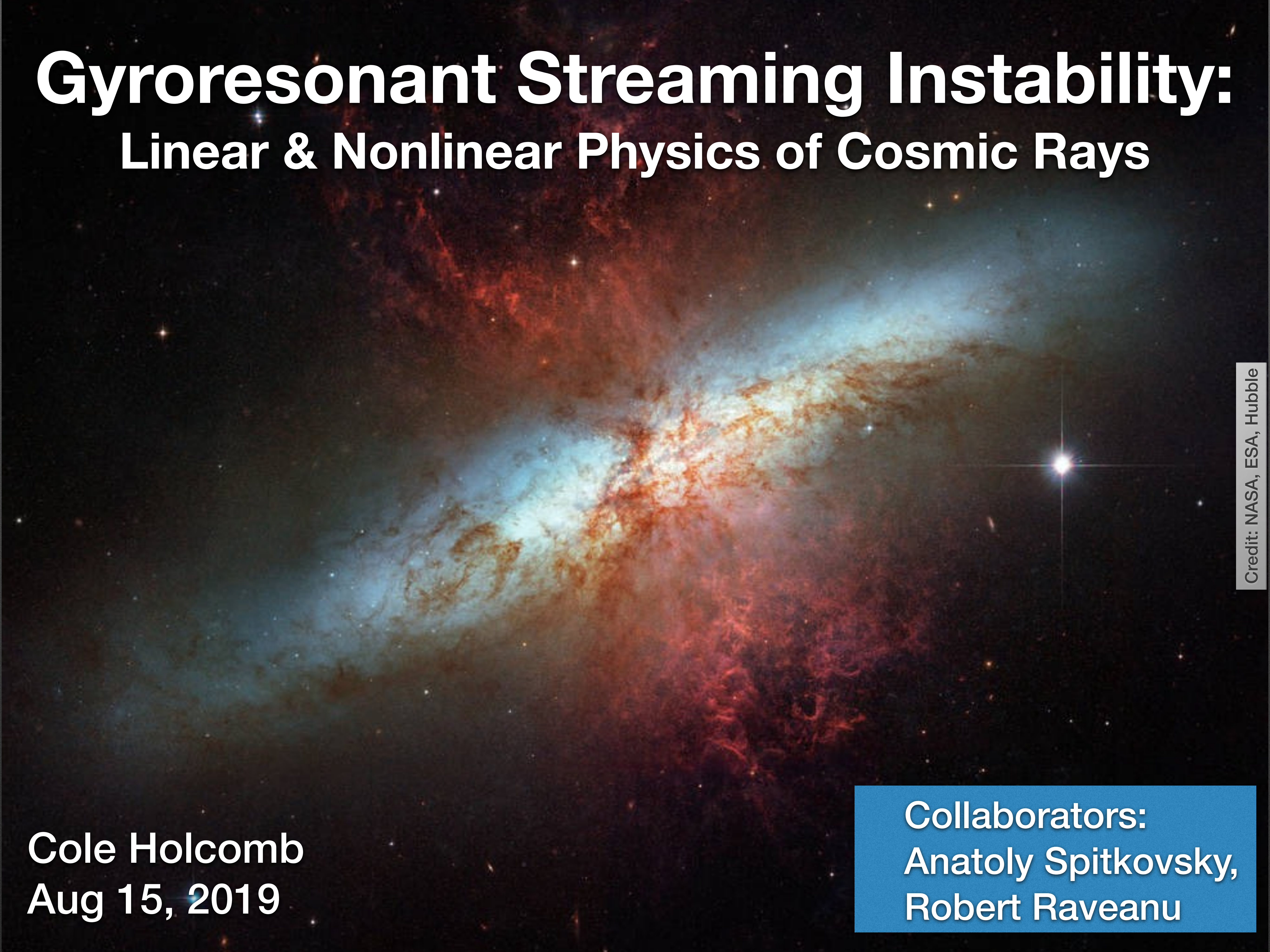
Gyroresonant Streaming Instability:
Linear & Nonlinear Physics of Cosmic Rays



Cole Holcomb
Aug 15, 2019

Collaborators:
Anatoly Spitkovsky,
Robert Raveanu

Life story of a CR

Acceleration

Confinement

Escape from the Galactic disk/ CR Halo

Point Source

Anisotropic Expansion

????
????

Isotropic/Fluid Transport

How do CR distributions evolve to isotropy from anisotropic initial conditions?

CR Self-Confinement

- CRs originate from point sources, but are somehow isotropized
- Need fluctuations to scatter on

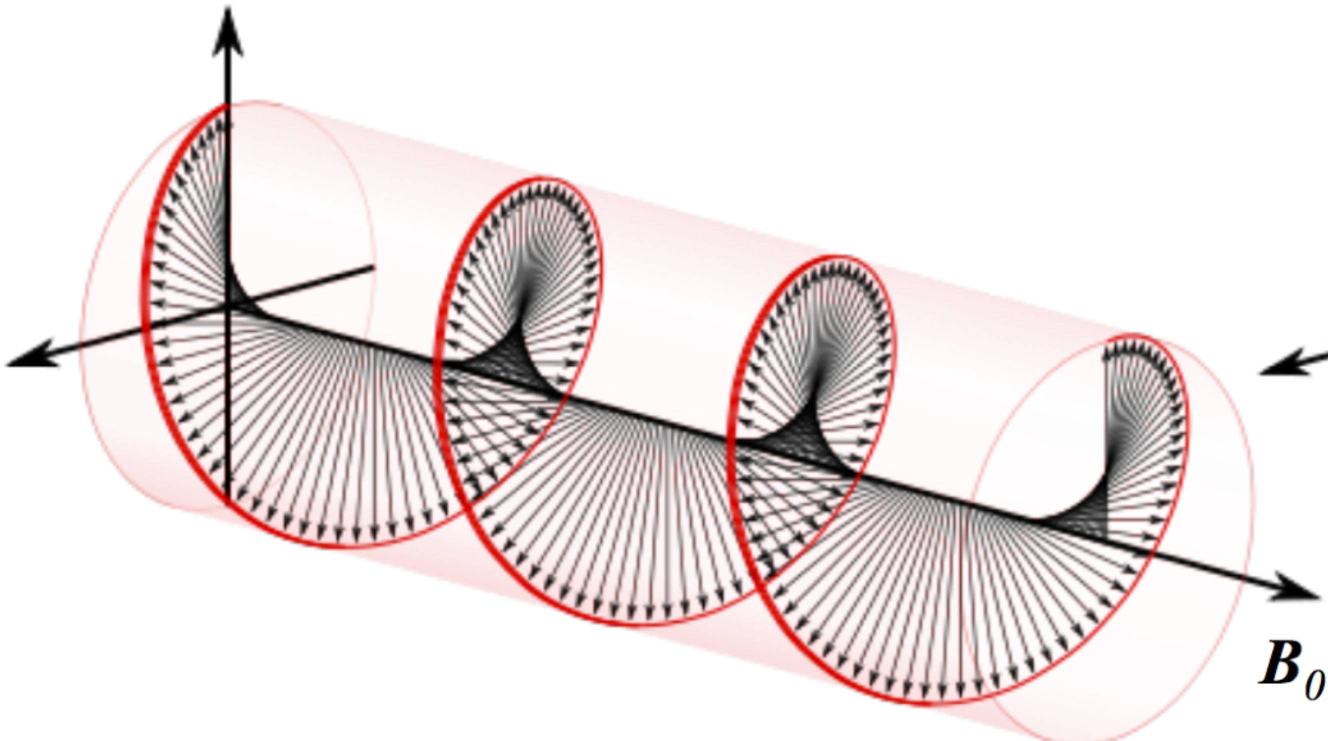
Solution: **Self-Confinement Paradigm via Gyroresonant Streaming Instability**

CRs create the waves that they subsequently scatter on

Wentzel 68, **Kulsrud** & Pearce 69

Resonant interactions with Alfvén waves

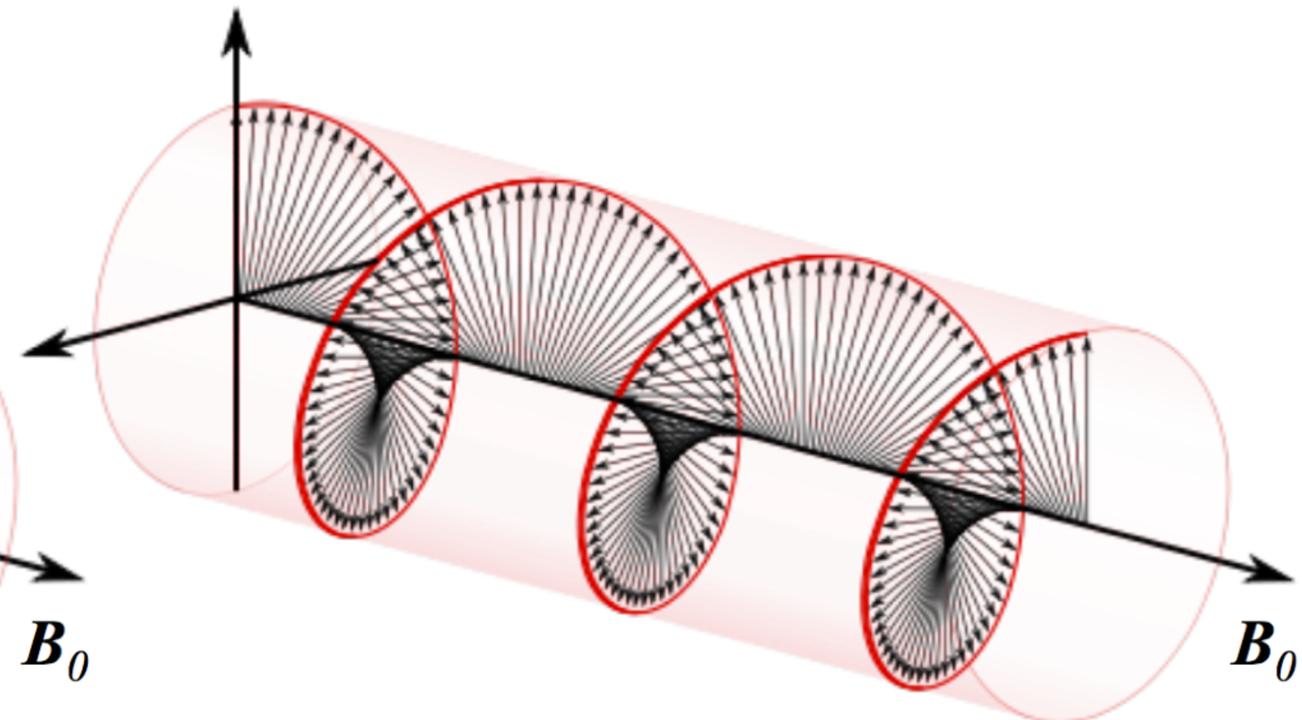
Left polarization: (right/positive helicity)



Resonant with backward-traveling ions.

(In the wave frame)

Right polarization: (left/negative helicity)



Resonant with forward-traveling ions.

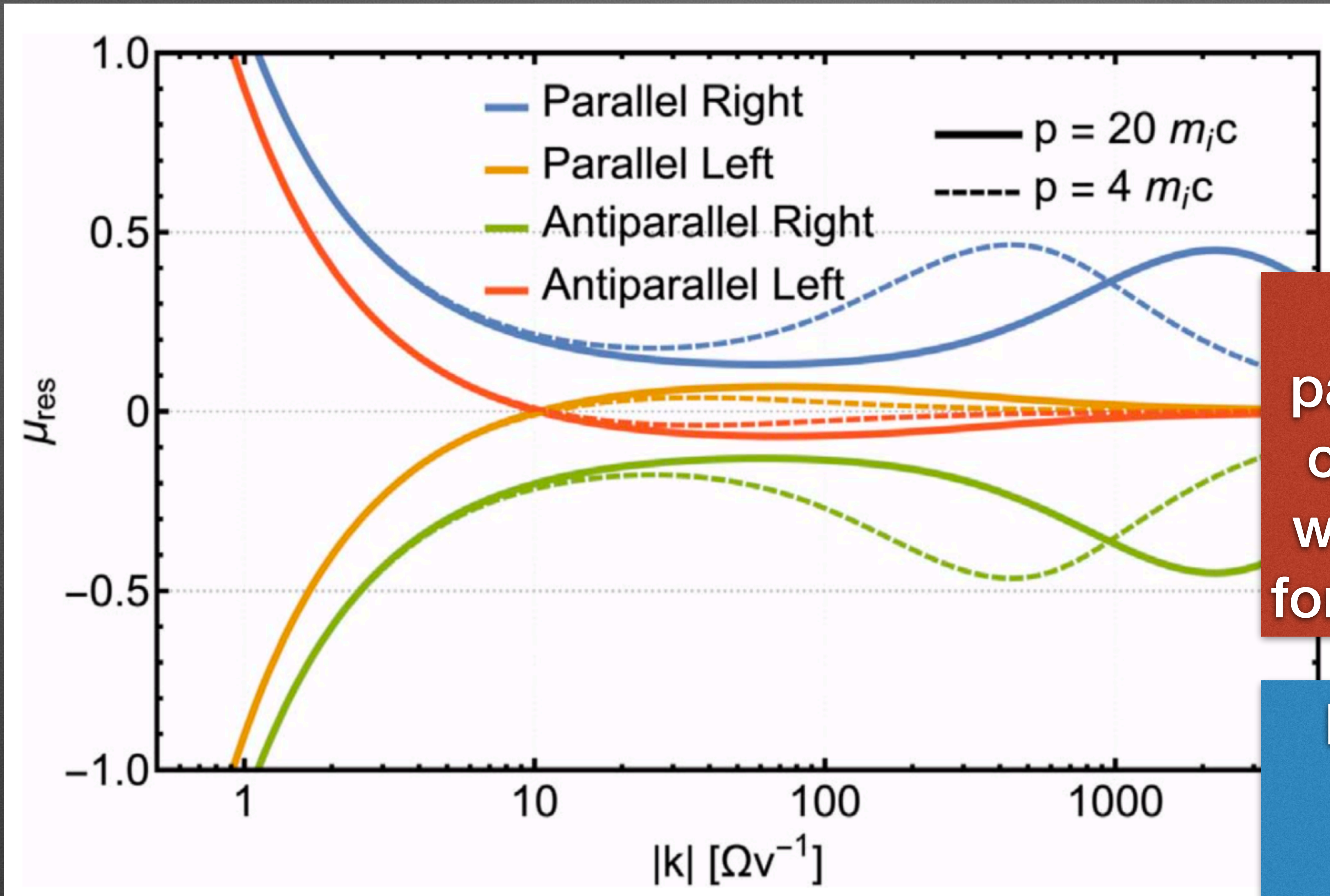
(In the wave frame)

Gyro resonance: $\omega - kv_z = \pm\Omega$

Doppler-Shifted Wave Frequency = Gyrofrequency

~~In general~~, $\omega << \Omega$: $v_z = \pm\Omega/k$

Slide adapted from X. Bai



For fixed momentum p

A spectrum of parallel-propagating circularly polarized waves is insufficient for achieving isotropy

Linearly polarized spectra are necessary

$$\mu_{\mathrm{res}}(k, p) = \frac{v_{\mathrm{ph}}(k)}{v(p)} - \frac{\Omega_0}{k\gamma(p)v(p)}$$

$$\mu \equiv \frac{\vec{p} \cdot \vec{B}}{|\vec{p}||\vec{B}|}$$

CR Streaming Instability: Linear Theory

$$\mu \equiv \frac{\vec{p} \cdot \vec{B}}{|\vec{p}||\vec{B}|}$$

$$\Gamma_{\rm cr}^{\pm}(k) = \frac{\pi^2 q^2}{2} \frac{v_A^2}{c^2} \sum_{\pm} \iint \delta\big(\omega - k\mu v \pm \Omega(p)\big) \left(\frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu \right) \frac{1}{p} \frac{\partial f}{\partial \mu} \right) v p^2 (1 - \mu^2) dp d\mu$$

Left and Right Polarization

Gyroresonance Condition

CR anisotropy

Physical picture: unstable waves are “pushed” by net force of CR anisotropy

CR Streaming Instability: Linear Theory

$$\Gamma_{\rm cr}(k) = \frac{1}{2}\frac{\pi}{4}\frac{\alpha-3}{\alpha-2}\frac{n_{\rm cr}}{n_i}\Omega_0\left(\frac{v_{\rm dr}}{\omega/k}-1\right)$$

"Streaming"

Assuming:

1. Power-law CRs
2. Small bulk drift v_{dr}
3. Distribution to infinite p
4. Wave frequency << Gyrofrequency

Notes:

