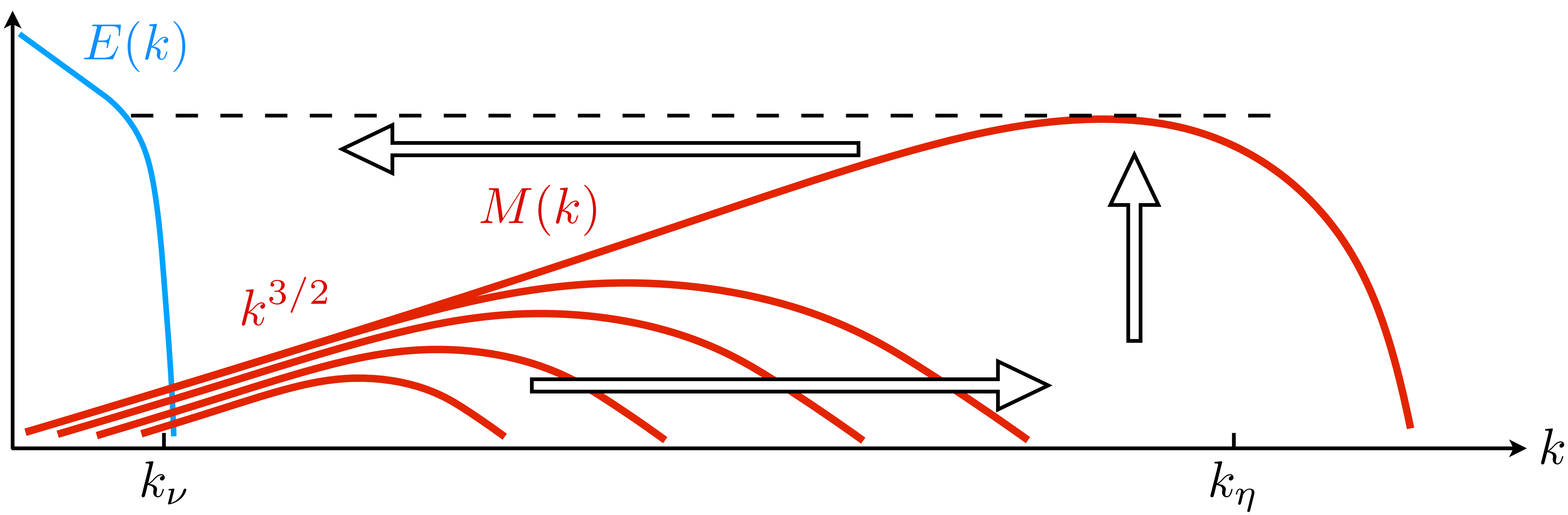
slower, larger-scale eddies take over stretching

secular growth $\langle B^2\rangle$ (Sch02&04, Cho+09, Beresnyak12)

3) saturation at $\langle B^2\rangle \sim \langle u^2\rangle$

not scale-by-scale! suppression of $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\nabla\boldsymbol{u}$

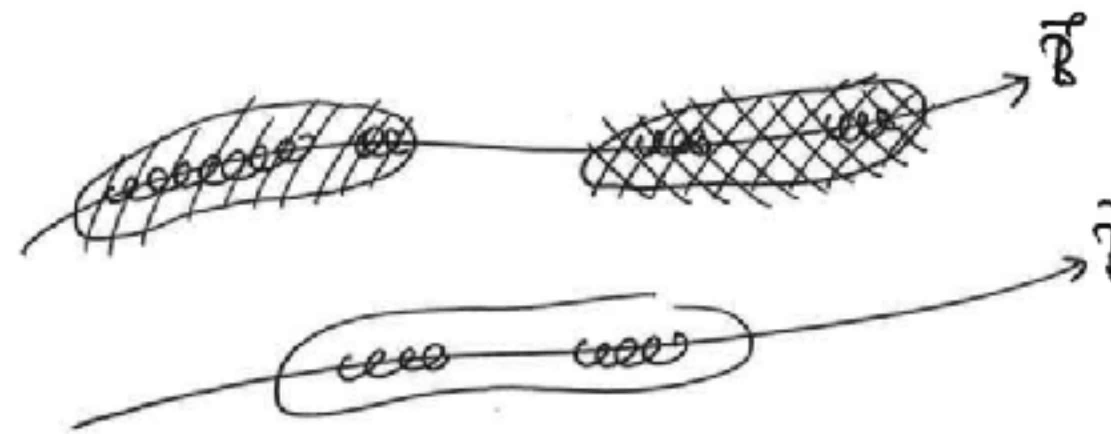




Issues with fluctuation dynamo in the ICM:

- ICM is **well magnetized**, even at $\beta \sim 10^{22}$; implies that viscous transport is anisotropic:

$$\frac{\lambda_{\rm mfp}}{\rho_{\rm i}} \sim 0.1 \left(\frac{\rm Pm}{\beta_{\rm i}}\right)^{1/2}$$



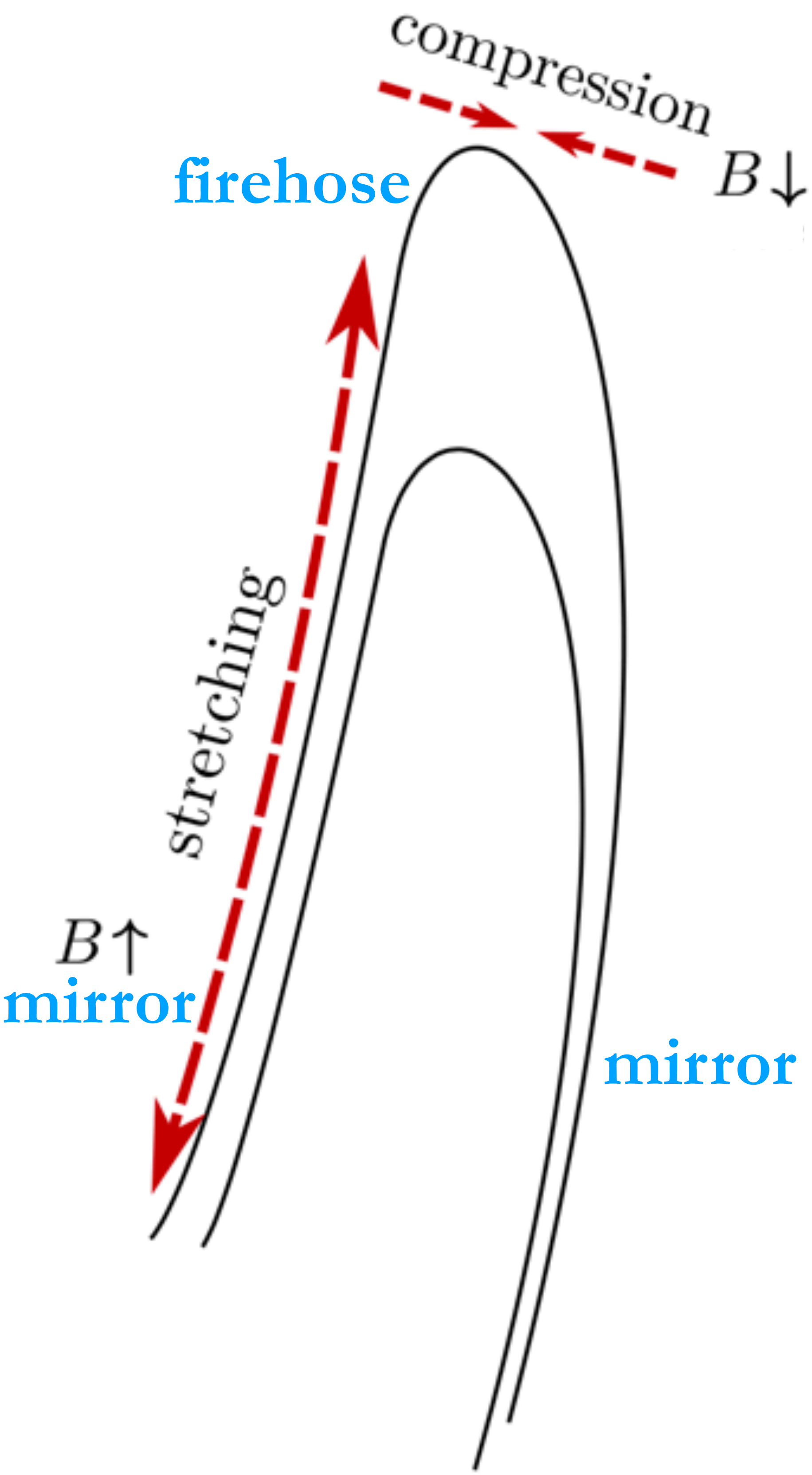
$$\Downarrow$$

$$\mathrm{Re}_{\|} \doteq M \frac{L}{\lambda_{\rm mfp}} \ll \mathrm{Re}_{\perp} \sim M \frac{L}{\rho_{\rm i}}$$

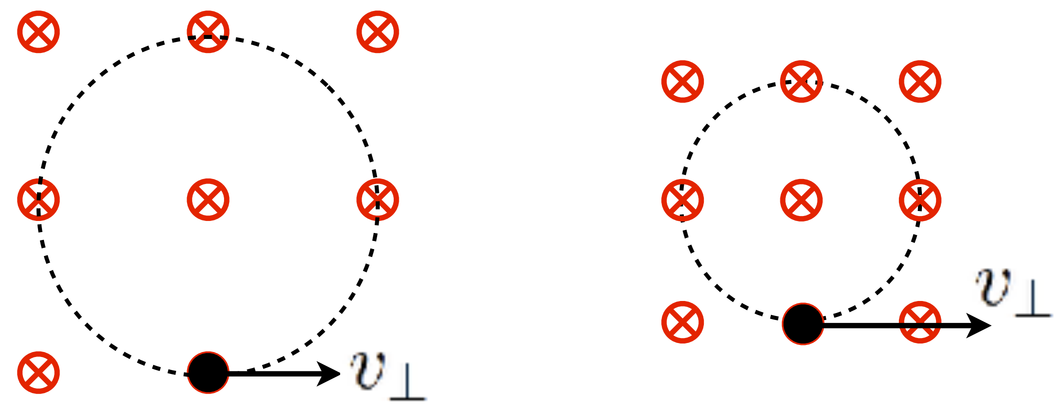
(M is Mach number)

- ICM is **weakly collisional**, i.e., not rigorously a fluid on all but the largest scales. Deviations from LTE expected. Why?

can't move a plasma differentially without stretching/compressing B



μ-conservation implies pressure anisotropy:



$$\frac{p_\perp}{p} \propto \frac{B}{B_0}$$

appreciable dynamo growth is *impossible* if μ is conserved; there's just not enough free energy (Helander *et al.* 2016)

implies (at least) two things:

1) μ must be broken, e.g., by **kinetic instabilities** that feed off $p_\perp \neq p_\parallel$

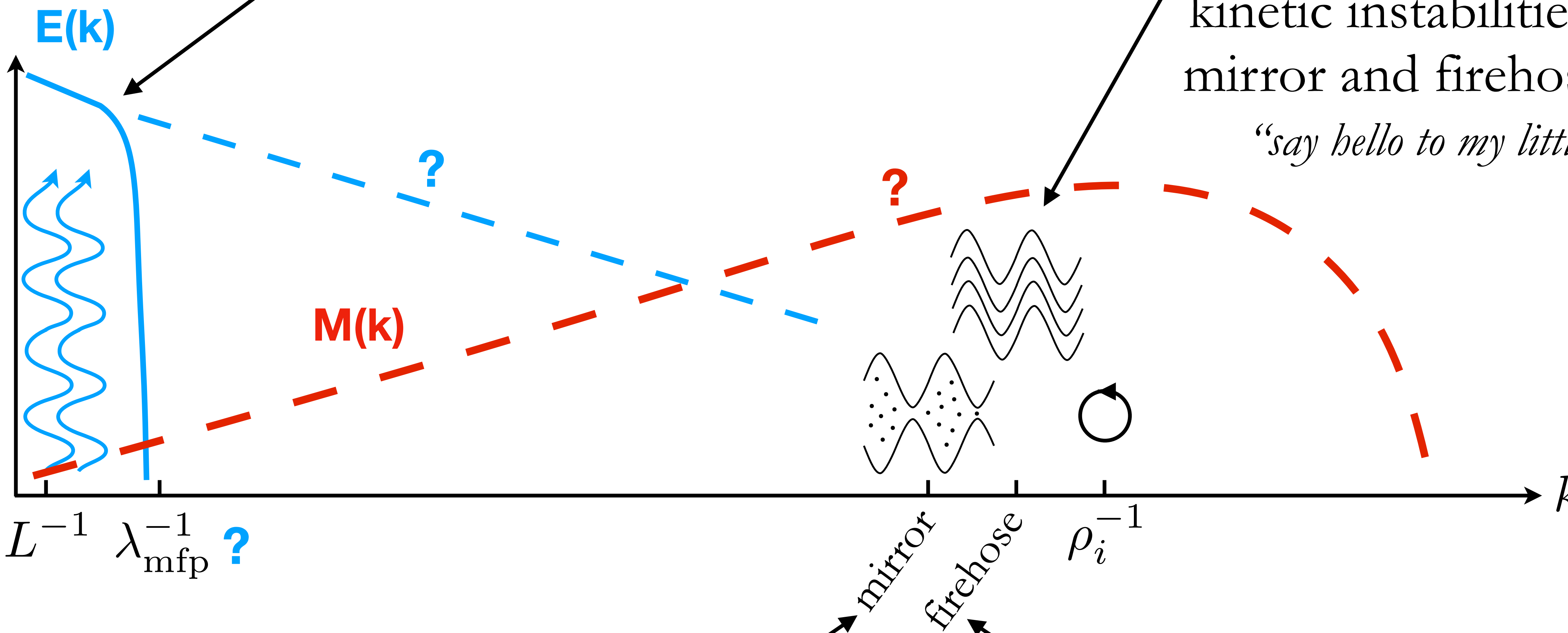
2) no "kinematic" phase… B, no matter how weak, influences the flow

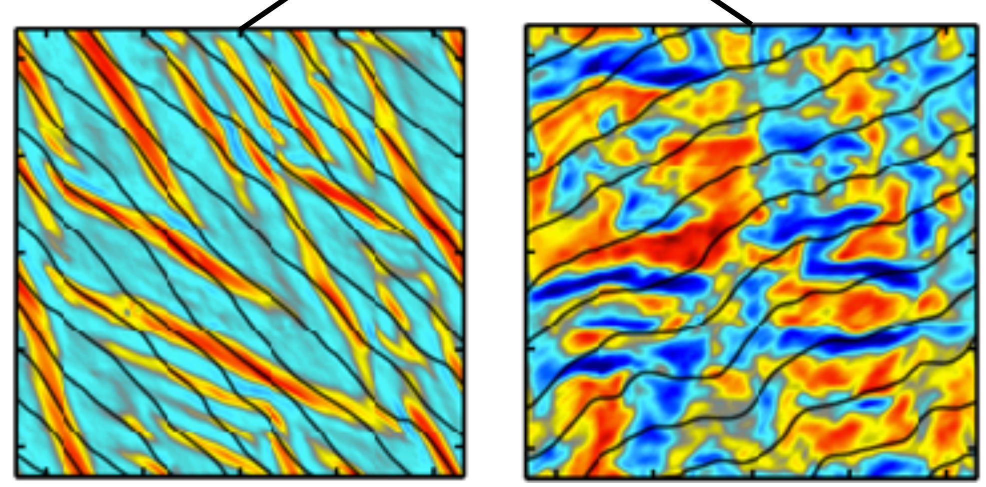
fastest stretching motions at parallel-viscous scale

(recall $\mathrm{d}\ln B/\mathrm{d}t = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}$)

parallel-viscous scale set by effective scattering rate of kinetic instabilities: mirror and firehose

"say hello to my little friends"





Kunz, Stone & Schekochihin 2014, PRL

these try to maintain

$$-\frac{B^2}{4\pi} \lesssim p_\perp - p_\parallel \lesssim \frac{B^2}{8\pi}$$

fastest stretching motions at parallel-viscous scale

(recall $\mathrm{d}\ln B/\mathrm{d}t = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla u}$)

parallel-viscous scale set by effective scattering rate of kinetic instabilities: mirror and firehose

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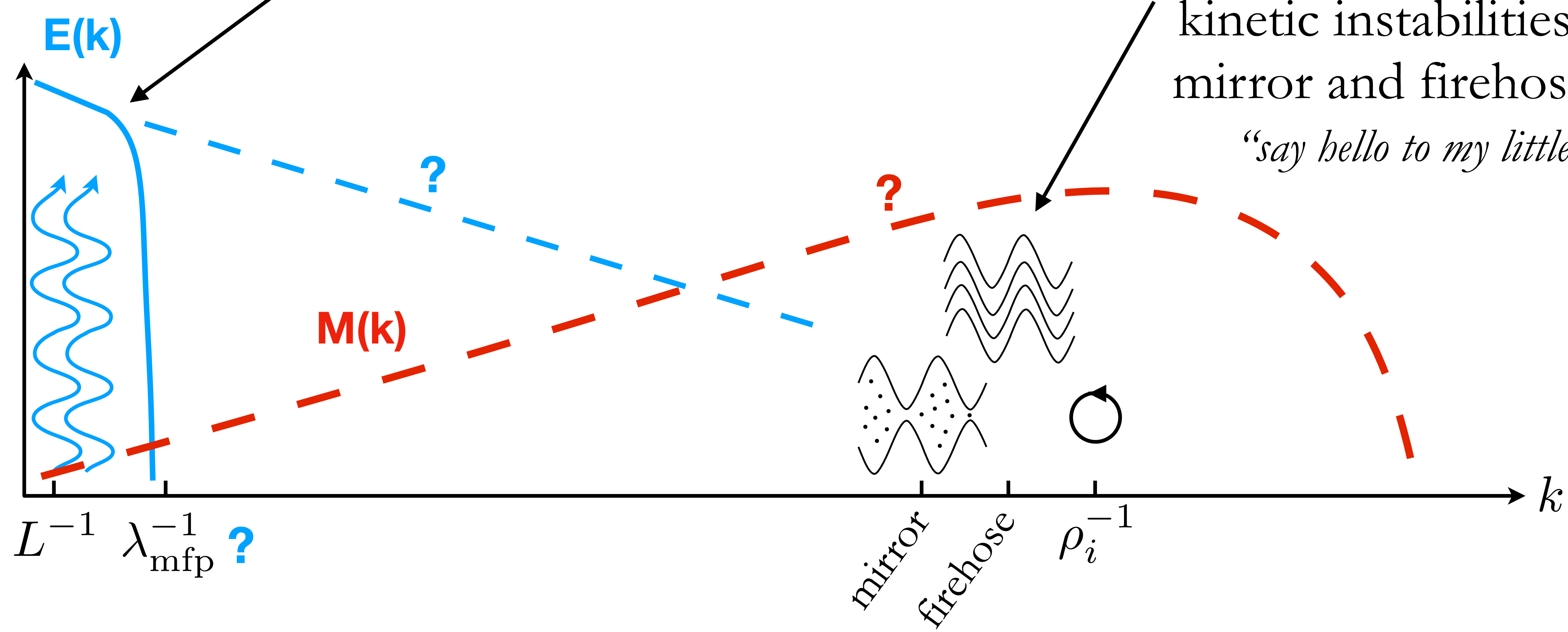


$$\mathrm{Re}_{\|} \sim M^2\,\nu_{\rm eff}\tau_{{\rm eddy},L} \longrightarrow \tau_{{\rm eddy},\ell_{\rm visc}} \sim \tau_{{\rm eddy},L}\,\mathrm{Re}_{\|}^{-1/2} \propto \nu_{\rm eff}^{-1/2}$$

the faster the instabilities scatter, the faster is the dynamo

namely, if firehose/mirror instabilities yield $\nu_{\rm eff}$ large enough to keep Δp tightly at marginal mirror/firehose instability (which can occur in ICM for $B \gtrsim$ nG), then

$$\mathrm{Re}_{\|,\mathrm{eff}} = \frac{U^4}{v_\mathrm{A}^4} \qquad \text{and} \qquad \frac{\ell_{\|,\mathrm{visc}}}{L} = \frac{v_\mathrm{A}^3}{U^3}$$

$$\Longrightarrow \mathrm{Re}_{\|,\mathrm{eff}} \sim 10^3 \left(\frac{M}{0.3}\right)^4 \left(\frac{n}{10^{-3}\ \mathrm{cm}^{-3}}\right)^2 \left(\frac{T}{10^8\ \mathrm{K}}\right)^2 \left(\frac{B}{1\ \mu\mathrm{G}}\right)^{-4}$$

supplants Coulomb-collision $\mathrm{Re}_{\|}$ ($\lesssim$10) for

$$B \lesssim 2\ \mu\mathrm{G} \left(\frac{n}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{1/4} \left(\frac{T}{5\ \mathrm{keV}}\right) \left(\frac{M}{0.2}\right)^{3/4} \left(\frac{L}{100\ \mathrm{kpc}}\right)^{-1/4}$$

