Cosmic Ray Transport: The Role of Pressure Anisotropy

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Working from an advanced draft

Caveat Emptor

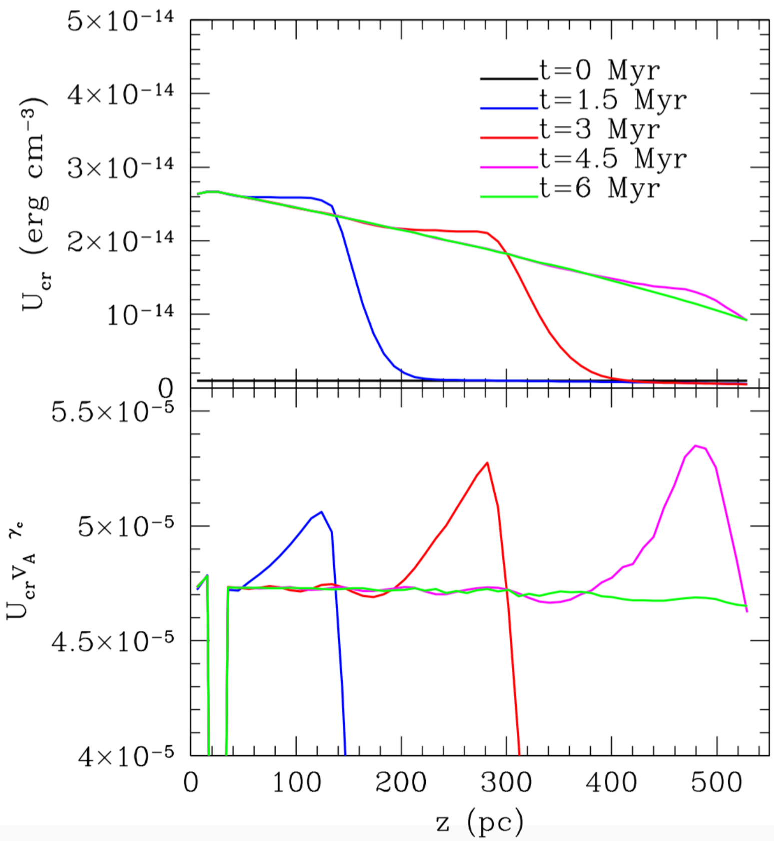
Cosmic Ray Confinement in a Nutshell

- Mediated by gyroscale (~AU) scale small amplitude waves.
- Wave are driven by a turbulent cascade (extrinsic turbulence) or generated by cosmic ray streaming anisotropy (self confinement)
- Cosmic rays exert a pressure gradient force in both cases and collisionlessly heat the gas by transferring energy to waves in the self confinement case ($H=|v_A \cdot \nabla P_c|$).

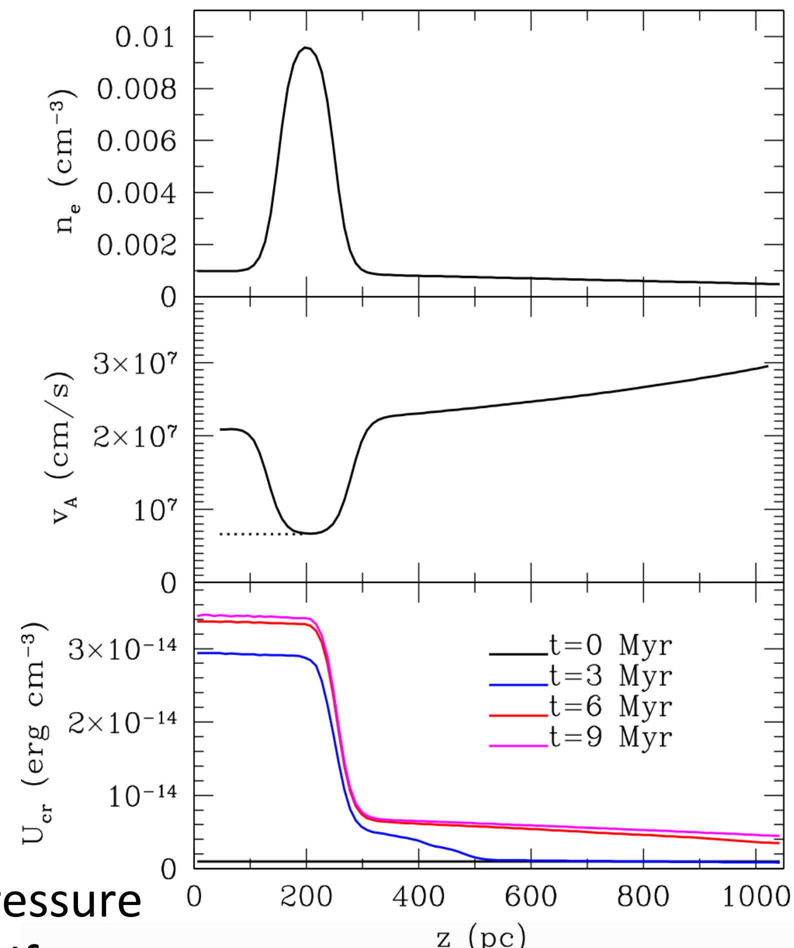
adsabs.harvard.edu/abs/2017PhPl...24e5402Z/abstract

