Radiative kinetic turbulence

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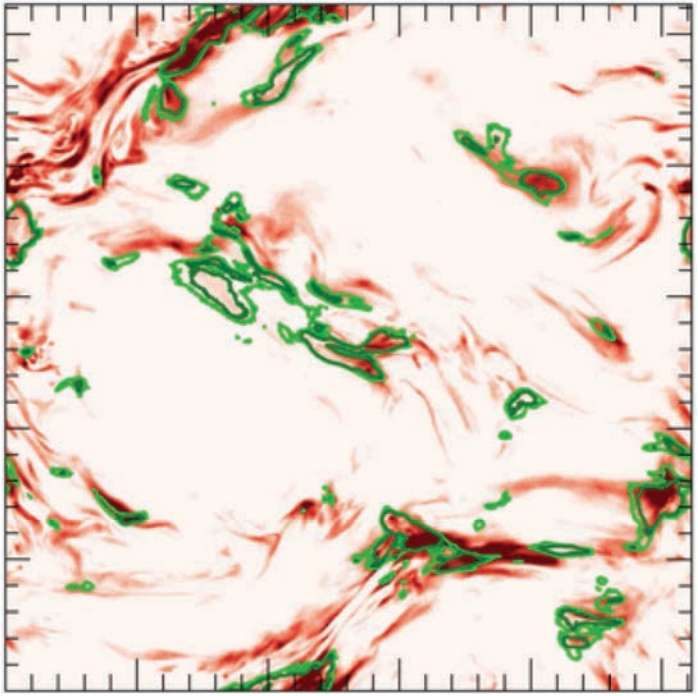
KITP program, 10/8/2019



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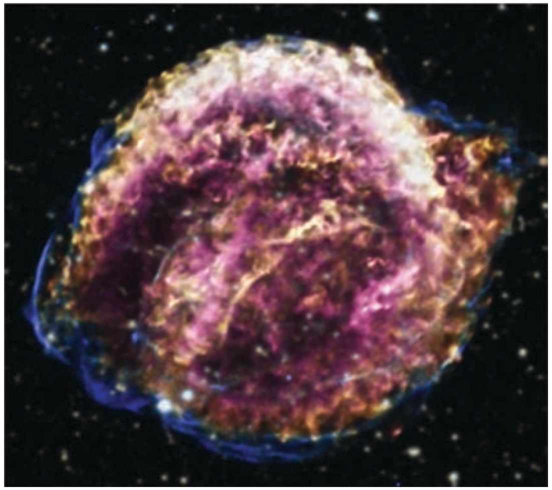
Outline

I. Overview of relativistic kinetic turbulence
II. Radiative turbulence in pair plasmas
III. Radiative turbulence in electron-ion plasmas
IV. Conclusions

High-energy astrophysical turbulence



Kepler's supernova (SNR)

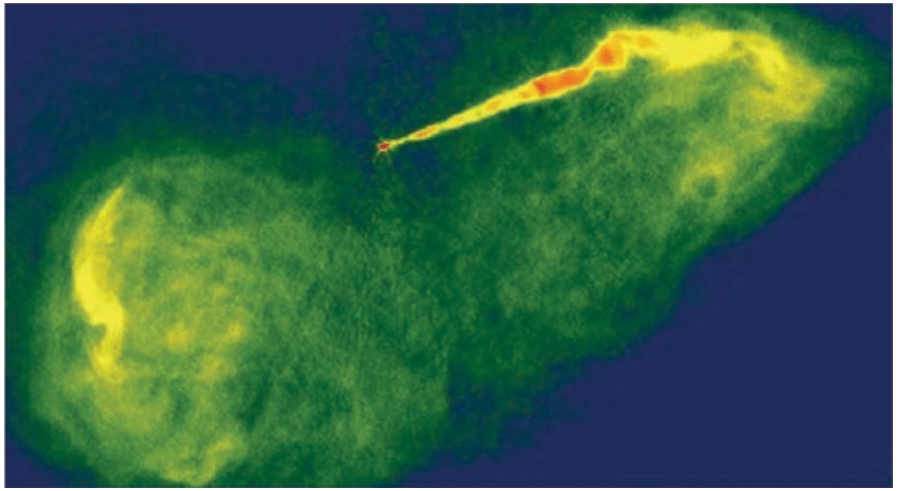
Turbulence is a **ubiquitous process** in high-energy astrophysics

Systems often comprise **relativistic, radiative, collisionless plasmas**

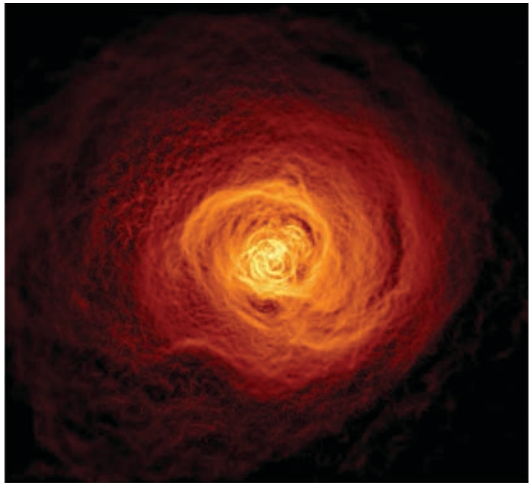
Small-scale turbulence important for understanding structure, spectra, etc.

M87 (accretion flow)

Crab nebula (PWN)



M87 (AGN jet)

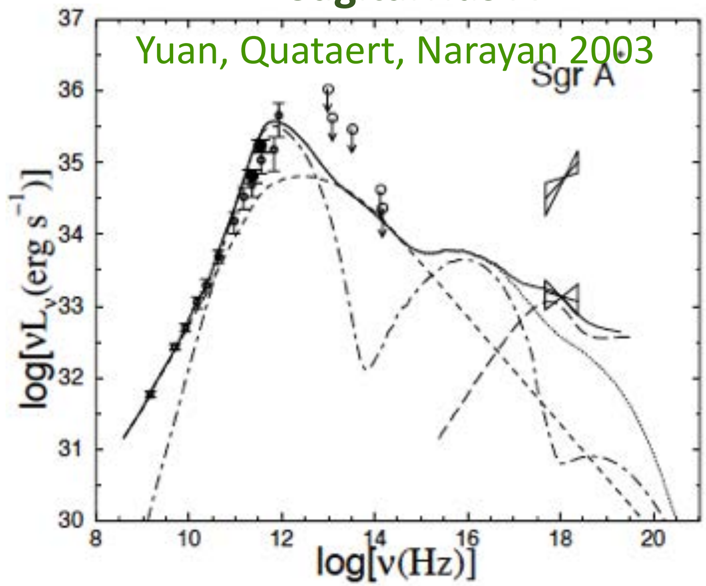


Perseus cluster (ICM)

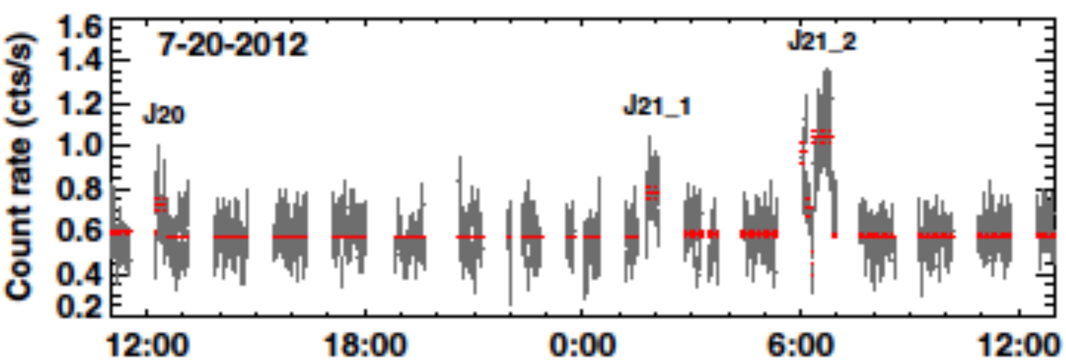
Some plasma astrophysics

- Radiatively inefficient accretion flows onto black holes (e.g., at galactic center) are a prototypical example of collisionless plasma turbulence
- Radiation spectra indicate highly nonthermal electrons, spanning orders of magnitude in energy
- Global models require "two-temperature" plasma in which ions are much hotter than electrons, to prevent collapse into collisional thin disk
- Rapid X-ray flares are occasionally observed
- Is our knowledge of turbulence sufficient to explain?

Sagitarrius A*

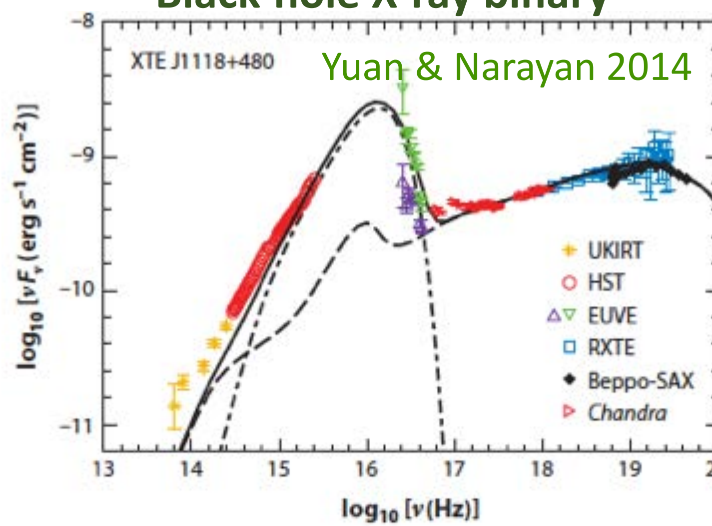


Sagitarrius A* X-ray flares



Barriere et al. 2014

Black-hole X-ray binary



Relativistic kinetic turbulence

Relativistic plasma: $\theta \equiv T/mc^2 \gg 1$ $(\bar{\gamma} \sim 3\theta \gg 1)$

Relativistic turbulence: $\sigma \equiv \frac{B_{\rm rms}^2}{4\pi h} \gtrsim 1$ $(h \sim 4n_0\bar{\gamma}mc^2/3)$

$$\delta v/c \sim v_A/c = \sqrt{\sigma/(\sigma+1)}$$

Four motivations:

- Ubiquitous in high-energy astrophysics (AGN, GRB, PWN, XRB, etc.)
- Unexplored frontier of turbulence
- Prototypical theoretical problem (nonthermal particle energization)
- Viable with first-principles particle-in-cell (PIC) simulations

Questions fall into two categories:

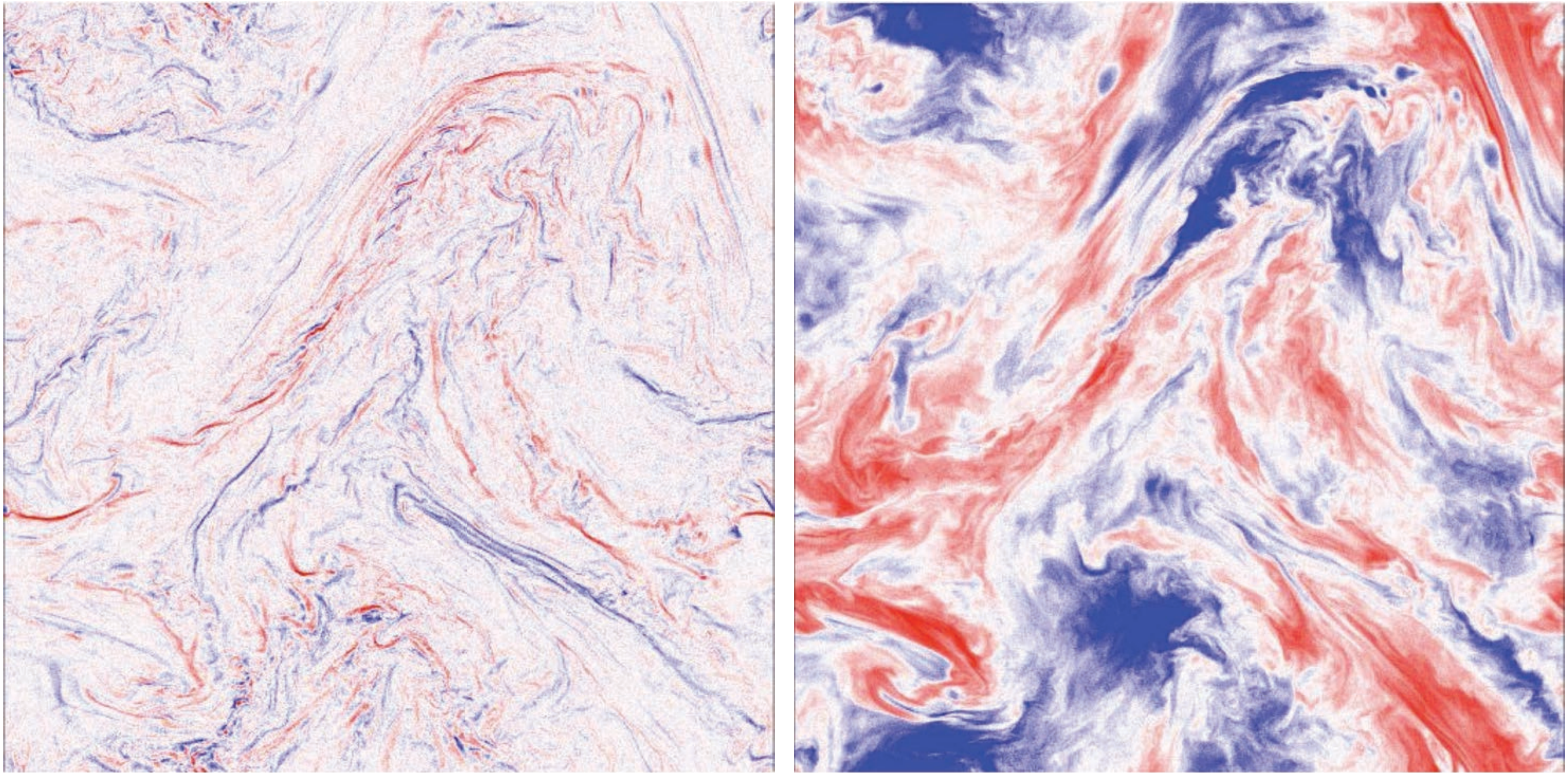
1. What are the statistical properties of the turbulence?
2. What are the kinetic properties of the particles?

What's known about relativistic kinetic turbulence

- 3D (and 2D) PIC pair plasma simulations appear to reproduce MHD inertial range

3D pair plasma turbulence - fixed-time fly-through

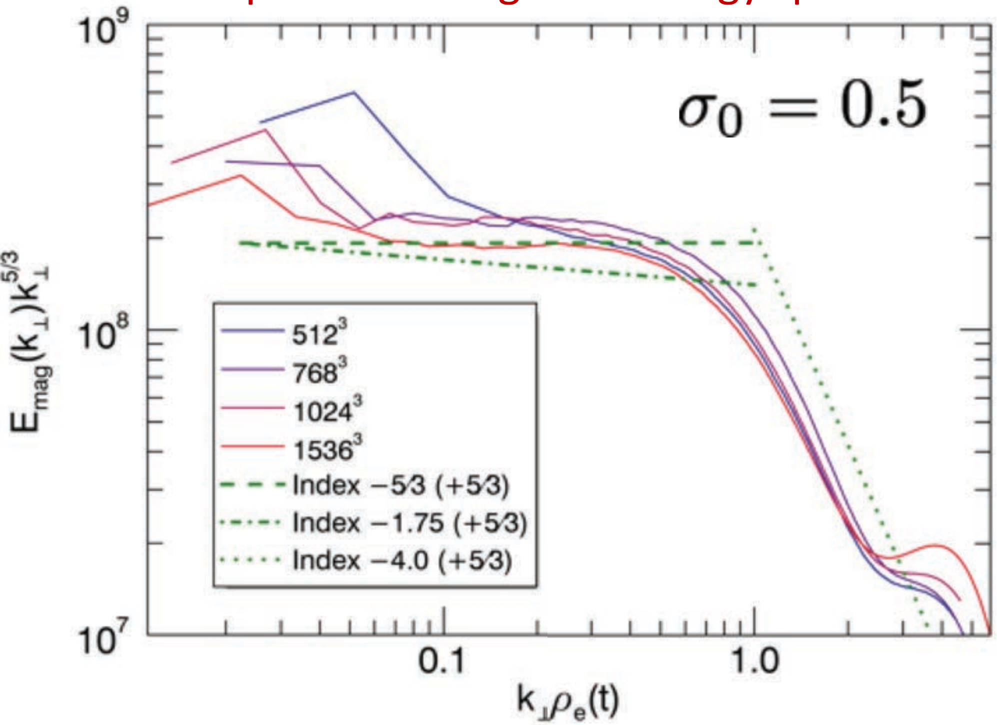
J_z δn



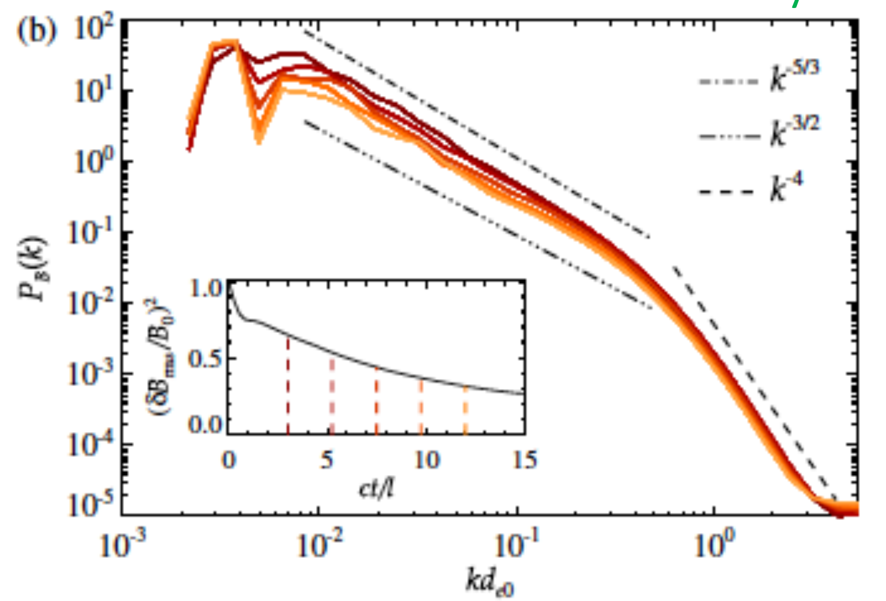
$\sigma_0 = 0.5$

Turbulence inertial range (pair plasma)