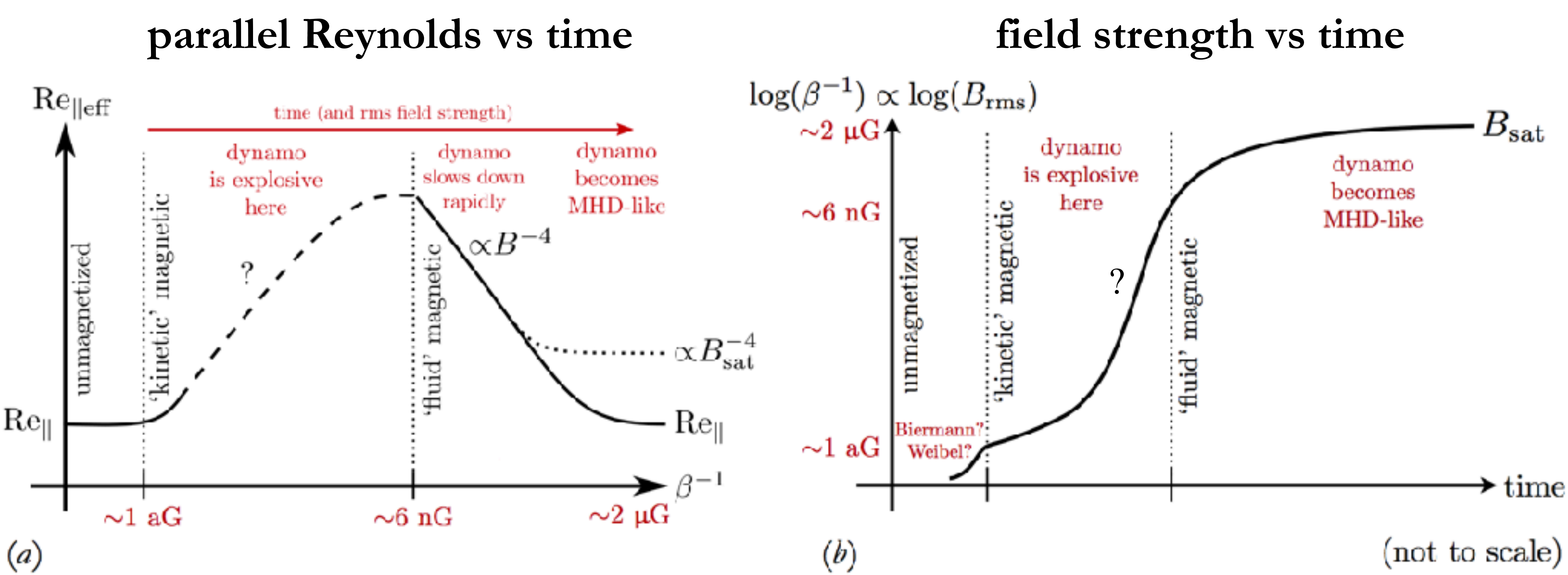
so, at least until the dynamo is approx. saturated, kinetic "collisionality" > Coulomb collisionality

If viscosity is regulated by ion-Larmor-scale instabilities, then dynamo was **much** faster in the past!

But, for 1 aG $\lesssim B \lesssim$ nG, the $\nu_{\rm eff}$ required to keep Δp marginally firehose/mirror unstable is $> \Omega_i$

This can't be achieved.

motivates the following idea of 3 dynamo regimes:



explosive growth? predicts ~nG fields in cosmologically short time.

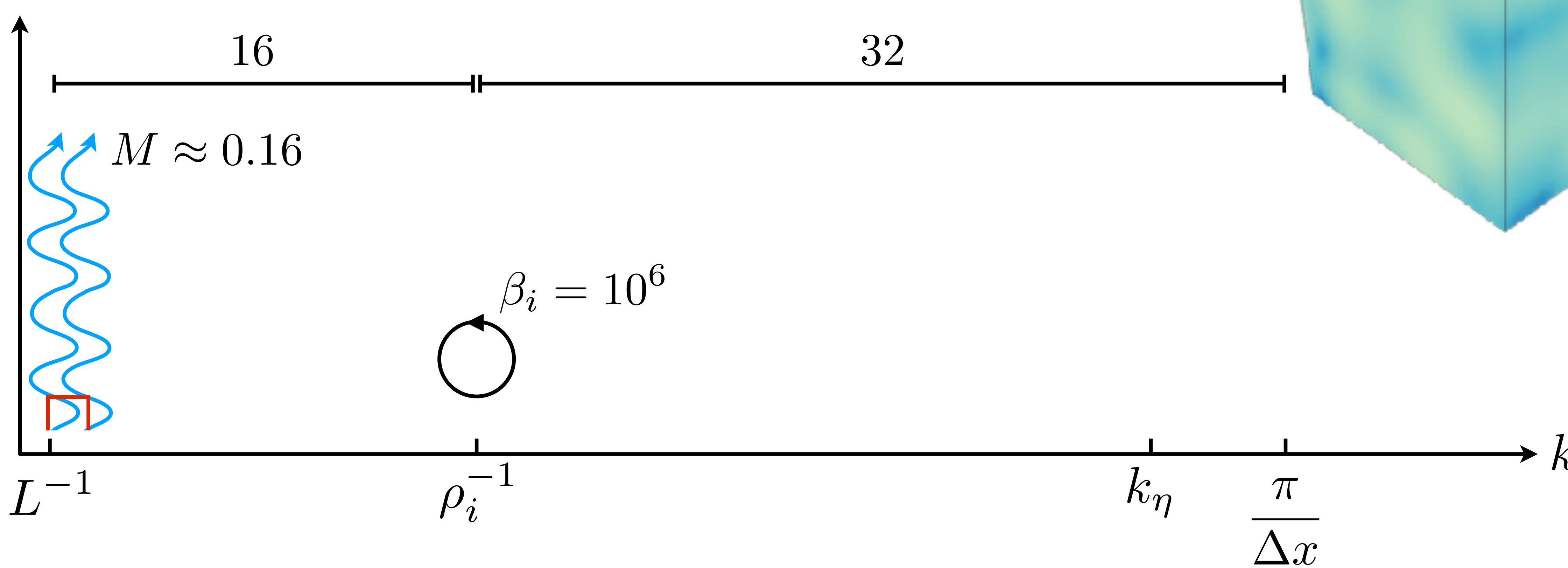
We studied fluctuation dynamo
in collisionless and weakly collisional plasmas
using hybrid-kinetic particle-in-cell simulations,
Braginskii-MHD simulations, and
analytic modeling.

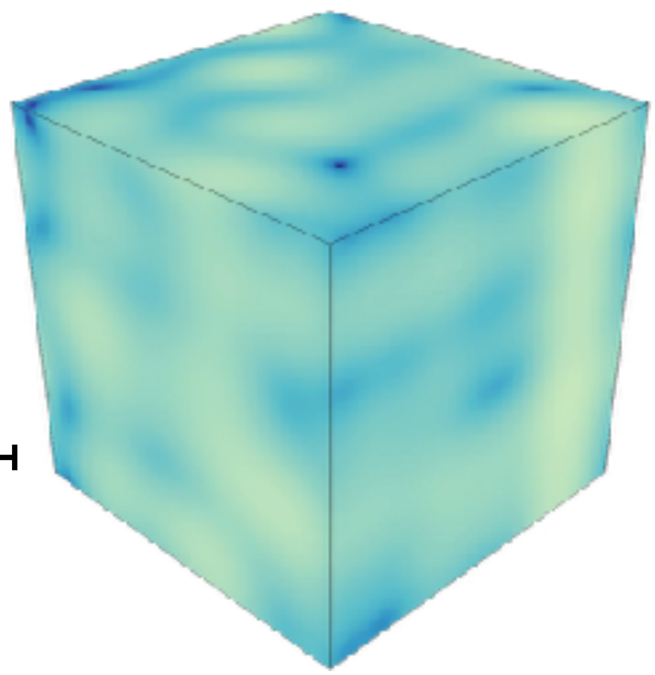
hybrid-kinetic particle-in-cell simulations using **Pegasus**

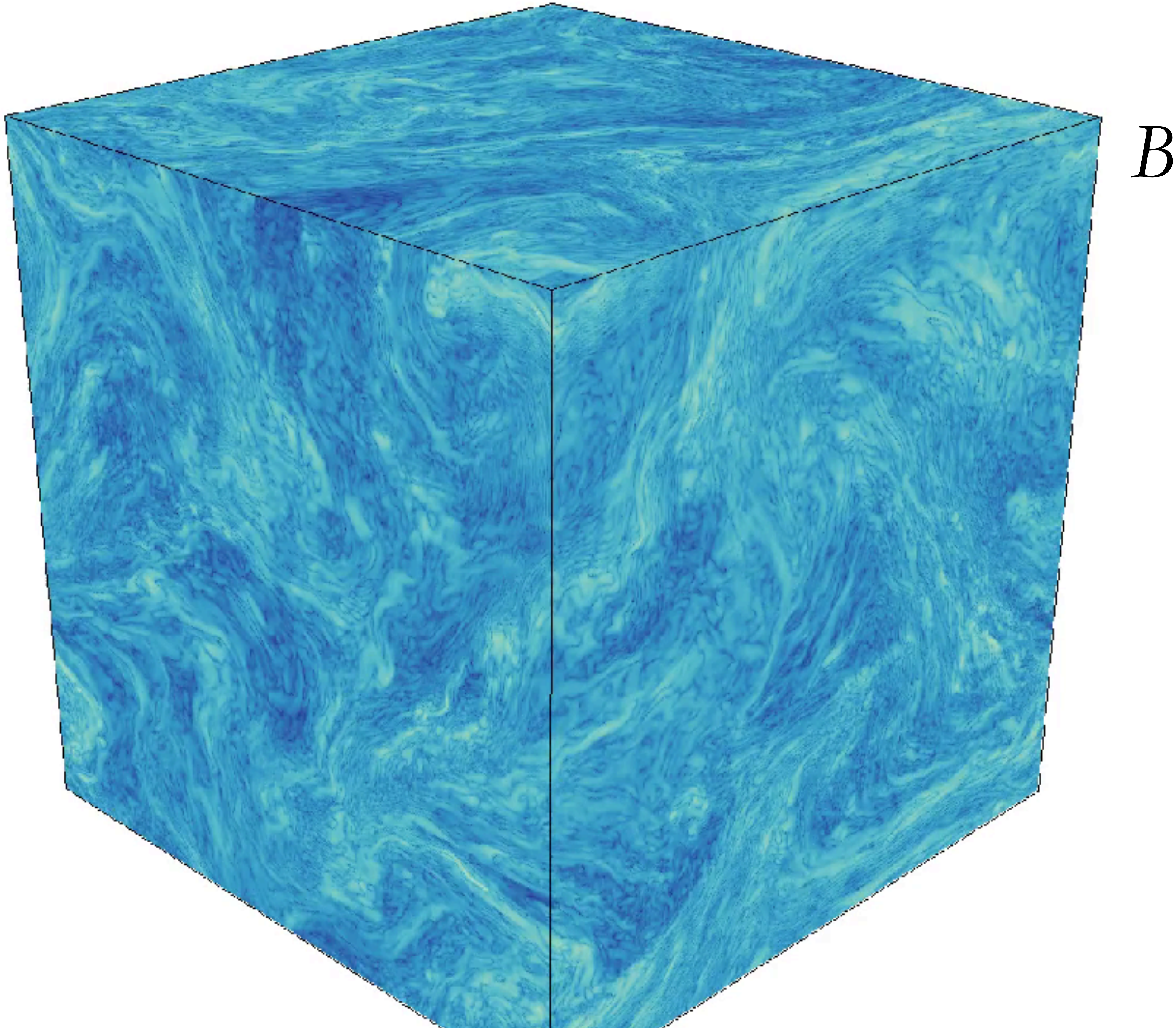
$$\frac{\partial f_{\rm i}}{\partial t} + \boldsymbol{v}\cdot\nabla f_{\rm i} + \left[\frac{Z_{\rm i}e}{m_{\rm i}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + \frac{\boldsymbol{F}}{m_{\rm i}}\right]\cdot\frac{\partial f_{\rm i}}{\partial \boldsymbol{v}} = 0$$ + fluid equation for massless, isothermal electrons

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{\nabla}\times\boldsymbol{E}$$ + hyper-resistivity

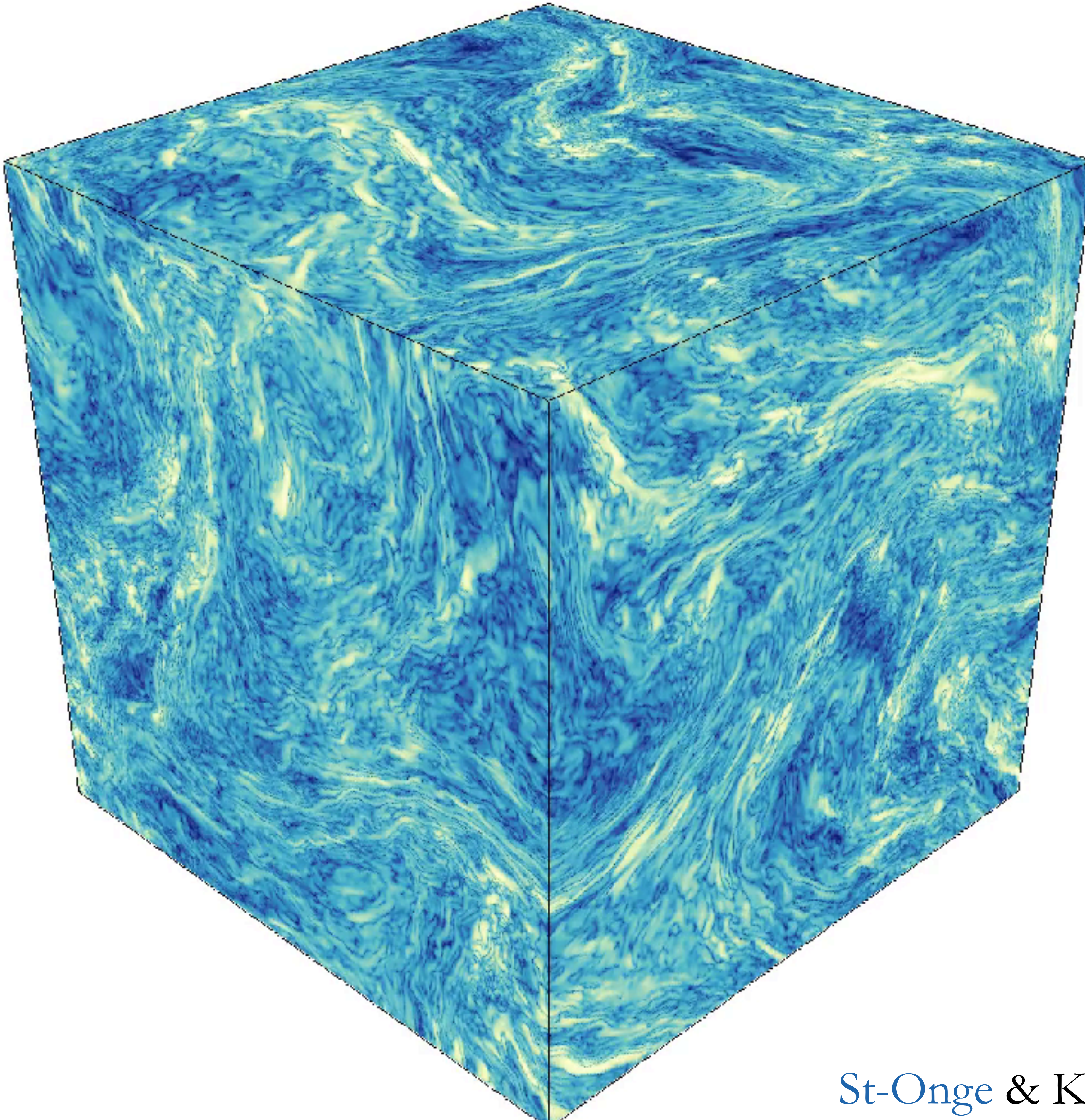
zero-net-flux magnetic field at the largest box scales, subject to time-correlated incompressible, subsonic stirring