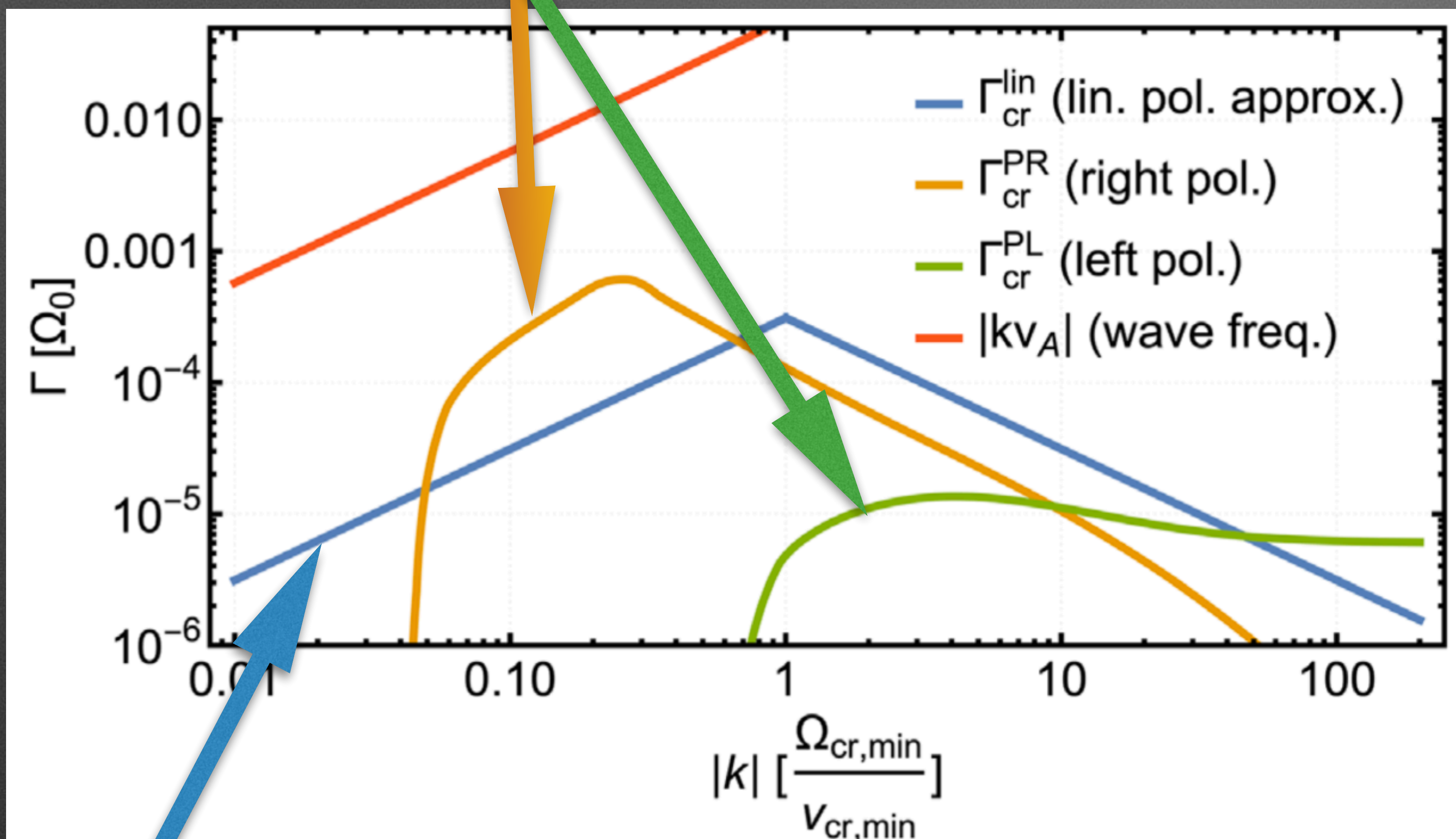
1. Instability requires $v_{dr} > v_A$
2. Polarization degeneracy

In general: solve the full growth rate integral numerically

Power-Law CR Dispersion Relation

Full numerical solution

$$\Gamma^{\pm}_{\rm cr}(k) = \frac{\pi^2 q^2}{2}\frac{v_A^2}{c^2}\sum_{\pm}\iint \delta(\omega - k\mu v \pm \Omega(p))\left(\frac{\partial f}{\partial p} + \left(\frac{kv}{\omega}-\mu\right)\frac{1}{p}\frac{\partial f}{\partial \mu}\right) v p^2(1-\mu^2)\,dp\,d\mu$$



$$\Gamma_{\rm cr}(k) = \frac{1}{2}\frac{\pi}{4}\frac{\alpha-3}{\alpha-2}\frac{n_{\rm cr}}{n_i}\Omega_0\left(\frac{v_{\rm dr}}{\omega/k}-1\right)$$

Approximate analytical solution

Typical PIC-friendly parameters

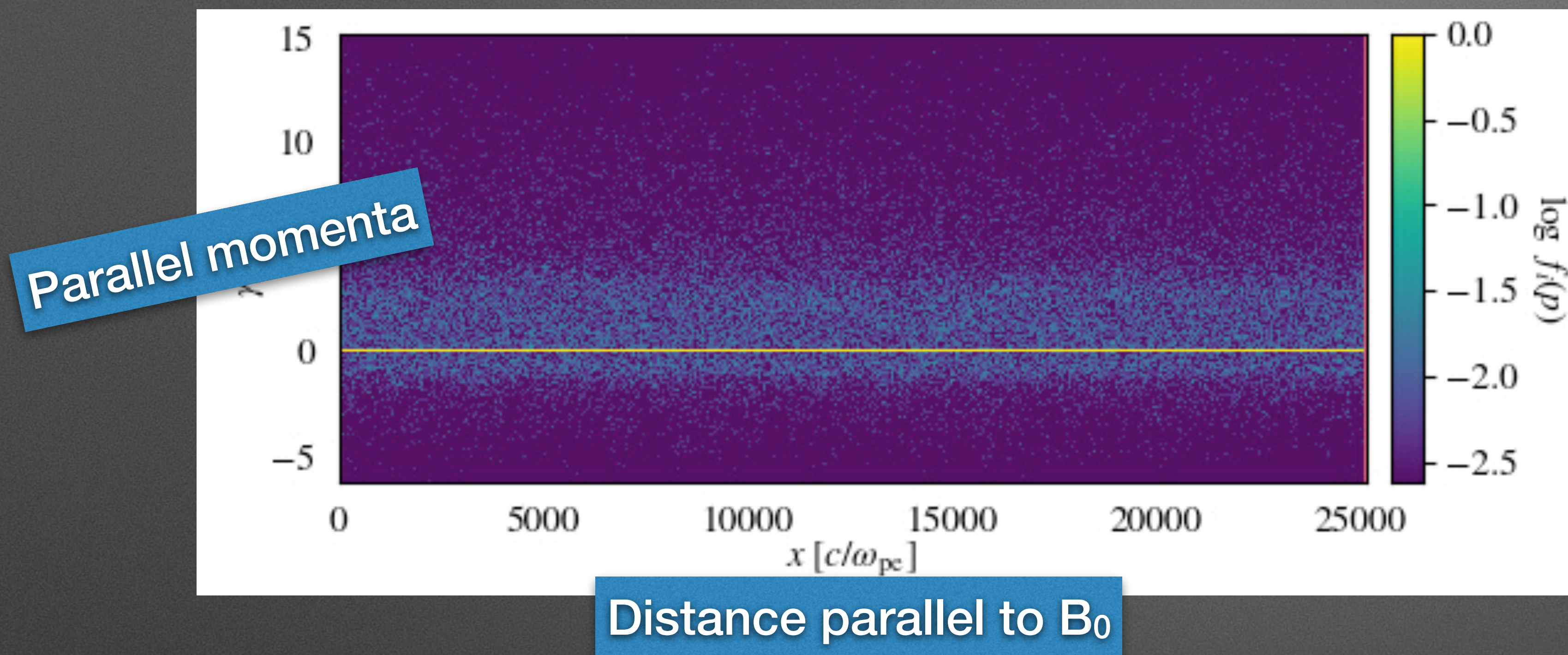
$n_{cr} = 2*10^{-4}\ n_i$

$v_D/v_A = .8c/.1c$

$\gamma = [2, 10]$

High anisotropy breaks degeneracy of left- and right-handed modes' growth rates

PIC Simulations

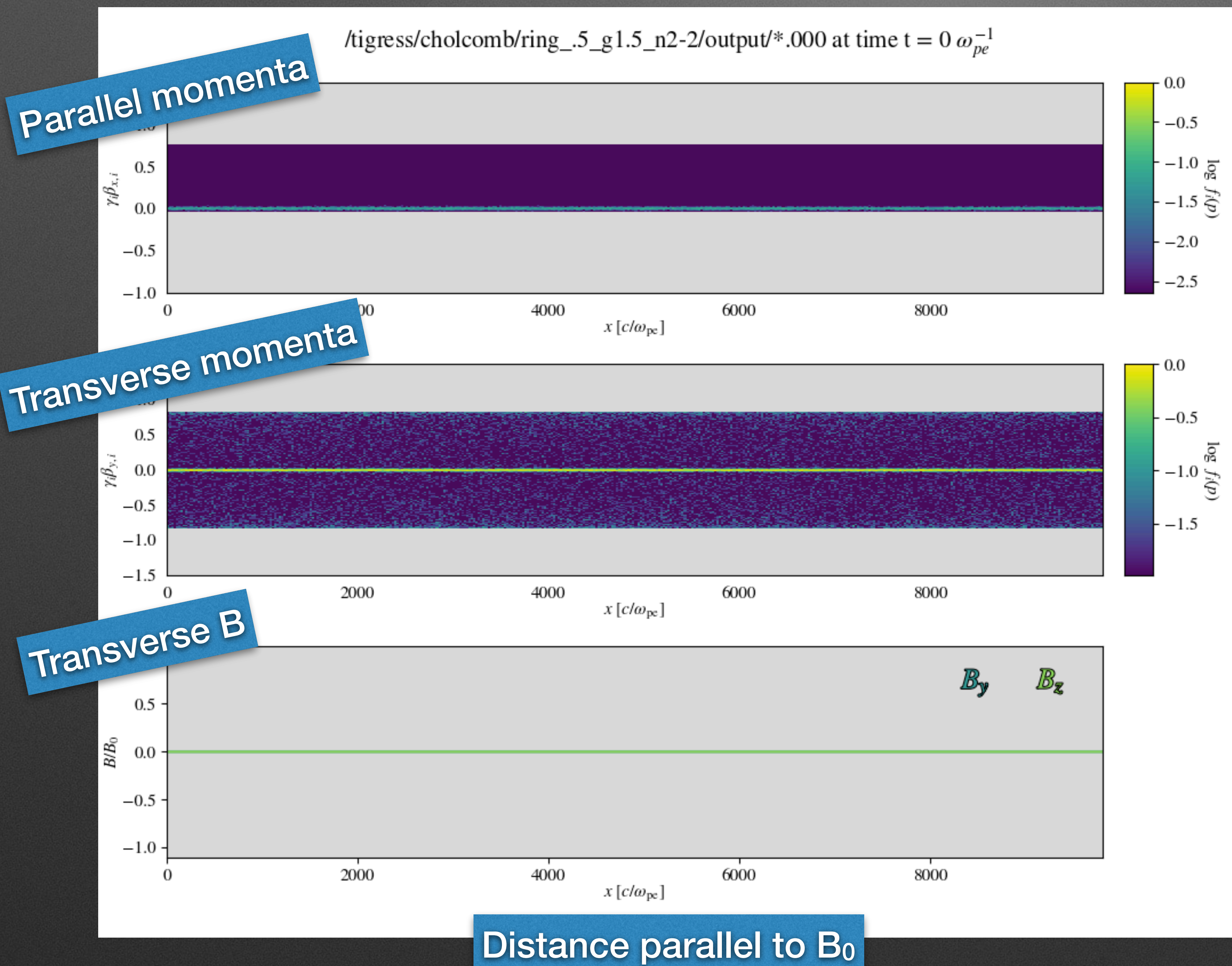


Example Initial Condition: Phase Space Density For Power-Law CRs

- 1D3V periodic particle-in-cell
- Cold Maxwellian background plasma
- $B_0 \parallel x$
- CRs with $v_{dr}/v_A > 1$
- $n_{cr} = \sim 10^{-4} - \sim 10^{-2}\ n_i$

Two families of CR distribution: Gyrotropic-Ring Distribution and Power-Law Distribution

Ring CR Distribution

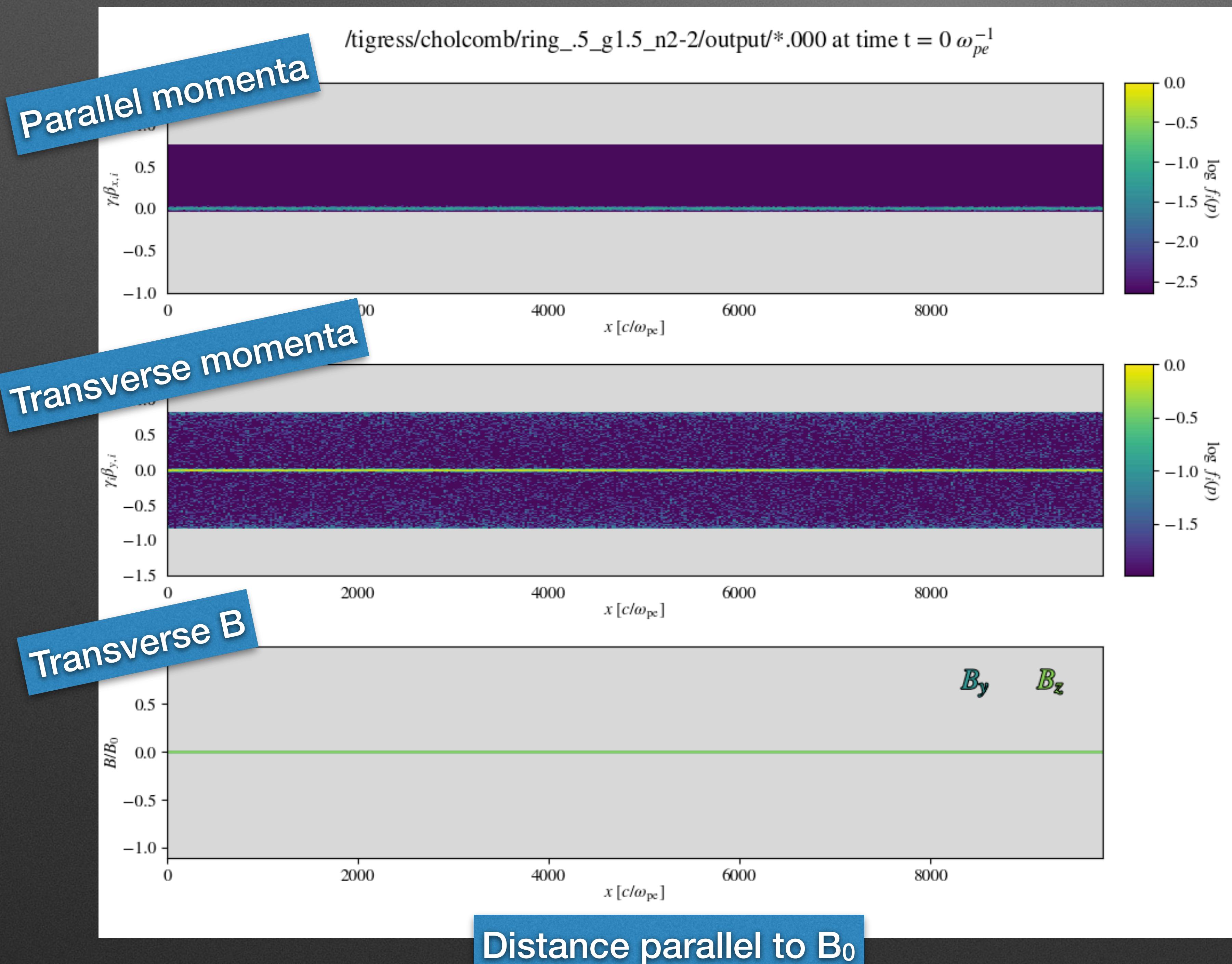


$$f_{\rm ring}(p,\mu) = \frac{n_{\rm cr}}{2\pi p^2}\delta(p-p_0)\delta(\mu-\mu_0)$$

All CRs occupy the same resonant bands

$$k_{\rm res}(p_0,\,\mu_0) = \frac{-\Omega(p_0)}{\mu_0 v(p_0) - v_{\rm ph}}$$

Ring CR Distribution



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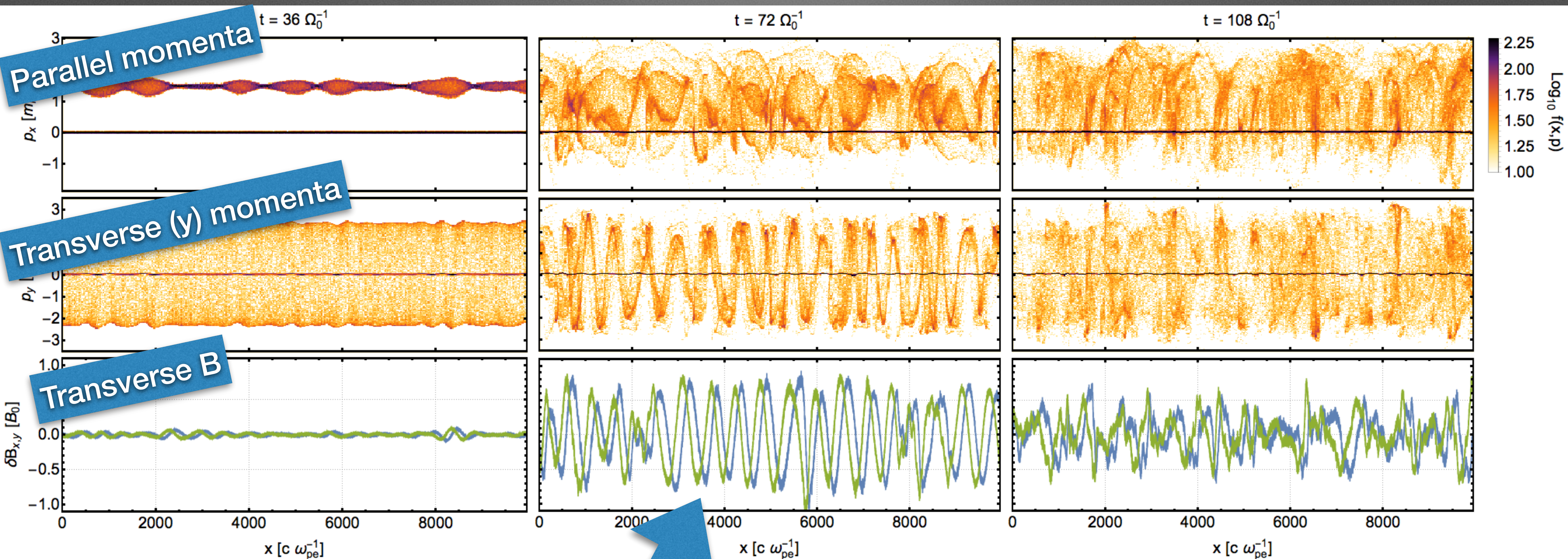
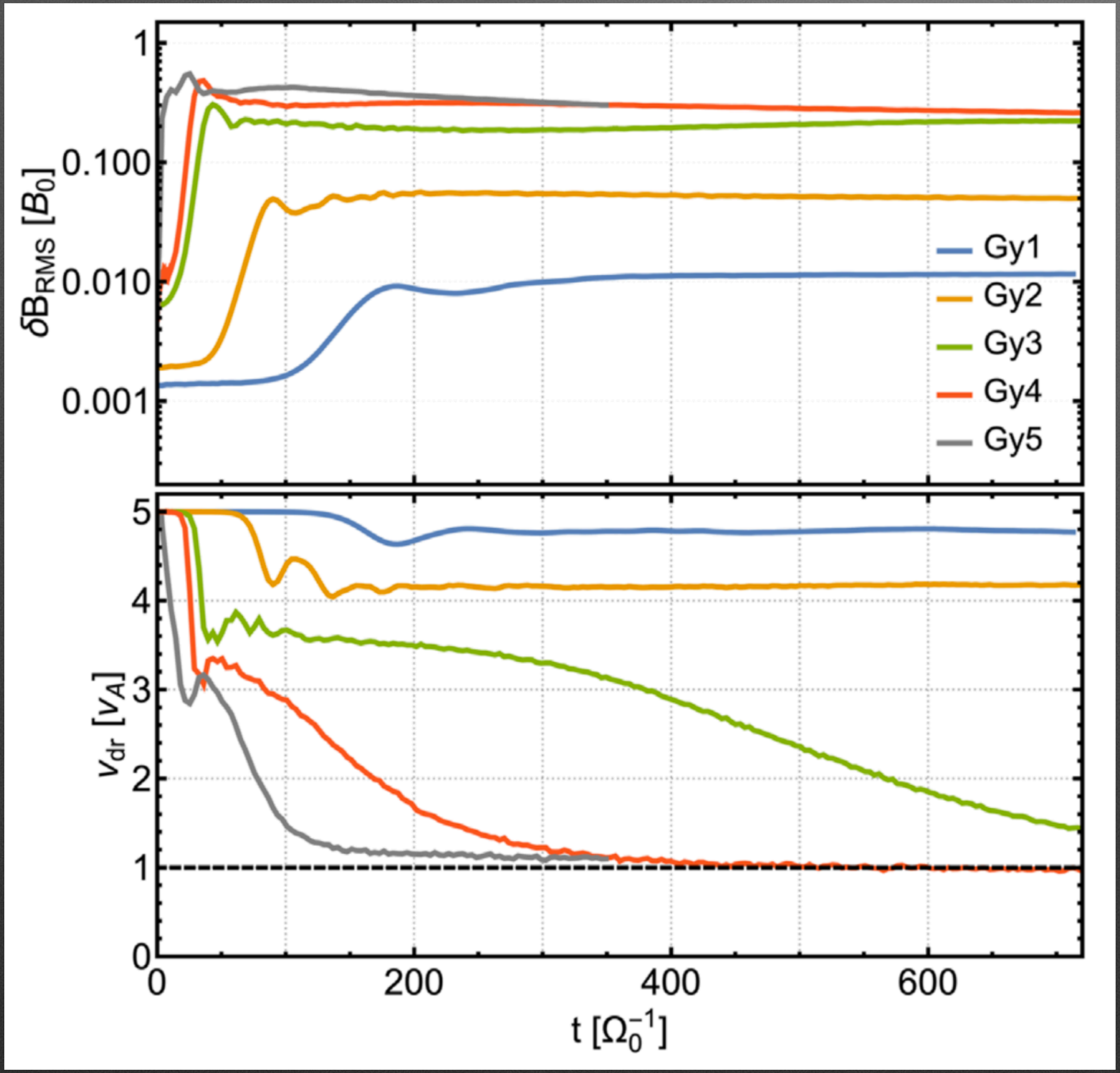
Ring CR Distribution



