Cosmic ray acceleration
and
magnetic field amplification

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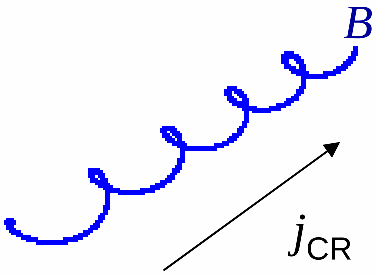
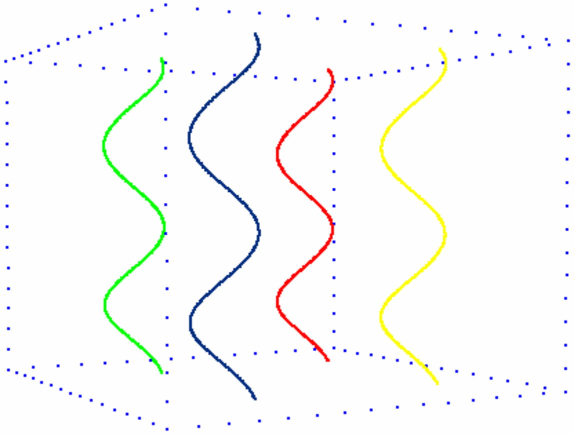
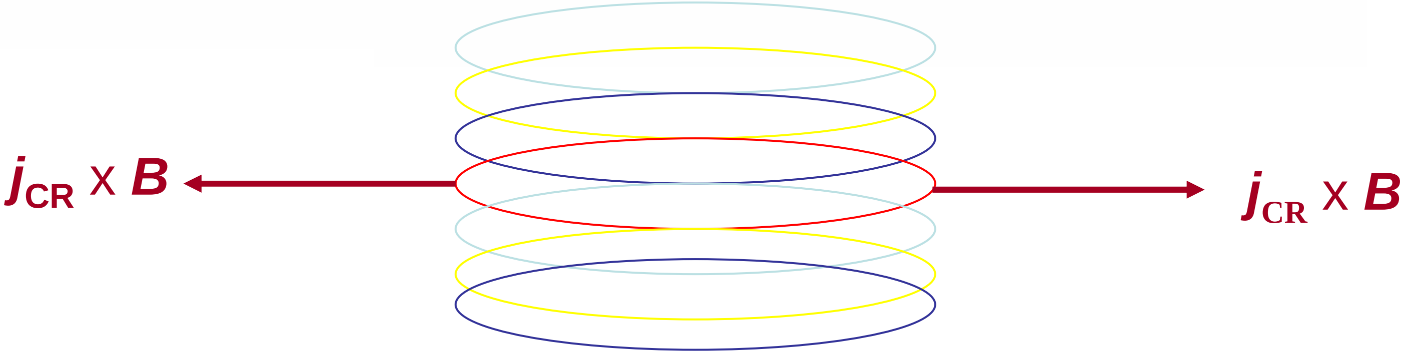
SN1006: A supernova remnant 7,000 light years from Earth
X-ray (blue): NASA/CXC/Rutgers/G.Cassam-Chenai, J.Hughes et al; Radio (red): NRAO/AUI/GBT/VLA/Dyer, Maddalena & Cornwell;
Optical (yellow/orange): Middlebury College/F.Winkler. NOAO/AURA/NSF/CTIO Schmidt & DSS

Linear instability







Purely growing, strong non-linear growth

Dispersion relation

Maxwell

$$\frac{\partial B}{\partial t} = -\nabla \times E \qquad E = -u \times B$$

Fluid acceleration

$$\rho \frac{du}{dt} = -j_{CR} \times B - \frac{1}{\mu_0} B \times \left(\nabla \times B \right)$$

Driving force

Growth limited by field tension at short wavelength

Growth rate

$$\gamma = \sqrt{\frac{kBj_{CR}}{\rho} - k^2 \mathrm{v}_A^2}$$

Dispersion relation

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Growth limited by field tension at short wavelength