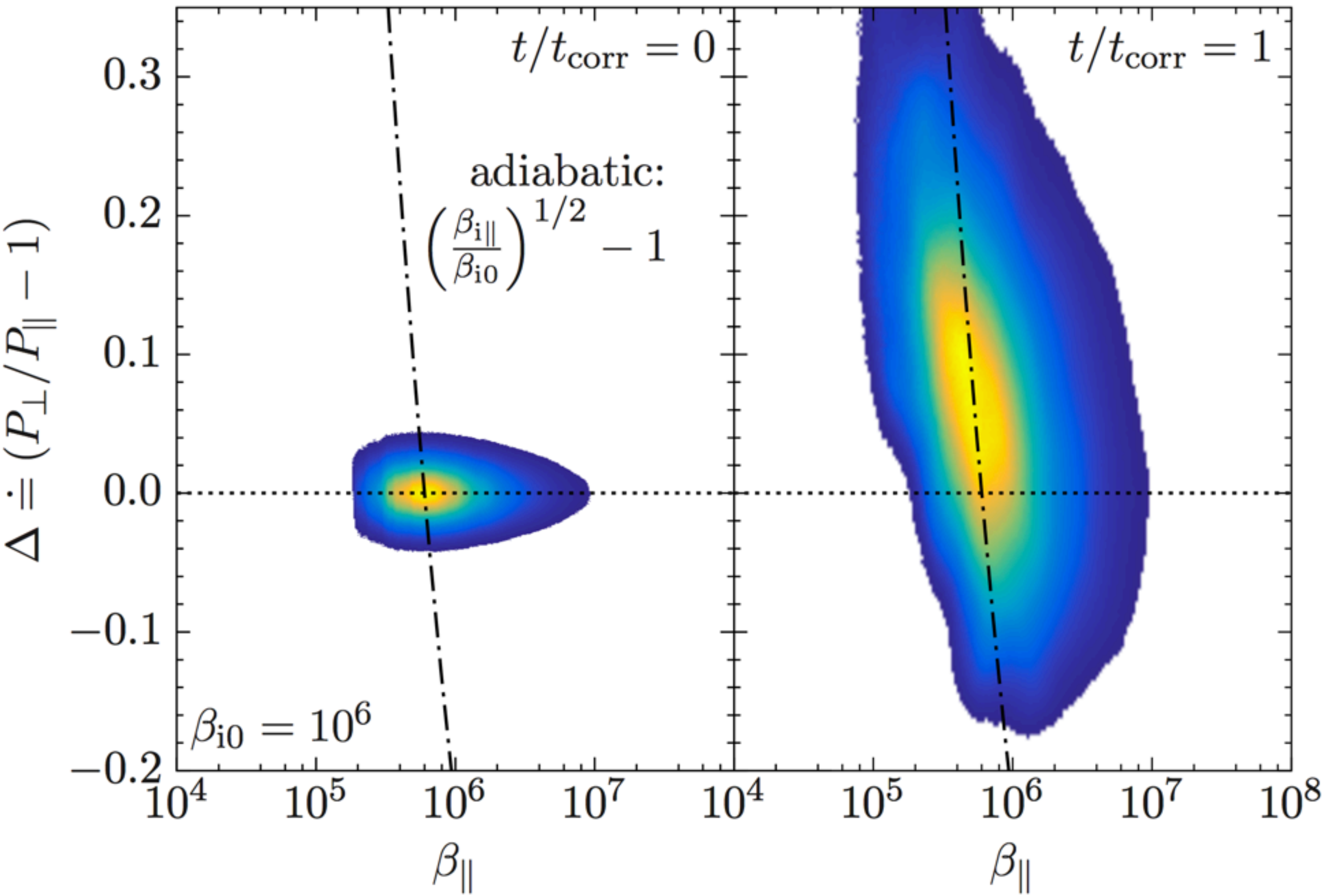


St-Onge & Kunz, 2018 *ApJL*

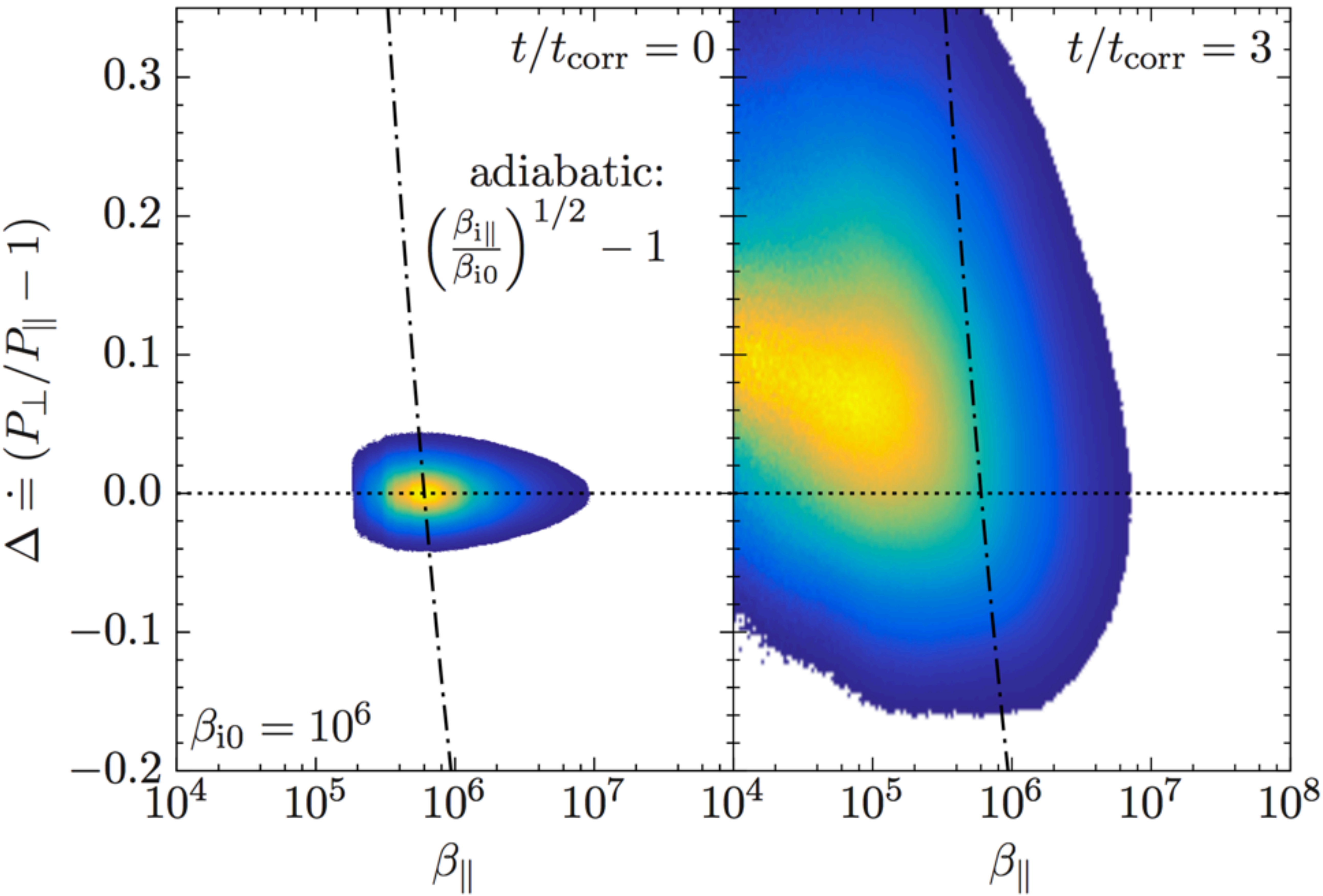


St-Onge & Kunz, 2018 *ApJL*

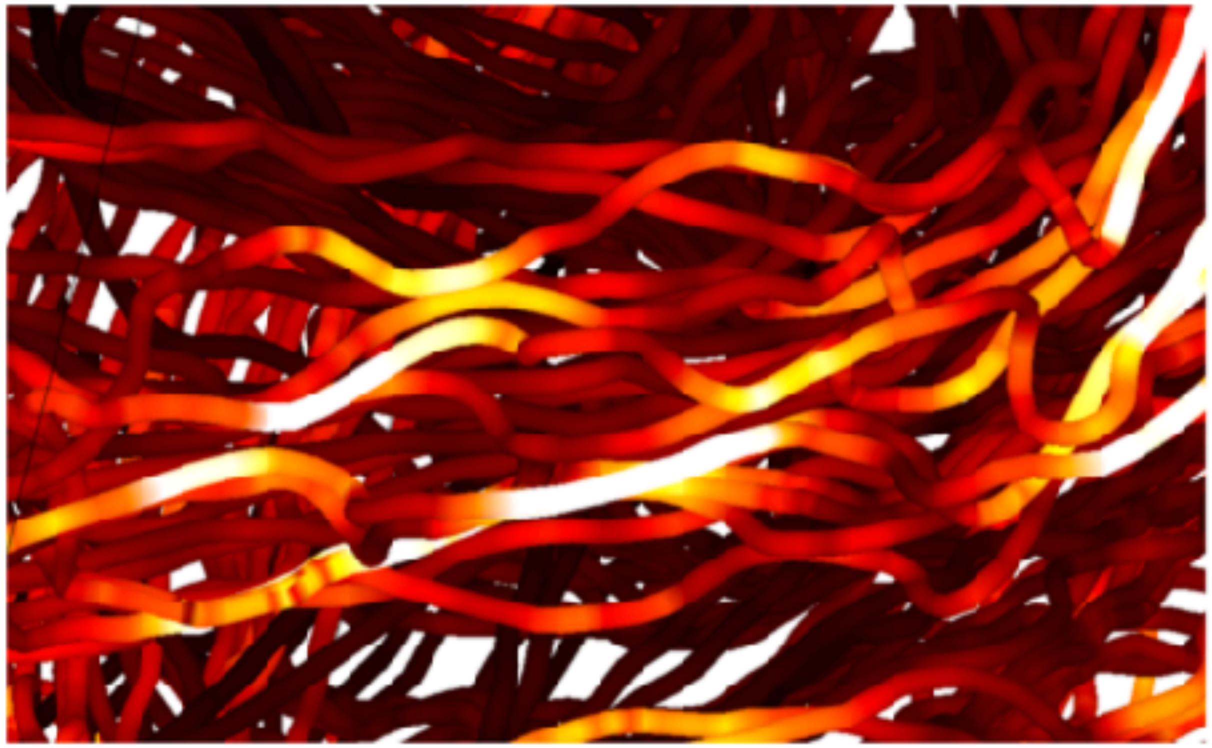
adiabatic evolution produces pressure anisotropy…

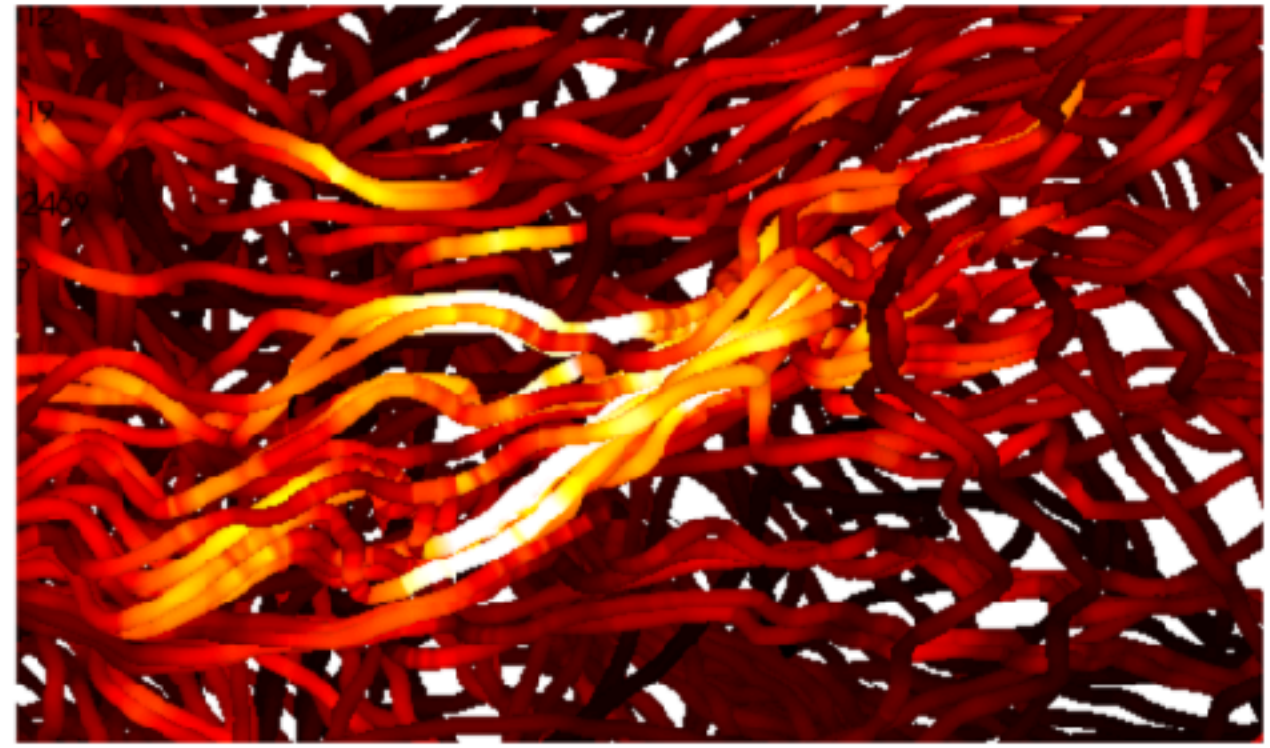


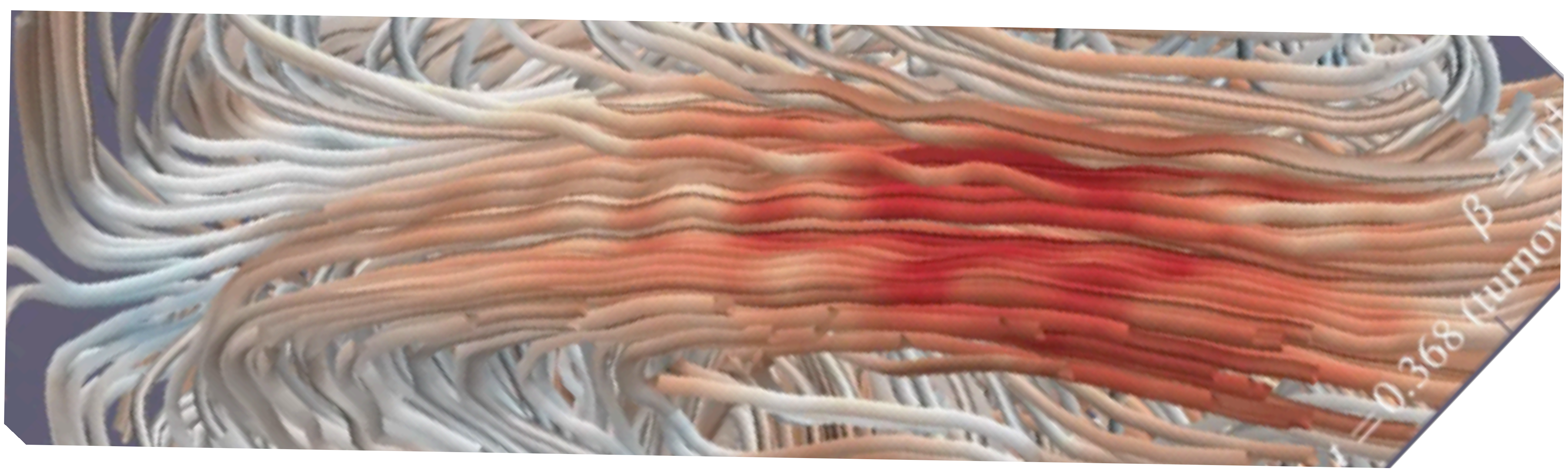
…which is relaxed by firehose/mirror instabilities.