$$k_{\rm res}(p_0, \mu_0) = \frac{-\Omega(p_0)}{\mu_0 v(p_0) - v_{\rm ph}}$$

Ring CR Drift Evolution

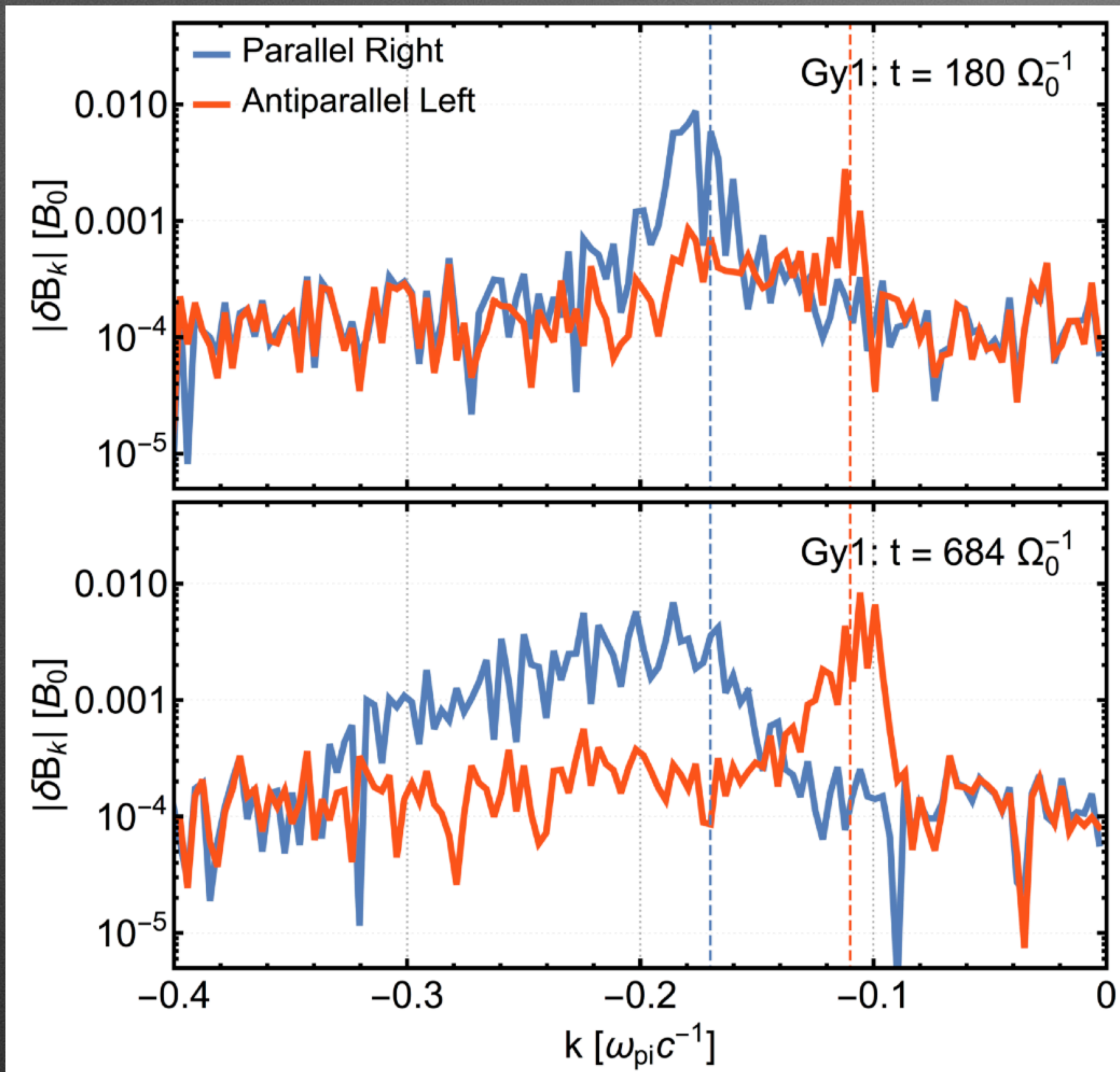


CR drift speed evolution depends on wave spectrum

Large amplitude waves quickly reduce $v_{dr} \Rightarrow v_A$

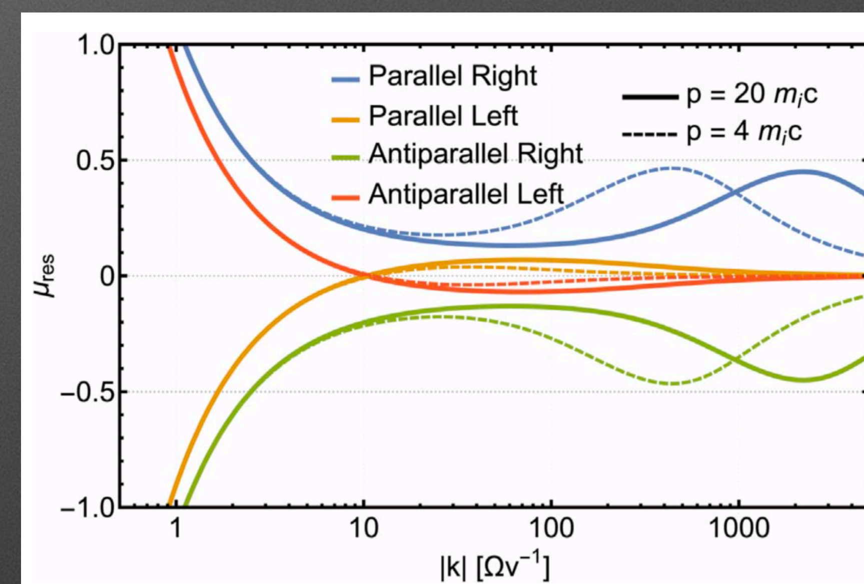
Resonant trapping in small amplitude waves results in indefinite super-Alfvenic drift

Ring Wave Spectra



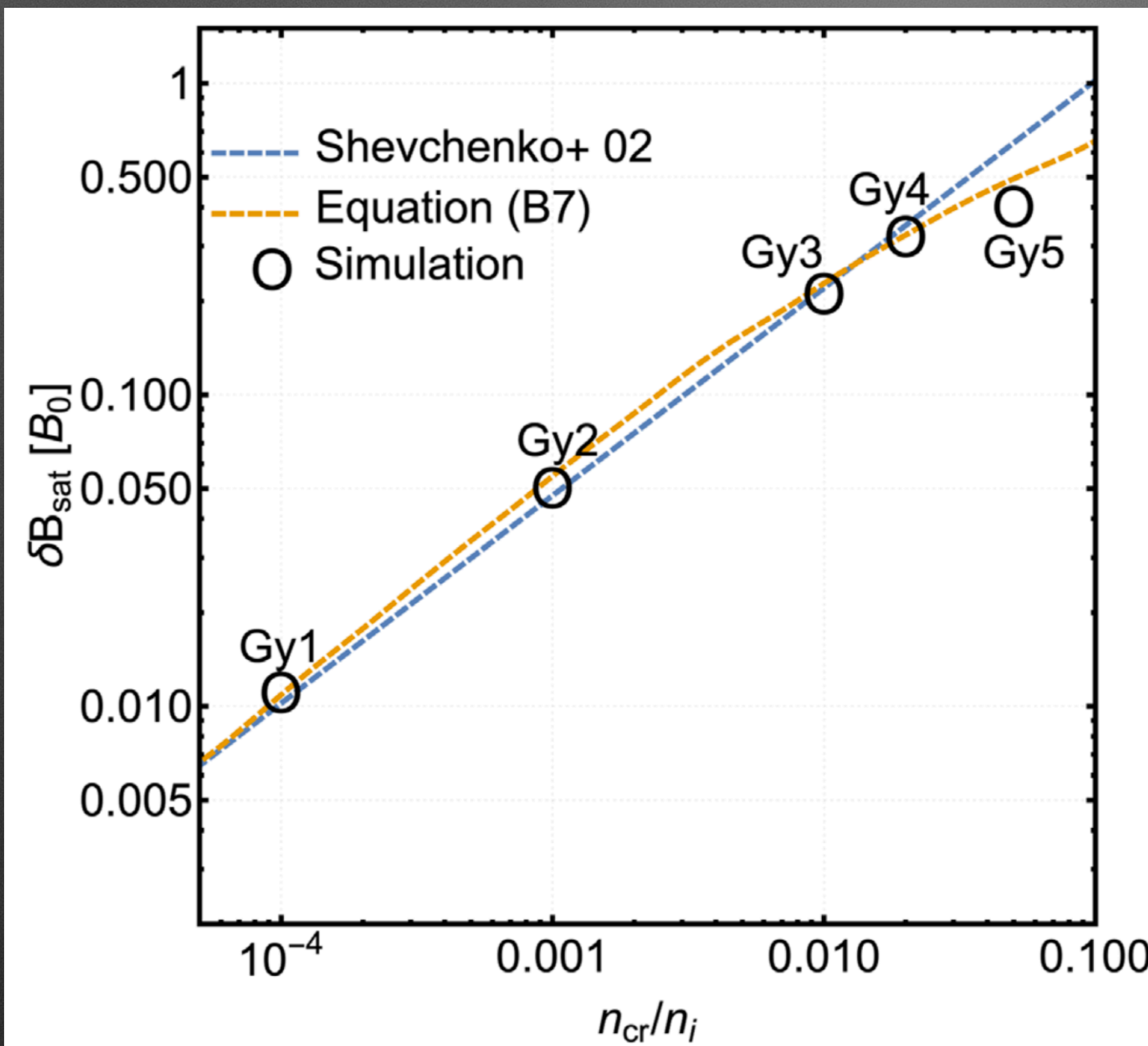
$$k_{\rm res}(p_0, \mu_0) = \frac{-\Omega(p_0)}{\mu_0 v(p_0) - v_{\rm ph}}$$

All CRs are resonant with Parallel-Right and Antiparallel-Left modes



Quasimonochromatic spectrum
➡
Trapped Particle Dynamics

Ring CR Instability Saturation



Growth Rate == Trapping Frequency

$$\Omega_{\rm trap} = \sqrt{\frac{\delta B}{B_0} k v_\perp \Omega},$$

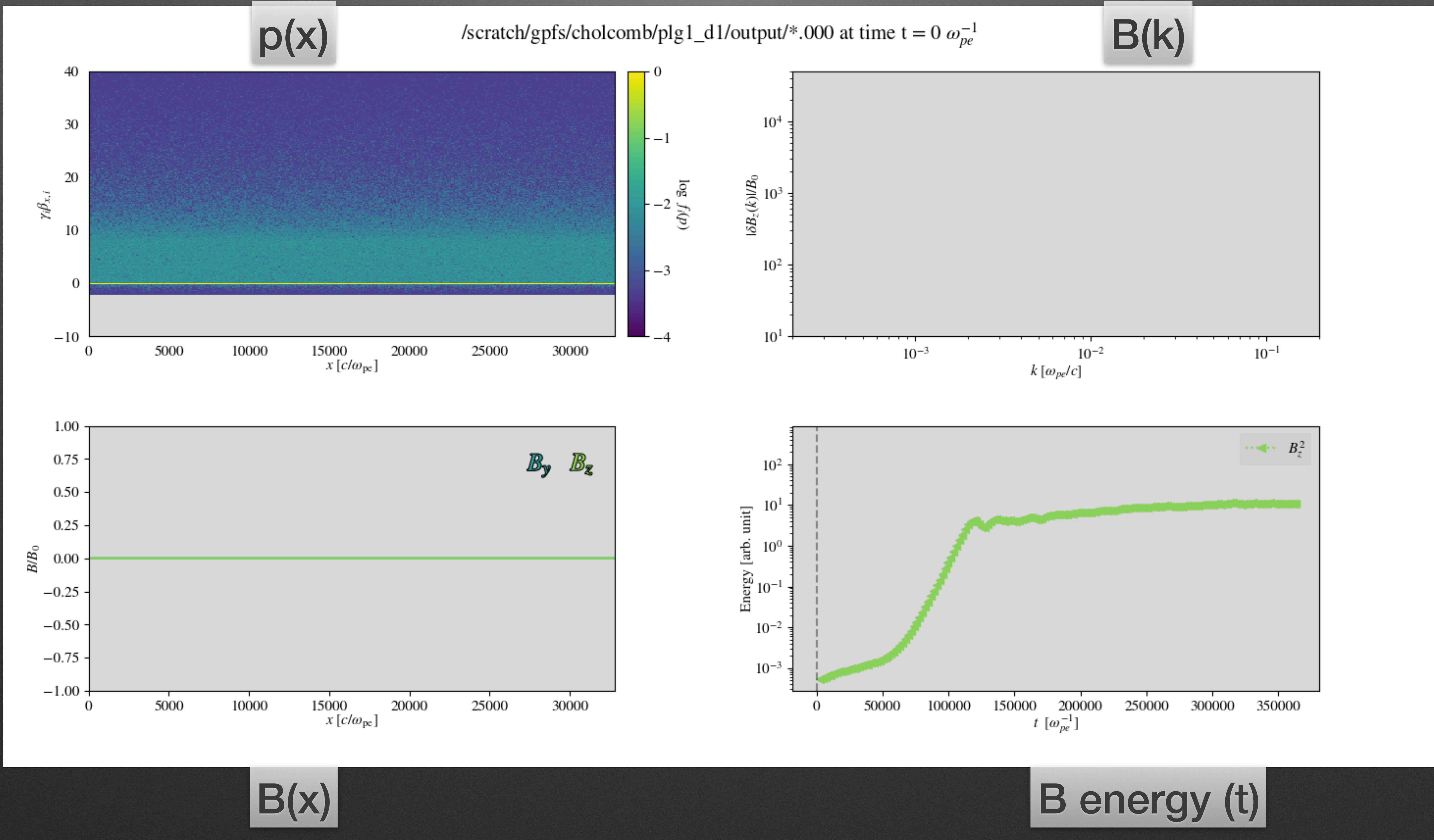
Sudan & Ott 71

$$\left(\frac{\delta B}{B_0}\right)_{\rm trap} \approx \left(\frac{n_{\rm cr}}{n_{\rm i}}\right)^{2/3} \left(\frac{v_\perp (v_{\rm dr,0} - v_{\rm ph})}{v_{\rm ph}^2}\right)^{1/3}$$