Compensated magnetic energy spectrum



Comisso & Sironi 2018 - Thursday



MHD range: -5/3 index (Goldreich & Sridhar 1995, Thompson & Blaes 1998)

Kinetic range: -4 index or steeper (kinetic cascade? Schekochihin+ 2009)

Turbulence statistics: VZ+ MNRAS 2018

What's known about relativistic kinetic turbulence

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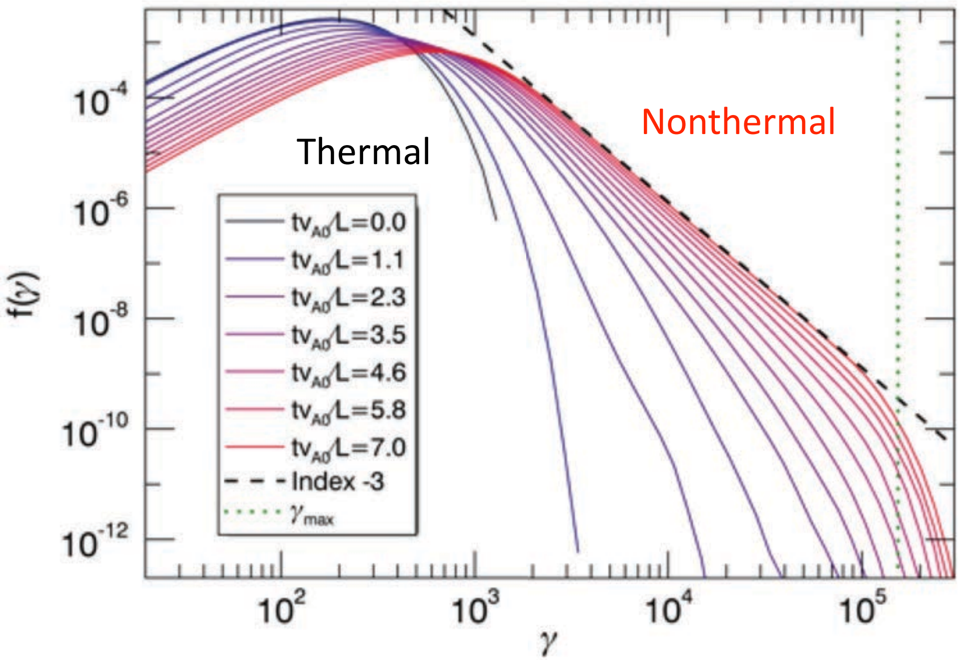
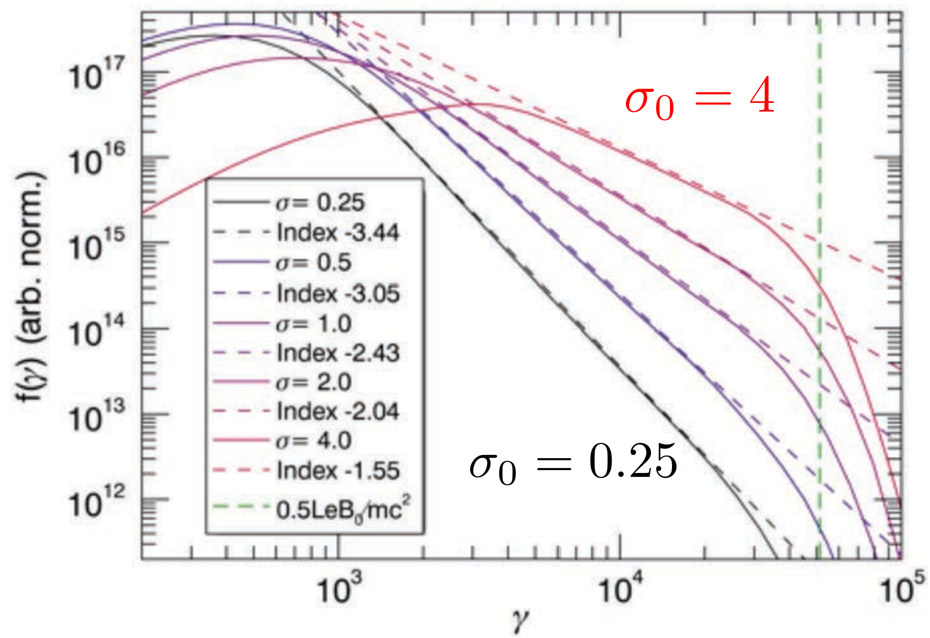
- Unknown: nature of sub-kinetic scale cascade
- Efficient particle acceleration (driven - VZ+ 2017; decaying - Comisso & Sironi 2018, Nattila 2019; see also kink instability – Alves+ 2018)

Nonthermal particle acceleration

Energy distribution evolution (1536^3)



Magnetization scan



Power law tail: $f(\gamma) \sim \gamma^{-\alpha} \quad (\gamma = E/m_e c^2)$

Spans from mean energy $\langle \gamma \rangle$ to system-size limited energy $\gamma_{\max} = LeB/2mc^2$

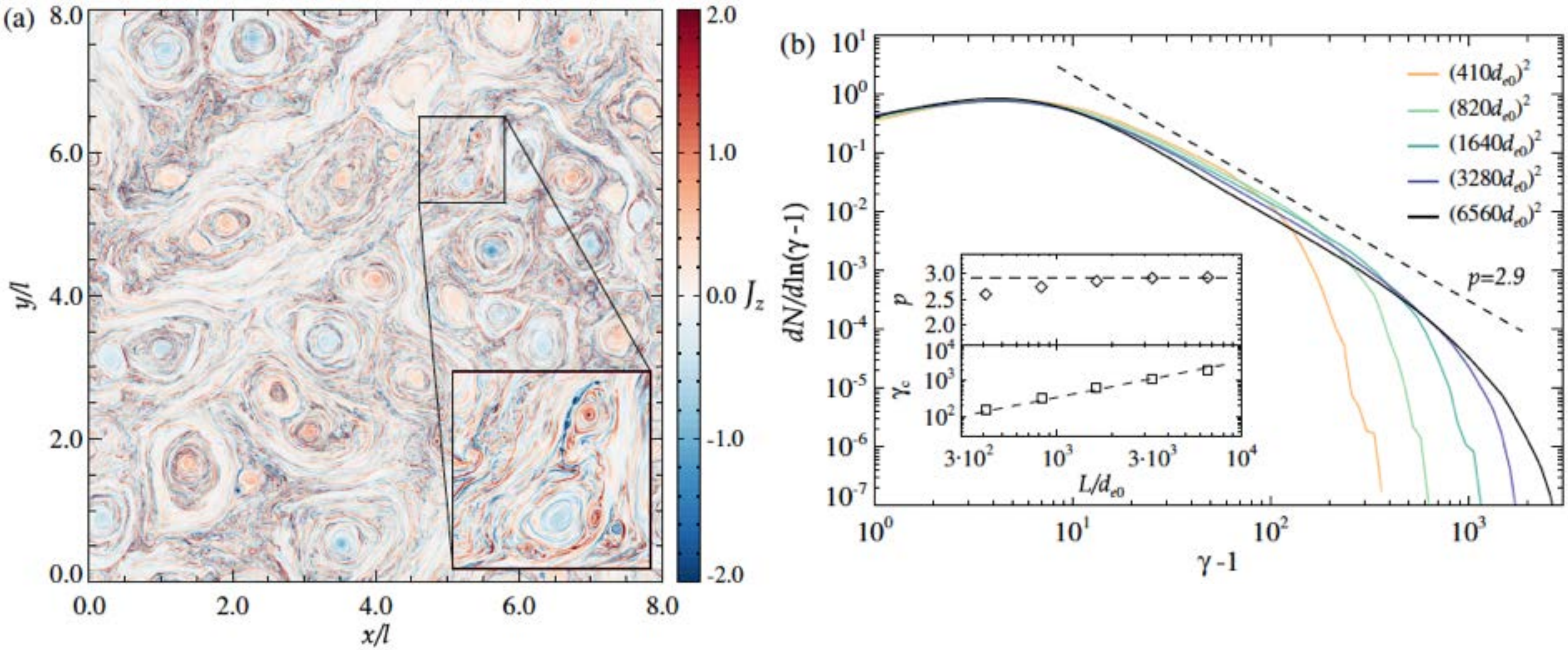
Hardens with increasing magnetization, converges with system size (but time-dependent)

VZ+ PRL 2017; VZ+ ApJL 2018 (also see Comisso & Sironi PRL 2018; Nattila 2019)

Complementary work

- 2D (& 3D) PIC simulations of decaying relativistic turbulence (Comisso & Sironi PRL 2018) confirm efficient particle acceleration at large sizes

Comisso & Sironi PRL 2018; arXiv 2019



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- Power-law index hardens with increasing magnetization
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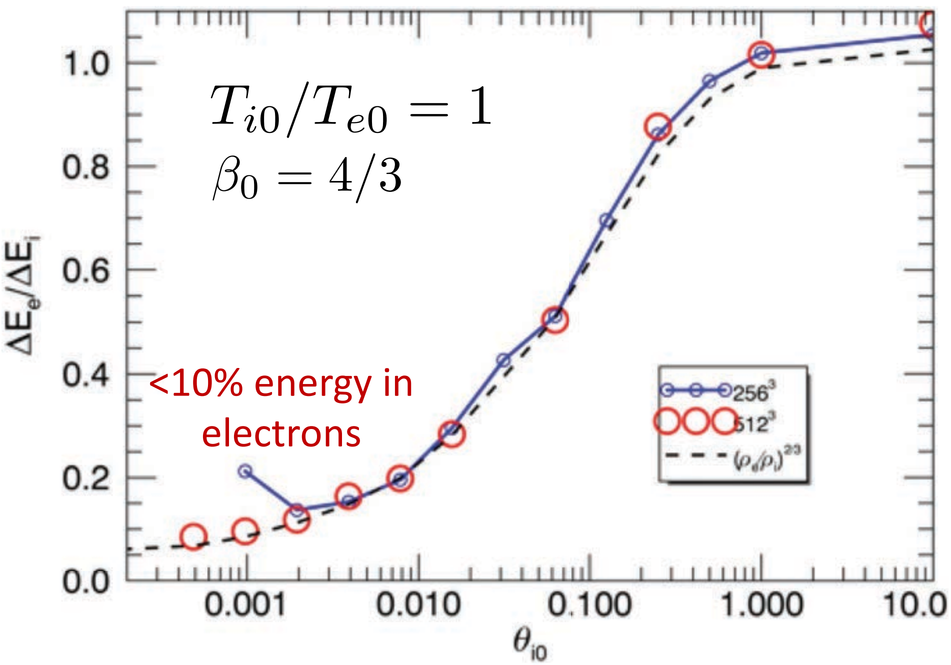
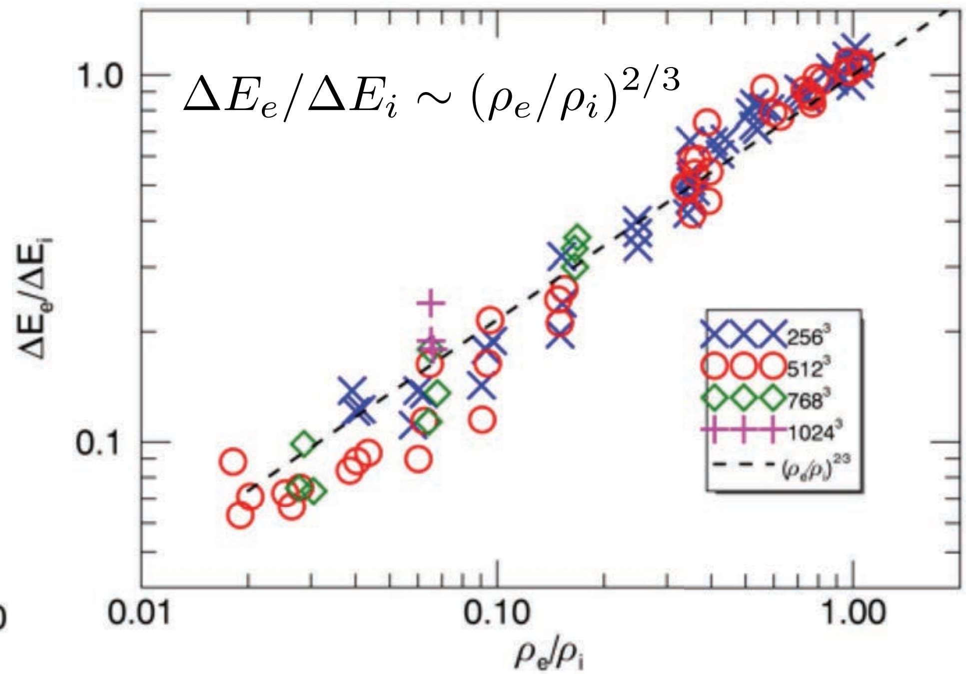
- Ions are preferentially heated and accelerated, at beta order unity (VZ+ 2019)

* System-size dependent formation time (VZ+ 2018); simulation sizes are limited

Electron-ion heating in trans-relativistic plasma





Trans-relativistic: $m_e c^2 < T < m_i c^2$ (ultra-relativistic electrons, sub-rel. ions)

Scale separation set by $\theta_i \equiv T/m_i c^2$ $(\rho_e/\rho_i \sim \theta_i^{1/2})$

Ions are preferentially heated (up to 90% of energy)

Broad parameter scan yields empirical fitting formula: $\Delta E_e/\Delta E_i \sim (\rho_e/\rho_i)^{2/3}$

VZ, Uzdensky, Werner & Begelman PRL 2019

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- Mechanism consistent with gyroresonant scattering by Alfvenic modes (Wong+ 2019); magnetic reconnection may play role at high magnetization (Comisso & Sironi 2019)

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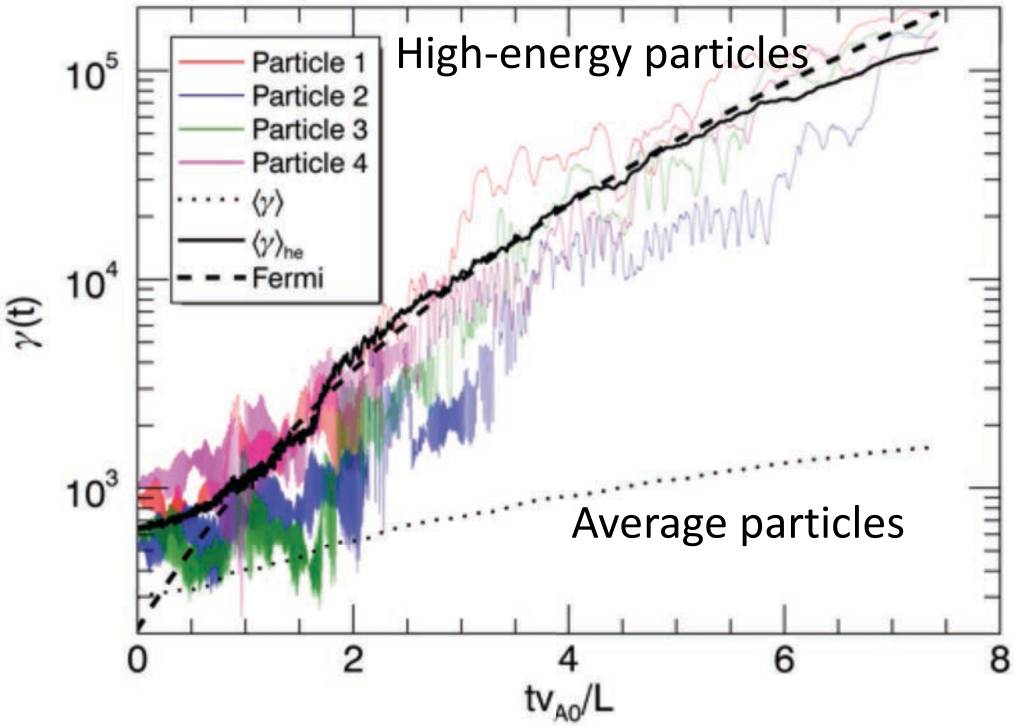
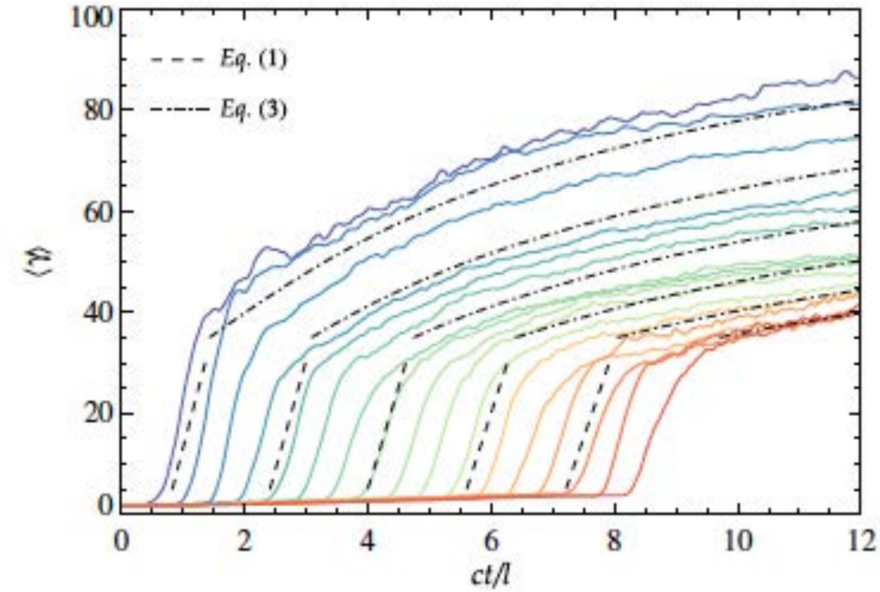
Acceleration mechanism

