General-Relativistic Kinetic Simulations of Black-Hole Magnetospheres

Kyle Parfrey

Princeton

with Alexander Philippov (CCA)
and Benoit Cerutti (Grenoble)

Connecting Micro and Macro Scales… — KITP — September 2019

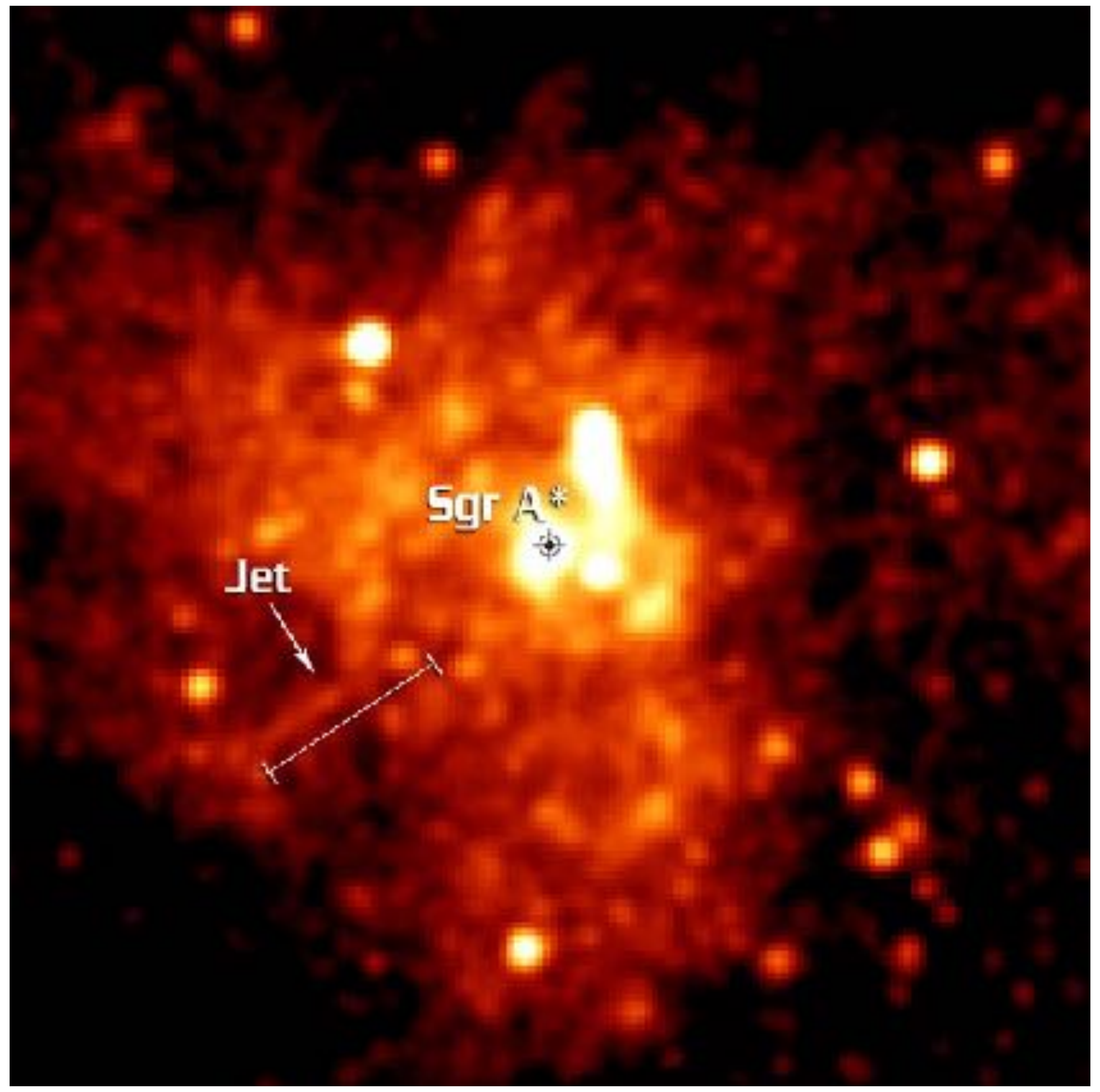
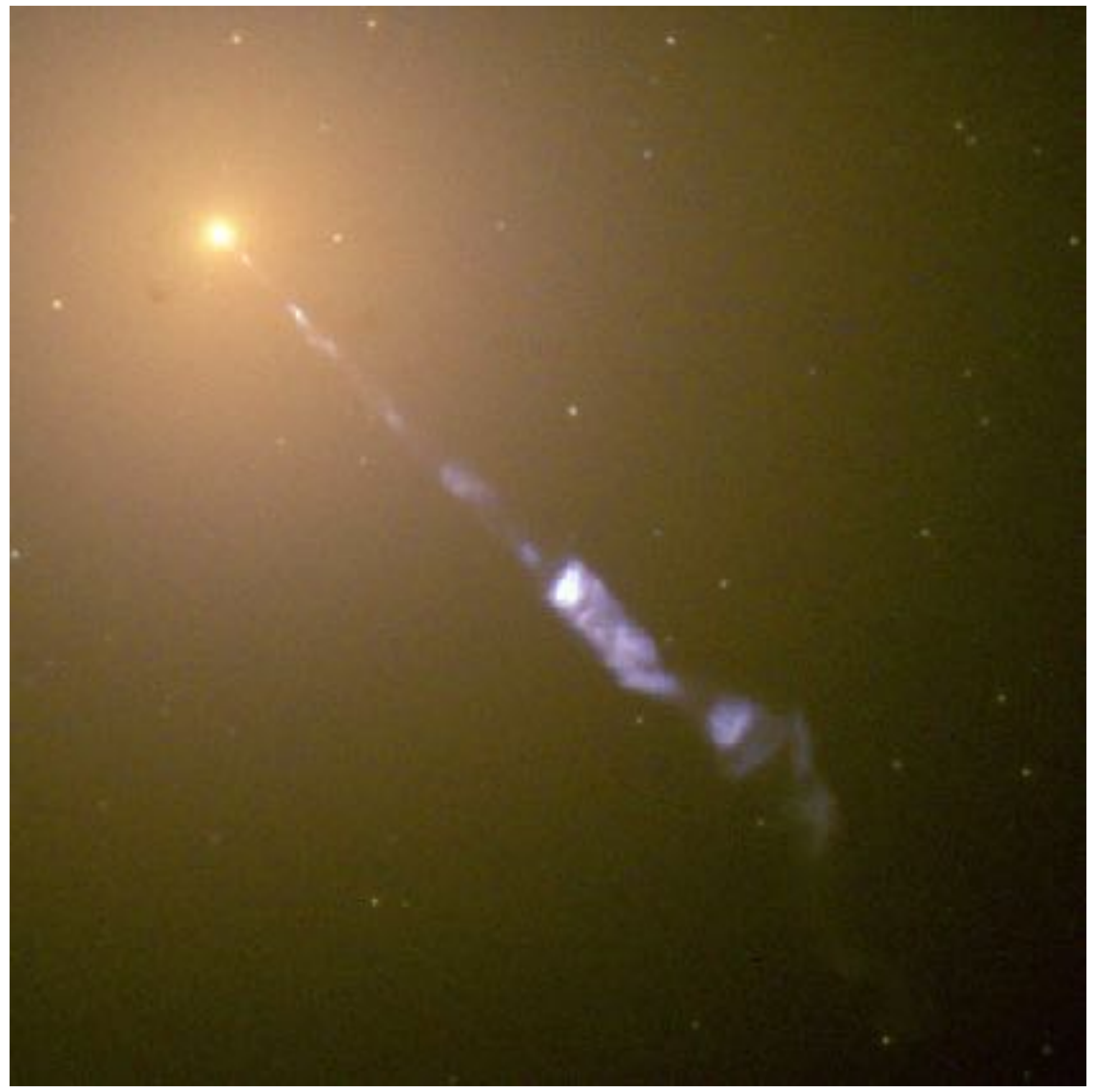
M87 central black hole at 1.3 mm

EHT Collaboration 2019

Event Horizon Telescope — targets

mm VLBI imaging of accretion flows on horizon scales

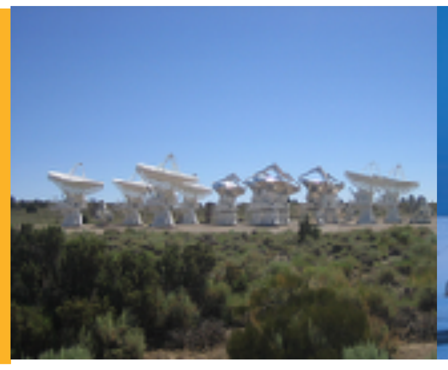
Also: GRAVITY on the VLT

	Galactic Center (Sgr A*)	M87	
Chandra (X-ray)			*Hubble* (visible)
$r_g = GM_{BH}/c^2$	0.05 AU	30 AU	~ horizon scale
apparent size	5 μas ~ 10^{-9} °	2 μas	


EventHorizonTelescope

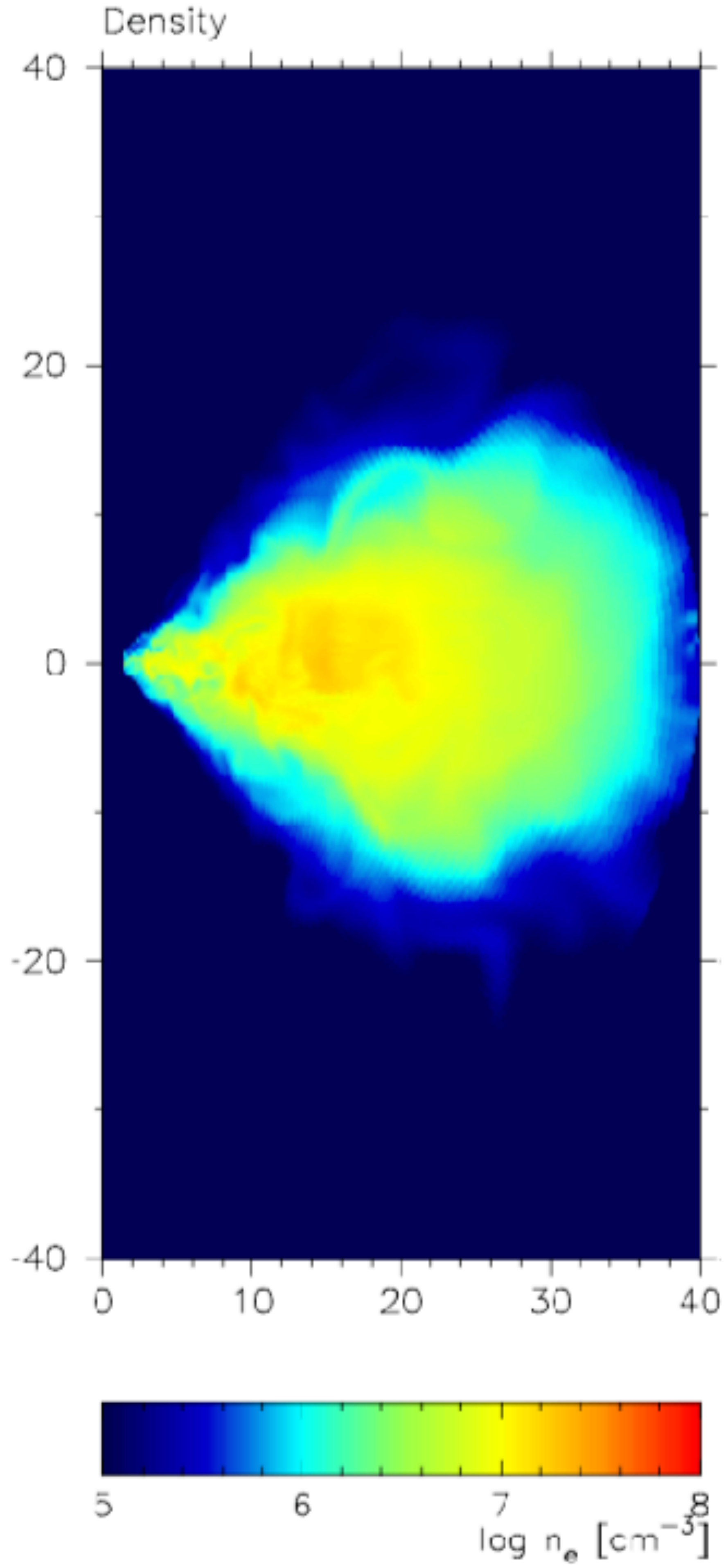
BlackHoleCam

Imaging the Event Horizon of Black Holes

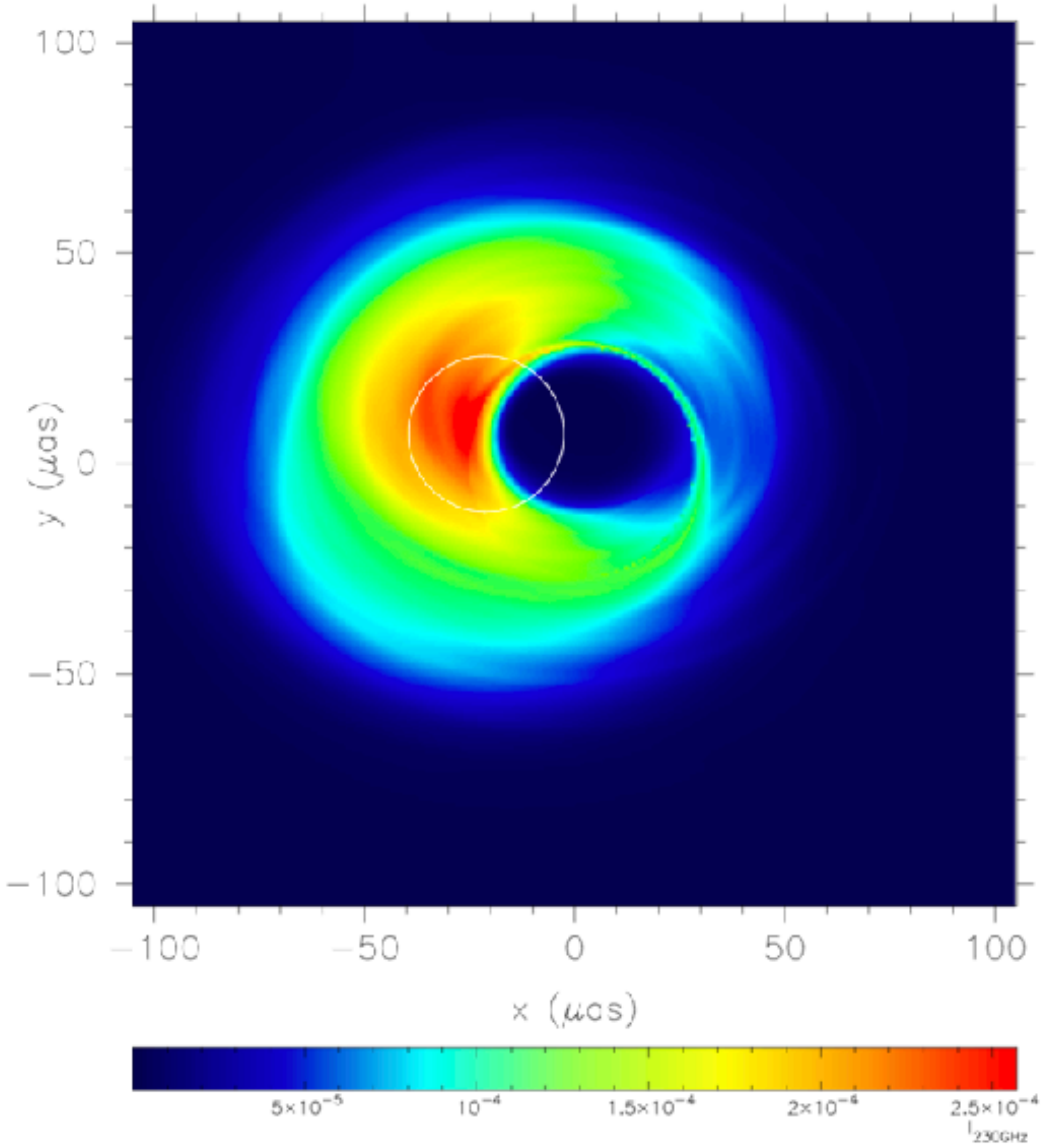



State of the art: GRMHD simulations

GRMHD
simulation



Moscibrodzka+ 2009



observational prediction
via ray-tracing

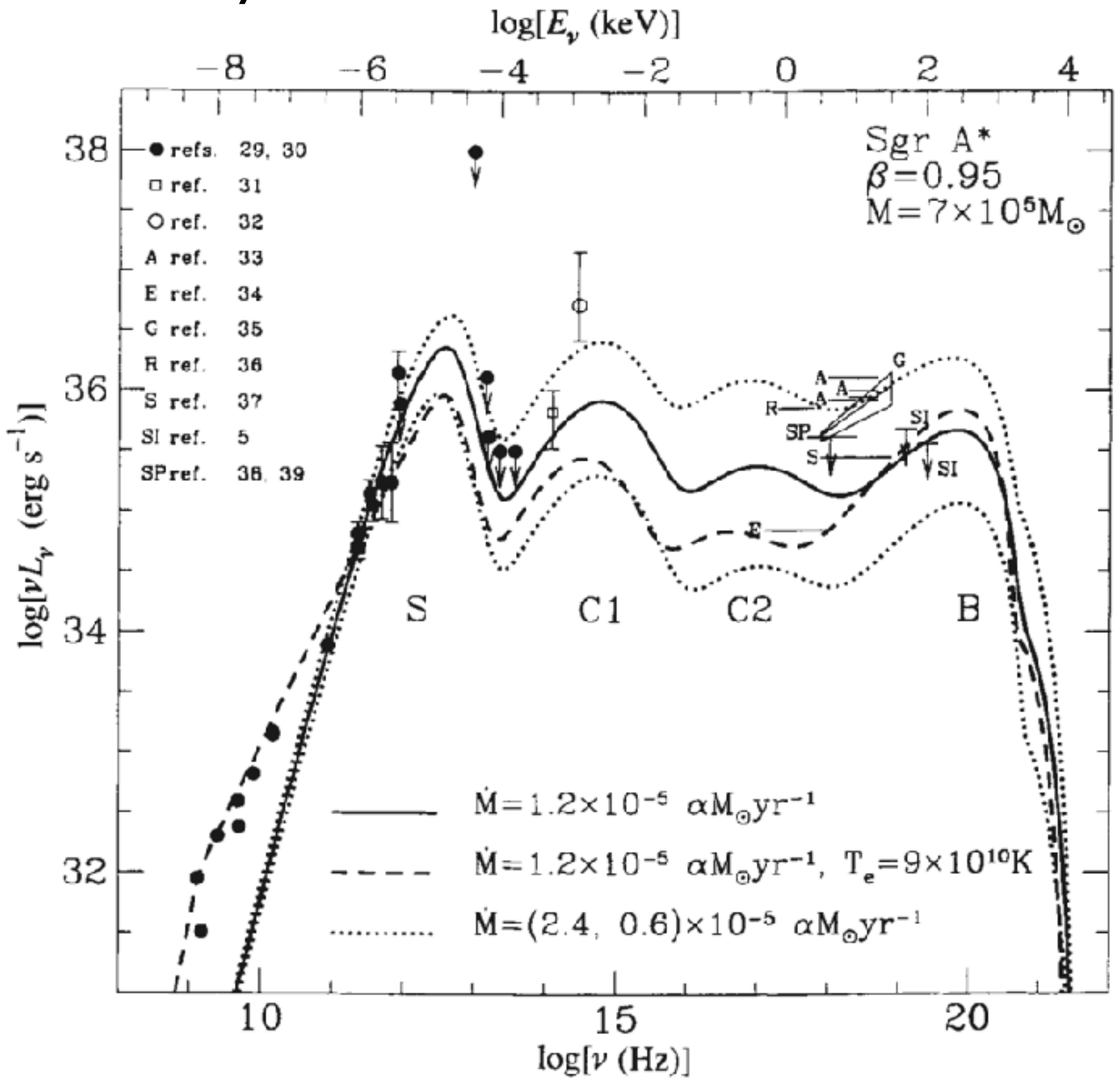
Includes all general-relativistic effects…

…but treats the accretion flow as a ***fluid***

These flows expected to be collisionless

At low accretion rate, flow can have ***low-density*** structure where:

Narayan+ 1995



fit of low-density *ADAF* model to Sgr A*

1. Collision mean free path ≫ system scales ~ r_g
2. Collision timescale ≫ accretion timescale

↓

electrons & protons have different T_{eff},
and generally *non-thermal distribution*

+

no single bulk velocity (anisotropic)

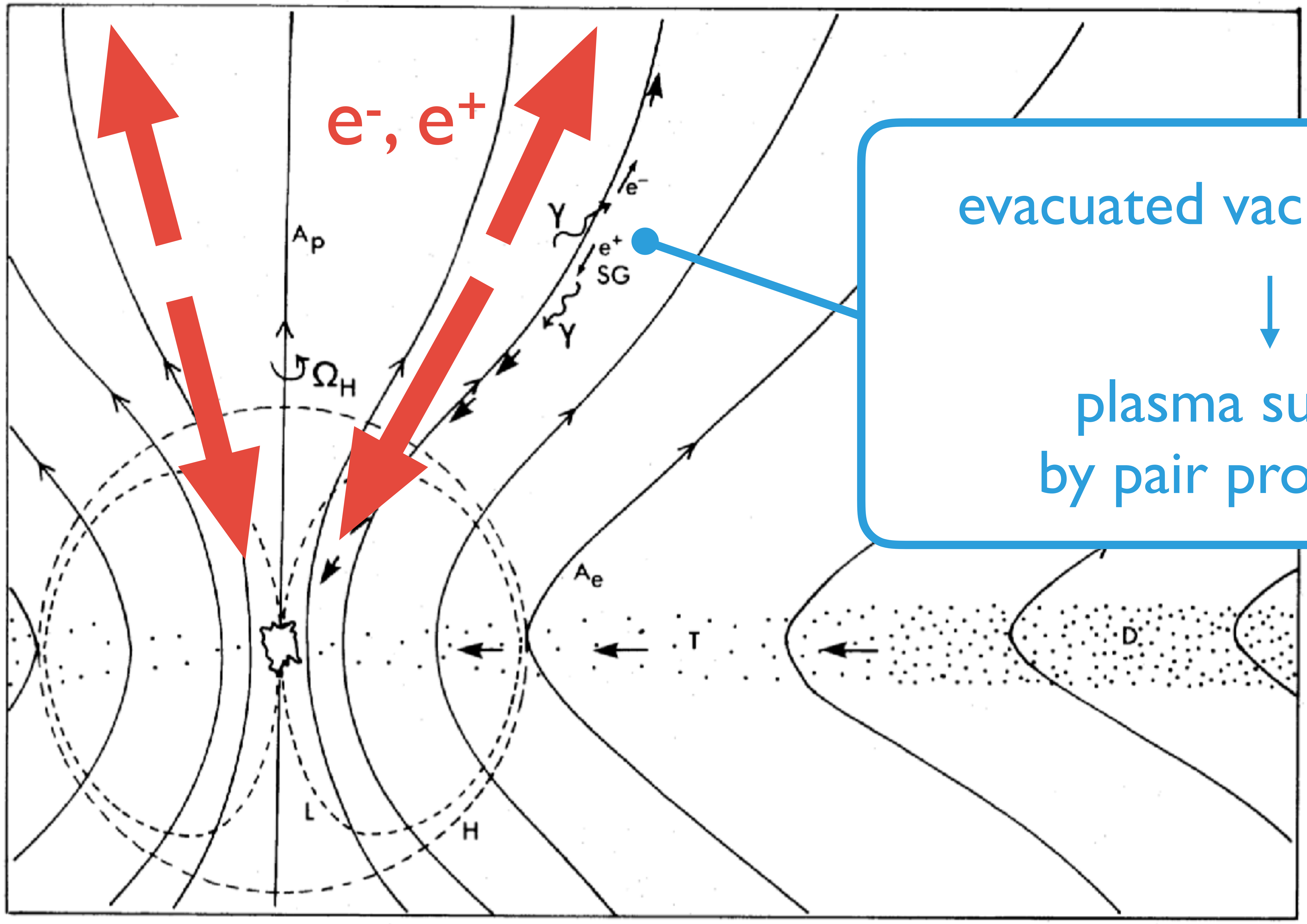
True for

Sgr A* **M87**

many AGN & XRBs

disc coronae jets

Jet launching by rotating black holes



Blandford & Znajek 1977

evacuated vacuum "gap"