Shevchenko+ 02

Power Law Distribution

$n_{cr} = 2*10^{-3}\, n_i$
$v_D/v_A = .8c/.1c$



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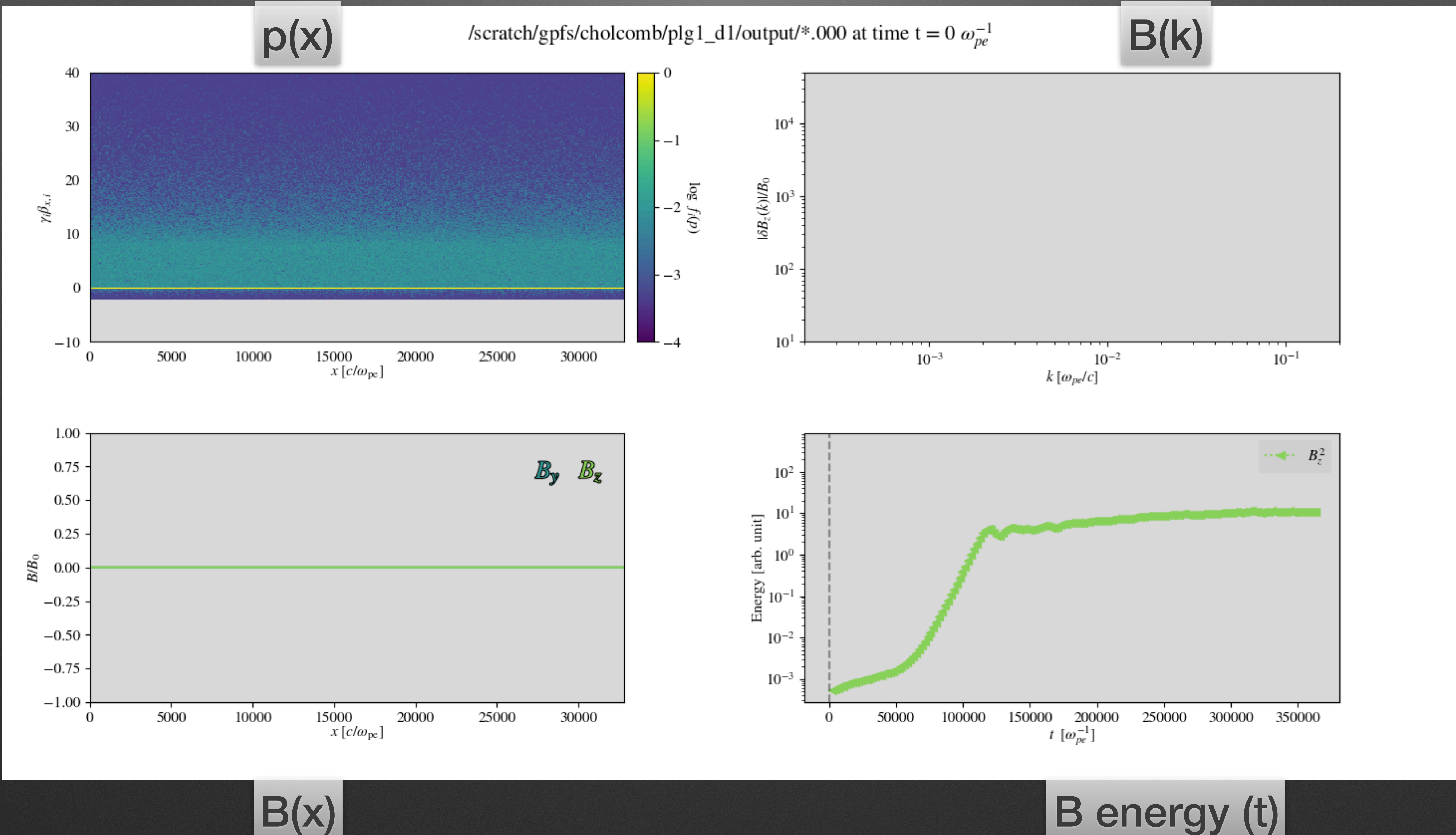
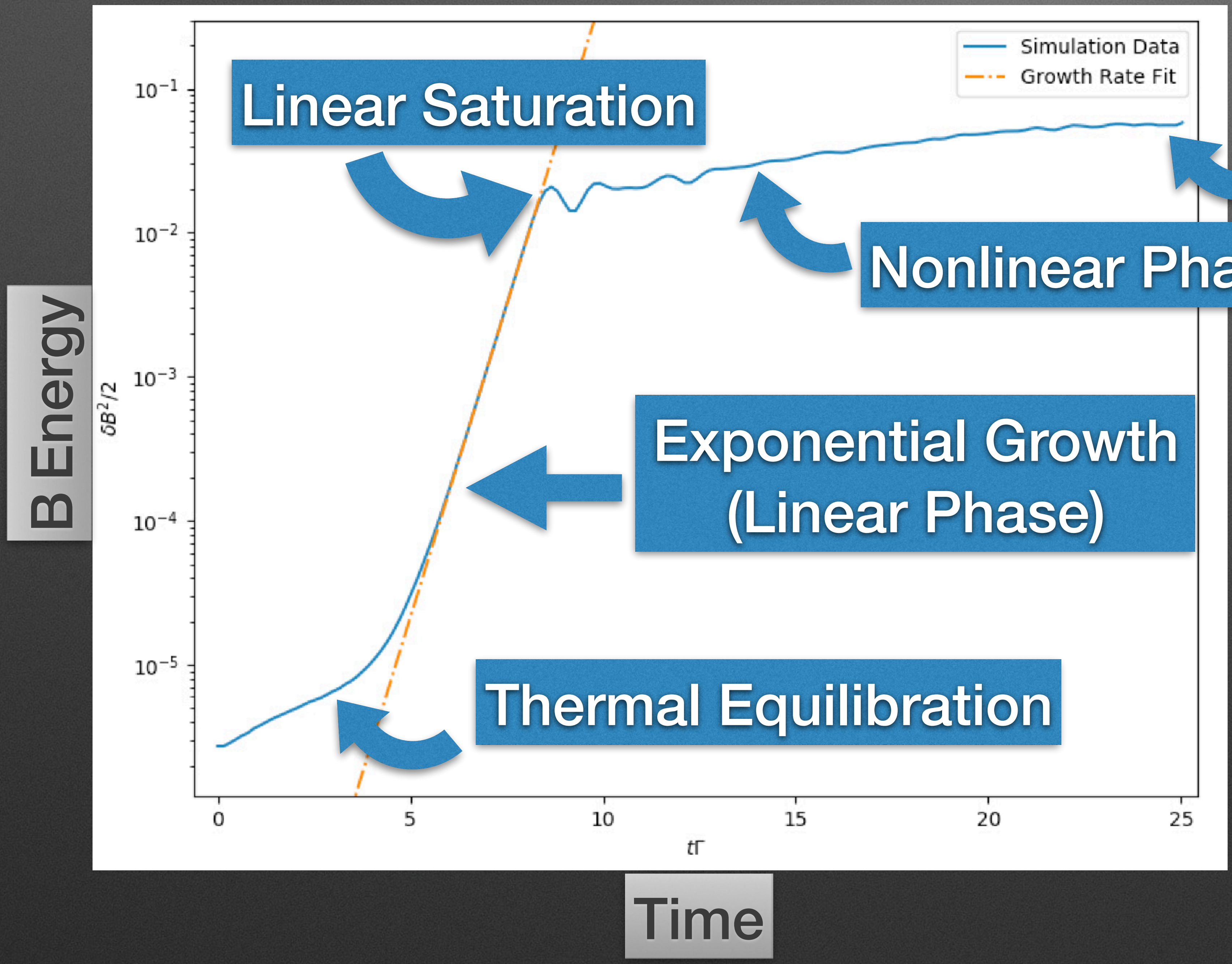


Growth Rate

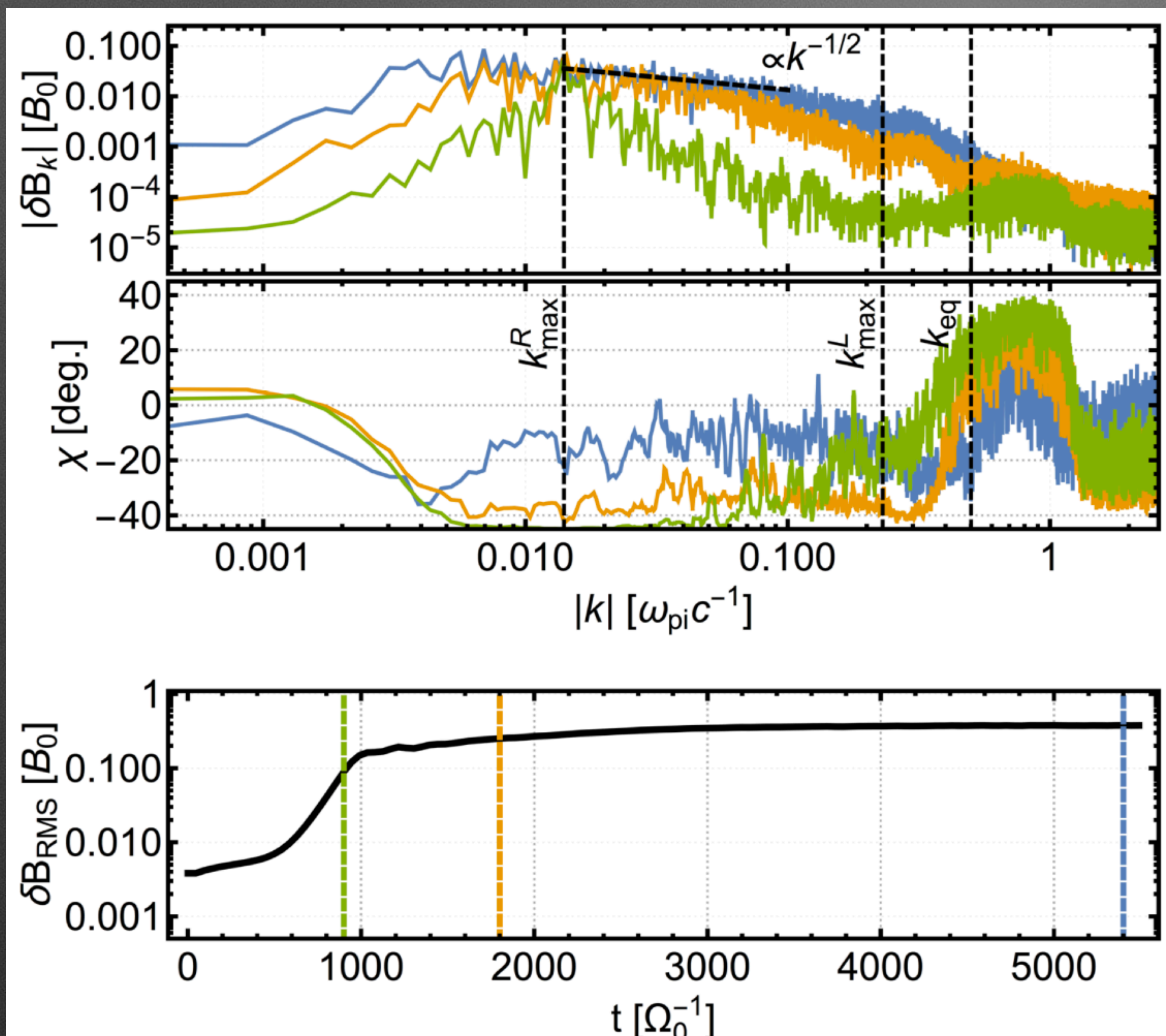
$n_{cr} = 2*10^{-3}\ n_i$
$v_D/v_A = .8c/.1c$



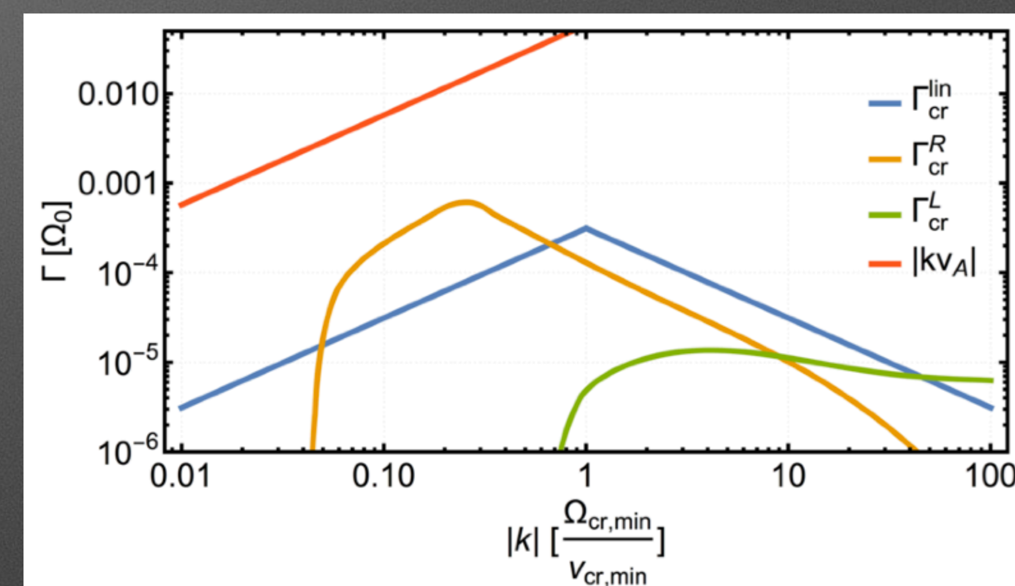
Measured growth rates match expectation to factor of ~2 or better