Bottleneck Effect

No Cloud



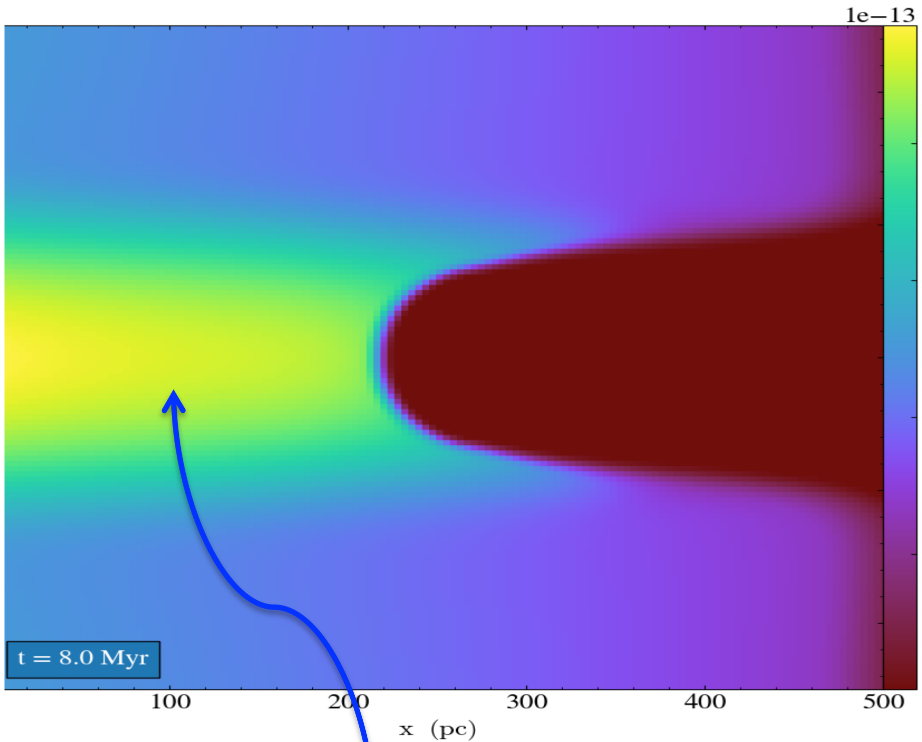
Cloud

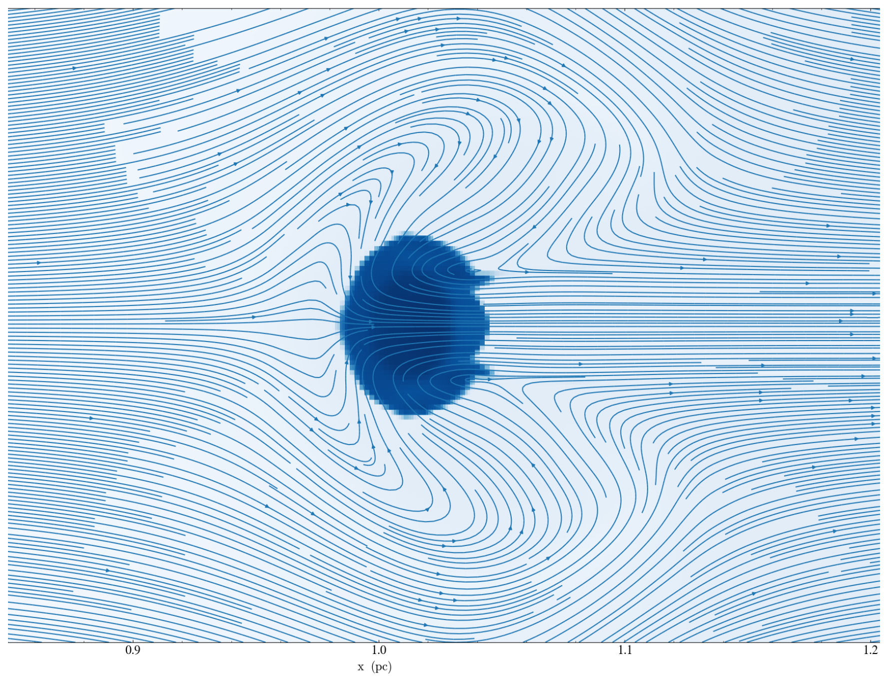


Left: Cosmic rays streaming down their pressure gradient at v_A ***evolve to constant $P_c/\rho^{\gamma_c/2}$***. If ρ has a maximum, P_c behind it must go flat (Right).

Wiener, Oh, EZ (2017), Wiener, EZ, Ruszkowski (2018)