VZ+ ApJL 2018



Comisso & Sironi 2018



Energy gain is consistent with second-order Fermi acceleration/gyroresonant scattering (Fermi 1949; Schlickeiser 1989, Miller et al. 1990, Chandran 2000):

$$\frac{d\gamma}{dt} \sim \frac{\gamma}{\tau_{\rm acc}(t)} \implies \gamma \sim \gamma_i \exp\left(t/\tau_{\rm acc}\right)$$

$$\tau_{\rm acc} \propto \frac{Lc}{v_A^2(t)}$$

(Alfvenic scatterers)

Particle acceleration is diffusive

Statistical evolution of tracked particles is described by Fokker-Planck equation:

$$\partial_t f = \partial_\gamma (D \partial_\gamma f) - \partial_\gamma (A f)$$ (advection-diffusion in energy space)

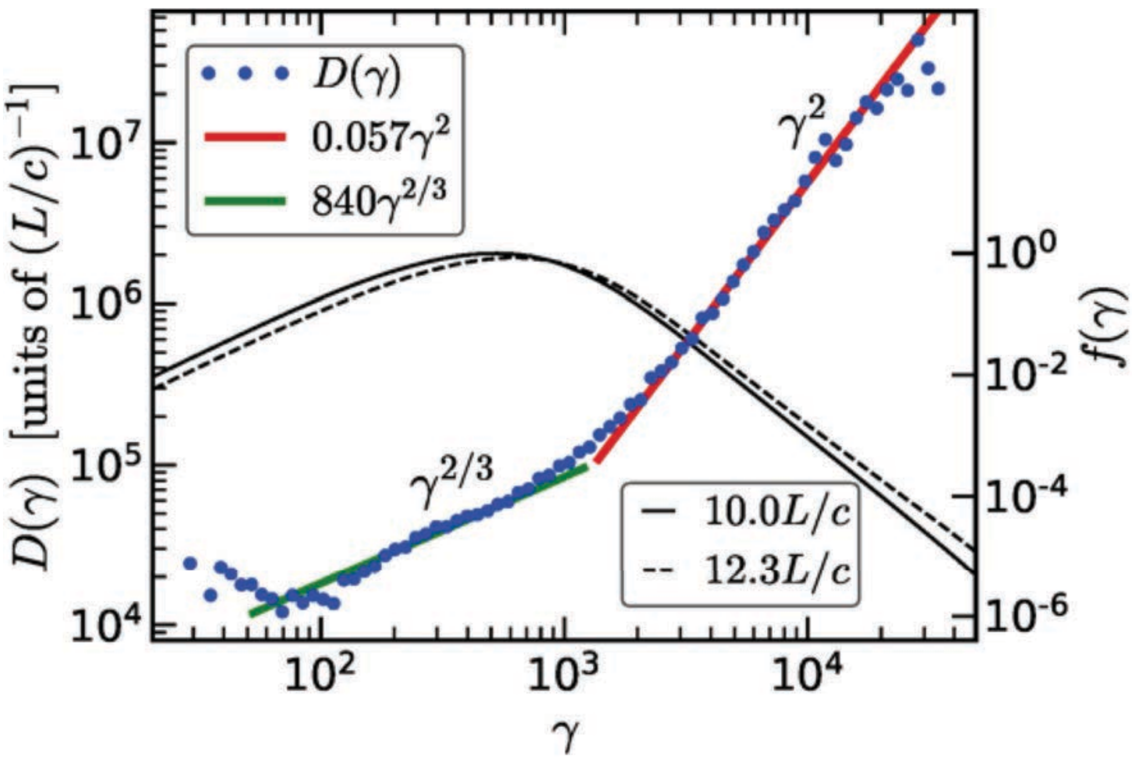
2nd order Fermi acceleration / gyroresonance by Alfven waves (e.g., Blandford & Eichler 1987):

$$D(\gamma) \sim \frac{u_A^2}{3c\lambda_{\rm mfp}} \gamma^2$$

Scattering by large-scale waves is sufficient to explain results:

$$\lambda_{\rm mfp} \sim L$$

Measured diffusion coefficient



Wong, VZ, Uzdensky, Werner & Begelman, *submitted* [arXiv:1901.03439]

(See also Dmitri Uzdensky's KITP talk online; Comisso & Sironi 2019)

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- Mechanism consistent with gyroresonant scattering by Alfvenic modes (Wong+ 2019); magnetic reconnection may play role at high magnetization (Comisso & Sironi 2019)
- Unknowns: how to predict power-law index, fraction of nonthermal particles, etc.
- Future direction 1: building and validating theory
- Future direction 2: adding complexity (radiation, pair production, driving, geometry, ...)
- Future direction 3: applying to model observations

* System-size dependent formation time (VZ+ 2018); simulation sizes are limited

Radiative turbulence

- In many high-energy astrophysical systems, electrons/positrons emit copious amounts of energy in radiation (synchrotron, inverse Compton, etc.)
- Understanding role of radiative cooling on turbulence and particle distributions is thus important
- Focus on *strong* radiative cooling, of sufficient strength to balance energy injection from external driving

Key questions:

1. Does radiative cooling influence turbulent cascade/structures?
2. Does the system attain a steady state? (equilibrium temperature)
3. How does radiative cooling influence nonthermal particle distributions?
4. What are observable radiative signatures? (spectra, beams)

Radiation implementation

- Implement external inverse Compton (IC) cooling by adding radiation reaction force to Lorentz force acting on relativistic electrons/positrons:

$$\boldsymbol{F}_{\mathrm{IC}} = -\frac{4}{3}\sigma_T U_{\mathrm{ph}} \gamma^2 \frac{\boldsymbol{v}}{c}$$

  σ_T is Thomson cross section, U_{ph} is (external) photon energy density
- Assume uniform, constant bath of external photons
- Assume optically thin medium (radiation escapes box)

Pair plasma radiative steady state

- Convenient feature of radiative turbulence in pair plasma is possible existence of a statistical steady state
- Energy injection rate from external driving: $\dot{\mathcal{E}}_{\rm inj} = \eta_{\rm inj} \frac{B_0^2}{8\pi} \frac{v_A}{L}$

  (assumes turbulent field $\delta B_{\rm rms} \sim B_0$)

- Radiative energy loss rate: $\dot{\mathcal{E}}_{\rm rad} = 16 n_0 \sigma_T c U_{\rm ph} \theta^2$

  (assumes Maxwell-Juttner distribution with temperature $T = \theta m_e c^2$)

- Predicted steady state temperature:

$$\dot{\mathcal{E}}_{\rm rad} \sim \dot{\mathcal{E}}_{\rm inj} \implies \boxed{\theta_{ss} = \frac{\eta_{\rm inj}}{16} \frac{m_e c^2}{\sigma_T U_{\rm ph} L} \sigma \frac{v_A}{c}} \qquad \bar{\gamma}_{ss} = 3\theta_{\rm ss}$$

- In simulations, $U_{\rm ph}$ chosen to give $\overline{\gamma}_{ss} \sim 300$

Numerical simulation setup

- Externally driven turbulence with 3D PIC code *Zeltron* (Cerutti+ 2013)
- Periodic cubic box (no particle escape)
- Initialize thermal plasma, apply large-scale driving (TenBarge+ 2014)
- Uniform background field $B_0 \sim \delta B_{\mathrm{rms}}$

- First consider relativistic pair plasma: $T/m_e c^2 \sim 100$
- Two physical parameters:

1) Magnetization (ratio of magnetic energy to total particle energy):

$$\sigma = \frac{B_{\mathrm{rms}}^2}{4\pi h} = \frac{3B_{\mathrm{rms}}^2}{16\pi n_0 \bar{\gamma} m_e c^2}$$

Alfvenic turbulence: $\frac{\delta v}{c} \sim \frac{v_A}{c} = \sqrt{\frac{\sigma}{\sigma+1}}$

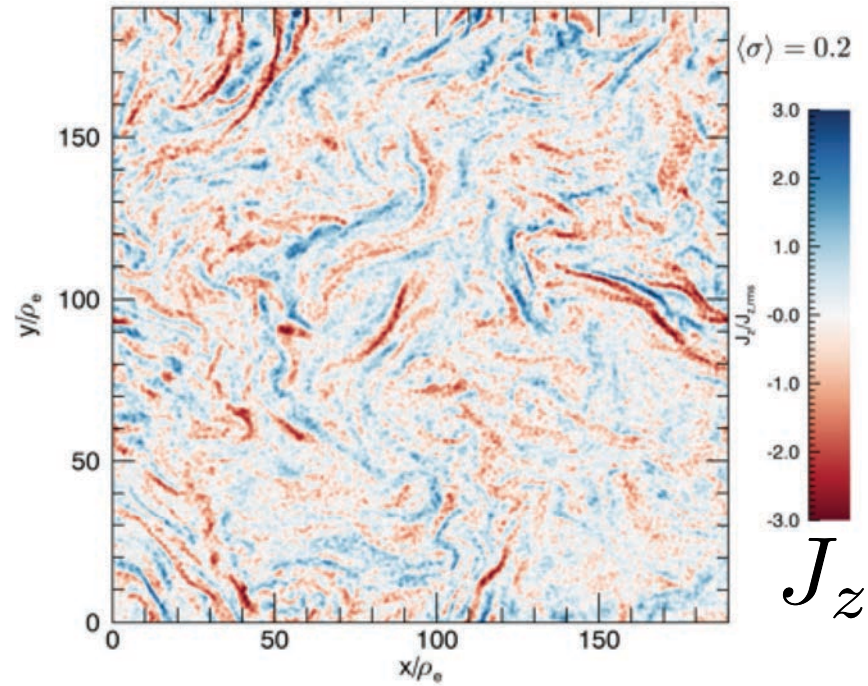
2) System size (ratio of driving scale to particle gyroradius):

$$L/2\pi\rho_e$$

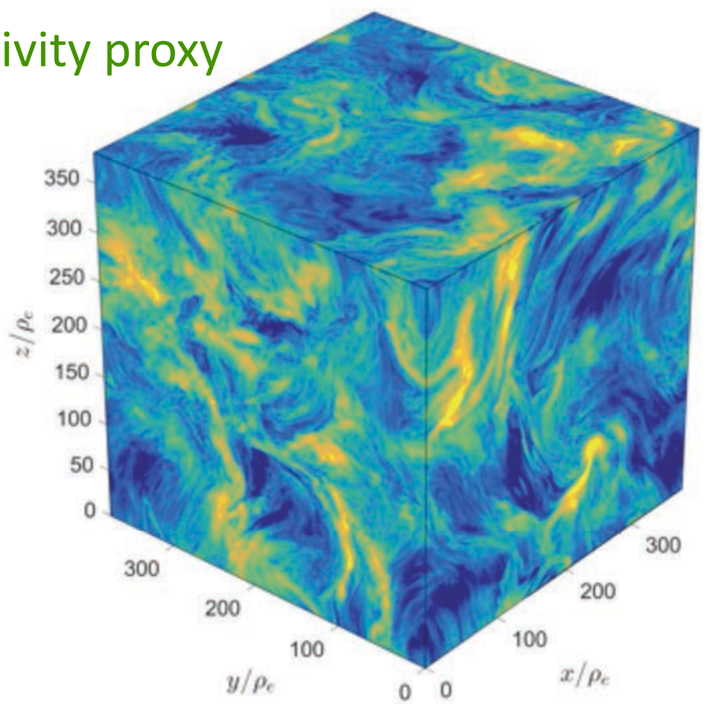
$$\rho_e = \frac{\bar{\gamma} m_e c^2}{e B_{\mathrm{rms}}}$$

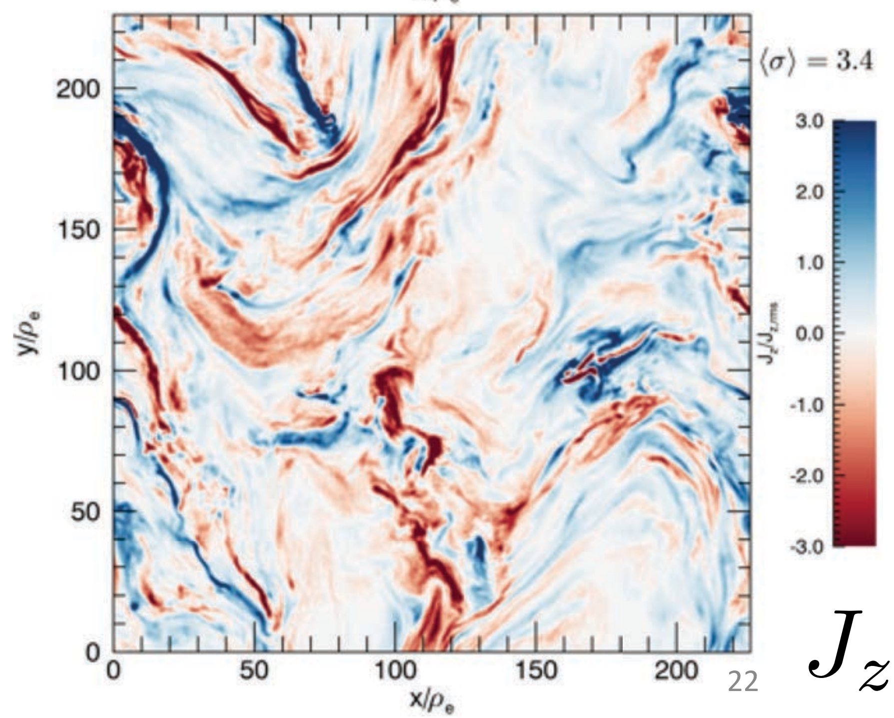
Parameter scan

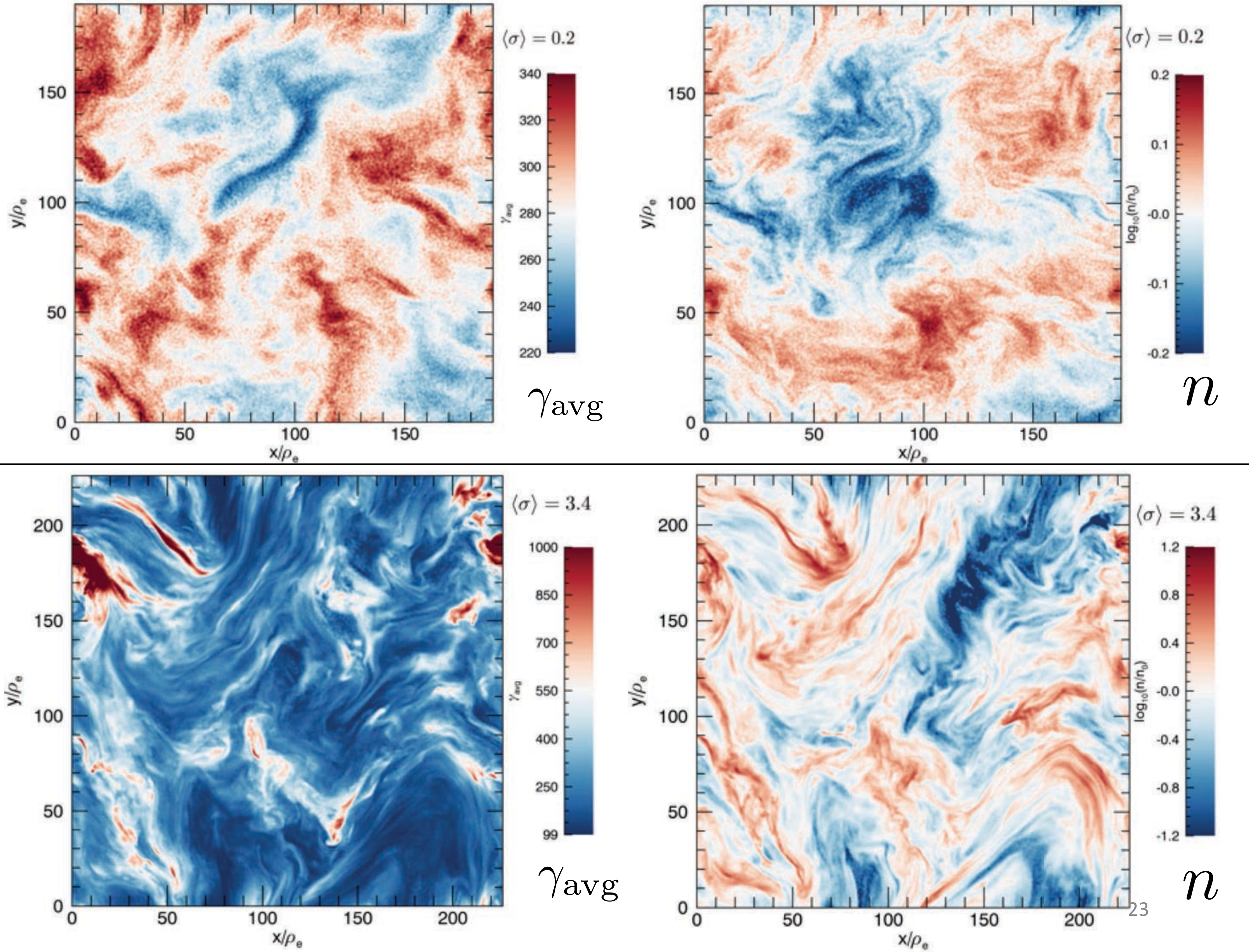
Case	N^3	$L/2\pi\langle\rho_e\rangle$	$\langle\sigma\rangle$	tv_A/L
rL1	768^3	60.4	0.90	24.6
rM1d4	512^3	29.6	0.20	34.1
rM1	512^3	39.4	0.86	24.2
rM4	512^3	38.9	3.4	35.8
rM4*	512^3	39.1	3.4	29.5
rS1d16	384^3	10.4	0.041	35.4
rS1d4	384^3	21.4	0.19	35.9
rS1	384^3	28.3	0.82	32.4
rS1*	384^3	28.3	0.83	60.1
rS4	384^3	28.1	3.3	29.7
rS16	384^3	24.9	11.0	32.1