mirror instability observed

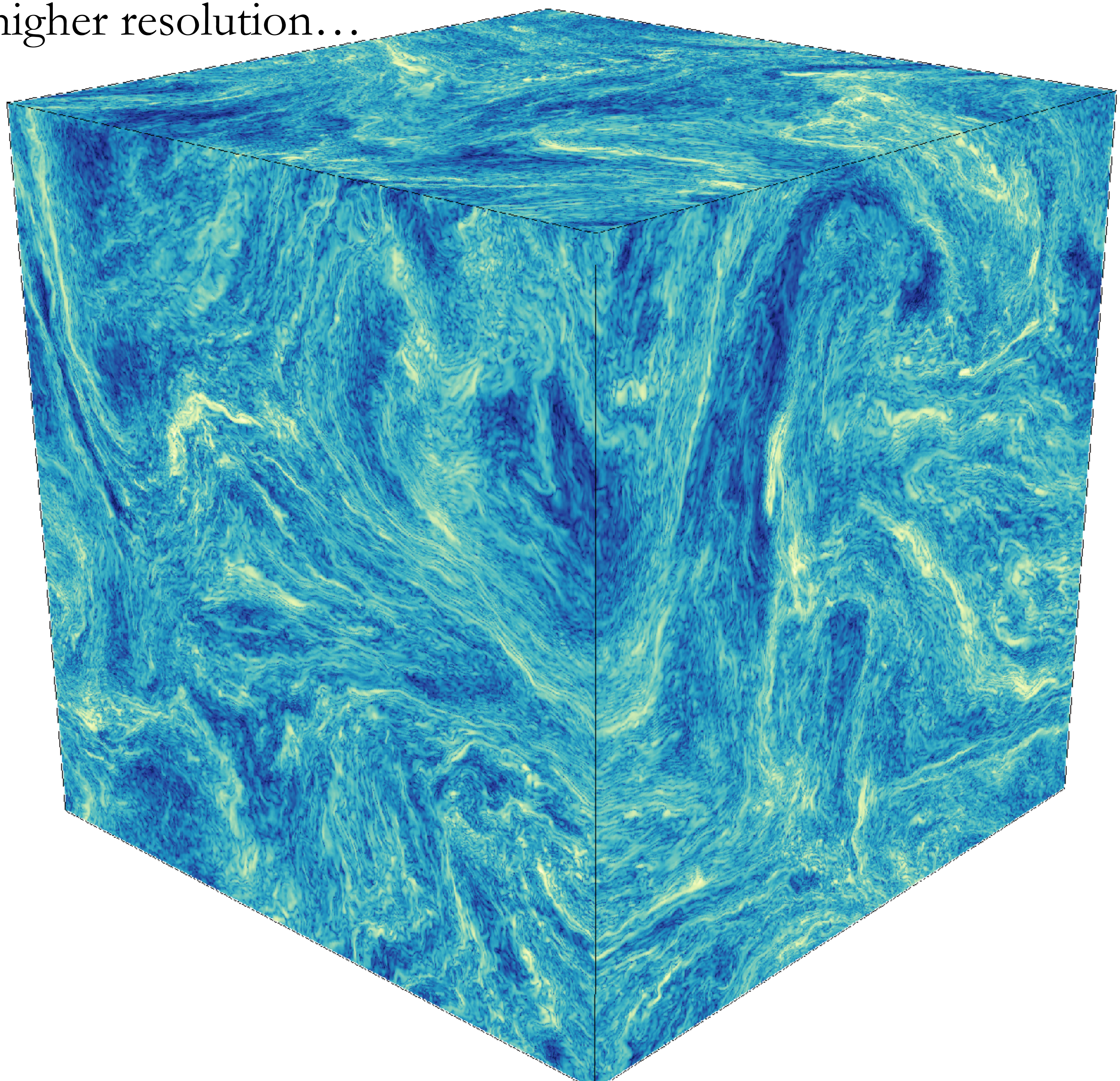


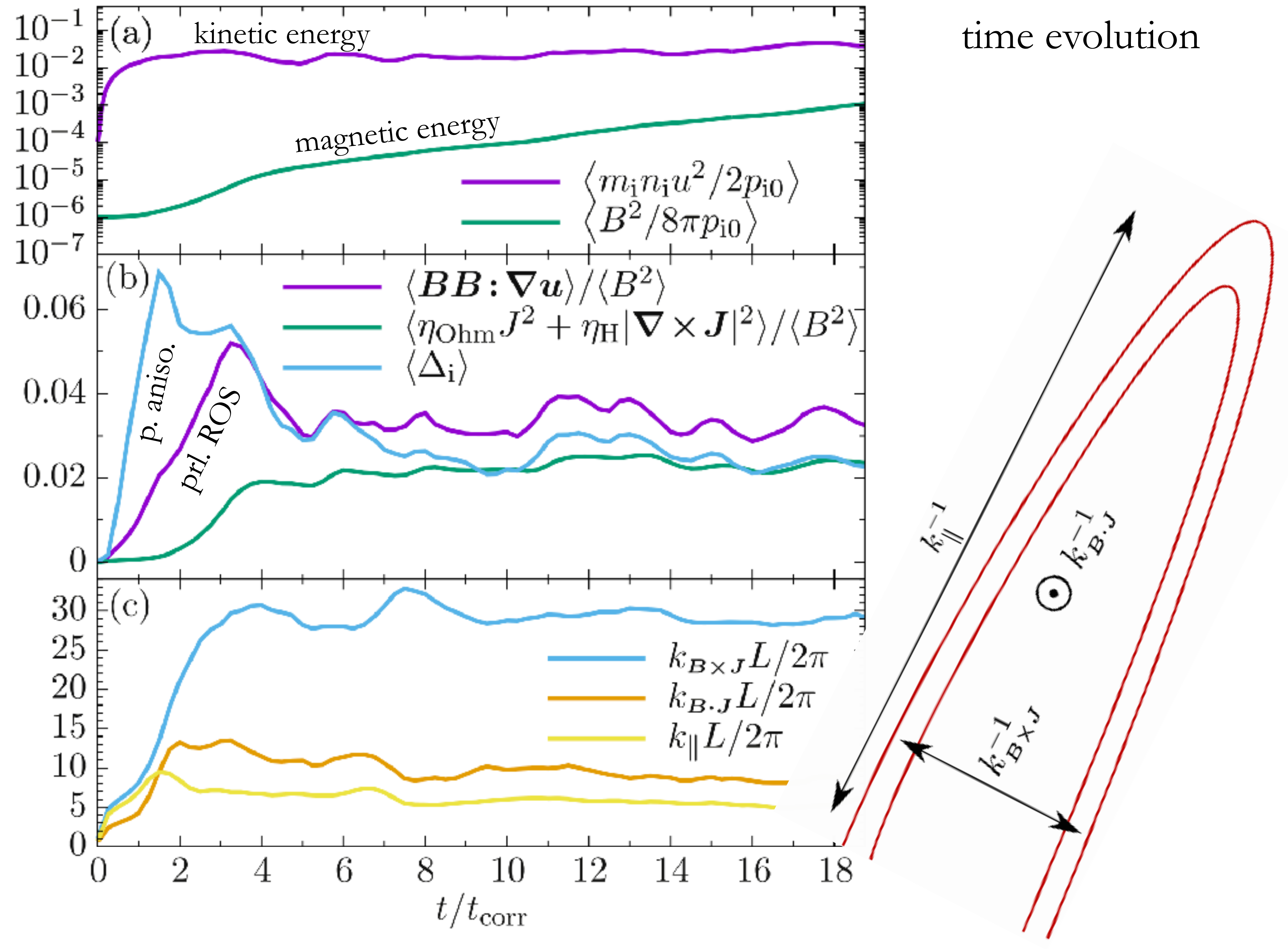




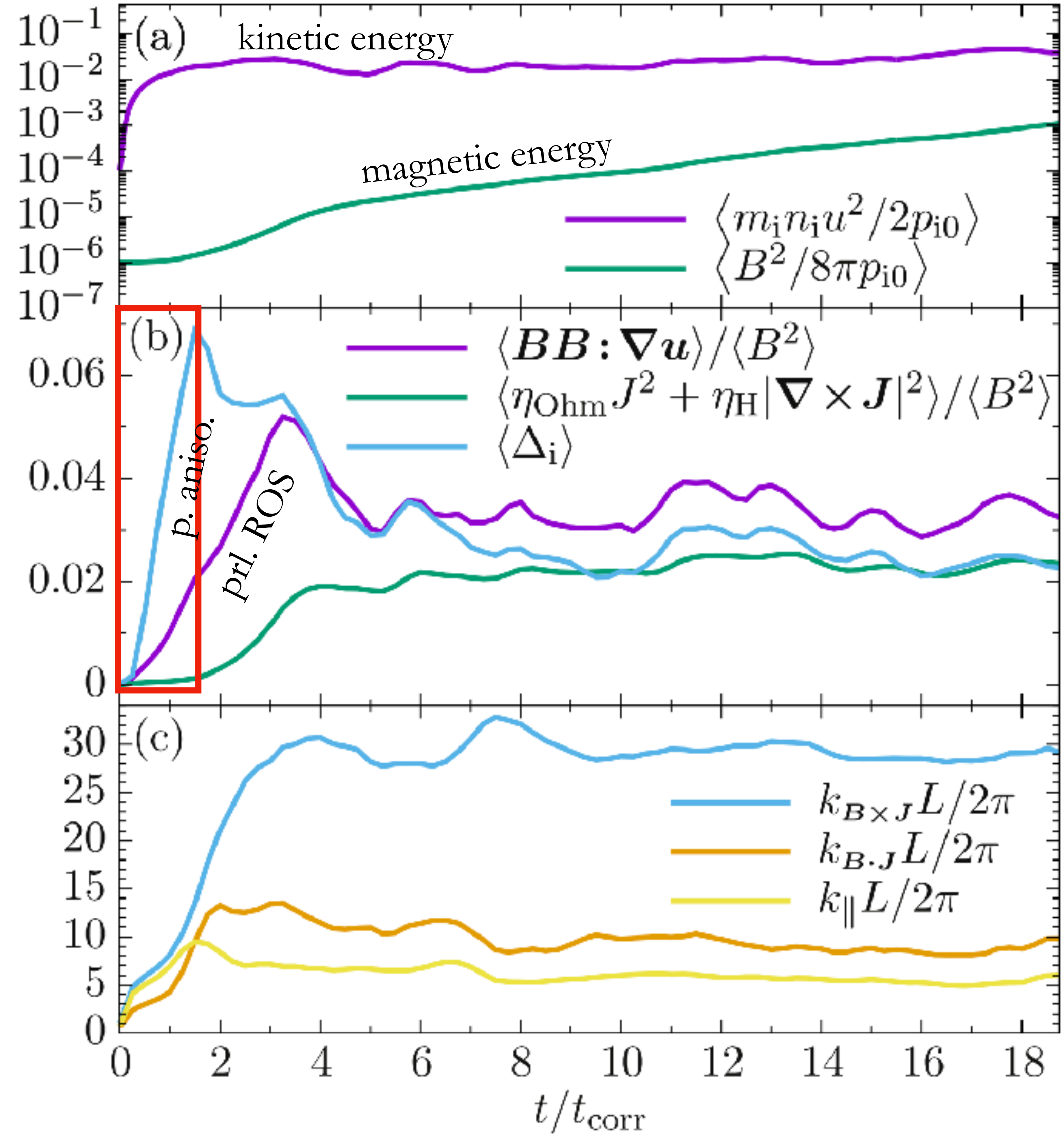
also seen in François' plasma dynamo simulations

at even higher resolution…
1008^3





time evolution
(a)
kinetic energy
magnetic energy
$\langle m_i n_i u^2/2p_{i0}\rangle$
$\langle B^2/8\pi p_{i0}\rangle$
(b)
$\langle \boldsymbol{BB}:\nabla\boldsymbol{u}\rangle/\langle B^2\rangle$
$\langle \eta_{\rm Ohm}J^2+\eta_{\rm H}|\nabla\times\boldsymbol{J}|^2\rangle/\langle B^2\rangle$
$\langle\Delta_i\rangle$
p. aniso.
prl. ROS
(c)
$k_{\boldsymbol{B}\times\boldsymbol{J}}L/2\pi$
$k_{\boldsymbol{B}.\boldsymbol{J}}L/2\pi$
$k_{\parallel}L/2\pi$
$t/t_{\rm corr}$
$k_{\parallel}^{-1}$
$k_{\boldsymbol{B}.\boldsymbol{J}}^{-1}$
$k_{\boldsymbol{B}\times\boldsymbol{J}}^{-1}$



"rapid growth phase"

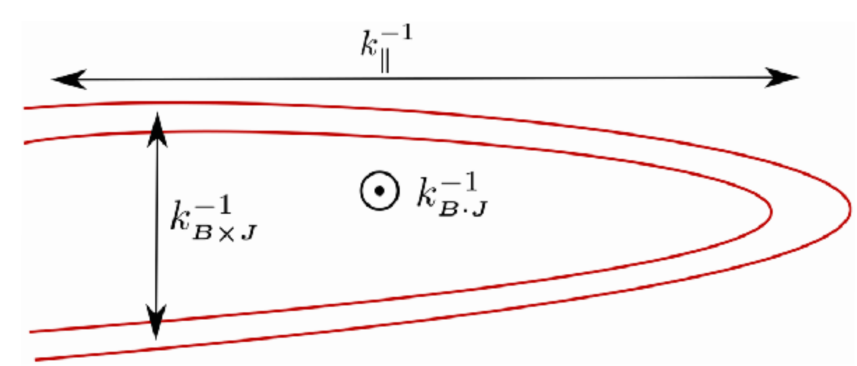
production of p. aniso.

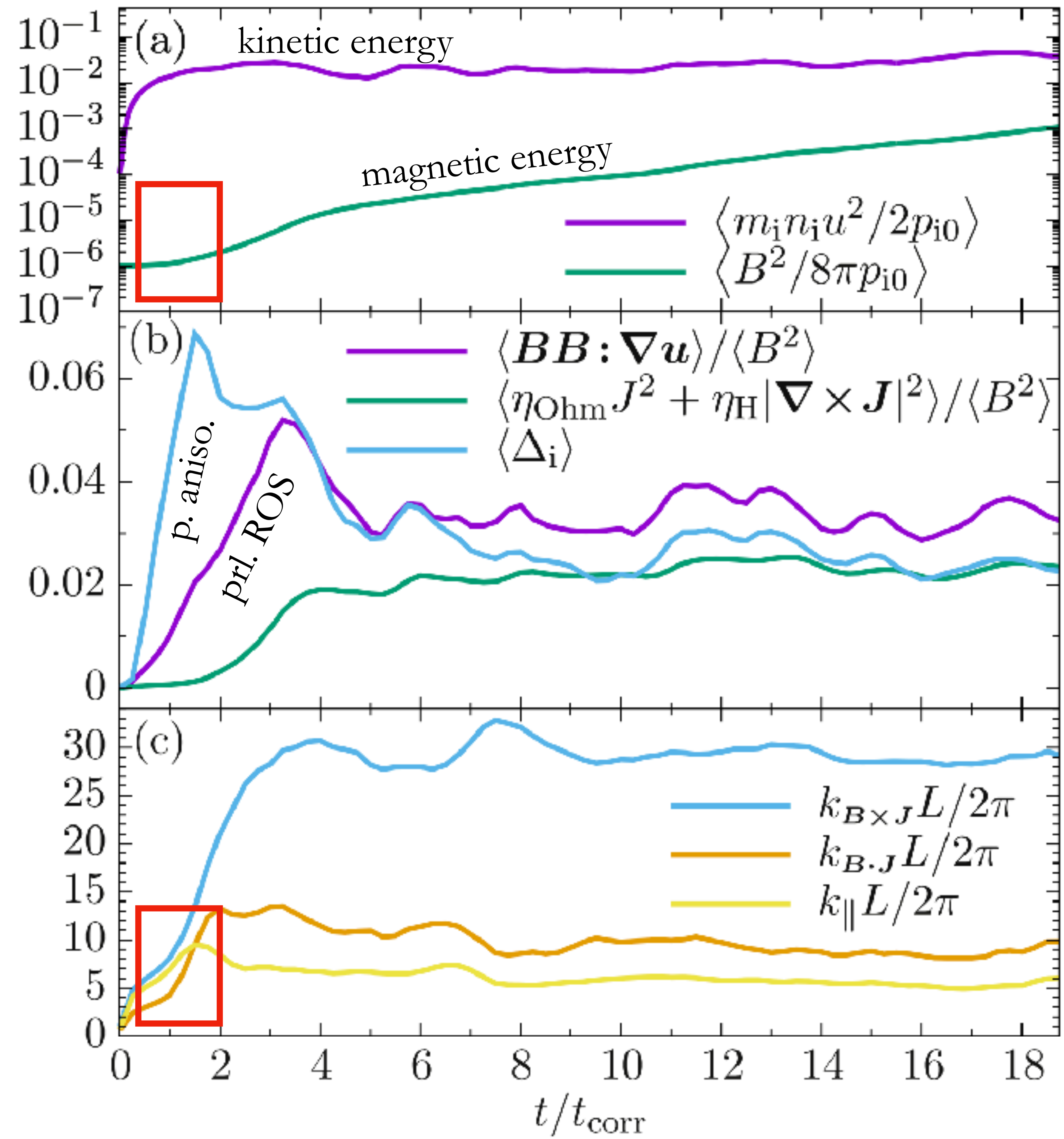
mirror/firehose growth

slight regulation of rate-of-strain

regulation of p. aniso., with Braginskii-esque closure:

$$\Delta \sim \frac{\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla u}}{\nu_\mathrm{eff}}$$





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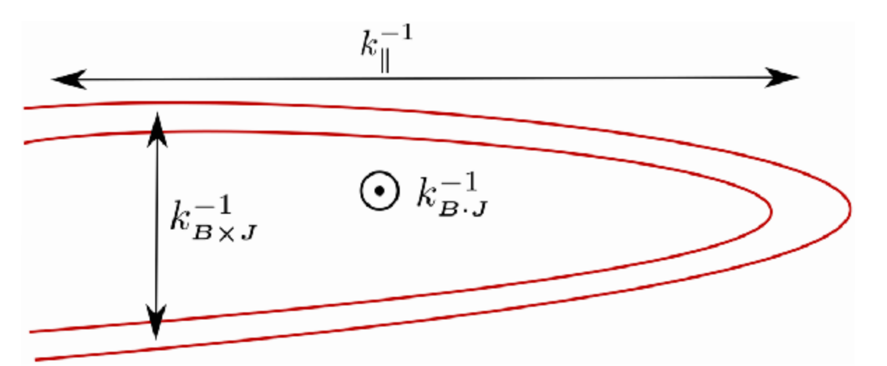

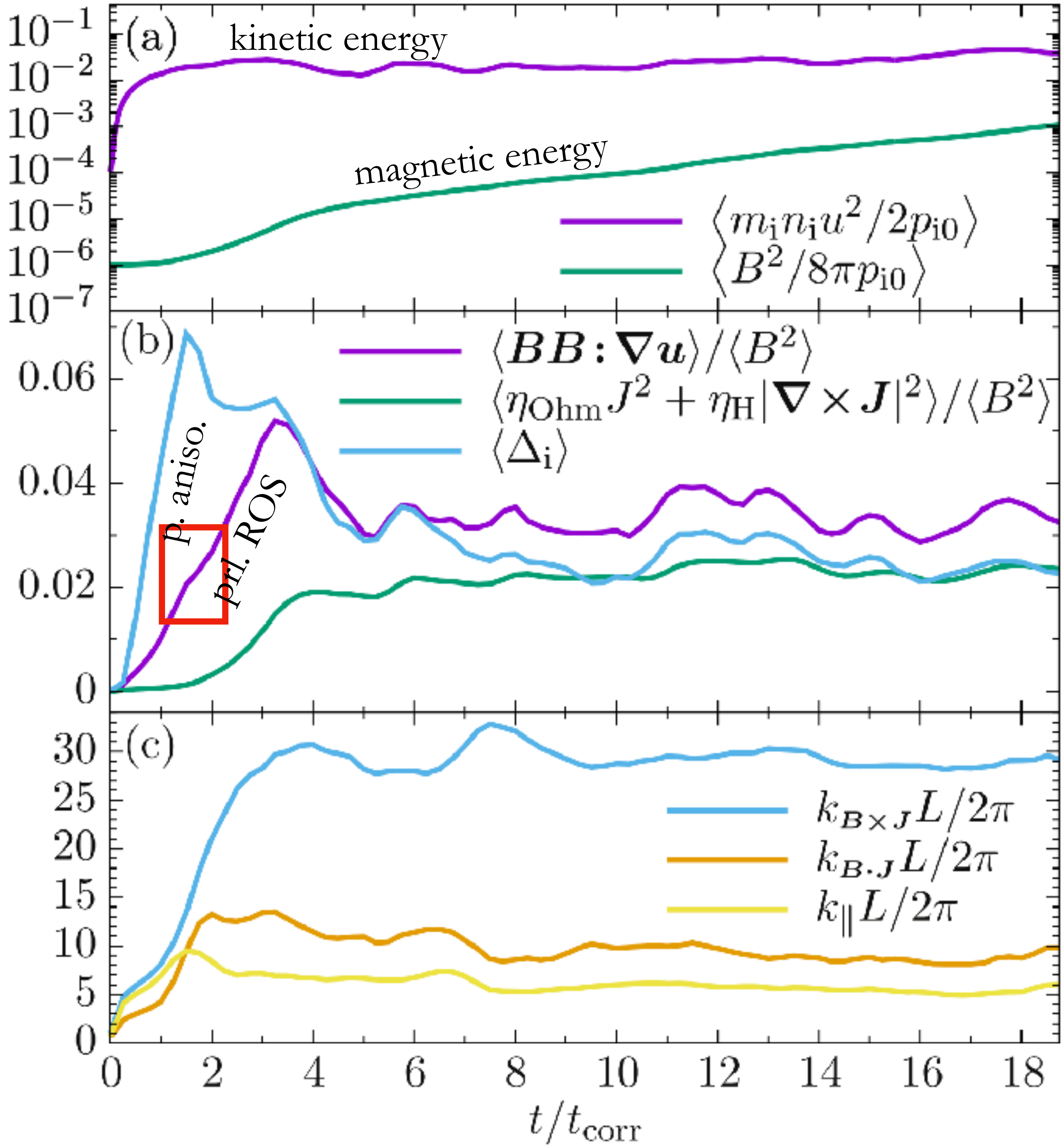

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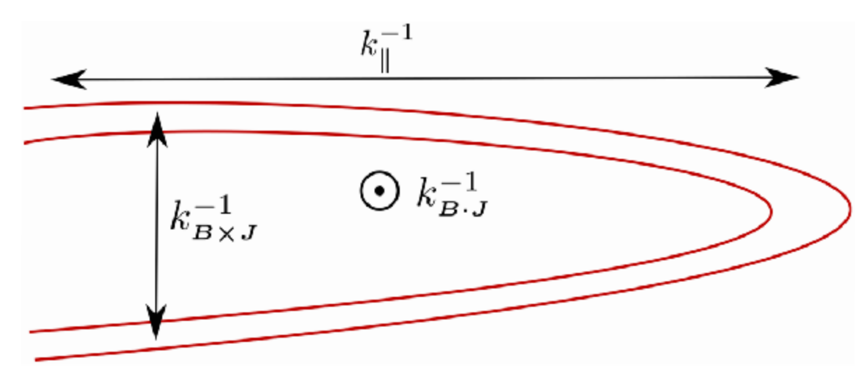