2D Bottlenecks



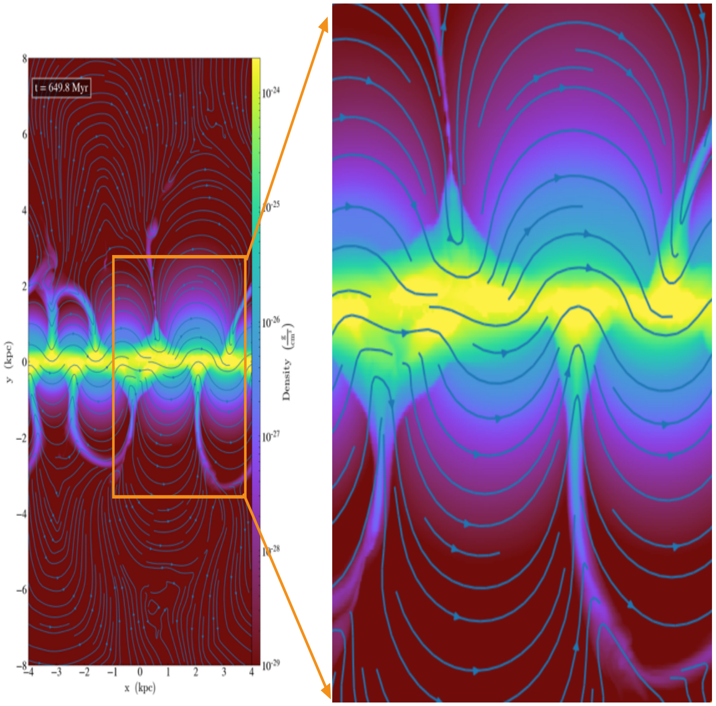


No cosmic ray – thermal gas coupling here

A weak magnetic field is strongly distorted by cosmic ray pressure buildup

Josh Wiener; JW & EZ 2019

Other Examples



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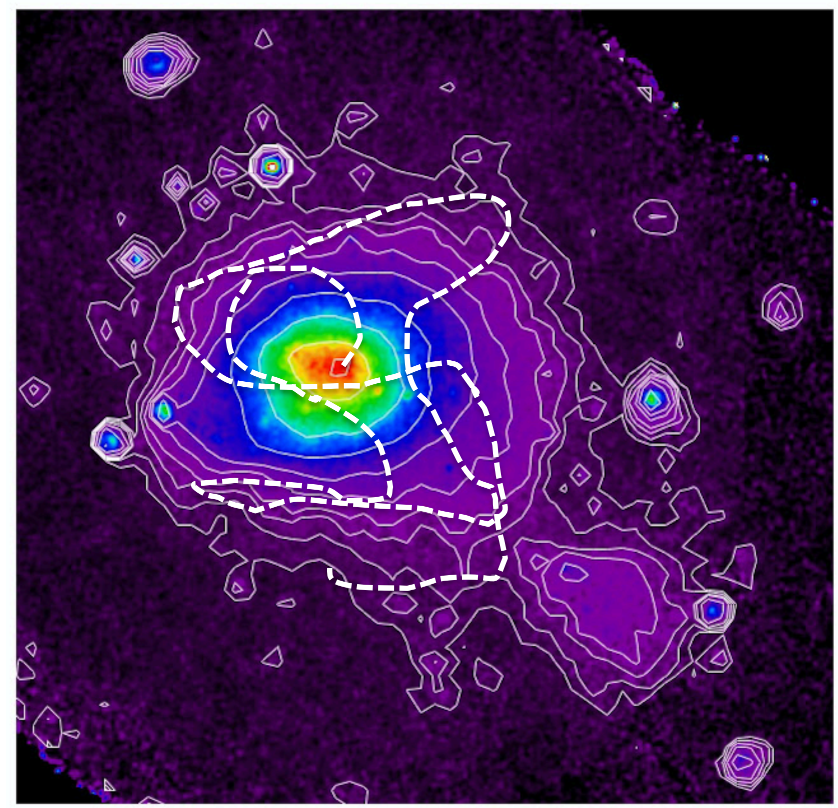