Emissivity proxy

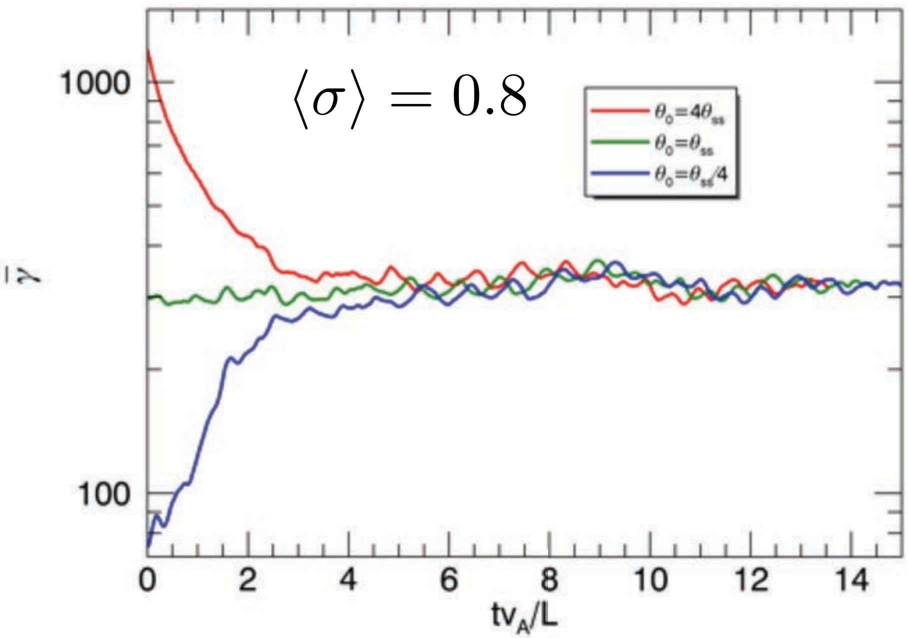




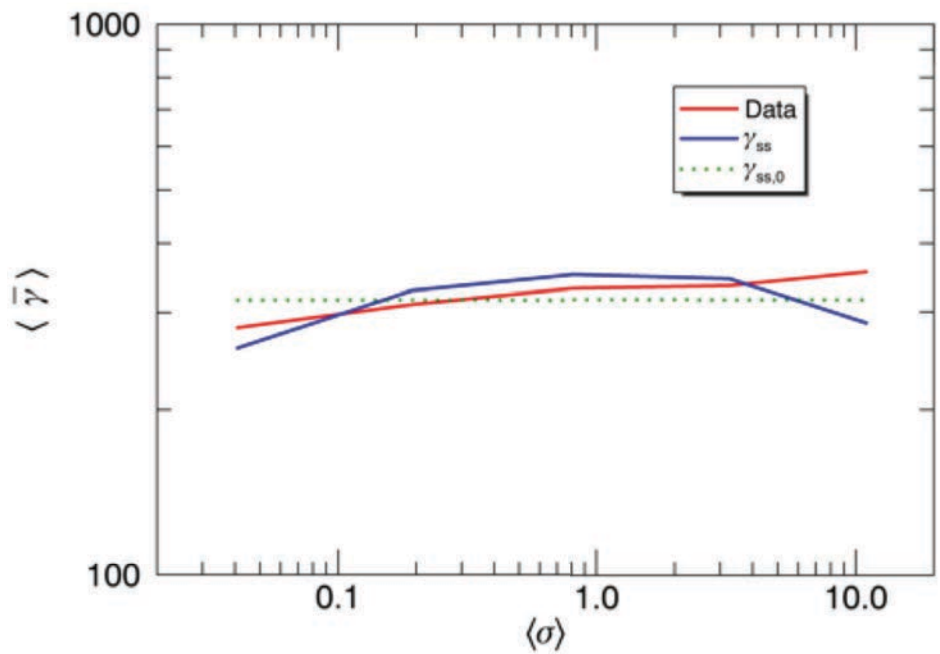

⟨σ⟩ = 0.2
γavg
n
⟨σ⟩ = 3.4
γavg
n
x/ρe
y/ρe
log10(n/n0)

Confirmation of steady state (pair plasma)

Temperature evolution



Temperature vs magnetization

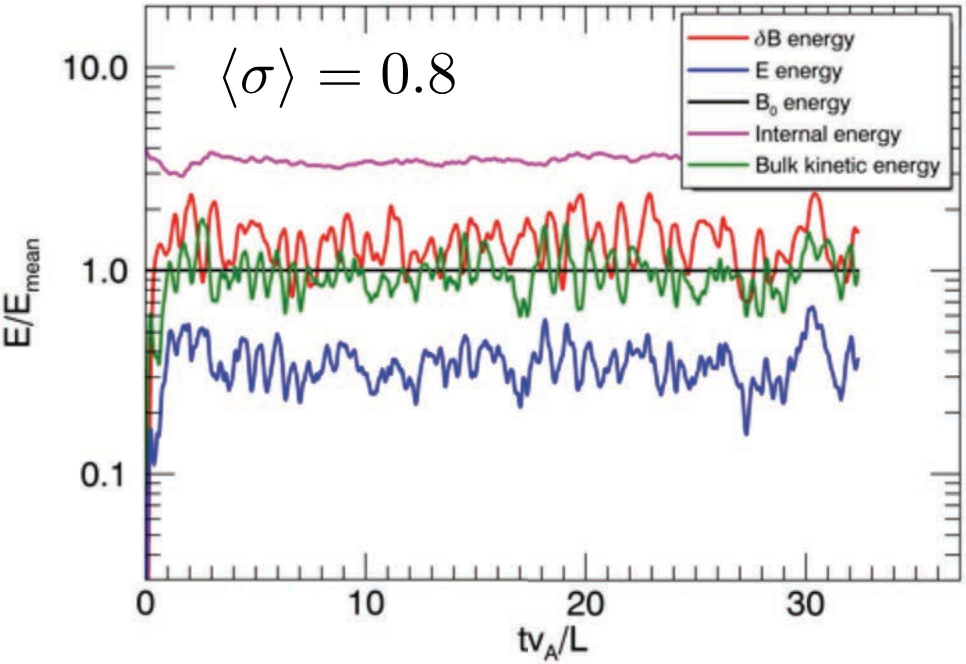


Steady state regardless of initial temperature

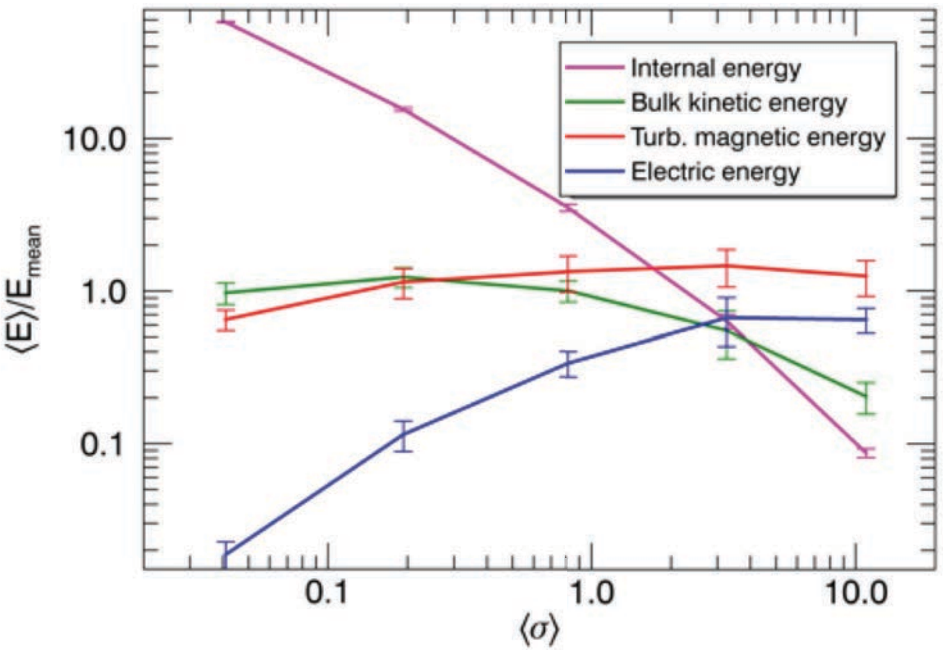
Steady-state particle energy agrees well with analytic estimate $(\bar{\gamma}_{ss} \sim 300)$

Steady-state energetics

Energy evolution



Energy vs magnetization



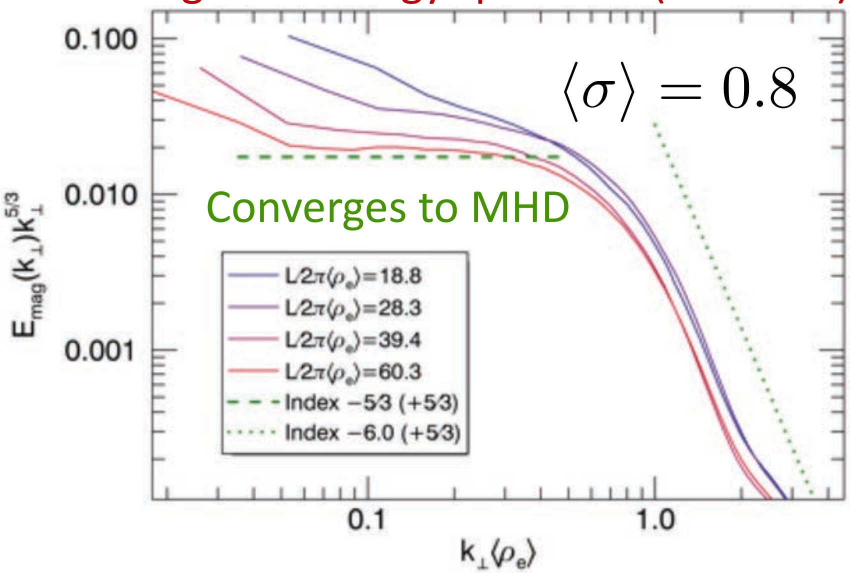
Low magnetization: Internal energy dominates; sub-relativistic flow ("classical MHD")

High magnetization: Electromagnetic energy dominates; relativistic flow

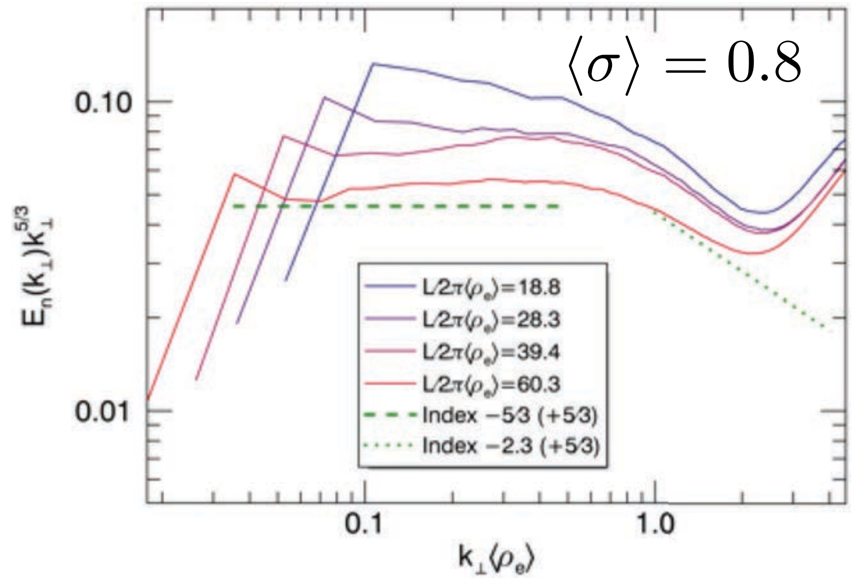
("force-free MHD"; see Thompson & Blaes 1998, Cho 2005, Zrake & East 2016)

Turbulence spectra

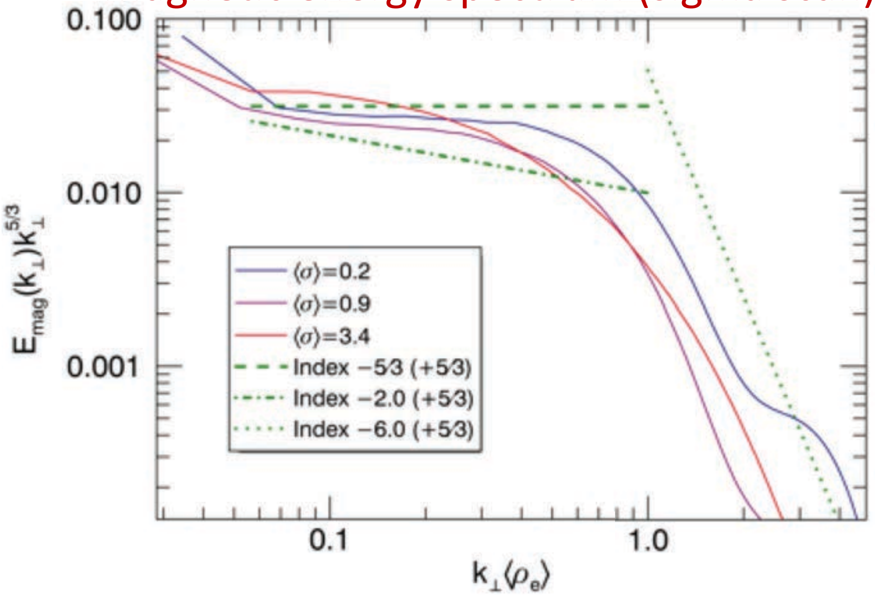
Magnetic energy spectrum (size scan)



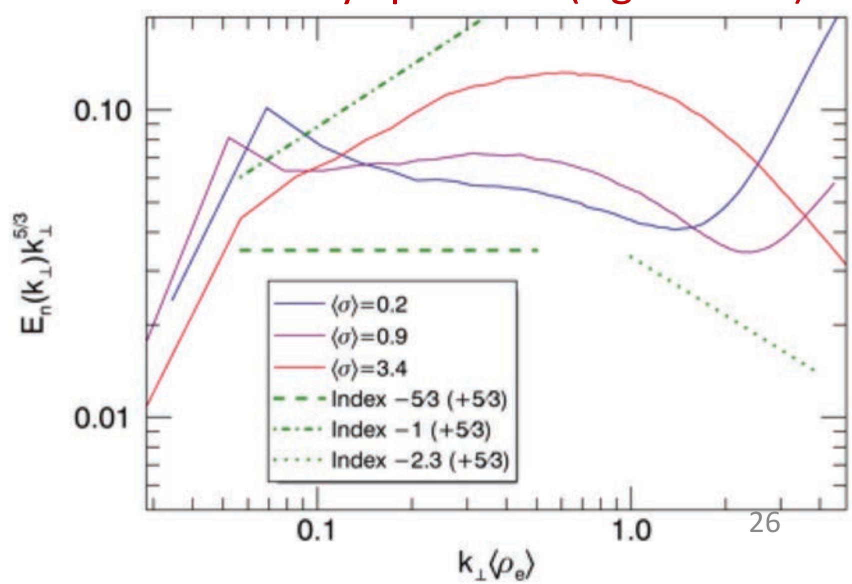
Density spectrum (size scan)



Magnetic energy spectrum (sigma scan)

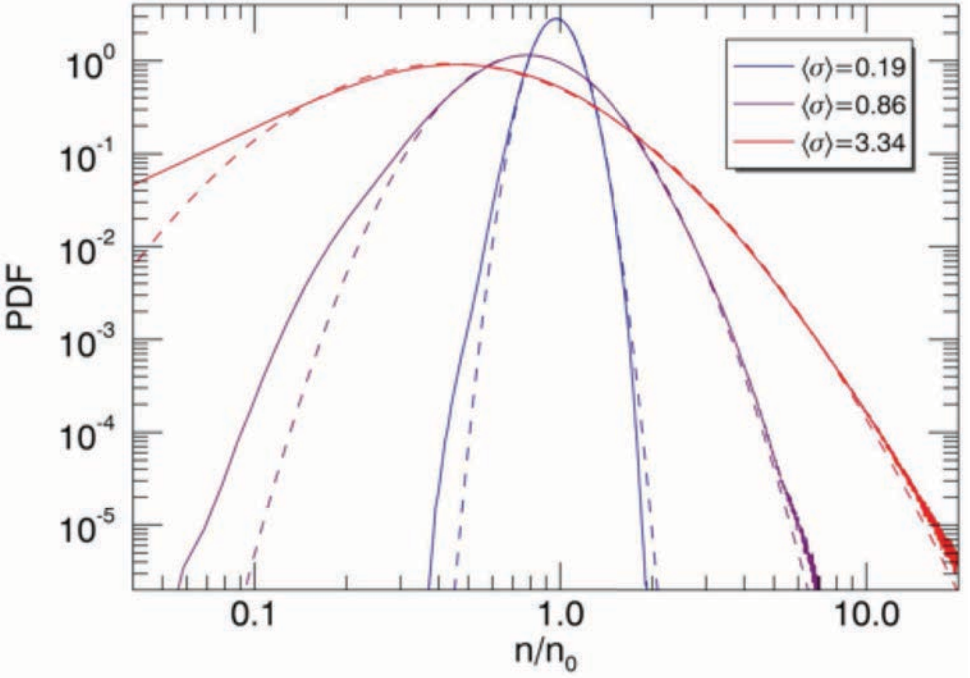


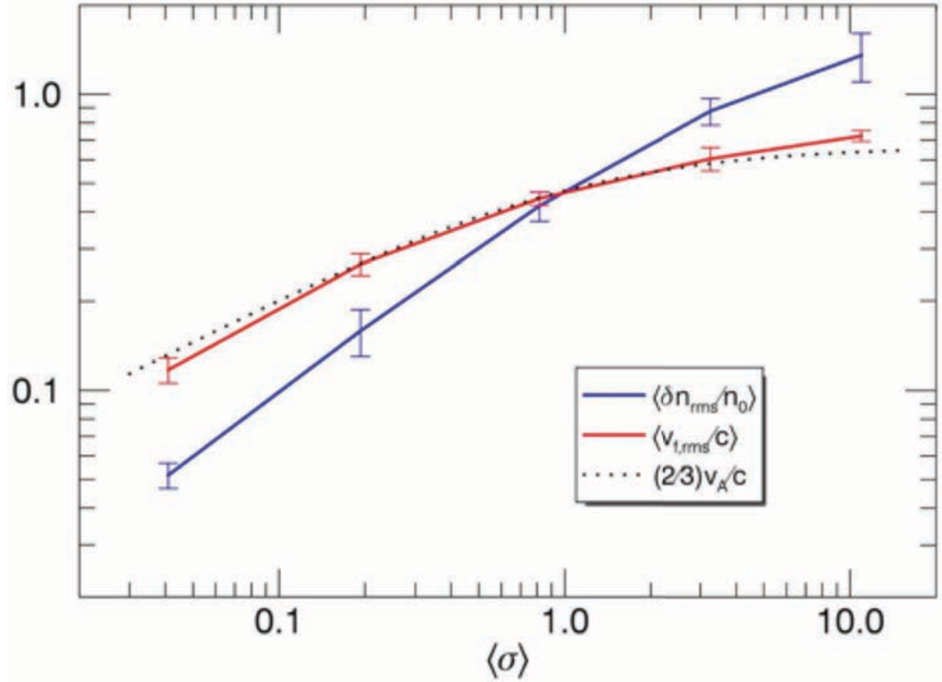
Density spectrum (sigma scan)