Power Law Spectra

$n_{cr} = 2{*}10^{-3}\ n_i$
$v_D/v_A = .8c/.1c$

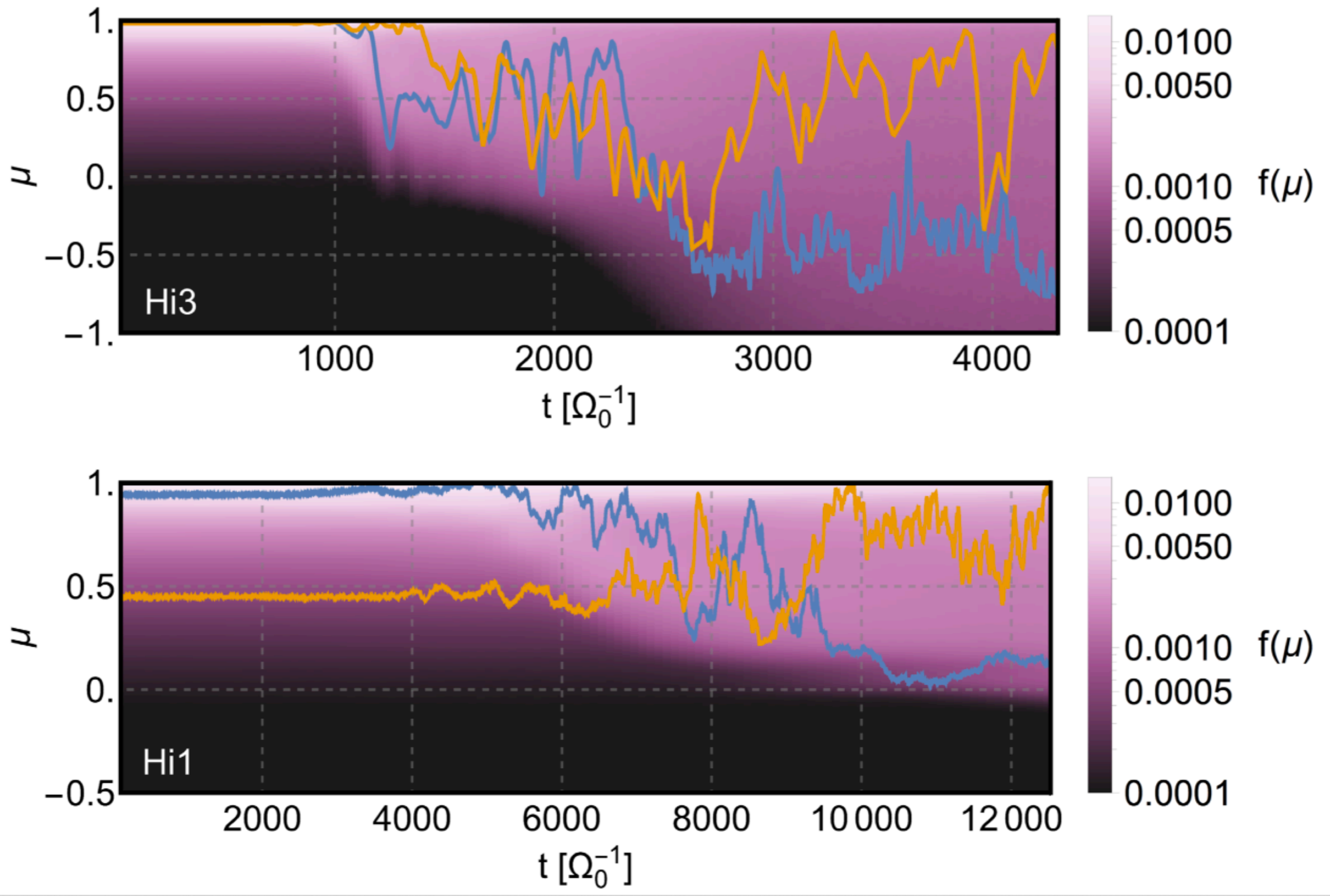


High Anisotropy
➡
Right-Handed Spectrum



Broad Spectrum
➡
Pitch-Angle Diffusive Particle Dynamics

Power-Law Distribution Evolution

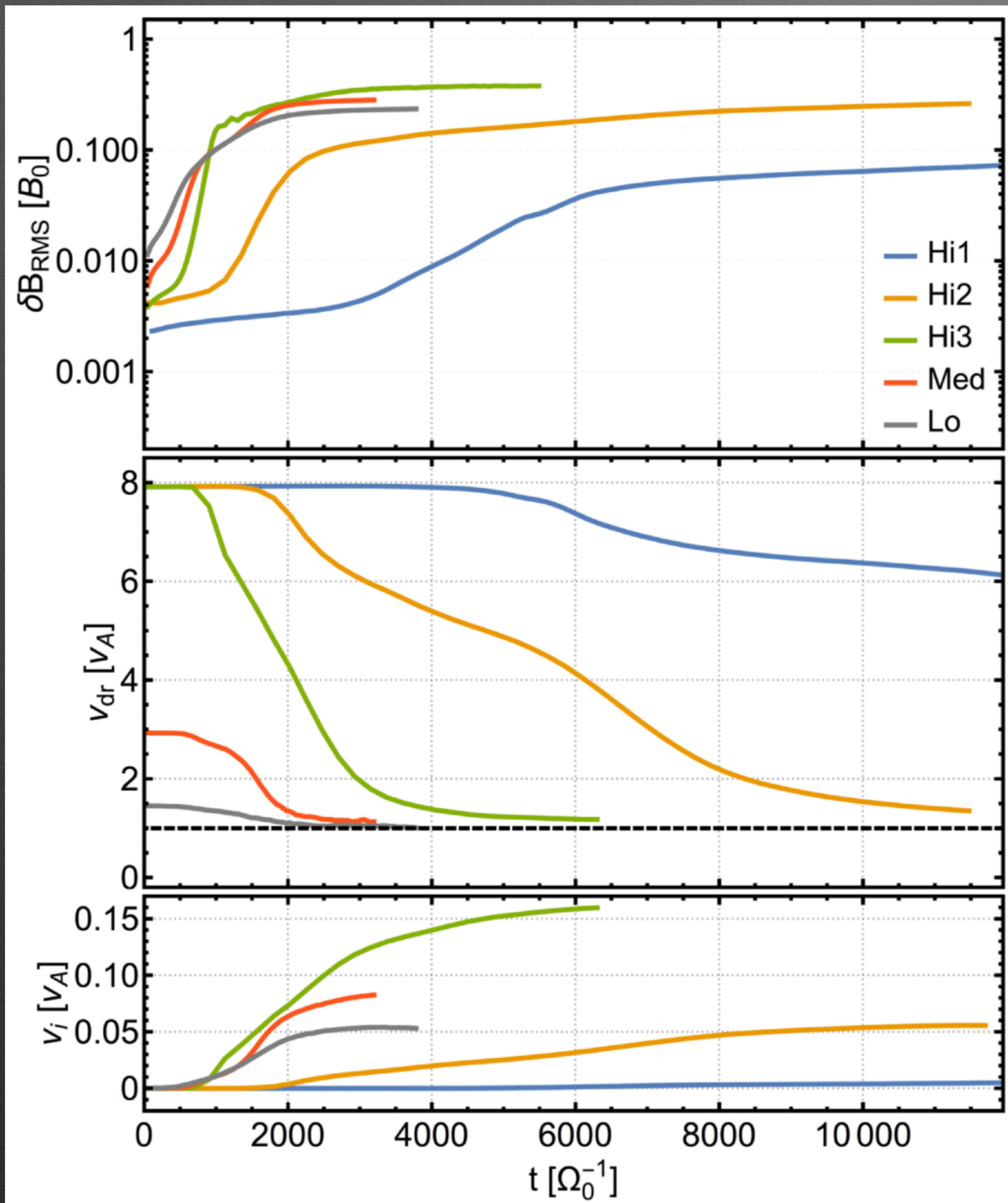


High CR Density
dB/B ~ 0.3

Low CR Density
dB/B ~ 0.1

Isotropy is not achieved unless left-handed modes are generated

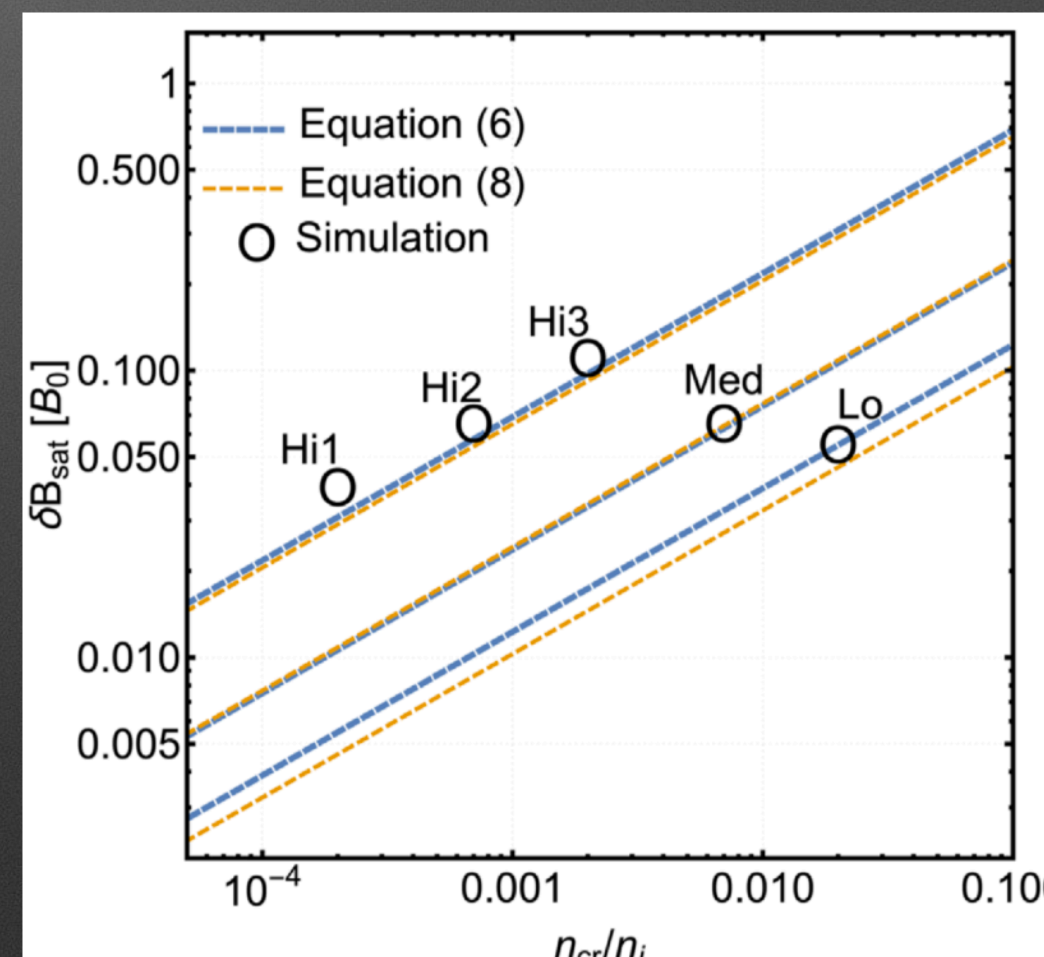
Power-Law Wave Evolution



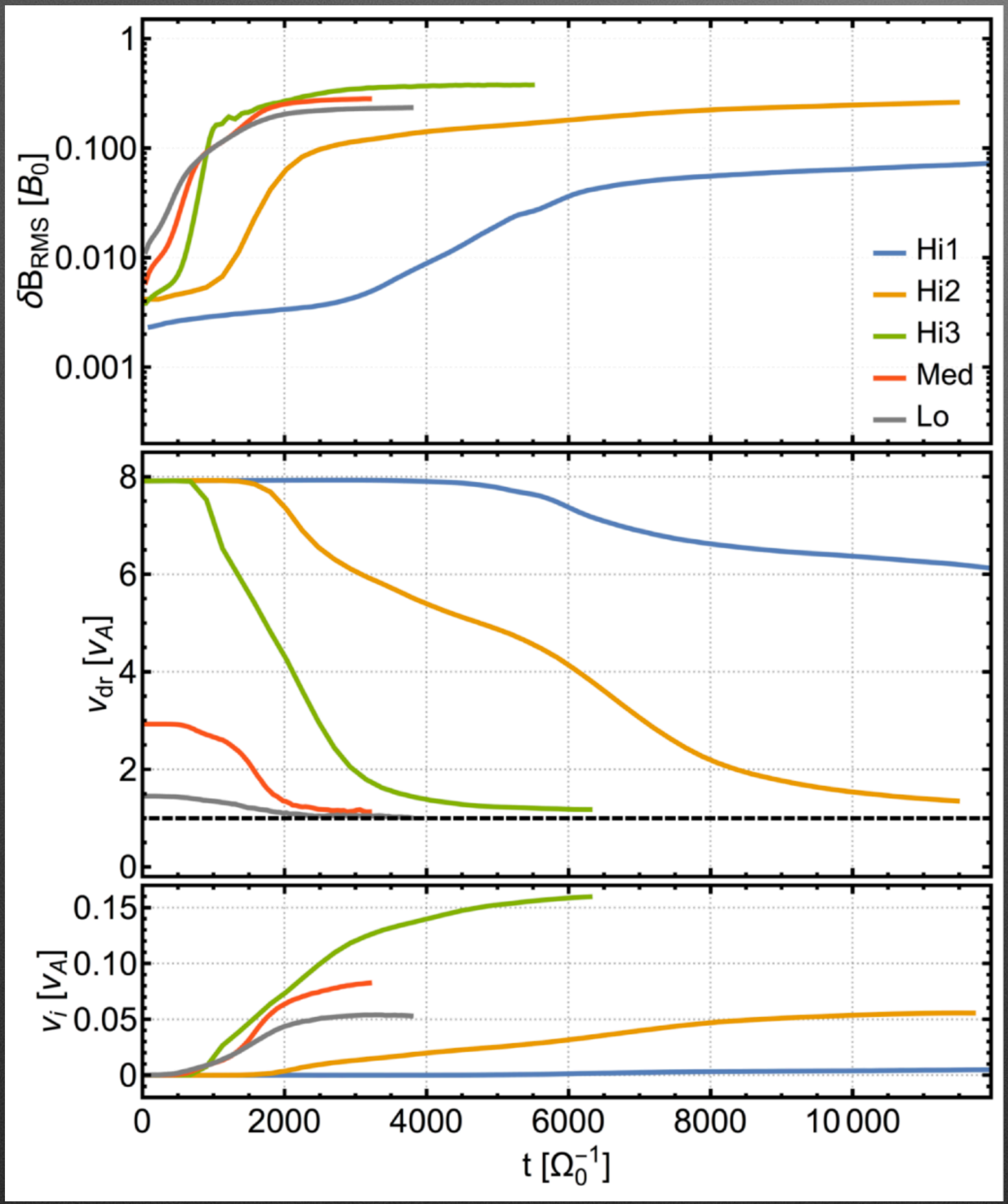
Linear Phase Saturation:

$$\nu_{\rm QLT} \approx \Gamma_{\rm cr}^{\rm lin}$$

$$\left(\frac{\delta B}{B_0}\right)_{\rm diff} \approx \sqrt{\gamma_{\rm dr}\gamma_{\rm cr}\frac{n_{\rm cr}}{n_i}\left(\frac{v_{\rm dr}}{v_{\rm A}}-1\right)}$$



Power-Law Drift Evolution



Linear Phase Saturation:

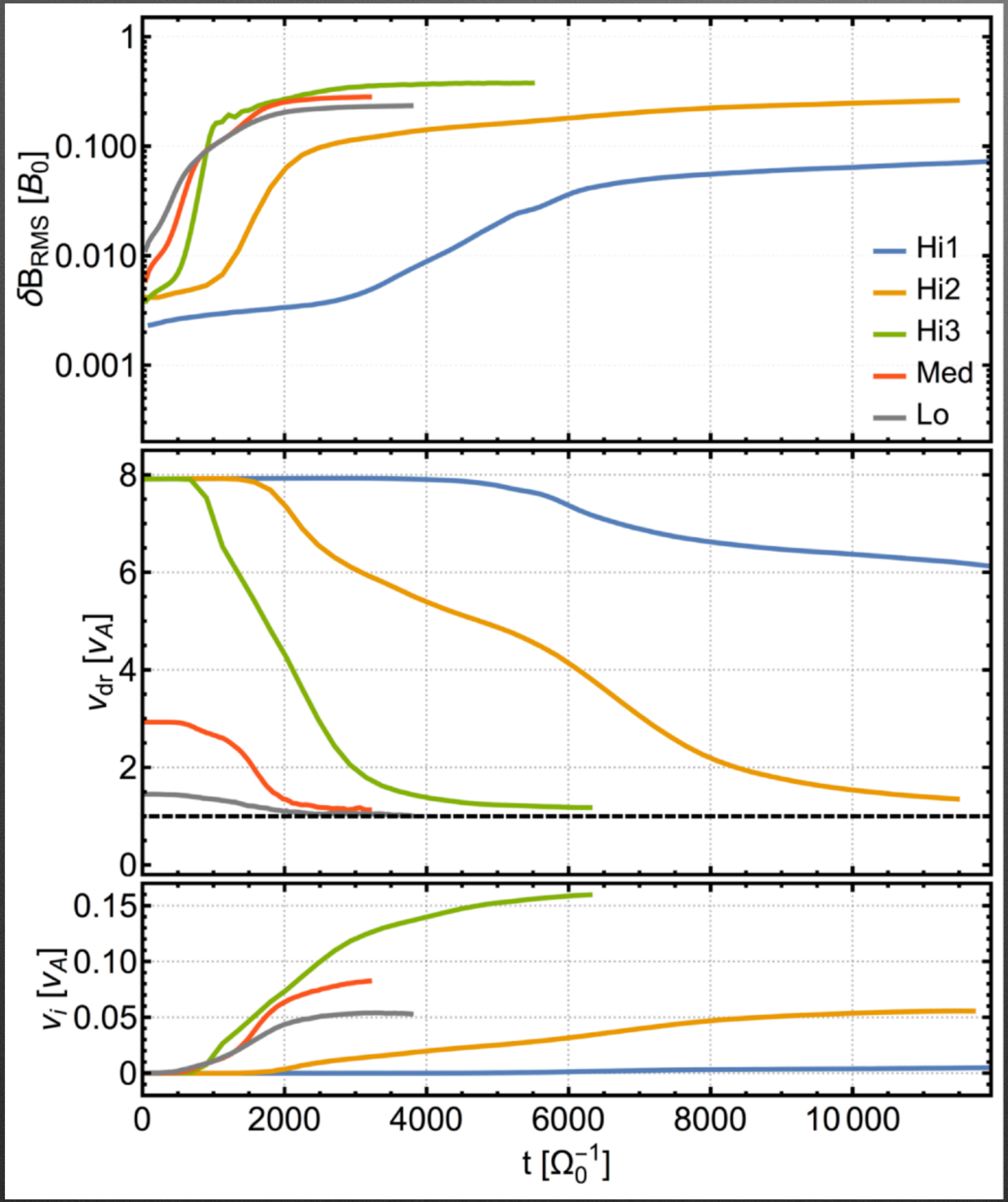
$$\left(\frac{\delta B}{B_0}\right)^2(t) \approx \gamma_{\rm cr}\frac{n_{\rm cr}}{n_i}\frac{v_{\rm dr,0} - v_{\rm dr}(t)}{v_{\rm ph}}$$

Total Saturation:

$$t_\mu = \frac{3}{8}\int_{\mu_{\rm M}}^{\mu_0} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

$$\approx \frac{3}{4\pi}\left(\frac{\delta B}{B_0}\right)^{-2}_{\rm diff}\Omega^{-1}C_\mu$$

CR-Driven Winds



CR-driven flow:
ExB drifts propel the background plasma

Periodic Simulations

- Linear theory predicts the instability growth rate to a satisfactory level in PIC simulations
    - (MHD-PIC codes perform better in this task; Lebiga+18, Bai+19)
- Can predict the nonlinear behavior of instability based on simple quasilinear scaling relations and conservation laws
- Initial anisotropy of CRs can have important impact on nonlinear evolution — Right-handed modes cannot isotropize CRs (unless amplitudes are dB~B)
- If left-handed modes never appear (either because of anisotropy or damping), CRs will not be self-confined
- v_{dr} ~ 0.8c is pretty extreme and probably unrealistic, why bother?
    - We'd like to understand the qualitative features of highly anisotropic CR instability in a way that is continuous with the (low anisotropy) standard models, i.e., push the power-law to the limit
    - We'd like to understand the qualitative features of highly anisotropic CR instability in a periodic setting before moving on to aperiodic simulations, where anisotropy is a natural consequence of expansion away from a source

Aperiodic CR Simulations

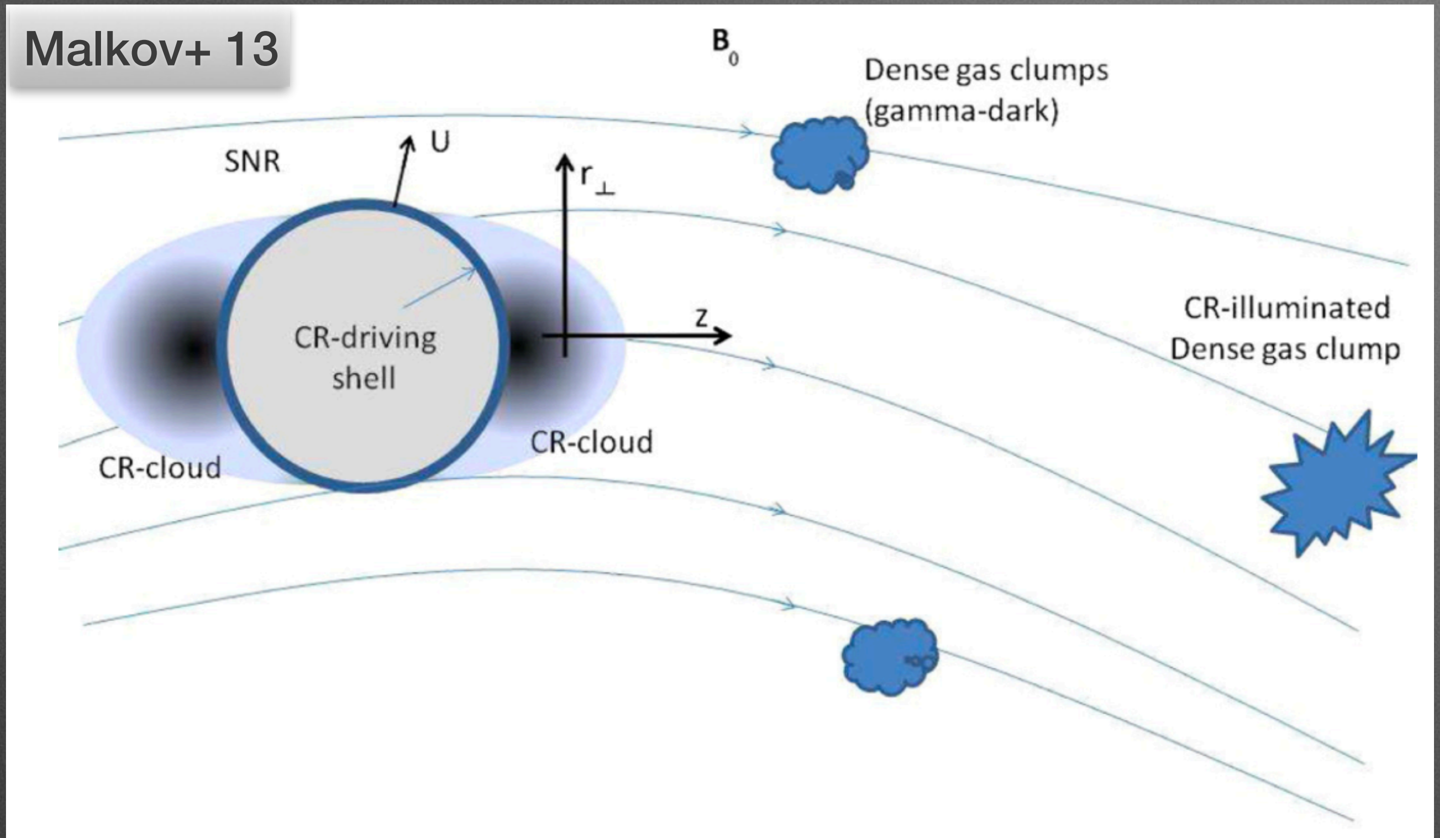
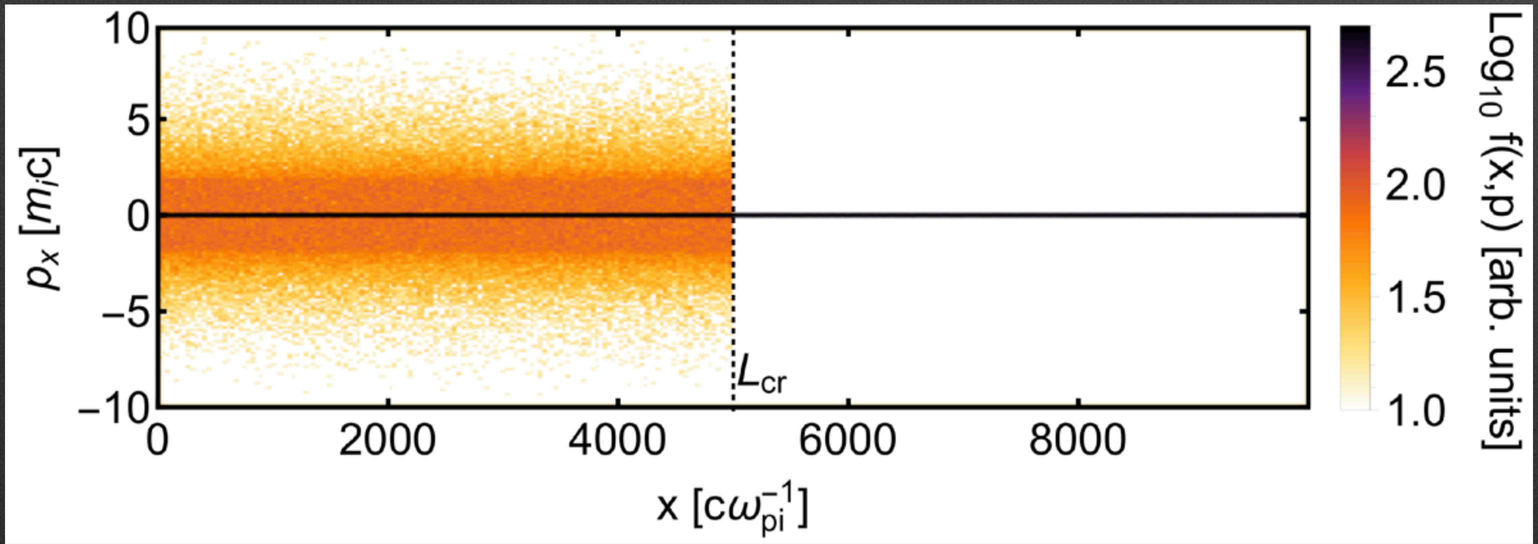


Figure 1. CR escape along the magnetic field $\mathbf{B}_0$ from the two polar cusps of SNR with a stalled blast wave. (A color version of this figure is available in the online journal.)