Left: Magnetic field & gas density structured by Parker Instability (Chad Bustard & Evan Heintz). Right: Rosat x-ray image of Coma Cluster; white dashed line represents tangled magnetic field .

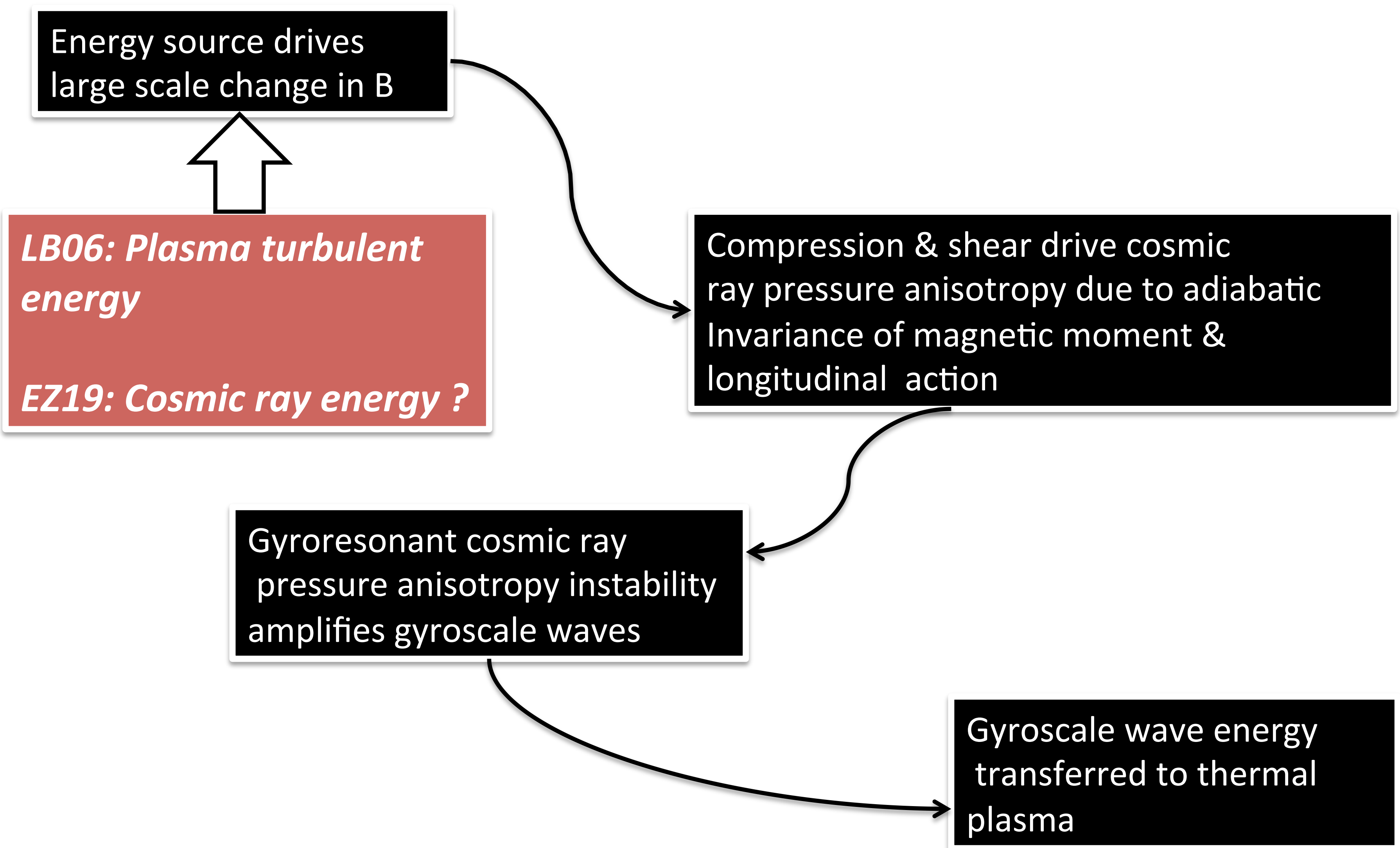
Is the Streaming Instability the Only Route to Self Confinement?