↓

plasma supplied
by pair production

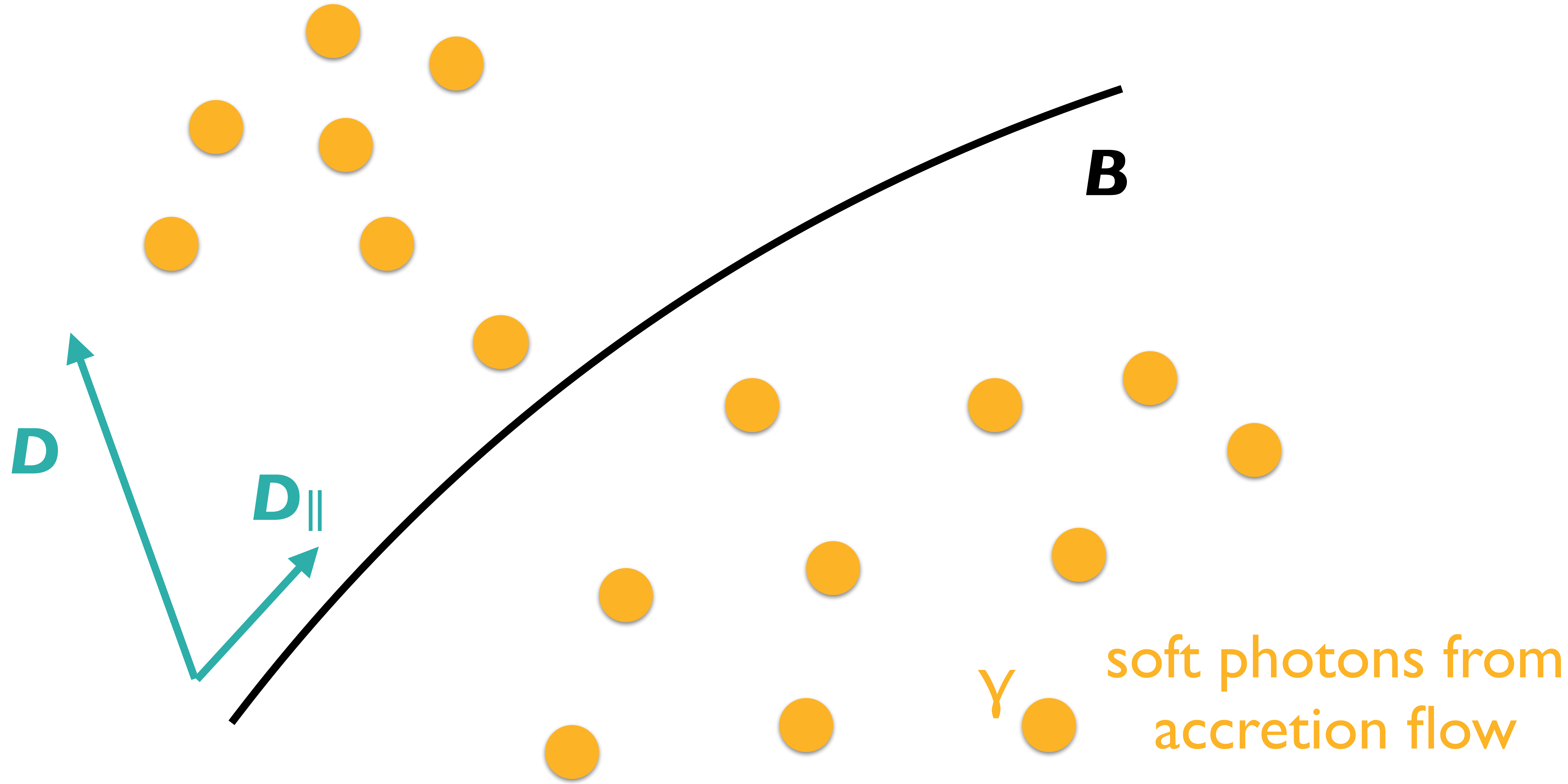
MHD cannot predict

gap location
jet density
jet composition
etc.

Two-photon pair creation

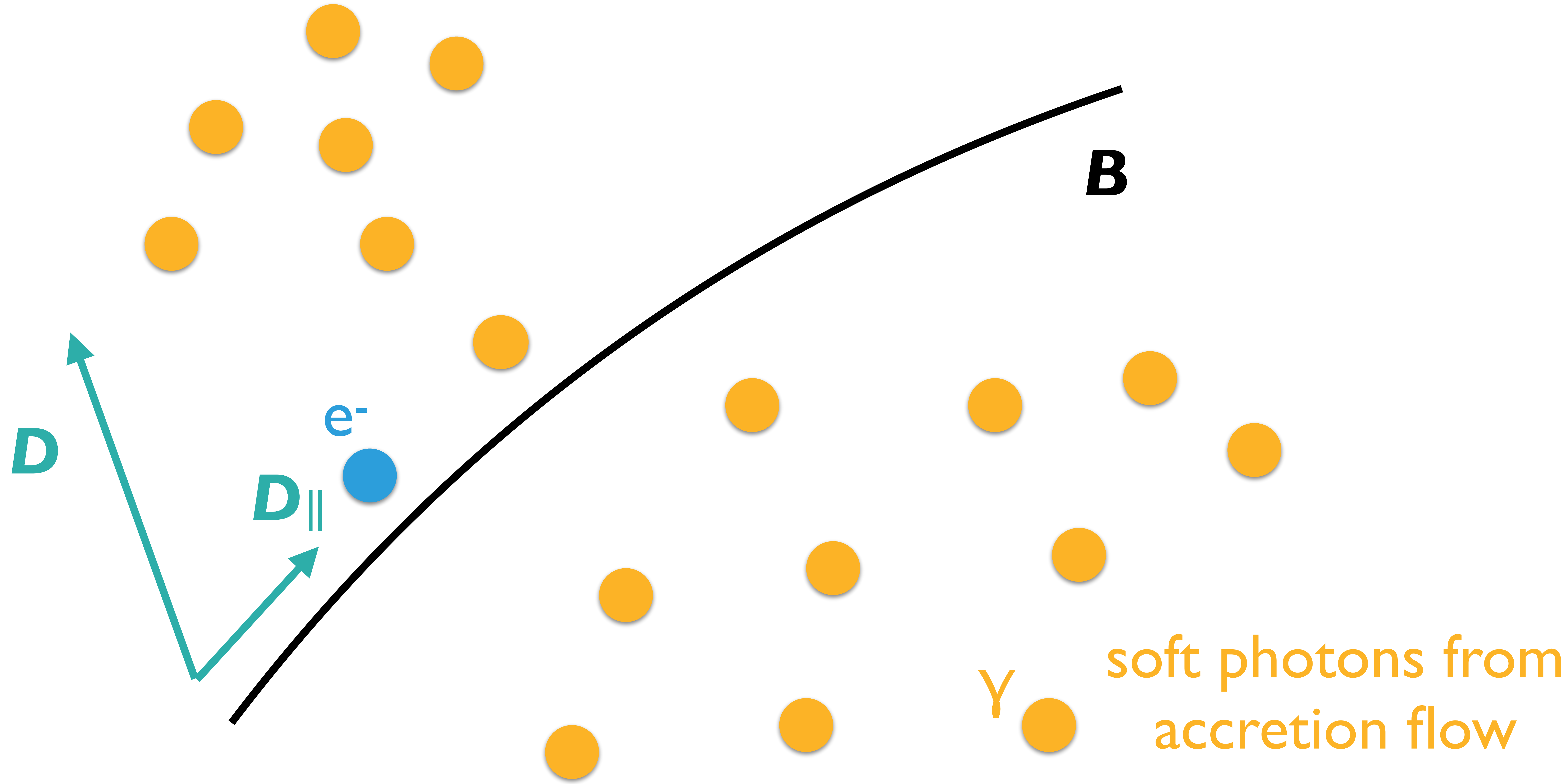
Start with parallel ("non-ideal") electric field: $\boldsymbol{D}_{\parallel}$

$\boldsymbol{D} \cdot \boldsymbol{B} \neq 0$



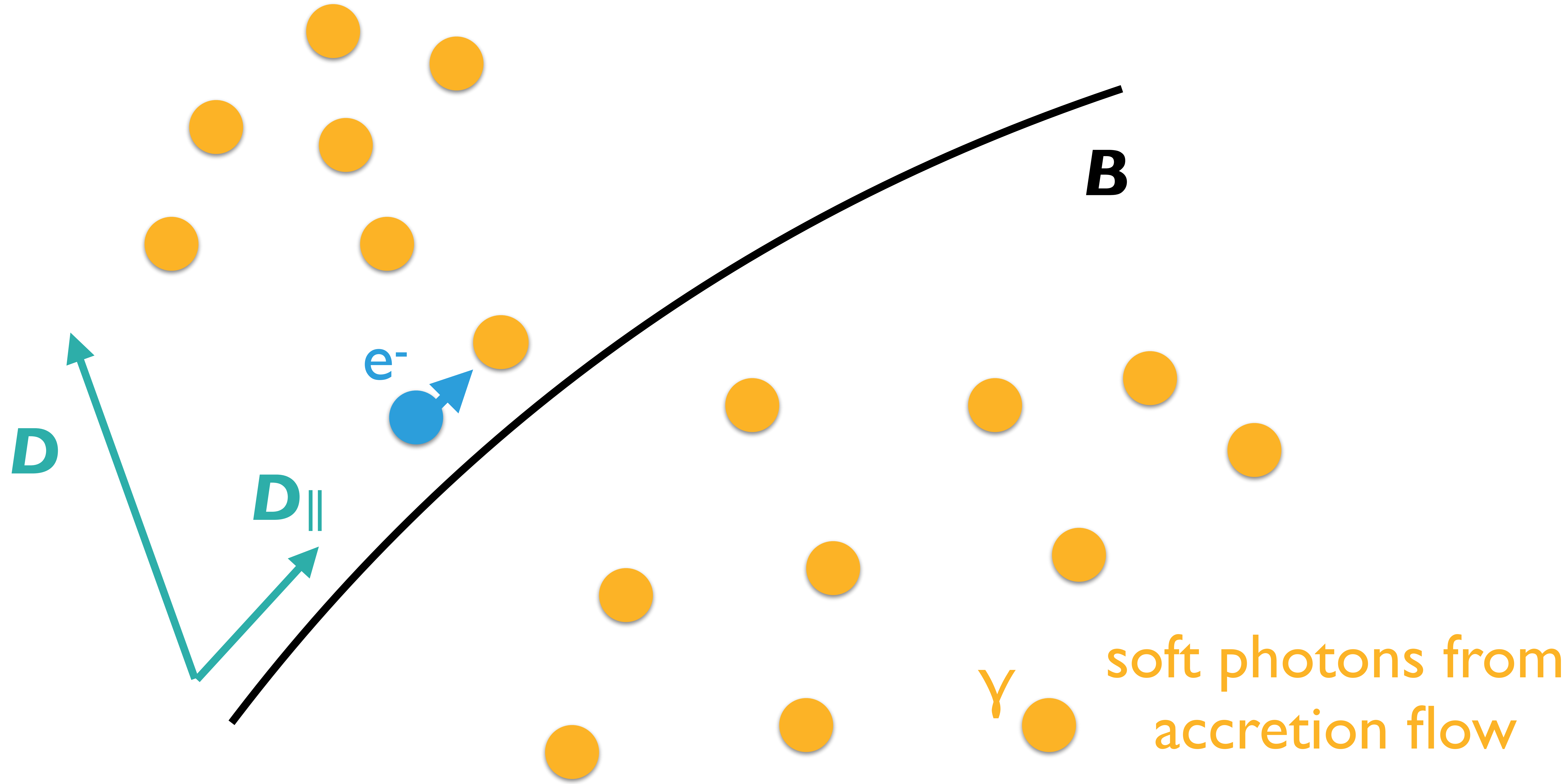
Two-photon pair creation

Introduce a stray lepton



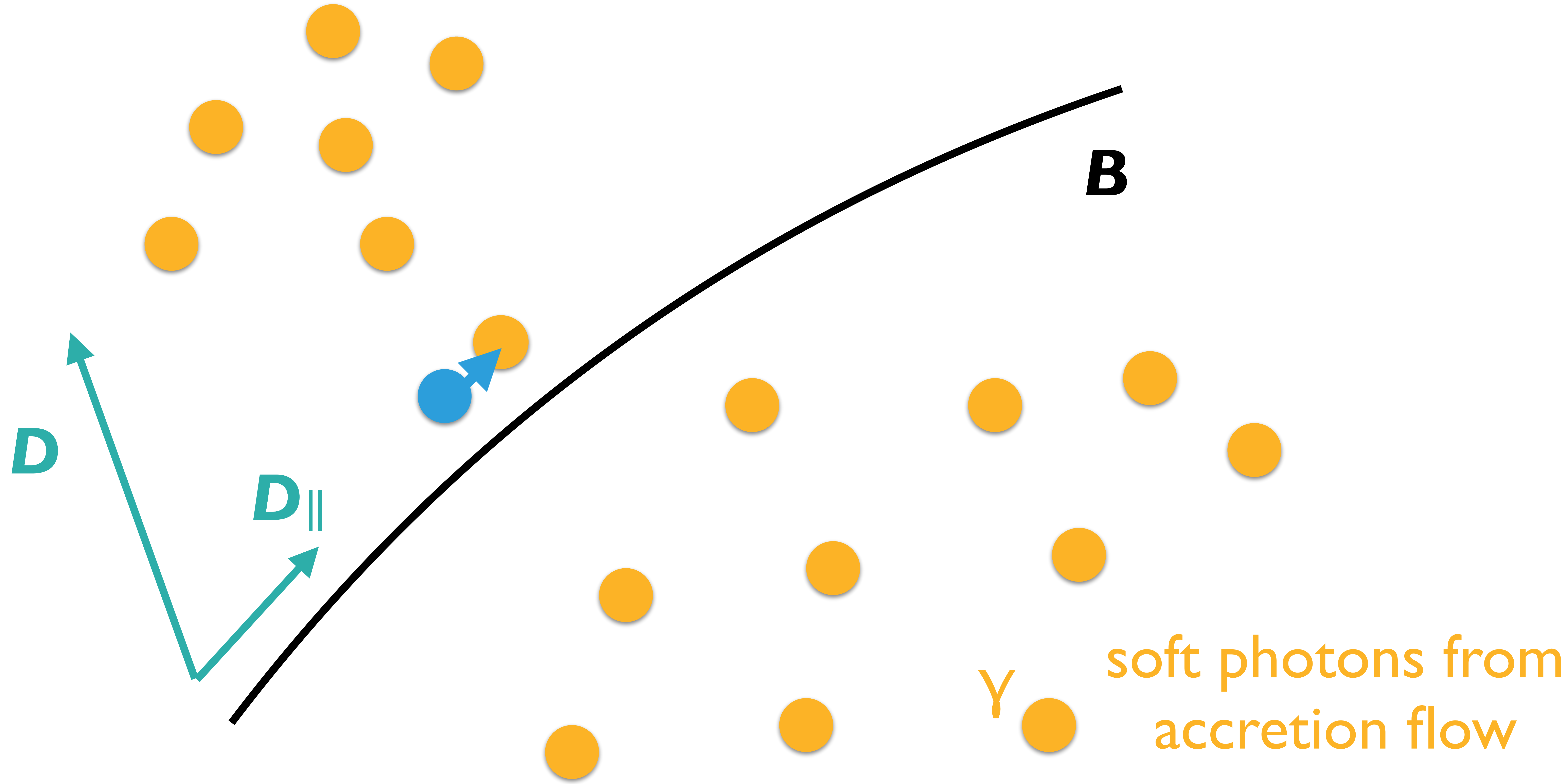
Two-photon pair creation

The lepton is accelerated along ***B*** by the electric field



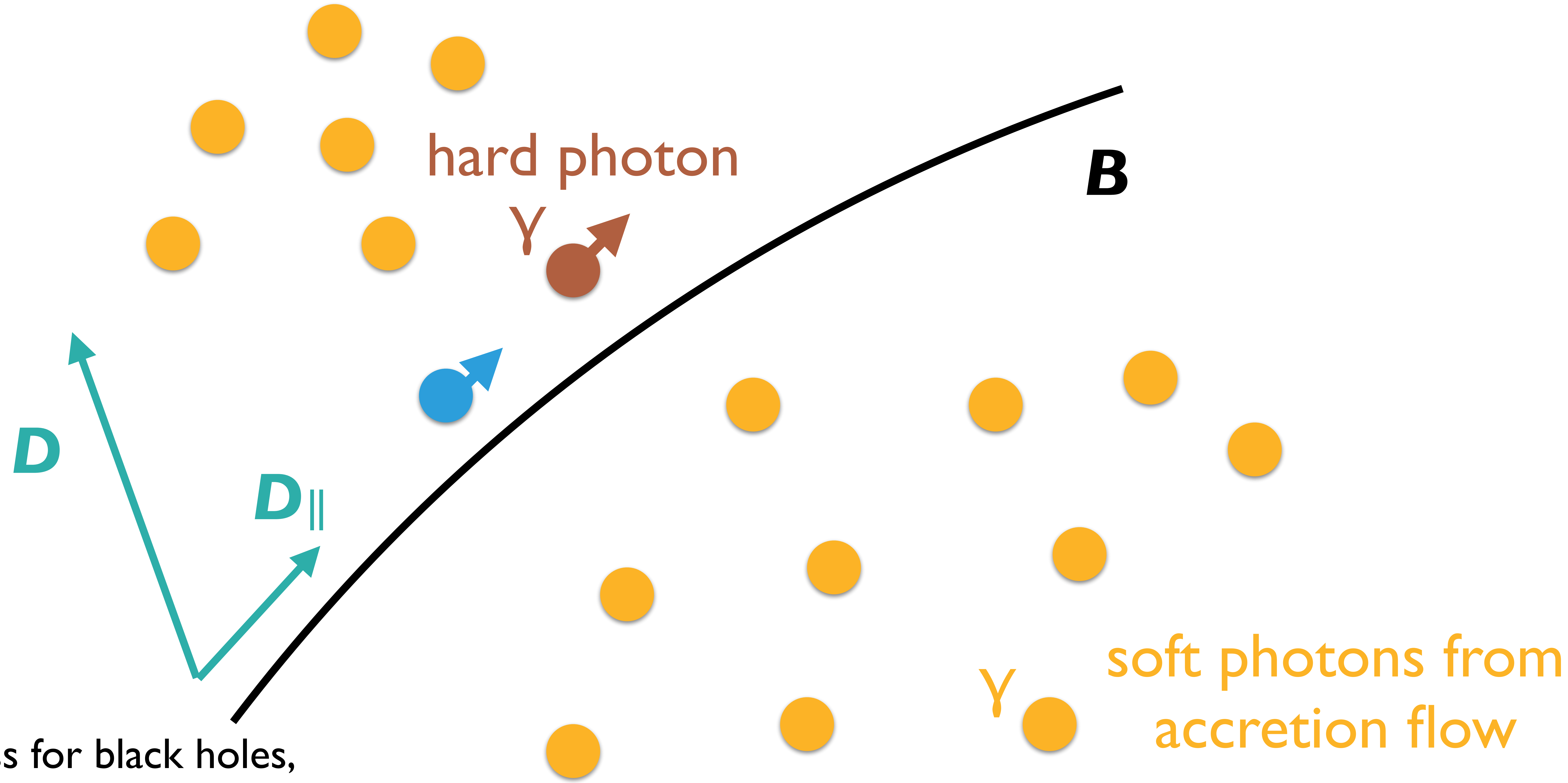
Two-photon pair creation

The lepton interacts with a soft background photon…



Two-photon pair creation

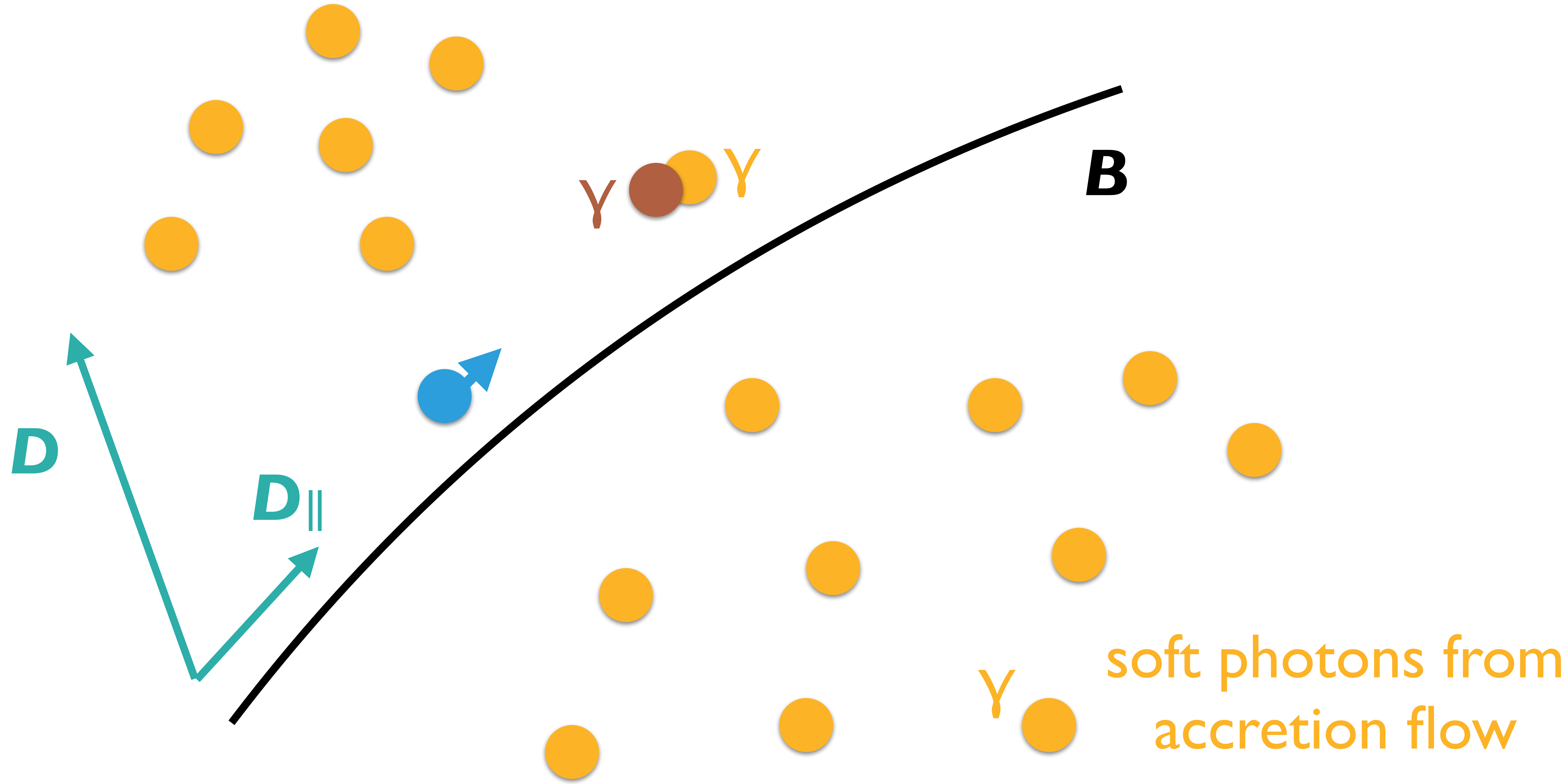
… and inverse-Compton upscatters it
to high energy