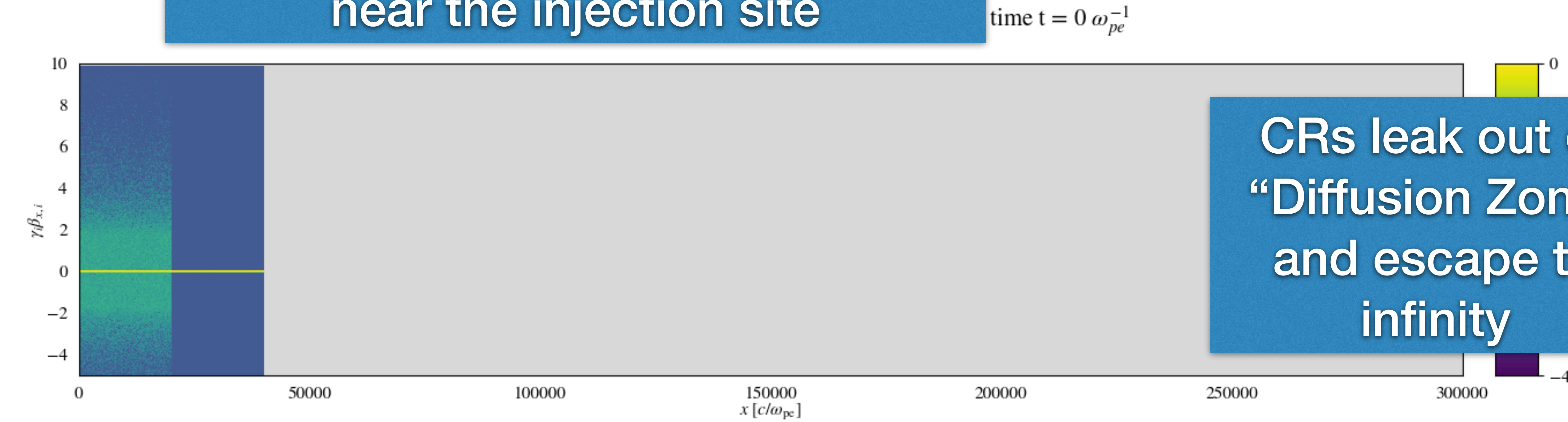
Performed 1D PIC simulations with reflecting and outflow boundary conditions

A "cloud" of CRs is initialized and allowed to freely expand into background plasma

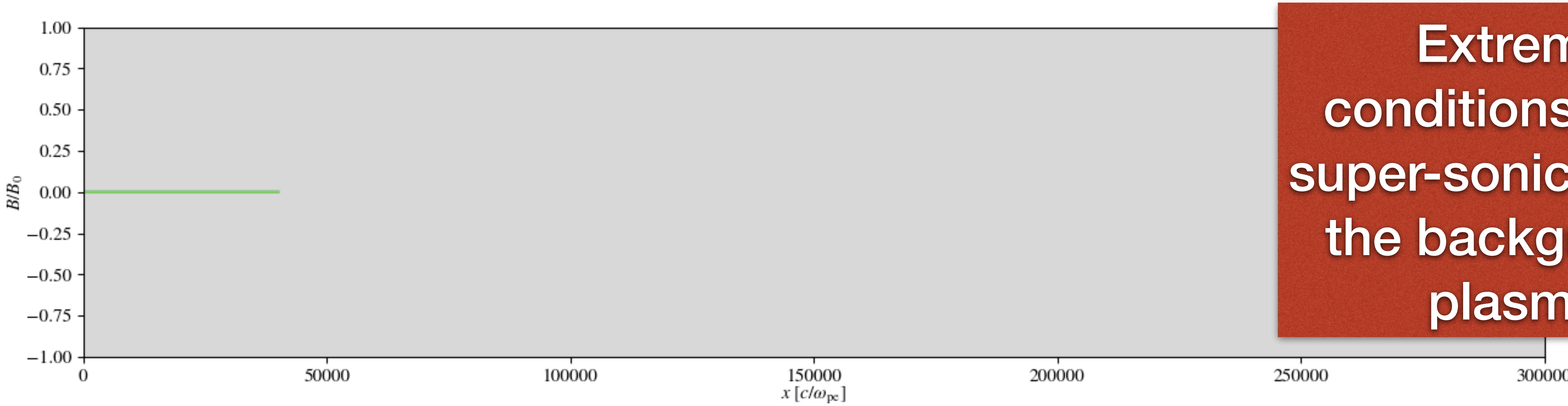
CR Cloud Simulations

High CR Density

Large amplitude waves trap CRs near the injection site



CRs leak out of "Diffusion Zone" and escape to infinity



Extreme conditions drive super-sonic flow in the background plasma

CR Cloud Simulations

High CR Density

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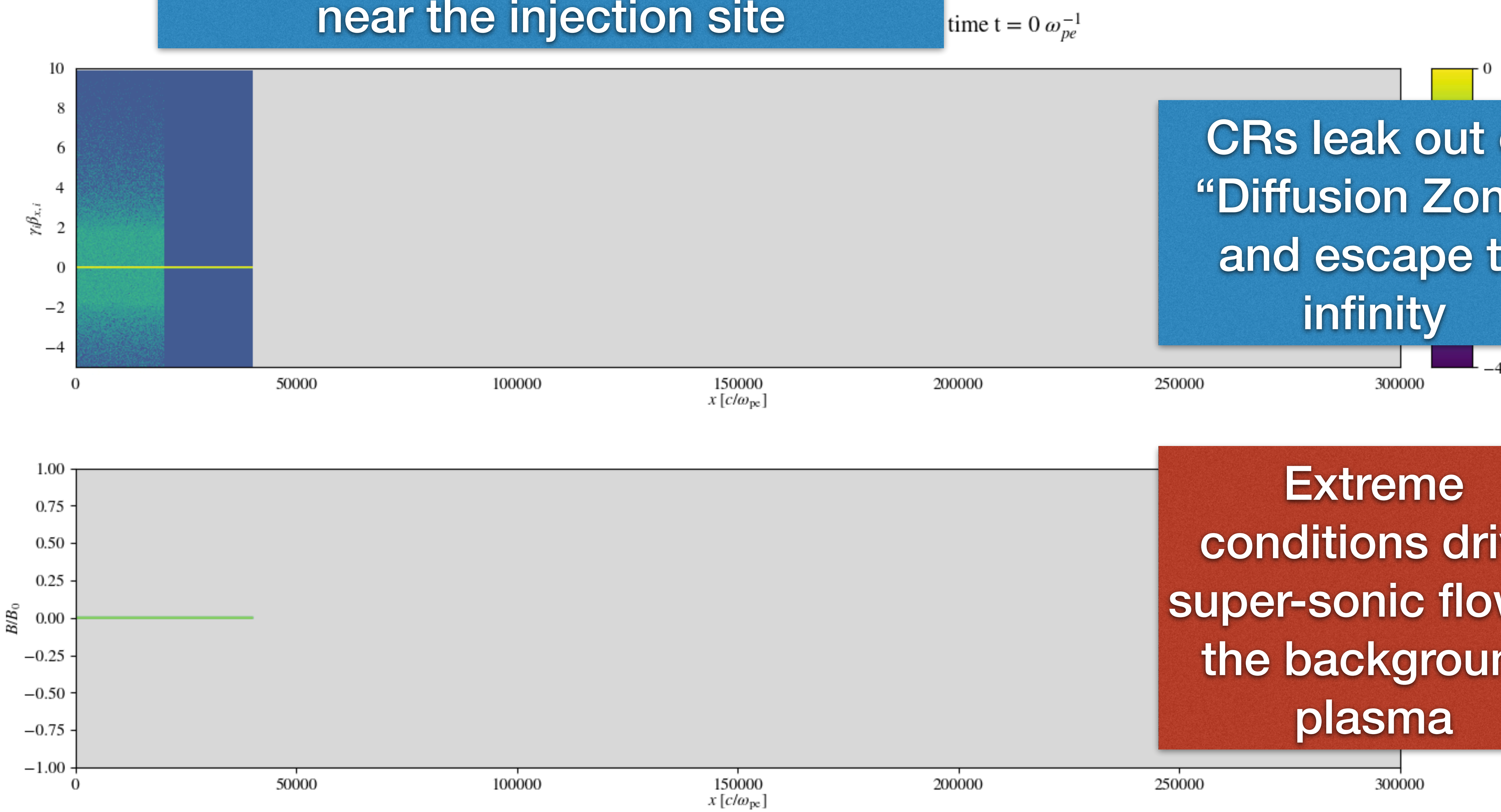

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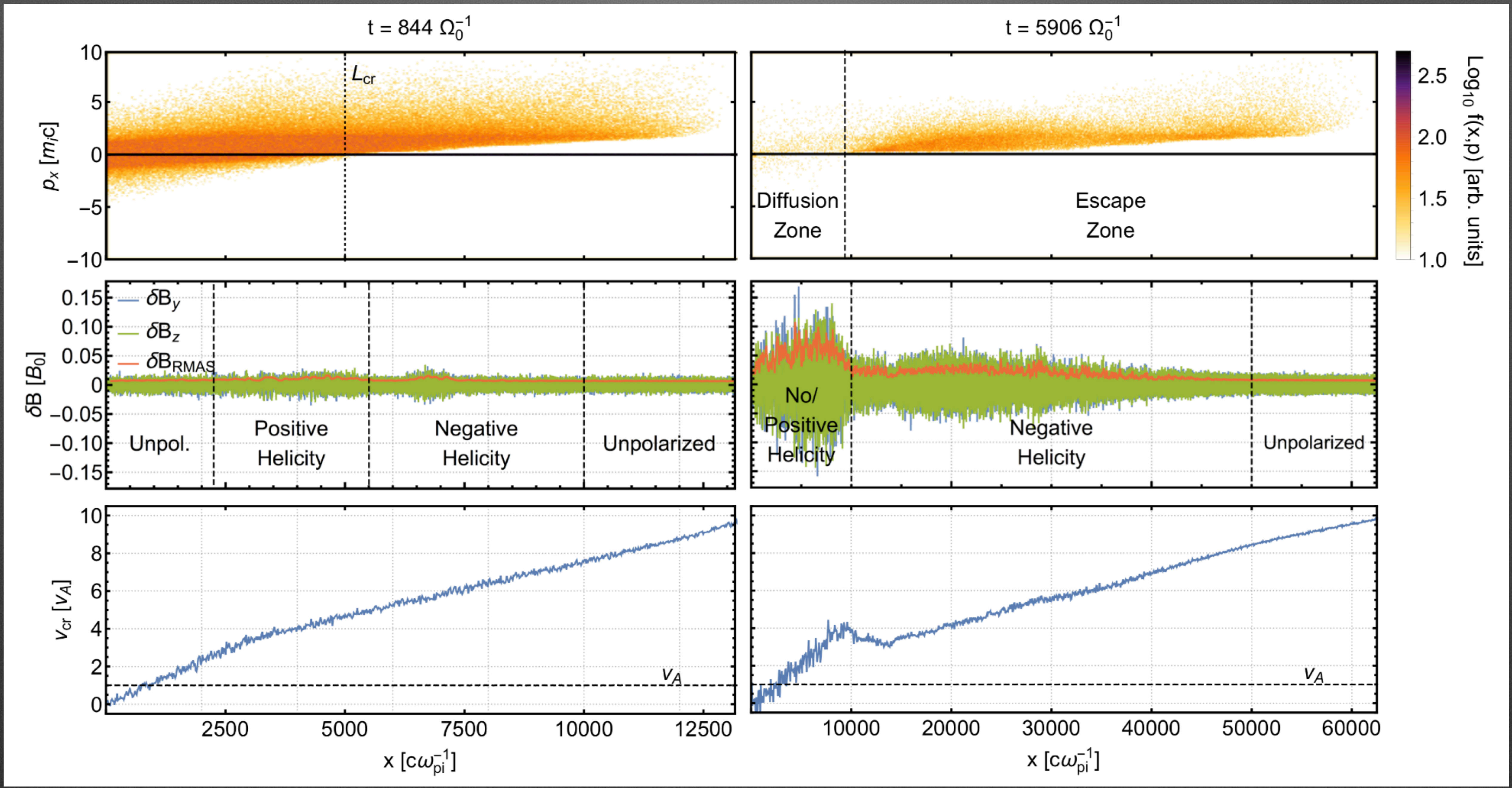
CR Cloud Simulations



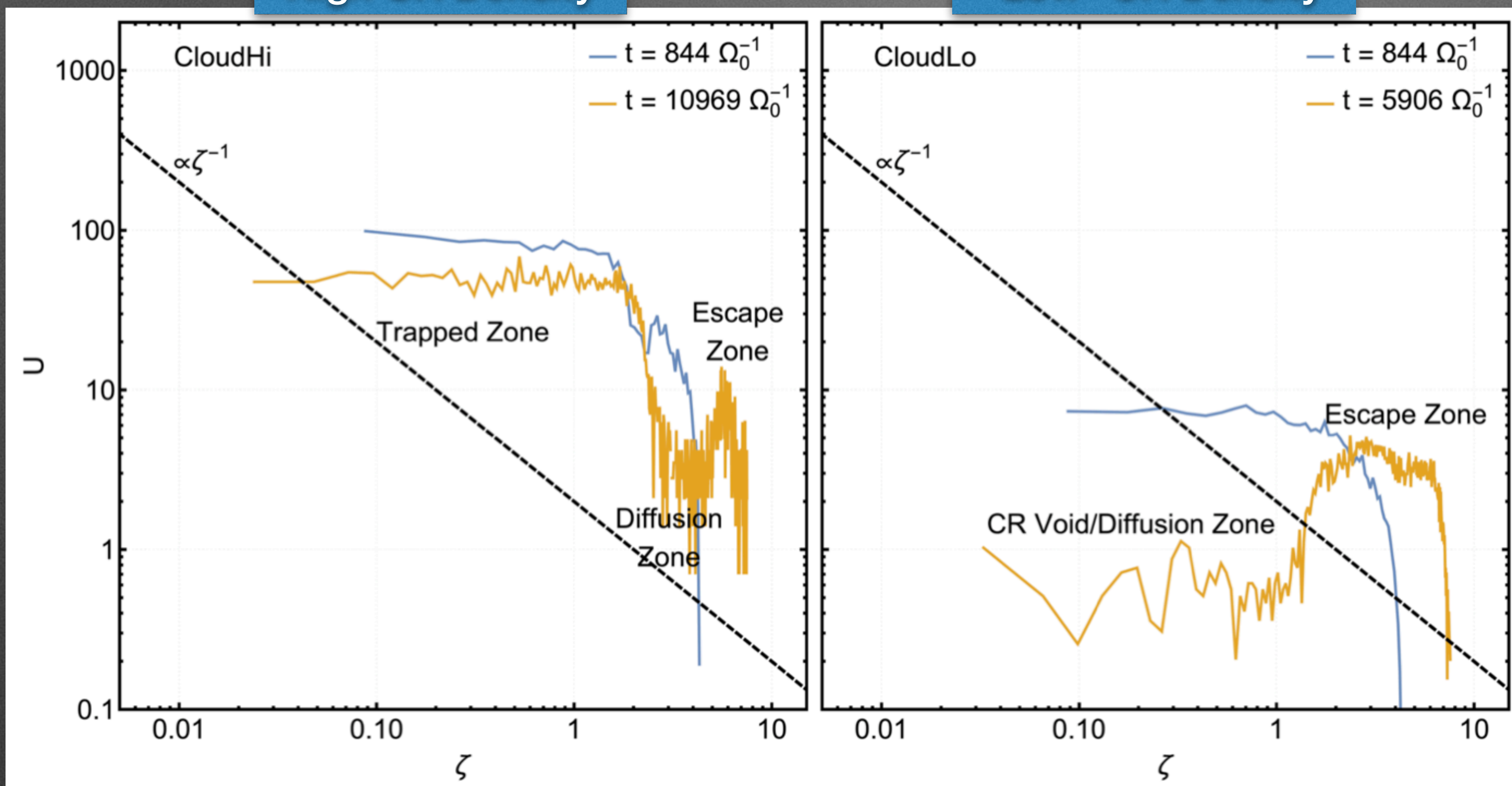
Low CR Density

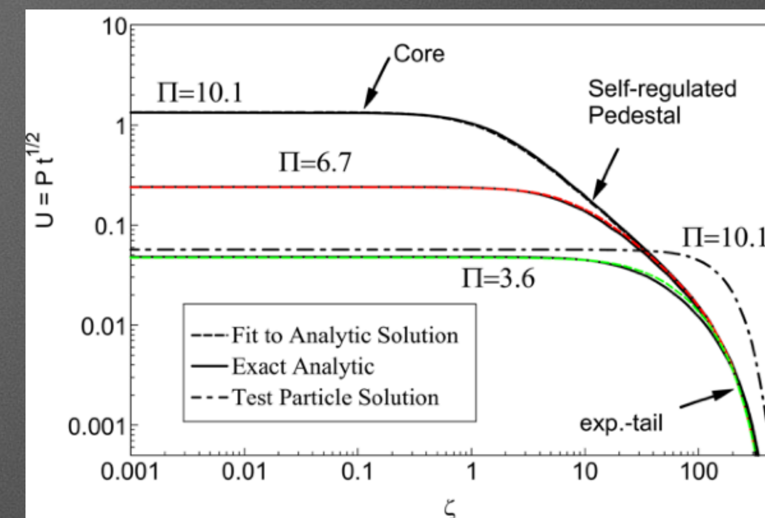
~95% of CRs escape before diffusive behavior is achieved