Growth rate

$$\gamma = \sqrt{\frac{kBj_{CR}}{\rho} - k^2 \mathrm{v}_A^2}$$

Equivalently

$$\gamma = \sqrt{\omega_{ci} k \mathrm{v}_{drift} - k^2 \mathrm{v}_A^2}$$

thermal ion Larmor frequency

$$\mathrm{v}_{drift} = j_{CR} / n_e e$$

return current drift velocity

Relation to Weibel instability

Three-fluid equations contain Weibel & non-resonant instabilities

CR momentum

$$\frac{dp_{CR}}{dt} = -e\mathrm{v}_{CR} \times B - eE$$

Thermal electron momentum

$$m_e \frac{d\mathrm{v}_e}{dt} = -e\mathrm{v}_e \times B - eE$$

Thermal ion momentum

$$m_i \frac{d\mathrm{v}_i}{dt} = e\mathrm{v}_i \times B + eE$$

Maxwell

$$\frac{\partial B}{\partial t} = -\nabla \times E \qquad \nabla \times B = \mu_0 \{ j_{CR} + j_i + j_e \}$$

Three-fluid equations contain Weibel & non-resonant instabilities

CR momentum

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$$\frac{\partial B}{\partial t} = -\nabla \times E \qquad \nabla \times B = \mu_0 \{ j_{CR} + j_i + j_e \}$$

Non-resonant instability: $\boxed{\text{fixed} \quad j_{CR}}$ (unmagnetised CR, magnetised ions & e)

$$\gamma = \sqrt{\omega_{ci} \; k\mathrm{v}_{drift}} \qquad \gamma_{\max} = k\mathrm{v}_{drift} = \omega_{ci}$$

Three-fluid equations contain Weibel & non-resonant instabilities

CR momentum $$\frac{dp_{CR}}{dt} = -e\mathrm{v}_{CR} \times B - eE$$

Thermal electron momentum $$m_e \frac{d\mathrm{v}_e}{dt} = -e\mathrm{v}_e \times B - eE$$

Thermal ion momentum $$\cancel{m_i \frac{d\mathrm{v}_i}{dt} = e\mathrm{v}_i \times B + eE}$$

Maxwell $$\frac{\partial B}{\partial t} = -\nabla \times E \qquad \nabla \times B = \mu_0 \{ j_{CR} + j_i + j_e \}$$

Non-resonant instability: fixed j_{CR} (unmagnetised CR, magnetised ions & e)

$$\gamma = \sqrt{\omega_{ci}\, k\mathrm{v}_{drift}} \qquad \gamma_{\max} = k\mathrm{v}_{drift} = \omega_{ci}$$

Weibel instability: fixed ions $\boxed{\mathrm{v}_i = 0}$ (partially magnetised CR & e)

$$\gamma = \frac{k\mathrm{v}_{drift}}{\sqrt{1 + k^2c^2/\omega_p^2}}$$

Instabilities both driven by drift, characterised by different magnetisation

Structure of turbulence

self-consistent CR/MHD

KALOS code

PPCF 48 R37 (2006)

Kinetic
a
Laser-plasma
o
Simulation

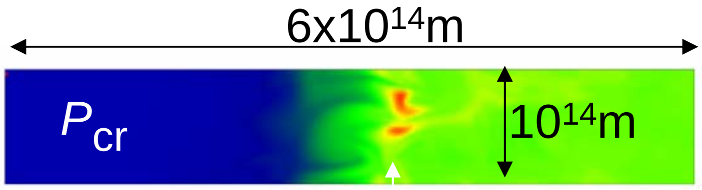
Hybrid kinetic/MHD

Kinetic COSMIC RAYS: Expand velocity dist^n in spherical harmonics

$$f(x,y,\underbrace{p,\theta,\phi}_{\text{momentum coordinates in 3D}},t) = \sum f_{nm}(x,y,p,t)\, P_n^{|m|}(\cos\theta)\, e^{im\phi}$$

CR/MHD interaction: non-relativistic shock

self-consistent CR/MHD



inject
plasma + CR + *B* (250μG)
3×10^{7}ms^{-1}

reflective wall

- 2D in space, 3D in momentum, 1 component of B (<u>out of plane</u>)

