Leibniz-Institut für Astrophysik Potsdam

AIP

BRAGINSKII VISCOSITY ON AN UNSTRUCTURED, MOVING MESH ACCELERATED WITH SUPER-TIME-STEPPING

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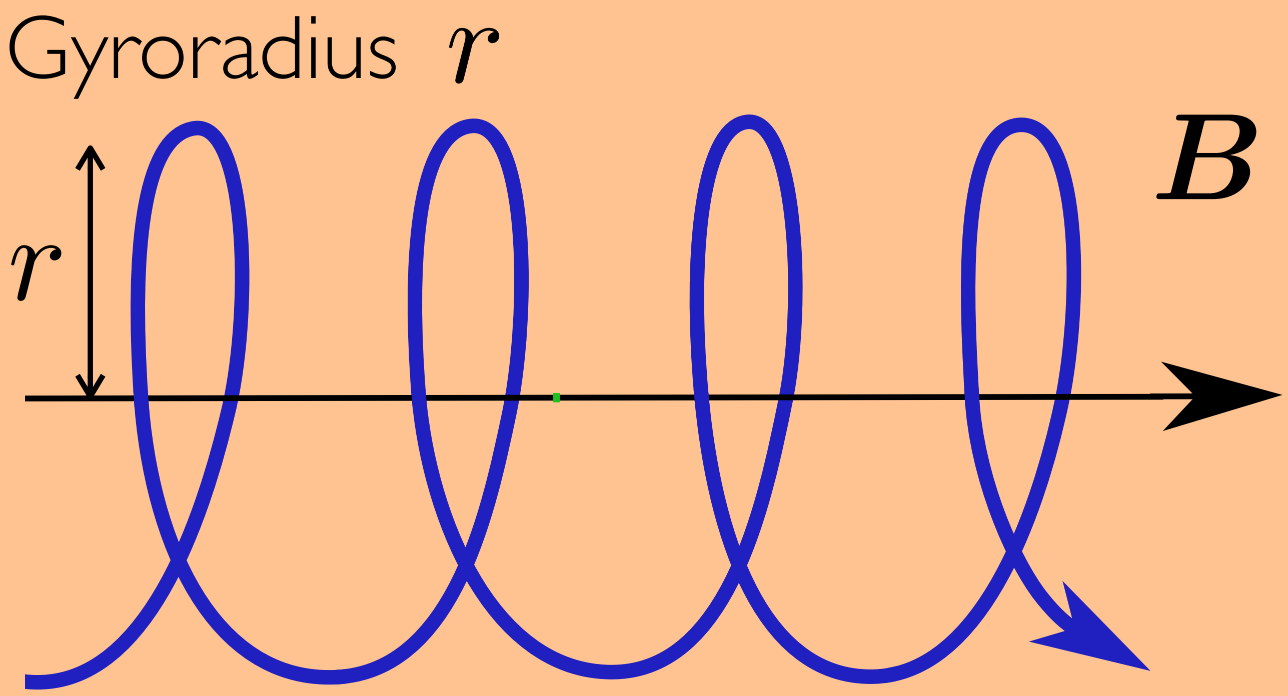
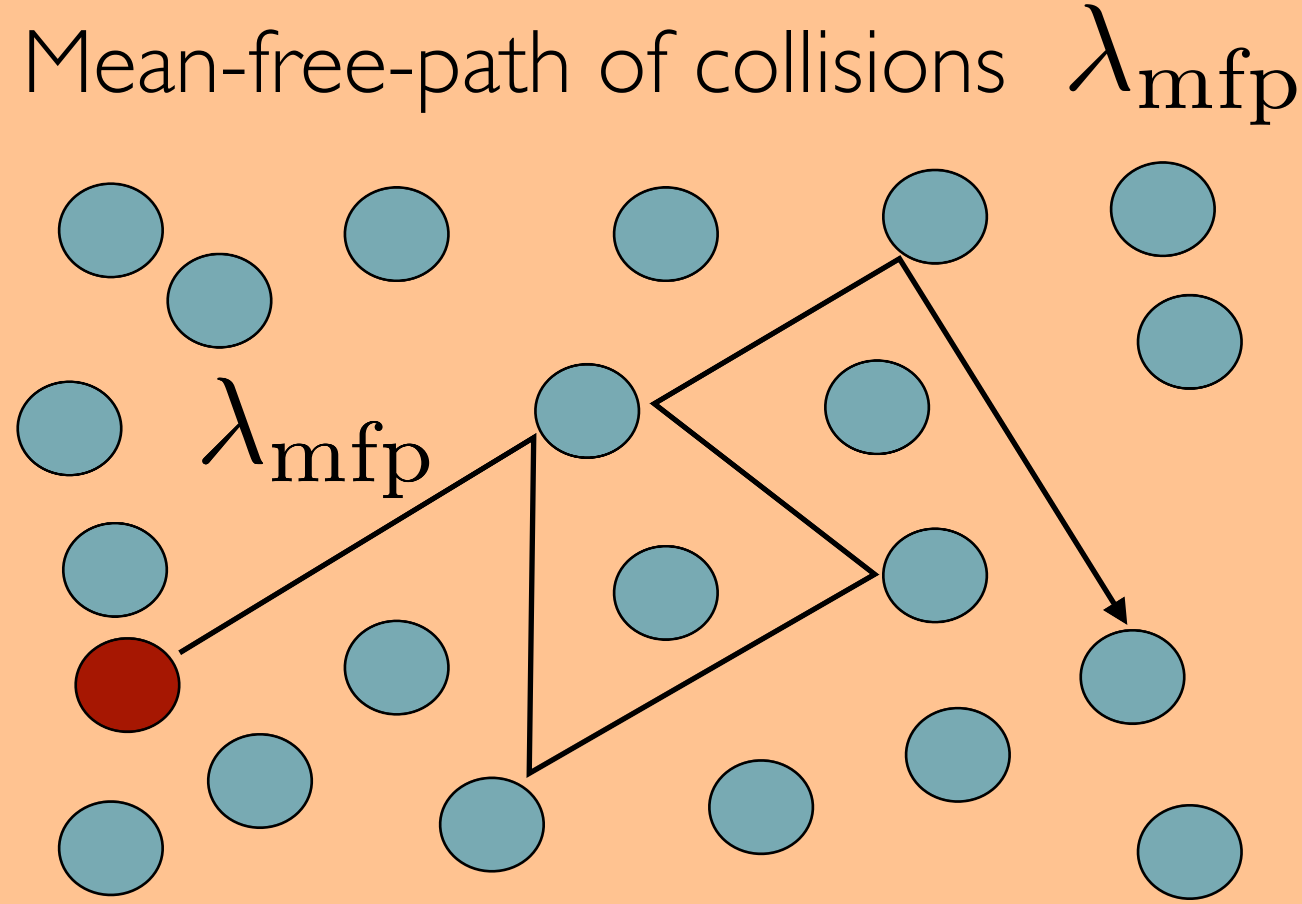
MULTISCALE PHENOMENA IN PLASMA ASTROPHYSICS, KITP 17/9/2019





Weakly collisional and magnetized

$$r \ll \lambda_{\rm mfp} \ll L$$

→ Transport of heat and momentum is along magnetic field lines.

Heat conduction

$$\boldsymbol{Q} = -\chi_{\|} \boldsymbol{b}(\boldsymbol{b} \cdot \nabla T)$$

Kannan+ 2016 in Arepo

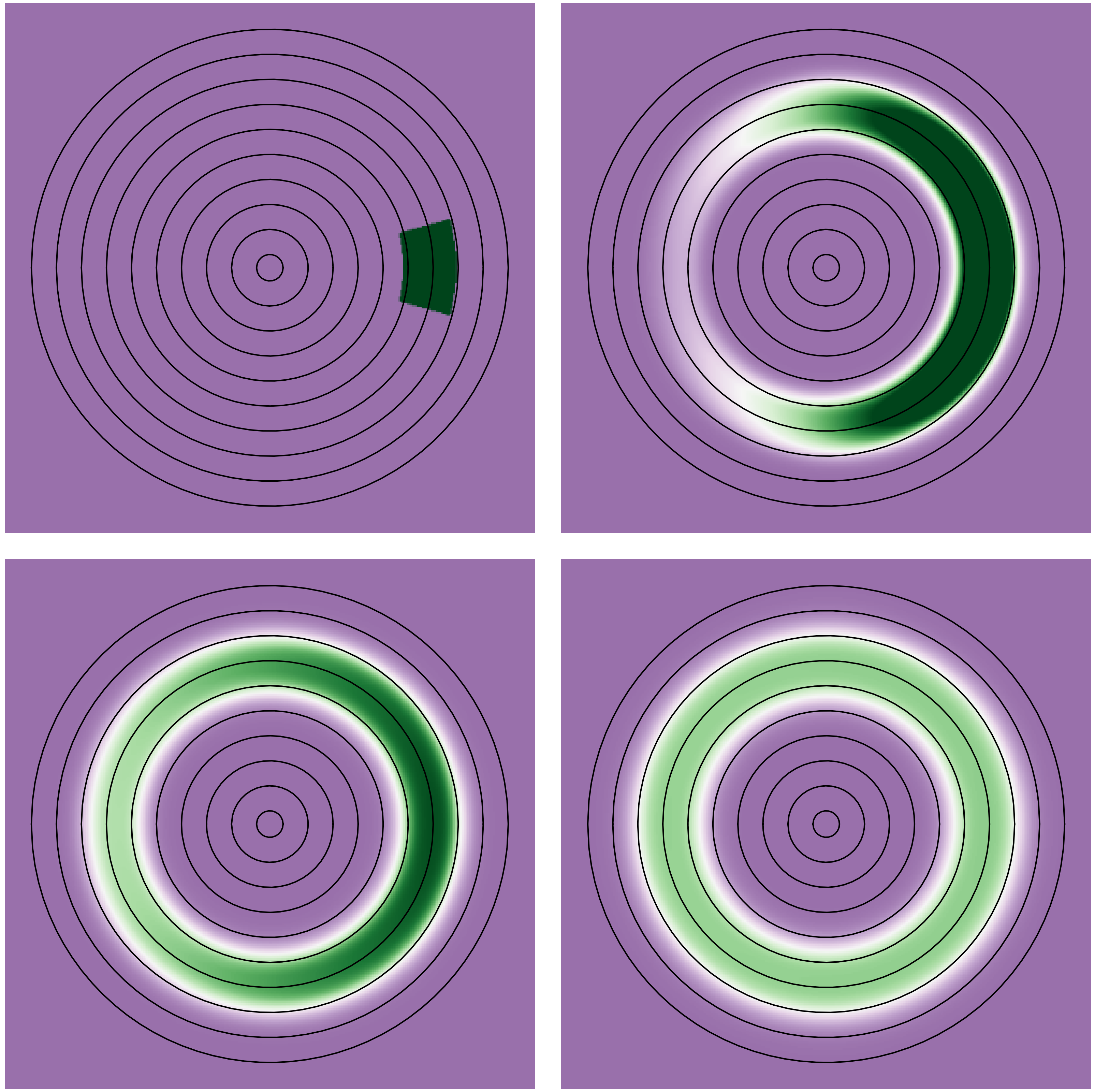
Sharma & Hammett 2007

Braginskii viscosity

$$\mathbf{\Pi} = -\Delta p \left(\boldsymbol{bb} - \frac{\mathbf{1}}{3} \right) ,$$

$$\Delta p = \rho \nu_{\|} (3\boldsymbol{bb} : \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{v}) .$$

ANISOTROPIC DIFFUSION

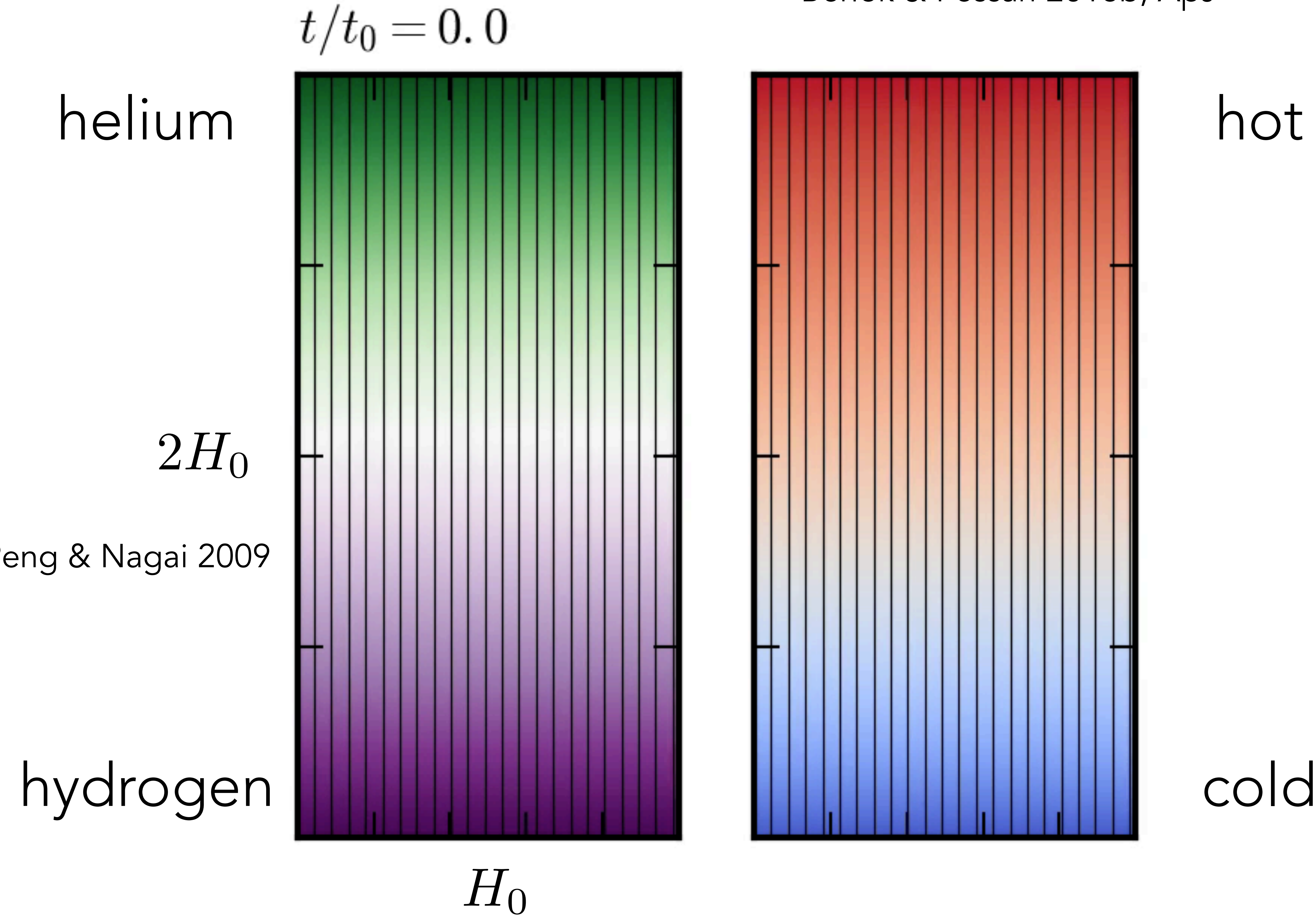


ATHENA (Stone et al. 2008)

(Parrish & Stone 2005, Sharma & Hammett 2007, Berlok & Pessah 2016a)

QUASI-GLOBAL SIMULATIONS

Berlok & Pessah 2016b, ApJ

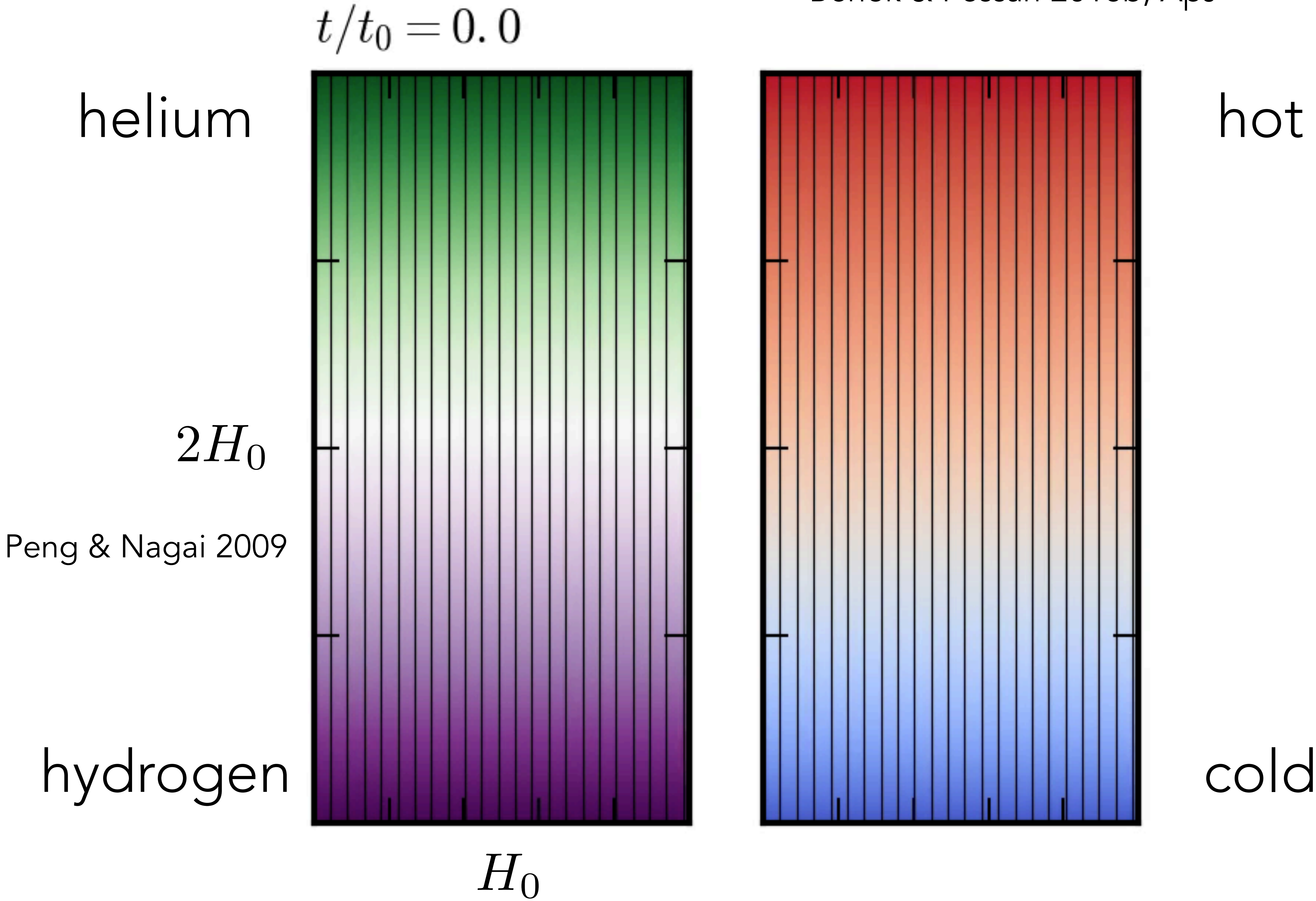


Peng & Nagai 2009

See Balbus 2000, 2001; Parrish & Stone 2005, 2007; Quataert 2008; Parrish & Quataert 2008; Parrish et al. 2008, 2009; Bogdanovic et al. 2009; Parrish et al. 2010; Ruszkowski & Oh 2010; McCourt et al. 2011, 2012; Latter & Kunz 2012, Kunz et al. 2012; Parrish et al. 2012a,b

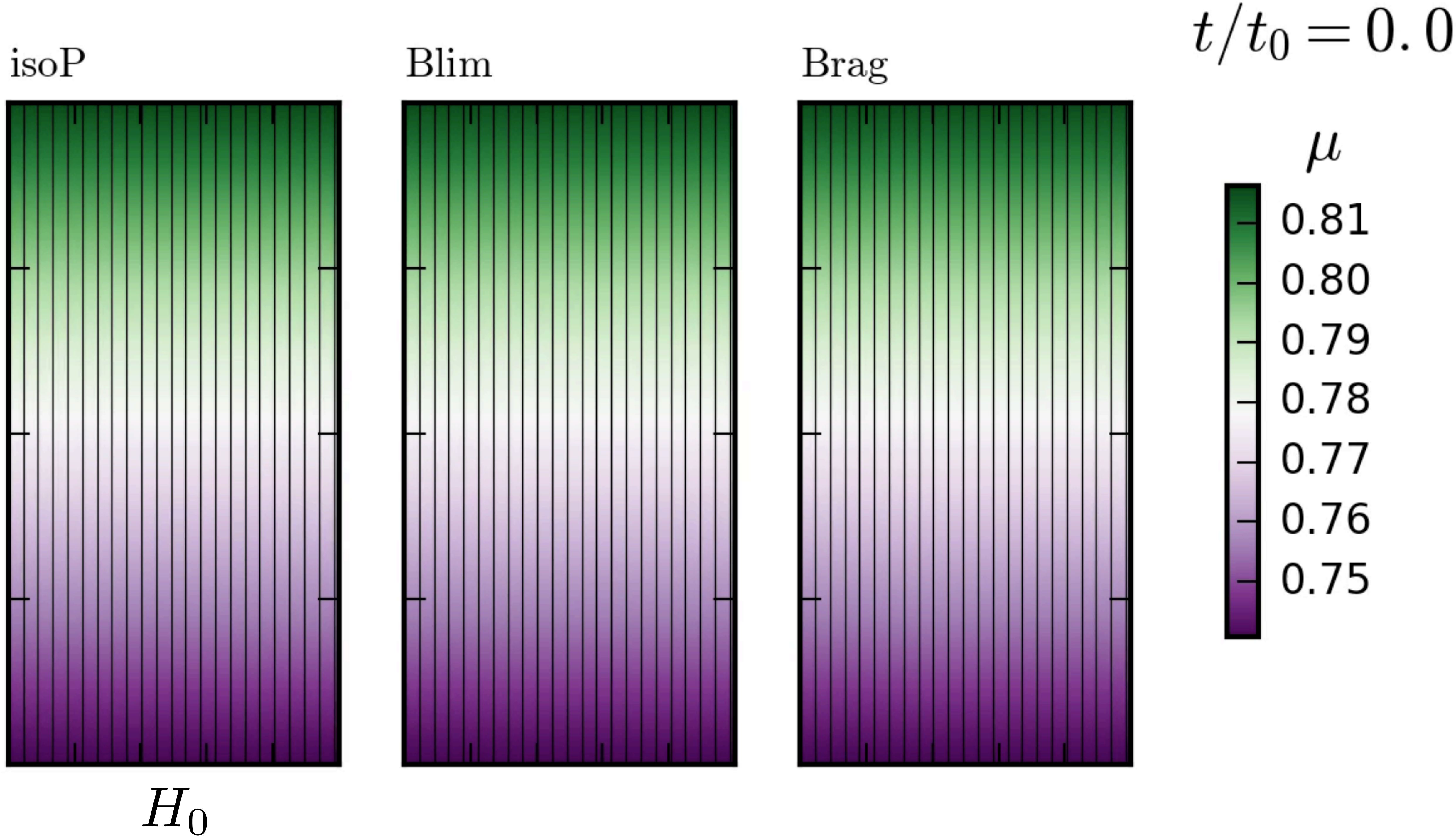
QUASI-GLOBAL SIMULATIONS

Berlok & Pessah 2016b, ApJ



See Balbus 2000, 2001; Parrish & Stone 2005, 2007; Quataert 2008; Parrish & Quataert 2008; Parrish et al. 2008, 2009; Bogdanovic et al. 2009; Parrish et al. 2010; Ruszkowski & Oh 2010; McCourt et al. 2011, 2012; Latter & Kunz 2012, Kunz et al. 2012; Parrish et al. 2012a,b

QUASI-GLOBAL SIMULATIONS

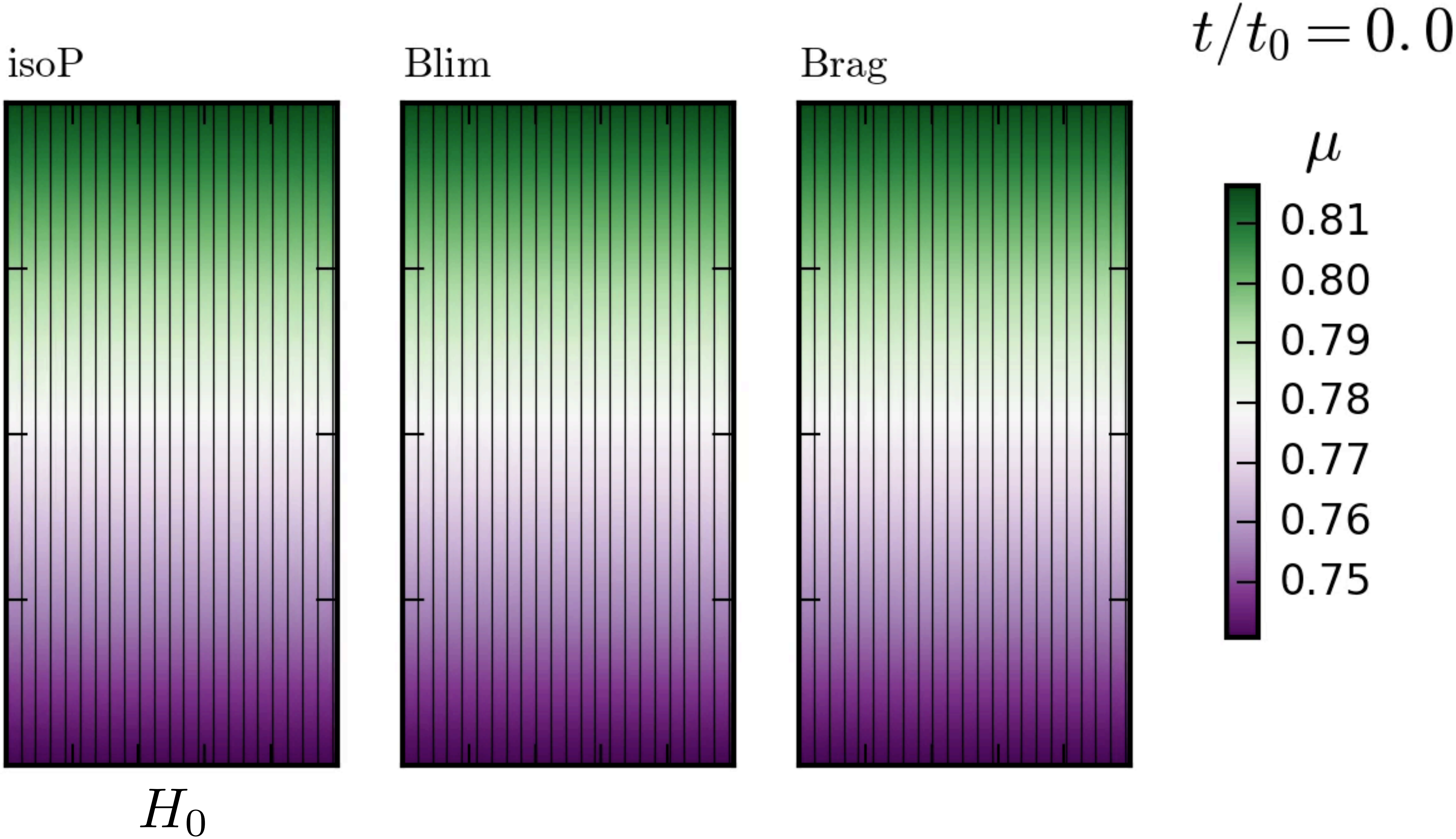


Berlok & Pessah 2016b
See also Kunz+ 2012

$$-\frac{B^2}{\mu_0} < \Delta p < \frac{B^2}{2\mu_0}$$

See Sharma+ 2006,
Schekochihin+ 2008, Kunz+ 2014

QUASI-GLOBAL SIMULATIONS



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$$-\frac{B^2}{\mu_0} < \Delta p < \frac{B^2}{2\mu_0}$$