Density statistics

Density distribution



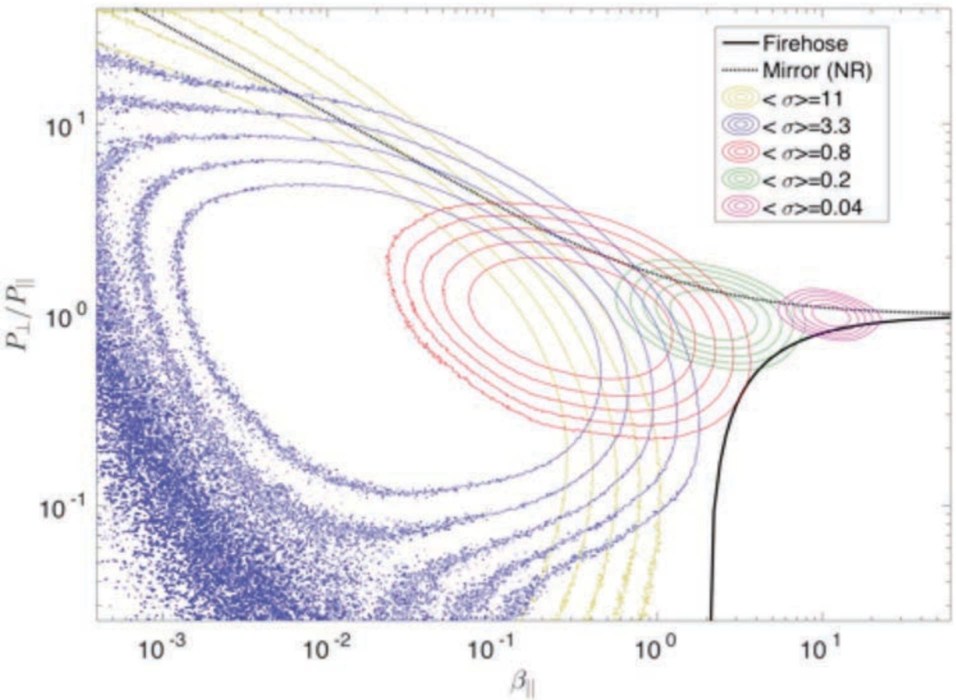


Density distribution close to lognormal

Density fluctuations increase with magnetization

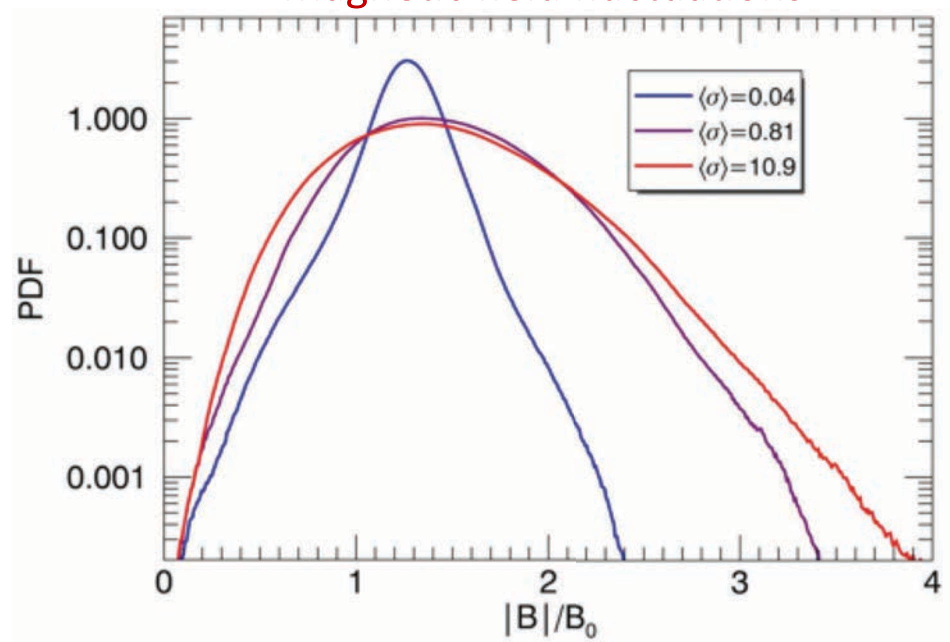
Non-MHD aspects: pressure anisotropy

Brazil plot



(thresholds from Cho & Hau 2004)

Magnetic field fluctuations



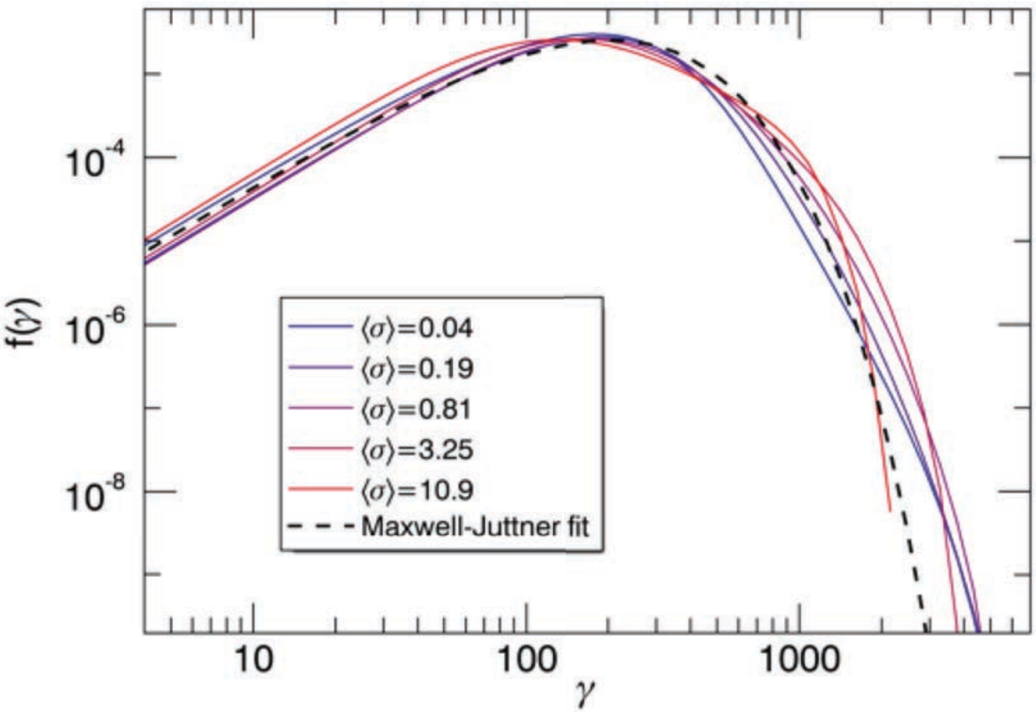
Pressure anisotropy bound by kinetic firehose, mirror instabilities (like solar wind)

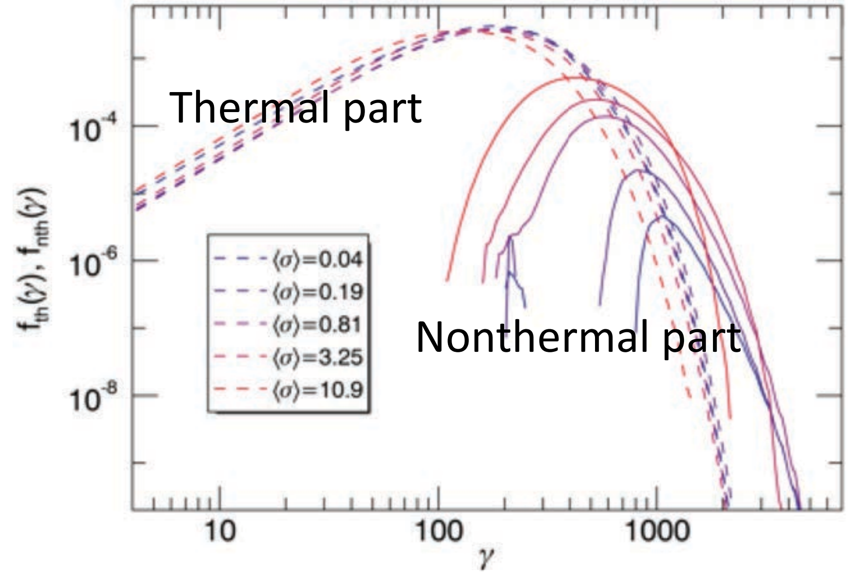
Magnetic field fluctuations become rotational at low magnetization

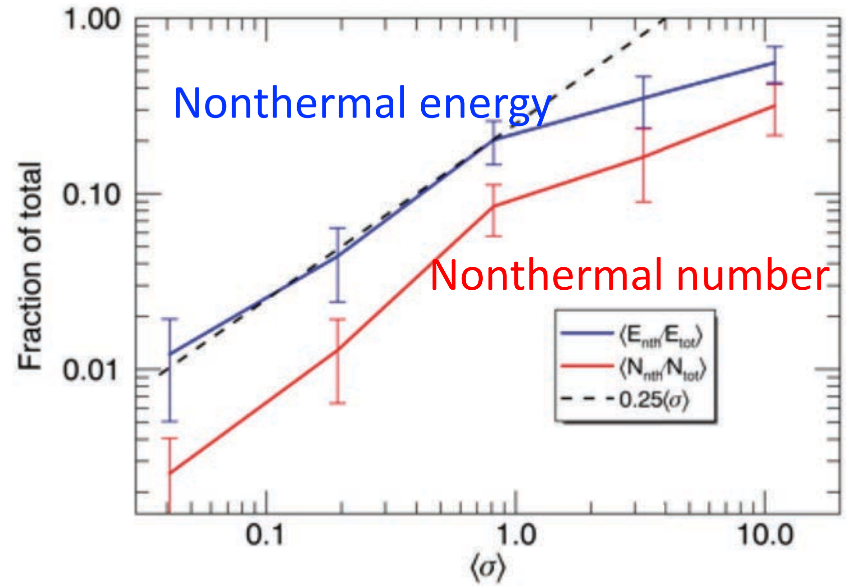
(magnetoimmutability? Squire+ JPP 2019)

Particle statistics

Steady-state particle distributions







Quasi-thermal particle distribution

Power-law tail quenched by cooling!

(anticipated by, e.g., Schlickeiser 1984)

Stochastic model for steady-state distribution

Fokker-Planck equation provides general model for stochastic particle energization:

$$\partial_t f = \partial_\gamma \left(\gamma^2 D_{pp} \partial_\gamma \frac{f}{\gamma^2} \right) - \partial_\gamma \left(A_p f - \frac{\gamma^2}{\gamma_0 \tau_c} f \right)$$

(diffusion + advection + cooling)

Diffusion coefficient (2nd order Fermi):

$$D_{pp} \sim \frac{\gamma_0^2}{\tau_0} + \frac{\gamma^2}{\tau_2}$$

Advection coefficient (1st order Fermi):

$$A_p \sim \frac{\gamma_0}{\tau_h} + \frac{\gamma}{\tau_a}$$

Analytic steady-state solution (four free parameters):

$$f(x) \propto x^2 \left(\Gamma_0 + \Gamma_2 x^2 \right)^{\Gamma_a / 2\Gamma_2} \exp \left[-\frac{x}{\Gamma_2} + \frac{\Gamma_0 + \Gamma_h \Gamma_2}{\Gamma_2^{3/2} \Gamma_0^{1/2}} \tan^{-1} \left(\sqrt{\frac{\Gamma_2}{\Gamma_0}} x \right) \right]$$

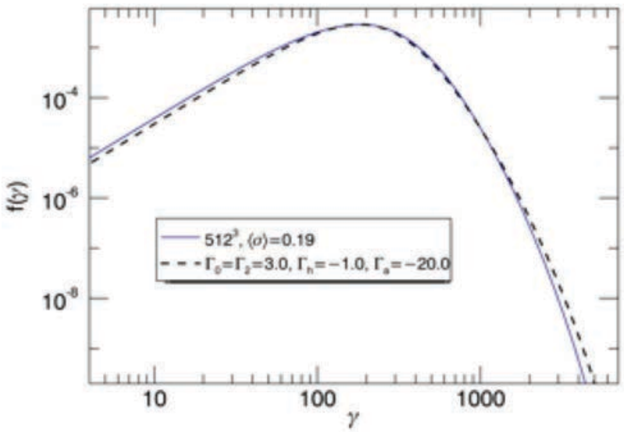
where $x = \gamma / \gamma_0$, $\Gamma_s = \tau_c / \tau_s$

Generically quasi-thermal

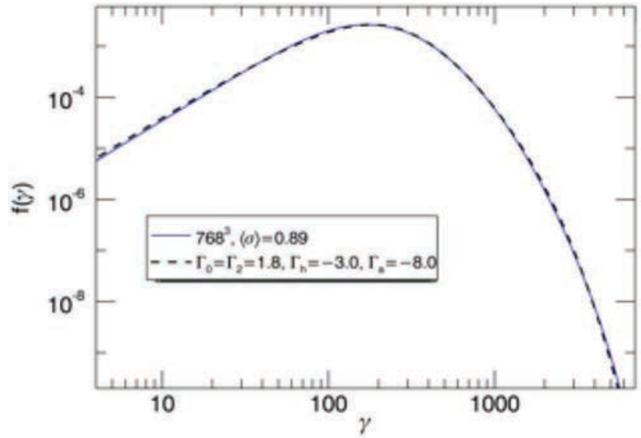
Note: recover Maxwell-Juttner distribution for sole diffusion $(\Gamma_0 = \Gamma_h = \Gamma_a = 0)$

Stochastic acceleration in simulations

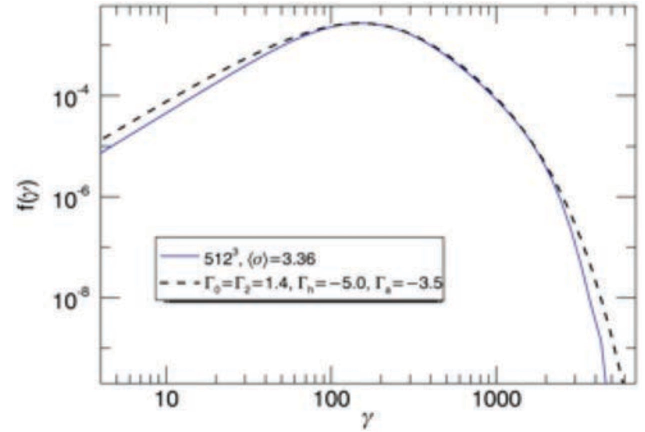
Low sigma



Moderate sigma



High sigma



Assumed scattering by Alfven modes: $\tau_2 \sim \dfrac{3\lambda_{\rm mfp}c}{u_A^2} \sim \dfrac{3L}{\sigma c}$

Other three parameters: choose by hand

Best fit parameters imply a first-order deceleration process

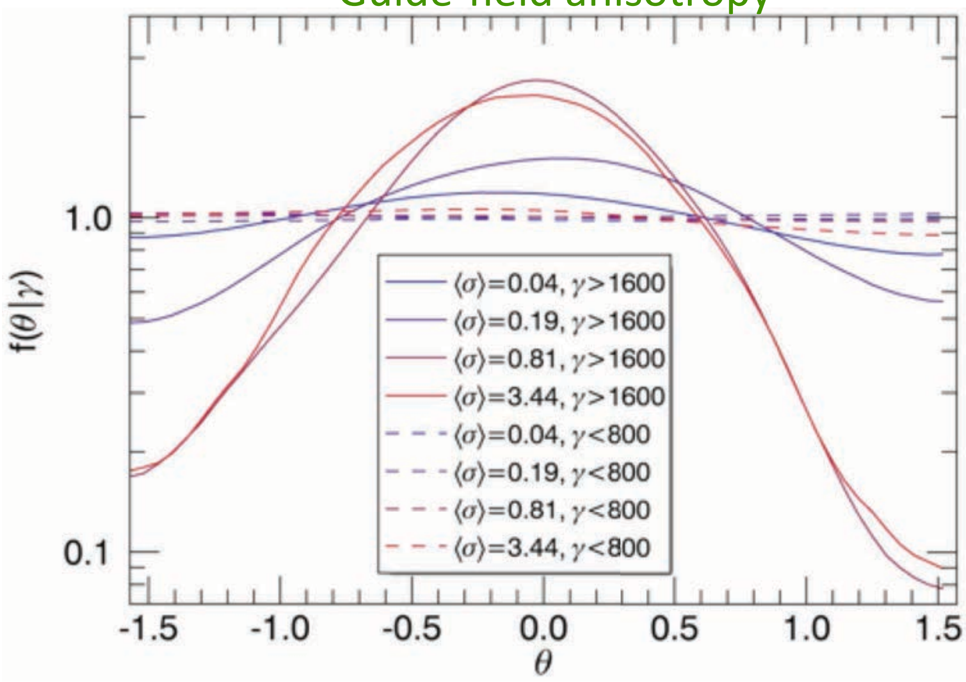
$\langle\sigma\rangle$	$\Gamma_0=\Gamma_2$	Γ_h	Γ_a
0.20	3.0	−1.0	−20.0
0.90	1.8	−3.0	−8.0
3.4	1.4	−5.0	−3.5