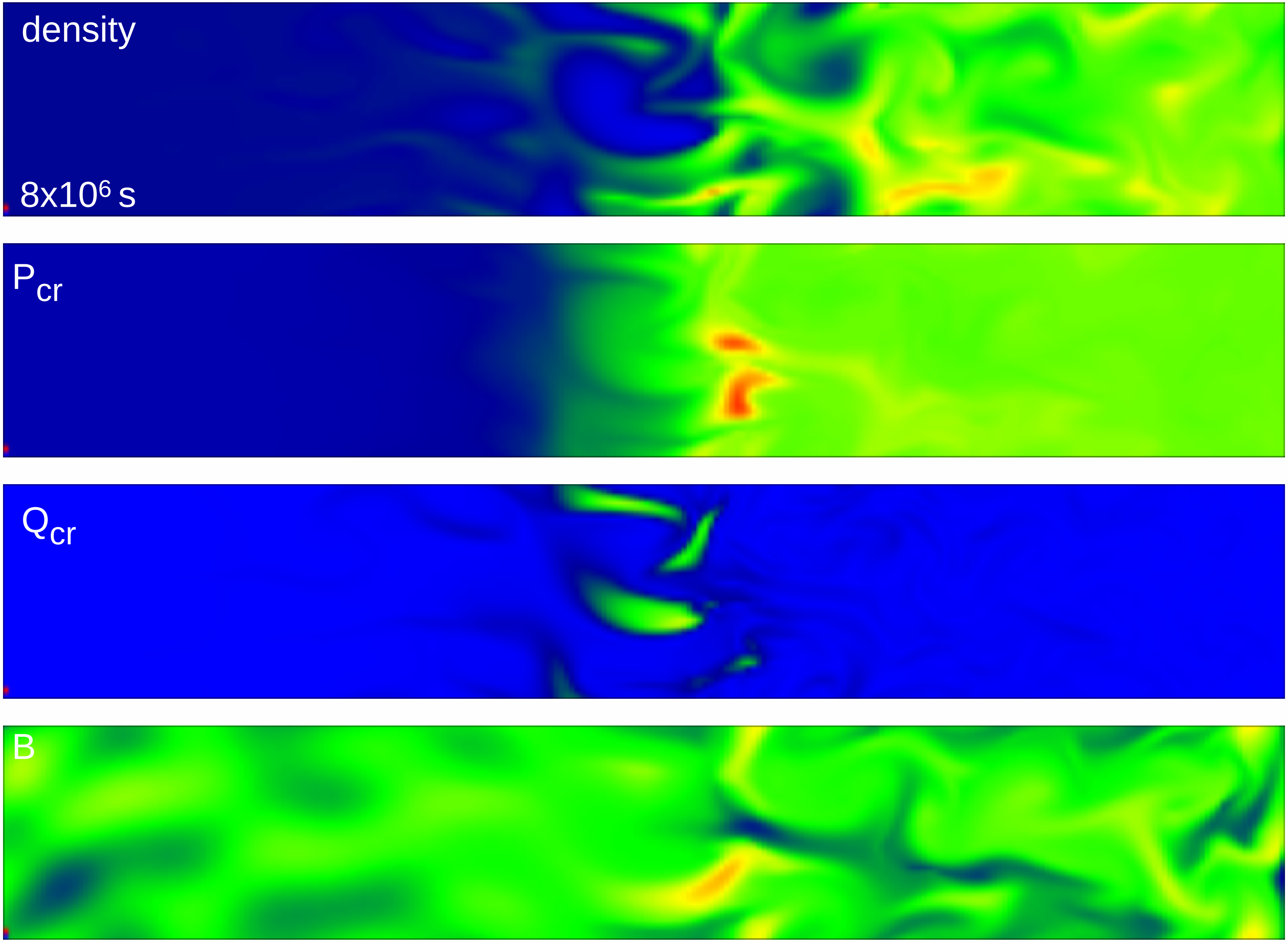
no instability

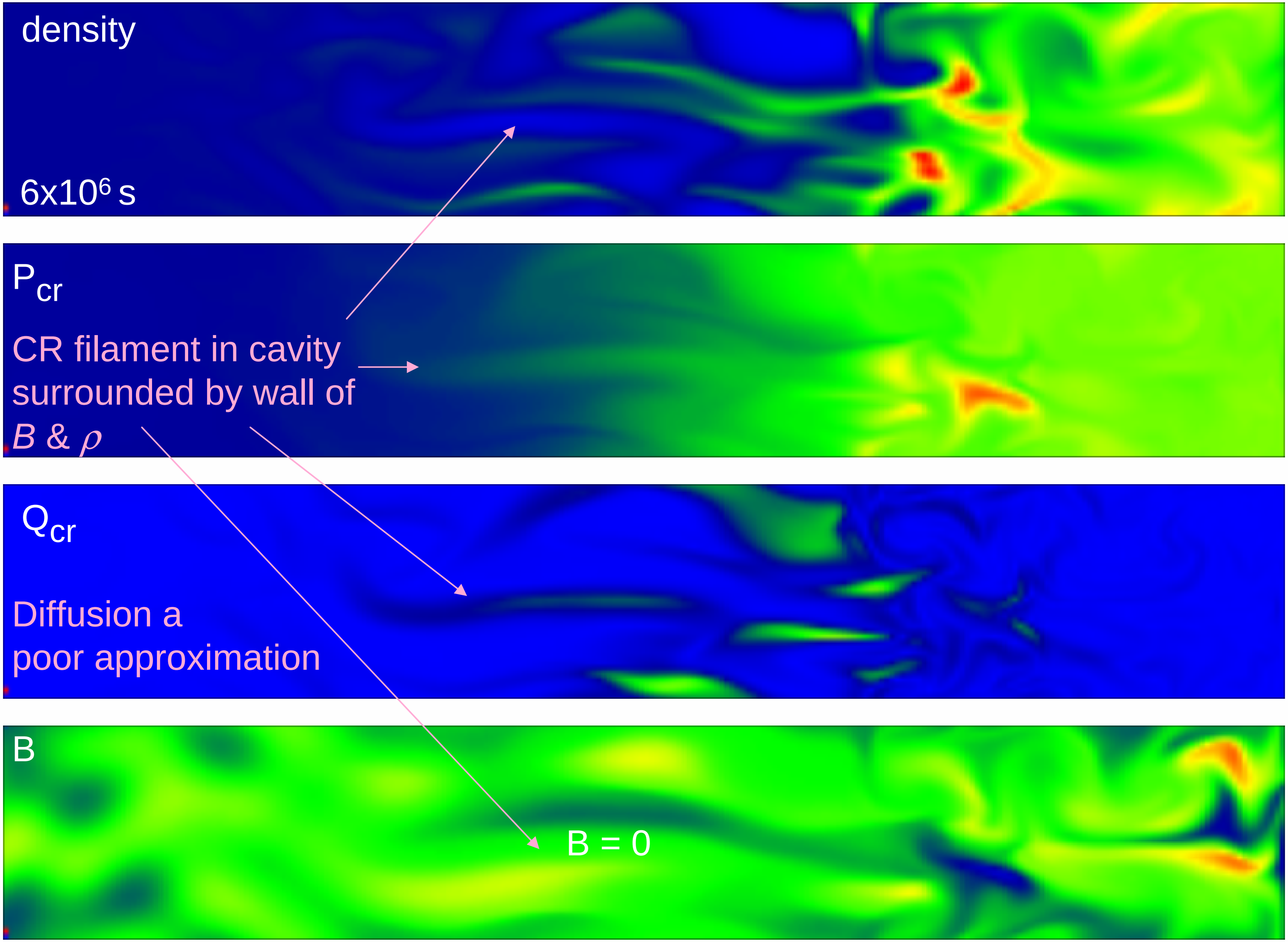
Perpendicular shock: B ~ 250 μG

Larmor radius: 1.3×10^{13}m

Disordered magnetic field on scale of few x 10^{13} m (changes direction)