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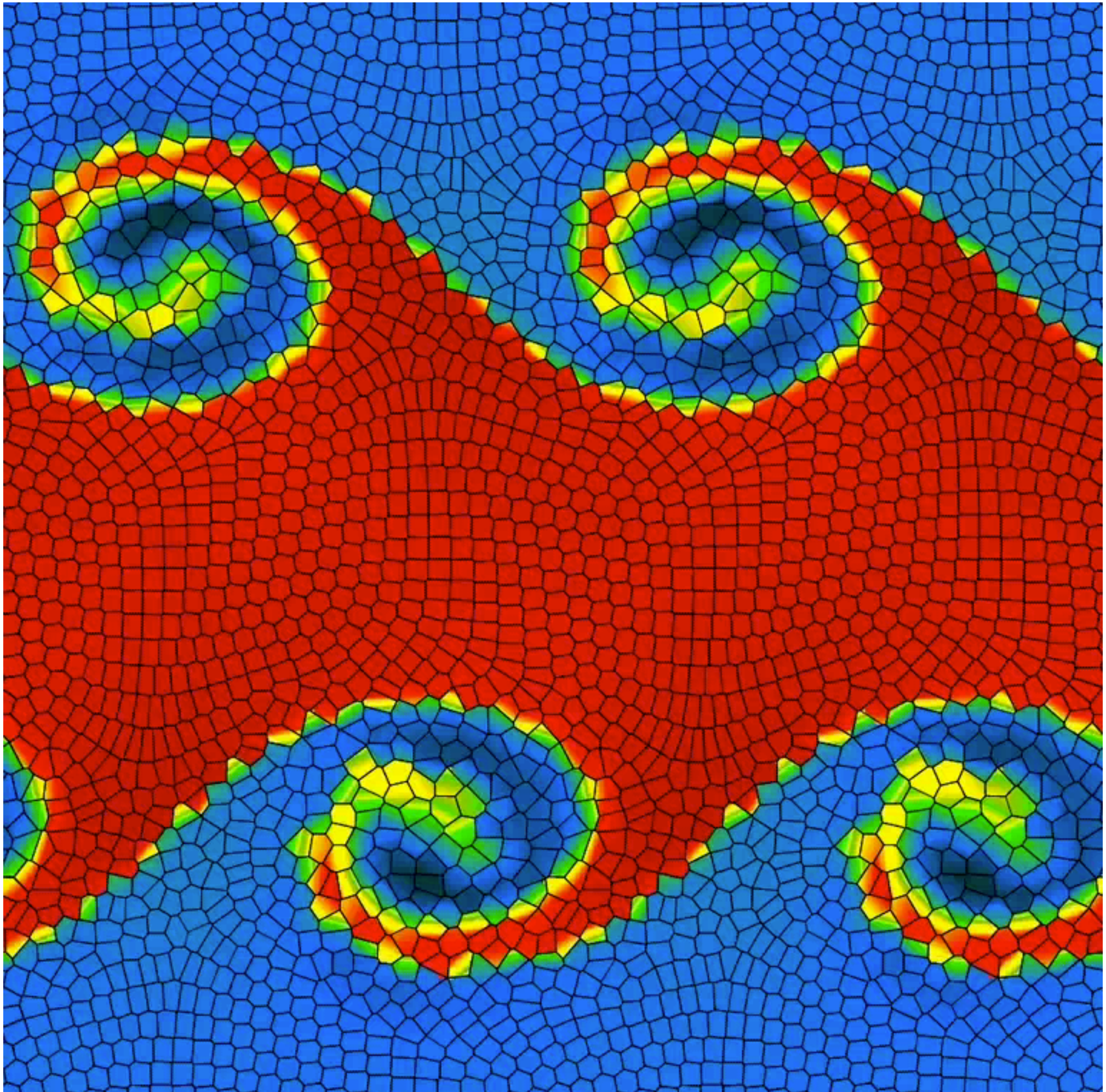
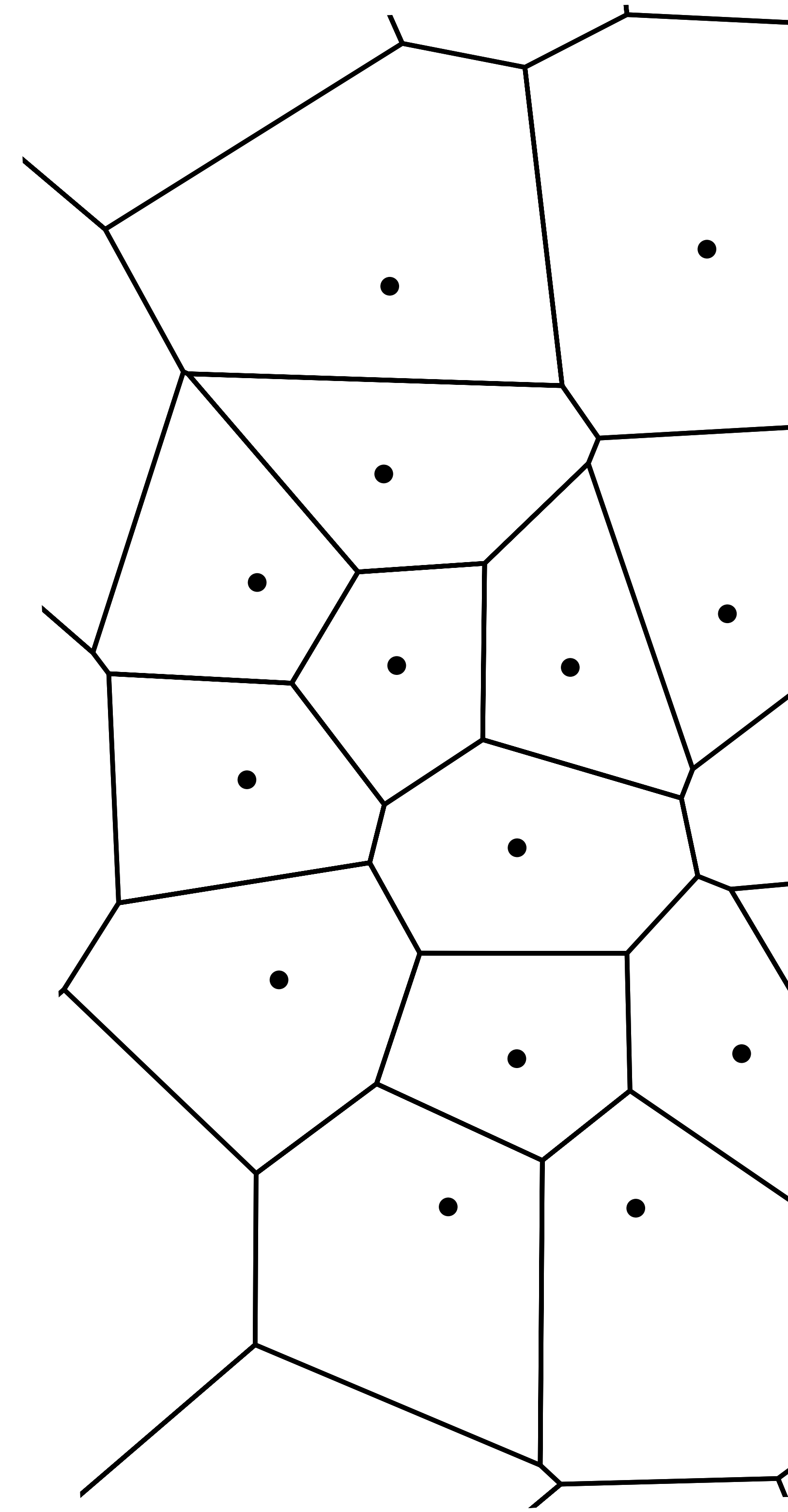
THE MOVING MESH CODE AREPO

Volker Springel (2010)

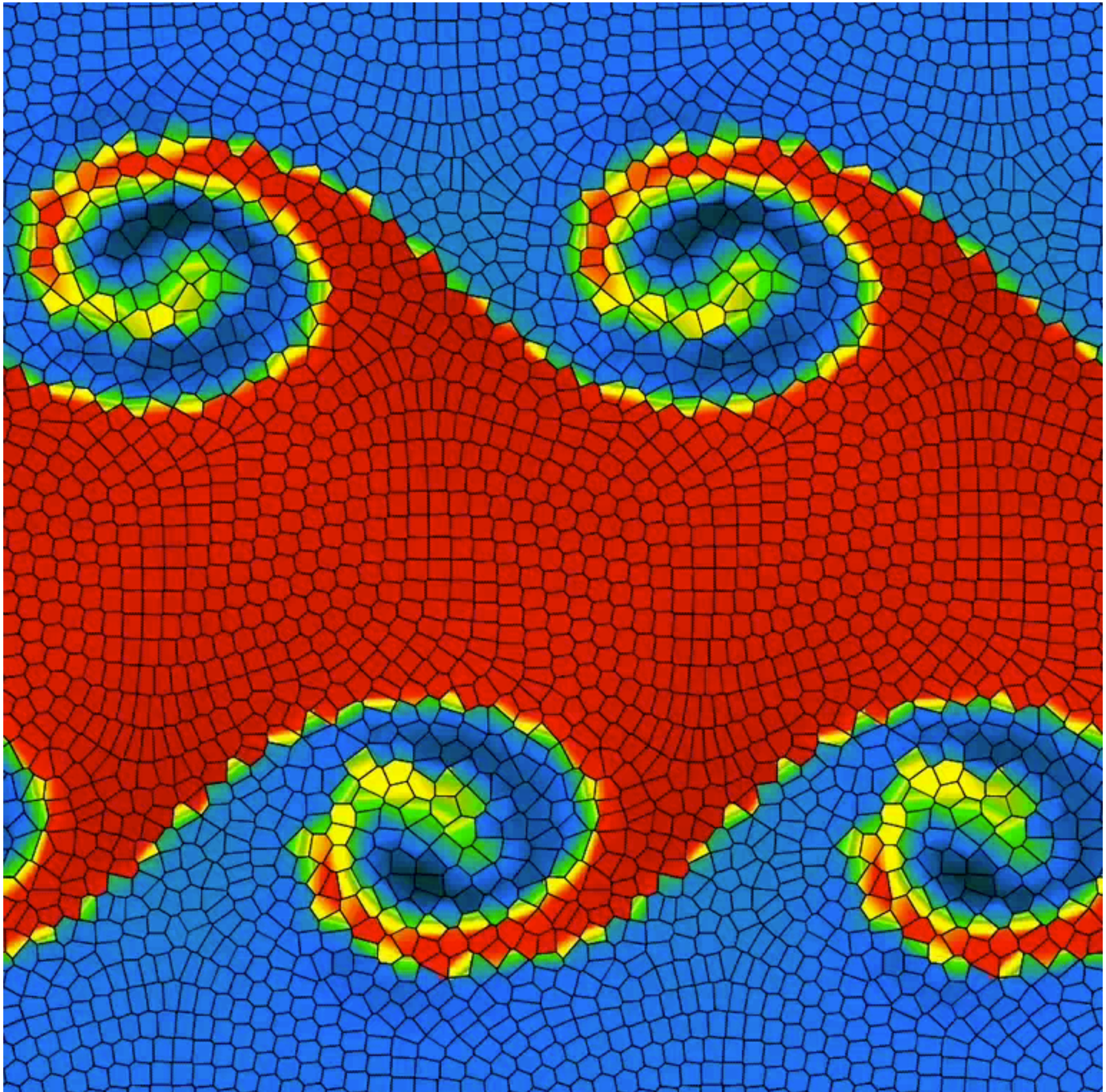


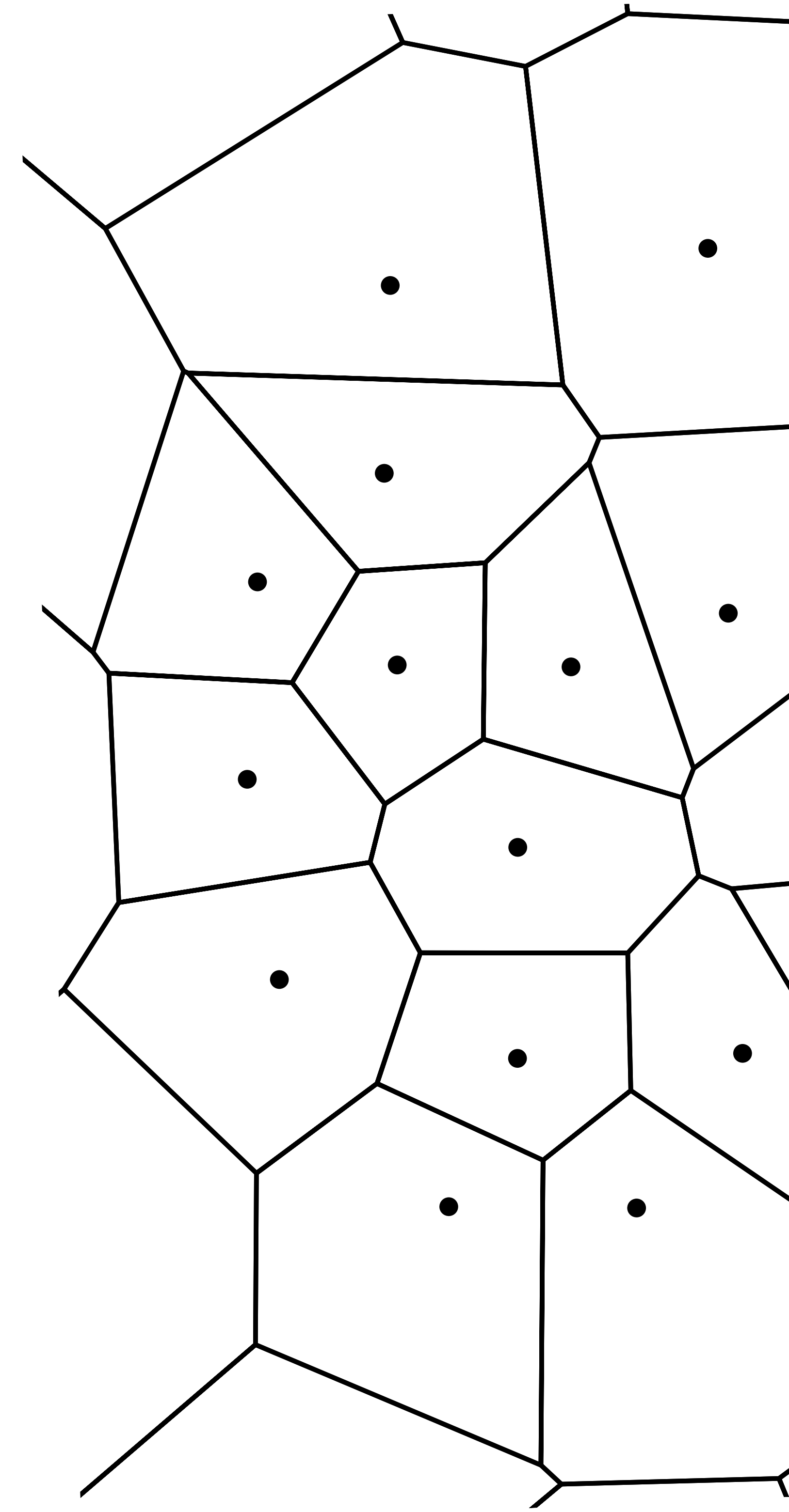


Pakmor+ 2011, 2013, 2016

THE MOVING MESH CODE AREPO

Volker Springel (2010)





Pakmor+ 2011, 2013, 2016

BRAGINSKII VISCOSITY IN AREPO

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot \mathbf{\Pi},$$
$$\frac{\partial E}{\partial t} = -\nabla \cdot (\mathbf{\Pi} \cdot \boldsymbol{v}) \ ,$$

$$\mathbf{\Pi} = -\Delta p \left(\boldsymbol{bb} - \frac{\mathbf{1}}{3} \right) \ ,$$
$$\Delta p = \rho \nu_{\parallel} (3 \boldsymbol{bb} : \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{v}) \ .$$

SECOND ORDER ACCURATE SUPER TIMESTEPPING

$$\Delta t \leq C \frac{(\Delta x)^2}{2 d \nu_{\parallel}}$$

$$\tau = \frac{\Delta t}{4} \left(s^2 + s - 2 \right)$$

$$\frac{\partial \boldsymbol{v}}{\partial t} = L(T, \boldsymbol{v})$$
$$\frac{\partial E}{\partial t} = \nabla \cdot \boldsymbol{F}_E(T, \boldsymbol{v})$$

Velocity update

$$\boldsymbol{Y}_0 = \boldsymbol{v}^n \ ,$$
$$\boldsymbol{Y}_1 = \boldsymbol{Y}_0 + \tilde{\mu}_1 \tau \boldsymbol{L}(T^n, \boldsymbol{Y}_0) \ ,$$
$$\boldsymbol{Y}_j = \mu_j \boldsymbol{Y}_{j-1} + \nu_j \boldsymbol{Y}_{j-2} + (1 - \mu_j - \nu_j) \boldsymbol{Y}_0 + \tilde{\mu}_j \tau \boldsymbol{L}(T^n, \boldsymbol{Y}_{j-1}) + \tilde{\gamma}_j \tau \boldsymbol{L}(T^n, \boldsymbol{Y}_0) \quad \text{for } 2 \leq j \leq s$$
$$\boldsymbol{v}^{n+1} = \boldsymbol{Y}_s$$

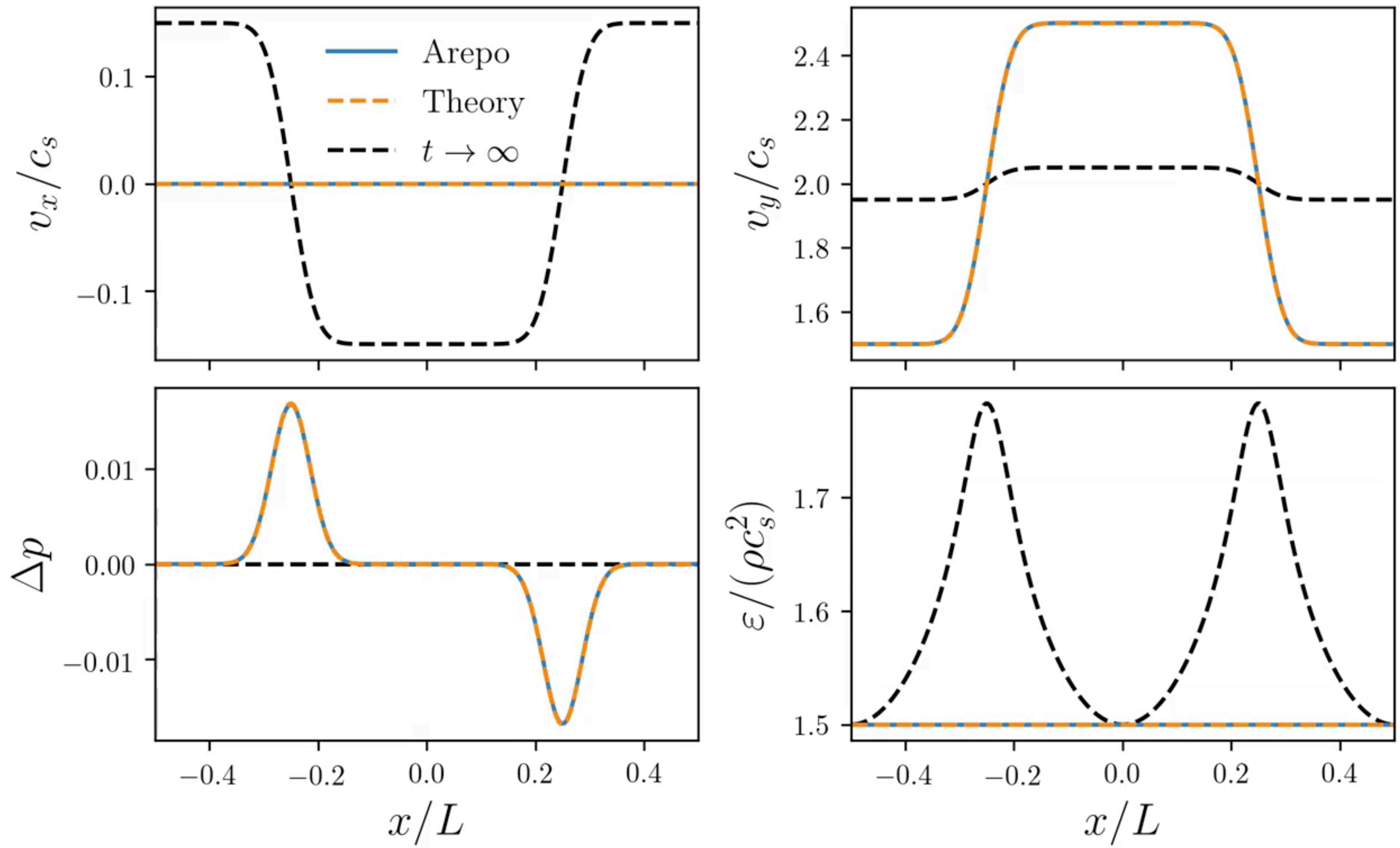
Energy update

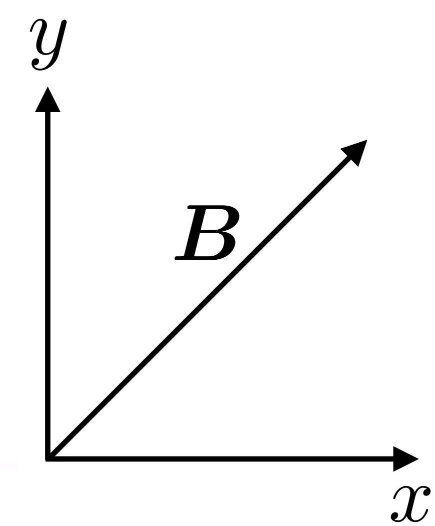
$$E^{n+1} = \frac{\tau}{2} \left[\nabla \cdot \boldsymbol{F}_E(T^n, \boldsymbol{v}^n) + \nabla \cdot \boldsymbol{F}_E \left(T^n, \boldsymbol{v}^{n+1} \right) \right]$$

RKL2 STS theory in Meyer+ 2012. See also Vaidya+ 2017

$$v_x(x,t) = -c_s \sum_{n=0}^{\infty} \frac{3a_n}{10} \cos(k_n x)\left(1 - e^{-\gamma_n t}\right) \qquad v_y(x,t) = c_s \sum_{n=0}^{\infty} \frac{a_n}{10} \cos(k_n x)\left(1 + 9e^{-\gamma_n t}\right)$$

$c_s t/L = 0.0$





$$\Delta p(x,t) = -\frac{3\rho c_s \nu_\parallel}{2} \sum_{n=1}^{\infty} k_n a_n \sin(k_n x)$$

$$\gamma_n = \frac{5\nu_\parallel}{6} k_n^2$$

$$\varepsilon(t) = \varepsilon_0 + \frac{9\rho c_s^2}{10} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_n a_m \frac{\sqrt{\gamma_n \gamma_m}}{\gamma_n + \gamma_m} \times \sin(k_n x)\sin(k_m x)\left(1 - e^{-(\gamma_n + \gamma_m)t}\right),$$

$$v_x(x,t) = -c_s \sum_{n=0}^{\infty} \frac{3a_n}{10} \cos(k_n x)\left(1 - e^{-\gamma_n t}\right) \qquad v_y(x,t) = c_s \sum_{n=0}^{\infty} \frac{a_n}{10} \cos(k_n x)\left(1 + 9e^{-\gamma_n t}\right)$$