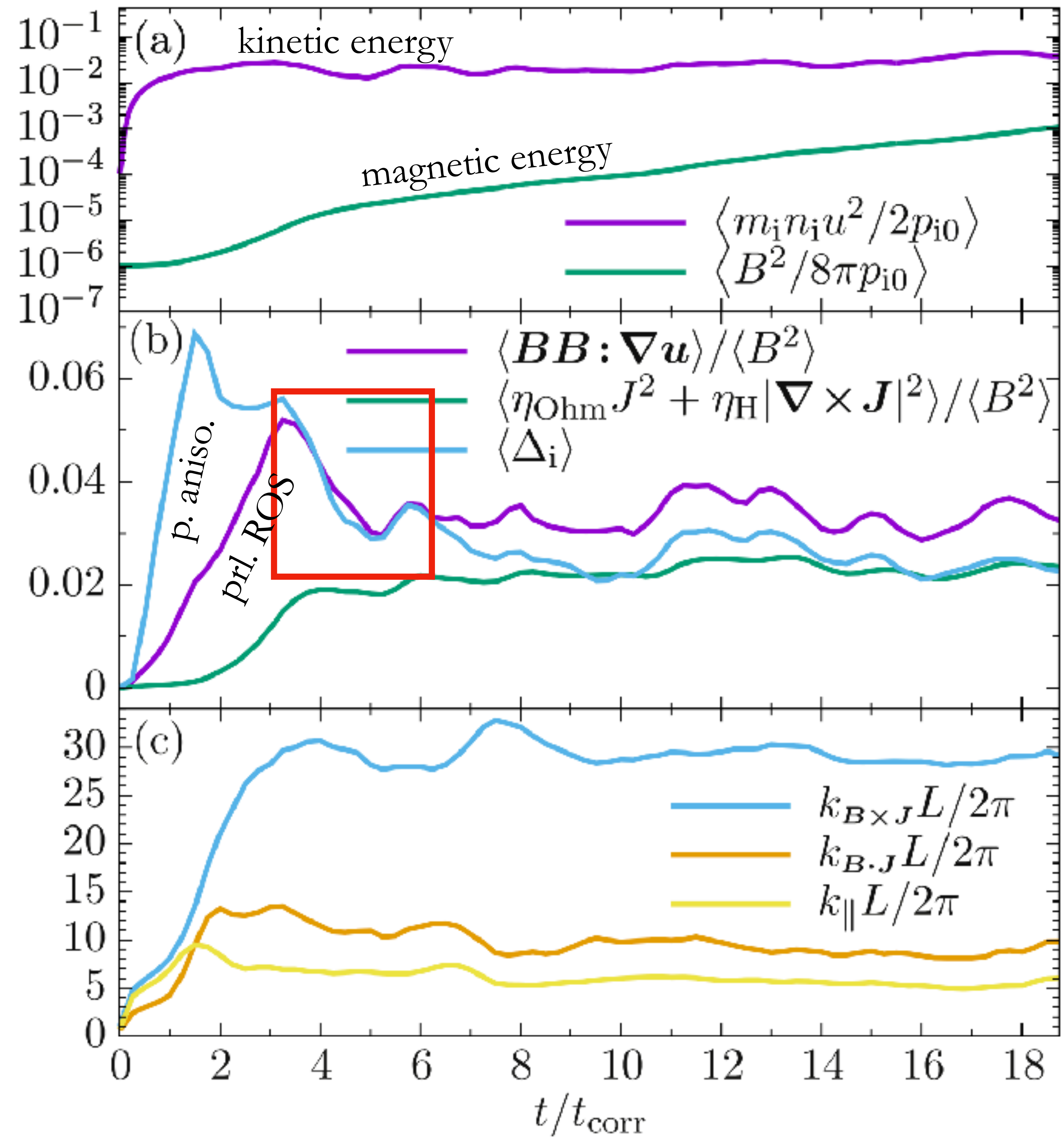

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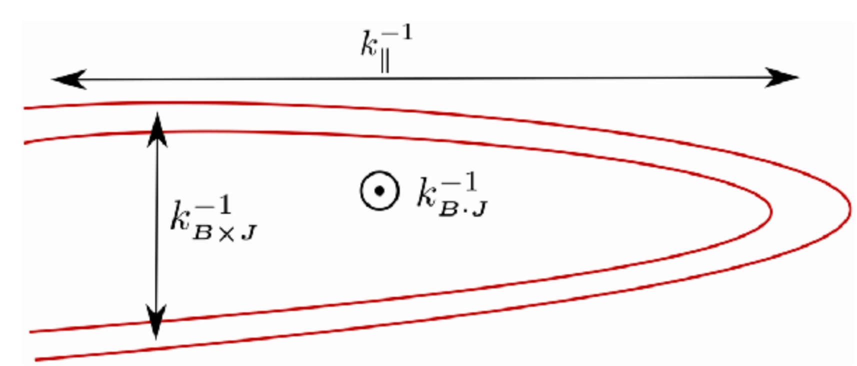
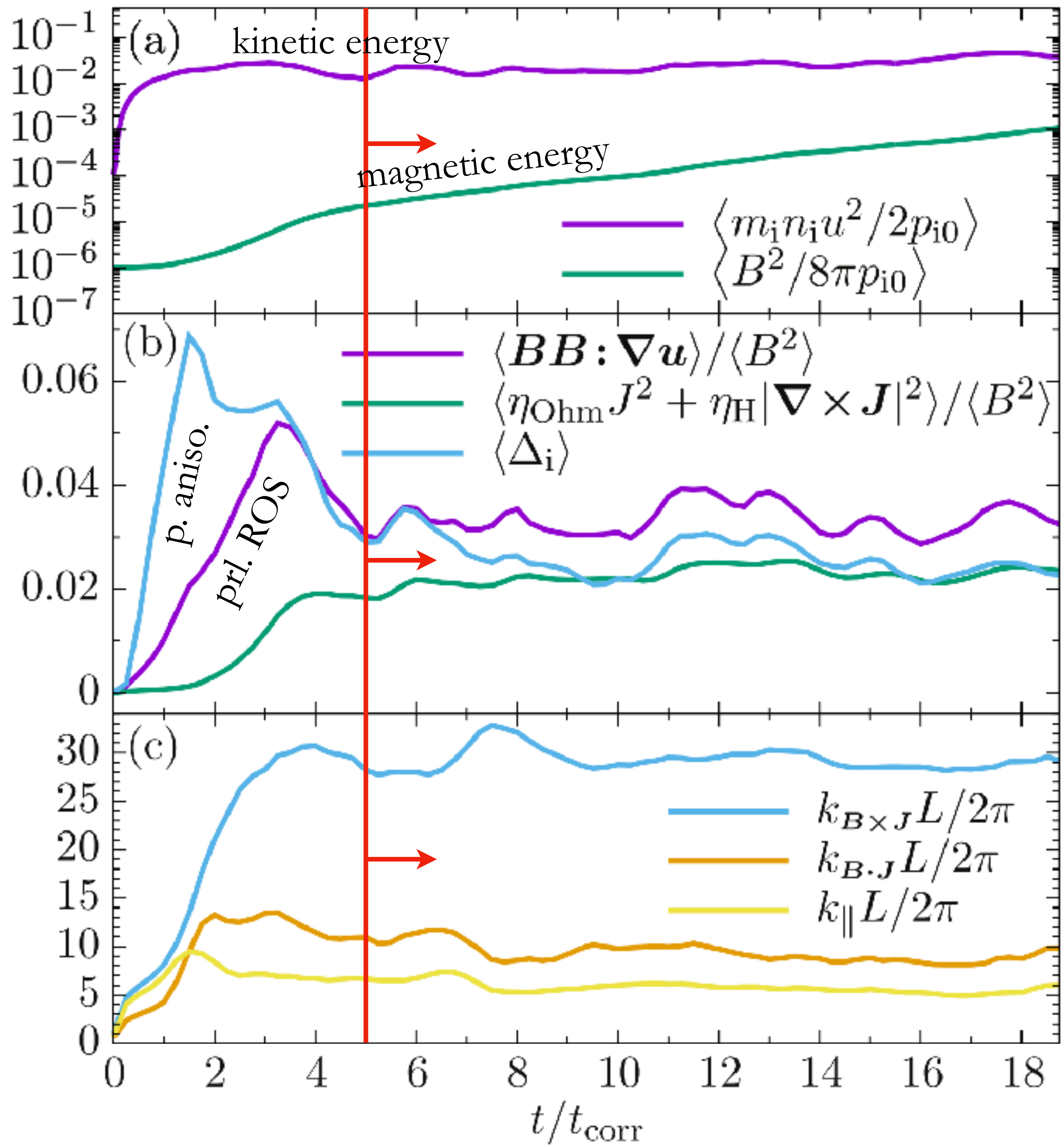


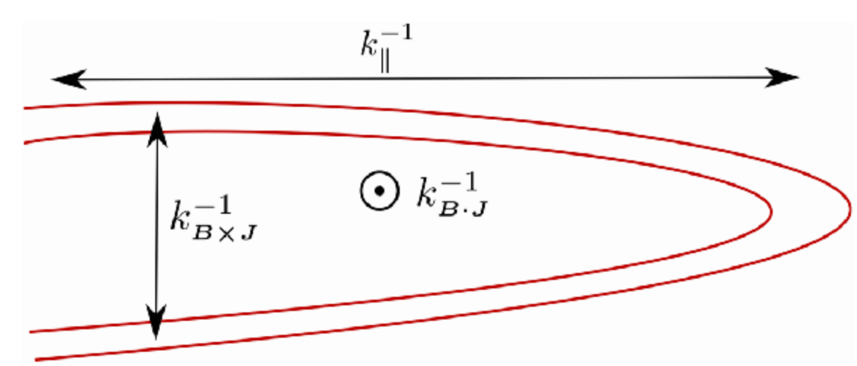


"kinematic phase"

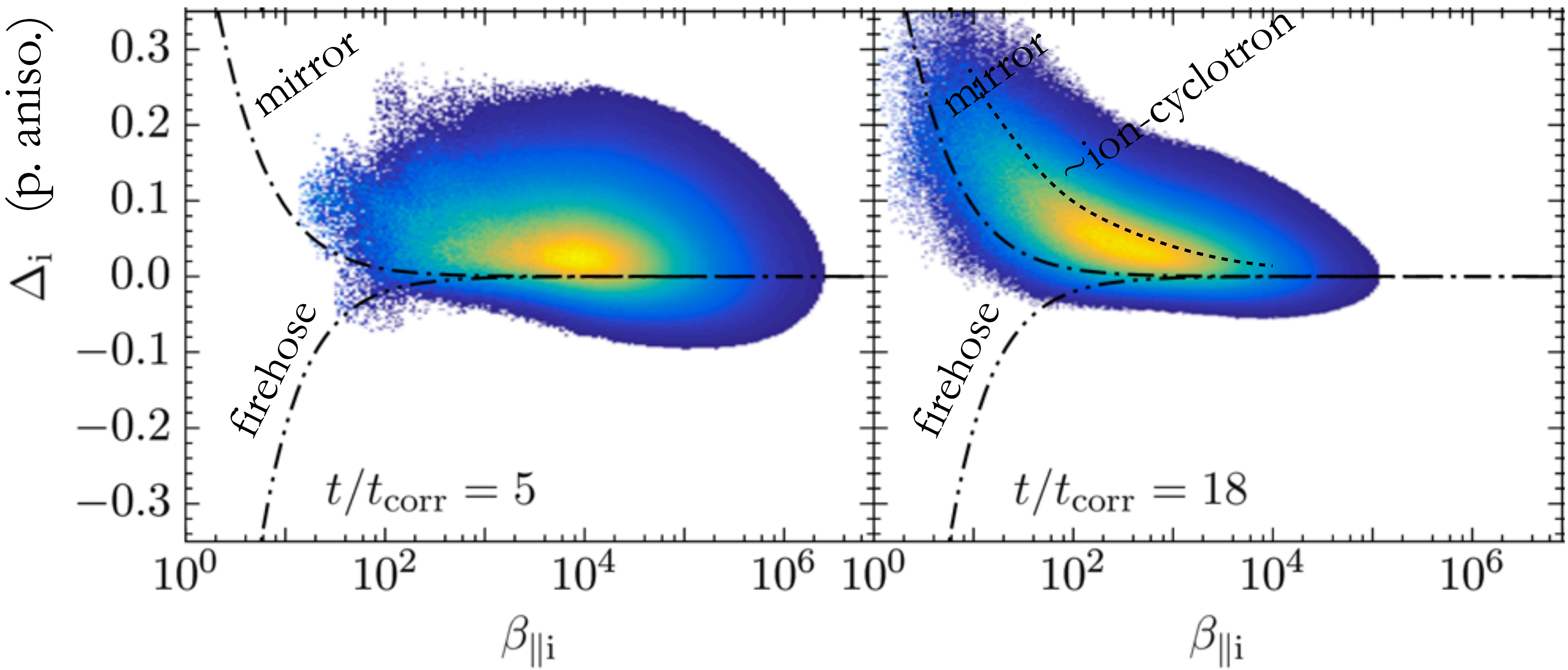
regulation of p. aniso., with Braginskii-esque closure:

$$\Delta \sim \frac{\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\nabla \boldsymbol{u}}{\nu_{\rm eff}}$$

exponential growth, folded fields, like Pm ≫ 1 dynamo!

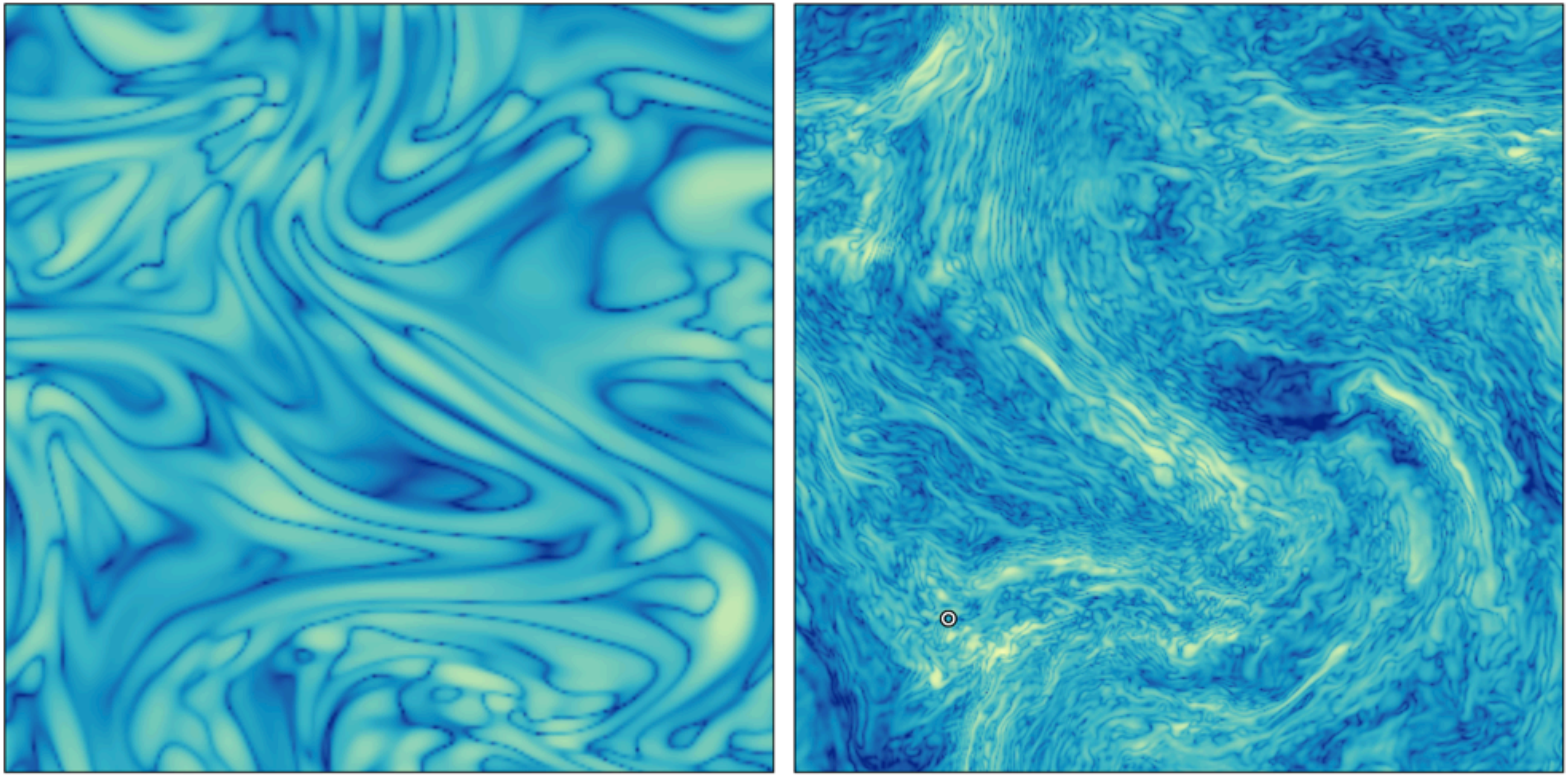


throughout exponential-growth phase, p. aniso. knows about thresholds



firehose/mirror instabilities limit
(though not completely)
departures from thermodynamic equilibrium