Anisotropy of particle distribution

- In addition to energy distribution of particles, can look at anisotropy of their momentum distribution
- Outgoing radiation is beamed in directions of anisotropy
- Define global conditional momentum anisotropy distribution:

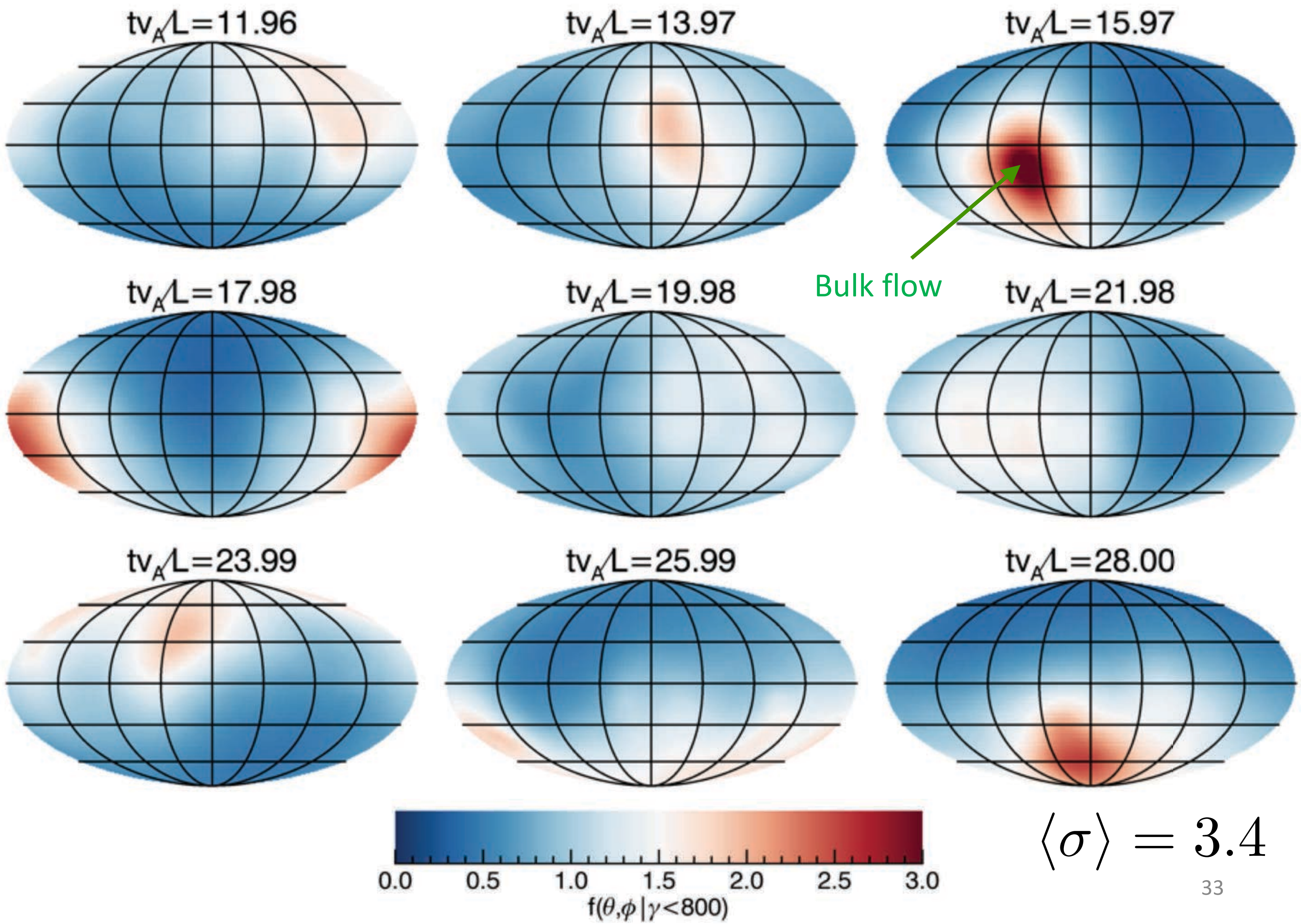
$$f(\theta, \phi | \gamma_{\text{thr},1} < \gamma < \gamma_{\text{thr},2}) \equiv \int_{\gamma_{\text{thr},1}}^{\gamma_{\text{thr},2}} d\gamma f(\theta, \phi, \gamma)$$

Guide-field anisotropy



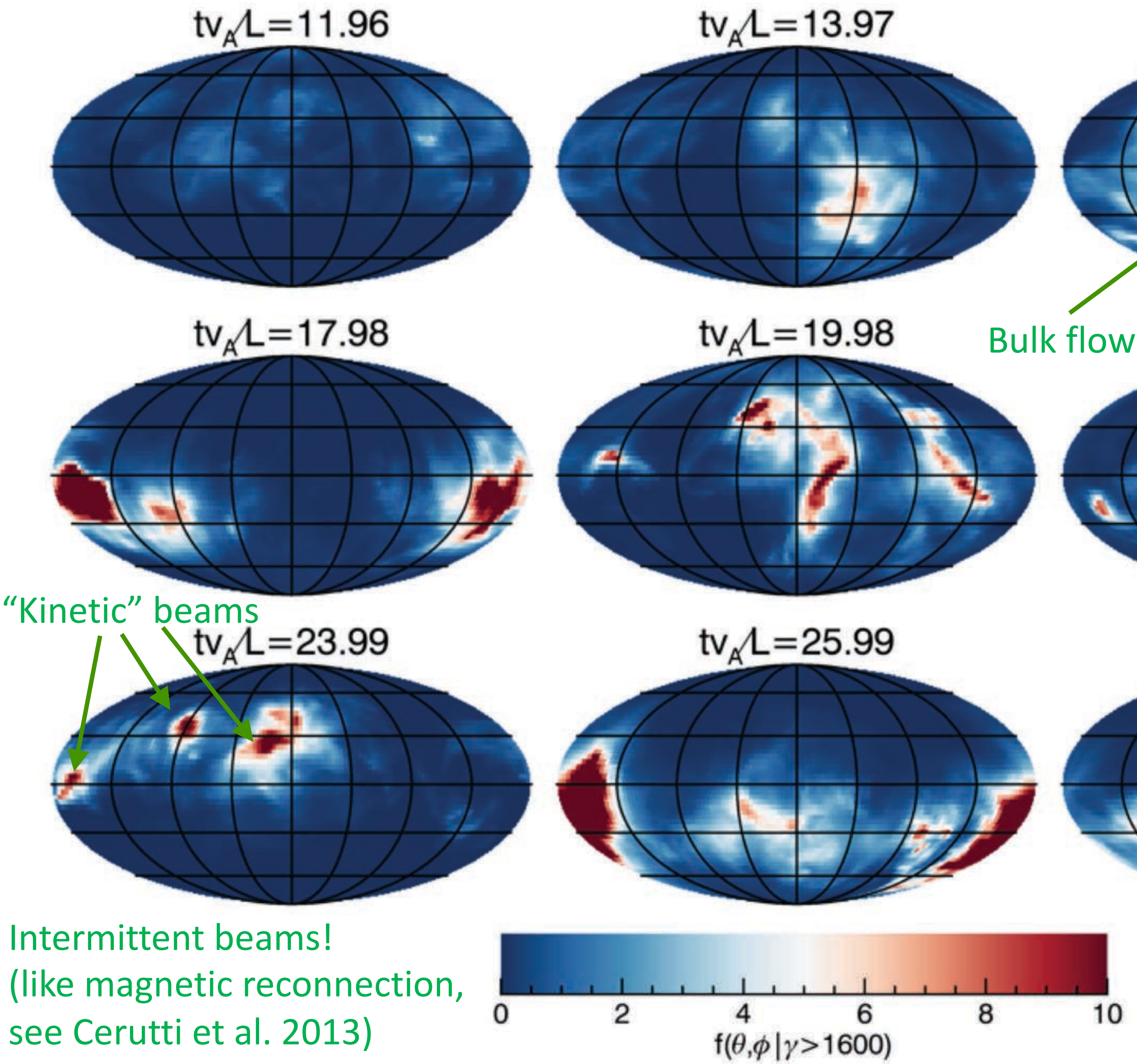
$$\theta = \cos^{-1}(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{z}})$$

Global momentum anisotropy – low energy



$$\langle \sigma \rangle = 3.4$$

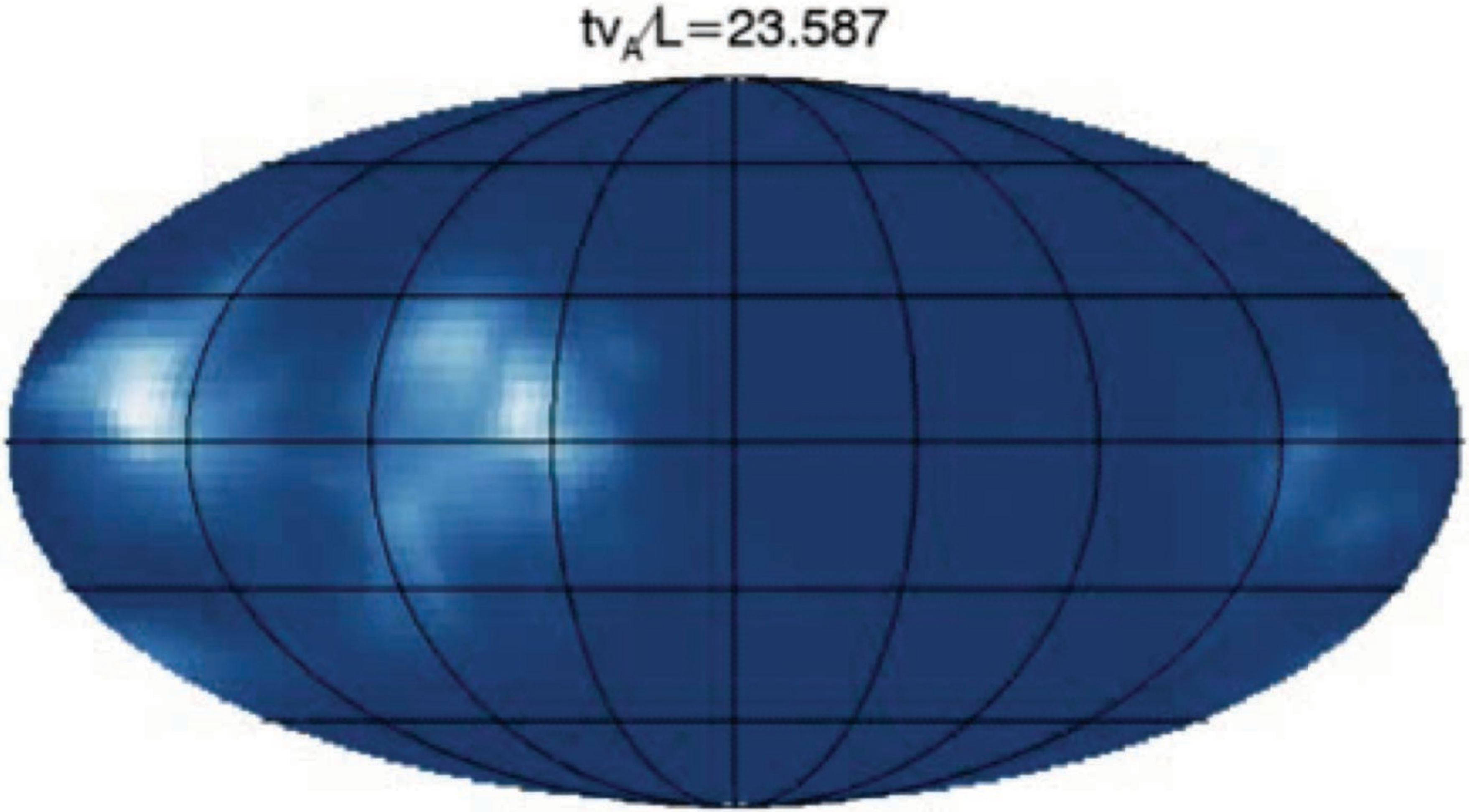
Global momentum anisotropy – high energy



Intermittent beams!
(like magnetic reconnection,
see Cerutti et al. 2013)

$$\langle \sigma \rangle = 3.4$$

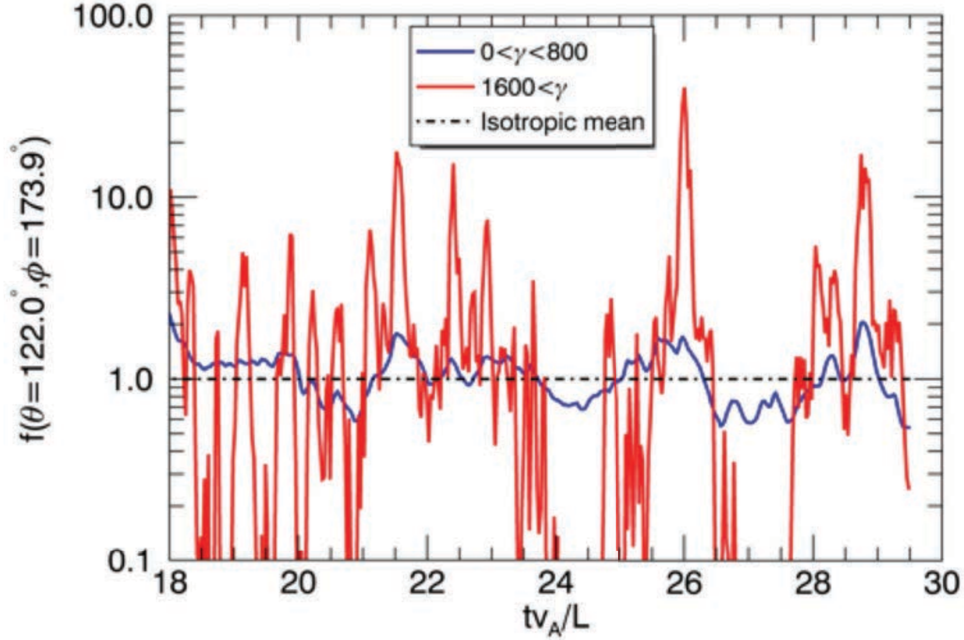
Angular distributions – high energy



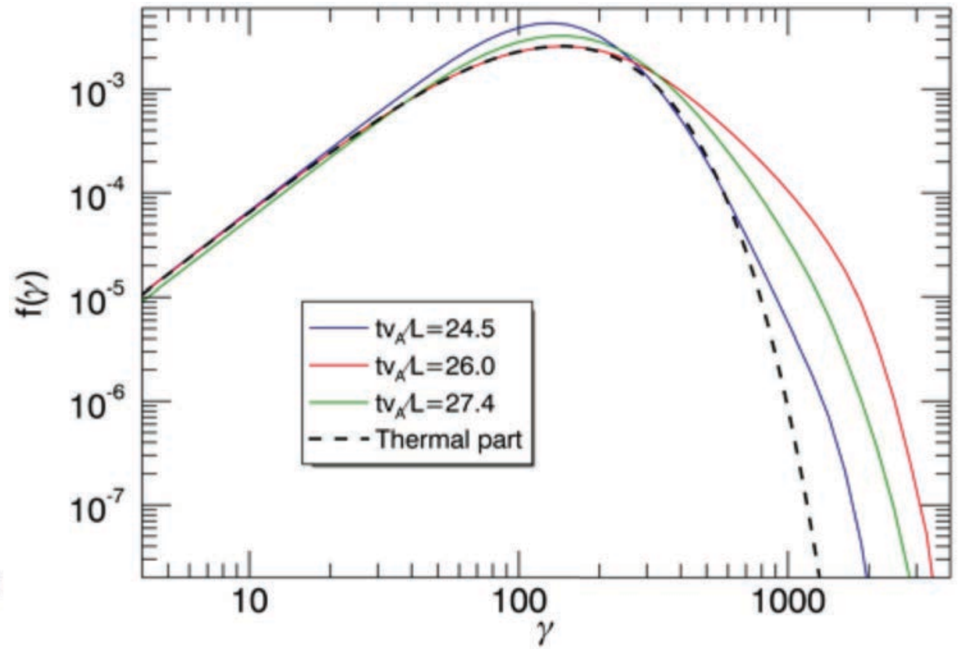
$$\langle \sigma \rangle = 3.4$$

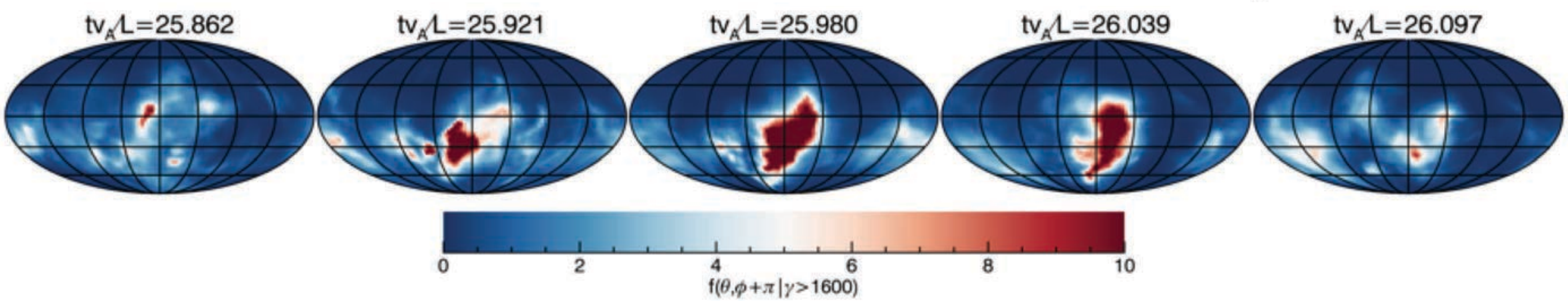
Beaming event in detail (tc/L = 26.0)