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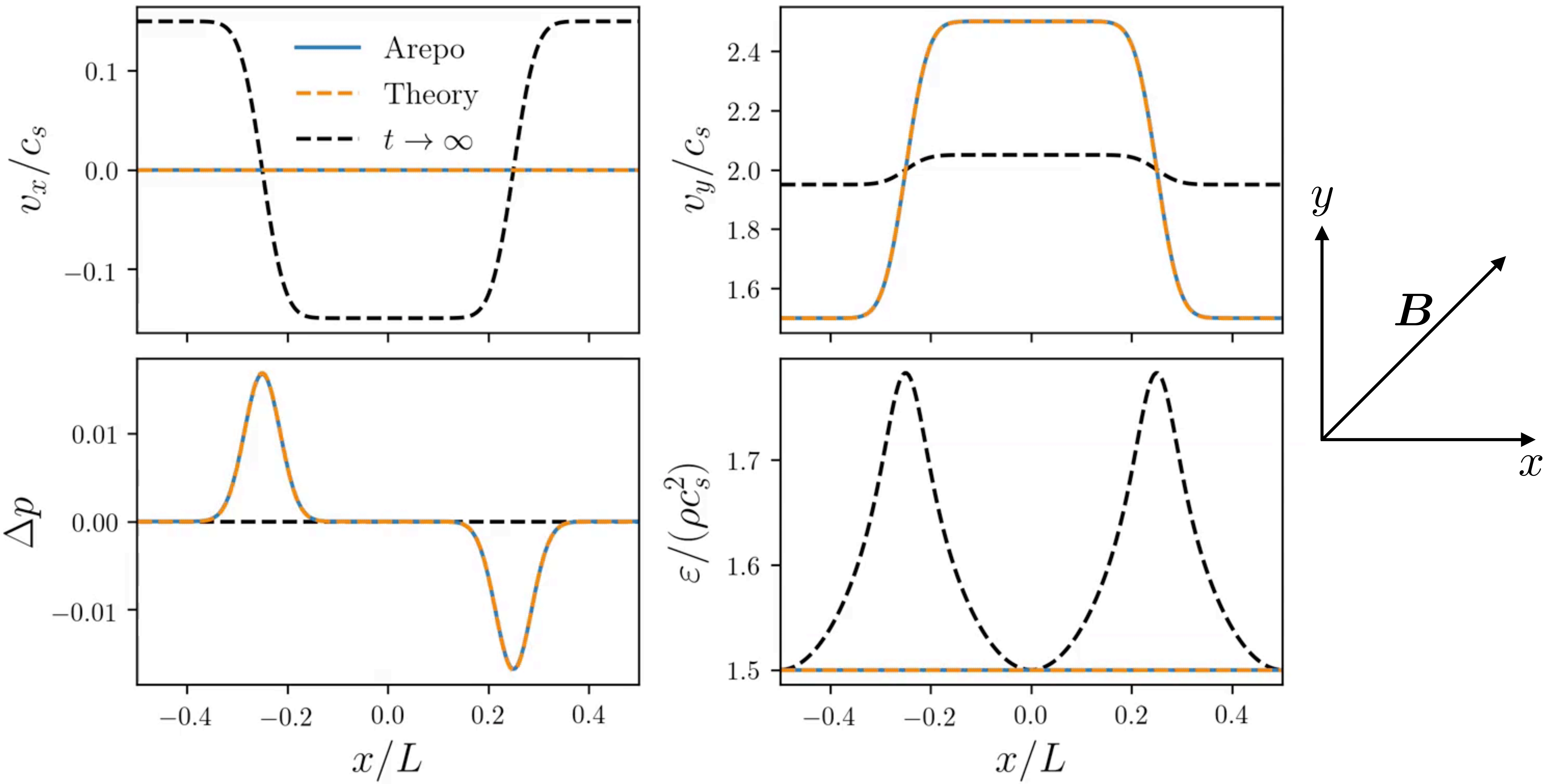

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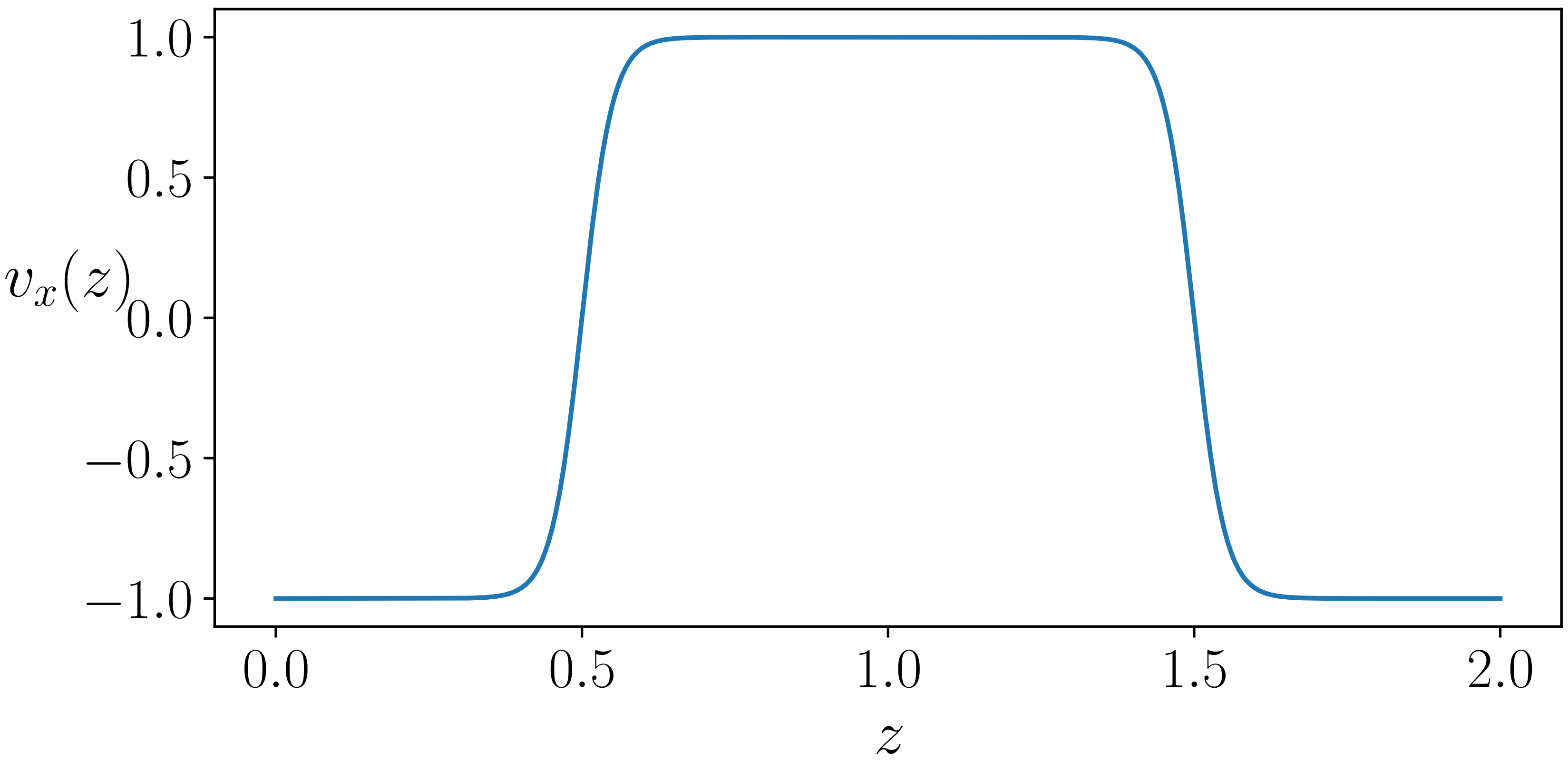
KELVIN-HELMHOLTZ INSTABILITY WITH BRAGINSKII VISCOSITY



Smooth equilibrium necessary for convergence of KHI. See e.g. McNally+ 2012 and Lecoanet+ 2016.

LINEAR THEORY FOR VISCOUS KELVIN-HELMHOLTZ INSTABILITY

Berlok & Pfrommer, 2019a, MNRAS
See also Suzuki+ (2013)

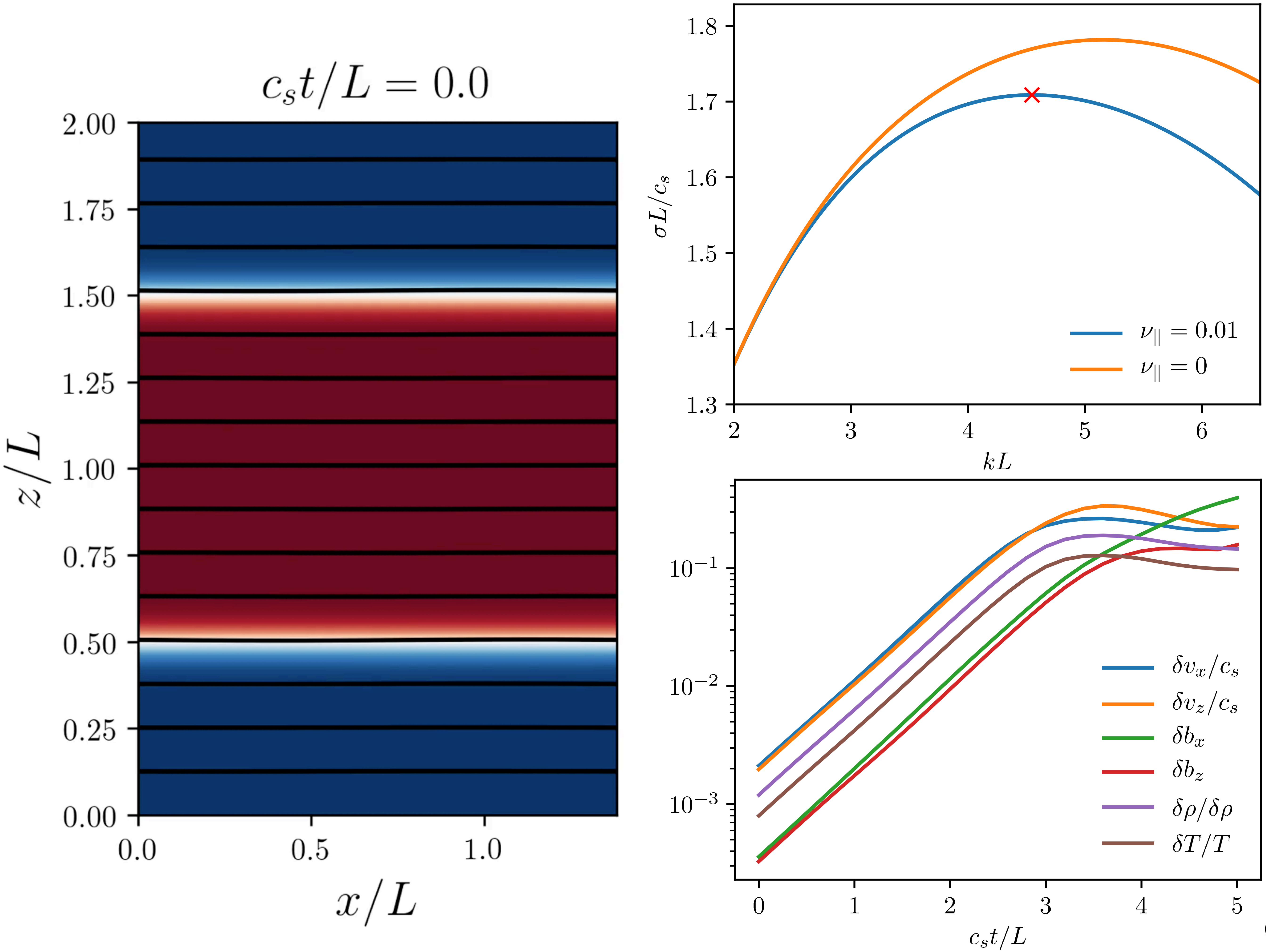
$$-i\omega\frac{\delta\rho}{\rho} = -ik\left(v_0\frac{\delta\rho}{\rho} + \delta v_x\right) - \frac{\partial \delta v_z}{\partial z} \ .$$

$$-i\omega\frac{\delta A}{B} = -ikv_0\frac{\delta A}{B} + \delta v_z \ ,$$

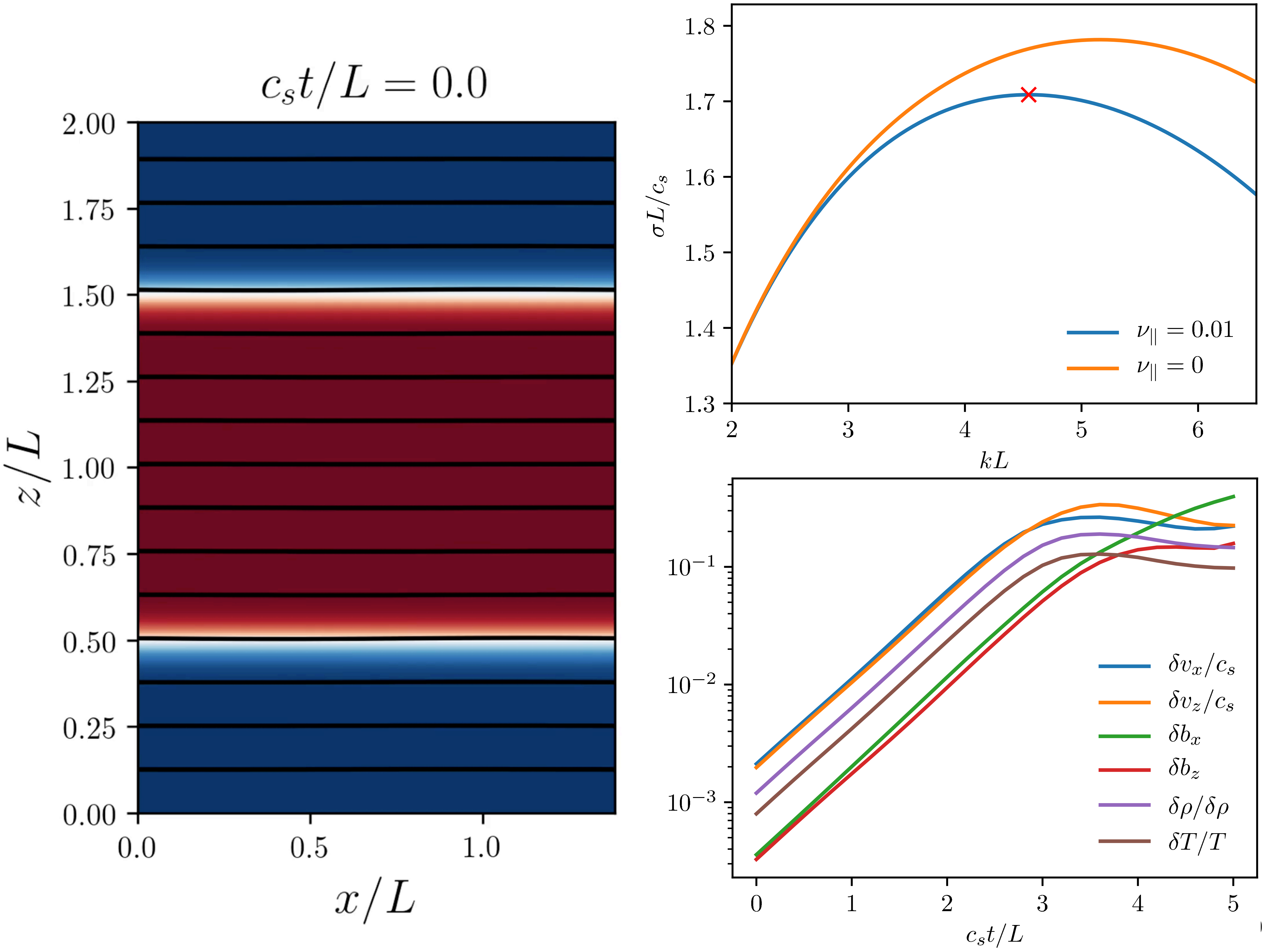
$$-i\omega\delta v_x = -ikv_0\delta v_x - \frac{\partial v_0}{\partial z}\delta v_z - ik\frac{\delta p}{\rho} - \nu_\parallel\left(\frac{4}{3}k^2\delta v_x + 2k^2\frac{\partial v_0}{\partial z}\frac{\delta A}{B} + ik\frac{2}{3}\frac{\partial \delta v_z}{\partial z}\right) \ ,$$

$$-i\omega\delta v_z = -ikv_0\delta v_z - \frac{1}{\rho}\frac{\partial \delta p}{\partial z} + v_a^2\left(\frac{\partial^2}{\partial z^2} - k^2\right)\frac{\delta A}{B} - \nu_\parallel\left(ik\frac{2}{3}\frac{\partial \delta v_x}{\partial z} + ik\frac{\partial^2 v_0}{\partial z^2}\frac{\delta A}{B} + ik\frac{\partial v_0}{\partial z}\frac{\partial}{\partial z}\frac{\delta A}{B} - \frac{1}{3}\frac{\partial^2 \delta v_z}{\partial z^2}\right) \ ,$$

$$-i\omega\frac{\delta T}{T} = -ik\left(v_0\frac{\delta T}{T} + \frac{2}{3}\delta v_x\right) - \frac{2}{3}\frac{\partial \delta v_z}{\partial z}$$

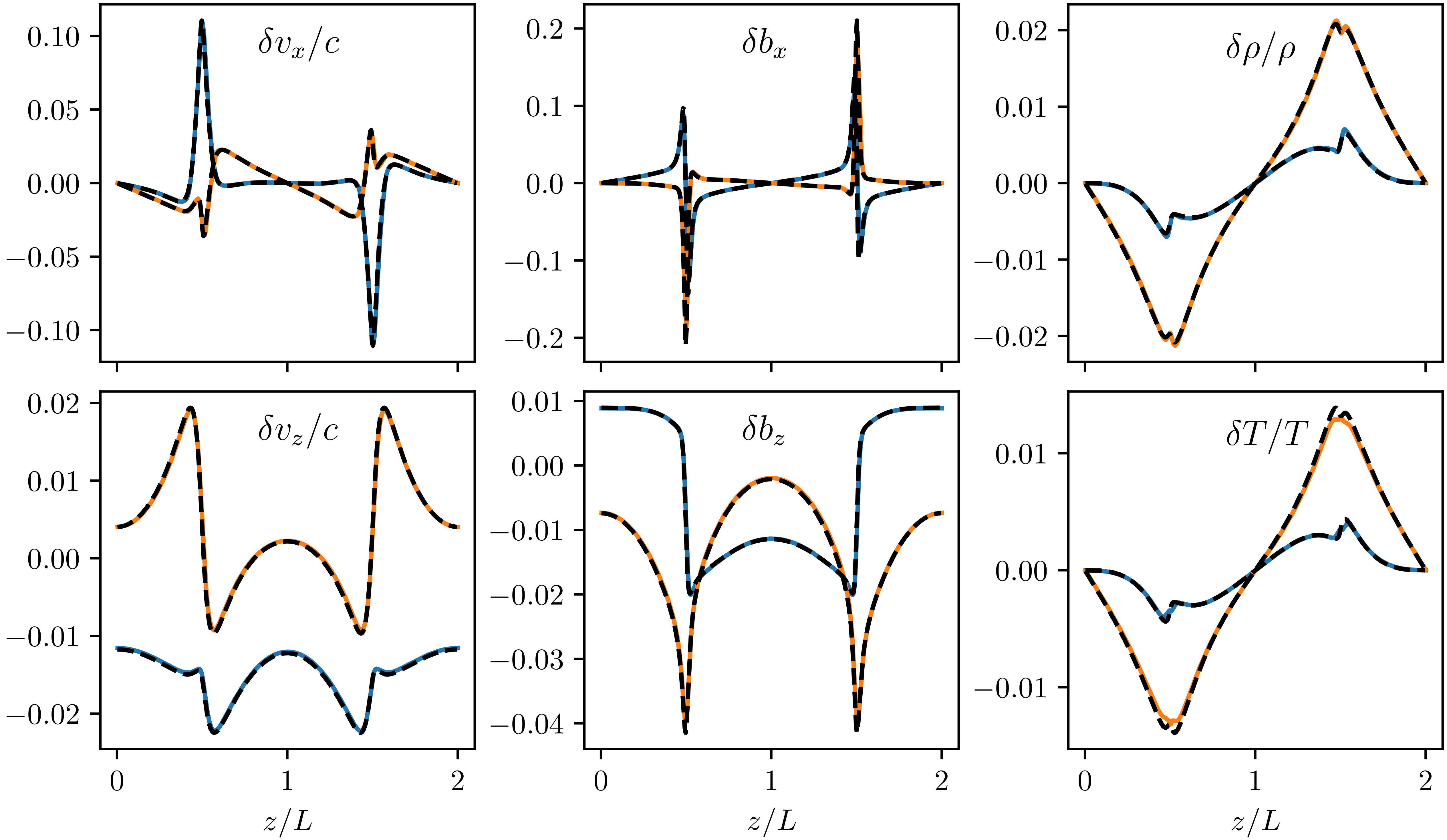


$c_s t/L = 0.0$
z/L
x/L
$\sigma L/c_s$
kL
$\nu_\parallel = 0.01$
$\nu_\parallel = 0$
$c_s t/L$
$\delta v_x/c_s$
$\delta v_z/c_s$
δb_x
δb_z
$\delta\rho/\delta\rho$
$\delta T/T$

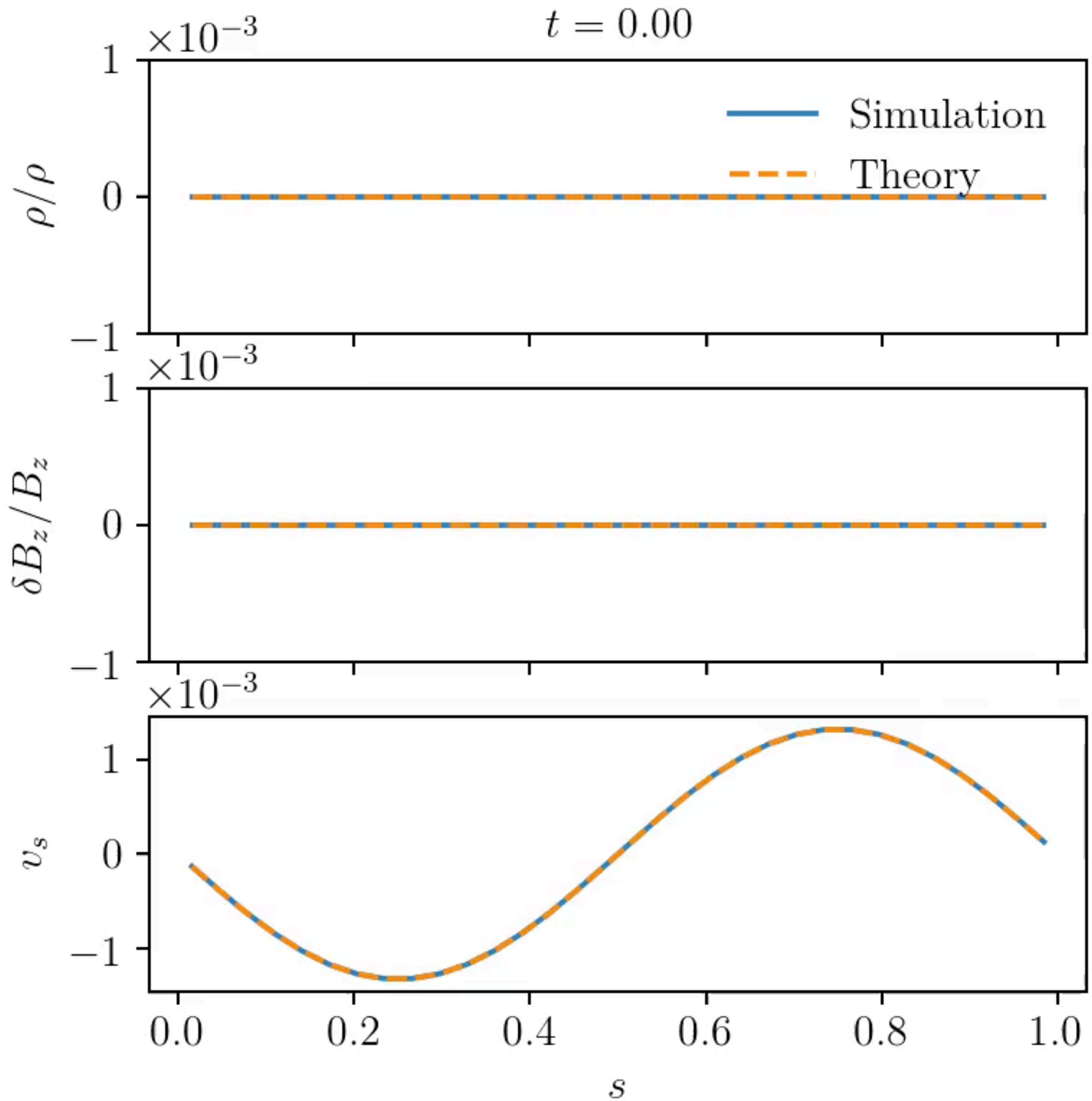


$c_s t/L = 0.0$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
z/L
0.0
0.5
1.0
x/L
1.8
1.7
1.6
1.5
1.4
1.3
$\sigma L/c_s$
2
3
4
5
6
kL
$\nu_\parallel = 0.01$
$\nu_\parallel = 0$
10^{-1}
10^{-2}
10^{-3}
$\delta v_x/c_s$
$\delta v_z/c_s$
δb_x
δb_z
$\delta\rho/\delta\rho$
$\delta T/T$
0
1
2
3
4
5
$c_s t/L$