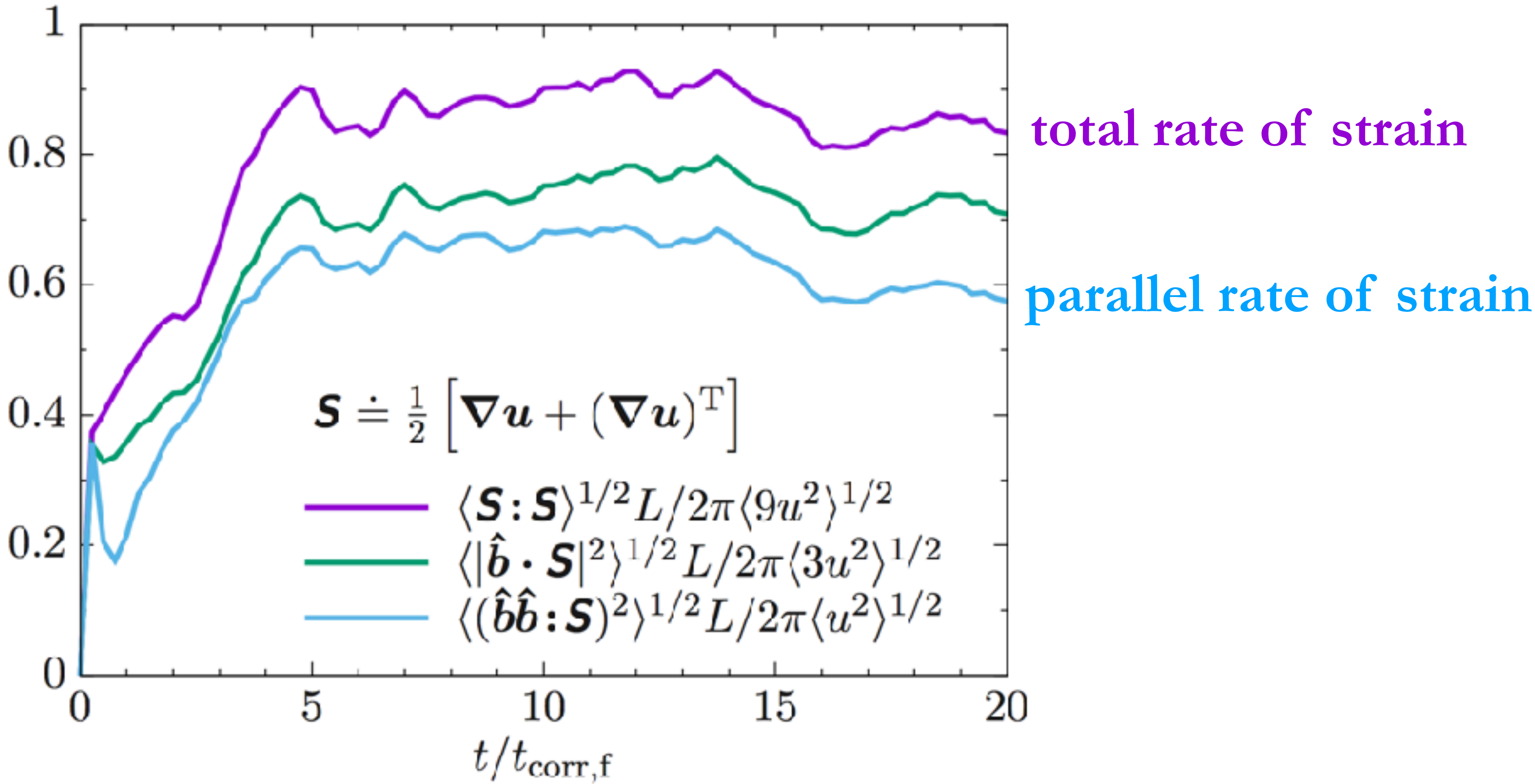
result is that collisionless plasma behaves like a Pm $\gg$ 1 fluid



MHD (Pm = 500) Kinetic

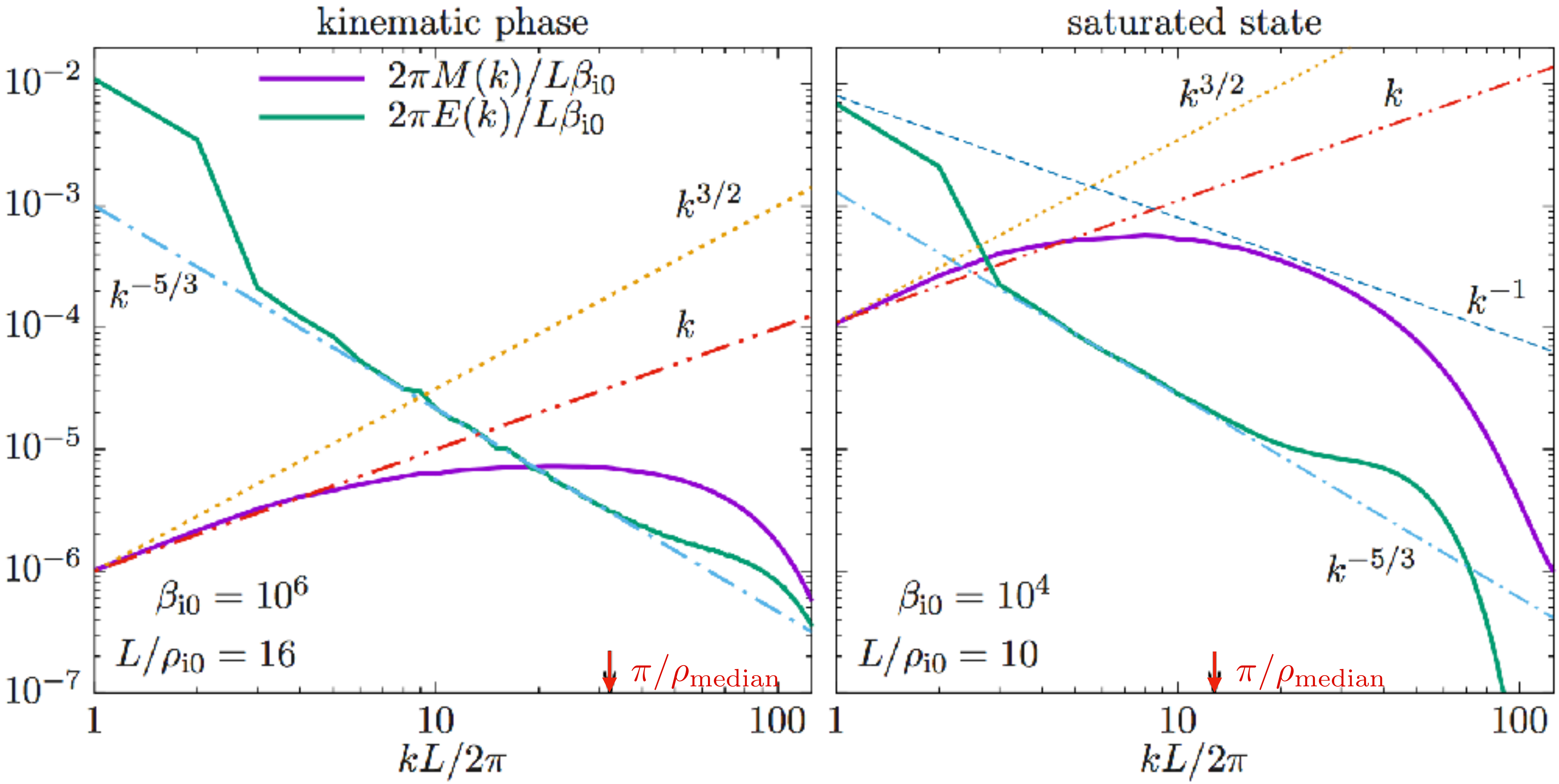
(take off your glasses)

but, the rate of strain is **anisotropic** w.r.t. B

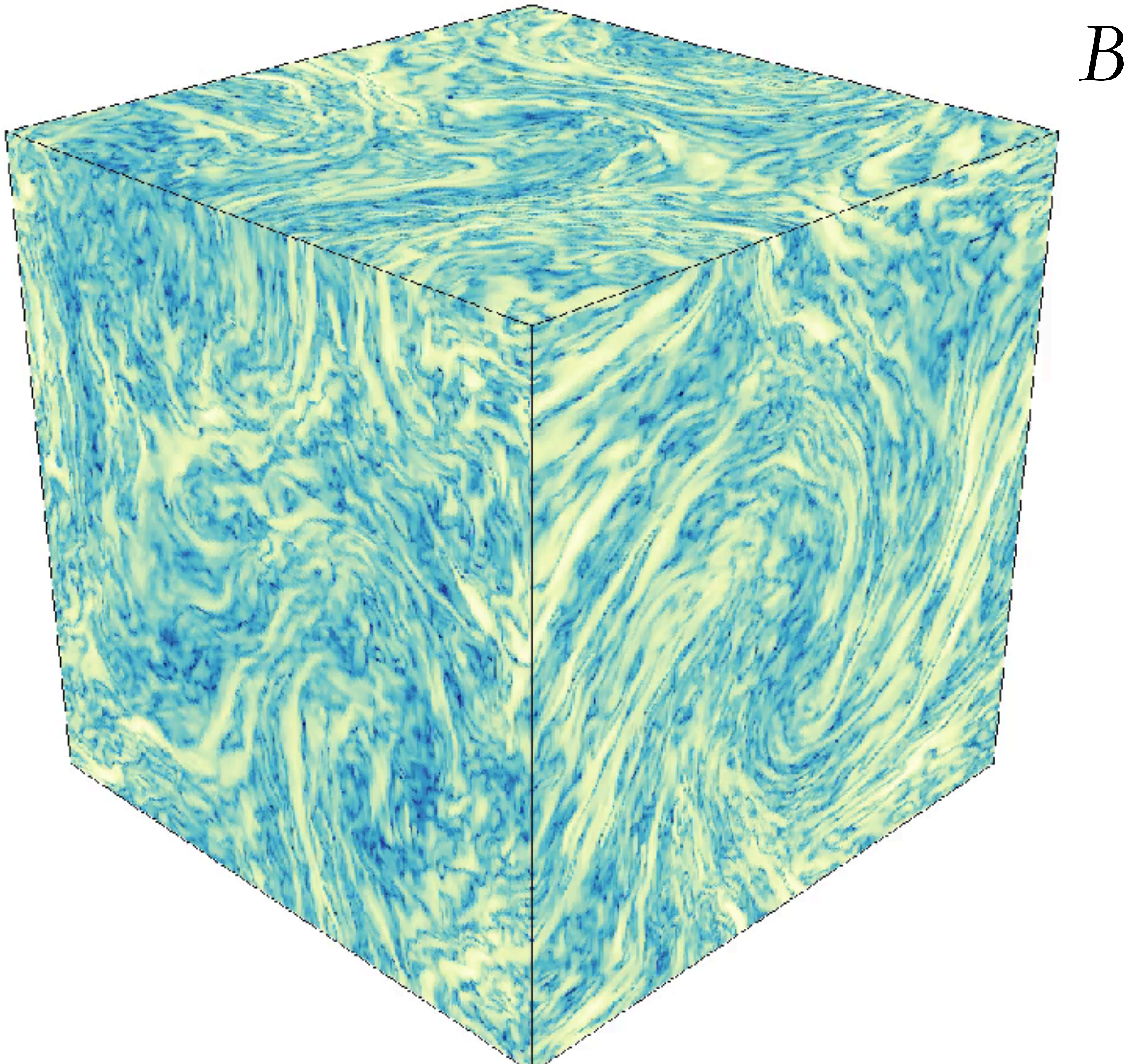


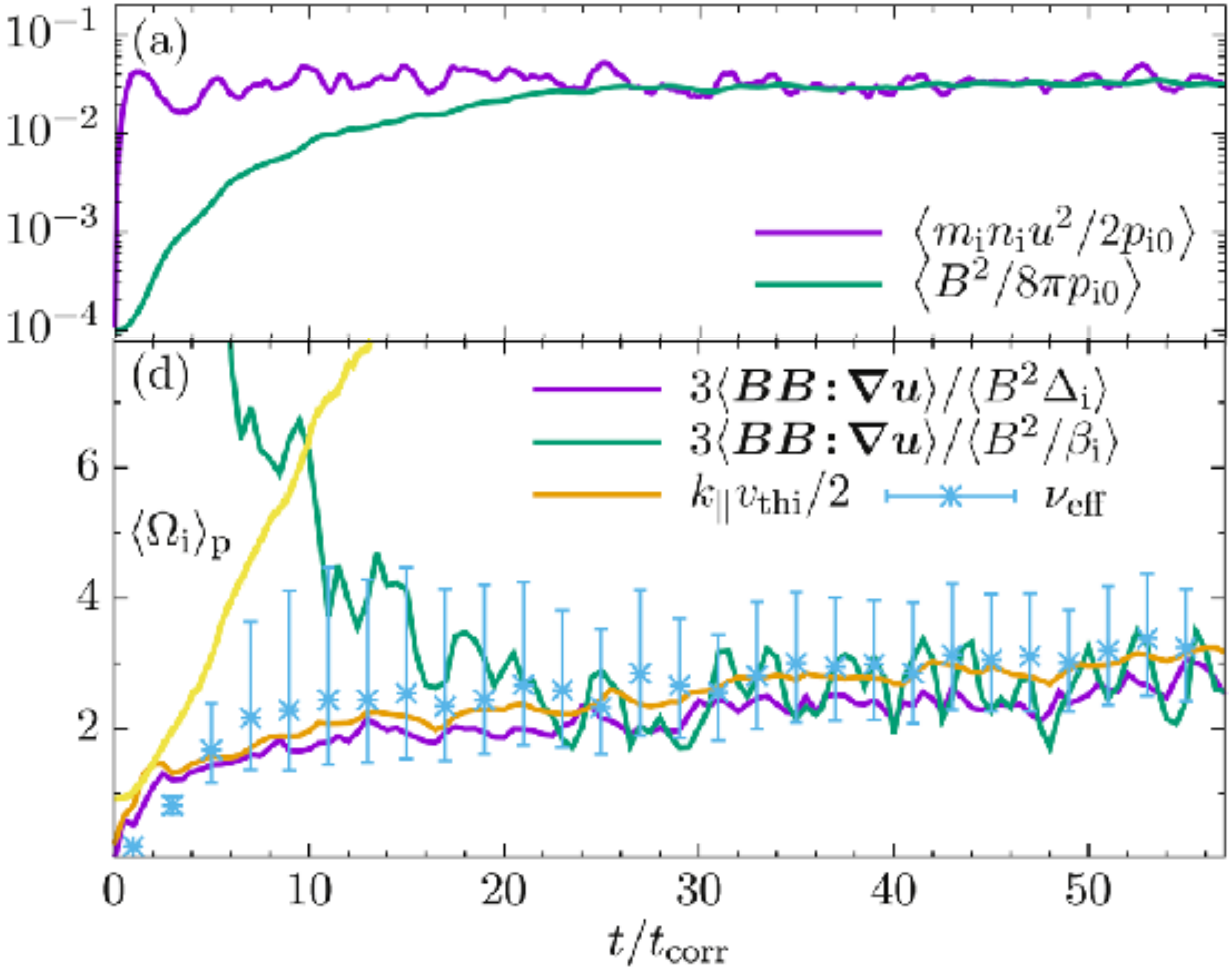
certain motions are preferred over others:
viscosity is **anisotropic**, as if it were a
weakly collisional, magnetized plasma (Braginskii 1965)