(most important process for black holes,
curvature radiation important for pulsars)

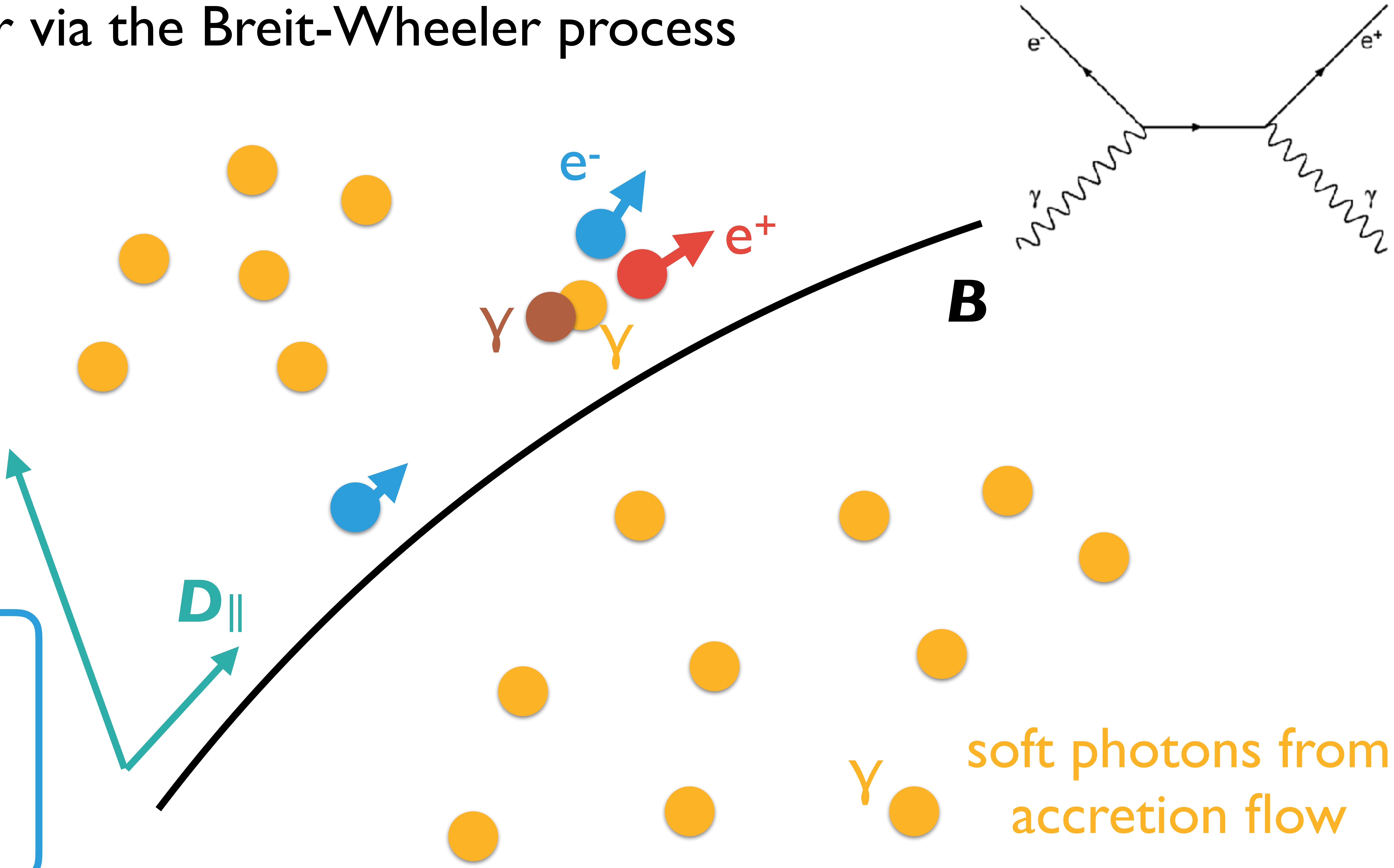
Two-photon pair creation

The hard photon interacts with a soft background photon…



Two-photon pair creation

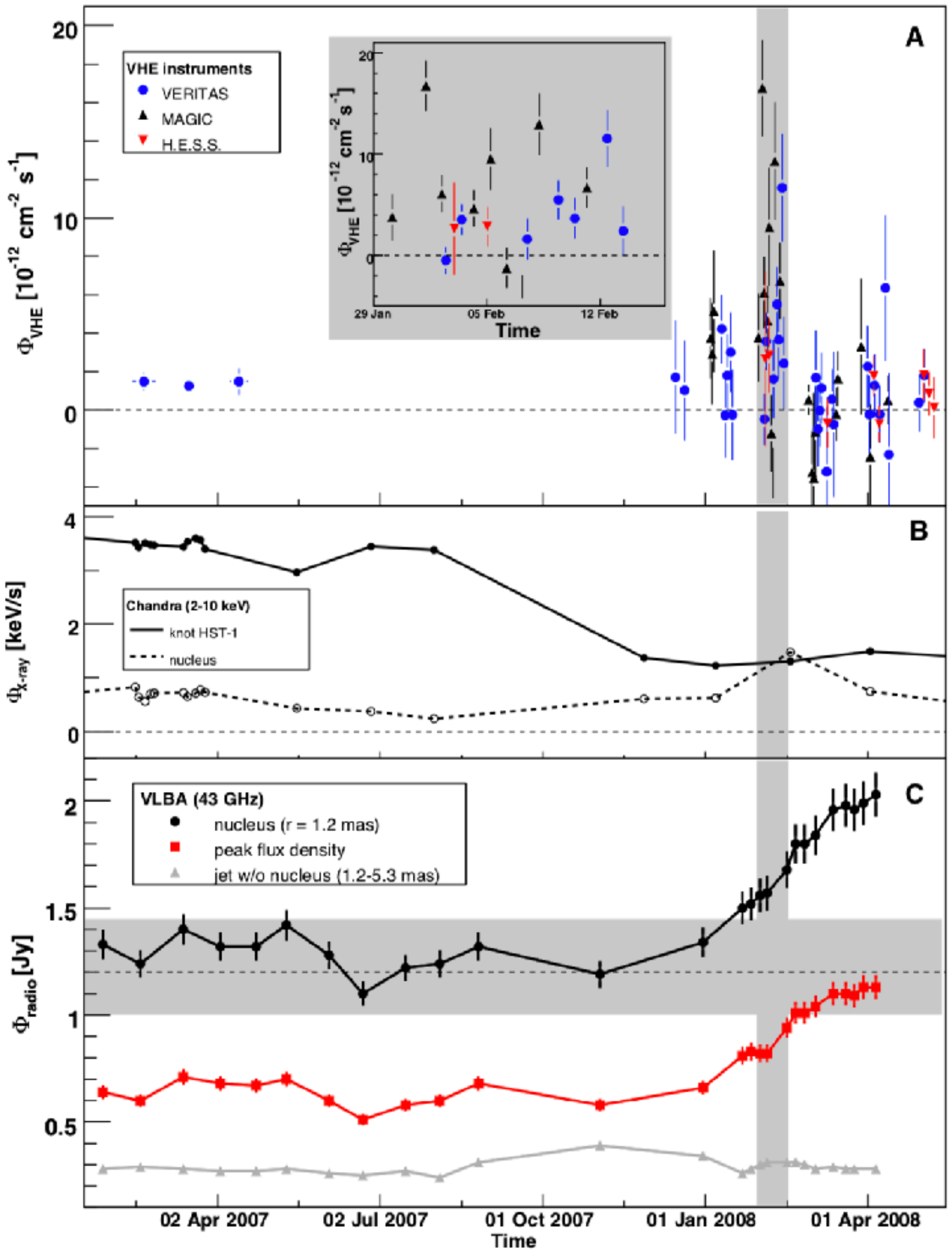
…giving an e$^{\pm}$ pair via the Breit-Wheeler process



$\boldsymbol{D} \cdot \boldsymbol{B} \neq 0$

→ pair plasma

Black-hole γ-ray flares — related to gap dynamics?



Veritas Collaboration 2009

γ-ray

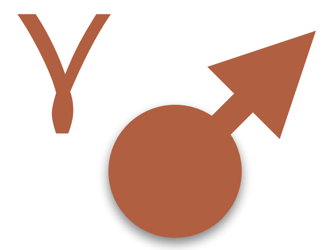
E > 0.35 TeV

X-ray

Radio

M87 TeV flares

Escaping hard
IC-upscattered photons?



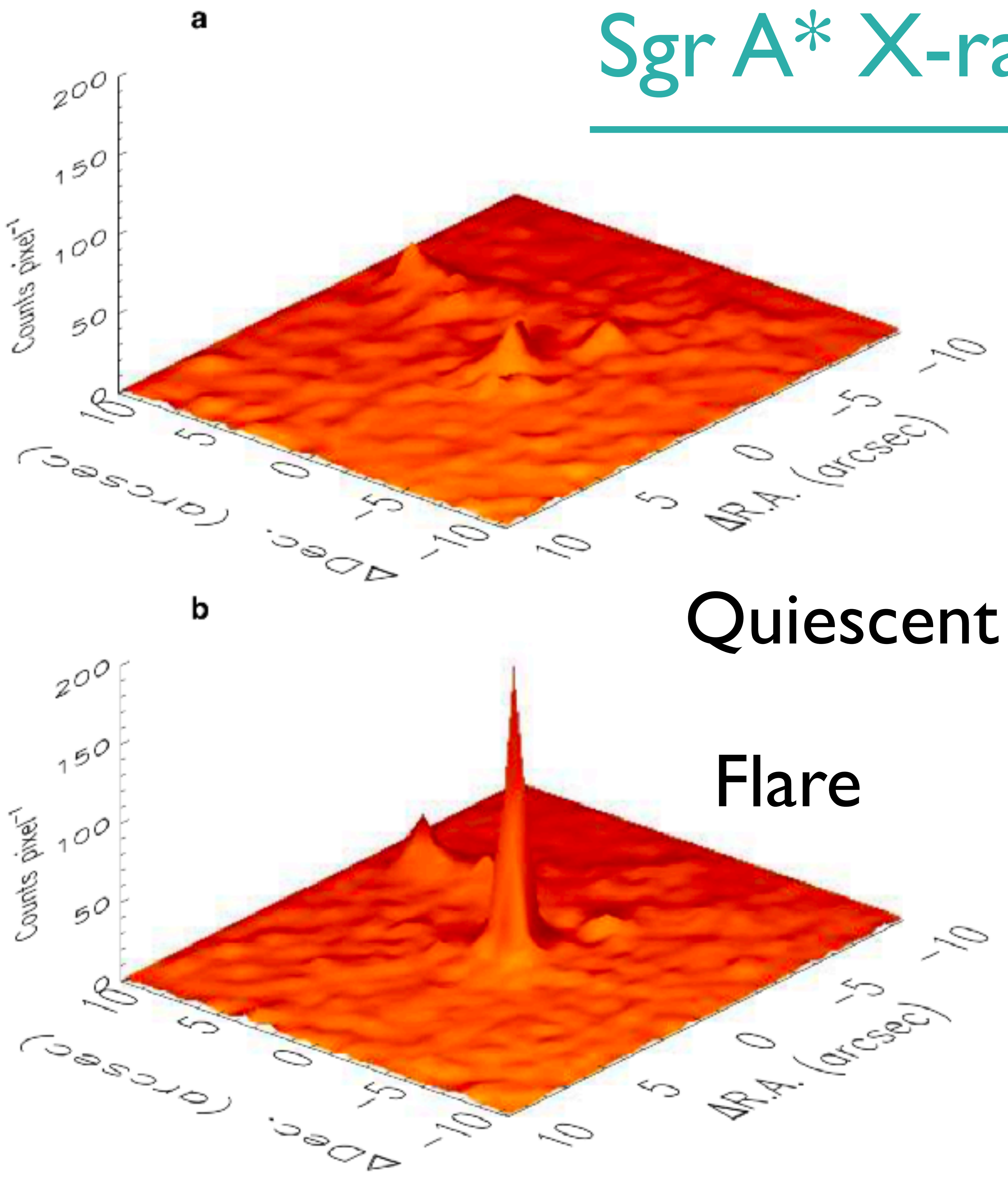
γ-ray emitting region < 100 r_g
from black hole

X-ray flares and coronae

Baganoff+ 2001

Chandra 2-8 keV

Sgr A* X-ray flares



Quiescent

Flare

High-energy emission from magnetic reconnection?

X-ray coronae in AGN & binaries

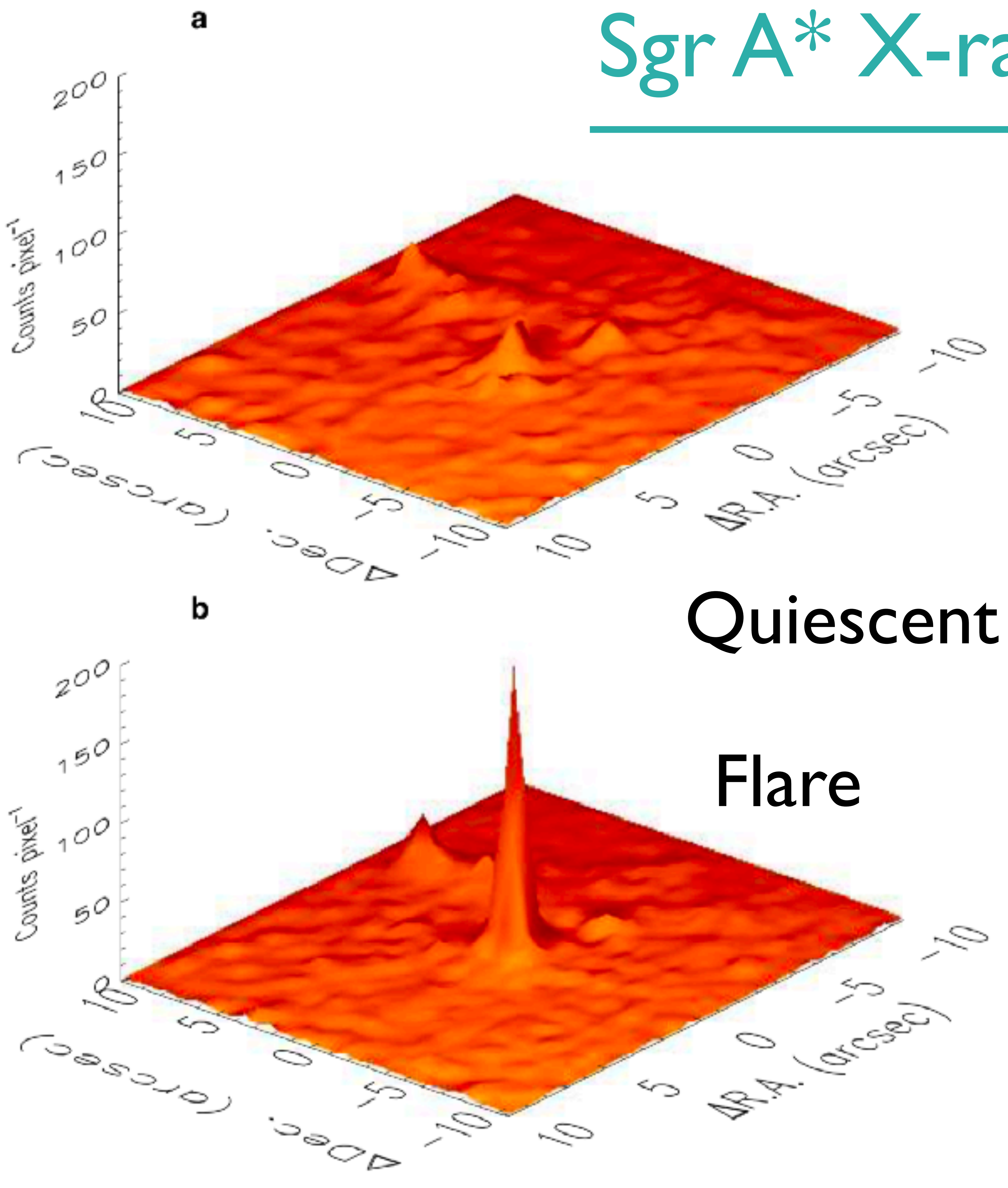
quasar microlensing limits size of emitting region to ~ 10 r_g

Morgan+ 08; Chartas+ 09; Chen+ 12

X-ray flares and coronae

Baganoff+ 2001
Chandra 2-8 keV

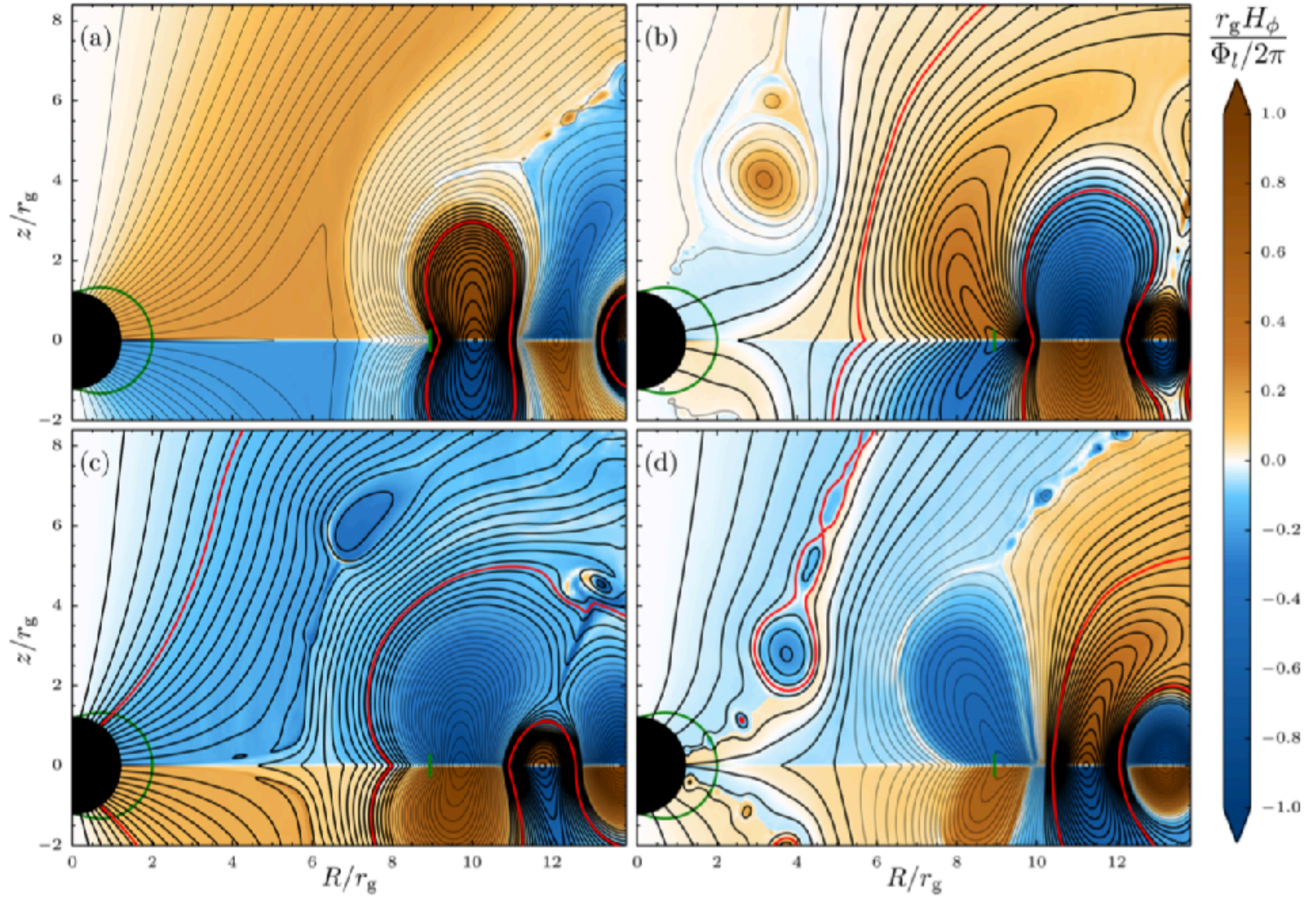
Sgr A* X-ray flares



Quiescent

Flare

Parfrey, Giannios, Beloborodov 2015



X-ray coronae in AGN & binaries

quasar microlensing limits size of emitting region to ~ 10 r_g

Morgan+ 08; Chartas+ 09; Chen+ 12

Collisionless physics: plasma kinetics

$$\partial_t \boldsymbol{E} = \nabla \times \boldsymbol{B} - \boldsymbol{J} \qquad \nabla \cdot \boldsymbol{E} = \rho_\mathrm{e}$$

Maxwell's equations

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E} \qquad \nabla \cdot \boldsymbol{B} = 0$$

+

(a) Continuum dynamics

$$\frac{\partial f_\mathrm{s}}{\partial t} + \boldsymbol{v}_\mathrm{s} \cdot \nabla f_\mathrm{s} + q_\mathrm{s} \left(\boldsymbol{E} + \boldsymbol{v}_\mathrm{s} \times \boldsymbol{B} \right) \cdot \frac{\partial f_\mathrm{s}}{\partial \boldsymbol{p}} = 0$$

s : electrons, ions

Solve for distribution function $f(\boldsymbol{x},\boldsymbol{p},t)$

—Vlasov-Maxwell system

or

(b) Particle dynamics

$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = q_i \left(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B} \right) \qquad \frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