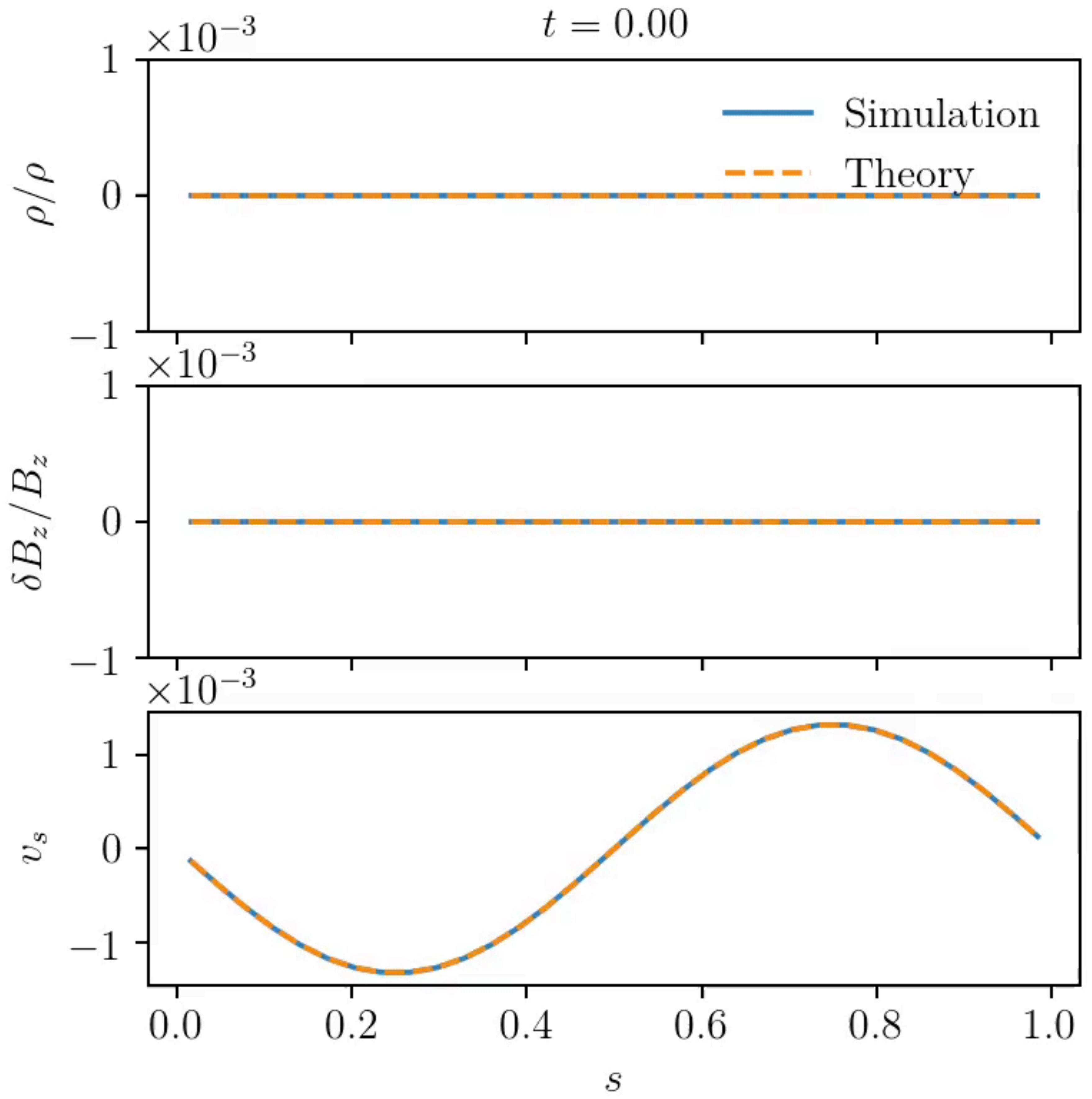
EIGENMODES OF THE INSTABILITY



DECAY OF 2D MAGNETO-SONIC WAVE



DECAY OF 2D MAGNETO-SONIC WAVE



DECAY OF 2D MAGNETO-SONIC WAVE

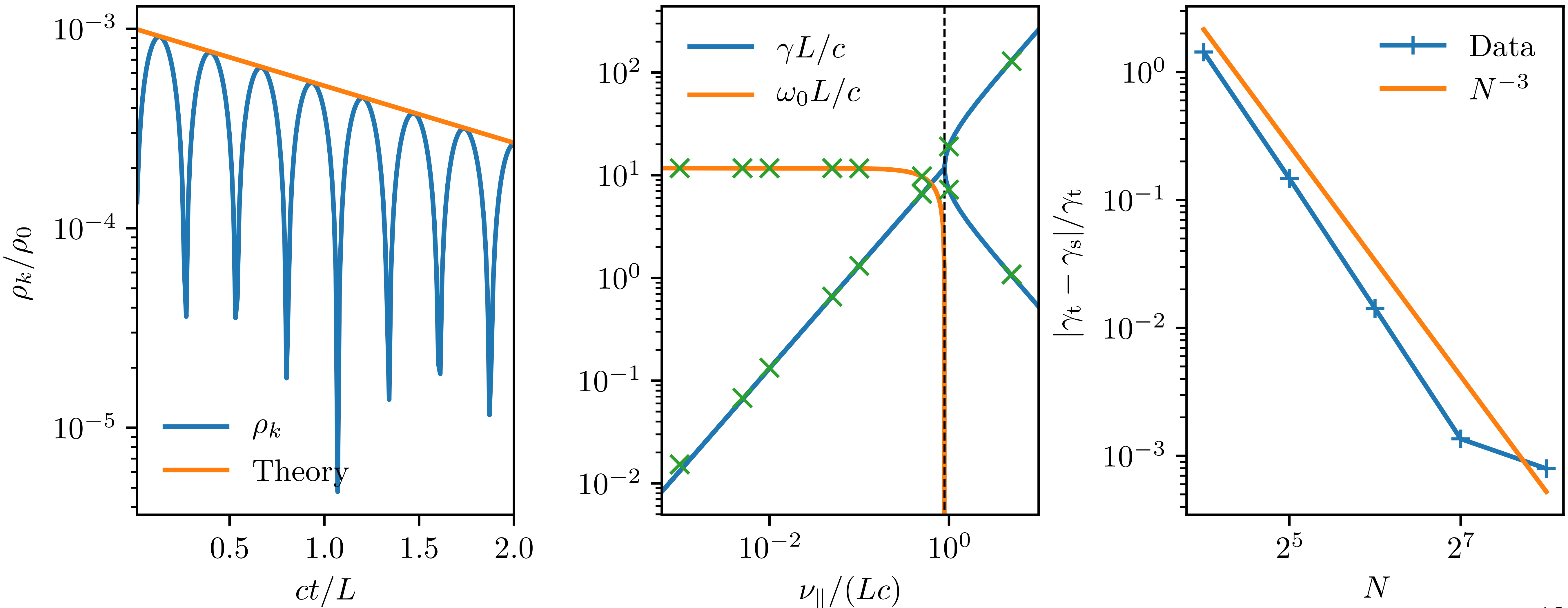
Alfvén waves

$$\left(\omega^2 - k_\parallel^2 v_a^2\right) = 0$$

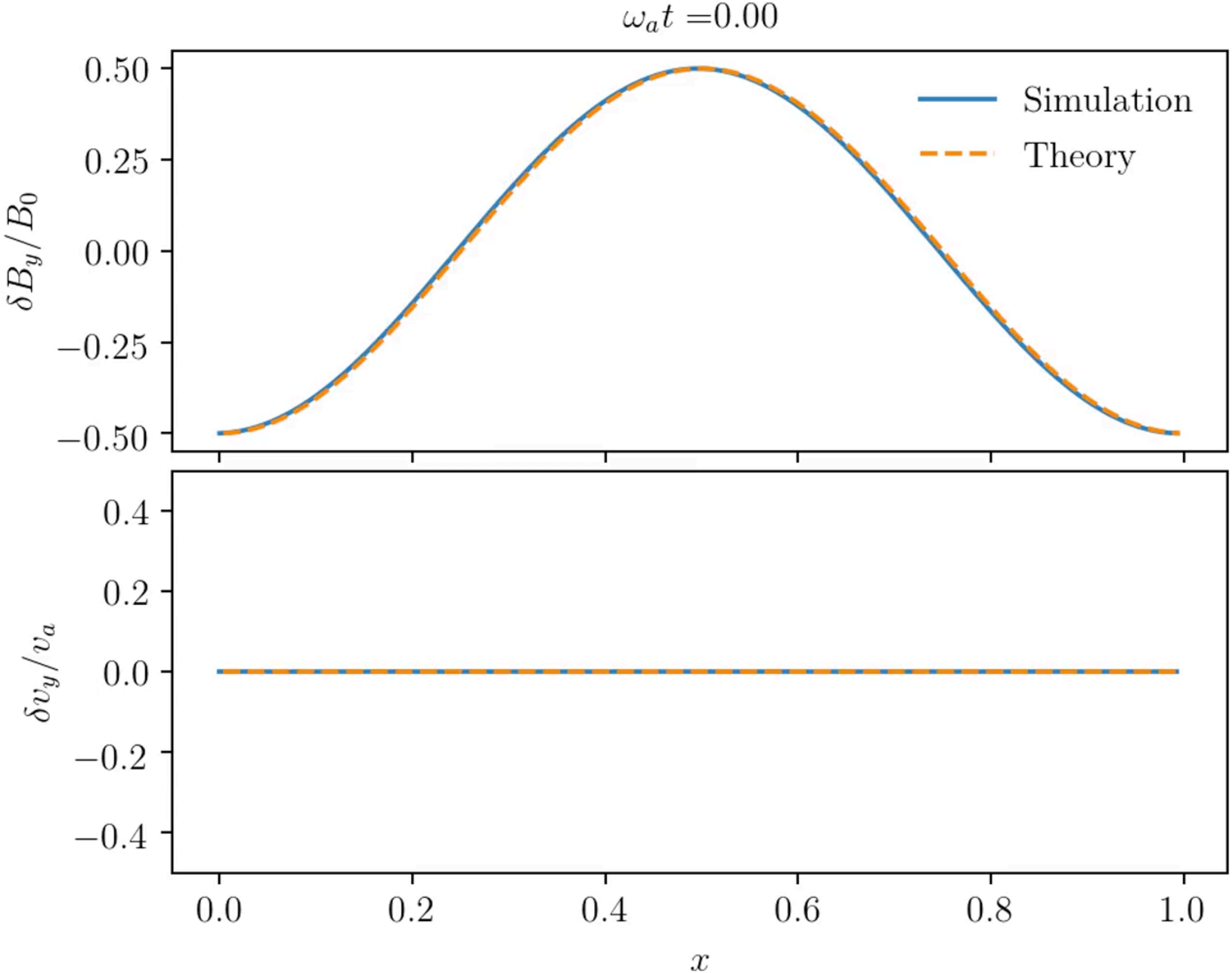
Fast and slow magnetosonic waves

$$\omega^4 + \mathrm{i}\omega^3 \frac{\nu_\parallel}{3}\left(4k_\parallel^2 + k_\perp^2\right) - \omega^2 k^2 \left(v_\mathrm{a}^2 + \tilde{c}^2\right) - \mathrm{i}\omega \frac{\nu_\parallel}{3} k_\parallel^2 \left(9k_\perp^2 \tilde{c}^2 + 4k^2 v_\mathrm{a}^2\right) + k_\parallel^2 k^2 \tilde{c}^2 v_\mathrm{a}^2 = 0$$

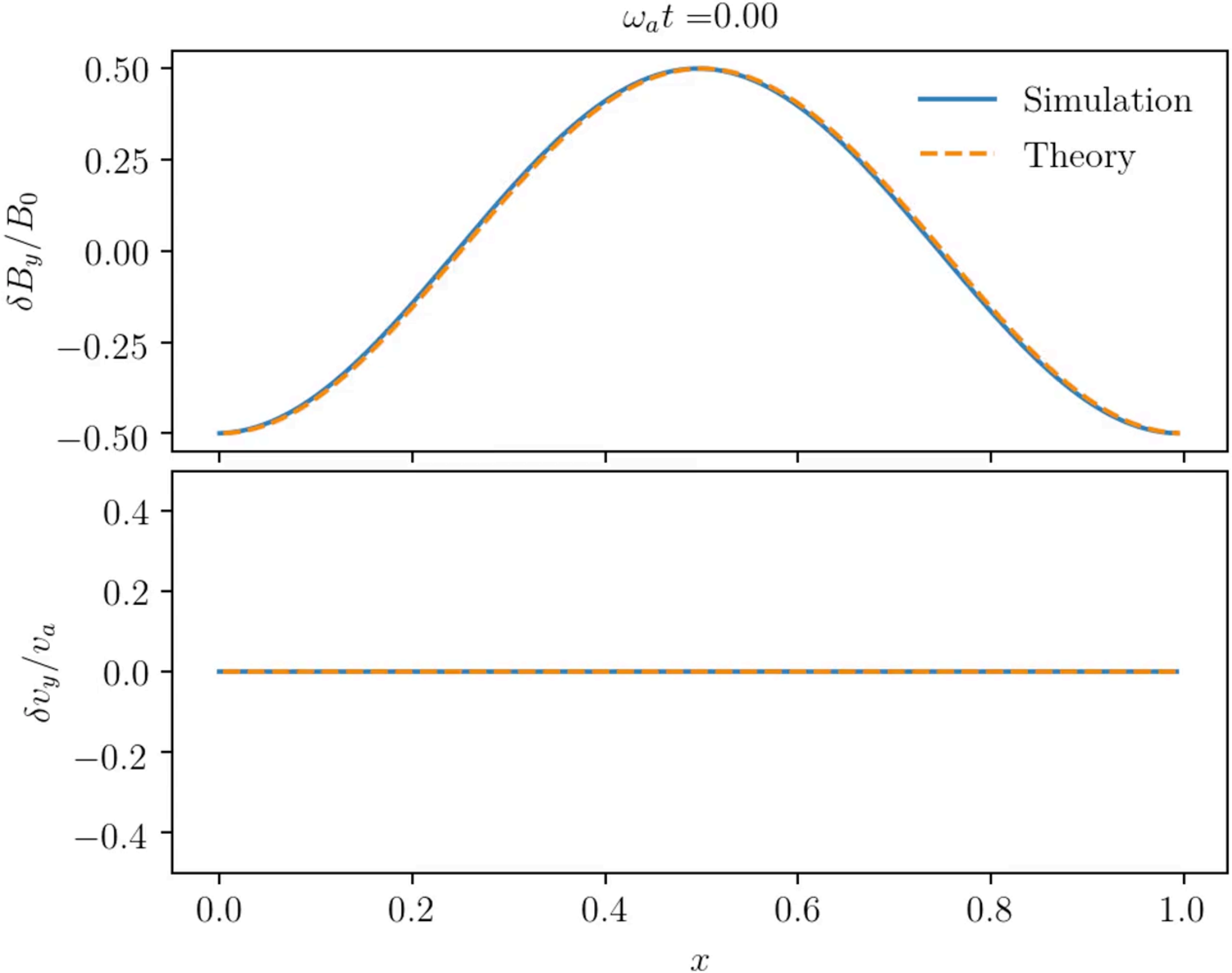
$$\text{if } k_\parallel = 0 \quad \text{then} \quad \omega = \pm k_\perp \sqrt{v_\mathrm{a}^2 + \tilde{c}^2 - \left(\frac{k_\perp \nu_\parallel}{6}\right)^2} - \mathrm{i}\frac{\nu_\parallel}{6} k_\perp^2$$



LINEARLY POLARIZED ALFVEN WAVE

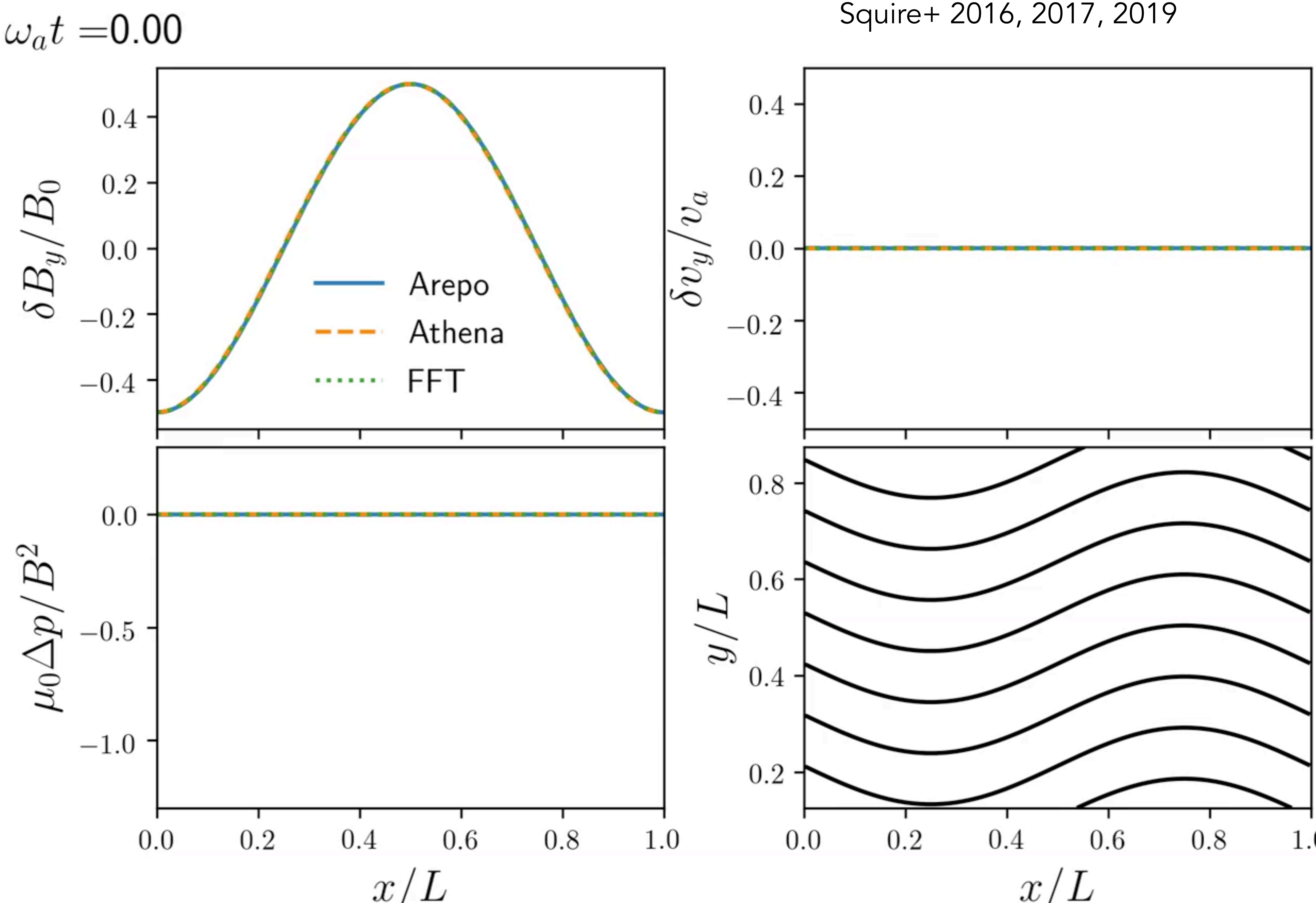


LINEARLY POLARIZED ALFVEN WAVE



INTERRUPTION BY THE FIREHOSE INSTABILITY

Squire+ 2016, 2017, 2019



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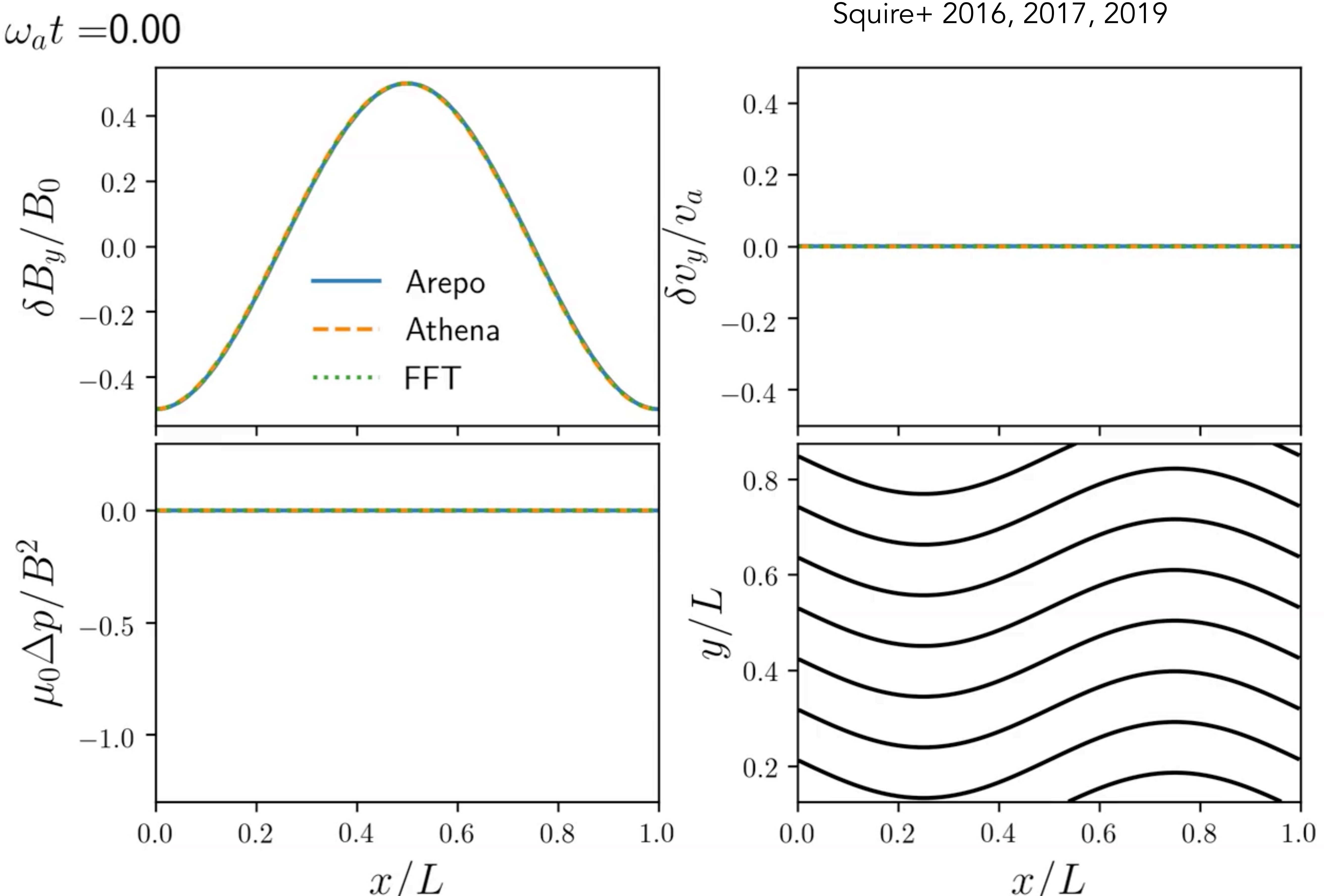
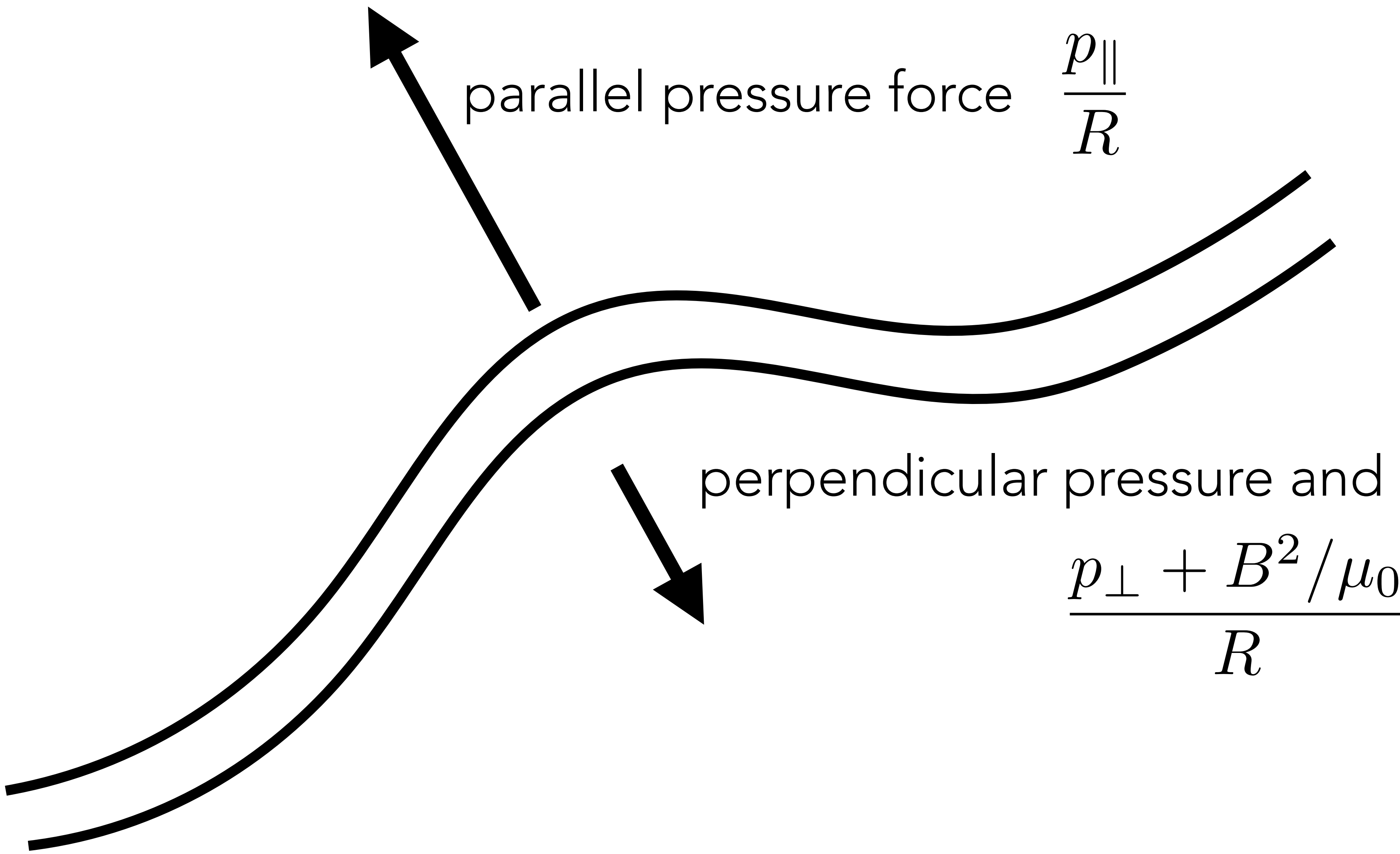
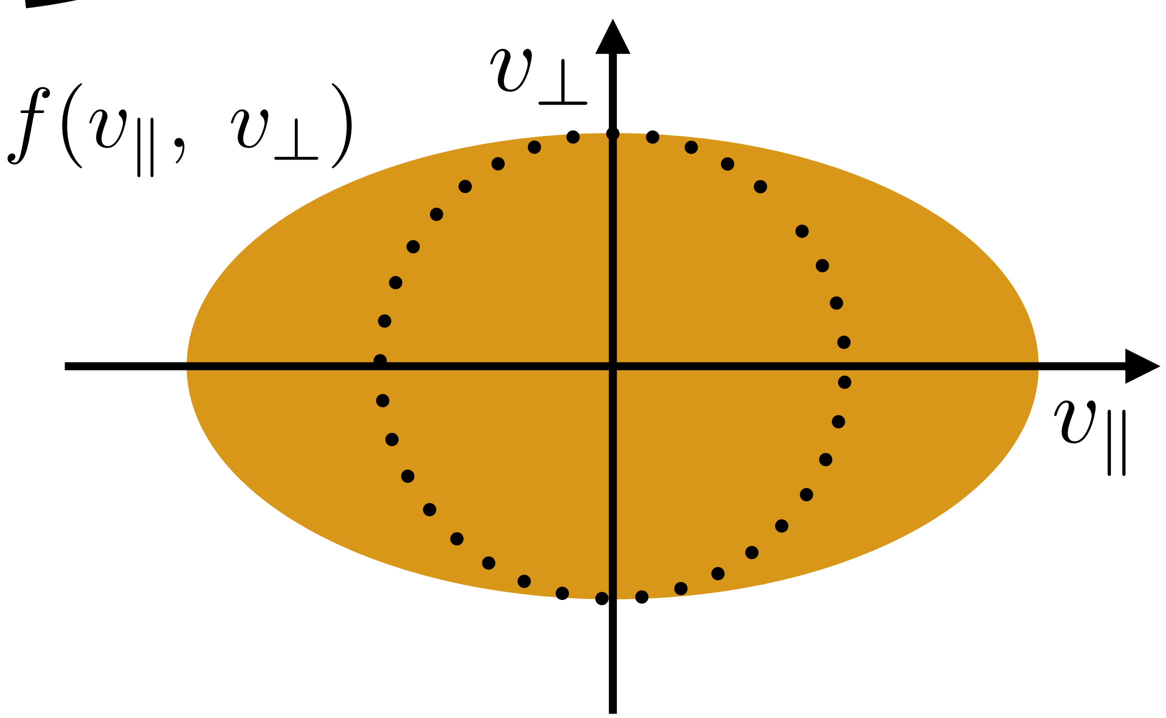