- Cosmic ray pressure anisotropy of order v_A/c *also* destabilizes hydromagnetic waves (Lazarian & Beresnyak 2006).

- Does pressure anisotropy provide self-confinement in the sense of short scattering mean free path?

- Does pressure anisotropy mediate cosmic ray work & heat transfer to the thermal gas?

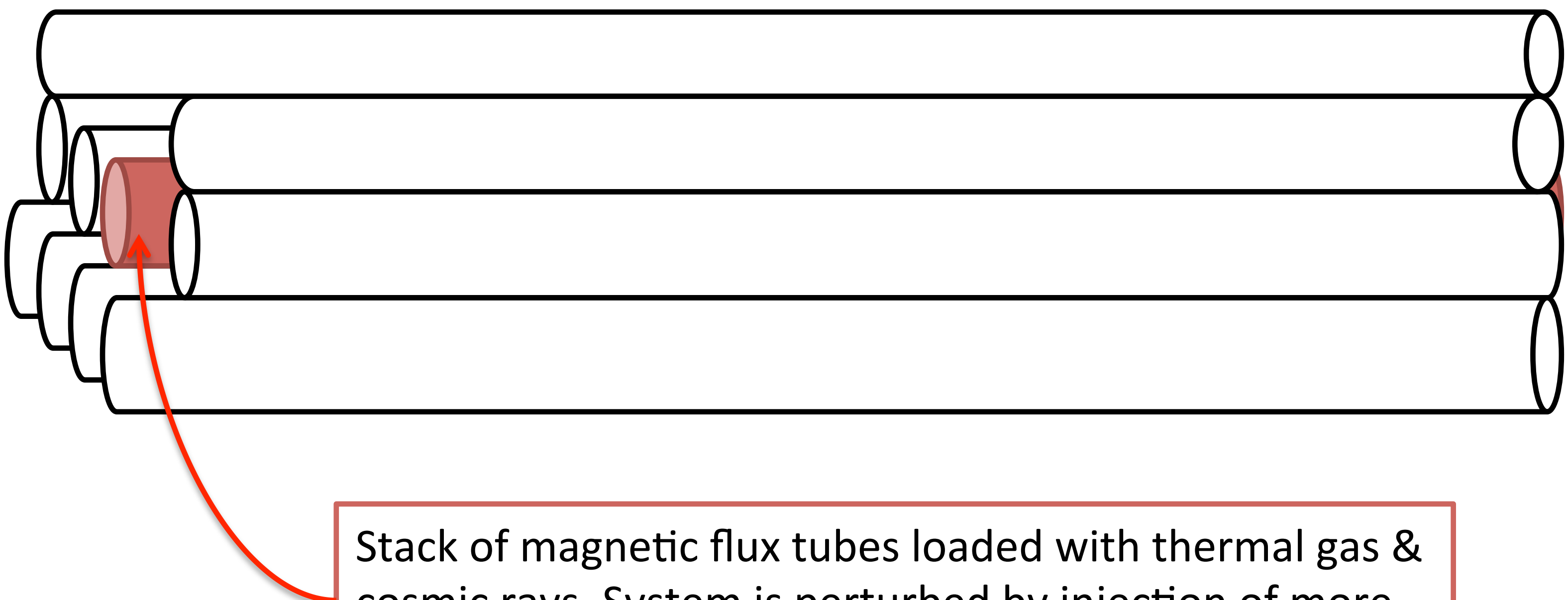
Overall Scheme



Plan of This Talk

- Basic setup
- Pressure anisotropy instability
- Tools for calculation
- Heating & scattering rates
- Conclusions
- Next steps.

Basic Setup



Stack of magnetic flux tubes loaded with thermal gas & cosmic rays. System is perturbed by injection of more cosmic rays. Zero-D for simplicity

Tube Expands to Maintain Total Pressure Balance

$$C_{ma}^2 \frac{d\rho_g}{dt} = -\Delta \dot{P}_c,$$

Injection term

$$C_{ma} \equiv \left(\frac{\gamma_g P_g + \gamma_c P_c + \gamma_m P_m}{\rho_g} \right)^{1/2}$$

$$\frac{\dot{\rho}_g}{\rho_g} = \frac{\dot{B}}{B} = -\frac{\Delta \dot{P}_c}{\rho_g C_{ma}^2} \equiv -\frac{1}{\tau_B}$$

Cosmic ray perpendicular pressure force is not in contention; it's the parallel force that needs wave-particle interactions.

Expansion of Tube Drives Cosmic Ray Pressure Anisotropy

- Assume magnetic moment & longitudinal action are preserved
  - Scattering will be added later
  - Assume *thermal plasma* is collisional
- Cosmic ray distribution function evolves according to