CR Cloud Pressure Profiles

High CR Density

"Low" CR Density





Malkov+ 13

$$U = 4L_{\rm cr}\sqrt{\frac{m_{\rm e}}{m_{\rm i}}}\left(\frac{p^3}{n_{\rm i}}\int\int f\,{\rm d}\mu{\rm d}\phi\right)$$

$$\zeta = x/\sqrt{t}$$

Analytic model fails because of isotropy assumption(s)

Trouble with Assumptions

Expansion via advection equation implies ballistic transport

Ballistic transport implies CR anisotropy

1. $$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial x} = 0$$

$$f(t, x) = f(t = 0, x - \mu v t)$$

2. "Sharpened" resonance does not describe wave-particle interaction; does not capture asymmetry between left and right-handed modes

$$k = \frac{\Omega}{\mu v} \neq \frac{\Omega}{v}$$

Aperiodic Simulations

- Models that do not account for anisotropy do not describe the simulated physics of CR expansion even qualitatively.
- CR structures can still be formed, but different from analytic predictions
- Large amplitude waves trap CRs, while right- and left-hand circularly polarized waves are required for fully diffusive behavior

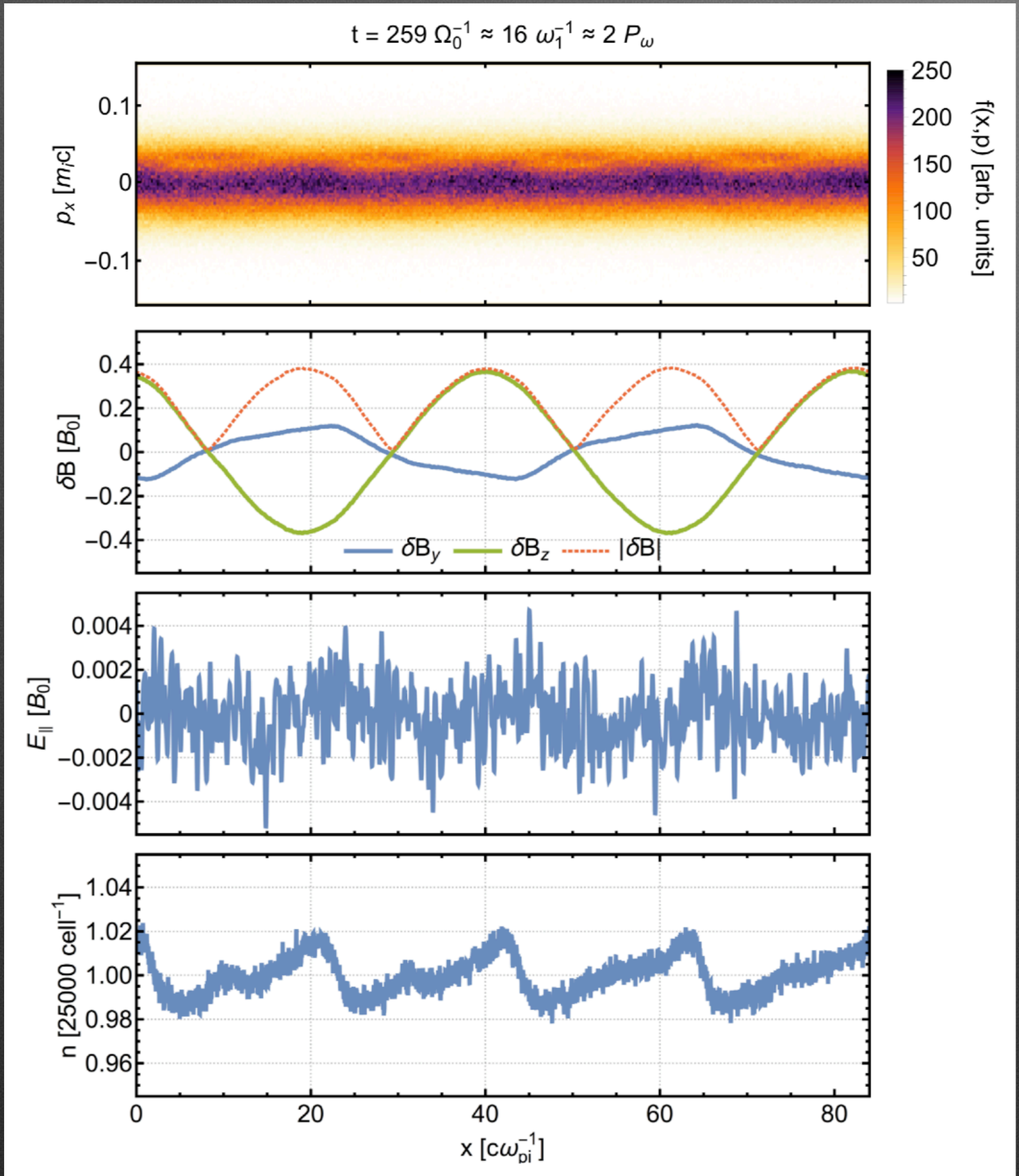
Wave Damping

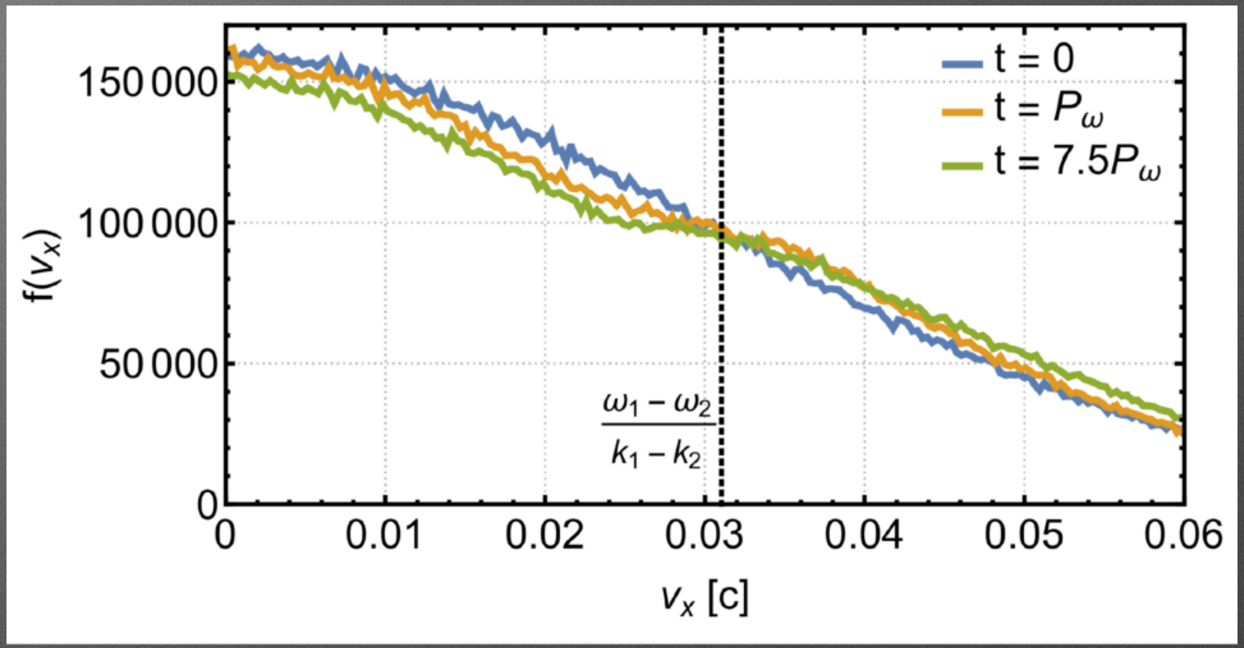
CRs give energy to waves via streaming instability, but other mechanisms exist that drain energy instead:

$$\Gamma = \Gamma_{\rm cr} + \Gamma_{\rm damp} \stackrel{?}{=} 0$$

• Ion-Neutral Friction	ISM
• Nonlinear Landau Damping	IGM, ISM (maybe)
• Turbulent Damping	Everywhere??

Nonlinear Landau Damping

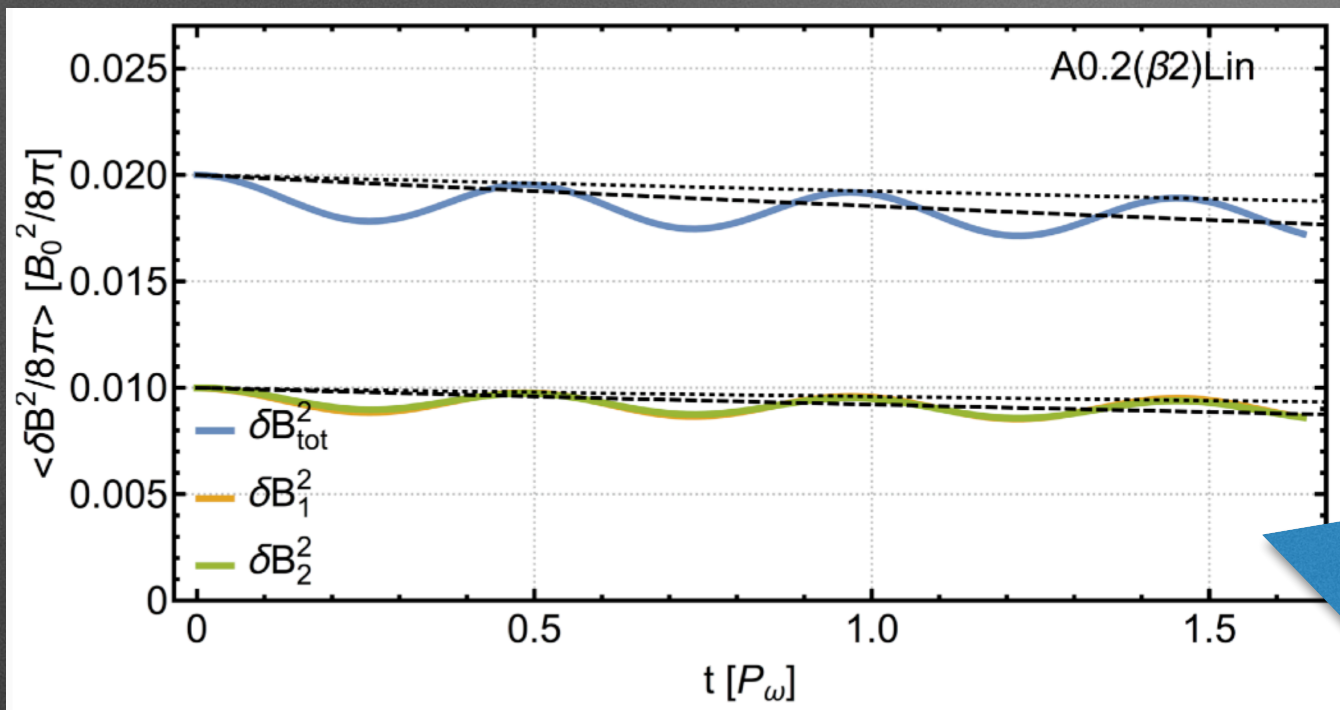




25,000 particles per cell, induced electric field is barely resolved
(70k cpu-hours on Perseus cluster at PU)

Self-consistent CR streaming + NLLD not feasible

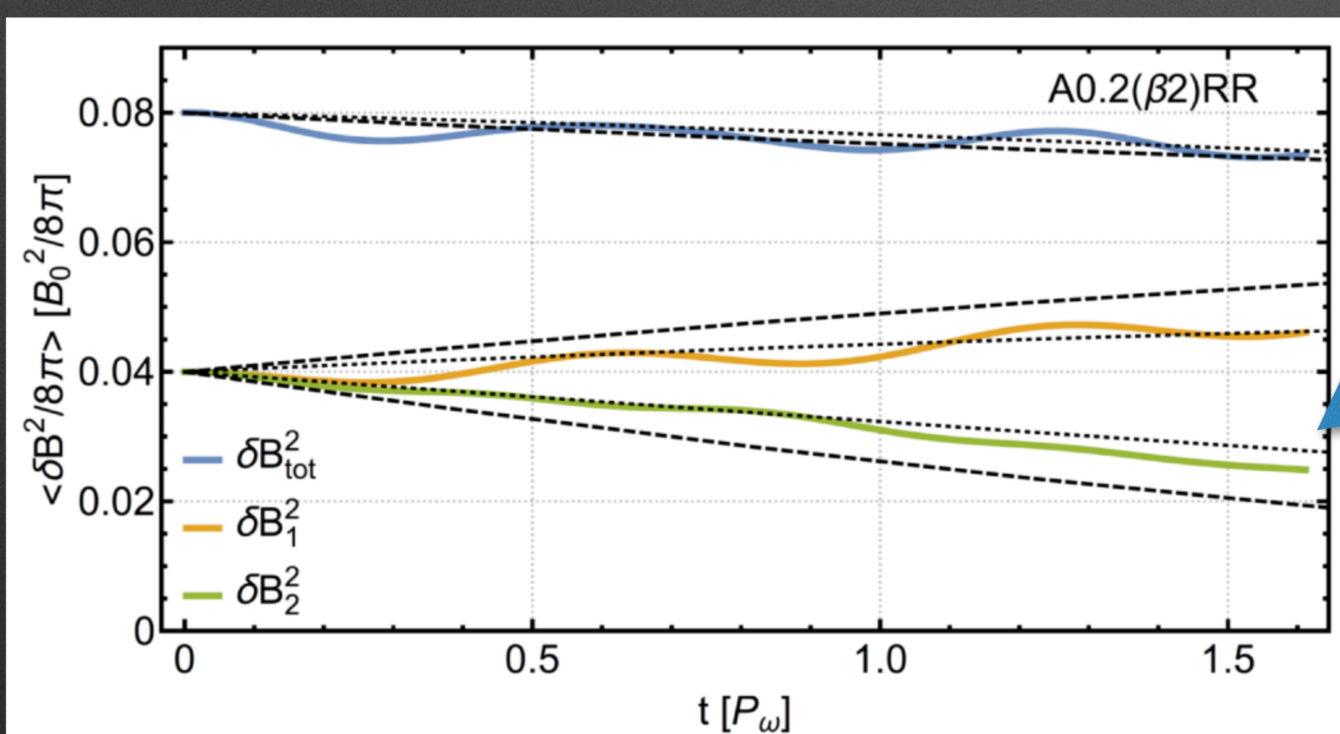
Nonlinear Landau Damping



$$\Gamma_{\rm NLLD,1} = -\frac{\sqrt{\pi}}{4}\left(\frac{\delta B_2}{B_0}\right)^2 \sqrt{\beta_{\rm i}}\frac{(\omega_1 - \omega_2)}{|\omega_1 - \omega_2|}\omega_1$$

$$\Gamma_{\rm NLLD,2} = -\frac{\sqrt{\pi}}{4}\left(\frac{\delta B_1}{B_0}\right)^2 \sqrt{\beta_{\rm i}}\frac{(\omega_2 - \omega_1)}{|\omega_2 - \omega_1|}\omega_2$$

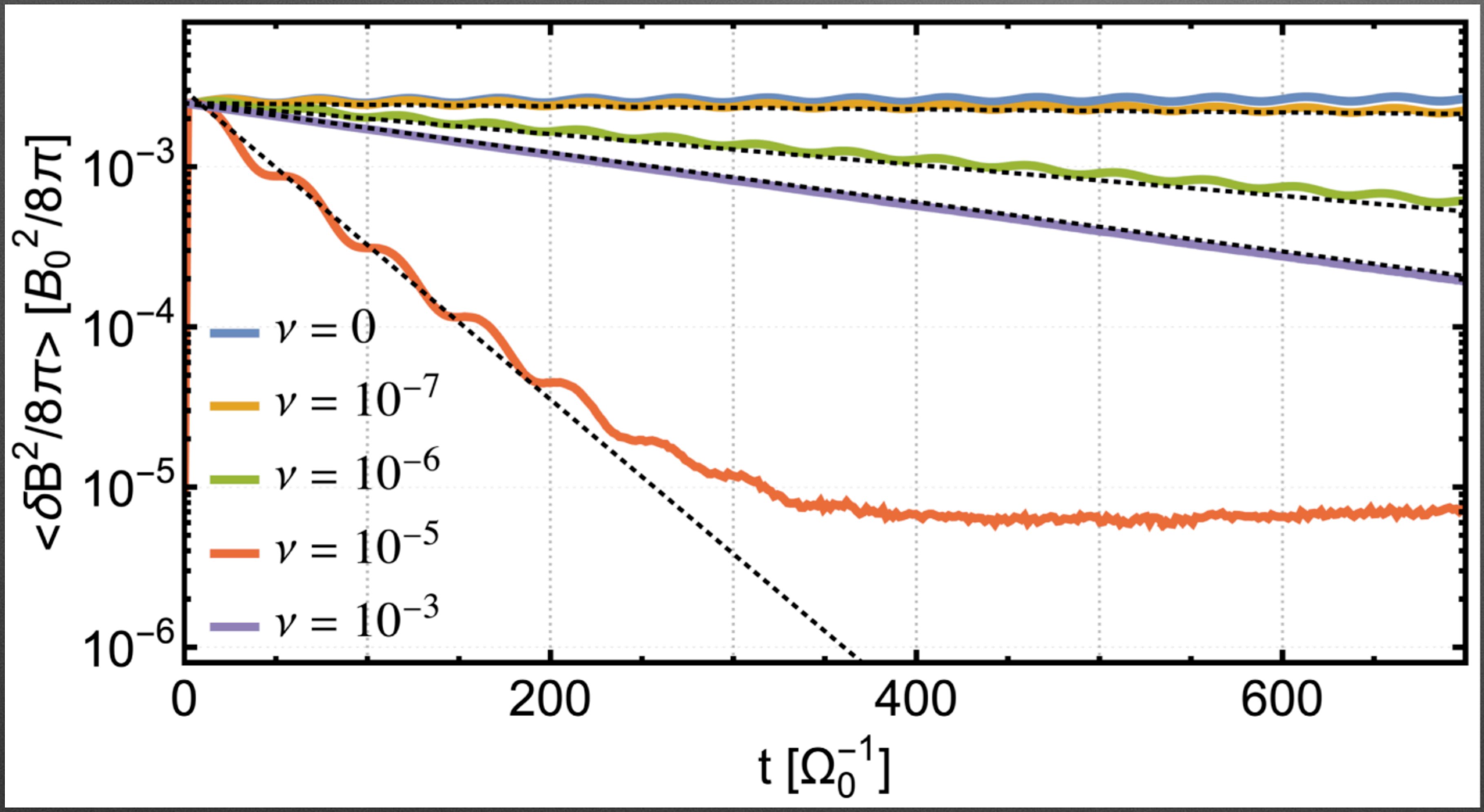
Linearly Polarized:
Both components damp