FIREHOSE INSTABILITY

Rosenbluth 1956

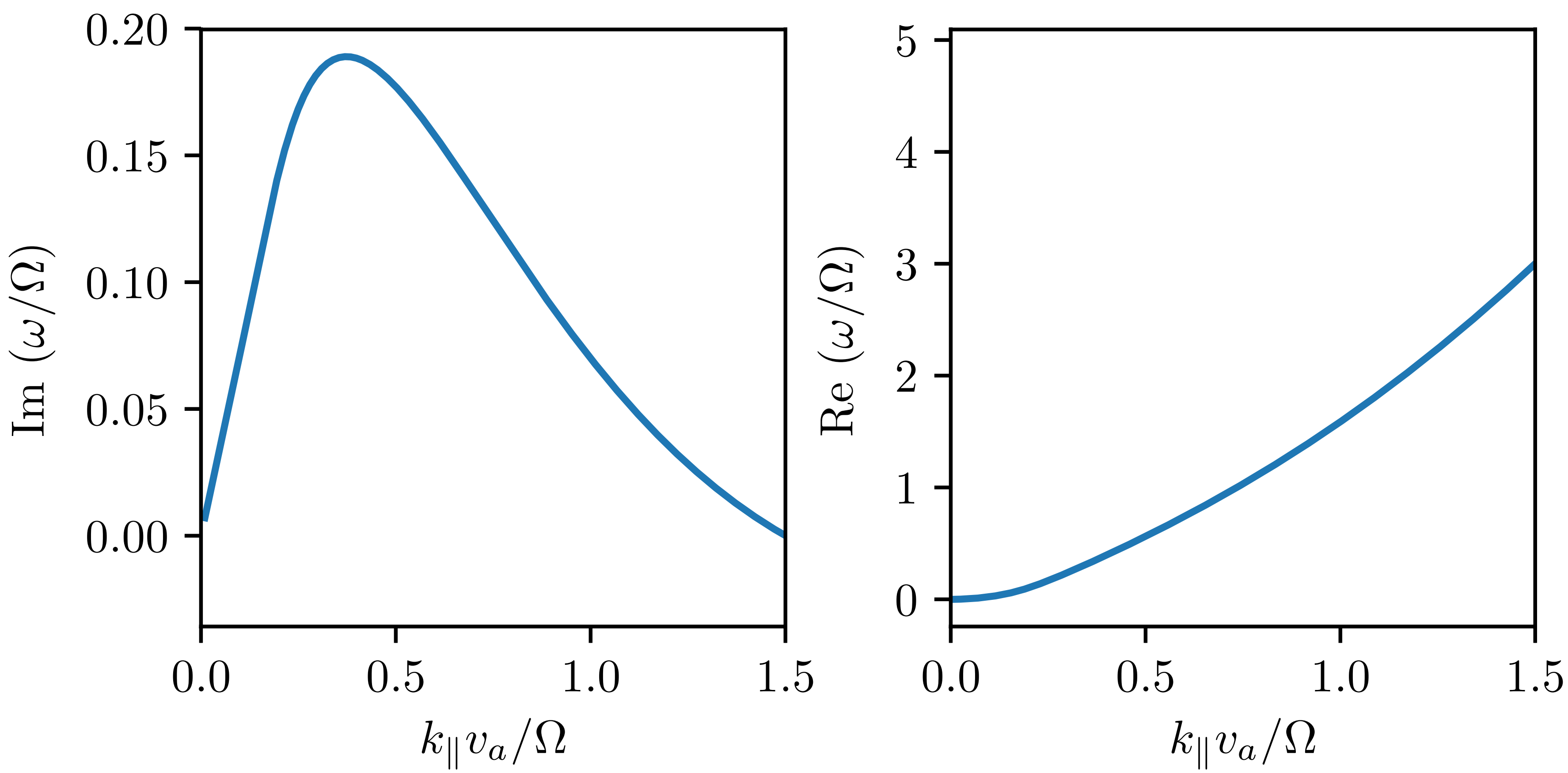




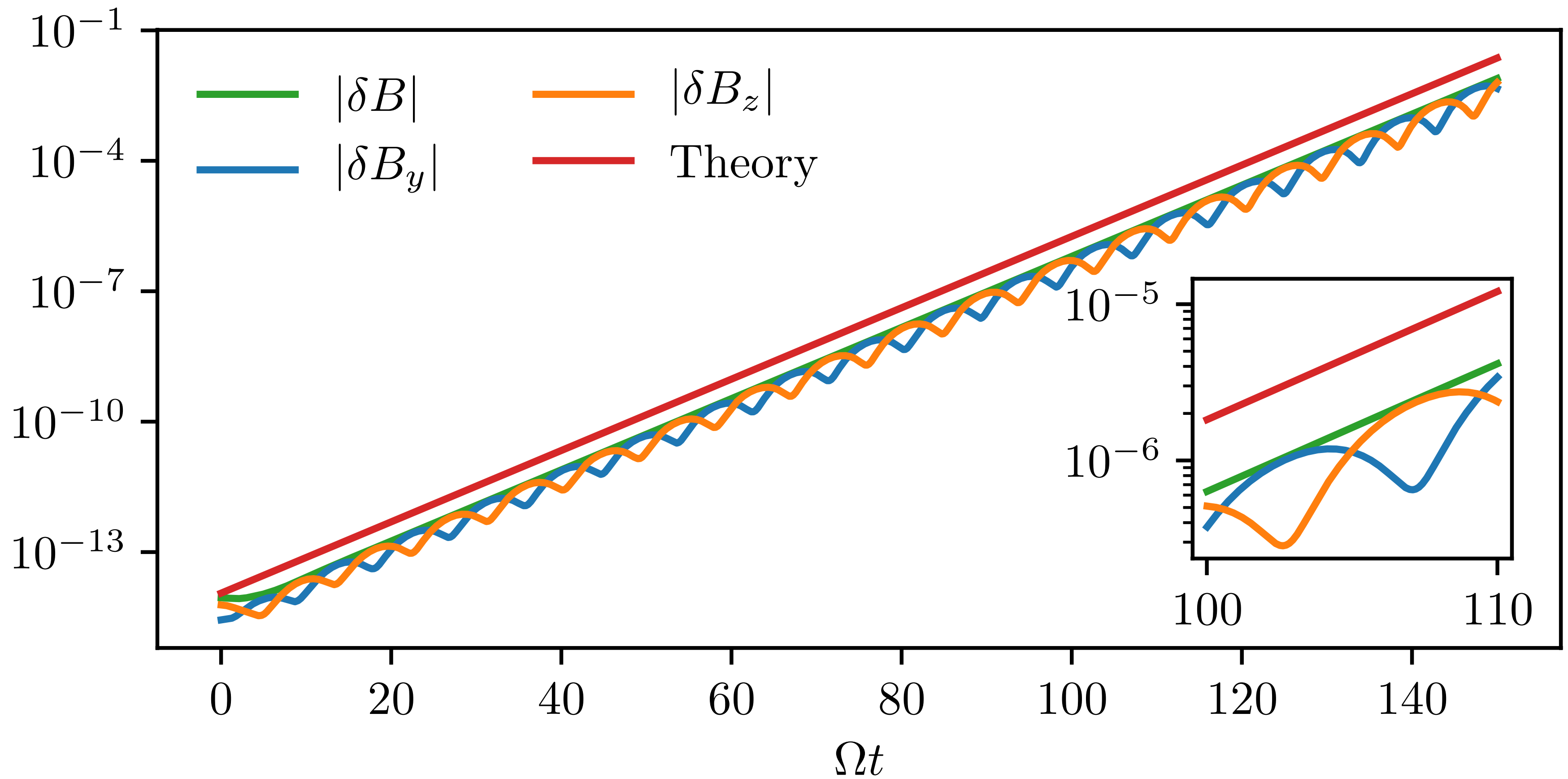
instability criterion

$$p_\parallel - p_\perp > \frac{B^2}{\mu_0}$$

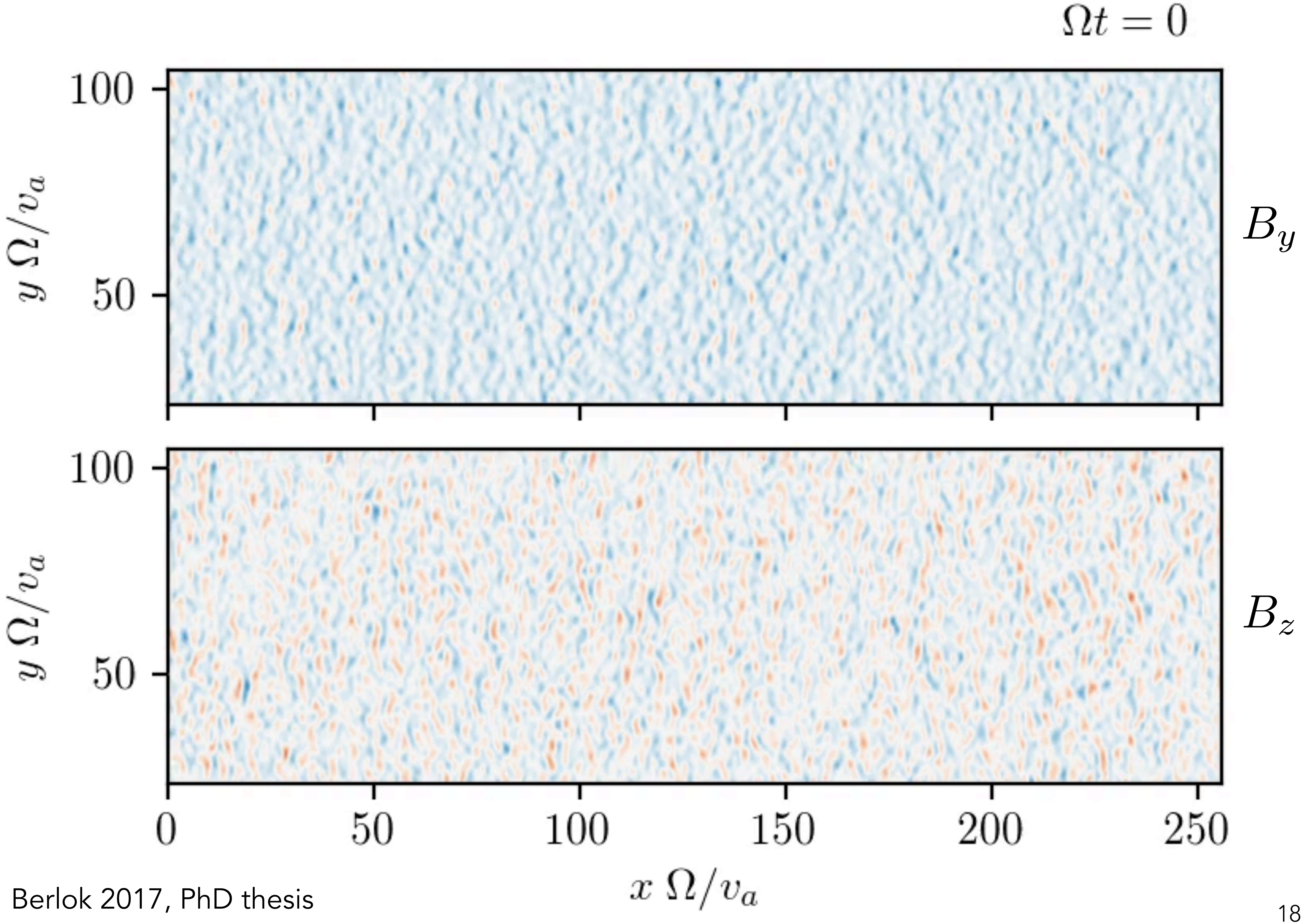
PARALLEL FIREHOSE INSTABILITY $\beta_{\parallel} = 4,\ \beta_{\perp} = 1,\ T_e = 0$



Berlok 2017, PhD thesis
Advisors:
Martin Pessah, Troels Haugbølle and Tobias Heinemann
http://www.nbi.dk/~berlok/

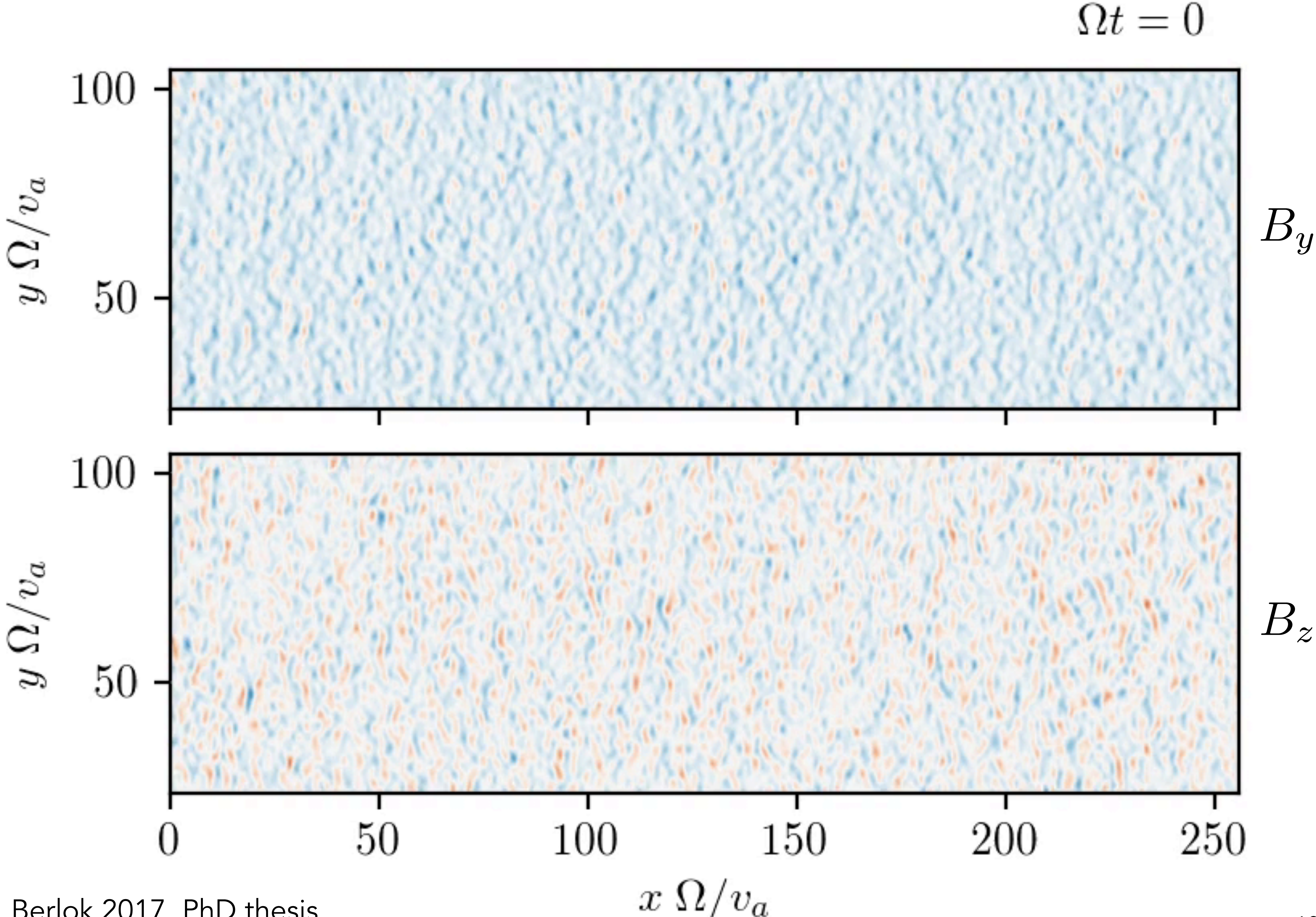


2D FIREHOSE INSTABILITY WITH 2D-3V HYBRID-KINETIC CODE



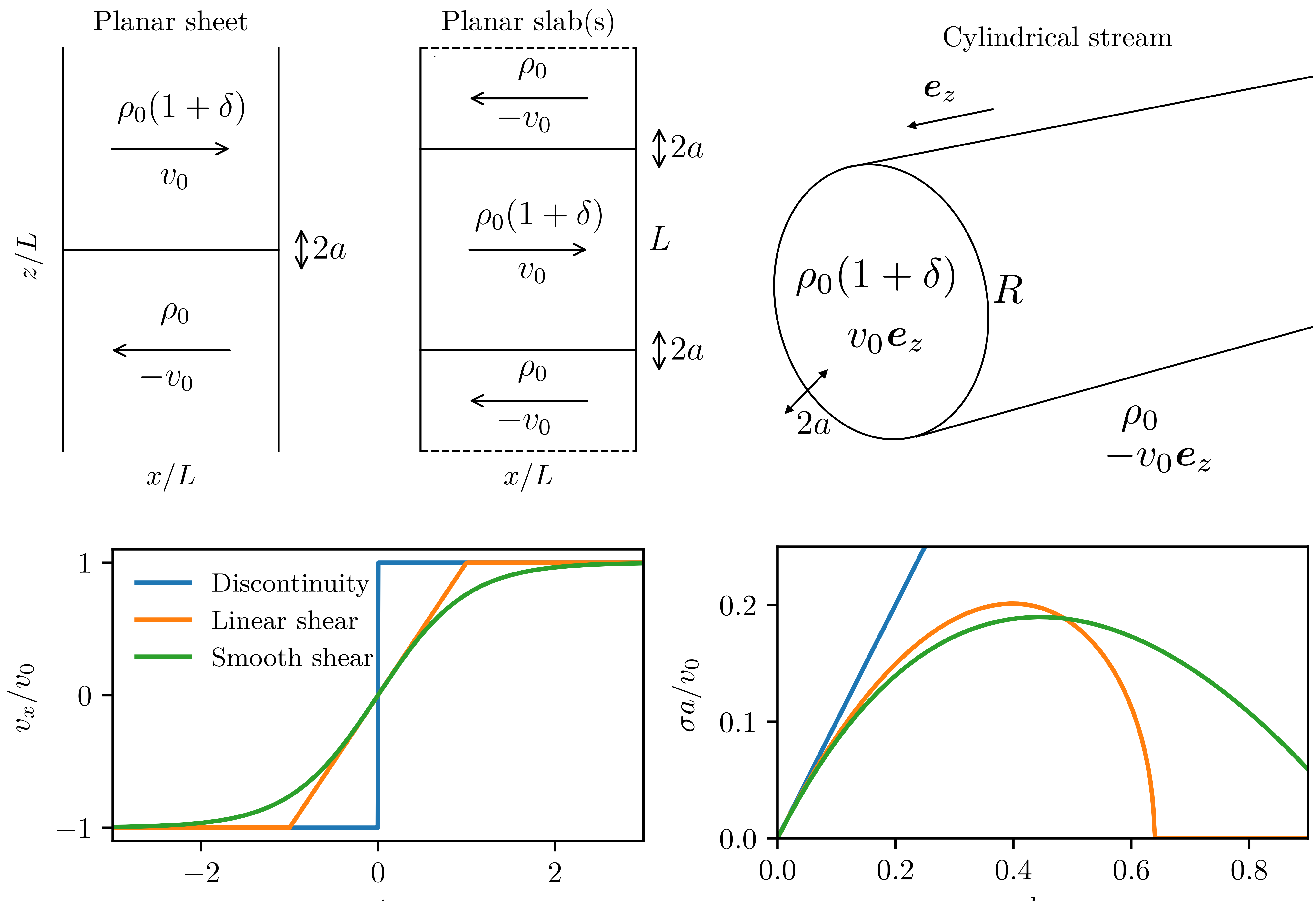
Berlok 2017, PhD thesis

2D FIREHOSE INSTABILITY WITH 2D-3V HYBRID-KINETIC CODE



Berlok 2017, PhD thesis

KELVIN-HELMHOLTZ INSTABILITY

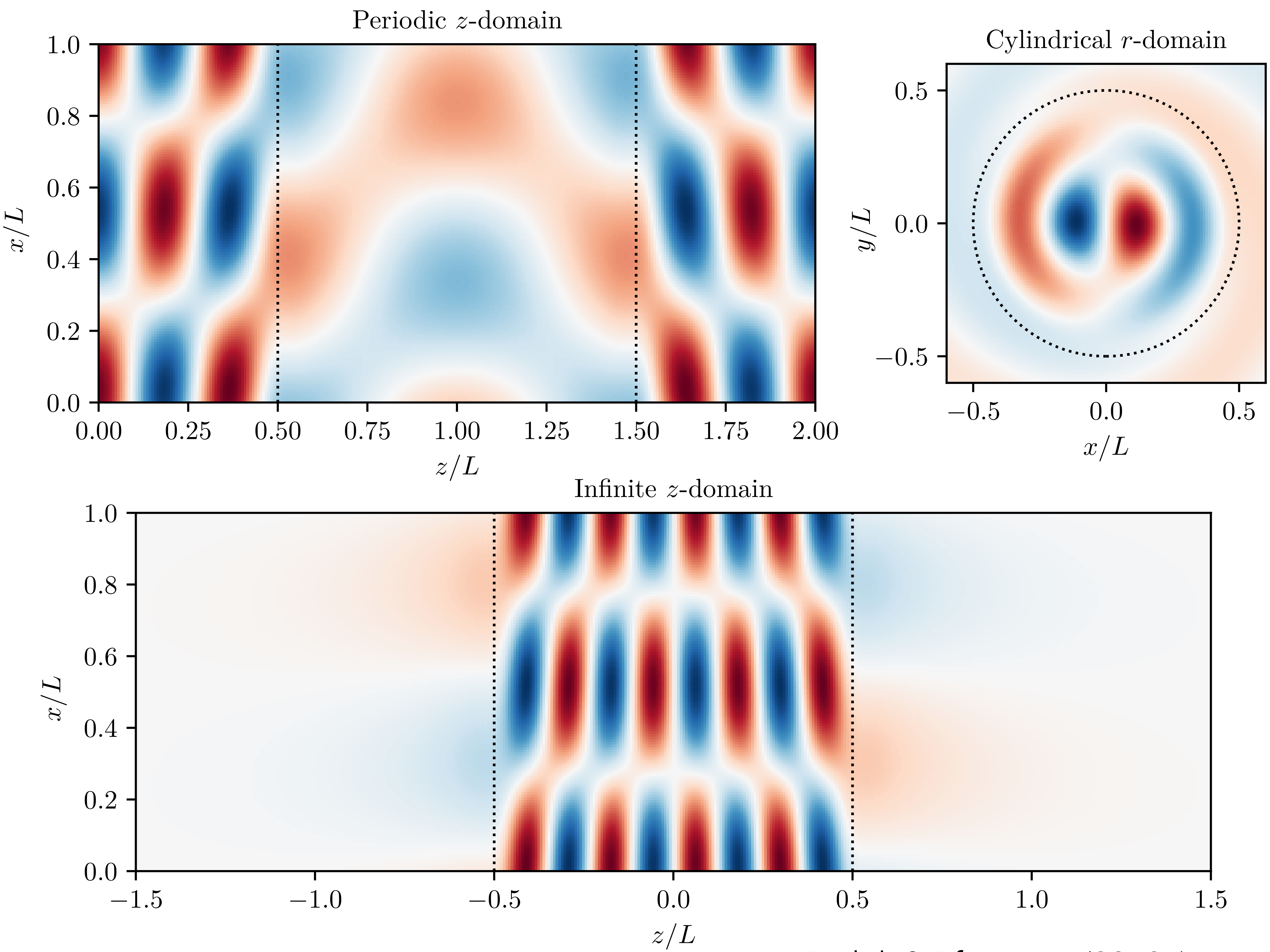


Planar sheet
$\rho_0(1+\delta)$
v_0
ρ_0
$-v_0$
$2a$
z/L
x/L
Planar slab(s)
ρ_0
$-v_0$
$\rho_0(1+\delta)$
v_0
$2a$
L
x/L
Cylindrical stream
$\boldsymbol{e}_z$
$\rho_0(1+\delta)$
$v_0\boldsymbol{e}_z$
R
$2a$
ρ_0
$-v_0\boldsymbol{e}_z$
Discontinuity
Linear shear
Smooth shear
v_x/v_0
z/a
$\sigma a/v_0$
ka