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{B}\frac{(1-\mu^2)}{2}\left(p\frac{\partial f}{\partial p} - \mu\frac{\partial f}{\partial \mu}\right) = \Delta\dot{f}(p,t)$$

Can be generalized to other forms of forcing.

Generates even anisotropy from an initially isotropic state.

Growth of Pressure Anisotropy

- Expand f in a Legendre series:

$$f(p,\mu,t) = \Sigma_{l=0}^{\infty} f_l(p,t) P_l(\mu)$$

- For small anisotropy,

$$\frac{\partial f_2}{\partial t} \approx \frac{\dot{B}}{3B} p \frac{df_0}{dp}$$

and

$$P_{||} - P_{\perp} \equiv \Delta P = \int p^2 dp d\mu p v P_2(\mu) f(p,\mu)$$

Gyroresonant Instability of Parallel – Propagating Alfven Waves

Gyroresonance Condition

$$\omega - kv\mu \pm \Omega = 0$$

Sign of ω indicates direction of propagation

Polarization

Relativistic gyrofrequency

Minimum resonant momentum: $p_1 \equiv m_i \Omega_0 / k$

Wave Growth/Damping Rate

$$\Gamma_c = \frac{\pi^2 q^2}{2} \frac{v_A^2}{c^2} \int_{p_{min}}^{p_{max}} p^2 dp \int_{-1}^{1} d\mu v(1-\mu^2)\delta(\omega - kv\mu \pm \Omega) A(f, \omega, k)$$

$$A\left[f, \omega, k\right] \equiv \frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu\right) \frac{1}{p} \frac{\partial f}{\partial \mu}.$$

A > 0 for instability

< 0

$\sim c/v_A$

Drift *vs* Pressure Anisotropy

Drift

- Unstable waves travel in same direction as drift.
- Particles resonate with opposite circular polarizations depending on direction of travel
- Instability threshold drift is v_A