Circularly Polarized:
One damps, one grows

Damping rates within
factor of 2 of prediction

Ion-Neutral Damping



Randomly scatter ions with rate nu to model the effect of ion-neutral collisions

$$\Gamma_{\rm in} \approx -\frac{\nu_{\rm in}}{2}$$

Very good agreement between model behavior and predicted damping rates

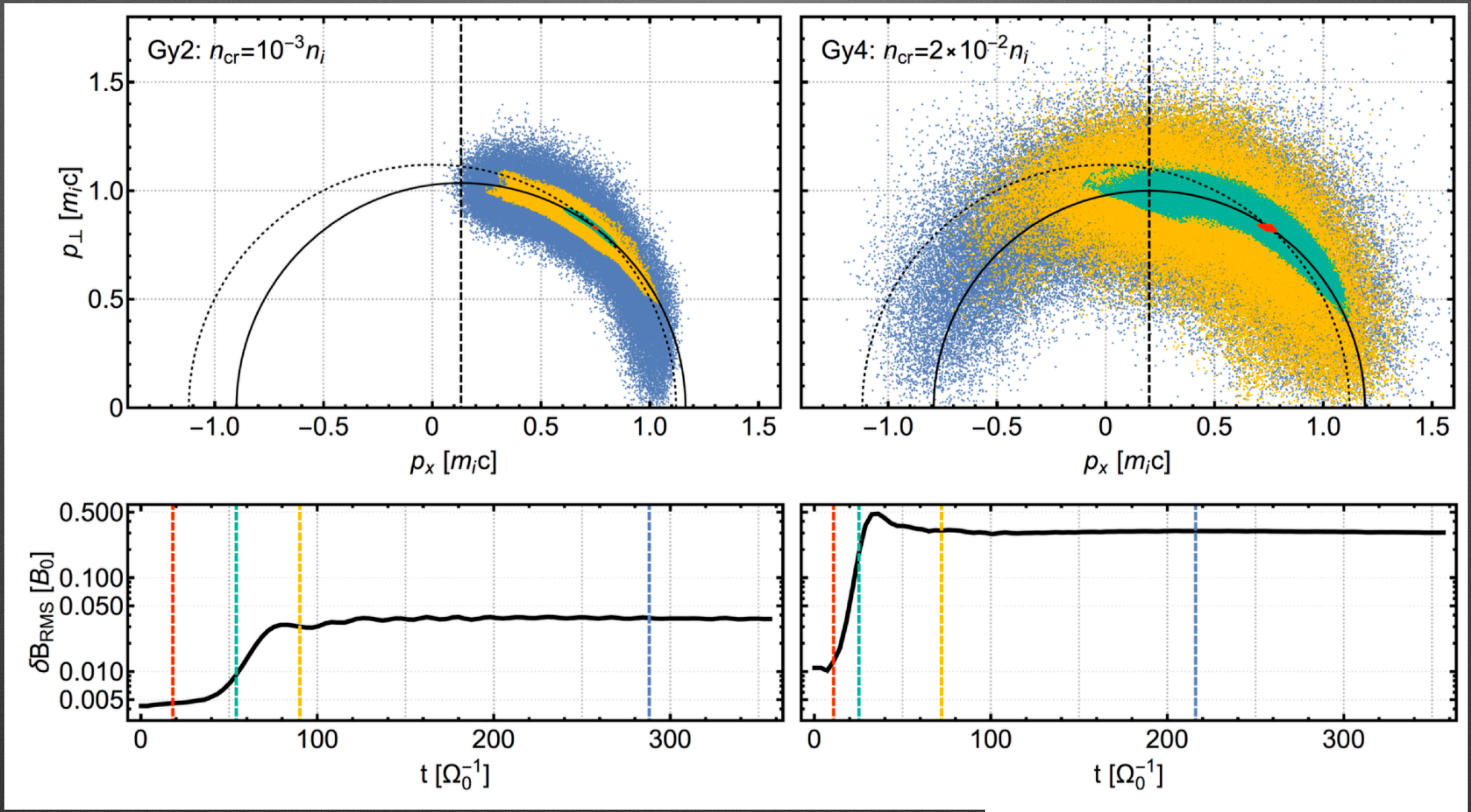
(except highest scattering rate, which is off by 2x)

Easy to implement in CR instability simulations

Summary

- Predictions of linear and quasi-linear theories are in satisfactory agreement with simulations
- CR anisotropy is important on micro- and meso-scales; not accounted for by analytical models
- Self-confinement will be more difficult in the presence of wave damping
- Ion-Neutral damping will be relatively easy to include in simulations, but NLLD does not look promising

Trapped Particle Dynamics

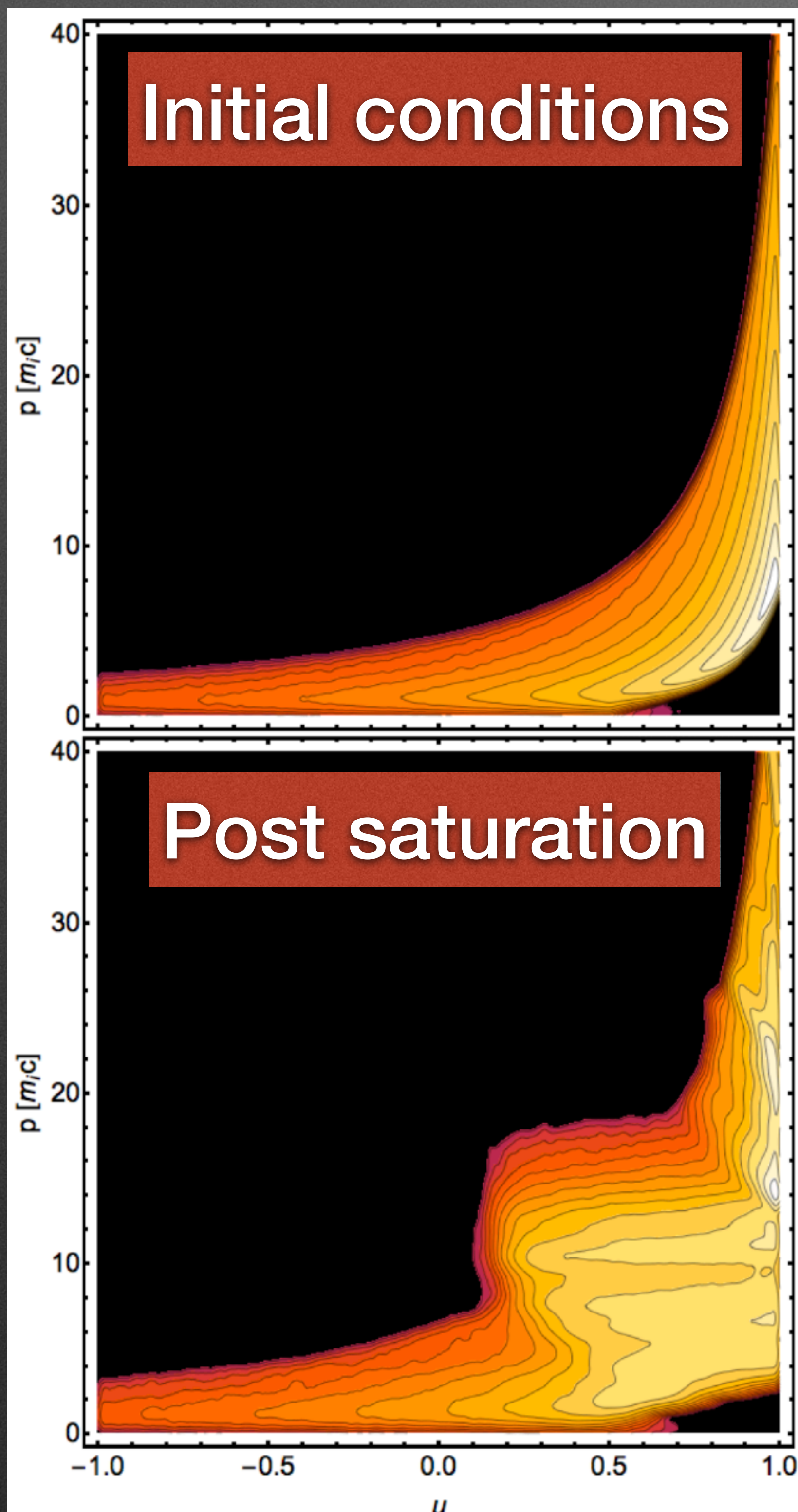


$$\gamma_{\rm ph}^2 (p_\parallel(t) - \gamma v_{\rm ph})^2 + p_\perp^2(t) = \text{constant}$$

$$\Delta v'_\parallel \approx 2^{\frac{3}{2}} \sqrt{\frac{\delta B'}{B_0} \frac{v_\perp}{k} \Omega'_{\rm cr}}$$

Sudan & Ott 71

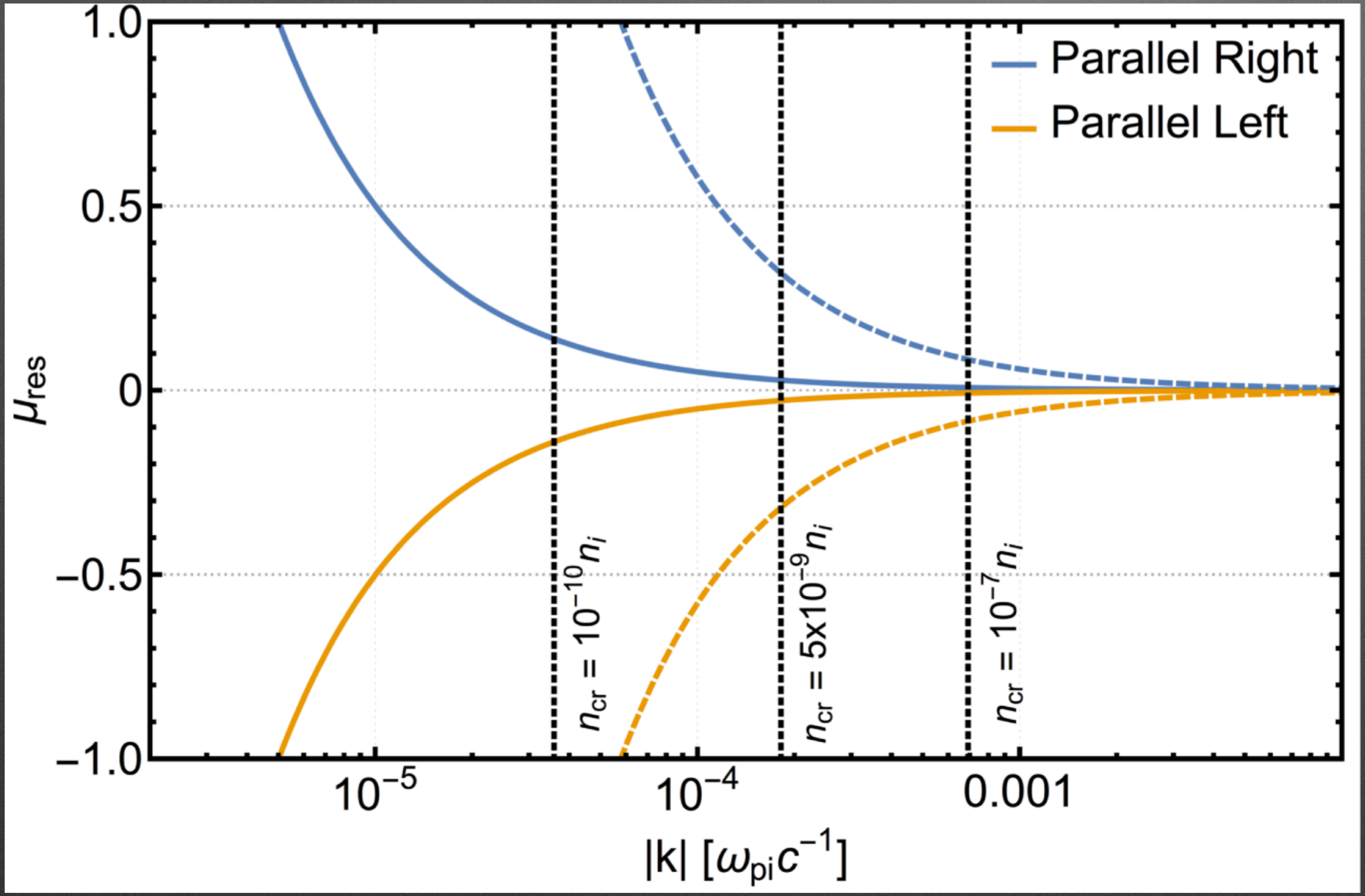
Saturation Mechanism



$$\Gamma^{\pm}_{\rm cr}(k) = \frac{\pi^2 q^2}{2}\frac{v_A^2}{c^2}\sum_{\pm}\iint \delta(\omega - k\mu v \pm \Omega(p))\left(\frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu\right)\frac{1}{p}\frac{\partial f}{\partial \mu}\right) v p^2(1-\mu^2)\,dp\,d\mu$$

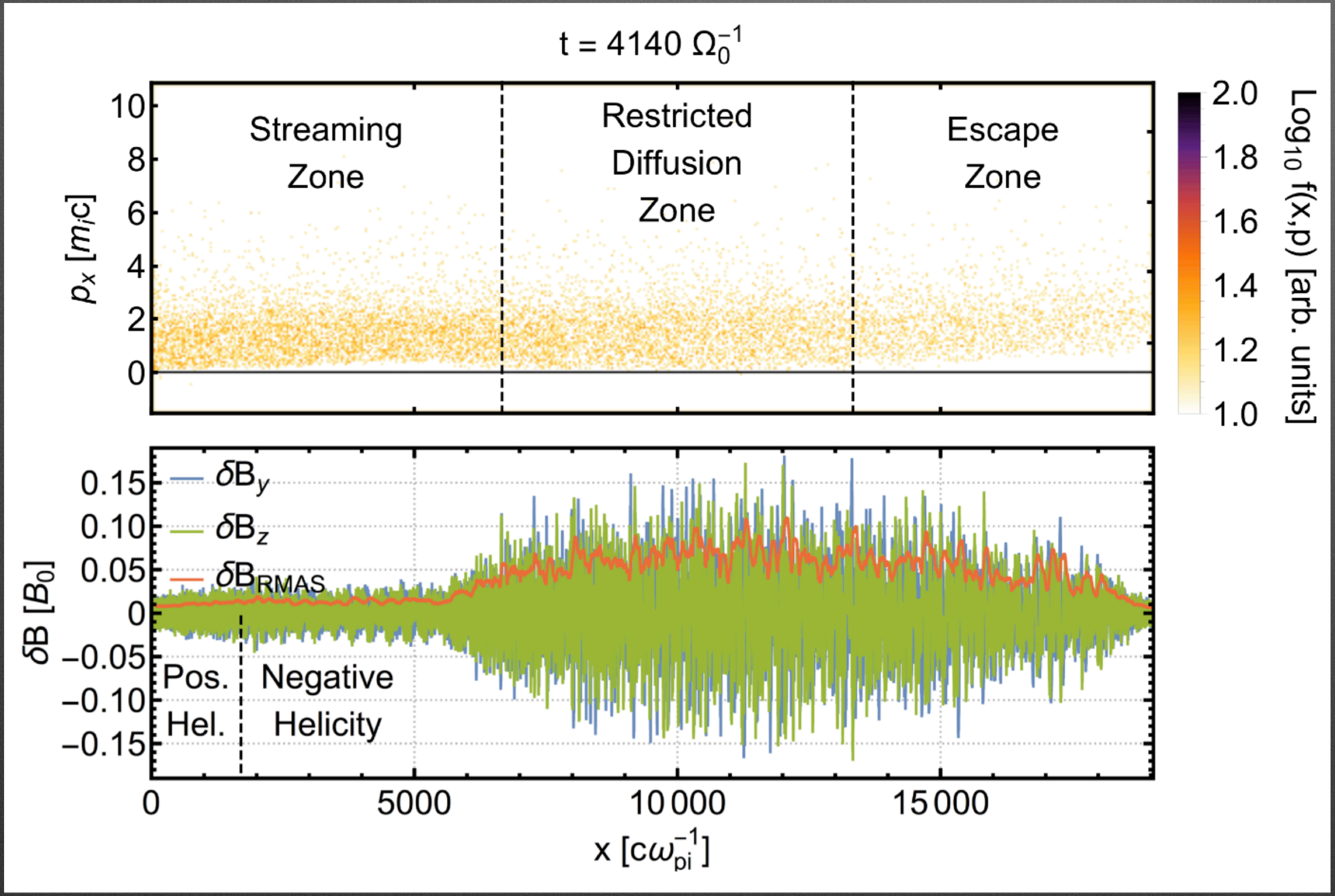
Instability is quenched when gradients are flattened

Ion-Neutral Damping



IN damping broadens resonance gaps

Continuous CR Injection



Low CR Density

Right-handed modes are unable to isotropize CRs