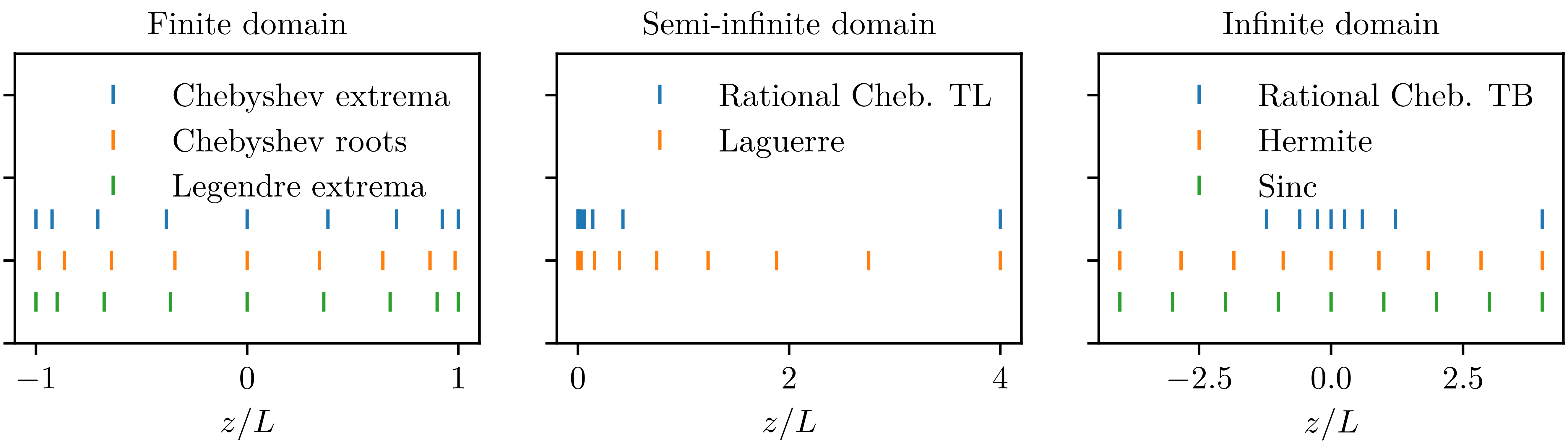
PSECAS

Pseudo-Spectral Eigenvalue Calculator with an Automated Solver

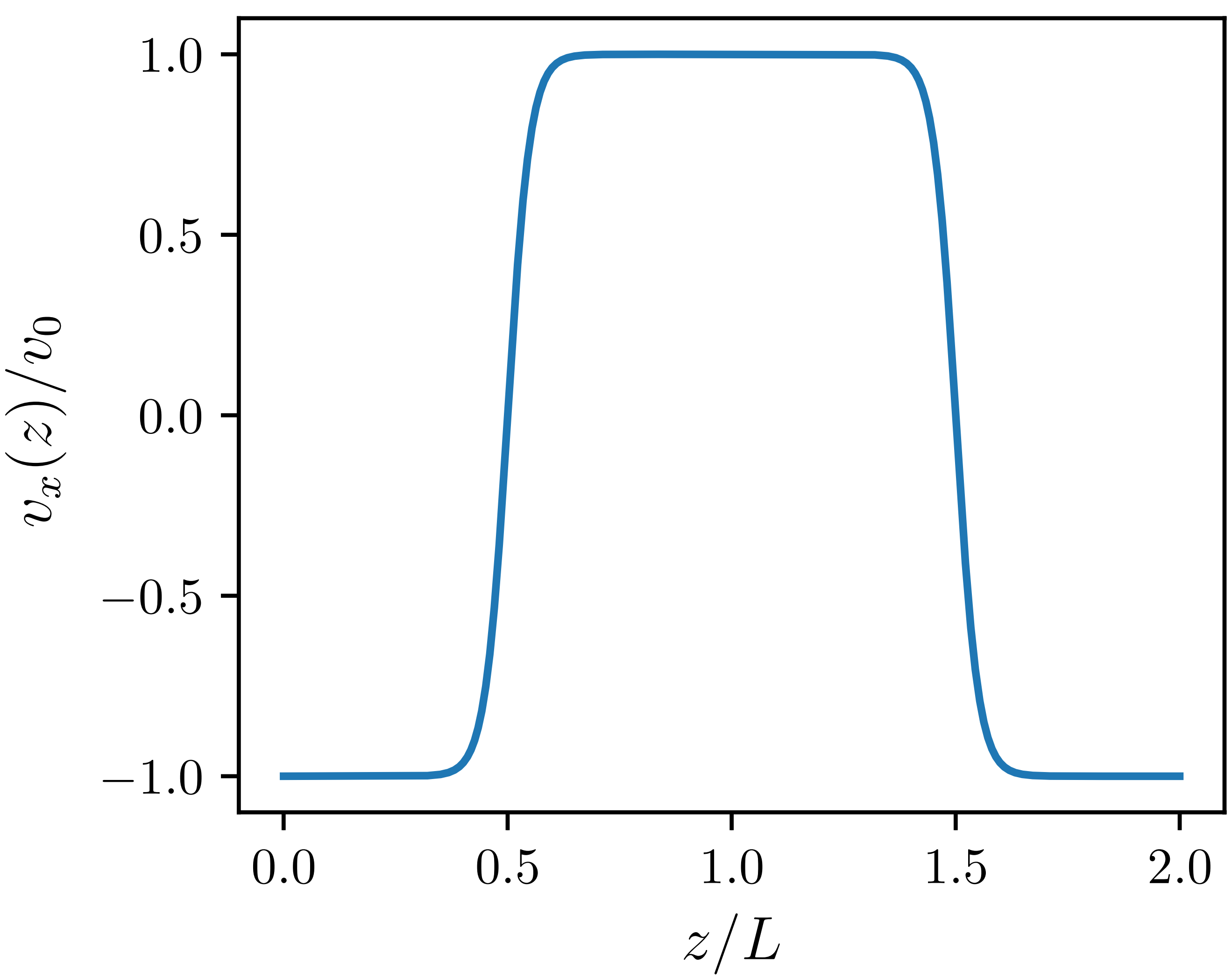
https://github.com/tberlok/psecas



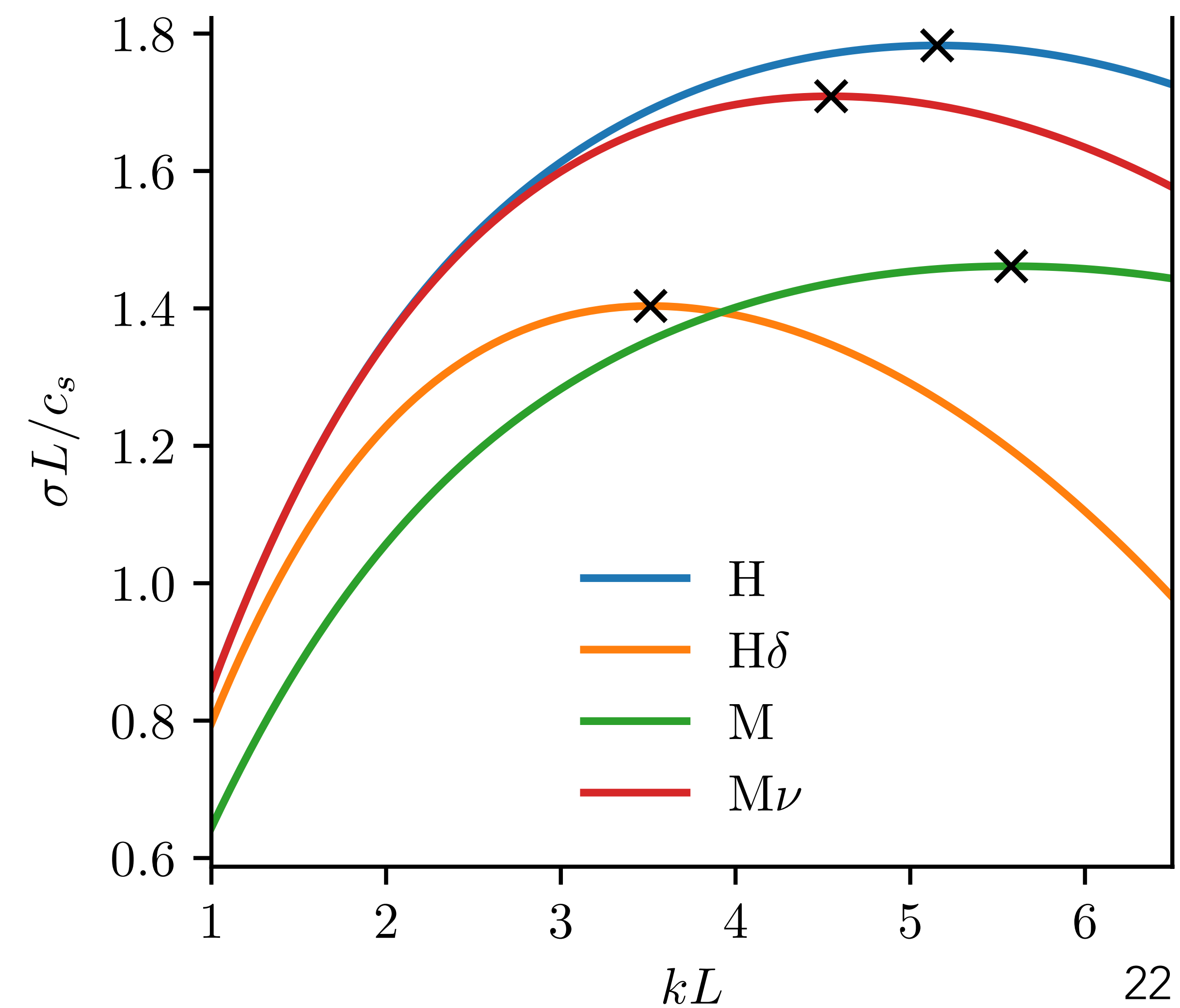
Berlok & Pfrommer (2019a), MNRAS



Finite domain
Chebyshev extrema
Chebyshev roots
Legendre extrema
−1
0
1
z/L
Semi-infinite domain
Rational Cheb. TL
Laguerre
0
2
4
z/L
Infinite domain
Rational Cheb. TB
Hermite
Sinc
−2.5
0.0
2.5
z/L

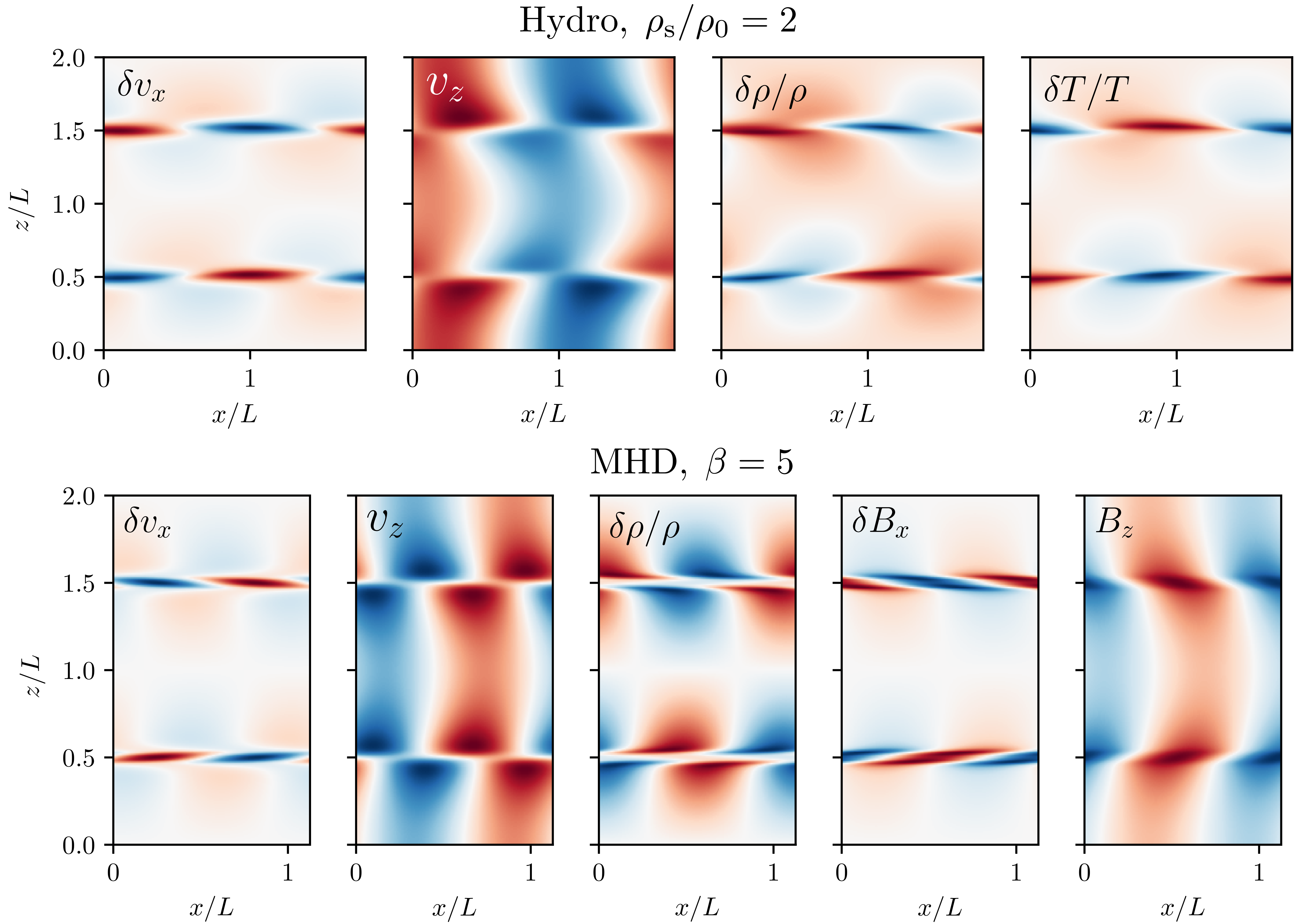


1.0
0.5
0.0
−0.5
−1.0
$v_x(z)/v_0$
0.0
0.5
1.0
1.5
2.0
z/L

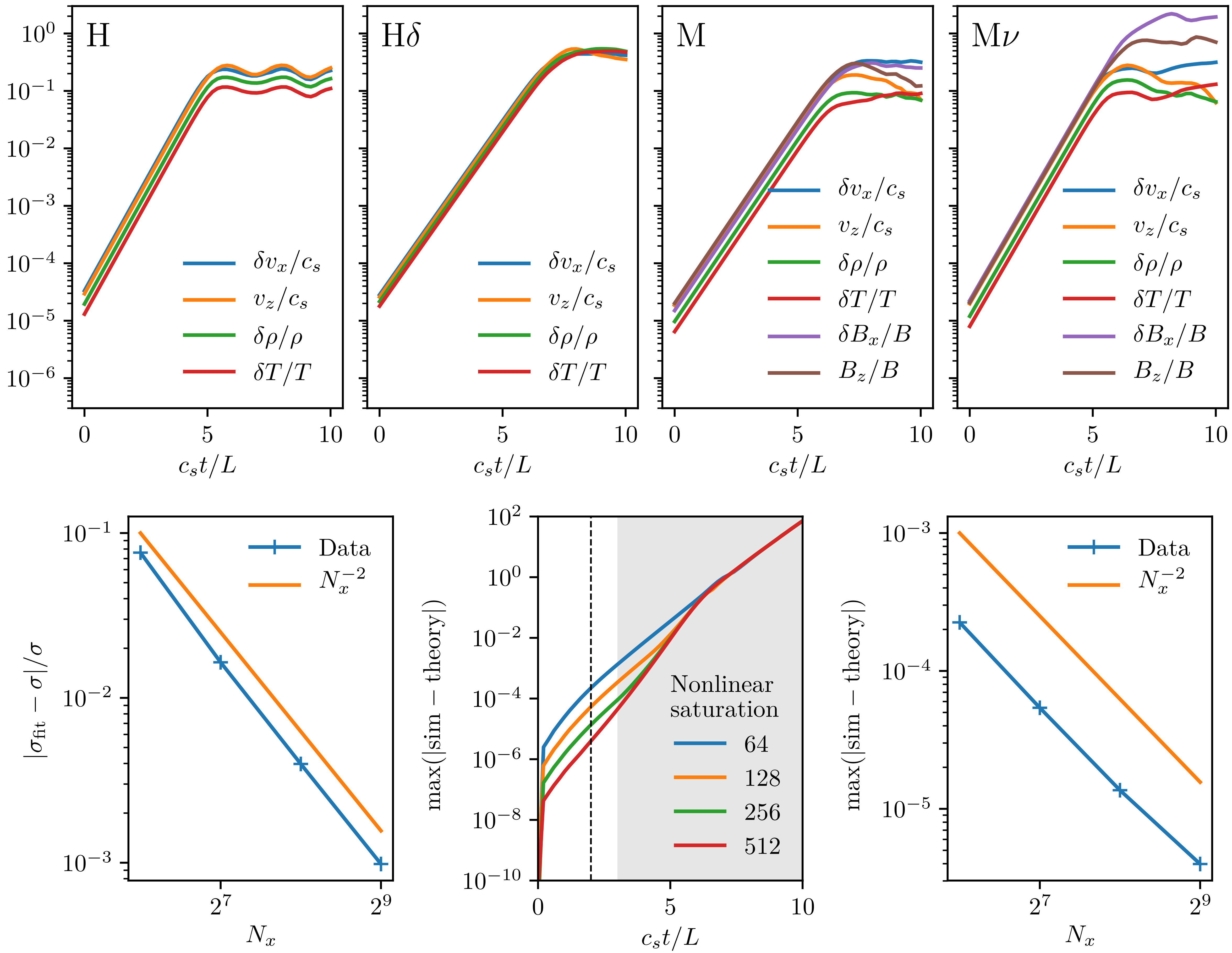


1.8
1.6
1.4
1.2
1.0
0.8
0.6
$\sigma L/c_s$
H
Hδ
M
Mν
1
2
3
4
5
6
kL

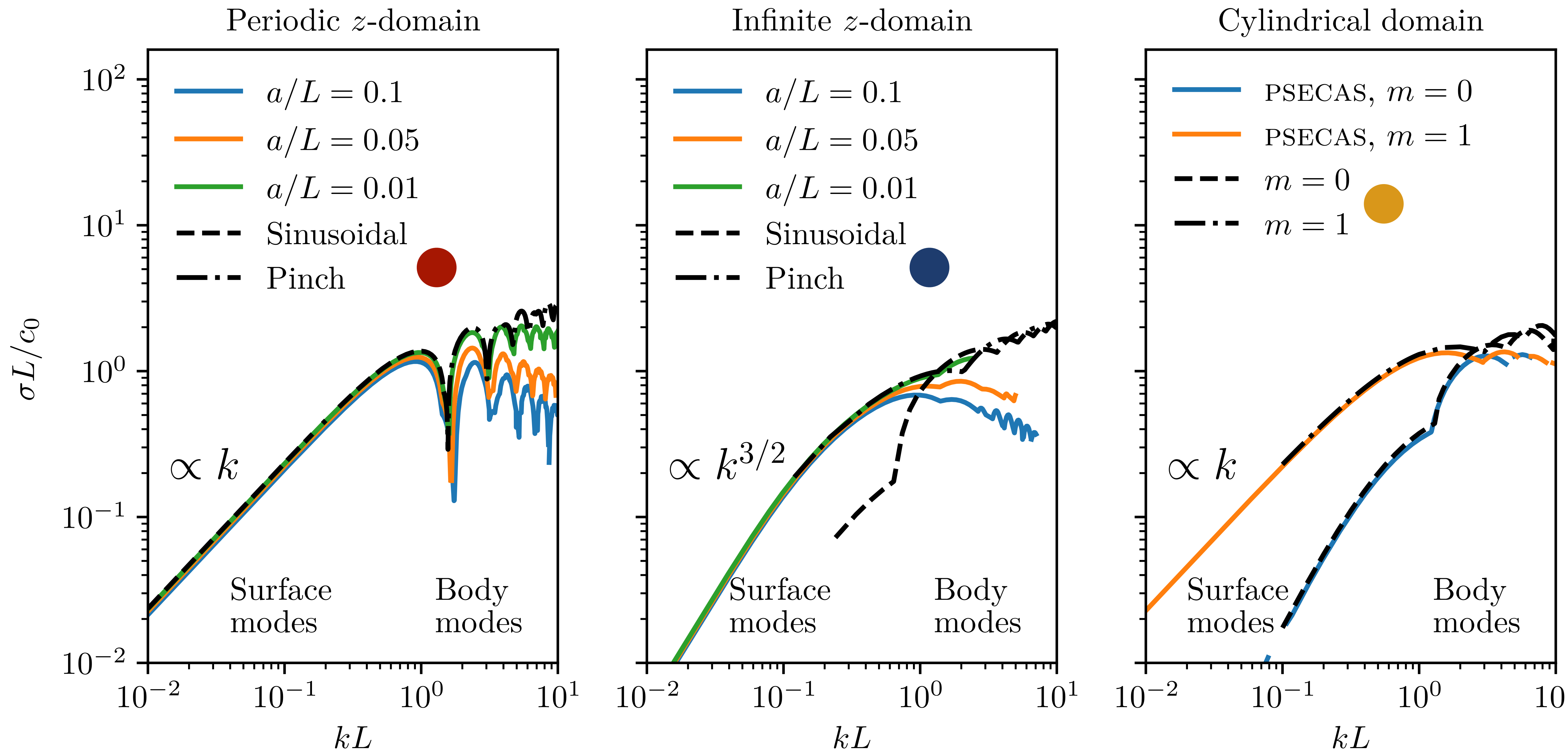
SURFACE MODE SIMULATIONS with Athena



KELVIN-HELMHOLTZ INSTABILITY with Athena



SUPERSONIC BODYMODE DISPERSION RELATION



$$\left(\frac{\omega - k v_2}{\omega - k v_1}\right)^2 \frac{\rho_2}{\rho_1}\frac{q_1}{q_2} = \bullet$$

Berlok & Pfrommer 2019a

$$\bullet \; \frac{\mathcal{T}(q_2 L/2)}{\mathcal{T}(q_1 L/2)}$$

Mandelker+ (2016)

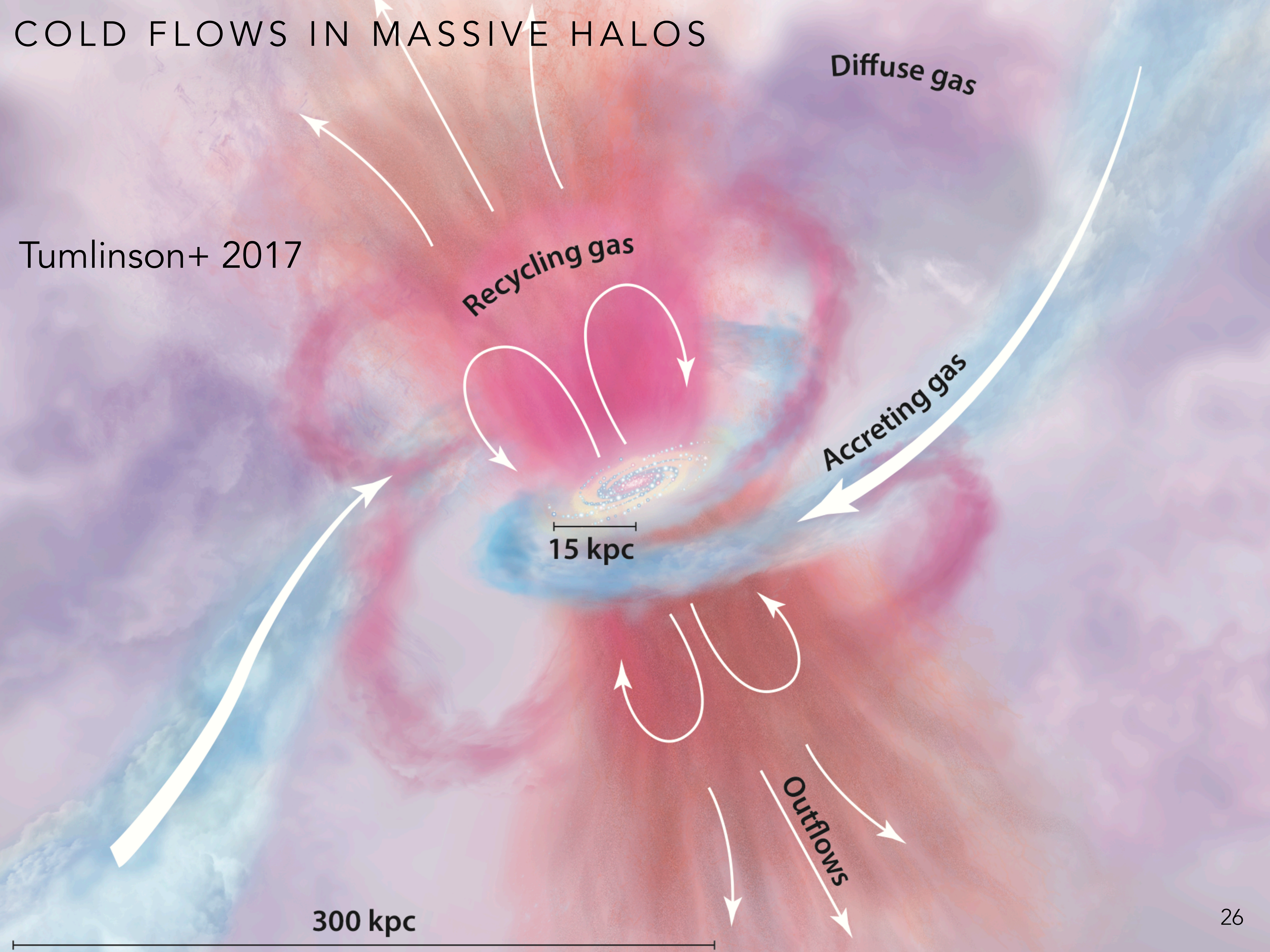
$$\bullet \; -\mathcal{T}(q_2 L/2)$$

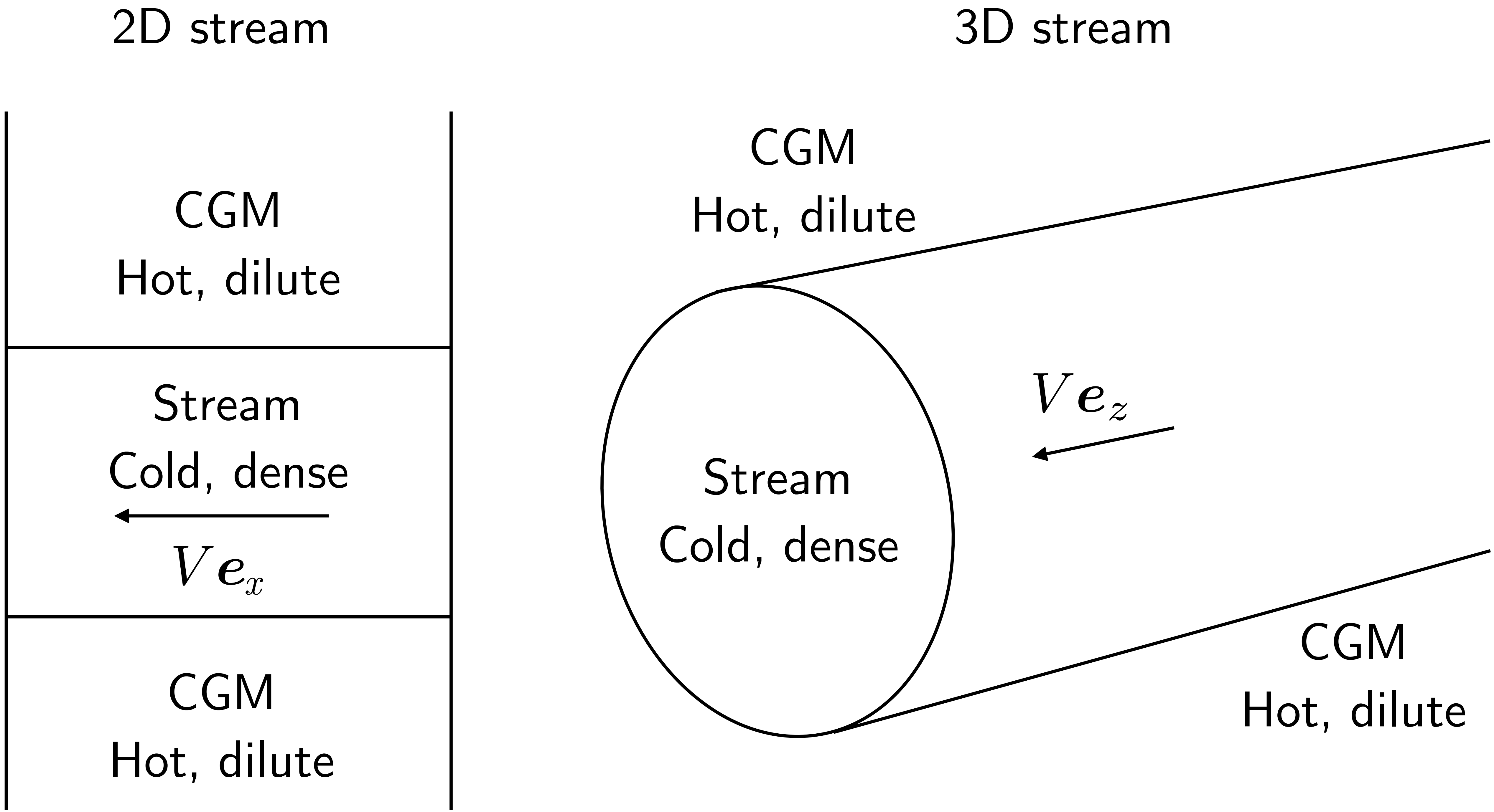
$$\bullet \; \frac{\mathcal{I}_m'(q_1 R)}{\mathcal{I}_m(q_1 R)} \frac{\mathcal{K}_m(q_2 R)}{\mathcal{K}_m'(q_2 R)}$$