shell-averaged energy spectra



we did a separate run that reached nonlinear and saturation regimes

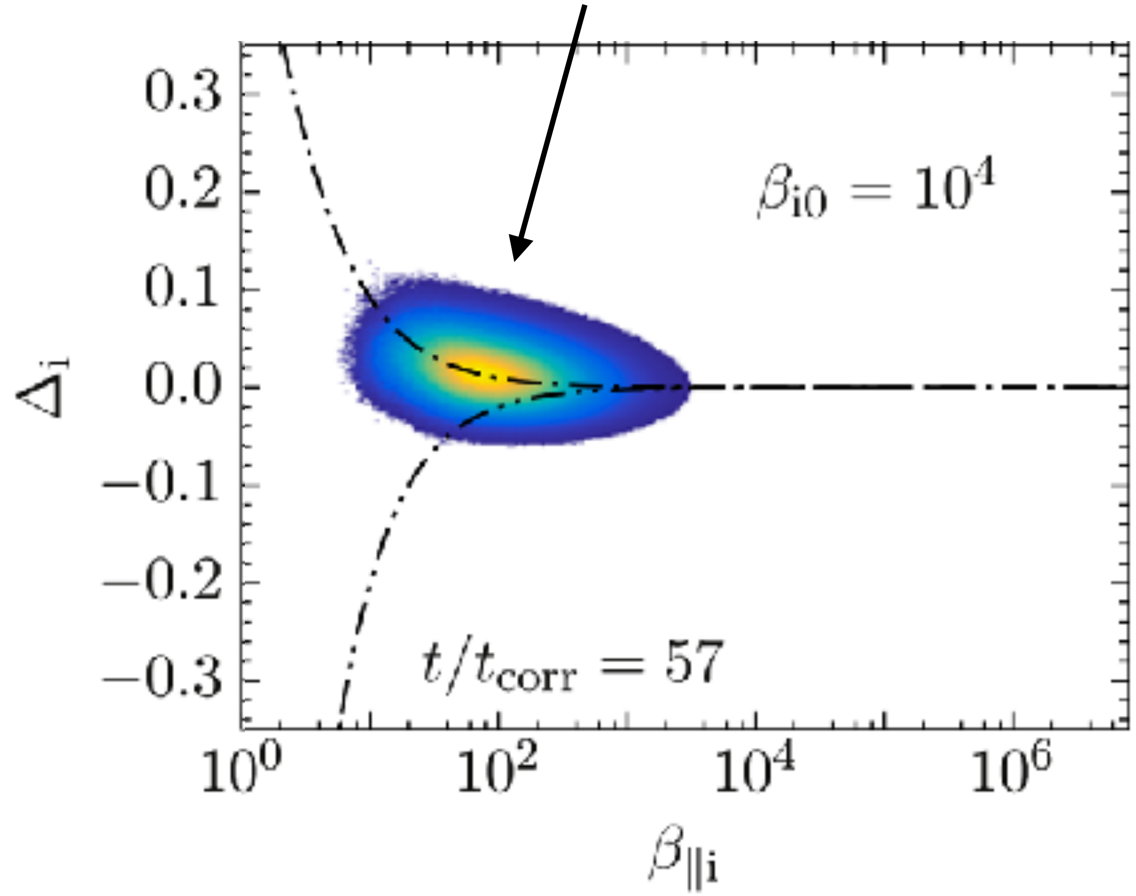




$\langle B^2\rangle \sim \langle u^2\rangle$

collisionality ~ shear × β

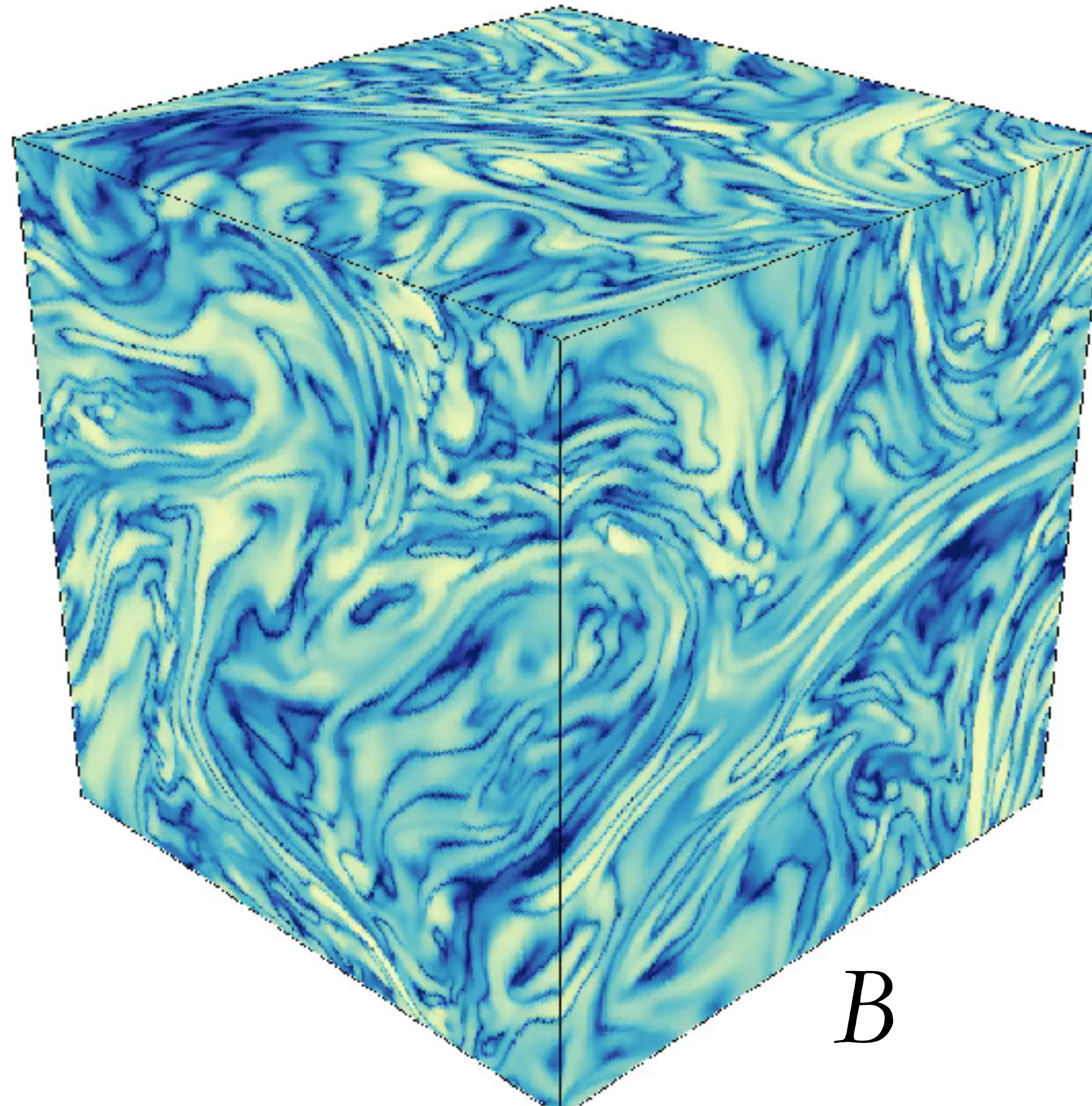
implies tight regulation of p. aniso. which is indeed seen



<u>problem for testing explosive growth</u>:
no room between "kinetic magnetized" and "fluid magnetized" regimes; explosive growth is predicted to onset in this run just as saturation occurs

Braginskii-MHD simulations using Snoopy

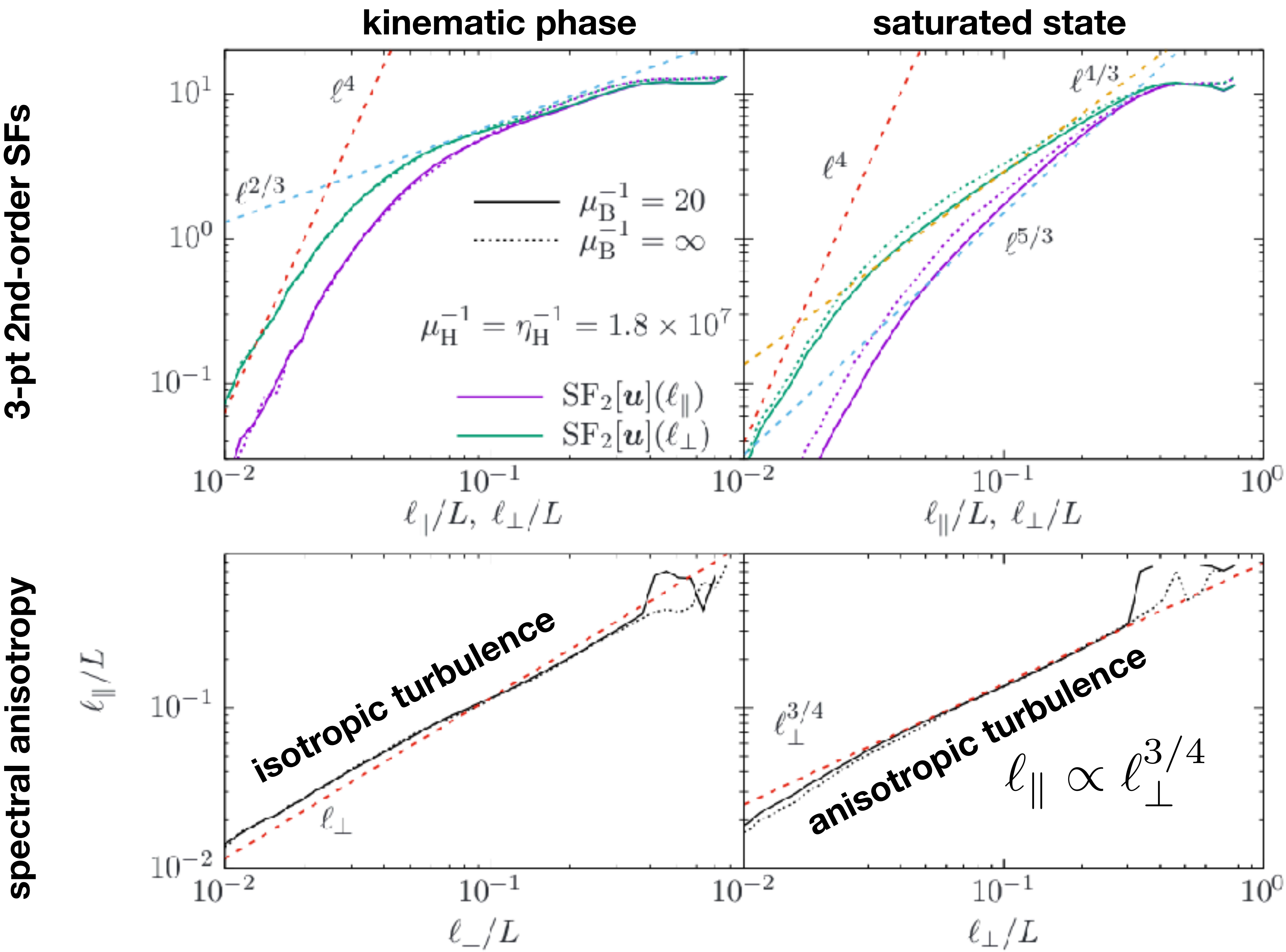
large parameter study with
hall-wall limited,
unlimited, and
"soft-wall" limited
closures on viscous stress



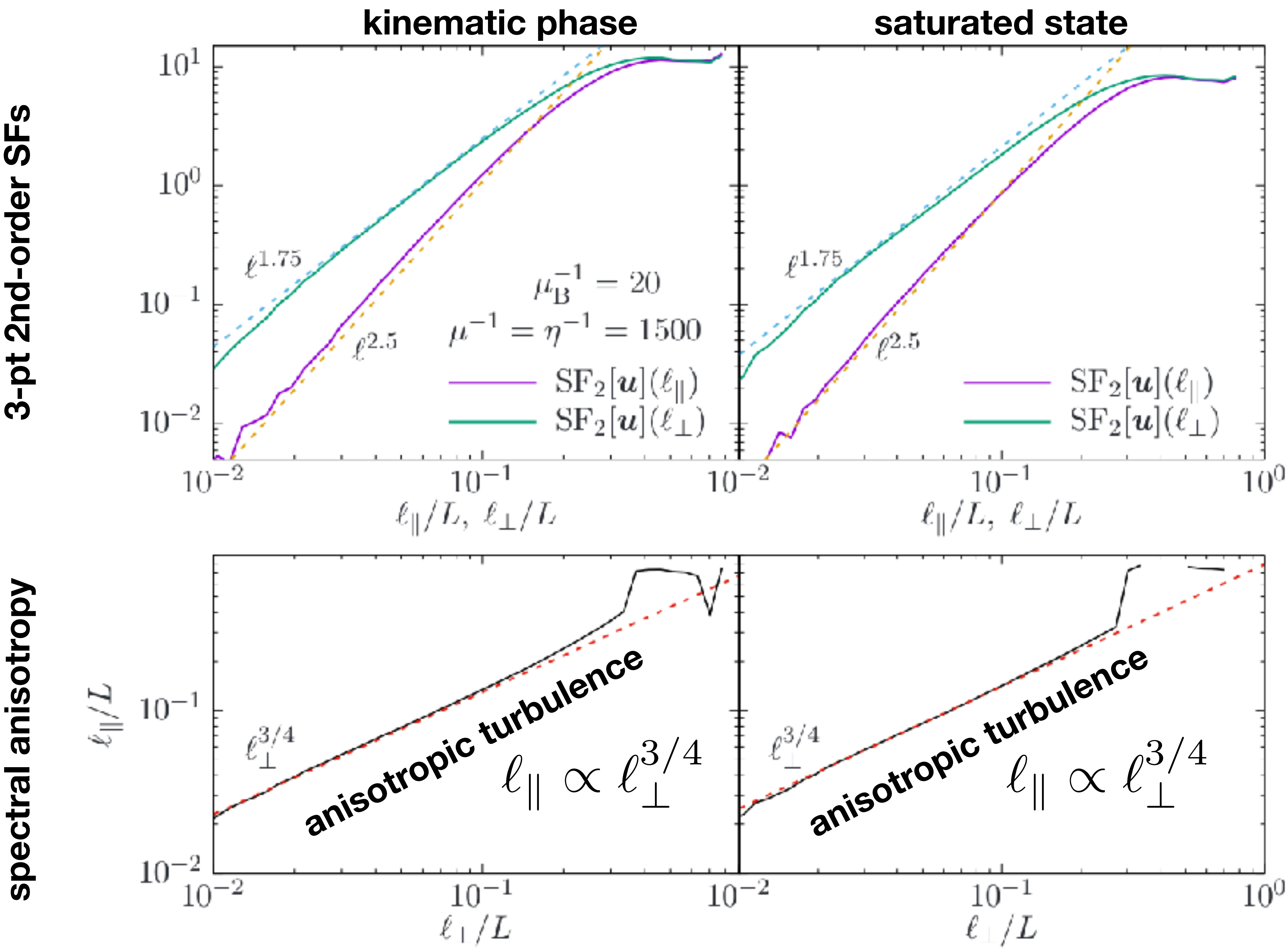
0th-order results:

- hard-wall limited Braginskii looks like Pm ≳ 1 MHD (see also Santos-Lima et al. 2014, who used CGL + anomalous collisionality motivated by fh/mr)

- unlimited Braginskii looks like saturated state of Pm ≳ 1 MHD ($B^2 \to \Delta p \propto \mathrm{d}_t B^2$ in tension) example of “hard-wall limited Braginskii looks like Pm ≳ 1 MHD”

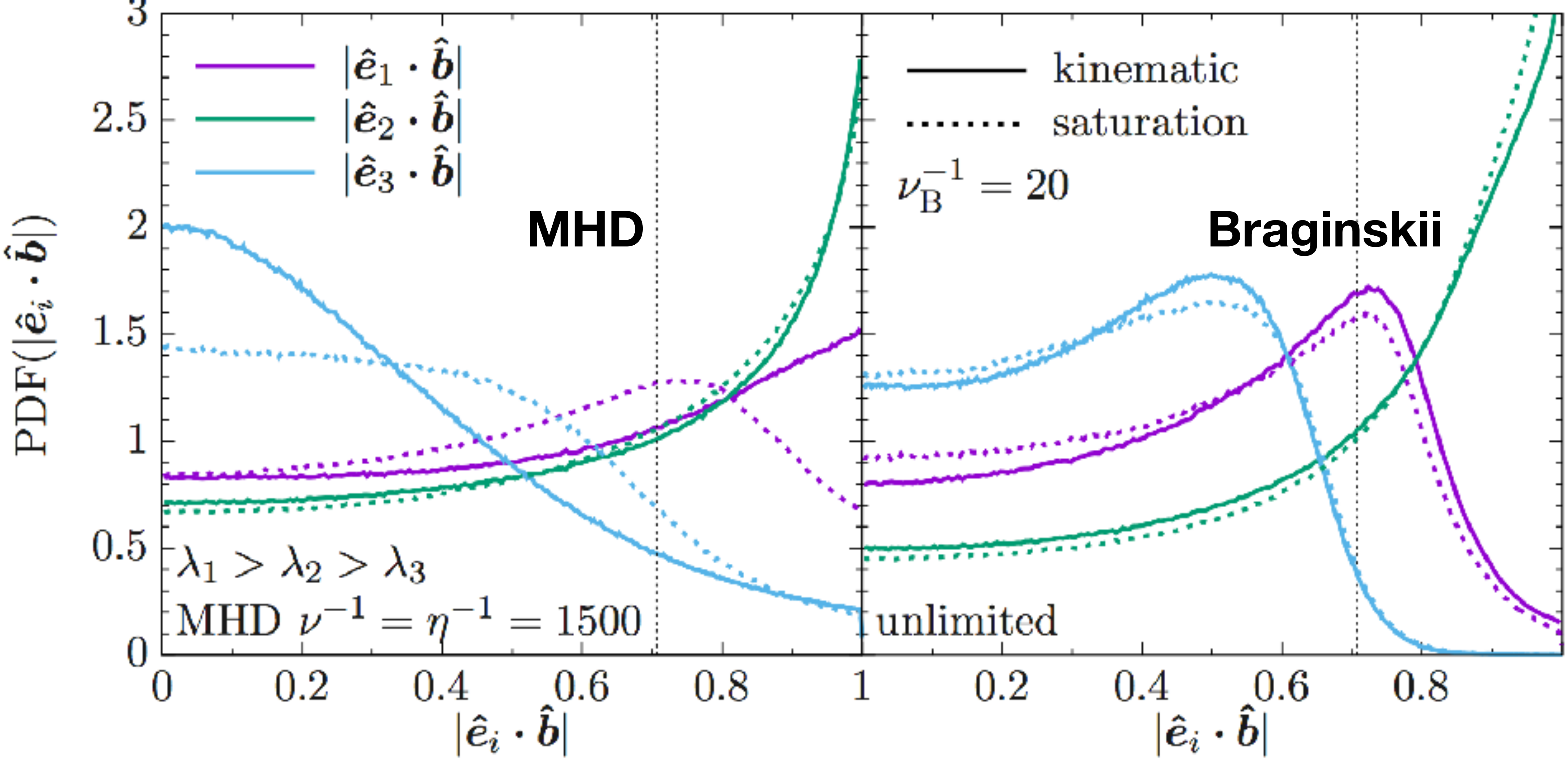


example of "unlimited Braginskii looks like saturated MHD"

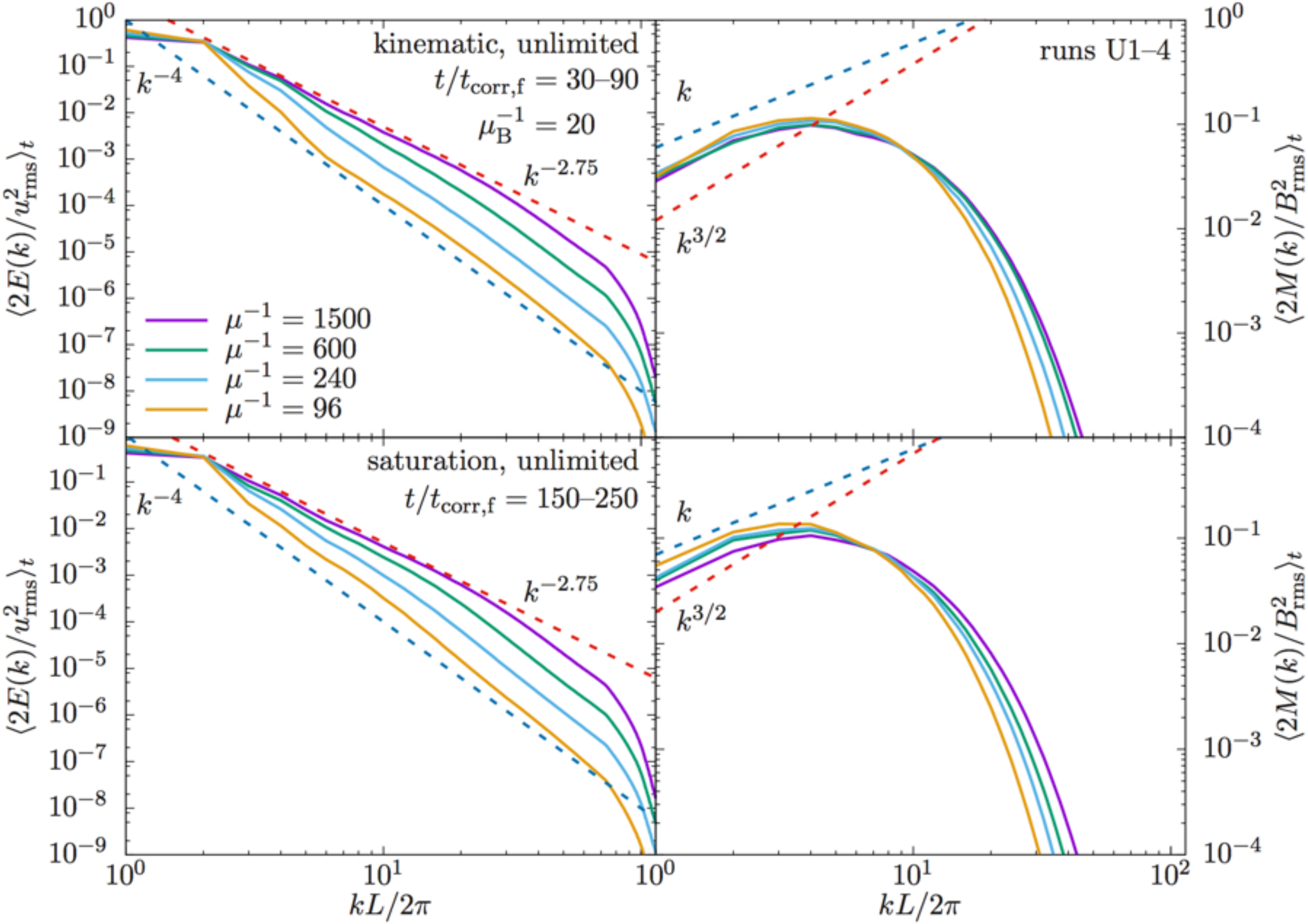


another example of "unlimited Braginskii looks like saturated MHD"

rate-of-strain eigenvectors projected onto local field direction



spectra in unlimited runs vs isotropic viscosity



calculate rate-of-strain tensor from sims,
find its eigenvectors and eigenvalues,
project eigenvectors onto magnetic-field direction $\hat{\boldsymbol{b}}$.
find a suppressed parallel ROS, $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla u}$.