$i = 1, \ldots, N$: particles

Solve for 6 fields: E, B + (3D momentum space **or** N particles)

Collisionless physics: plasma kinetics

$$\partial_t \boldsymbol{E} = \nabla \times \boldsymbol{B} - \boldsymbol{J} \qquad \nabla \cdot \boldsymbol{E} = \rho_e$$

Maxwell's equations

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E} \qquad \nabla \cdot \boldsymbol{B} = 0$$

+

(a) Continuum dynamics

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}_s \cdot \nabla f_s + q_s \left(\boldsymbol{E} + \boldsymbol{v}_s \times \boldsymbol{B} \right) \cdot \frac{\partial f_s}{\partial \boldsymbol{p}} = 0$$

s : electrons, ions

Solve for distribution function $f(\boldsymbol{x},\boldsymbol{p},t)$

—Vlasov-Maxwell system

or

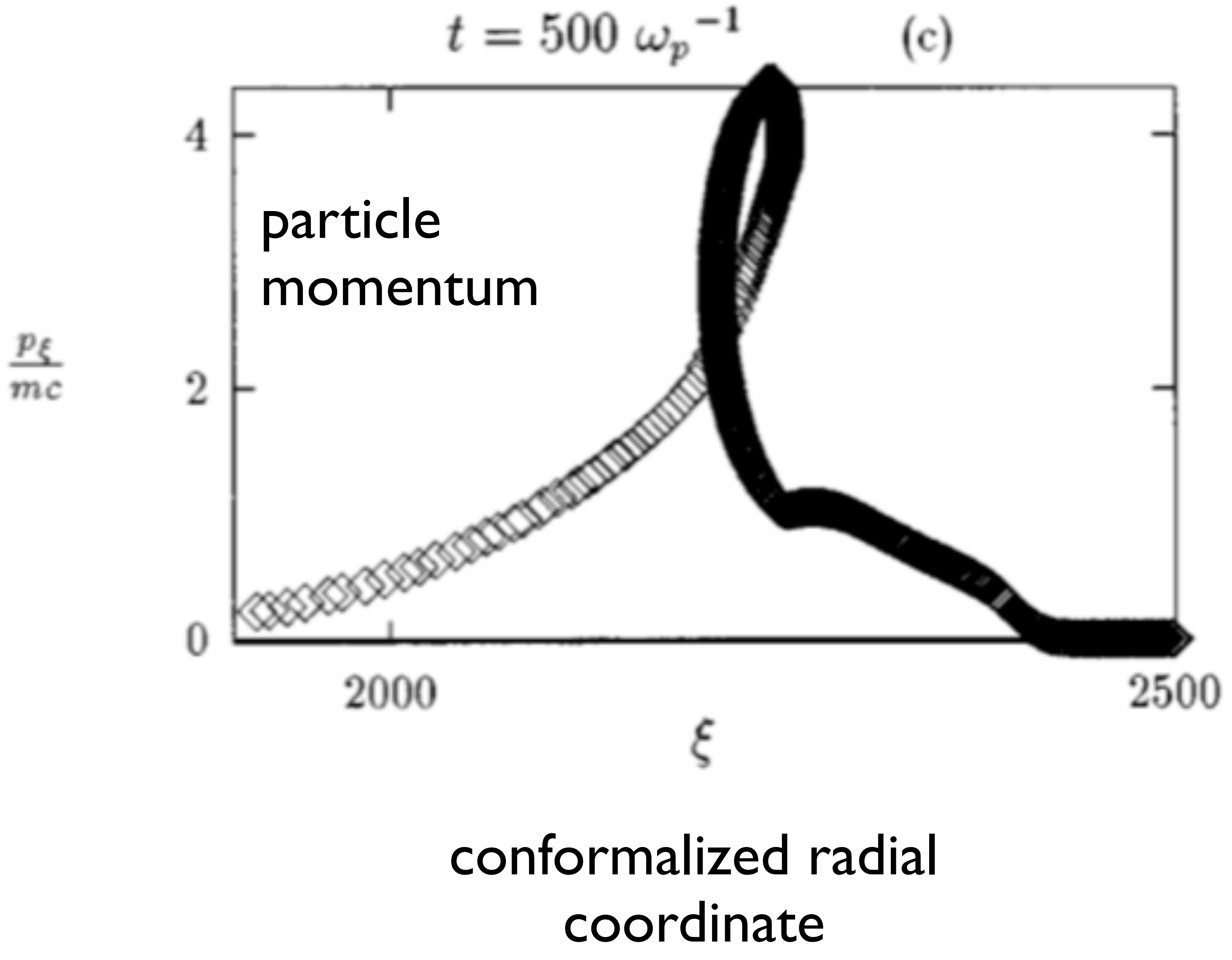
(b) Particle dynamics

$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = q_i \left(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B} \right) \qquad \frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = \boldsymbol{v}_i$$

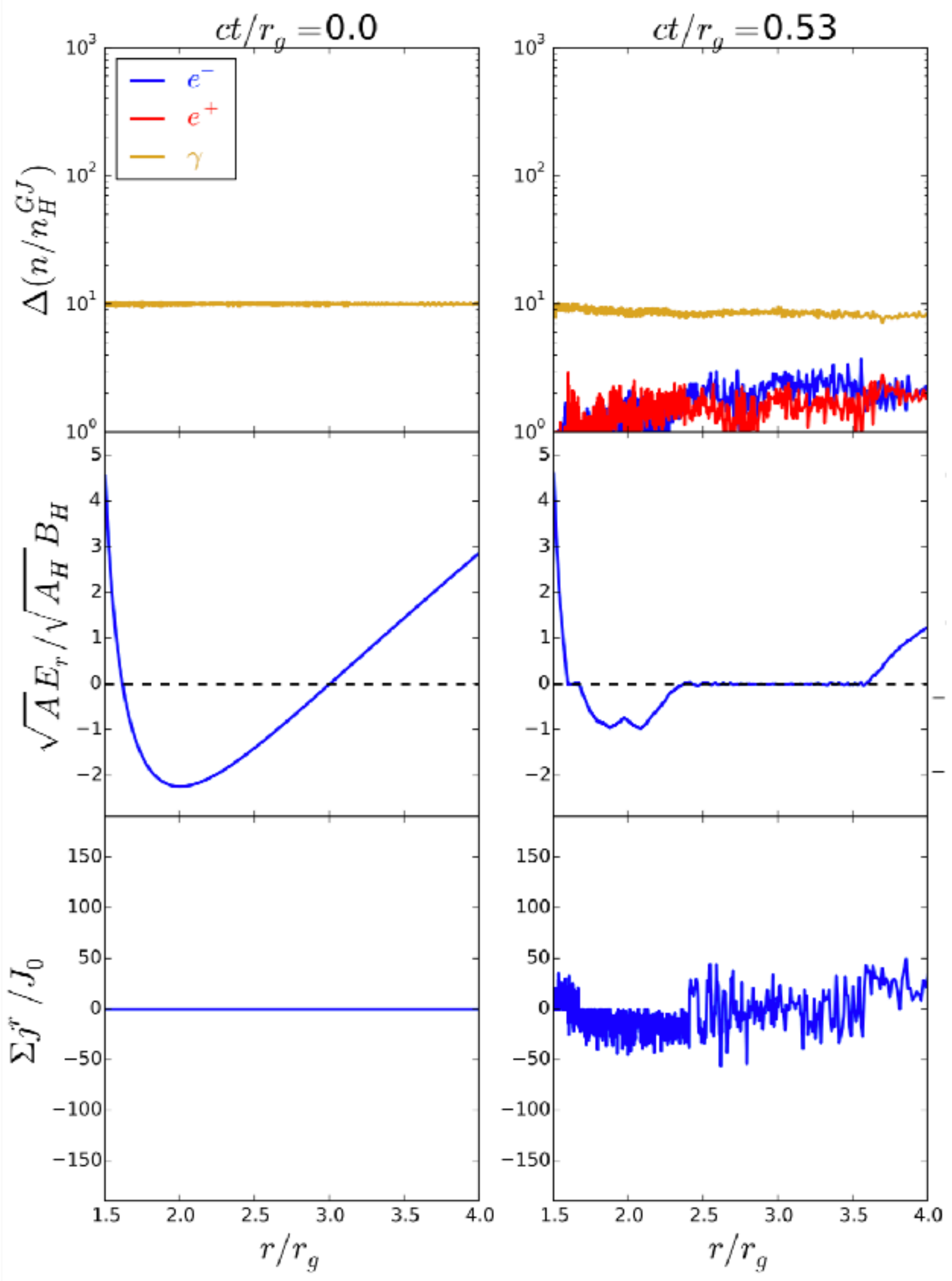
$i = 1, \ldots, N$: particles

Choose particle-in-cell (PIC) method

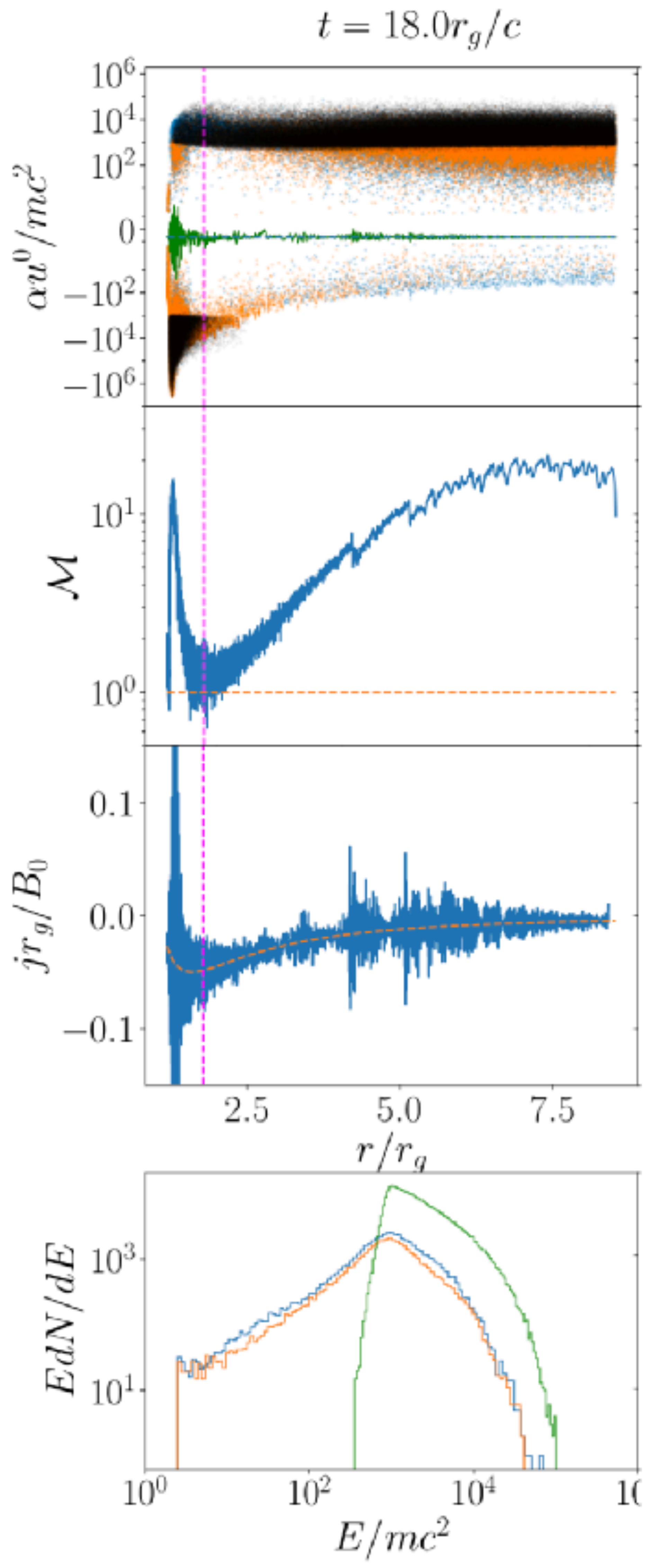
Collisionless plasma in general relativity — 1D simulations



Daniel & Tajima 1997



Levinson & Cerutti 2018



Chen & Yuan 2019

GRPIC 1. – electromagnetic field equations

4D spacetime form

$$\nabla_\nu F^{\mu\nu} = I^\mu \qquad \nabla_\nu {}^*F^{\mu\nu} = 0$$

$$\longrightarrow$$

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J} \qquad \partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

3+1 form

$$\nabla \cdot \boldsymbol{D} = \rho_\mathrm{e} \qquad \nabla \cdot \boldsymbol{B} = 0$$

constitutive relations:

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boxed{\boldsymbol{\beta} \times \boldsymbol{B}}$$
$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boxed{\boldsymbol{\beta} \times \boldsymbol{D}}$$

Komissarov 2004

and particles determine current density $\boldsymbol{J}$

α and β^i are metric functions: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \, \mathrm{d}x^\nu$

$$= \left(\beta^2 - \alpha^2\right)\mathrm{d}t^2 + 2\beta_i\,\mathrm{d}x^i\,\mathrm{d}t + \gamma_{jk}\,\mathrm{d}x^j\,\mathrm{d}x^k$$

GRPIC 2. – particles

Start from 4D action: $S = \int \left(-m\,\mathrm{d}s + qA_\mu\,\mathrm{d}x^\mu\right)$ where $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu$

find Lagrangian $L = -m\alpha/\Gamma + qA_j v^j + qA_t$

and Hamiltonian $H = \pi_i v^i - L$

canonical momentum: $\pi_i = p_i + qA_i$

kinetic momentum: p_i and $v^i = \dfrac{\mathrm{dx^i}}{\mathrm{dt}}$

Hamilton's equations give

$$\frac{\mathrm{d}x^i}{\mathrm{dt}} = \frac{\alpha}{m\Gamma}p^i - \beta^i$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -m\Gamma\partial_i\alpha + p_j\partial_i\beta^j - \frac{\alpha}{2\Gamma m}\partial_i(\gamma^{lm})p_l p_m + q\left\{\alpha D_i + \epsilon_{ijk}(v^j + \beta^j)B^k\right\}$$

gravitational acceleration

~ extrinsic curvature

Lorentz force

Particle integrator scheme

Requirements:

1. conserve phase-space volume, $|\boldsymbol{x} \wedge \boldsymbol{p}|$

2. time-symmetric

$\frac{1}{2}\,\Delta t$ | Δt | $\frac{1}{2}\,\Delta t$

Strang split: $\boxed{\text{Lorentz force}}$ — $\boxed{\text{coordinate terms + move particle}}$ — $\boxed{\text{Lorentz force}}$

Boris rotation in
local orthonormal frame

→ volume conserving

implicit symplectic integrator

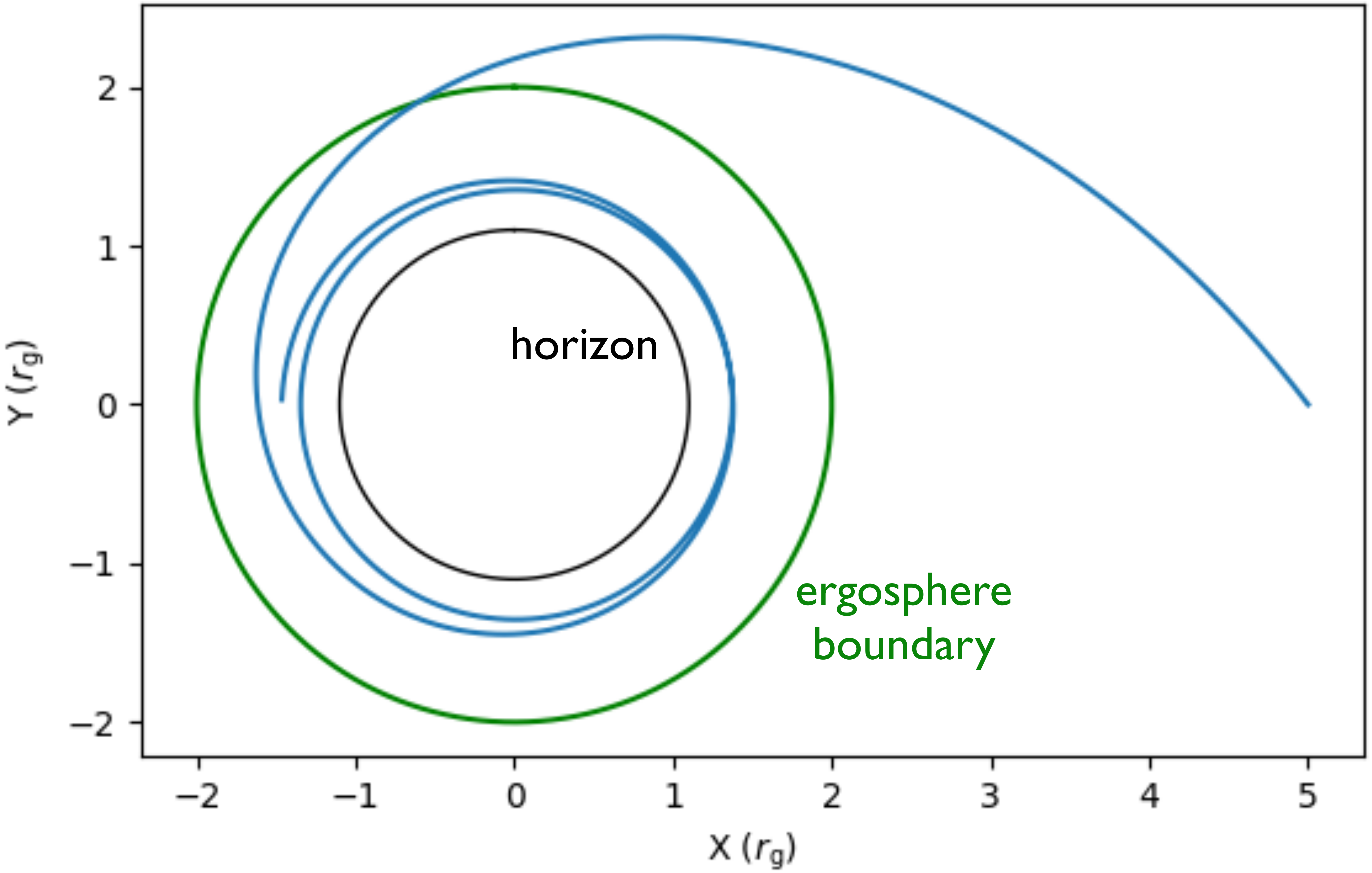
→ since $\boldsymbol{H}$ is non-separable