$\mathcal{T}(z) = \tanh(z) \text{ or } \cosh(z)$ $\quad q = k\sqrt{1 - \left(\frac{\omega - kv}{kc}\right)^2}$ $\quad \mathcal{I}_m, \mathcal{K}_m$ are modified Bessel functions

COLD FLOWS IN MASSIVE HALOS

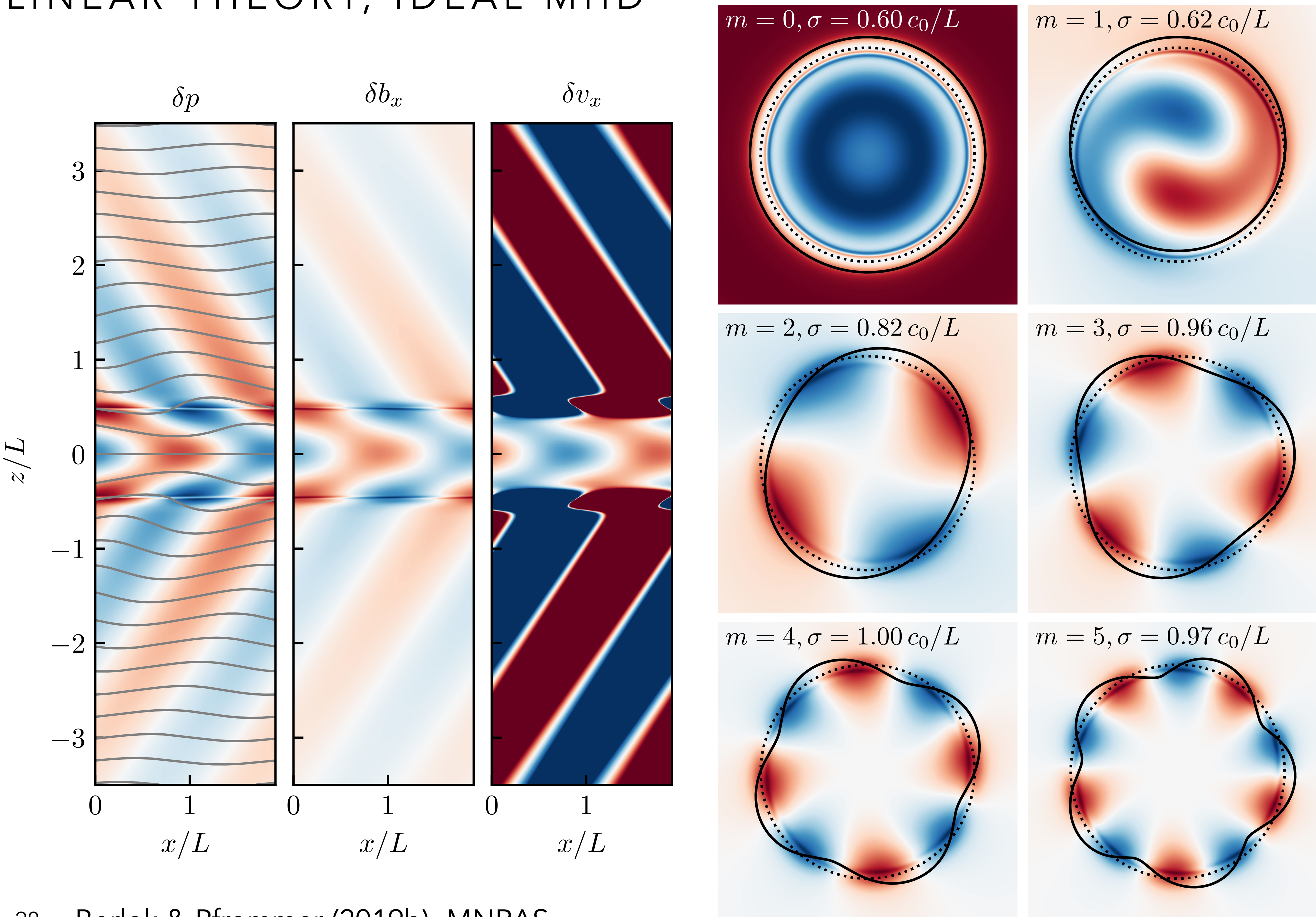
Tumlinson+ 2017





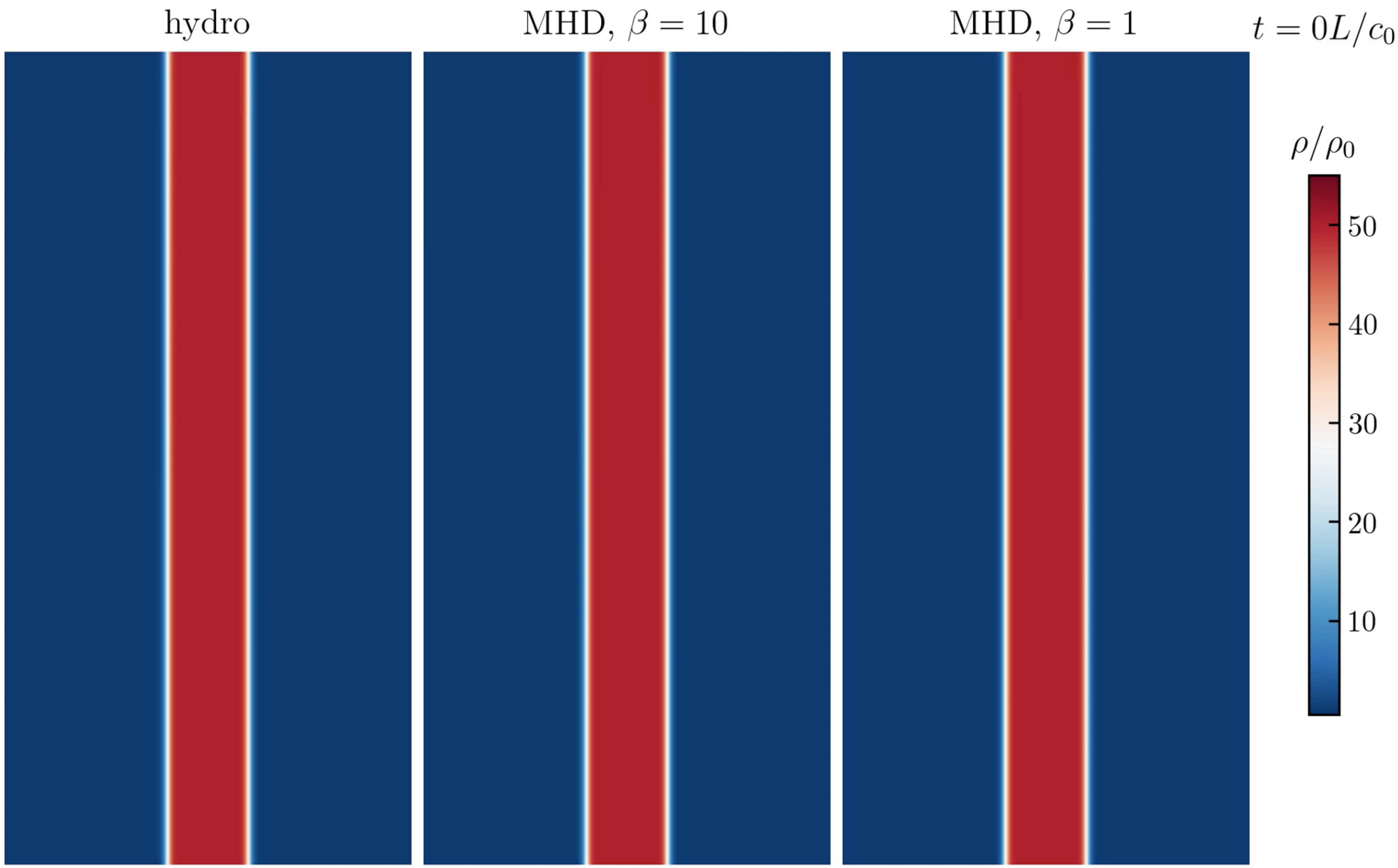
Hydrodynamic studies in Mandelker+ 2016, 2019 and Padnos+ 2019
MHD study in Berlok & Pfrommer 2019b