<u>modified Kazantsev-Kraichnan model for magnetized plasma dynamo</u>

$$\langle u^i(t,\boldsymbol{r})\rangle = 0, \quad \langle u^i(t,\boldsymbol{r})u^j(t',\boldsymbol{r}')\rangle = \delta(t-t')\,\kappa^{ij}(\boldsymbol{r}-\boldsymbol{r}')$$

use a rate-of-strain tensor that knows about magnetic-field direction:

$$\kappa^{ij}(\boldsymbol{k}) = \kappa^{(\mathrm{iso})}(k,|\xi|)\big(\delta^{ij} - \hat{k}_i\hat{k}_j\big) + \kappa^{(\mathrm{aniso})}(k,|\xi|)\big(\hat{b}^i\hat{b}^j + \xi^2\hat{k}_i\hat{k}_j - \xi\hat{b}^i\hat{k}_j - \xi\hat{k}_i\hat{b}^j\big)$$

$$\xi \doteq \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{b}}$$

(a la Schekochihin *et al.* 2002, 2004 for sat. MHD) after some effort, can derive several statistics of the magnetic field, e.g., its 1D spectrum $M(k)$:

$$\frac{\partial M}{\partial t} = \frac{\gamma_\perp}{8}\frac{\partial}{\partial k}\left[\left(1 + 2\sigma_{||}\right)k^2\frac{\partial M}{\partial k} - \left(1 + 4\sigma_\perp + 10\sigma_{||}\right)kM\right] + 2(\sigma_{||} + \sigma_\perp)\gamma_\perp M - 2\eta k^2 M$$

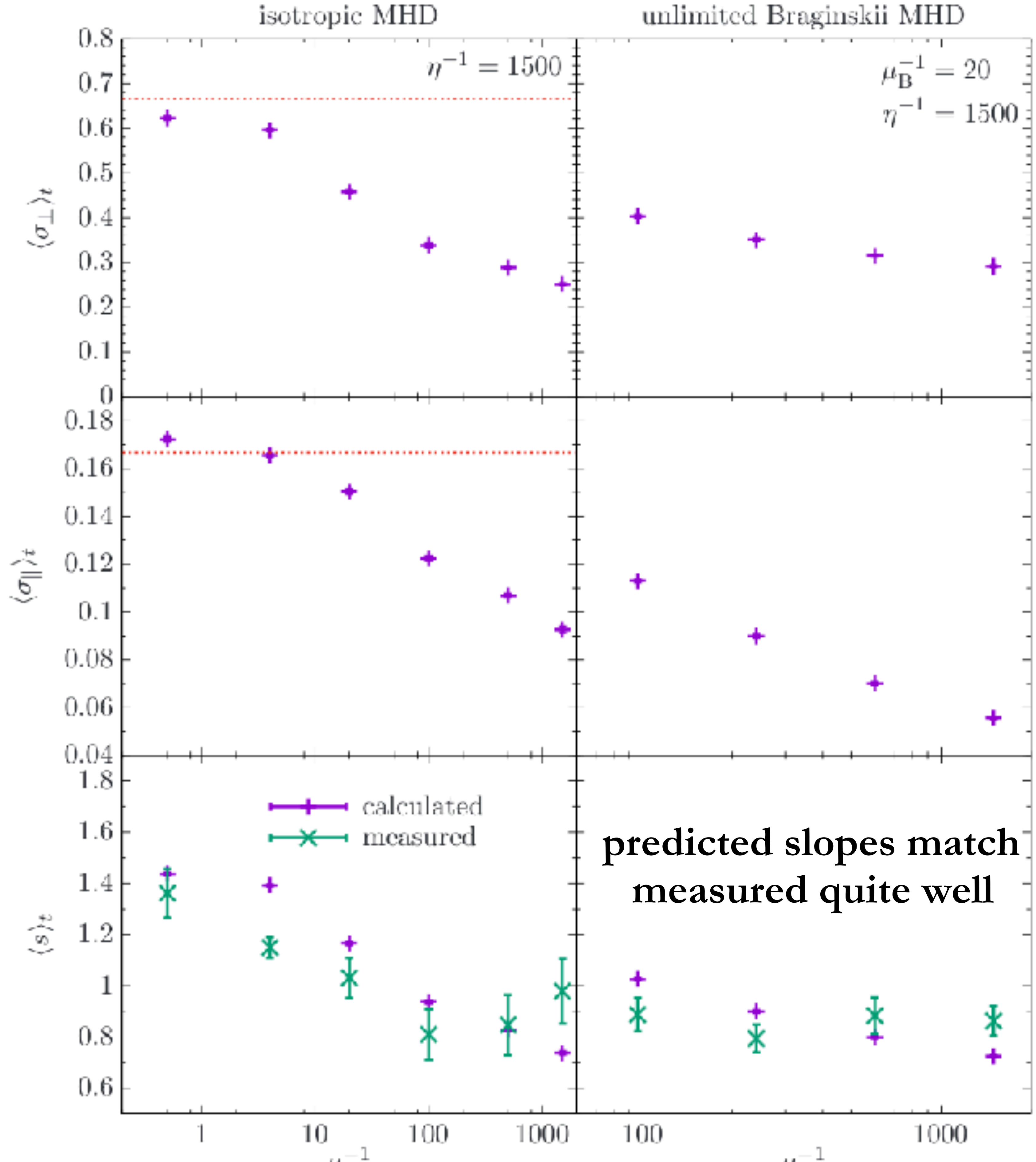
ratio of stretching to mixing (influences growth rate and spectral index)

modified Kazantsev-Kraichnan model works very well at describing both hybrid-kinetic and Braginskii simulations

relative amount of mixing

relative amount of stretching

M(k) slope in saturation



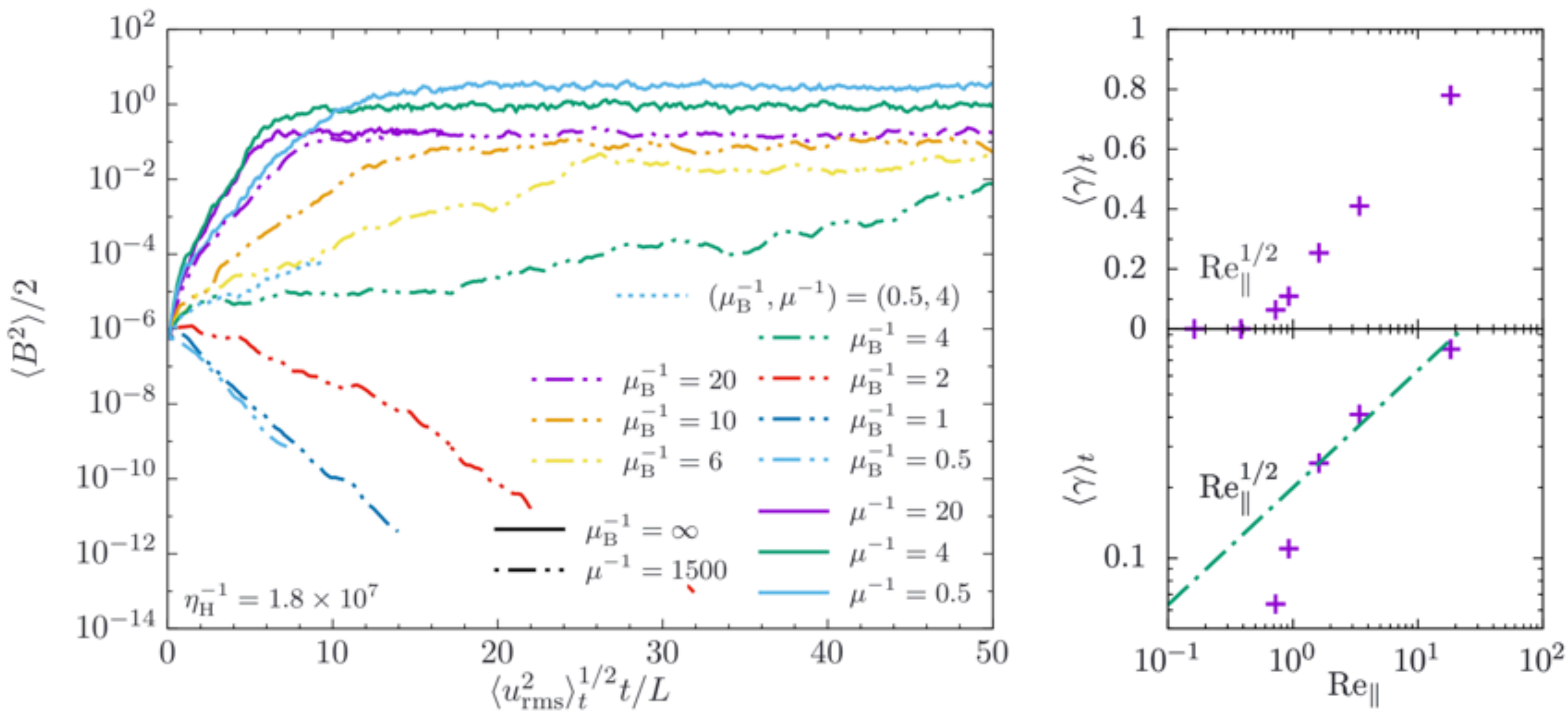
predicted slopes match measured quite well

no time to go through it here in detail, but
ratio of stretching and mixing is the key parameter

$$\mathrm{Re}\,,\ \mathrm{Pm} \quad \Rightarrow \quad \mathrm{Re}_{\parallel}\,,\ \mathrm{Re}_{\perp}\,,\ \mathrm{Pm}_{\parallel}$$

in particular, unlimited Braginskii dynamo is not viable if
mixing-to-stretching ratio is too large ($\mathrm{Re}_{\perp}/\mathrm{Re}_{\parallel}$)

(related to Zel'dovich anti-dynamo thm + Squire's magneto-immutability)



some take-aways on **plasma dynamo**

- Turbulent dynamo works in a collisionless plasma (see also Rincon *et al.* 2016), a non-trivial statement! Needs help from kinetic instabilities (little friends). Can amplify B to dynamically important strengths.

- In many respects, collisionless magnetized plasma behaves as though it were weakly collisional, magnetized fluid with $\mathrm{Re}_{\|} \sim 1$, $\mathrm{Re}_{\perp} \gg 1$ and $\mathrm{Pm}_{\|} \gg 1$

- …because firehose/mirror easily triggered, break μ, and limit departures from LTE; wave-particle interactions supplant particle-particle interactions

- Hybrid-kinetic and Braginskii-MHD simulations performed and analyzed: St-Onge & Kunz 2018, *ApJL*; St-Onge, Kunz, Squire, Schekochihin 2019

- Some aspects of unlimited Braginskii match behavior in kinetic runs (hard-wall pressure anisotropy limiters might not always be a good closure)

- possibility of **explosive growth** up to ~nG fields in ICM (in prep.)

now investigating impact of tearing & reconnection on $\mathrm{Pm} \gtrsim 1$ dynamo
w/ Alex Schekochihin and Alisa Galishnikova

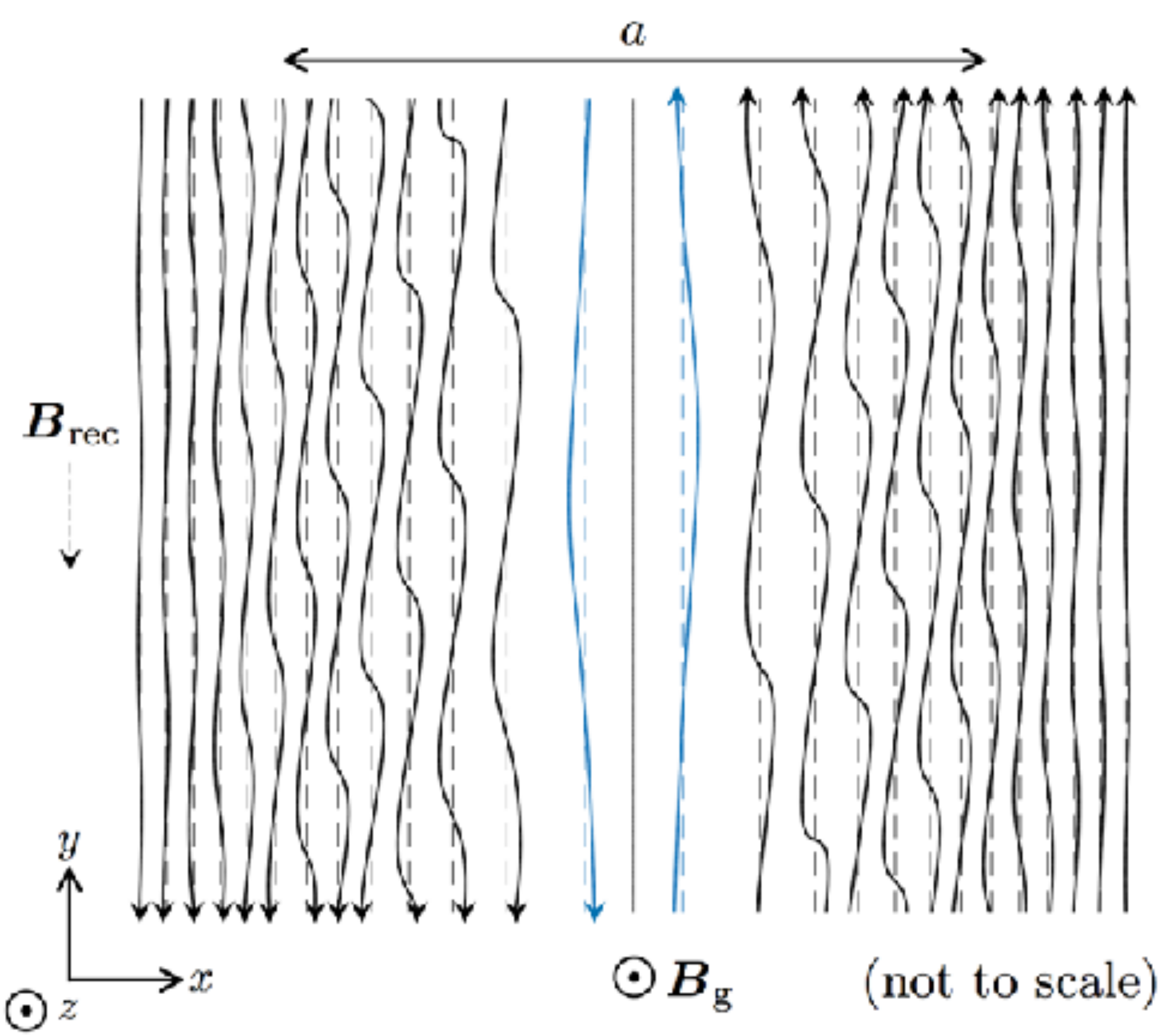
quick advertisement…

Alt & Kunz, 2019, JPP Letters, 85, 764850101

LETTER

Onset of magnetic reconnection in a collisionless, high-β plasma

Andrew Alt[1,2] and Matthew W. Kunz[1,2,†]



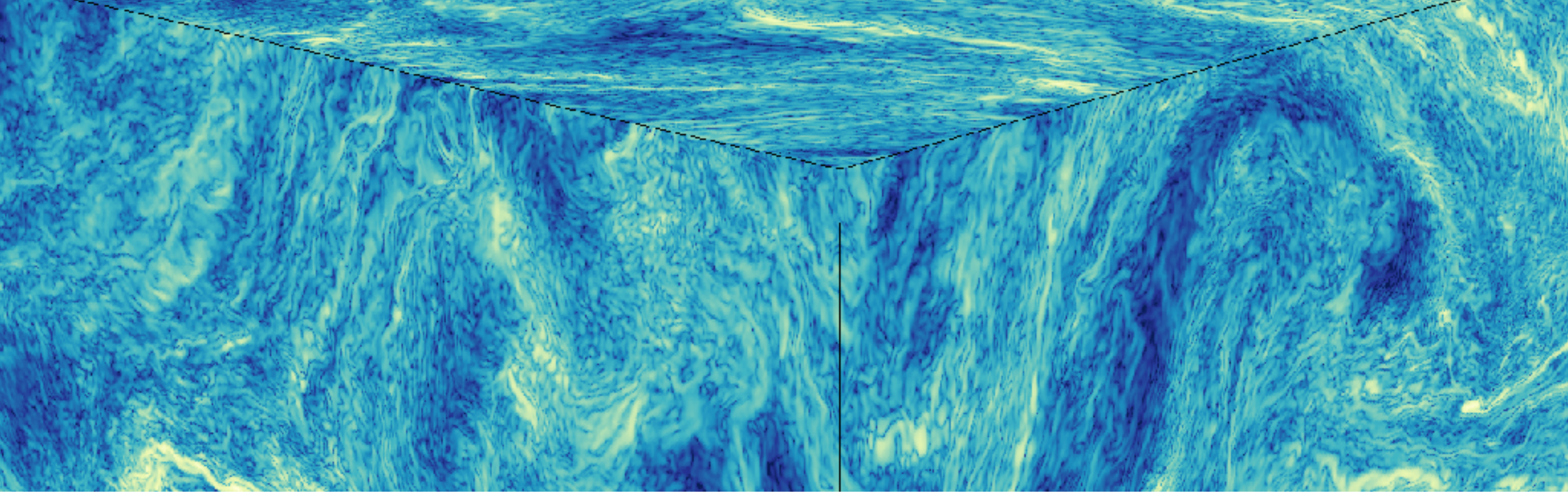
Consider a thinning current sheet in a collisionless, magnetized plasma

B will increase in inflowing fluid elements, driving $P_\perp > P_\parallel$

mirrors will rapidly grow and saturate above ion-Larmor scales, changing $\Delta'(k)$

tearing modes grow and disrupt CS formation earlier than they would otherwise.

quantitative theory worked out for this process



Thank you