$$\Gamma = \sqrt{1 + \gamma^{ij} p_i p_j / m^2}$$

fuses spatial & momentum
dependence everywhere

Geodesic tests — $\boldsymbol{B} = 0$

equatorial plane



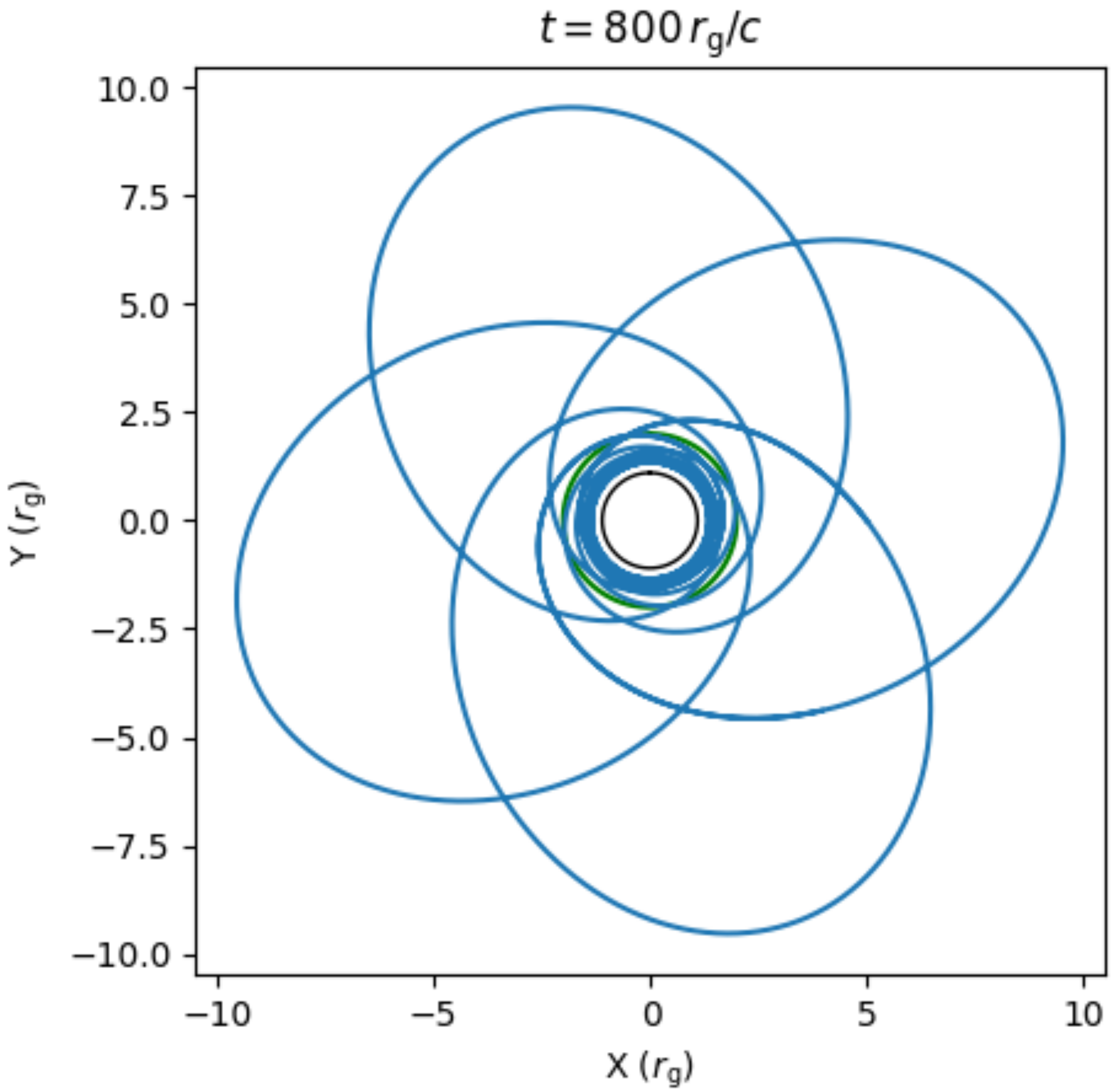
Rotating (Kerr) black hole
$a = 0.995$

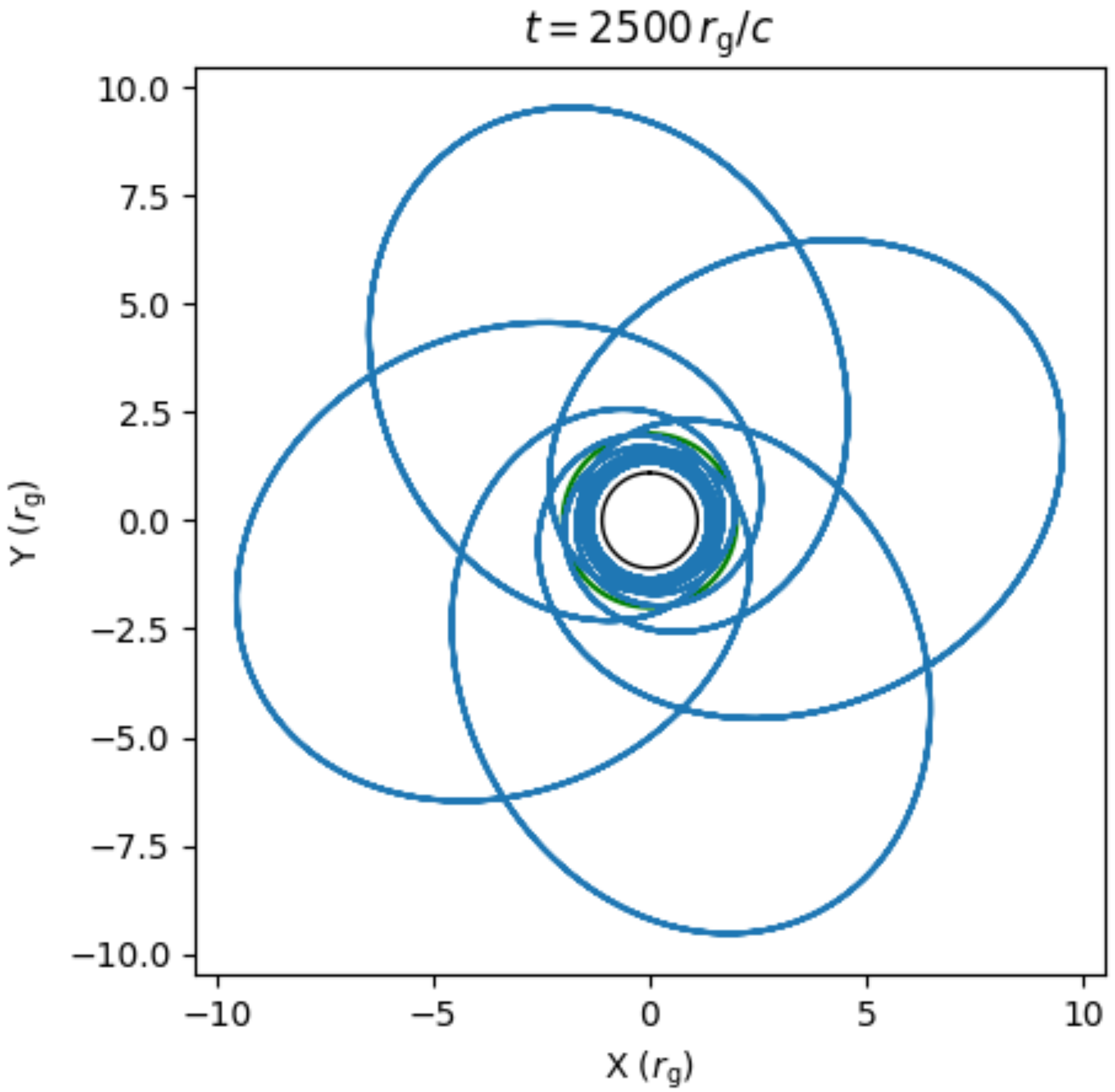
Prograde orbit with
$L = u_\varphi = 2$
$E = -u_t = 0.915082$

Symplectic integrator (implicit midpoint)
with dt = 0.1 r_g/c

(r_g/c ~ horizon light-crossing time)

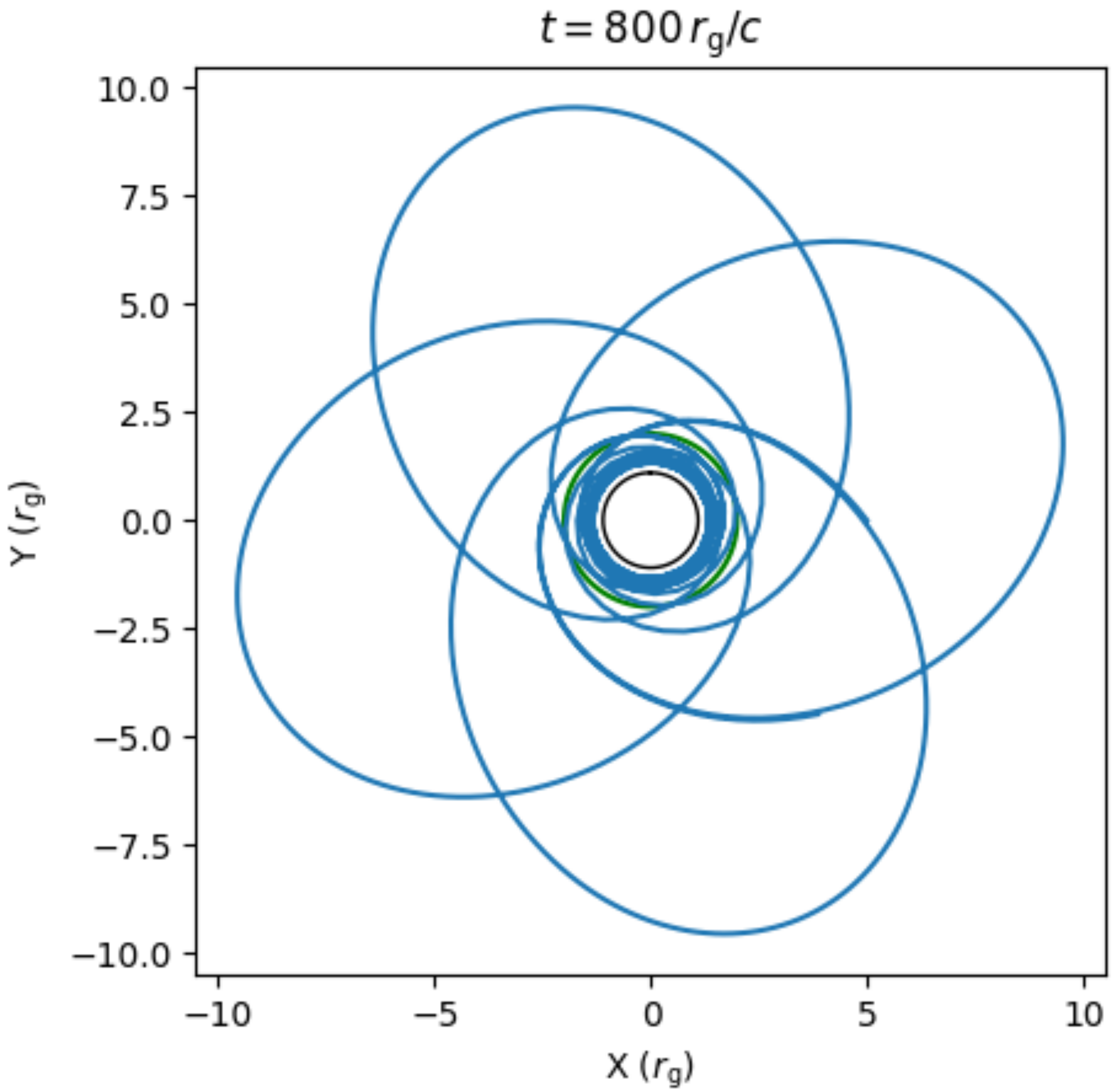
Geodesic tests — $\boldsymbol{B} = 0$ @ dt = 0.1 r_g/c

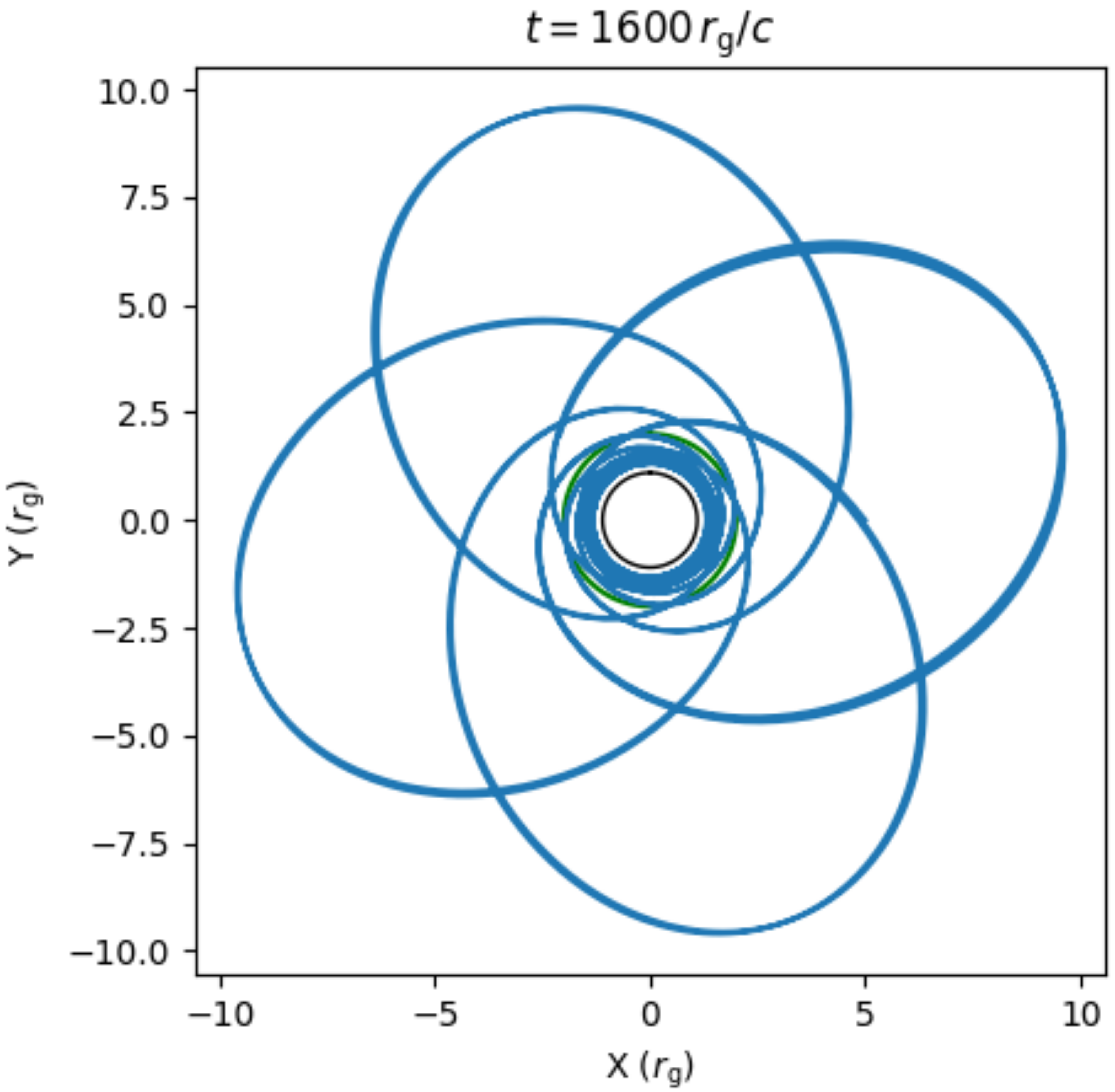




Periodic orbits: Levin & Perez-Giz 2008

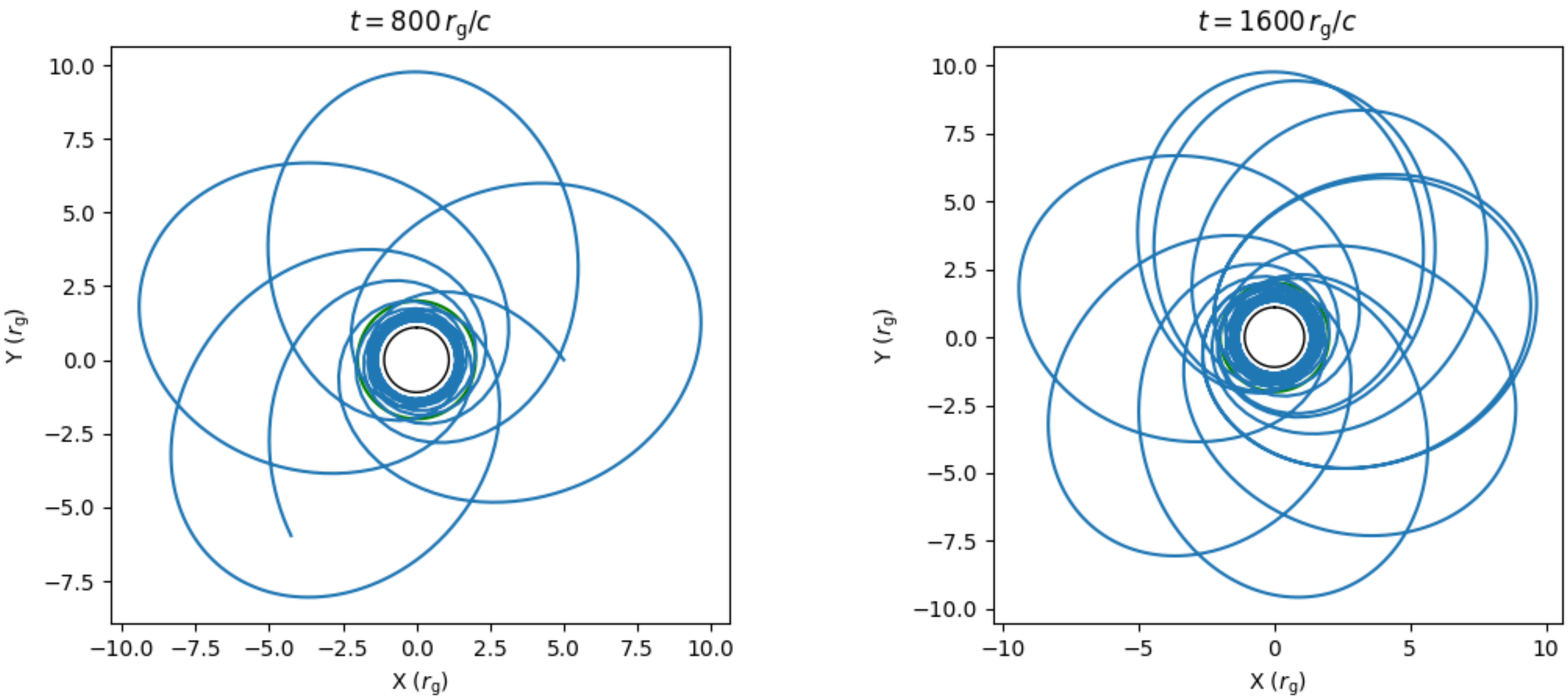
Geodesic tests — $\boldsymbol{B} = 0$ @ $\mathrm{d}t = r_g/c$



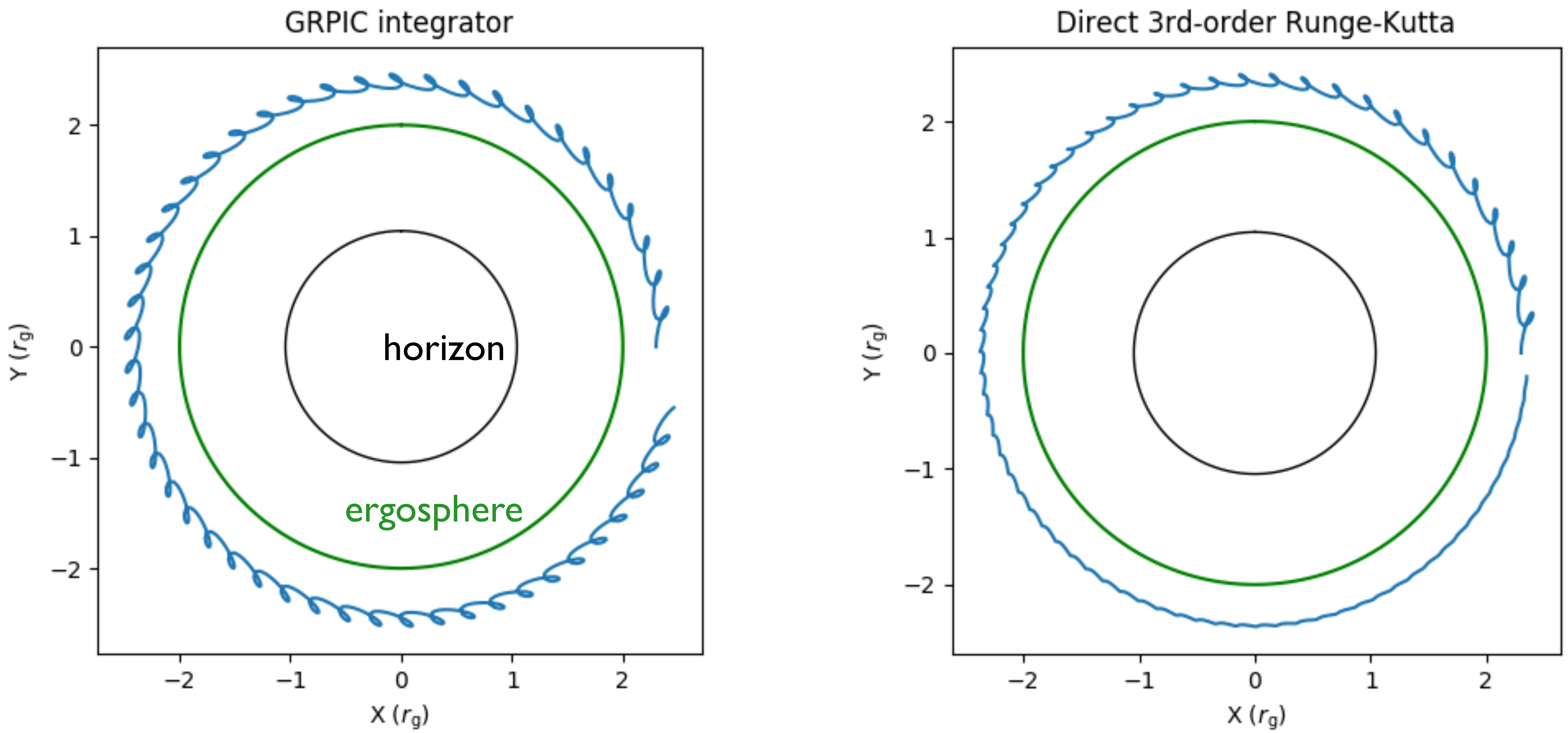


large timestep

3rd-order Runge-Kutta @ dt = r_g/c



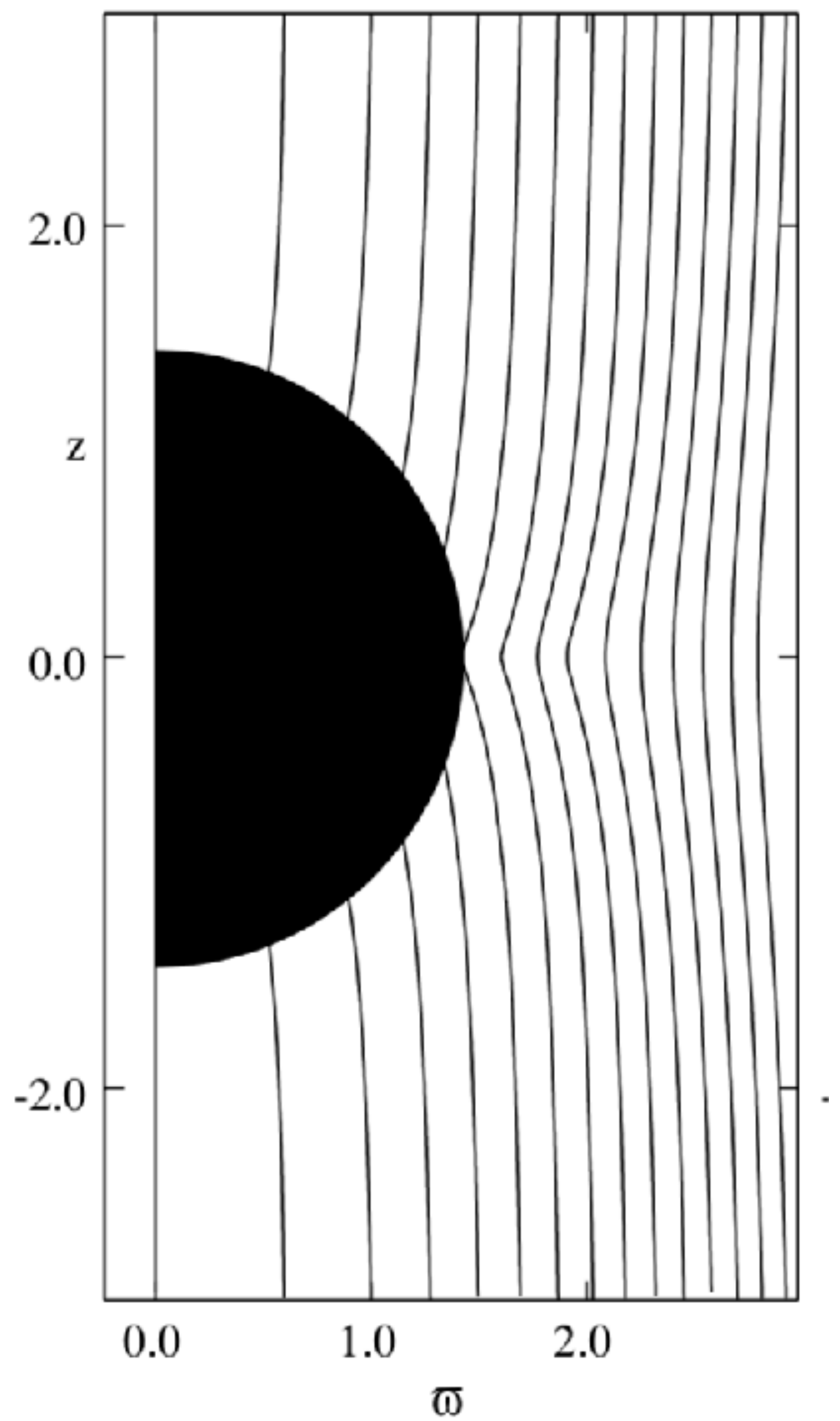
Add ***B*** and ***D*** – particle in uniform (Wald) vacuum field