Momentum distribution in beam direction



Nonthermal energization



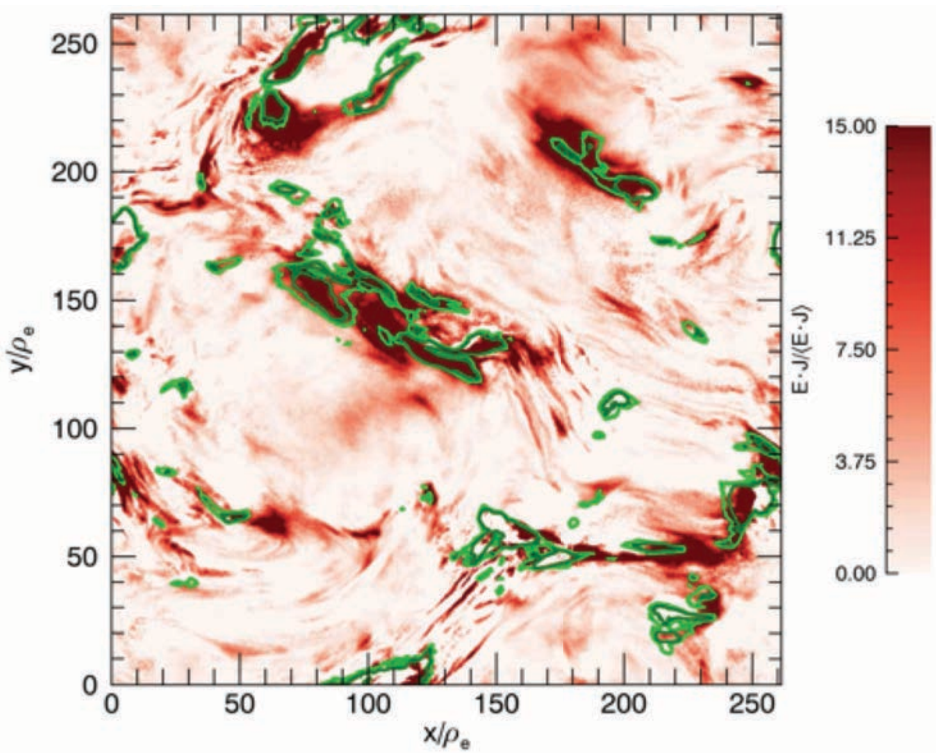


High-energy particle beam appears as rapid, intense flare in given line of sight

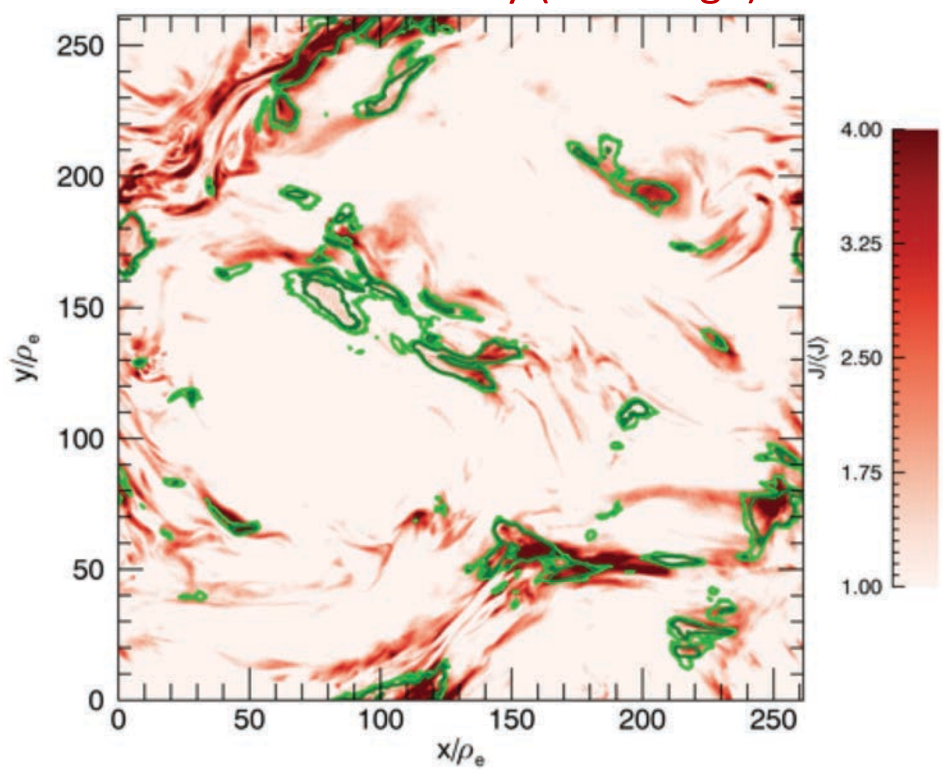
Mechanism of beaming

High-energy particle density (green contours)

Heating rate (red image)



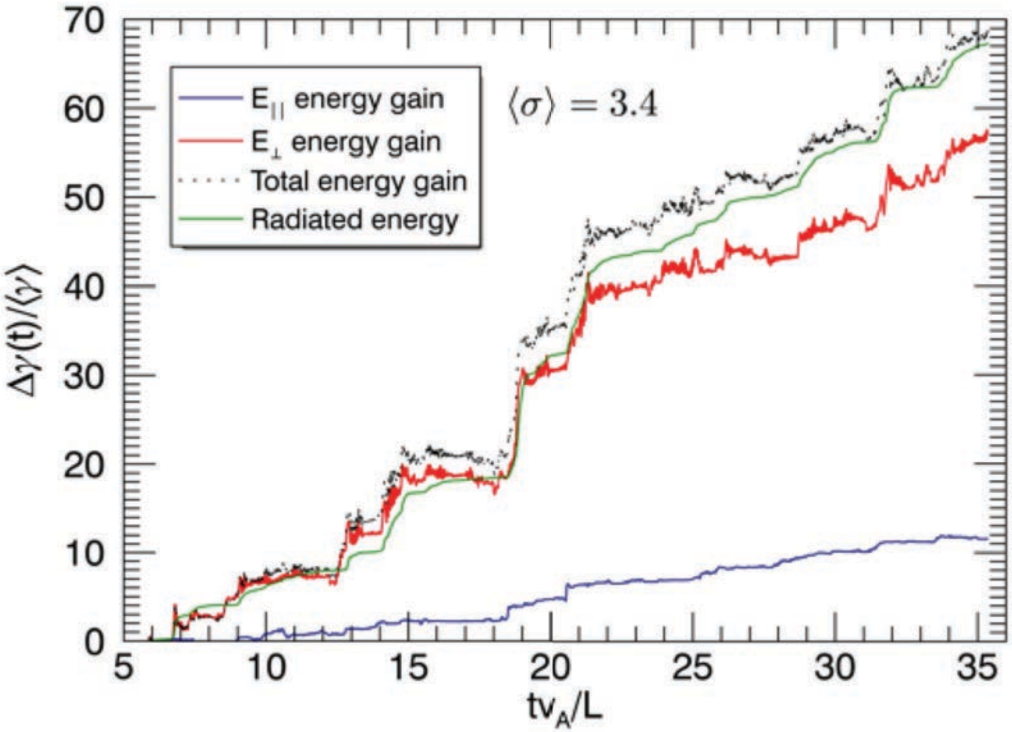
Current density (red image)



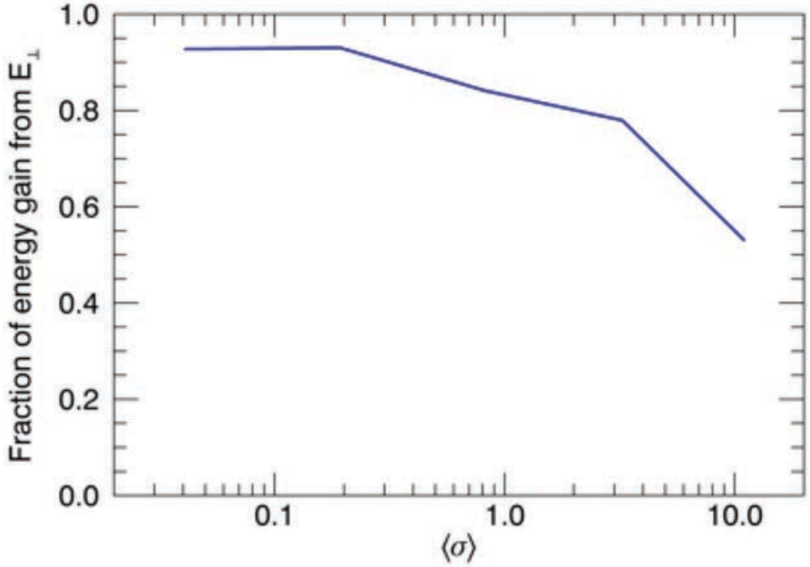
High-energy particles $(\gamma > 1200)$ correlated with intermittent current sheets, suggesting that magnetic reconnection may energize and beam particles

Tracked particle energization

Work done on a tracked particle



Work done by perpendicular field



Tracked particle energization dominated by perpendicular (ideal) electric fields, consistent with diffusive particle acceleration (e.g., gyroresonant acceleration)

Signatures of injection by parallel fields before most rapid acceleration events

Parallel fields important at high magnetization, possibly due to magnetic reconnection

Radiative electron-ion turbulence

- In contrast to pair plasma, the existence of a steady state for driven turbulence in radiative electron-ion plasma is nontrivial
- Since ions do not radiate, they will continuously heat up unless there is a mechanism to transfer thermal energy from ions to electrons
- No viable collisionless mechanism of thermal coupling (more efficient than Coulomb collisions) is known to exist (theoretically)

Key question:

Does there exist a sufficiently strong collisionless thermal coupling mechanism to prevent the ion-to-electron temperature ratio from growing very, very large?

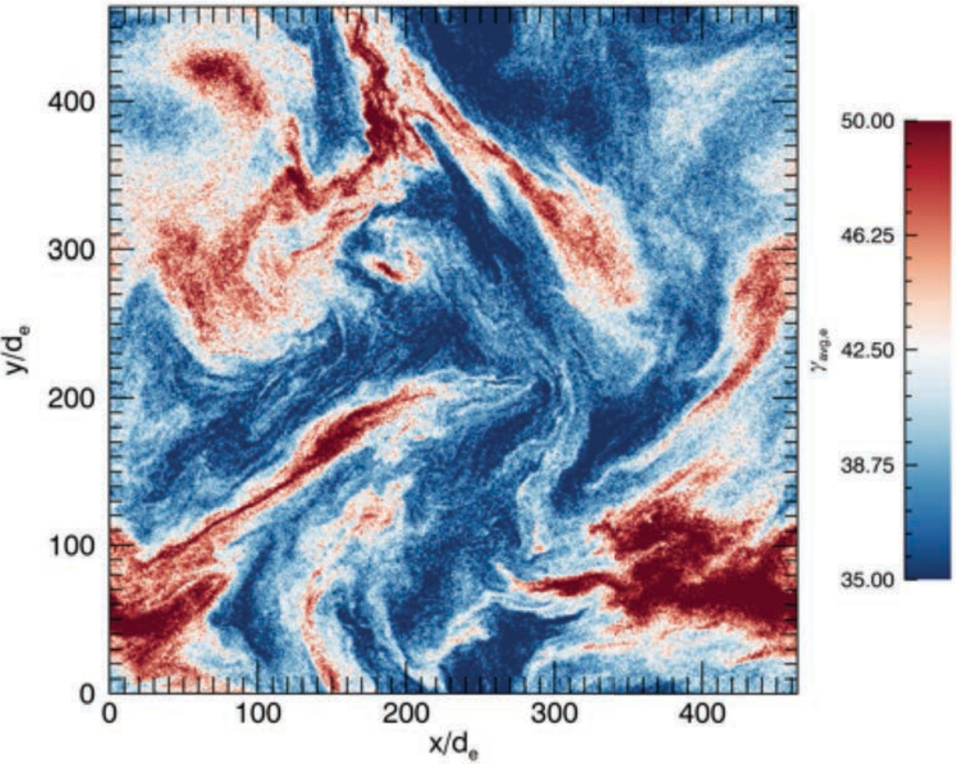
(in other words, do electrons and ions both attain a steady state energy?)

Important implications for radiatively inefficient accretion flows!

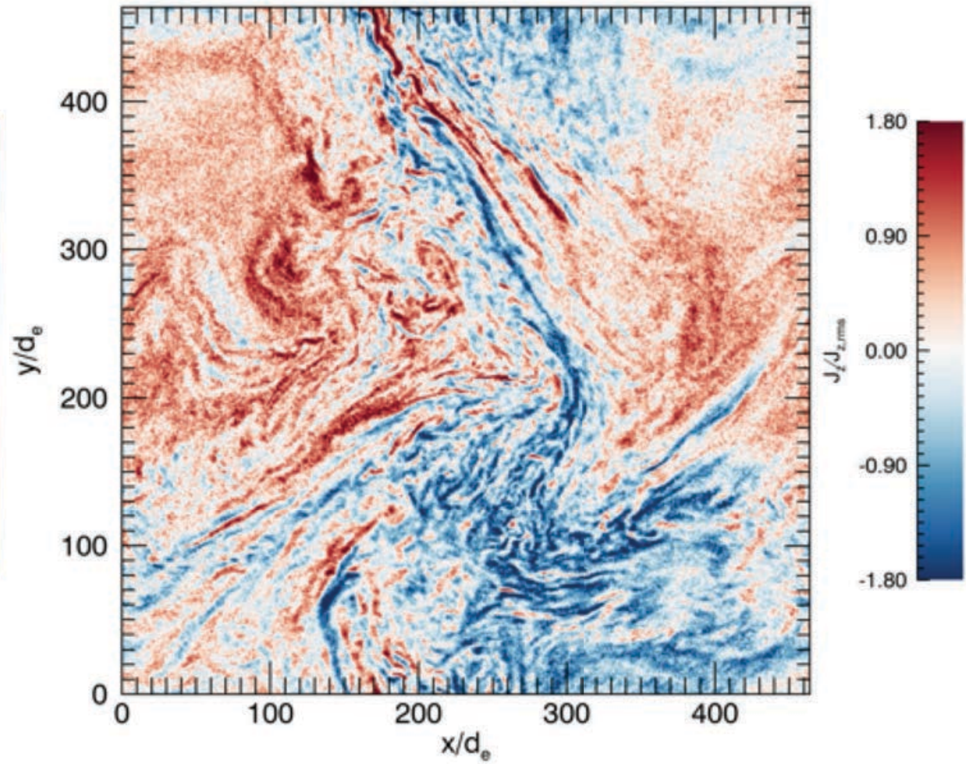
Radiative electron-ion setup

- Fiducial simulation: $\beta_0 = 4/3 \qquad L/2\pi\rho_{i0} = 27.2$
- Both species relativistically hot: $\theta_{i0} = 100$
- Photon energy density U_{ph} chosen near pair plasma steady state value

Electron energy density



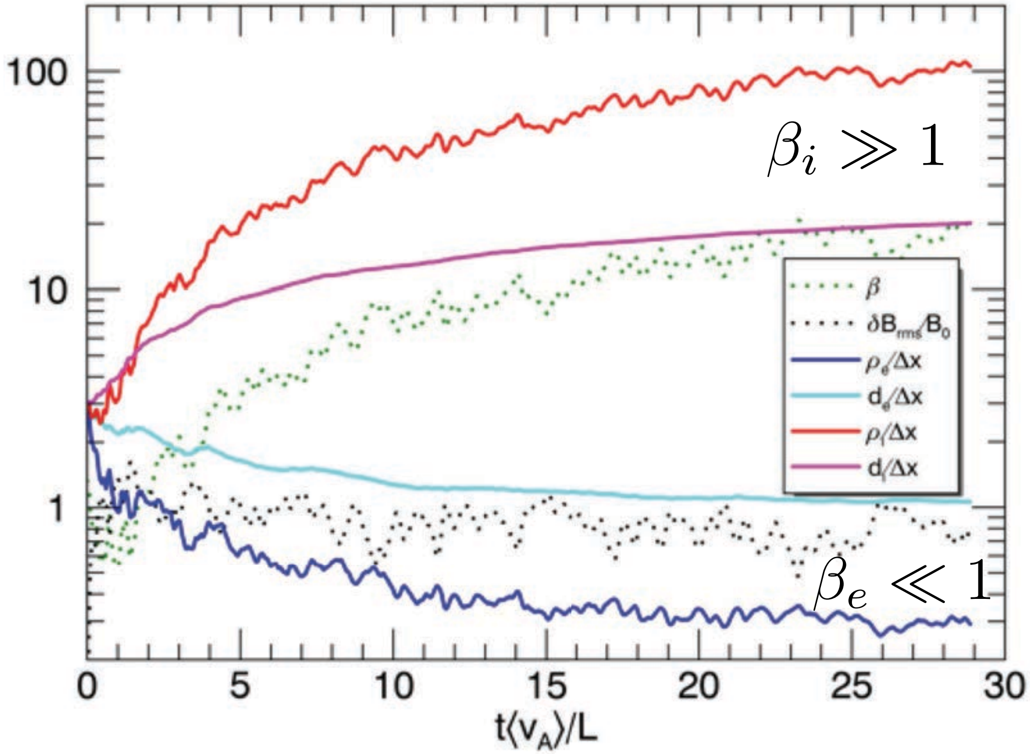
Current density



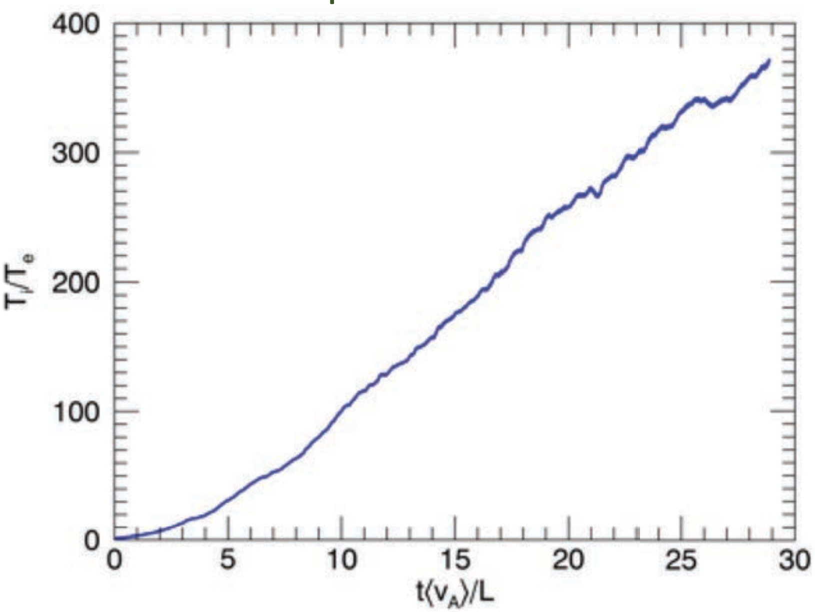
Unbounded growth of temperature ratio

- Electrons attain a steady-state temperature, ions do not (no coupling!)
- Temperature ratio builds up to $T_i/T_e \gtrsim 300$
- 8% of injected energy is lost to radiation