Downstream CR pressure 40% of total


density
8x10^6 s
P_cr
Q_cr
B



CR filament in cavity surrounded by wall of *B* & *ρ*

Diffusion a poor approximation

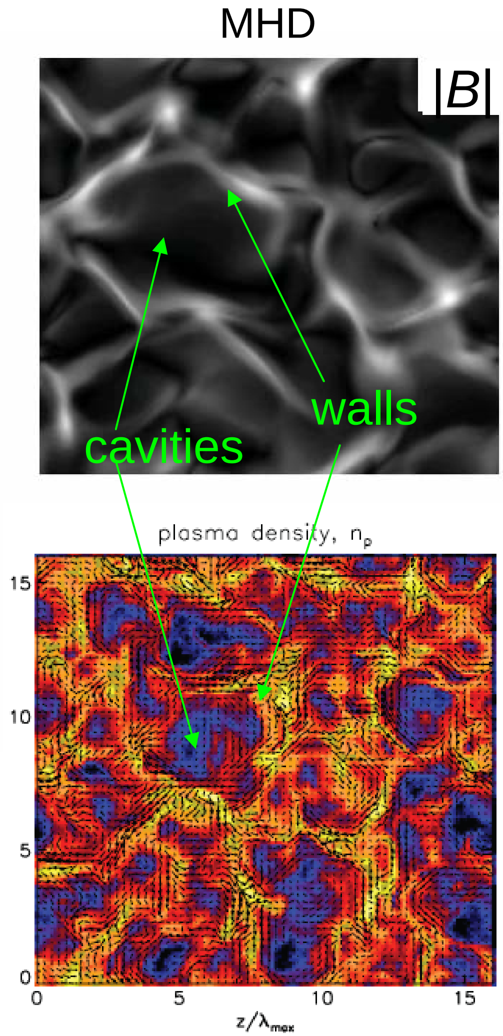
B = 0

Same except for smaller B (100μG – larger structures)