LINEAR THEORY, IDEAL MHD



Berlok & Pfrommer (2019b), MNRAS

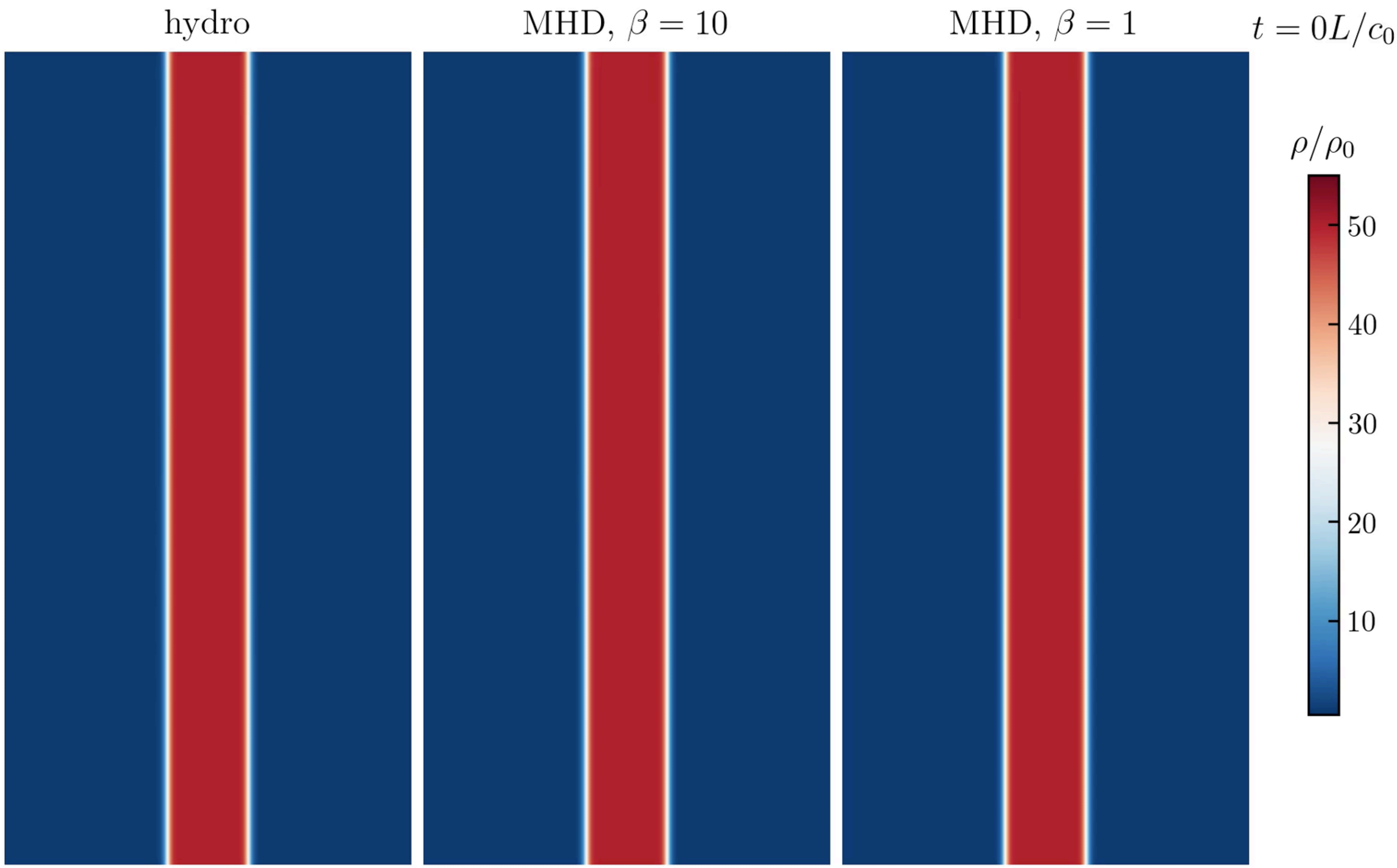
2D MAGNETIZED COLD STREAMS



Berlok & Pfrommer (2019b), MNRAS

with Athena++

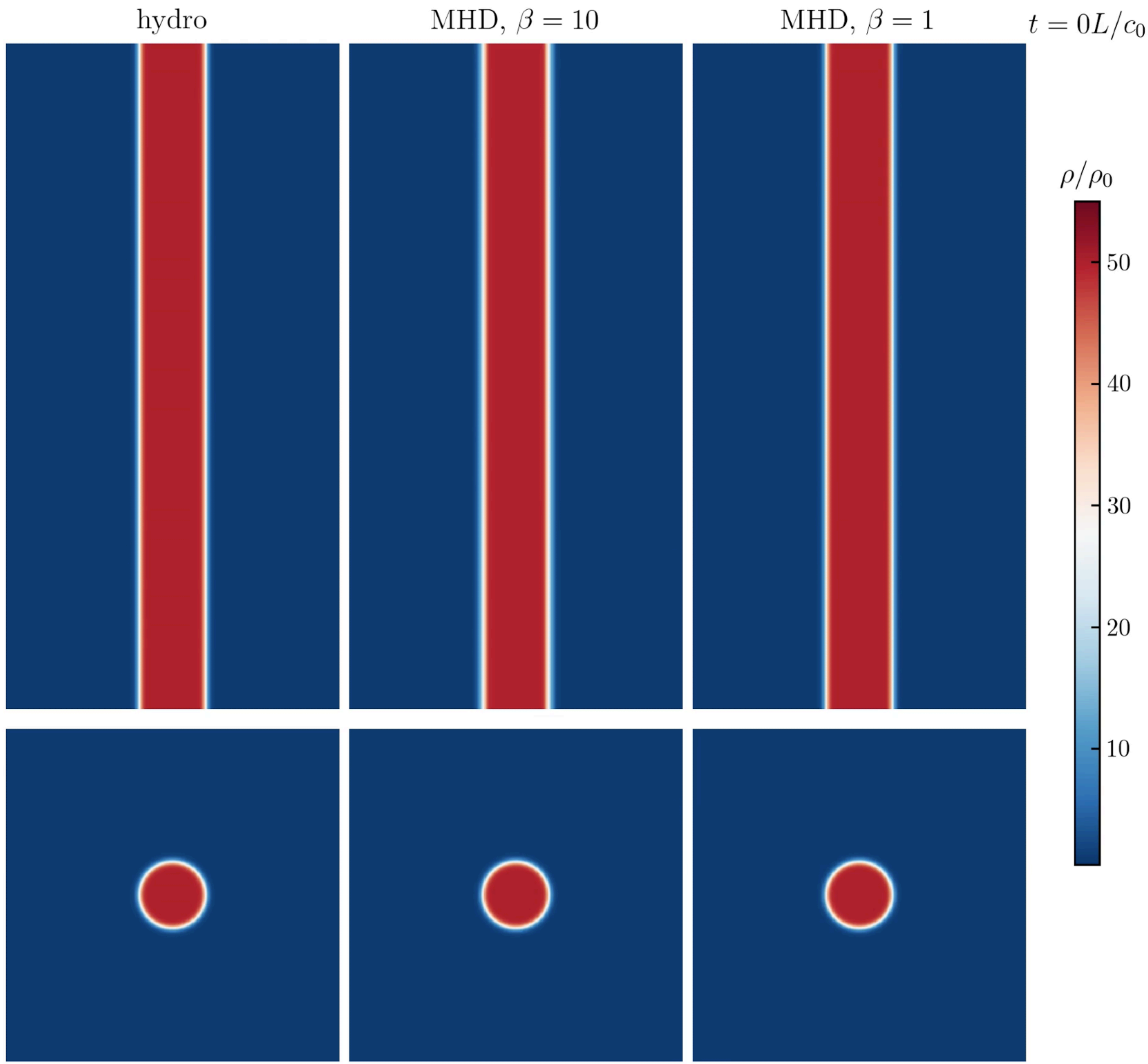
2D MAGNETIZED COLD STREAMS



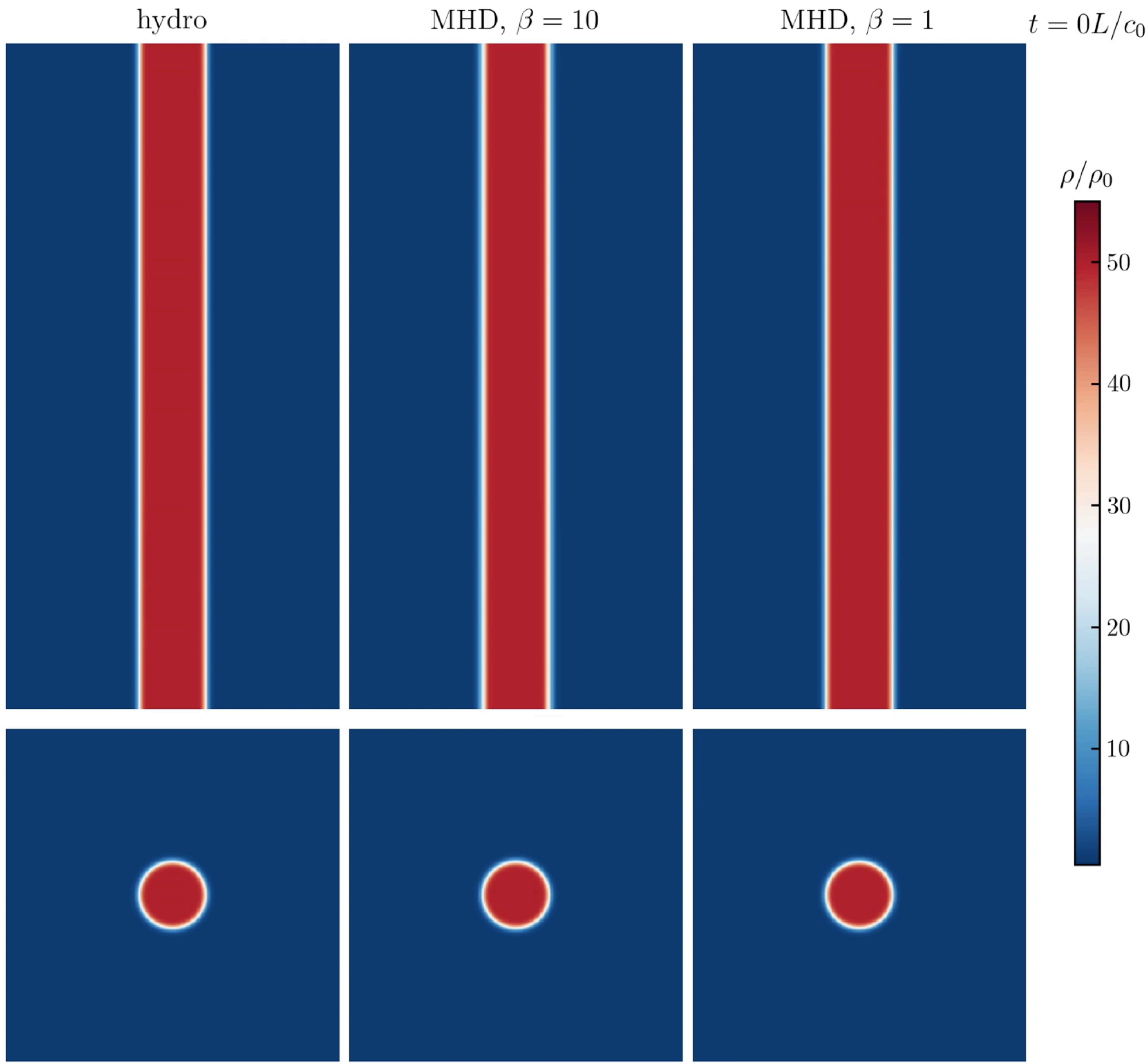
Berlok & Pfrommer (2019b), MNRAS

with Athena++

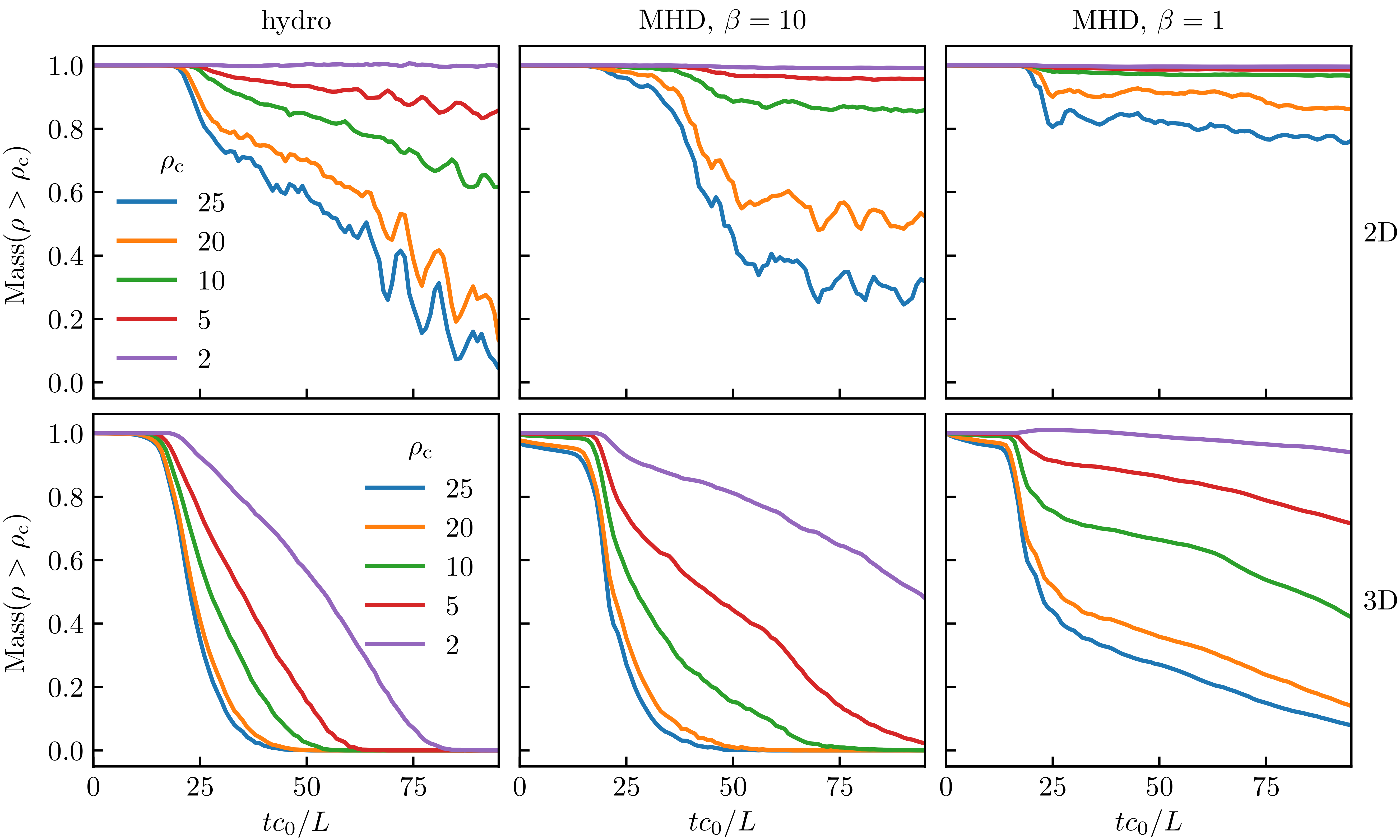
3 D



3 D

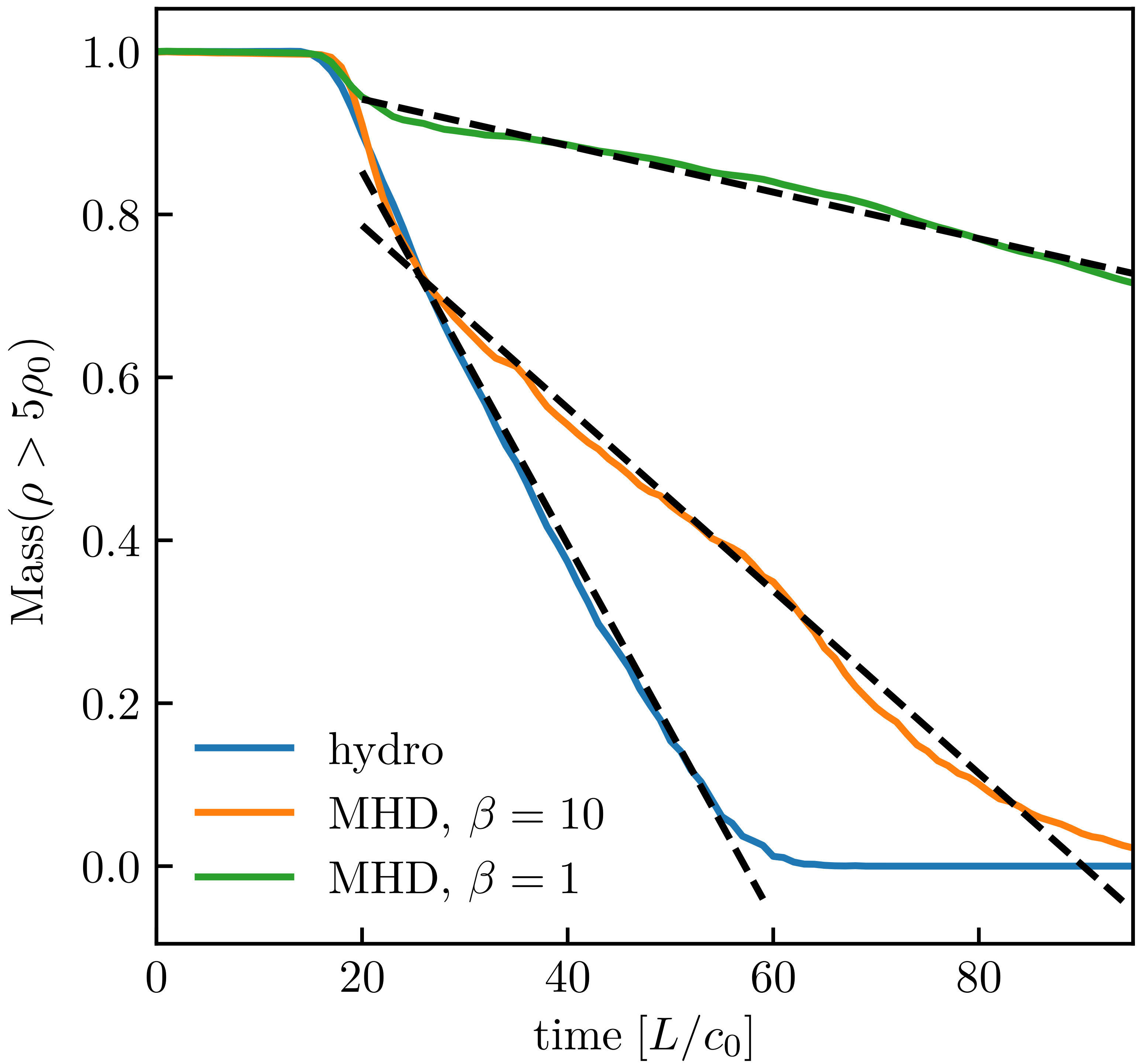


MIXING OF COLD STREAMS WITH CGM



Berlok & Pfrommer (2019b), MNRAS

MAGNETIC FIELDS SUPPRESS MIXING



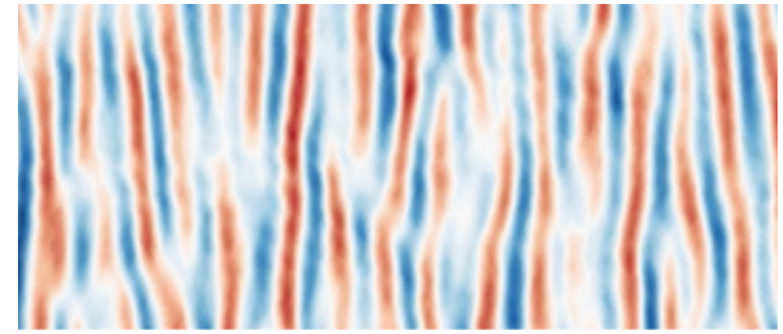
$$\frac{L}{R_{\mathrm{v}}} \lesssim \begin{cases} 2 \times 10^{-2} & \text{hydro} \\ 9 \times 10^{-3} & \beta = 10 \\ 2 \times 10^{-4} & \beta = 1 \end{cases}$$

$$B = \sqrt{\frac{2p\mu_0}{\beta}} = 0.8\left(\frac{n_{\mathrm{H}}}{10^{-4}\mathrm{cm}^{-3}}\right)^{1/2}\left(\frac{T}{10^6\mathrm{K}}\right)^{1/2}\beta^{-1/2}\mu\mathrm{G}$$

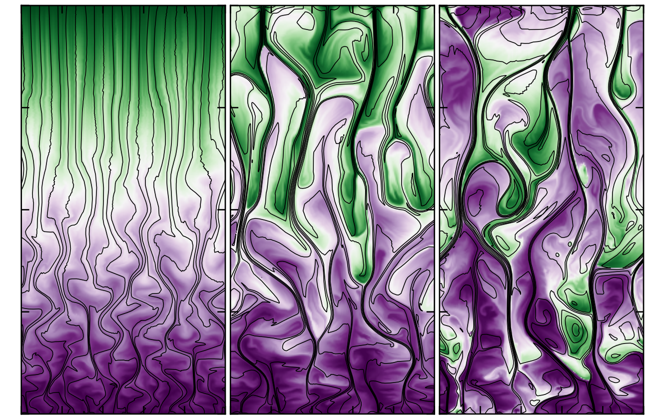
SUMMARY

- Weakly collisional and collisionless plasmas: Small scales with hybrid-kinetic codes, intermediate scales with Athena
- Large scales with Braginskii viscosity in Arepo
- Supersonic, magnetized Kelvin-Helmholtz instability in cold streams at high redshift

$r_i \sim 10^{-9}$ pc

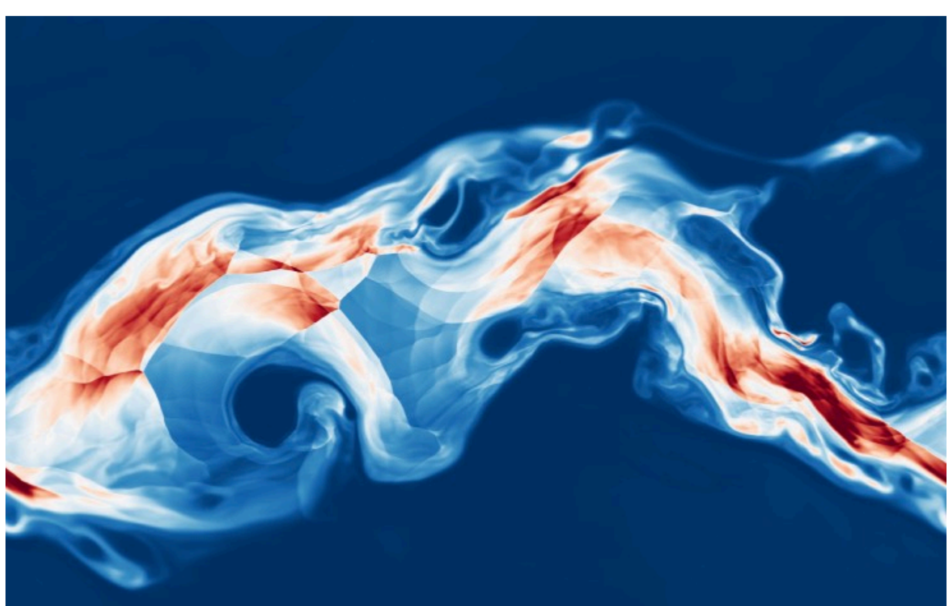


$H \sim 10^2$ kpc

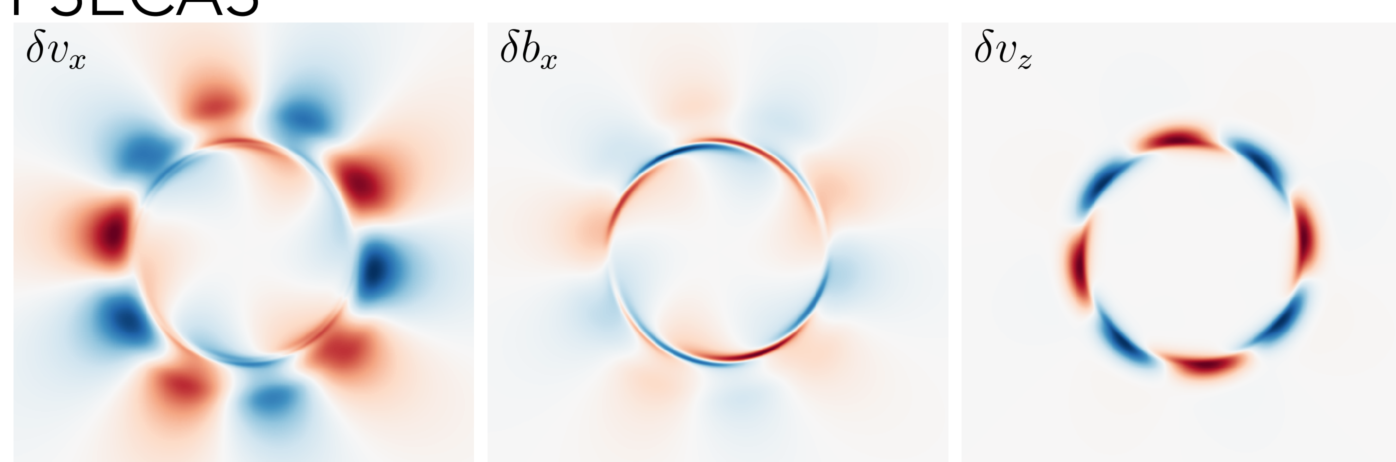


$L \sim$ Mpc

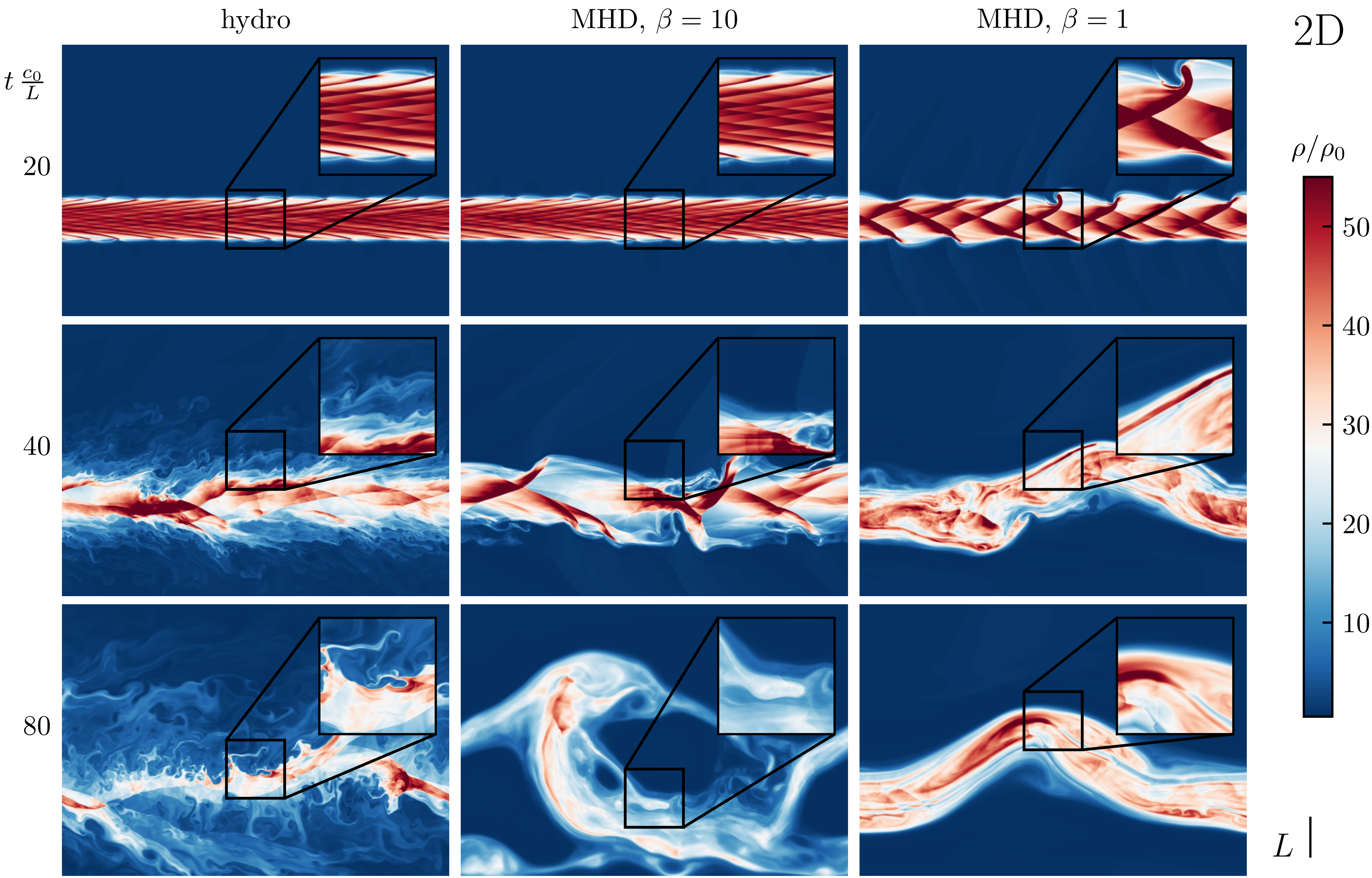
Magnetized cold streams

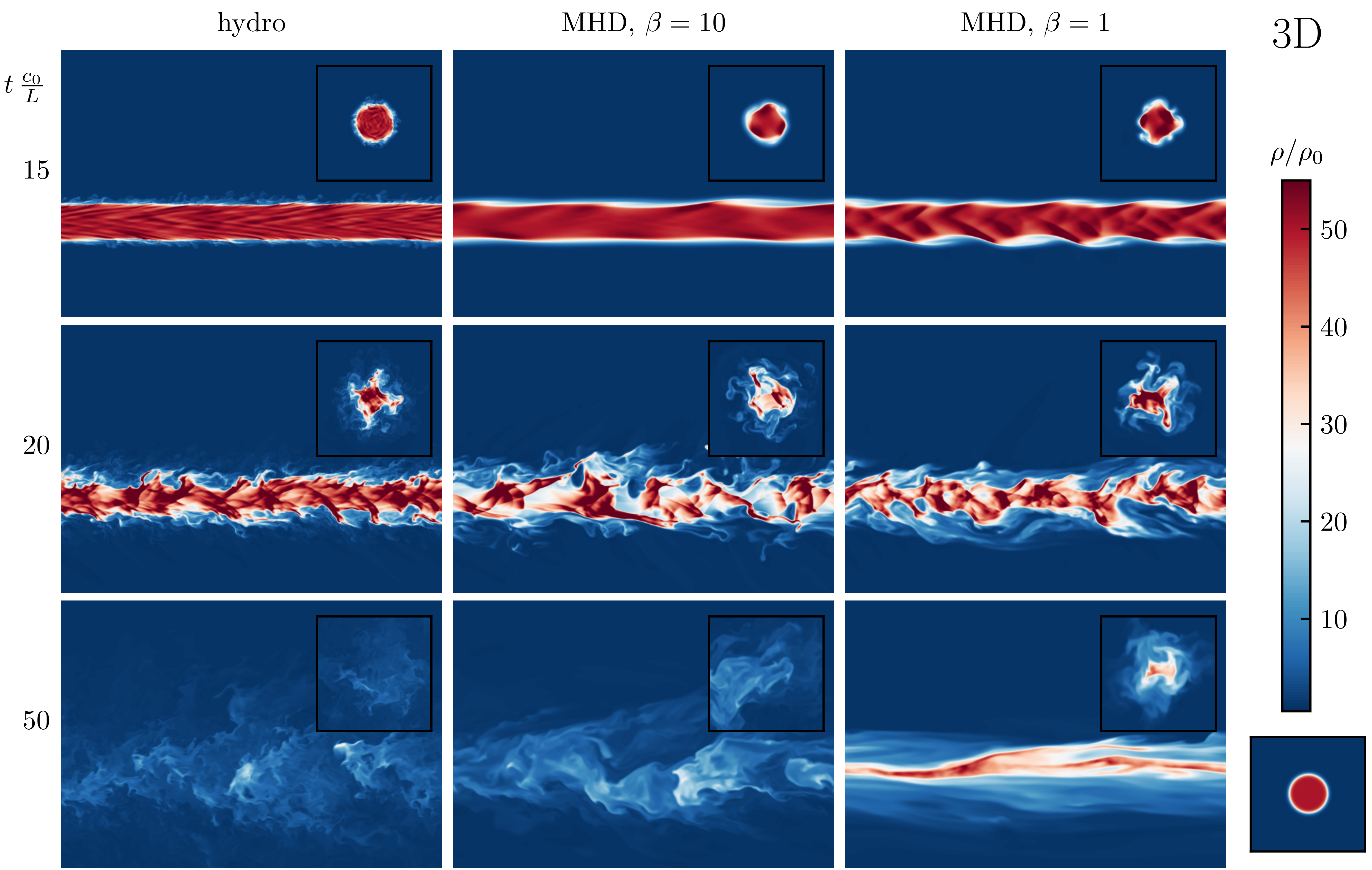


PSECAS

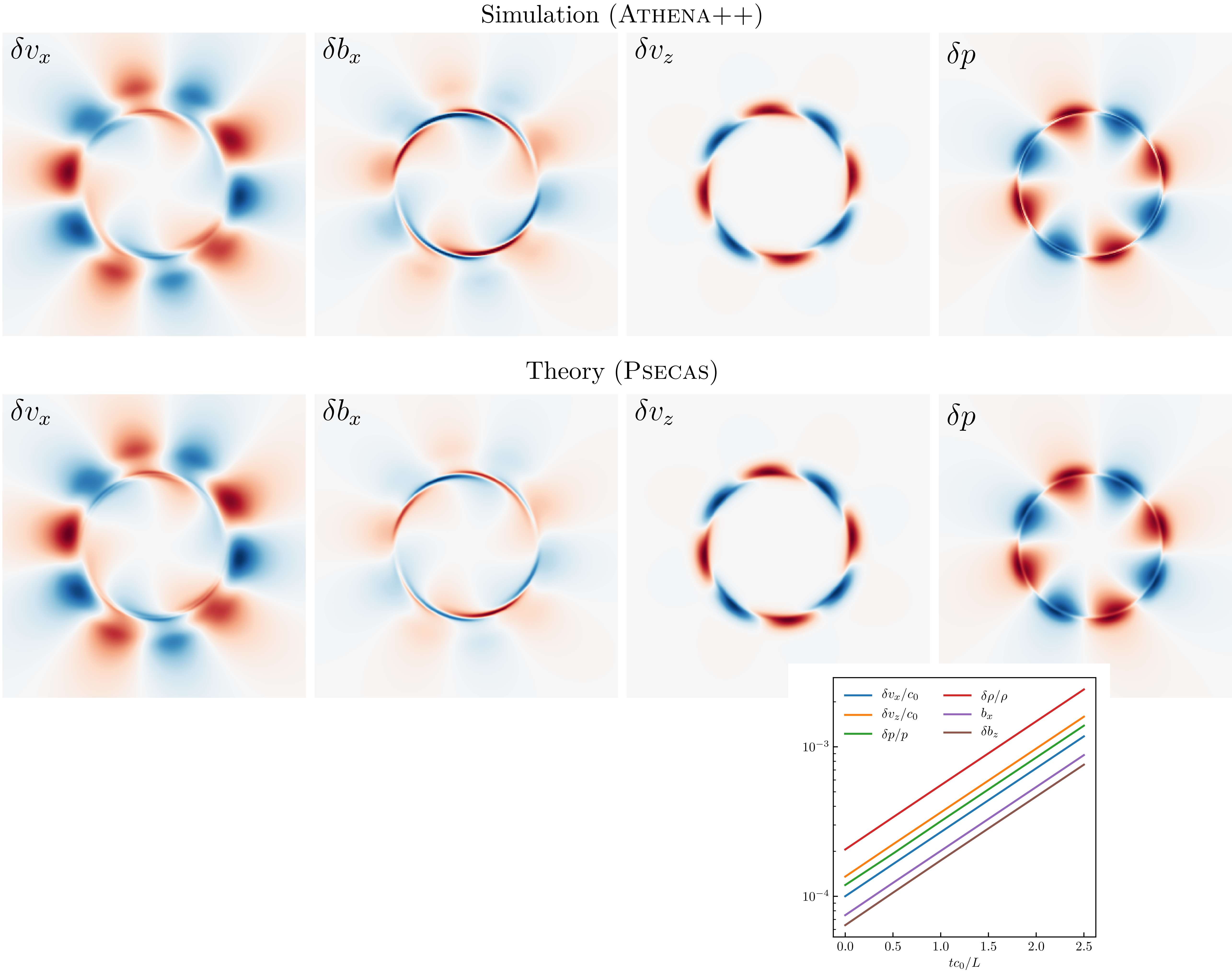


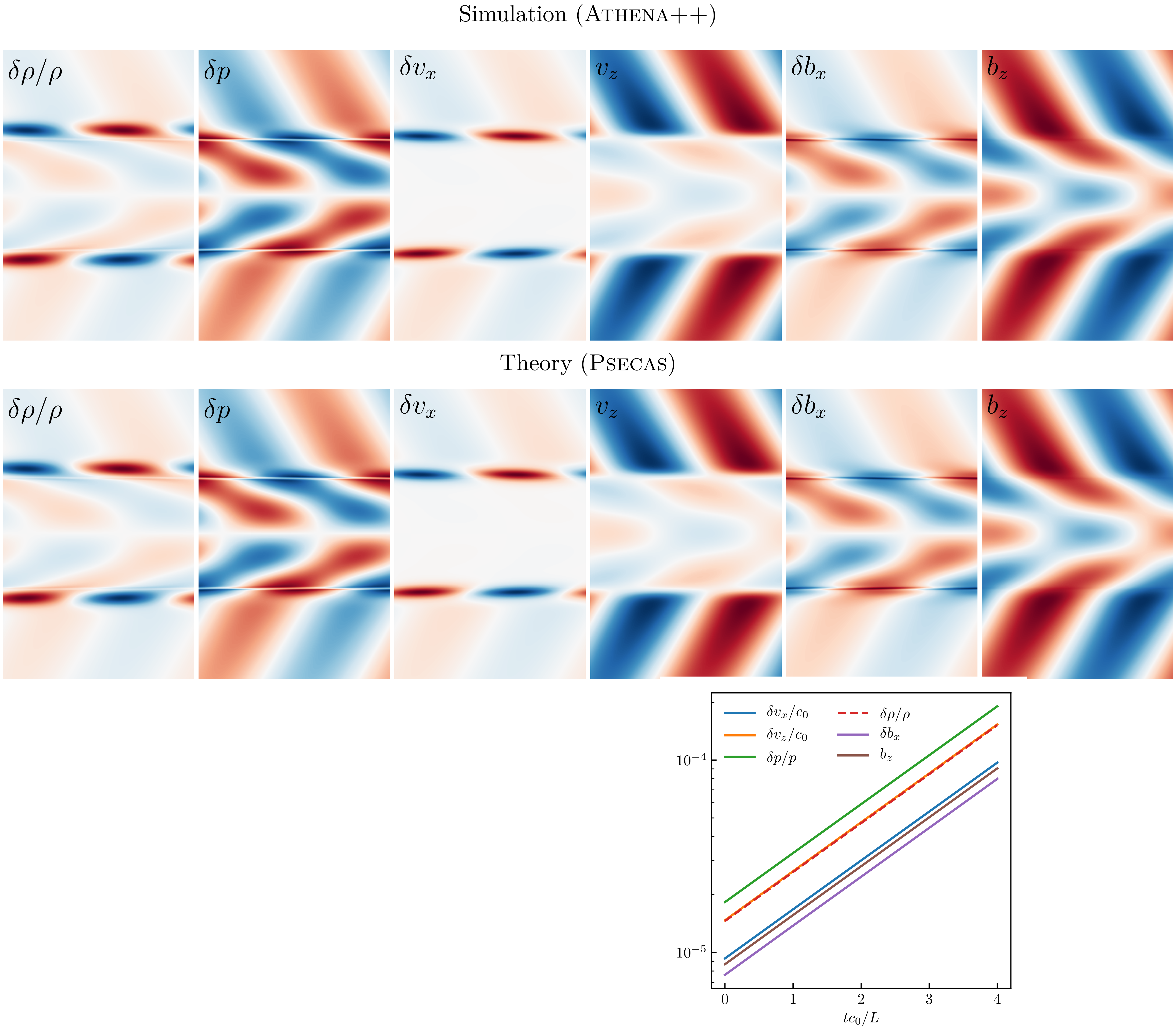
This project has received funding from the European Research Counsil (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No CRAGSMAN–646955).


hydro
MHD, $\beta = 10$
MHD, $\beta = 1$
2D
$t\,\frac{c_0}{L}$
20
40
80
ρ/ρ_0
50
40
30
20
10
L



hydro
MHD, $\beta = 10$
MHD, $\beta = 1$
3D
$t \frac{c_0}{L}$
15
20
50
ρ/ρ_0
50
40
30
20
10


Simulation (Athena++)
δv_x
δb_x
δv_z
δp
Theory (Psecas)
δv_x
δb_x
δv_z
δp
$\delta v_x/c_0$
$\delta v_z/c_0$
$\delta p/p$
$\delta \rho/\rho$
b_x
δb_z
10^{-3}
10^{-4}
0.0
0.5
1.0
1.5
2.0
2.5
tc_0/L


Simulation (Athena++)
$\delta\rho/\rho$
δp
δv_x
v_z
δb_x
b_z
Theory (Psecas)
$\delta\rho/\rho$
δp
δv_x
v_z
δb_x
b_z
$\delta v_x/c_0$
$\delta v_z/c_0$
$\delta p/p$
$\delta\rho/\rho$
δb_x
b_z
10^{-4}
10^{-5}
0
1
2
3
4
tc_0/L