Parameter evolution



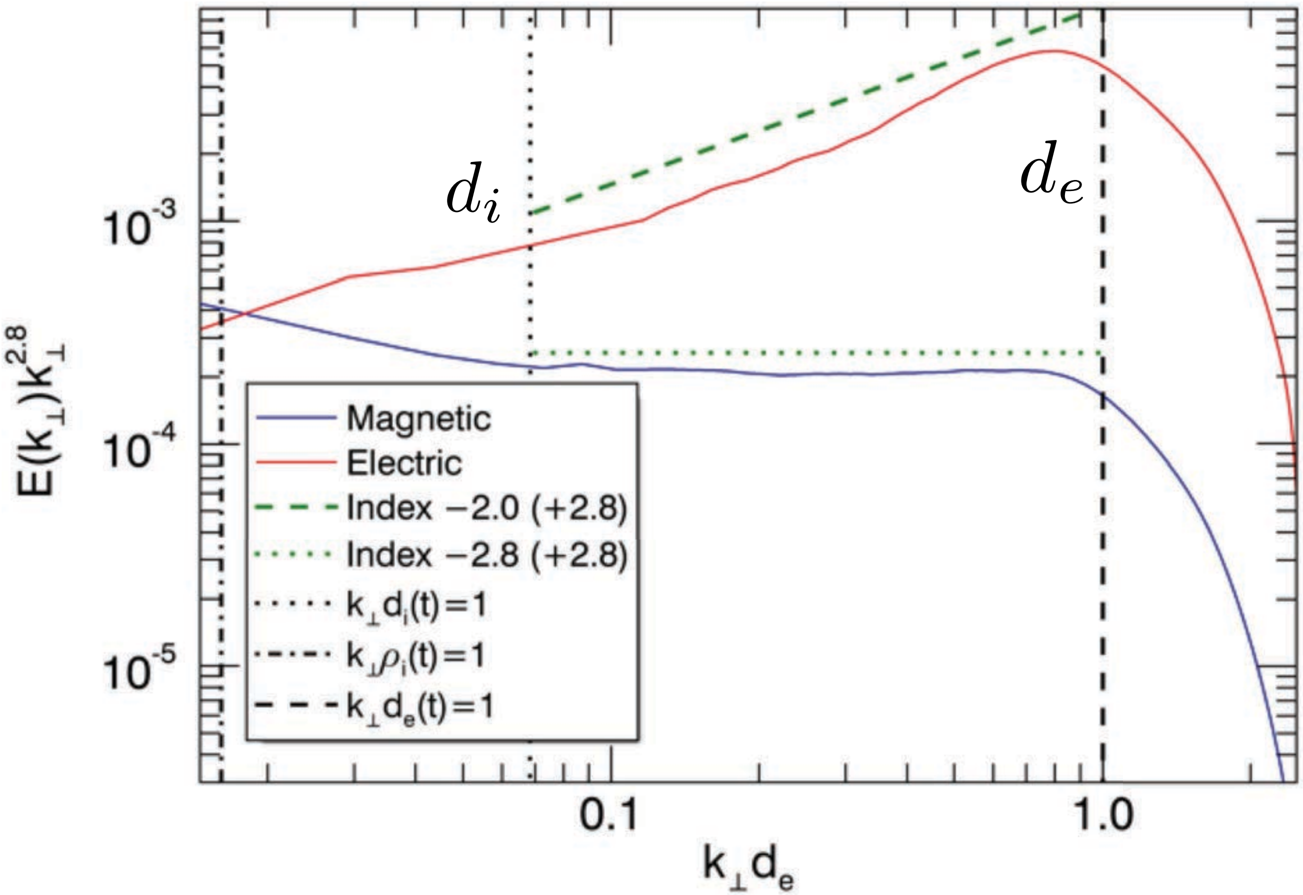
Temperature ratio



Sub-ion scale turbulence

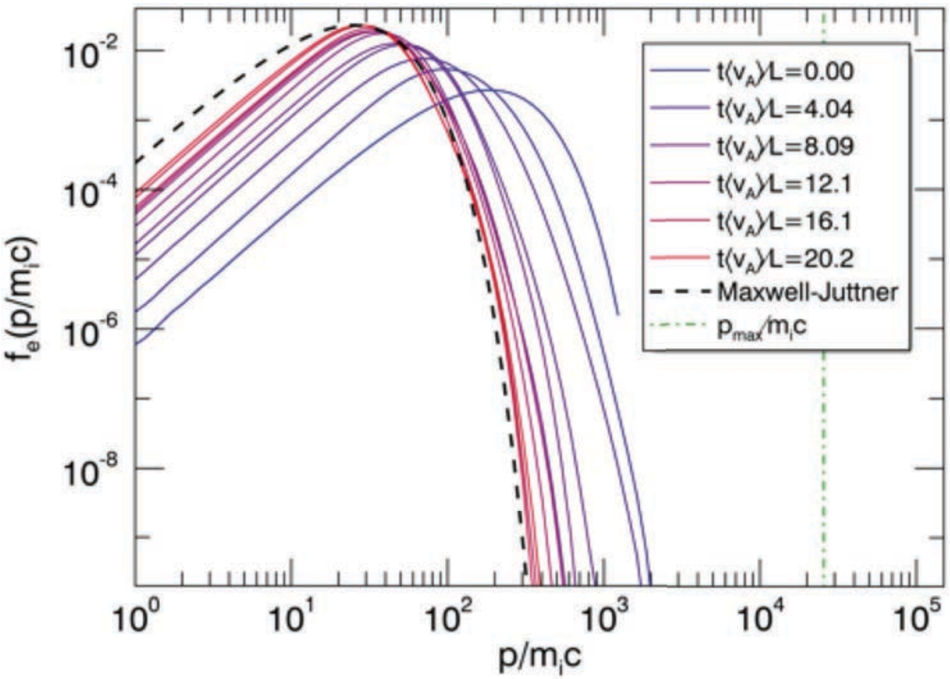
Clean -2.8 power law in kinetic range! (kinetic Alfven wave cascade?)

Turbulence spectrum

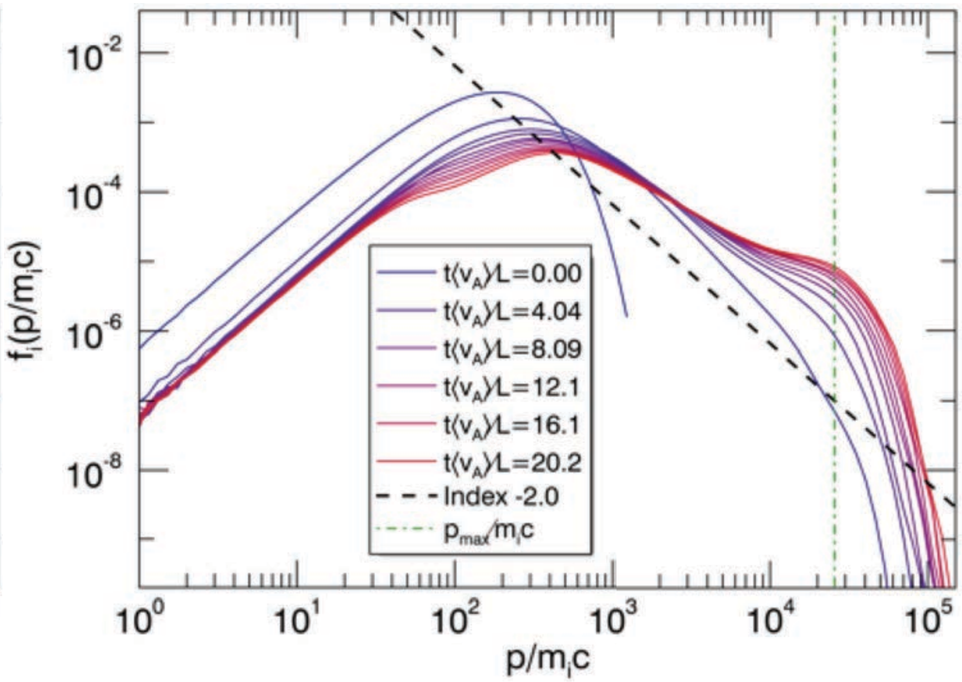


Electron-ion energy distributions

Electron distribution



Ion distribution

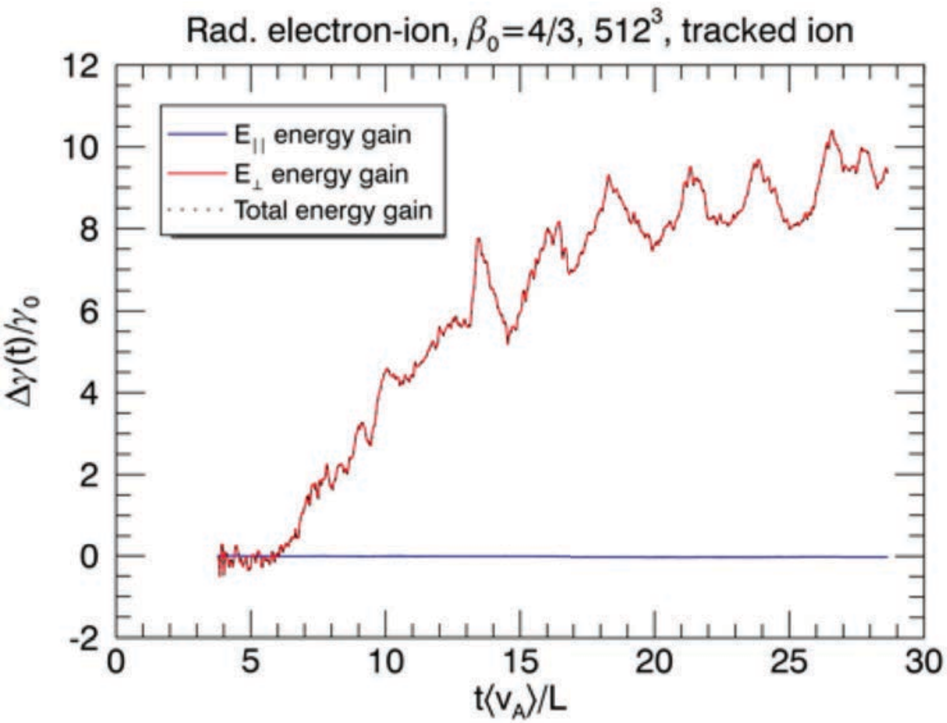


Electrons are thermalized by radiative cooling

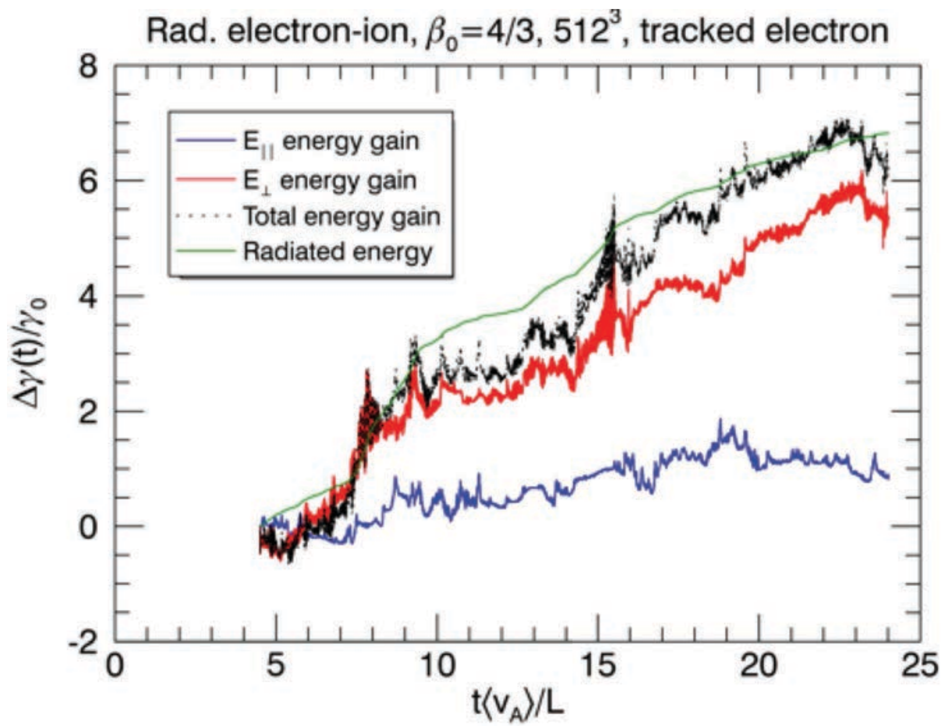
Ions are efficiently accelerated to hard power law, up to system size limit

Tracked particle statistics

Random tracked ion



Random tracked electron



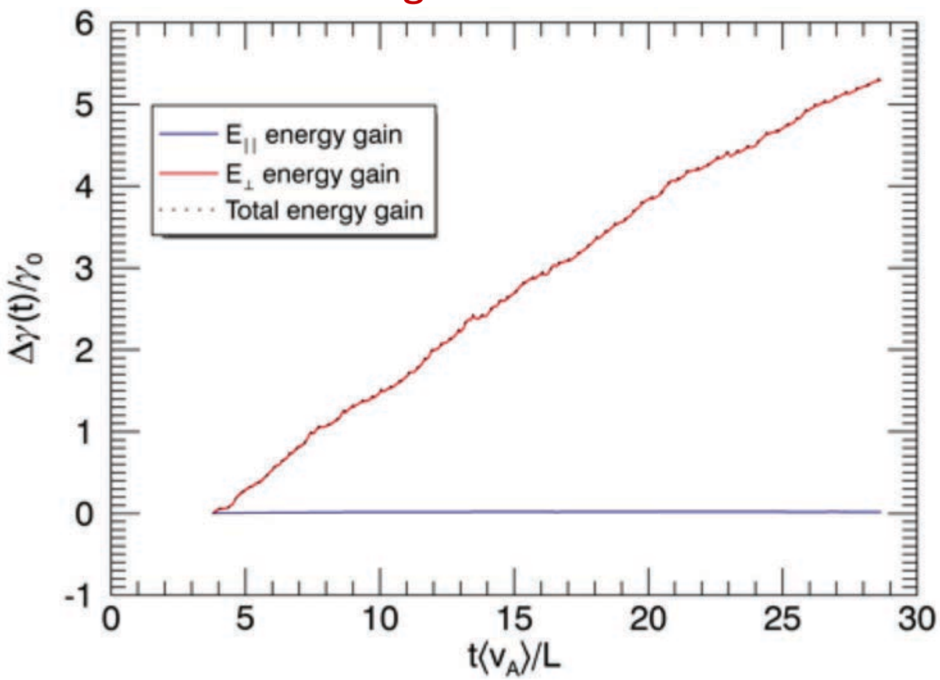
>99% of energy gain for ions is from perpendicular electric fields

Electrons gain from mixture of parallel and perpendicular fields

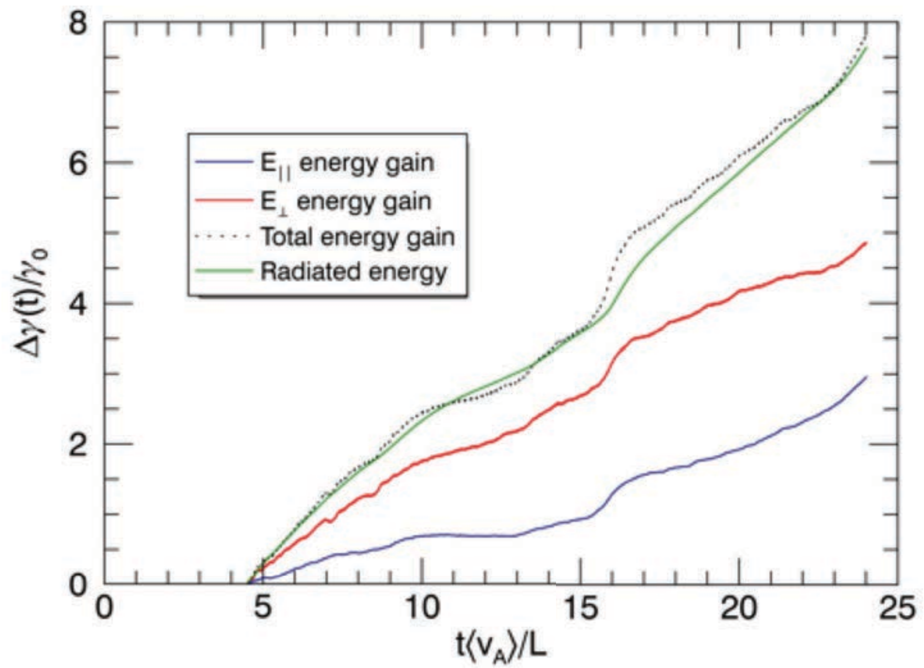
Broadly consistent with gyroresonant acceleration of ions

Tracked particle statistics

Average of tracked ions



Average of tracked electrons



>99% of energy gain for ions is from perpendicular electric fields

Electrons gain from mixture of parallel and perpendicular fields

Conclusions

- Applied 3D PIC simulations to study kinetic turbulence in relativistic and radiative plasmas
- In presence of strong radiative cooling, driven turbulence attains statistical steady state in pair plasmas but not in electron-ion plasmas (i.e., no indication of collisionless electron-ion thermal coupling)
- Radiative cooling efficiently thermalizes the plasma, leading to quasi-thermal distributions consistent with diffusive acceleration models
- At high magnetization, high-energy particles are intermittently beamed, evidently due to magnetic reconnection in current sheets
- Beams may explain rapid flares in high-energy astrophysical systems (e.g., blazar jets)

VZ, Uzdensky, Werner & Begelman, *submitted* **arXiv:1908.08032**