Kerr metric: a = 0.999

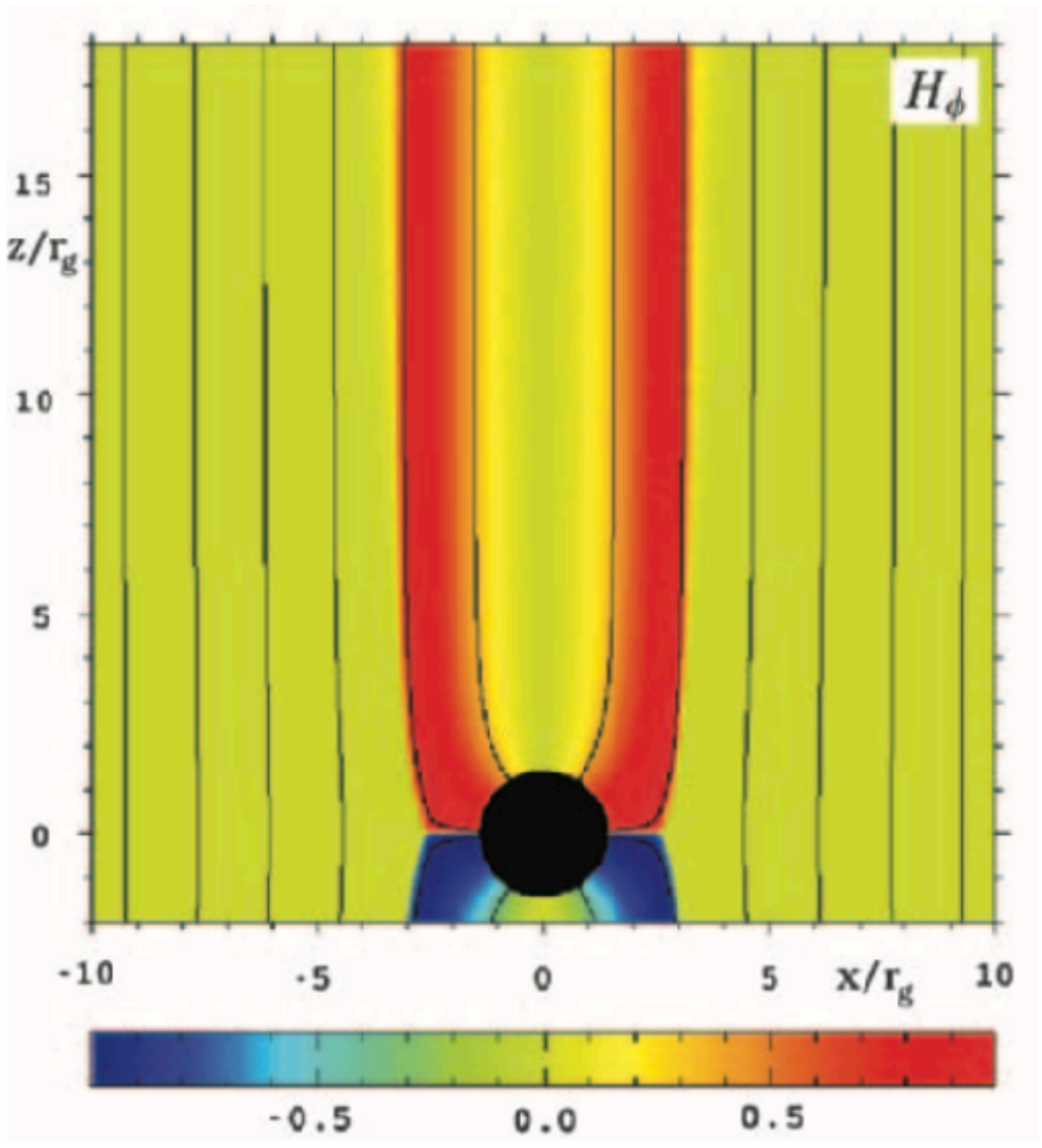
equatorial plane

A first application: “magnetospheric Wald” problem



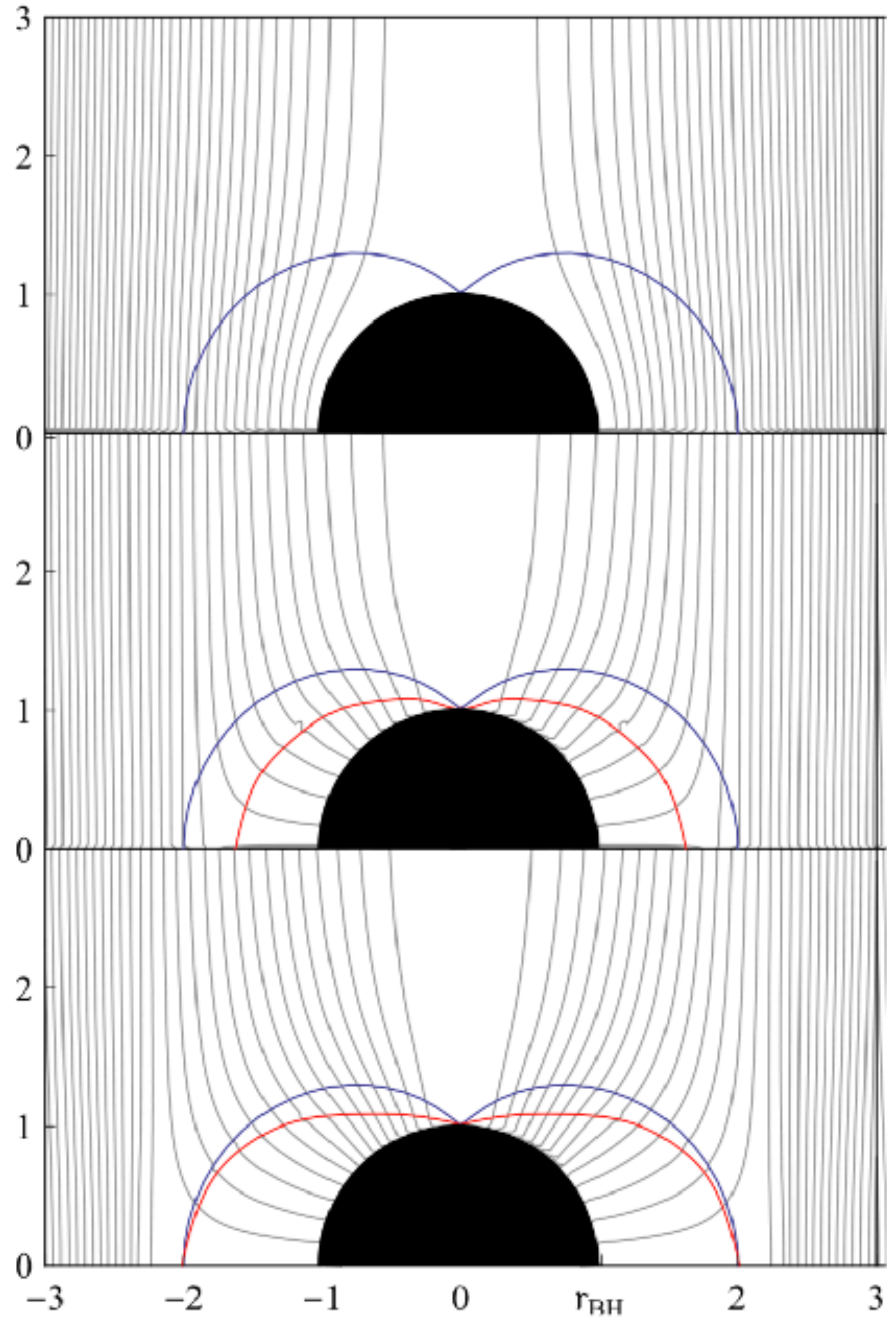
force-free electrodynamics

Komissarov 2004



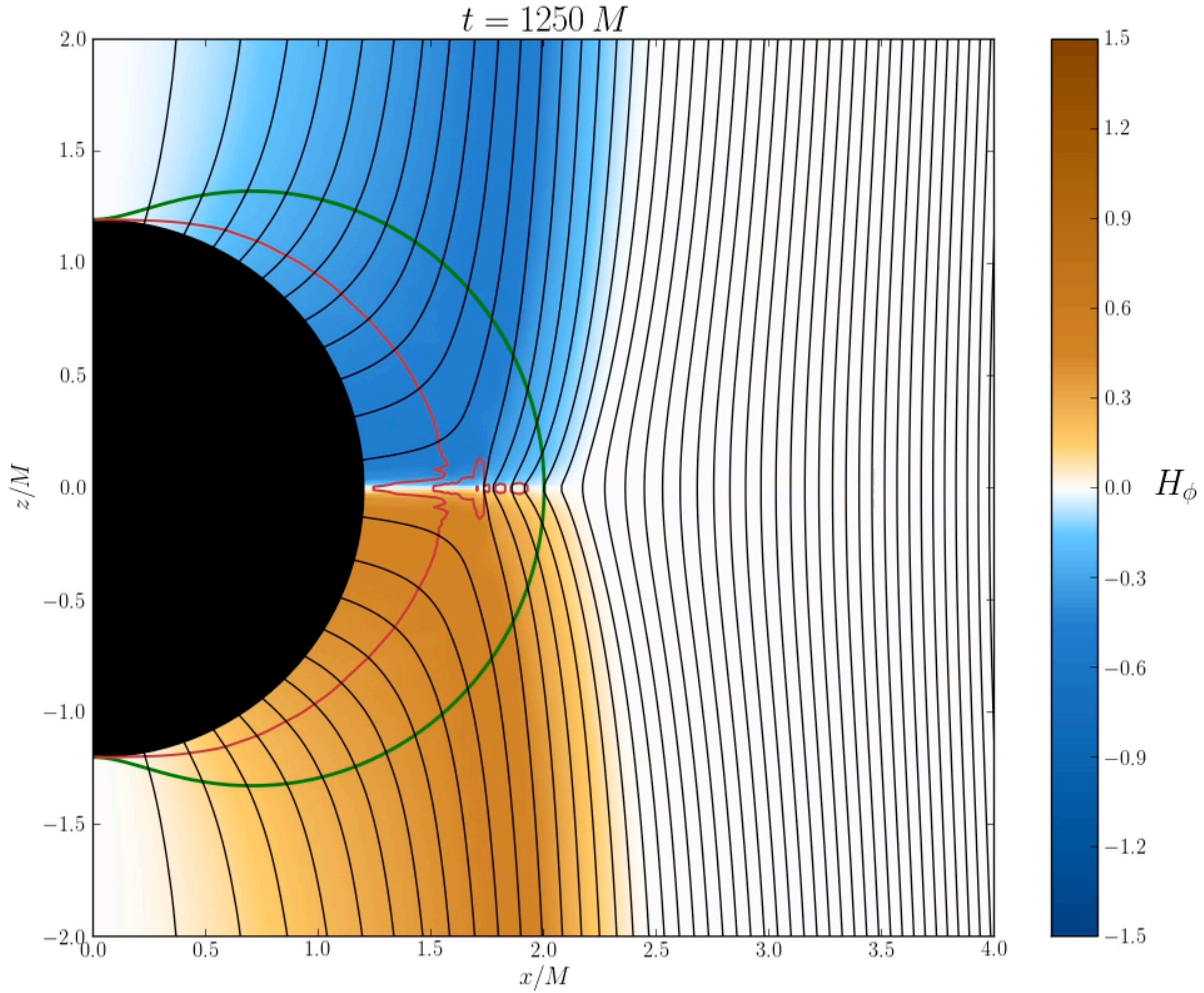
MHD

Komissarov 2005



Grad-Shafranov

Nathanail & Contopoulos 2014



force-free electrodynamic solution ($B^2 >> \rho c^2$)

Phaedra code – Parfrey+ 2012

a = 0.98

Energy flux through horizon

$L_{\mathrm{FFE}} \sim 0.2\ B_0^2$

Kinetic problem set-up

Characteristic quantities

Larmor (gyro) radius: $r_{\mathrm{L},0} = 10^{-3} r_{\mathrm{g}} \longrightarrow B_0$ uniform field strength at infinity

$$n_0 = \frac{\Omega_{\mathrm{H}} B_0}{4\pi c e} \longrightarrow \sigma_0 = \frac{B_0^2}{4\pi n_0 m c^2} \approx 2000$$ strongly magnetized

Scale hierarchy: $r_{\mathrm{L},0} \ll \delta_0 \ll r_{\mathrm{g}}$

plasma skin depth

Plasma Injection

If: $\dfrac{\vec{D}\cdot\vec{B}}{B^2} > \epsilon_{D\cdot B}$ then: inject particles $n \propto |\vec{D}\cdot\vec{B}|$

Two runs:
- $\epsilon_{D\cdot B} = 10^{-3}$ "high plasma supply"
- $\epsilon_{D\cdot B} = 10^{-2}$ "low plasma supply"