Non-diffusive behaviour

escape or sweep-out
either can steepen spectrum

Non-diffusive behaviour

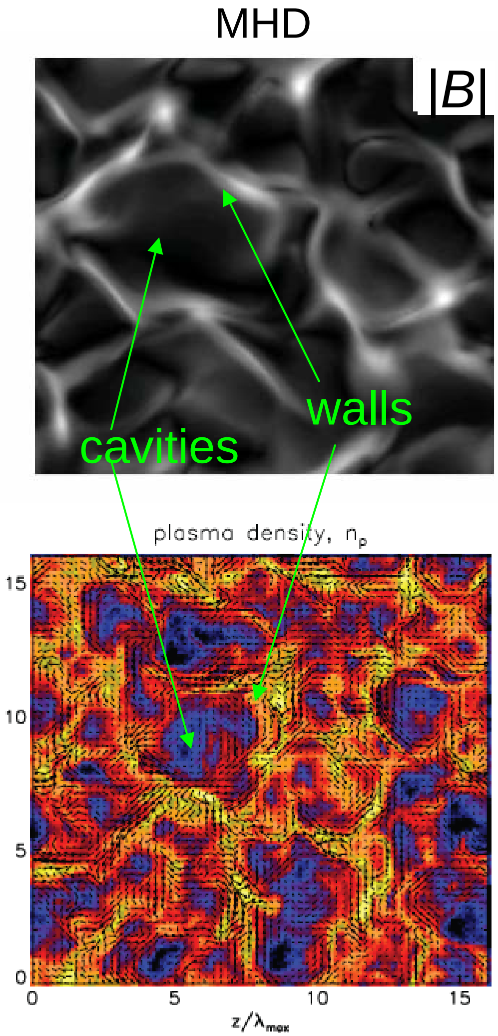


Riquelme & Spitkovsky
(PIC)

Escape upstream through cavities

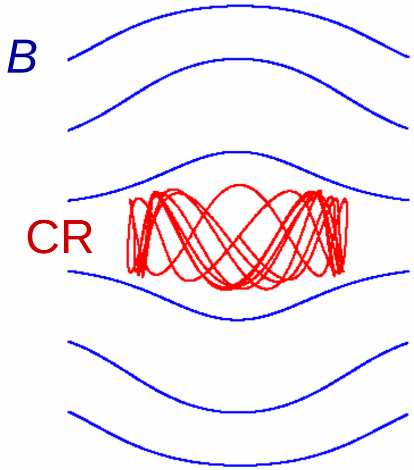
Allows some CR escape SNR
without adiabatic loss

Non-diffusive behaviour

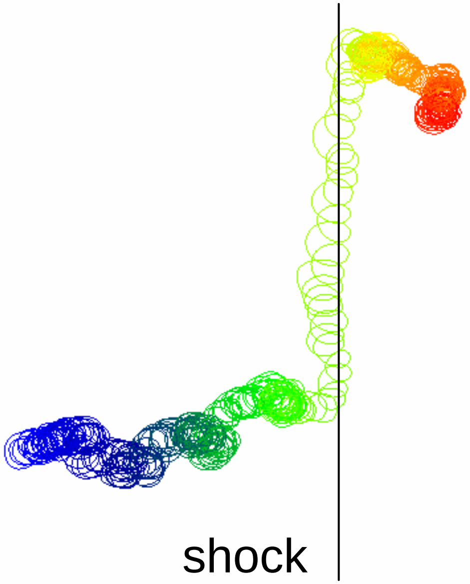


Riquelme & Spitkovsky
(PIC)