Pressure Anisotropy

- Unstable waves can travel in either direction; particles resonate with waves traveling in same direction
- Opposite senses of wave polarization are excited in each direction
- Instability threshold anisotropy $\Delta P/P \sim v_A/c$

Compact Expressions for Growth Rates:

$$\Gamma_{cd} \sim \Omega_0 \frac{n_c(> p_1)}{n_i} \left(\frac{v_D}{v_A} - 1 \right)$$

$$\Gamma_{cpa} \sim \Omega_0 \frac{n_c(> p_1)}{n_i} \left(\frac{c}{v_A} \frac{|\Delta P|}{P} - 1 \right)$$

Spectrum – dependent factors have been suppressed

Once Instability is Triggered…

- The waves grow, scatter the particles and transfer particle energy to the waves.
- A balance between driving anisotropy by flux tube expansion and removing it by scattering is achieved.
- The anisotropy must *also* be such that wave excitation is balanced by thermal damping.

Using these principles, we estimate the heating rate and scattering frequency.

Fokker-Planck Equation

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{B}\frac{(1-\mu^2)}{2}\left(p\frac{\partial f}{\partial p} - \mu\frac{\partial f}{\partial \mu}\right) = \Delta \dot{f}(p) + \frac{\partial F_\mu}{\partial \mu} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 F_p\right),$$

Momentum space diffusion

Diffusive Fluxes

$$F_\mu = \frac{\nu\left(1-\mu^2\right)}{2}\frac{pv_A}{v}A(f, kv_A, k),$$

$$F_p = \frac{\nu\left(1-\mu^2\right)}{2}\left(\frac{pv_A}{v}\right)^2 A(f, kv_A, k).$$

Use $-kv_A$ for particles with $\mu < 0$.

A is the same anisotropy factor we saw in Γ

Pitch angle scattering frequency

$$\nu(p, \mu) = \Omega\frac{8\pi k W_k}{B^2}$$

Fluctuation power spectrum

Energy Equation

- Multiply Fokker Planck eqn. by particle energy ε & integrate over momentum space

$$\frac{\partial U_c}{\partial t} - \frac{\dot{B}}{B}(U_c + P_c) = \frac{\dot{B}}{3B}(P_{c\perp} - P_{c\parallel}) + \Delta \dot{U}_c - \int v F_p p^2 dp d\mu$$

Adiabatic, isotropic part

Injection rate

Energy transfer to waves

Faux gyroviscous heating

We can show that

$$\int v F_p p^2 dp d\mu = -2 \int \Gamma_c(k) W_k dk,$$

Evaluate Heating Rate By Balancing Driving & Damping of Anisotropy

$$\int_0^1 \nu\mu\left(1-\mu^2\right)\left(\frac{\partial f}{\partial p}+\frac{v}{v_A p}\frac{\partial f}{\partial \mu}\right)d\mu =$$

$$\int_0^1 \nu\mu\left(1-\mu^2\right)A[f,\omega,k]d\mu$$

$$=\left(\frac{2}{45}\right)\frac{v}{v_A}\frac{\dot{B}}{B}\frac{\partial f}{\partial p}$$

RHS would involve particle motion down the gradient & LHS would not be an average over μ in the drift case.

Lower Bound on Heating Rate

$$\left|\int vF_p p^2 dp d\mu\right| \equiv H_{pa} \geq \left|\frac{2}{45} v_A \frac{\dot{B}}{B} \int p^4 \frac{df_c}{dp}\right|$$

$$= \left|\frac{4}{15}\frac{v_A}{c}\frac{\dot{B}}{B}P_c\right| = \frac{4}{15}\frac{v_A}{c}\frac{P_c \Delta \dot{P}_c}{\gamma_c P_c + \gamma_m P_m + \gamma_g P_g},.$$

How does this compare with heating by drift anisotropy?

Comparison of Heating by Parallel & Perpendicular Gradients

$$H_d = |v_A \nabla_{||} P_c| \sim \frac{P_c}{\tau_A}$$

$$H_{pa} \sim \frac{v_A}{c} \frac{P_c}{\tau_B}$$

Why is anisotropy heating so slow? Because scattering balances the magnetosonic time, not the light travel time.