Initial State

vacuum

Wald steady-state solution

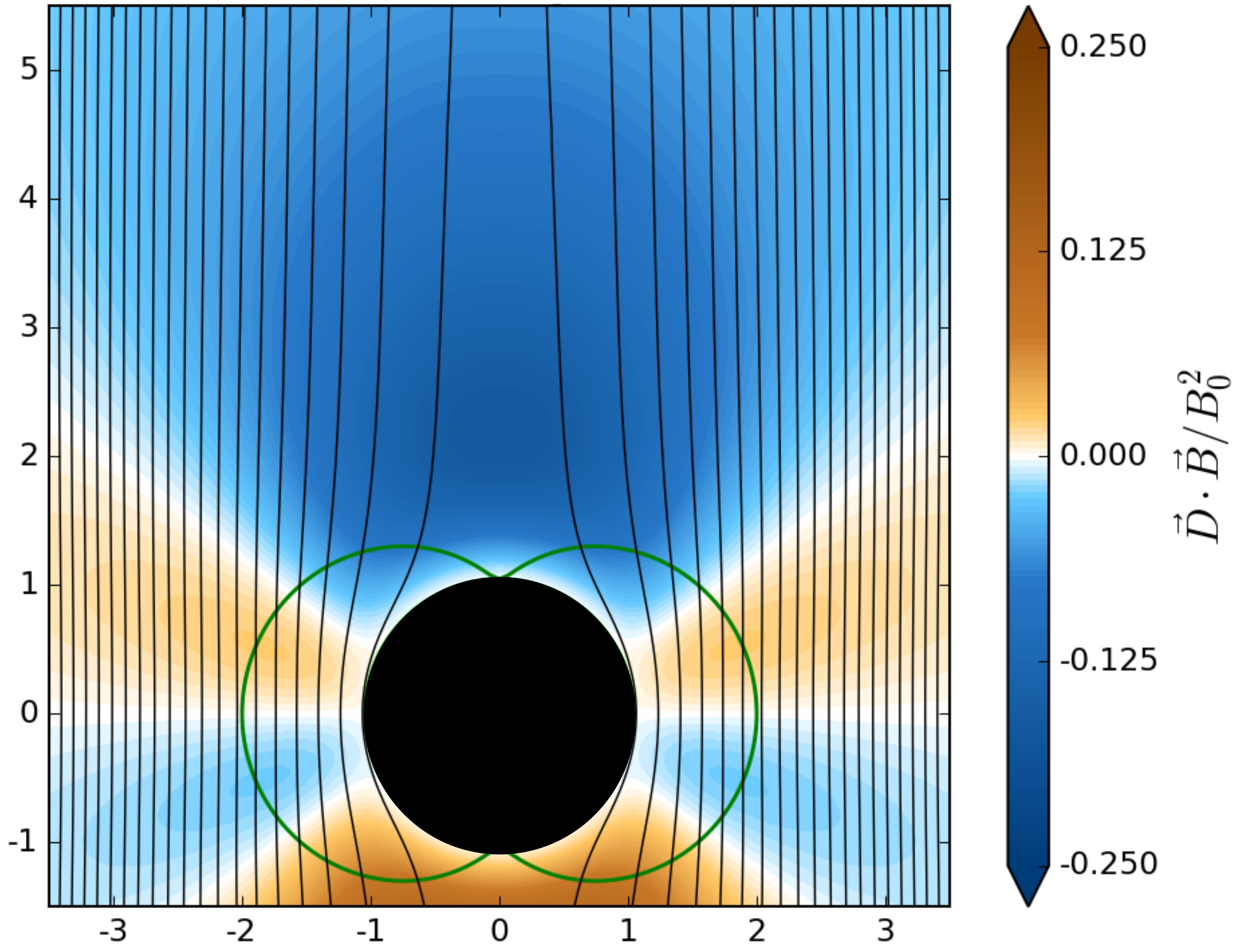
Wald 1974

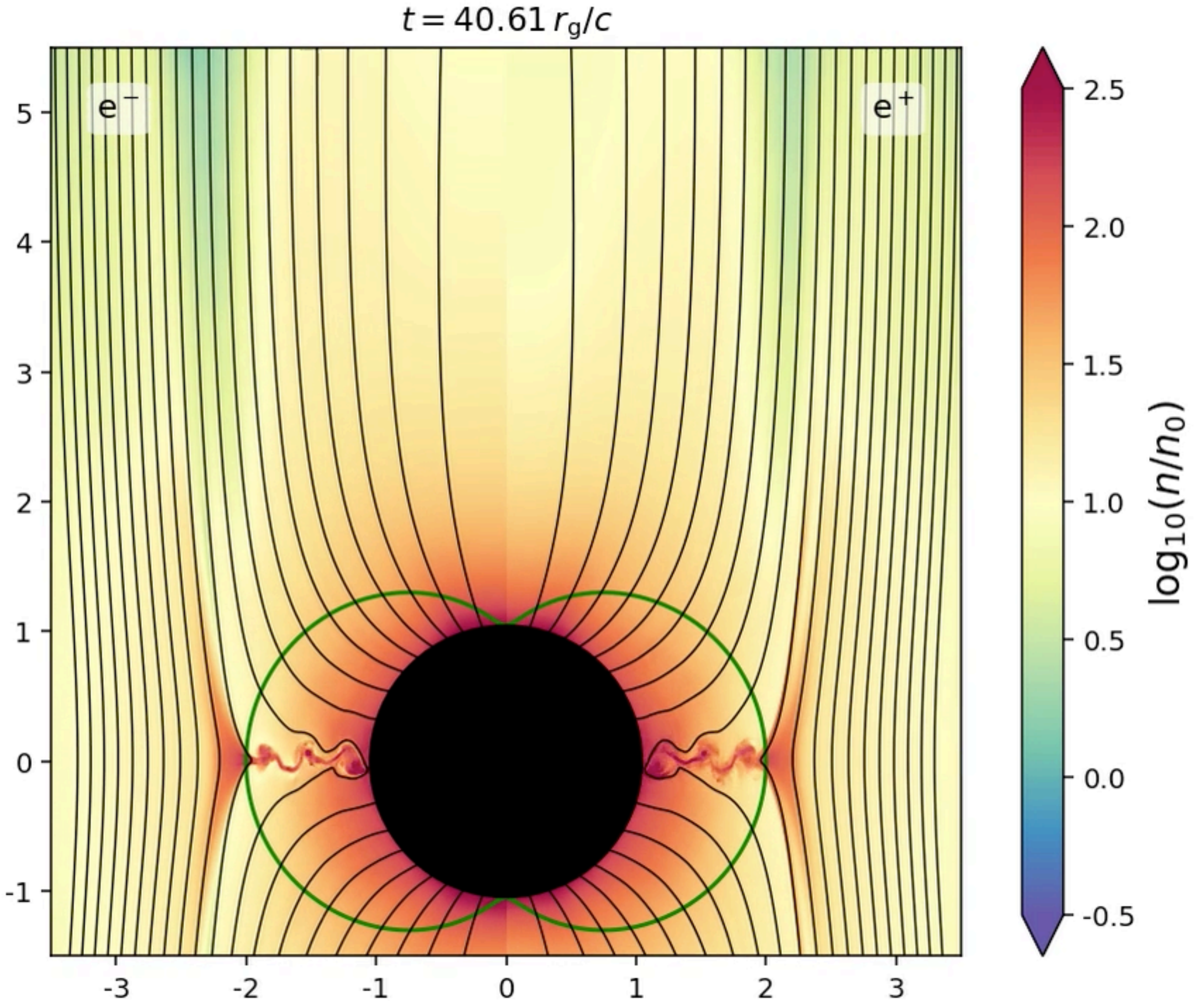
$a = 0.999$

$$A_\mu = m_\mu + 2ak_\mu$$

where $m^\mu = \partial_\phi$

$k^\mu = \partial_t$





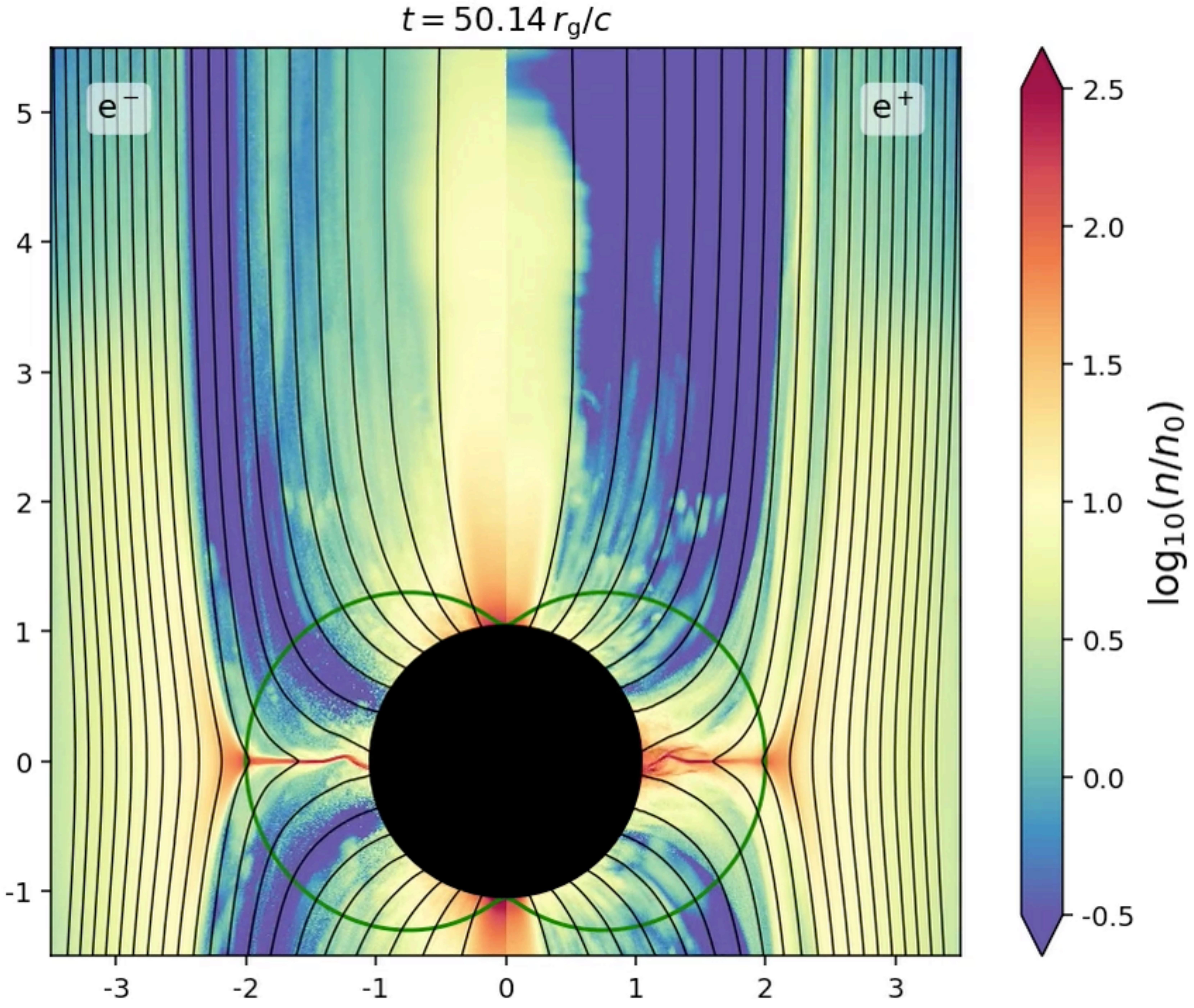
High Plasma Supply

$$\epsilon_{D\cdot B} = 10^{-3}$$

<u>colour</u>

electron & positron densities

Collisionless plasma instabilities in full GR



Low Plasma Supply

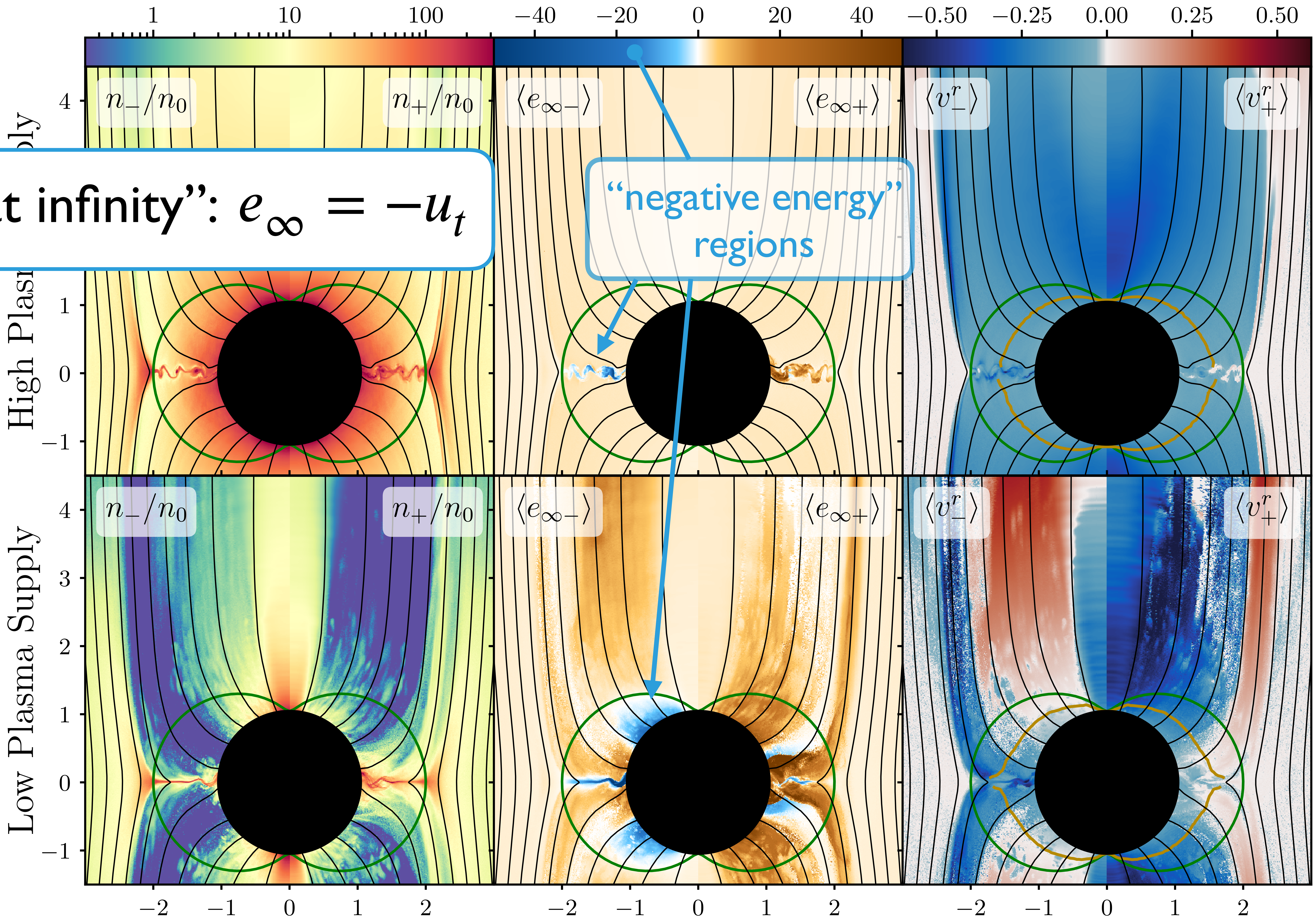
$\epsilon_{D\cdot B} = 10^{-2}$

colour

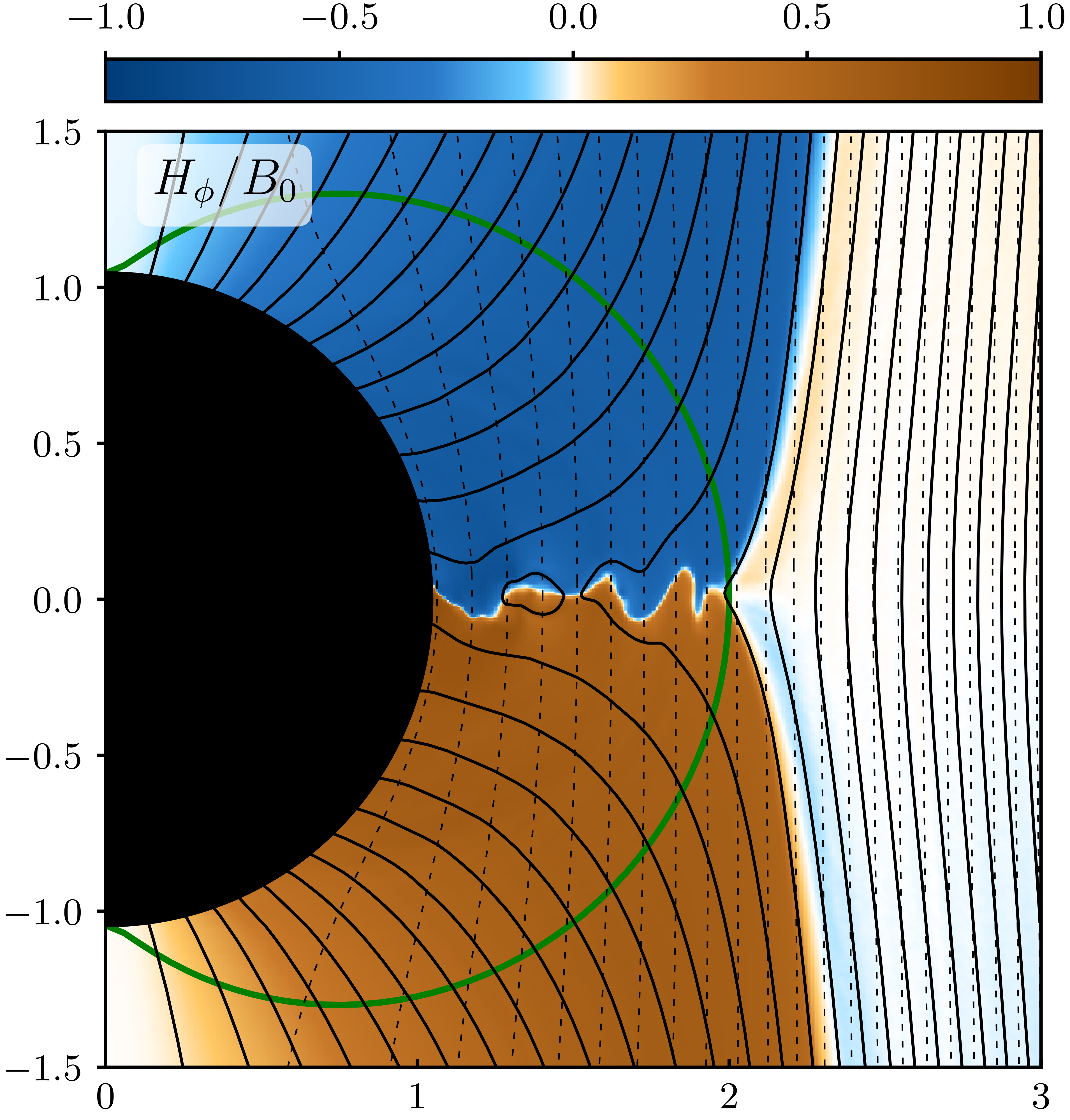
electron & positron densities

Parfrey, Philippov, Cerutti 2019

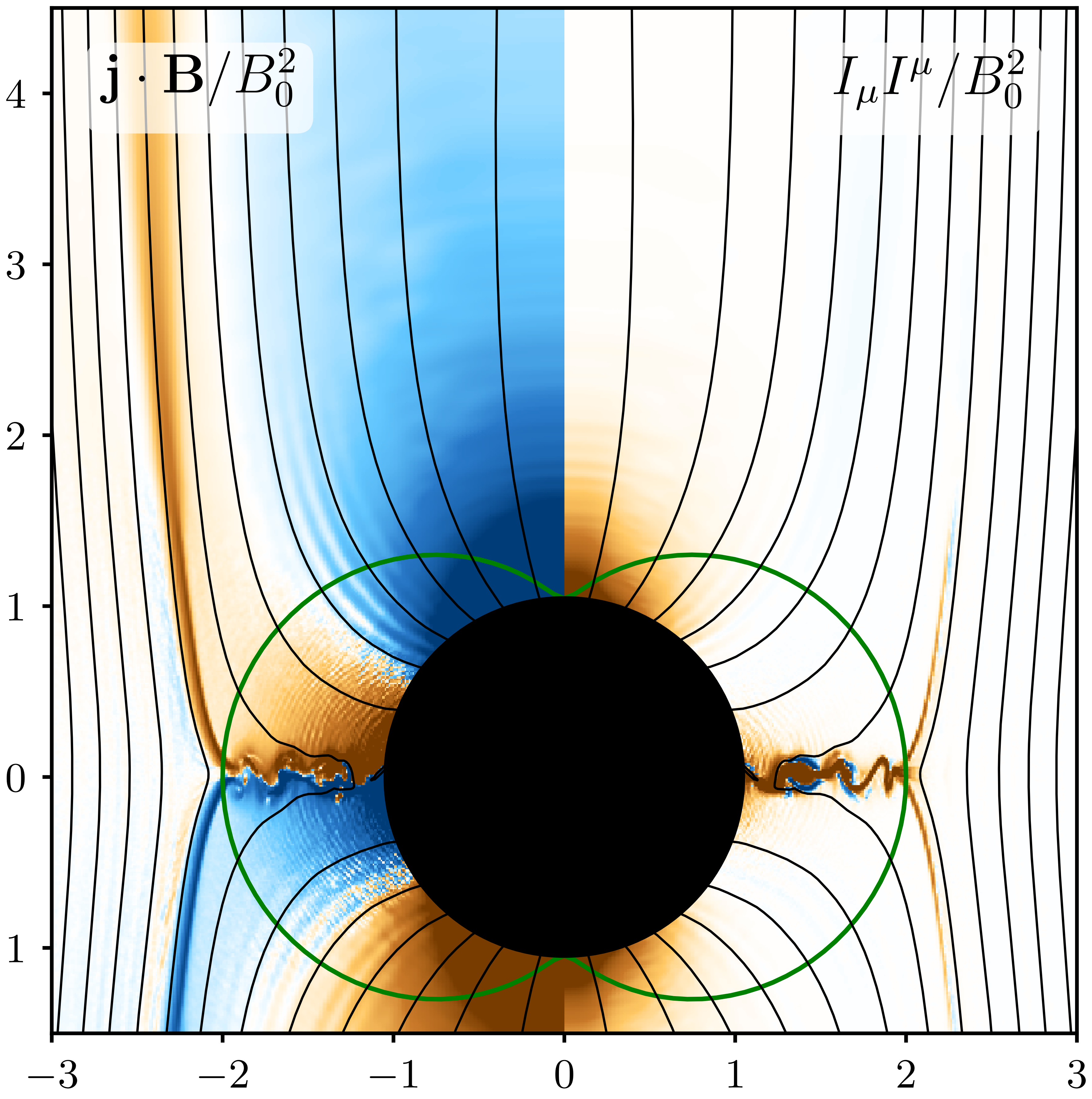
“energy at infinity”: $e_\infty = -u_t$



Gross electrodynamic solution

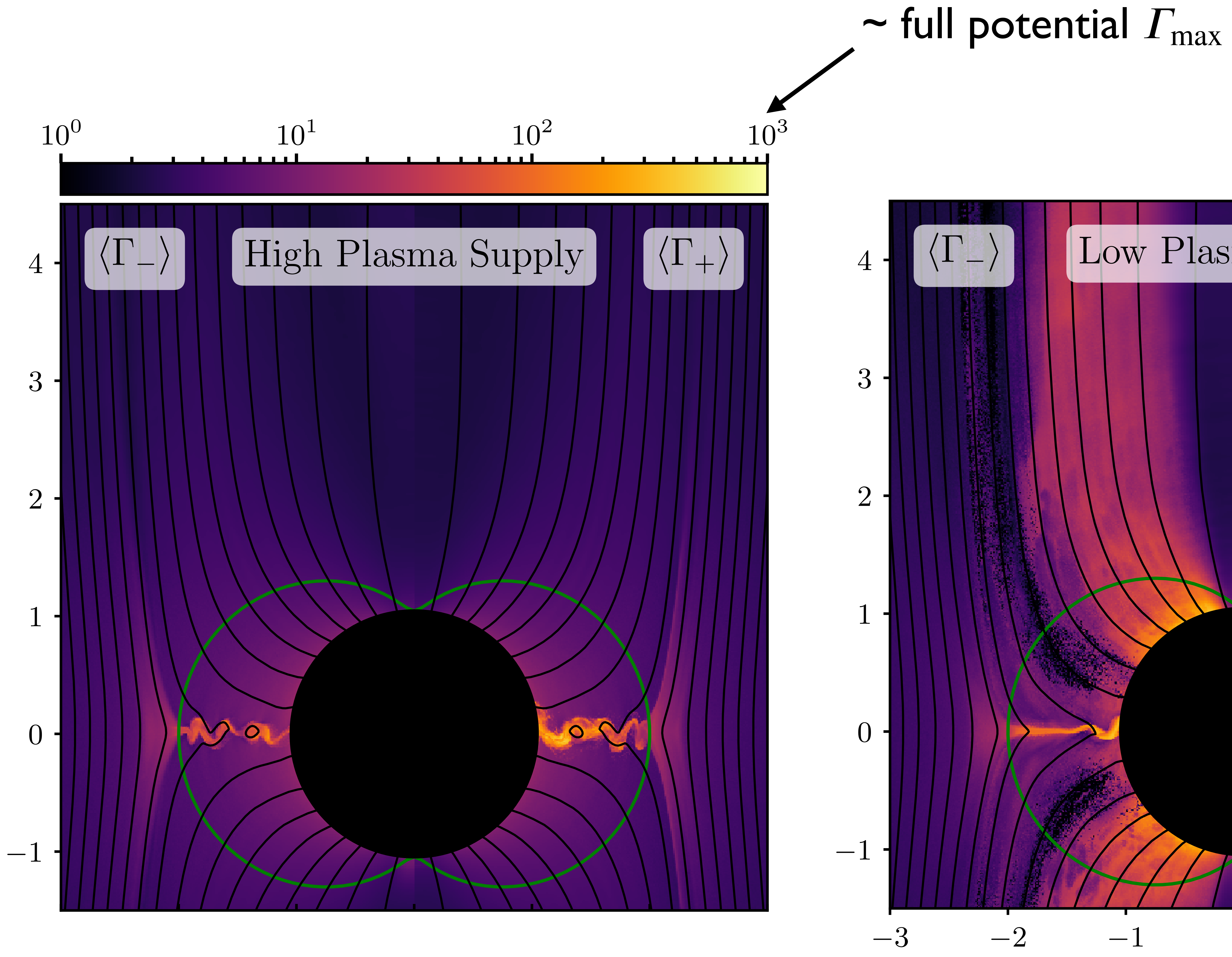


Toroidal magnetic field



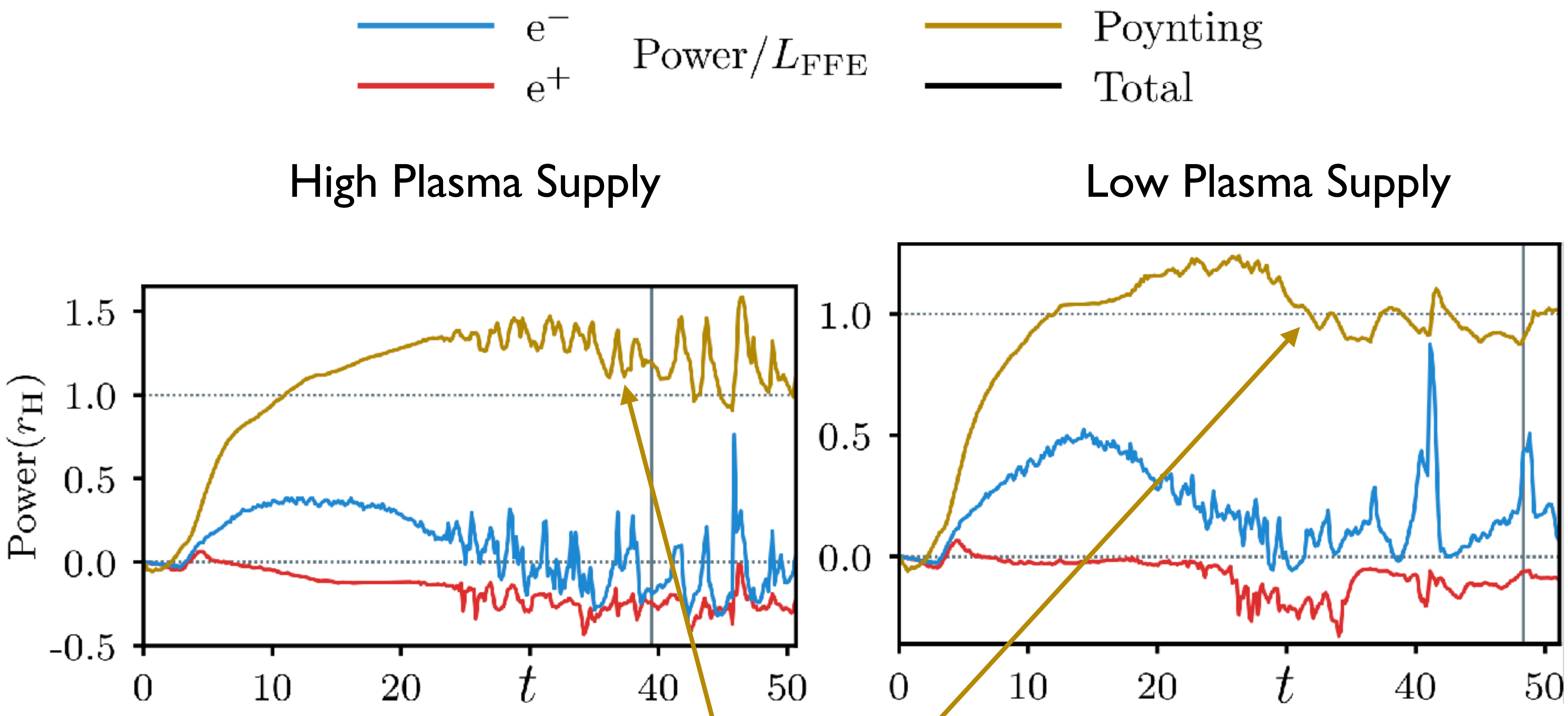
Current structure

Particle acceleration



Average particle Lorentz factor

Energy at infinity flux: integrated over horizon

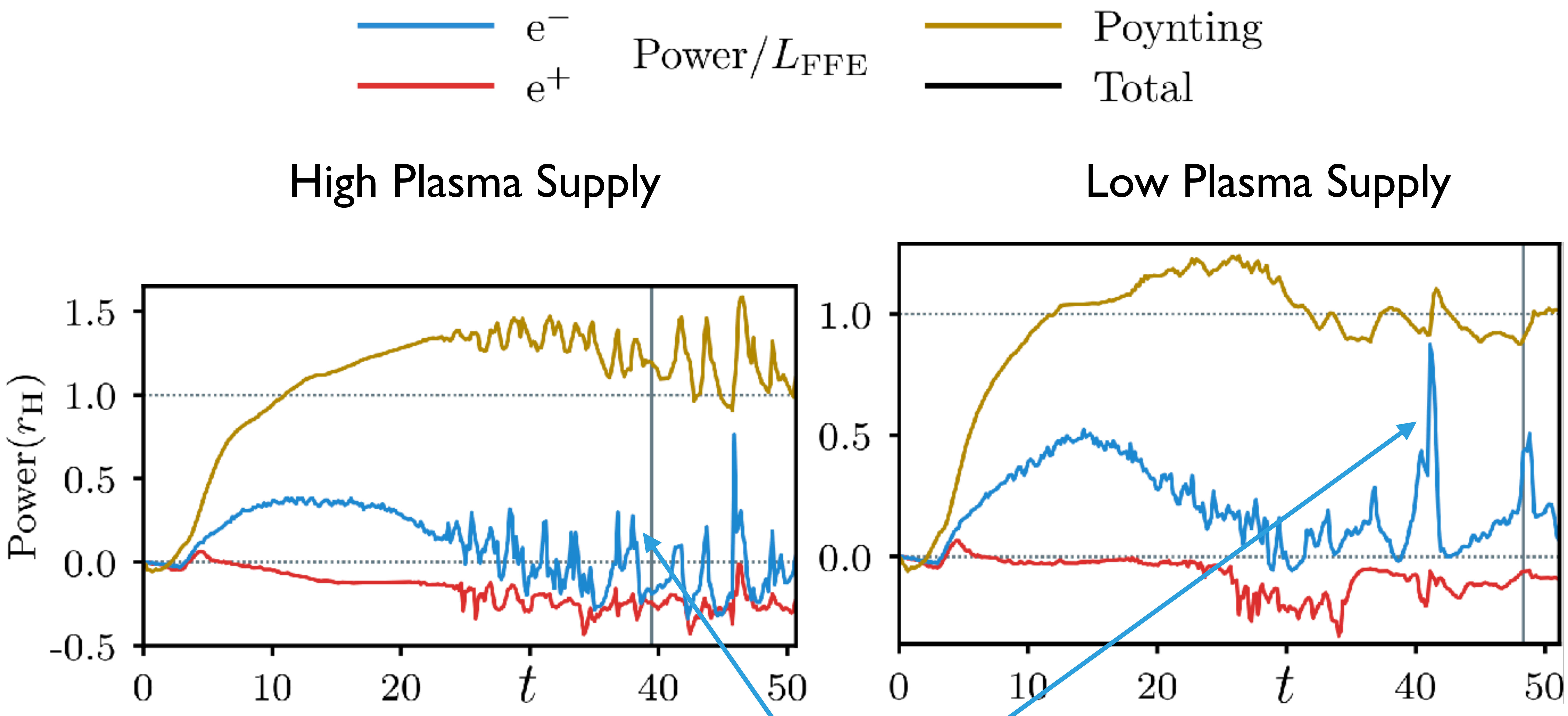


electromagnetic extraction of black hole energy

Blandford-Znajek (1977) mechanism for relativistic jets

Parfrey, Philippov, Cerutti 2019

Energy at infinity flux: integrated over horizon

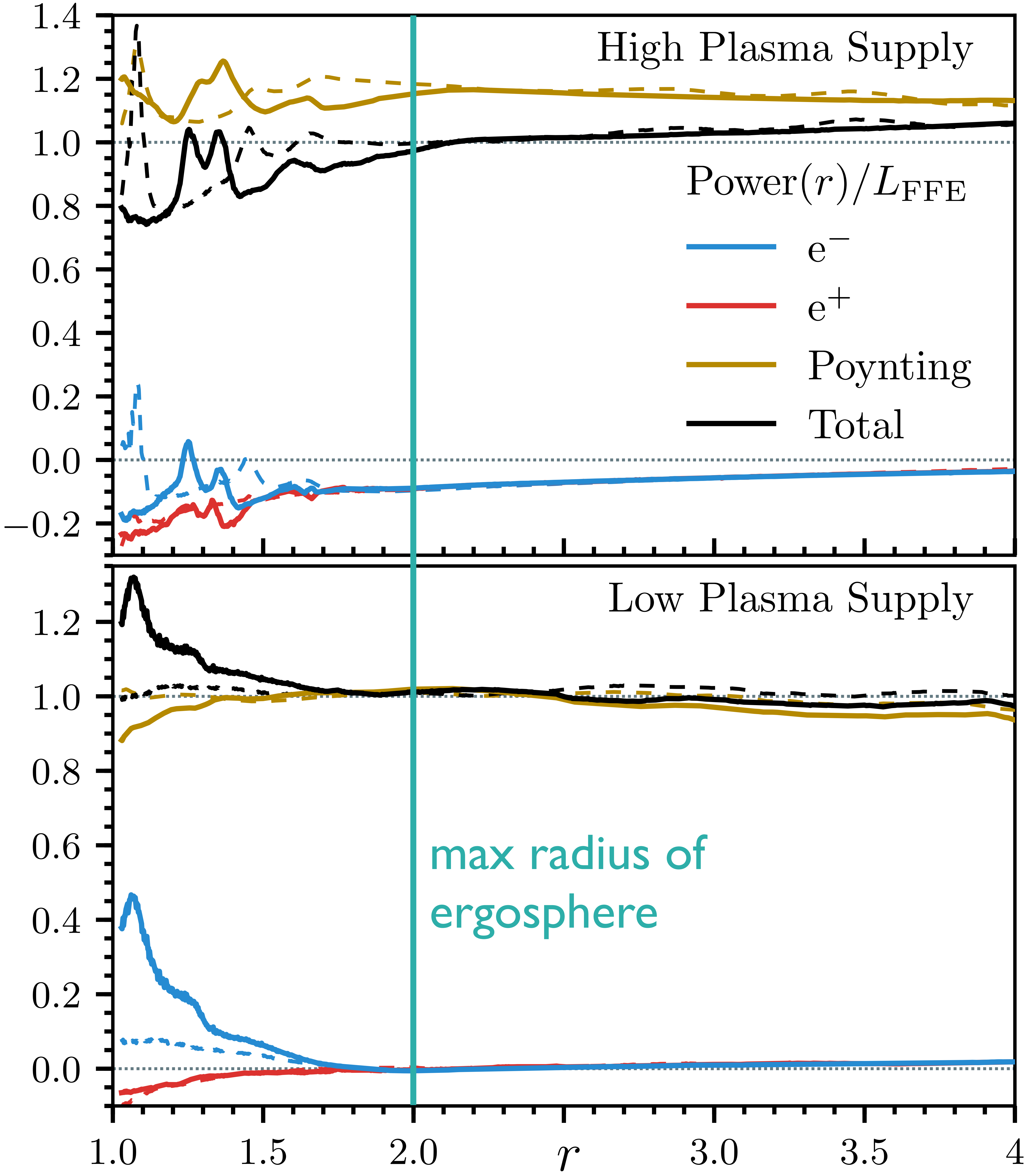


extraction of black hole
energy by **particles**

variant of *Penrose (1969) process*

Parfrey, Philippov, Cerutti 2019

Parfrey, Philippov, Cerutti 2019



Energy at infinity flux vs radius

time indicated by grey line in previous slide

t = 50 r_g/c

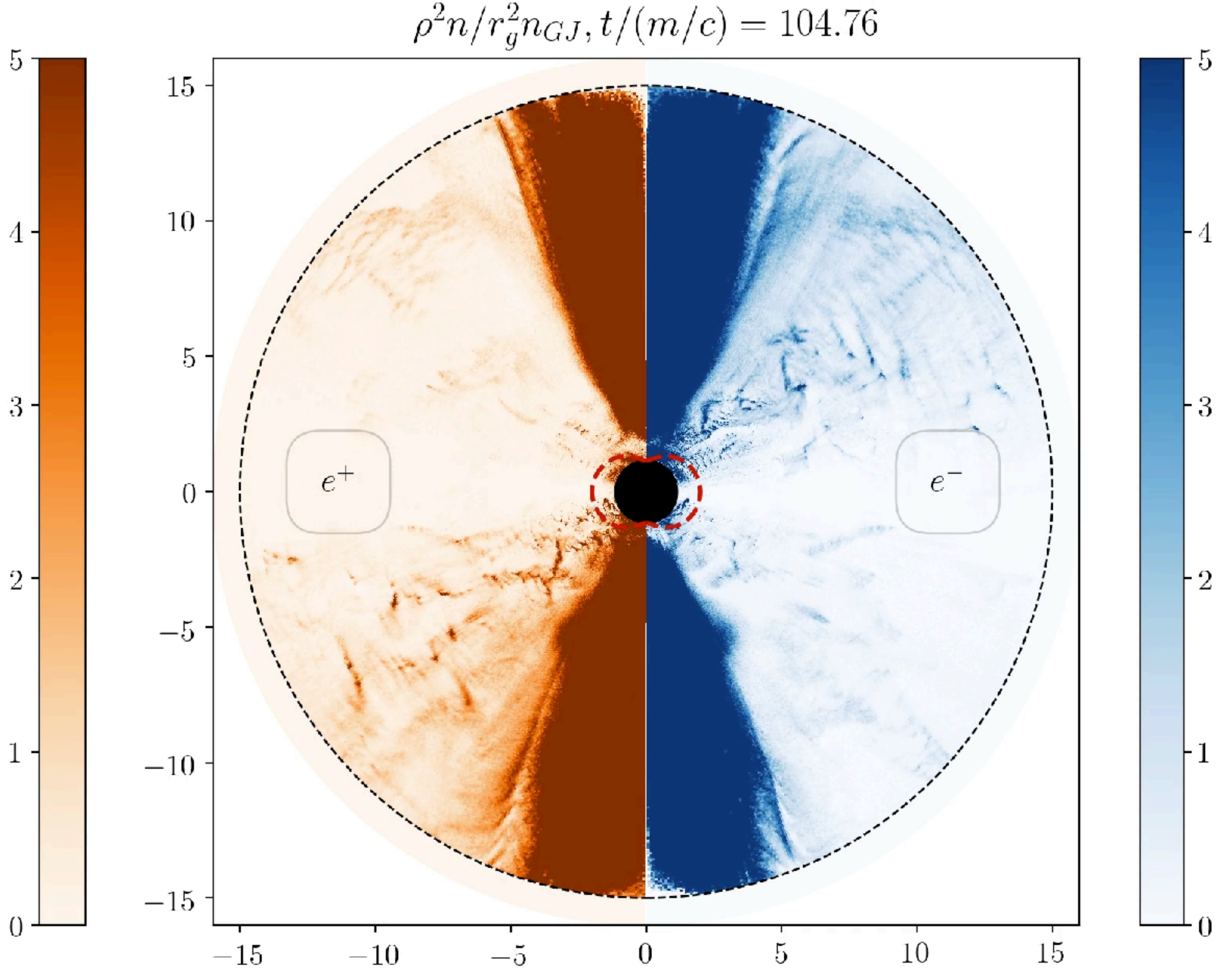