Magnetic walls/mirrors



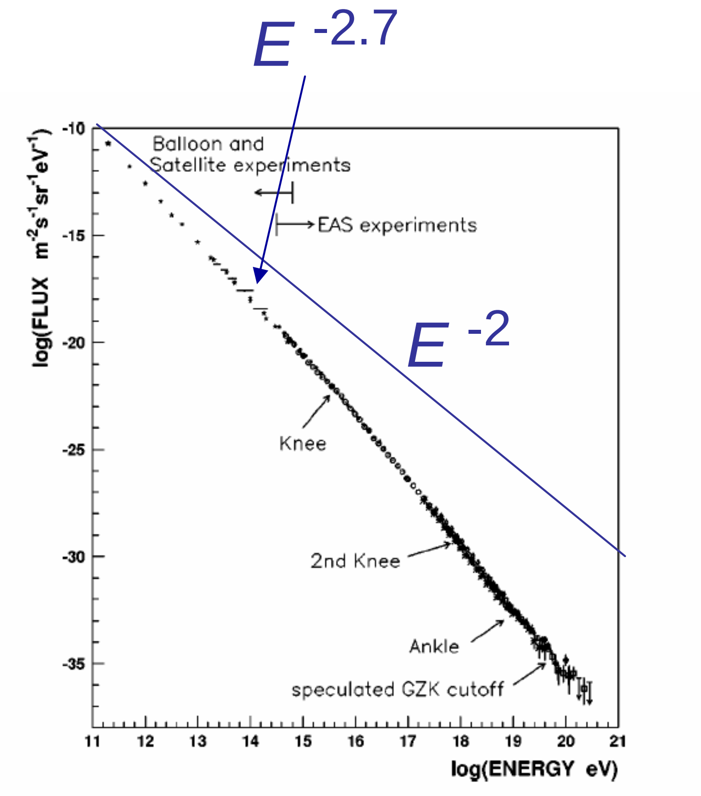
Locally
perpendicular shocks



CR loss downstream
Steepens spectrum

Cosmic Ray spectrum arriving at earth

Nagano & Watson 2000

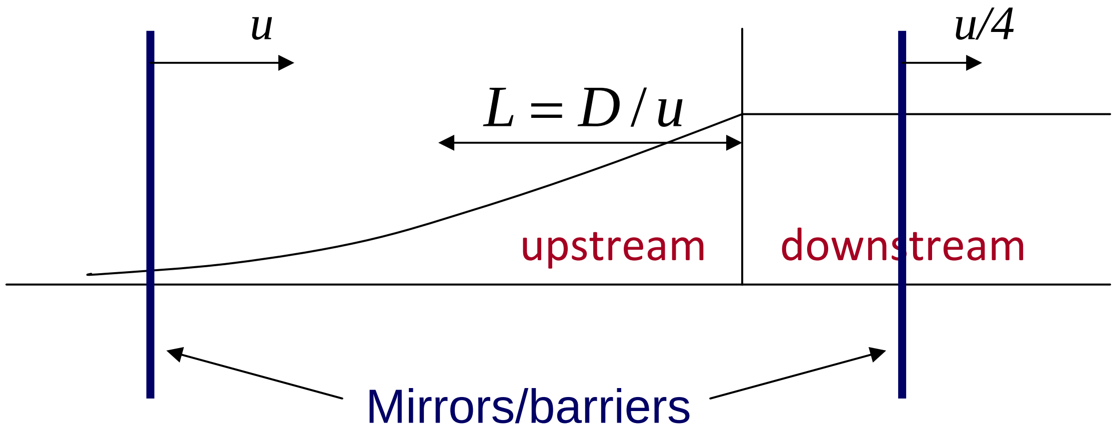


Leakage from galaxy accounts for some of difference

but not all: escape too rapid at high *E* (Hillas 2005)

Spectral steepening due to ‘sweep-out’ events

(mirrors, perpendicular field)



$$\frac{\partial f}{\partial t} + \frac{\partial (uf)}{\partial z} - \frac{\partial}{\partial z}\left(D\frac{\partial f}{\partial z}\right) - \frac{1}{3}\frac{\partial u}{\partial z}\frac{1}{p^2}\frac{\partial (p^3 f)}{\partial p} = S_{sweep-out}$$