Self Confinement by Drift vs Pressure Anisotropy

Recall that this is the heating rate:

$$\int vF_p p^2 dp d\mu = -2\int \Gamma_c(k) W_k dk,$$

Growth balances damping; fixes Γ

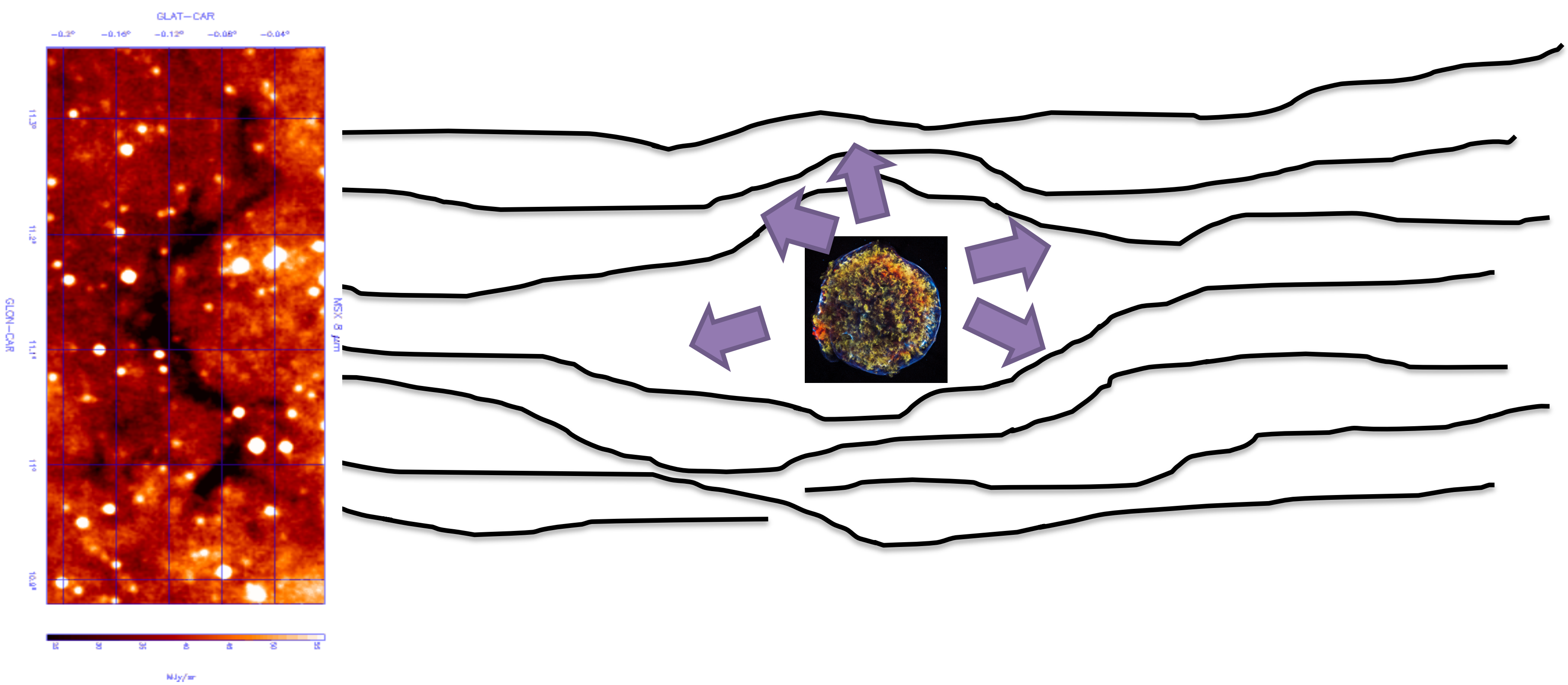
ν/Ω

If heating rate is lower by v_A/c, so is scattering rate

Outcomes of This Work

- Cosmic ray driven pressure anisotropy cannot substitute for drift anisotropy on **global, galactic scales**.
  - Scattering mean free path is longer by c/v_A.
  - Heating is weaker by a factor of v_A/c.
- The calculations presented here could be important when cosmic ray energy is injected on **local** scales.

Pressure & Drift Anisotropies Near Local Sources



Thank you!