At large r, all power carried by *Poynting flux*

Preliminary: magnetosphere with Monte Carlo pair creation

- Uniform, isotropic, monoenergetic background radiation field at energy ε_0
- High-energy photons γ_{HE} as a third particle species
- $e^\pm$ produce γ_{HE} via inverse-Compton scattering
- γ_{HE} can pair-produce off the background radiation
- Use the complete differential cross sections for post-interaction energies
- Monte Carlo: an interaction occurs if a random number $p \in [0,1]$ satisfies

$$p < 1 - e^{-\delta\tau}$$

where $\delta\tau$ is the optical depth traversed over a time step.



Fiducial optical depth

$\tau_0 = n_0\, r_g\, \sigma_T = 20$

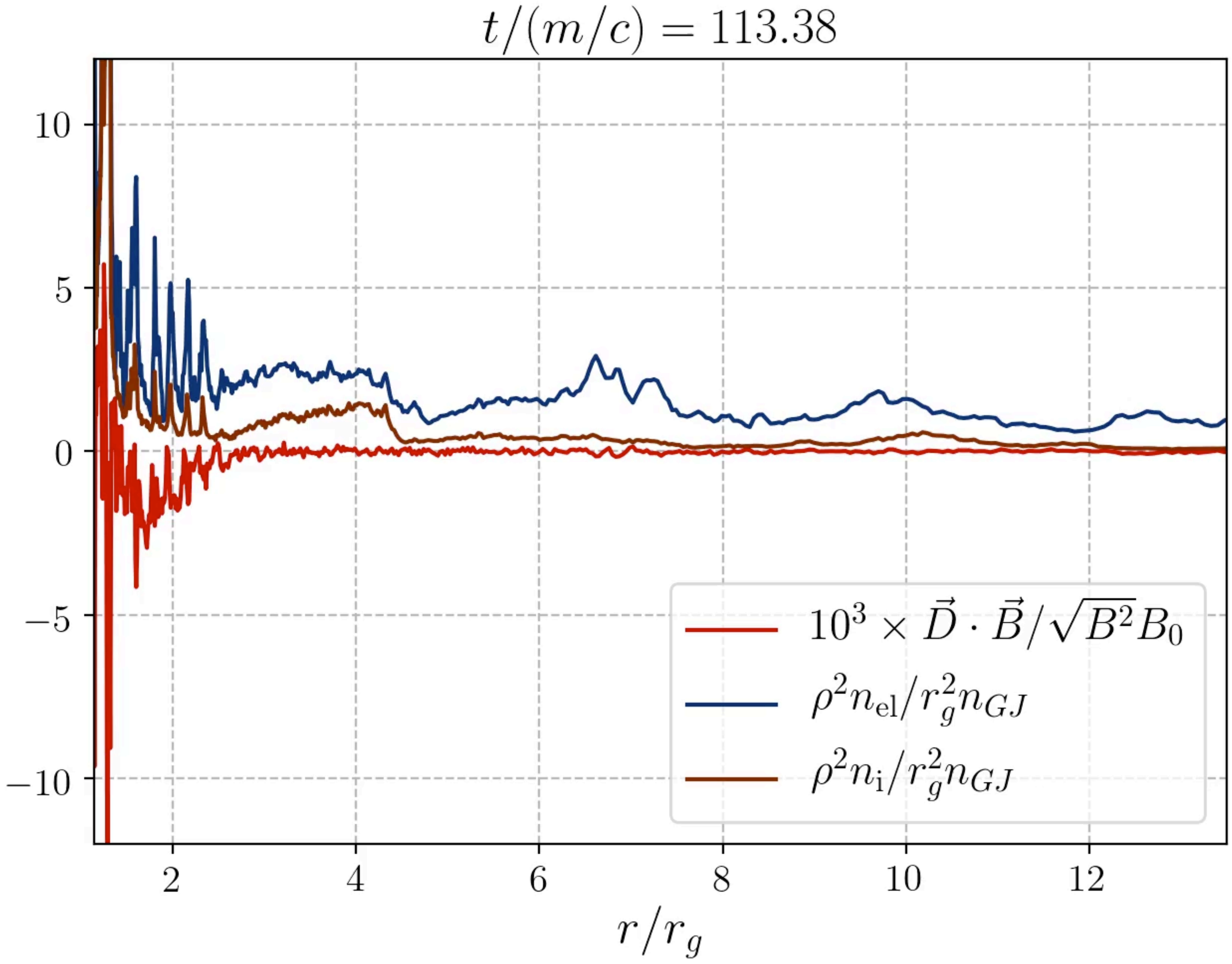
$B_0\, \varepsilon_0 = 10^3$

$\gamma_{escape} \ll \gamma_{sec} \ll \gamma_{rad}$

$\sim 1/\varepsilon_0$

$\gamma_{rad}/\gamma_{sec} \sim f\,(B_0\varepsilon_0)$

Crinquand et al. , *in prep*



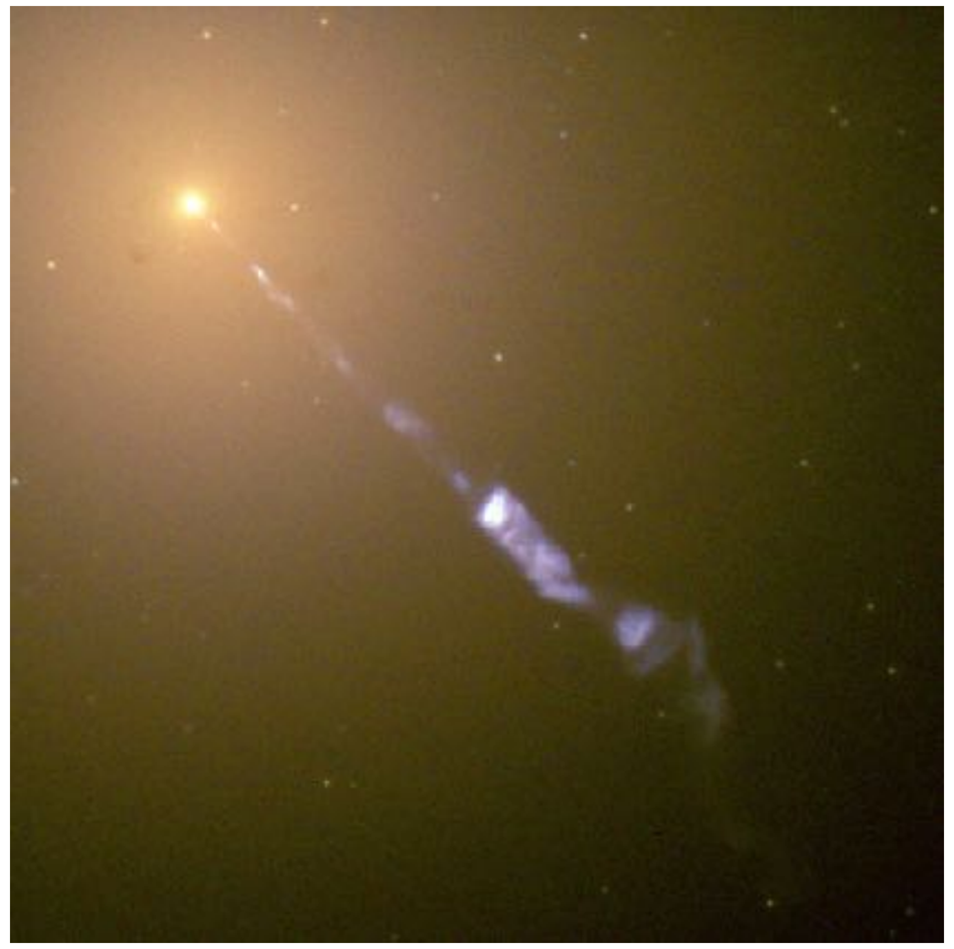
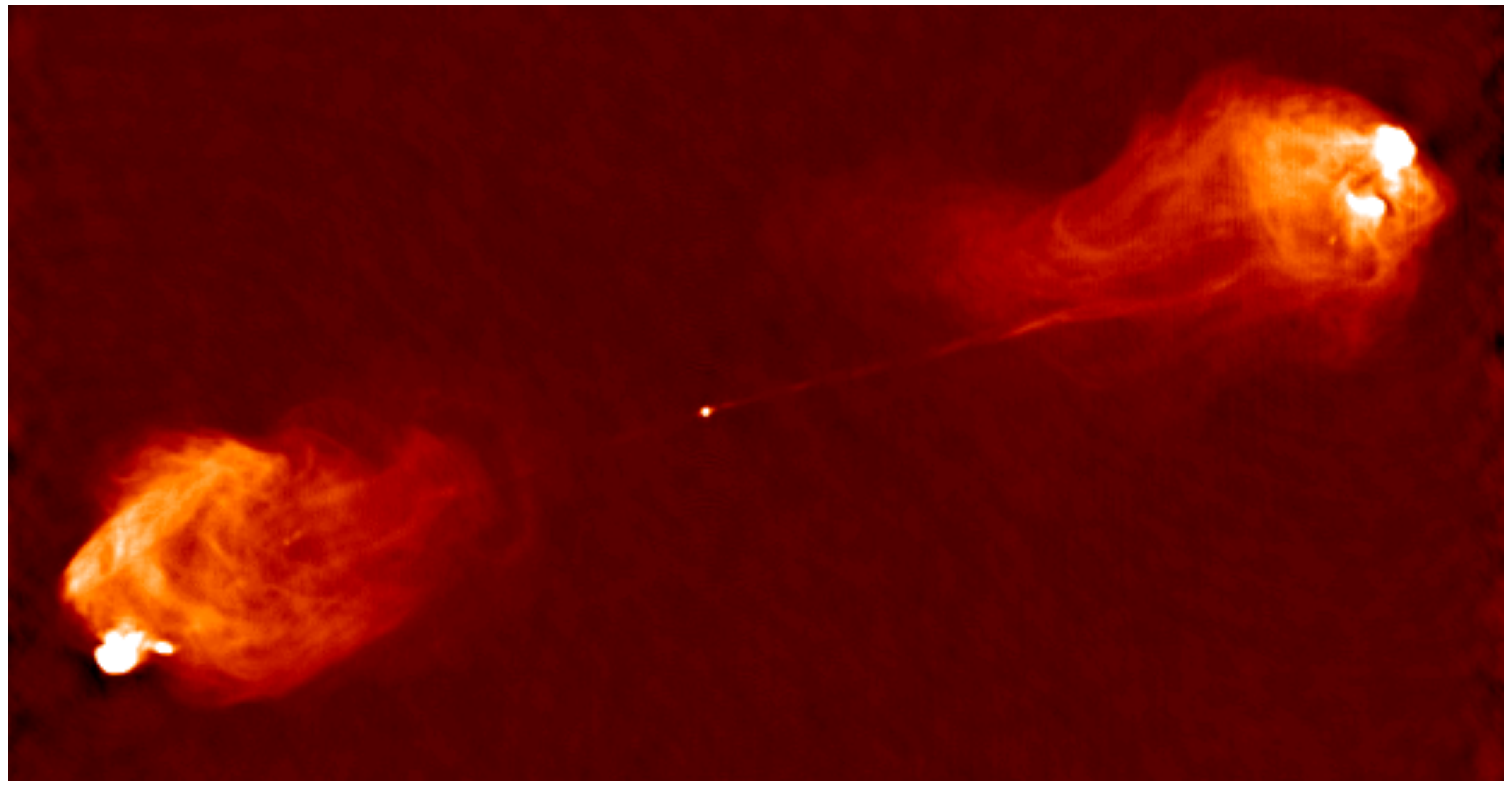
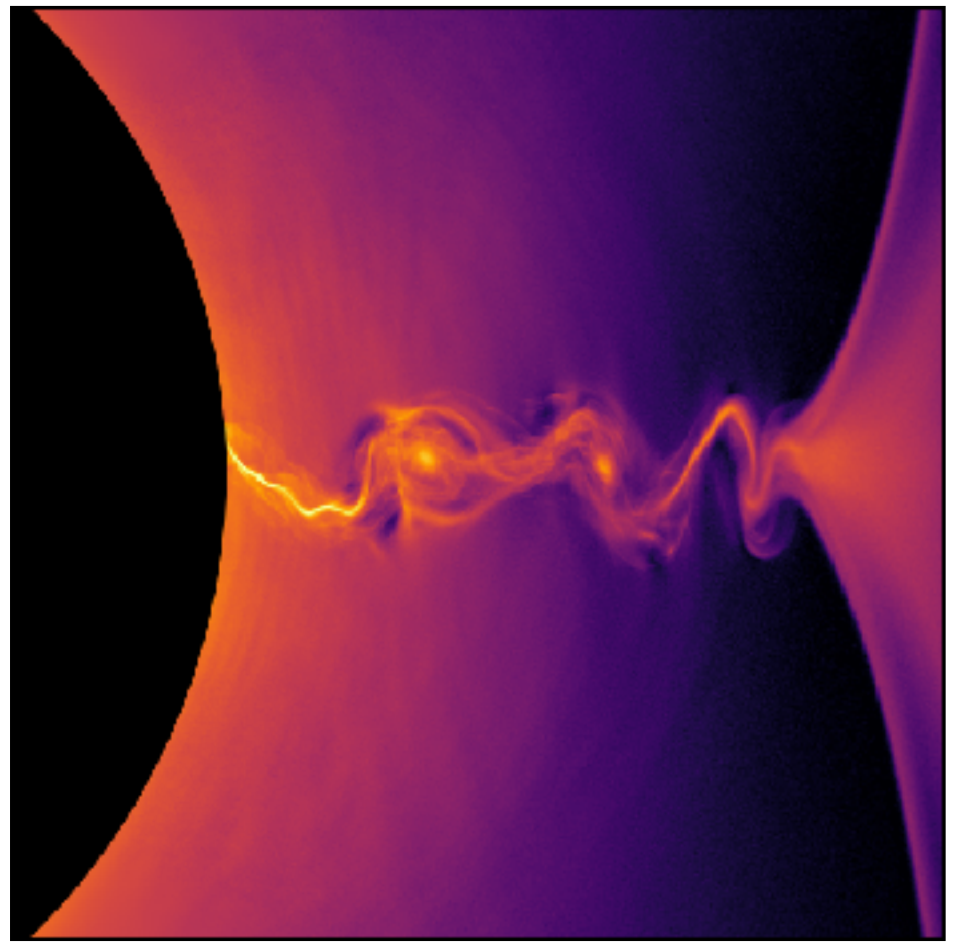
Fiducial optical depth

$\tau_0 = n_0\, r_g\, \sigma_T = 20$

$B_0\, \varepsilon_0 = 10^3$

$\gamma_{\rm escape} \ll \gamma_{\rm sec} \ll \gamma_{\rm rad}$

Crinquand et al., *in prep*

Summary

– First multidimensional collisionless plasma simulations in full GR

– Poynting-flux-dominated jet launching from first principles

– Negative-energy "Penrose" particles in both *jet* and *current sheet*

Work in progress

– Add realistic pair-creation physics: how do vacuum gaps behave?

– Complete simulation of a collisionless accretion flow…