advection

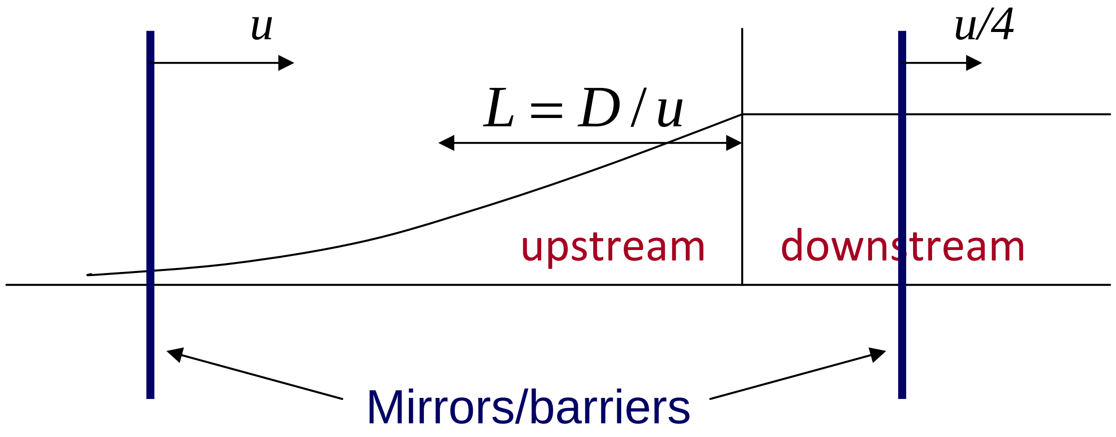
diffusion

acceleration at shock

Randomly placed mirrors

Spectral steepening due to ‘sweep-out’ events

(mirrors, perpendicular field)



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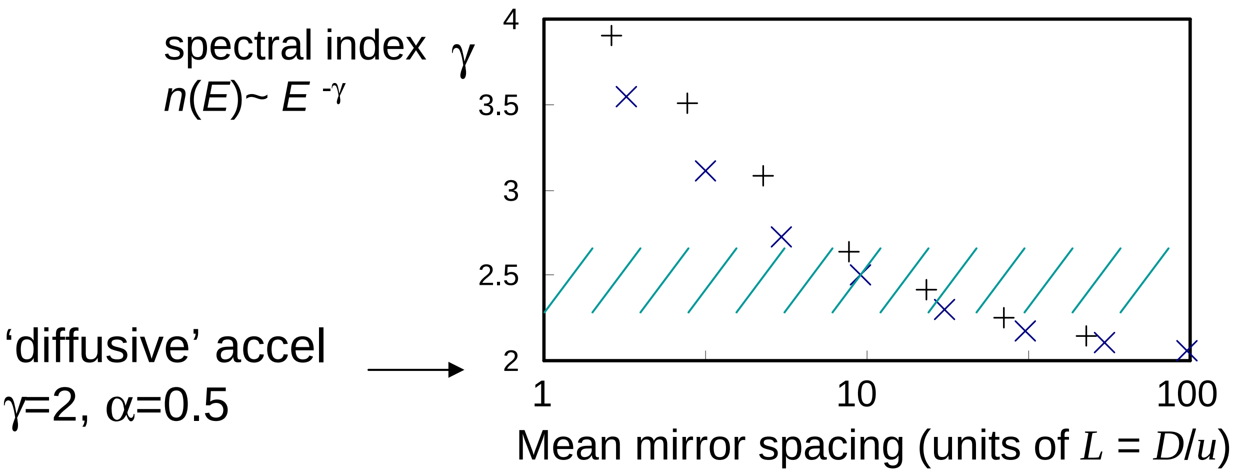
advection diffusion acceleration at shock

Randomly placed mirrors



Galactic CR: γ = 2.3-2.4

Young SNR: γ = 2.0-2.8

Historical SNR (Glushak 1985)

Cas A, Kepler, Tycho, SN1006, RCW86, RCW103, G319.7, 3C391, 0519-69.0

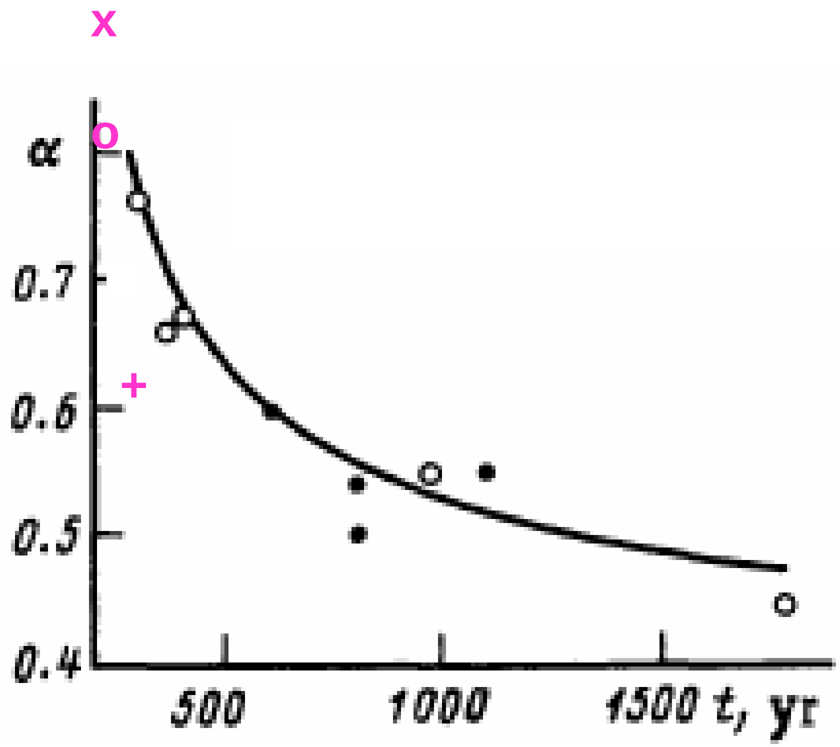
o SN1993J: $\alpha = 0.81$ (Weiler et al 2007)

x SN1987A: $\alpha = 0.9$, flattening to 0.8 (Manchester et al 2005)

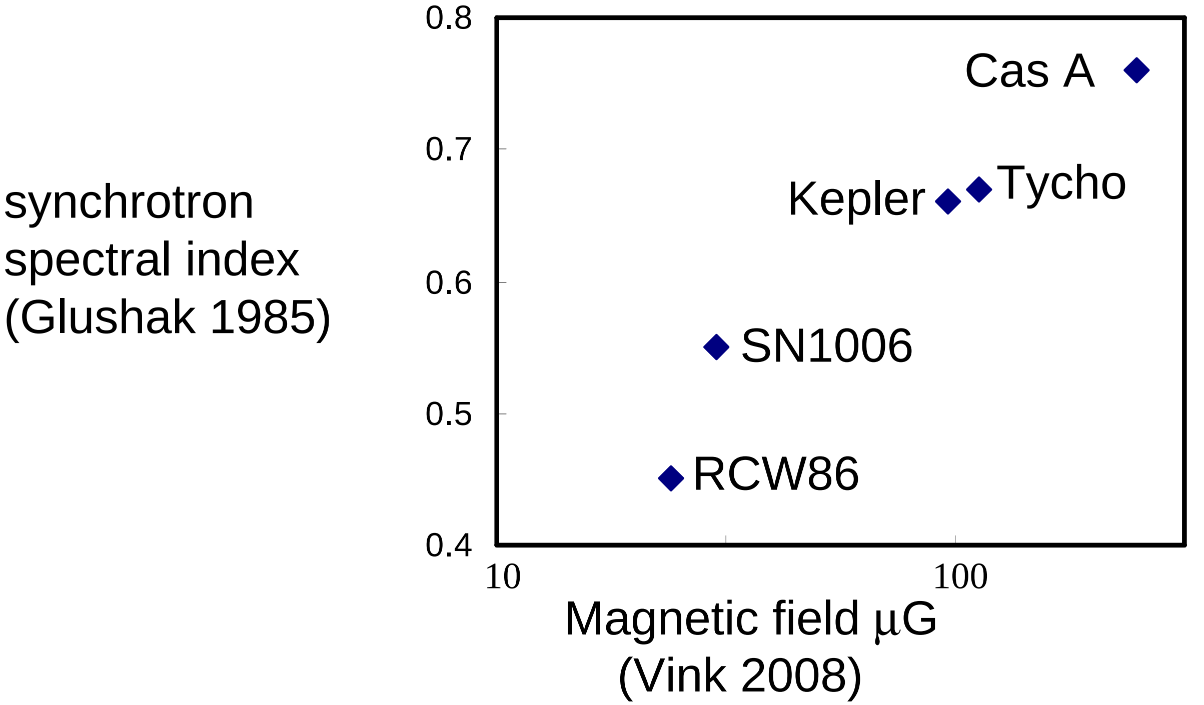
\+ G1.9+0.3: $\alpha = 0.62$ (Green et al 2008)

Radio spectral index vs. age

‘Diffusive’ shock acceleration →



Young SNR: spectral index vs magnetic field



Expect more mirroring in strongly amplified field

Possibility: strong field amplification produces steep spectrum

Allows straight power law spectrum in CR-dominated shock

Similar steepening in extragal. radio lobes – confused by synchrotron steepening

Magnetic field saturation

Saturation magnetic field

For unstable growth:

A) Driving force exceeds magnetic tension $$j_{CR} \times B > \left(\frac{\nabla \times B}{\mu_0} \right) \times B \approx \frac{B^2}{\mu_0 l}$$

B) Instability scalelength < CR Larmor radius $$l < \frac{p}{eB}$$

Eliminate l

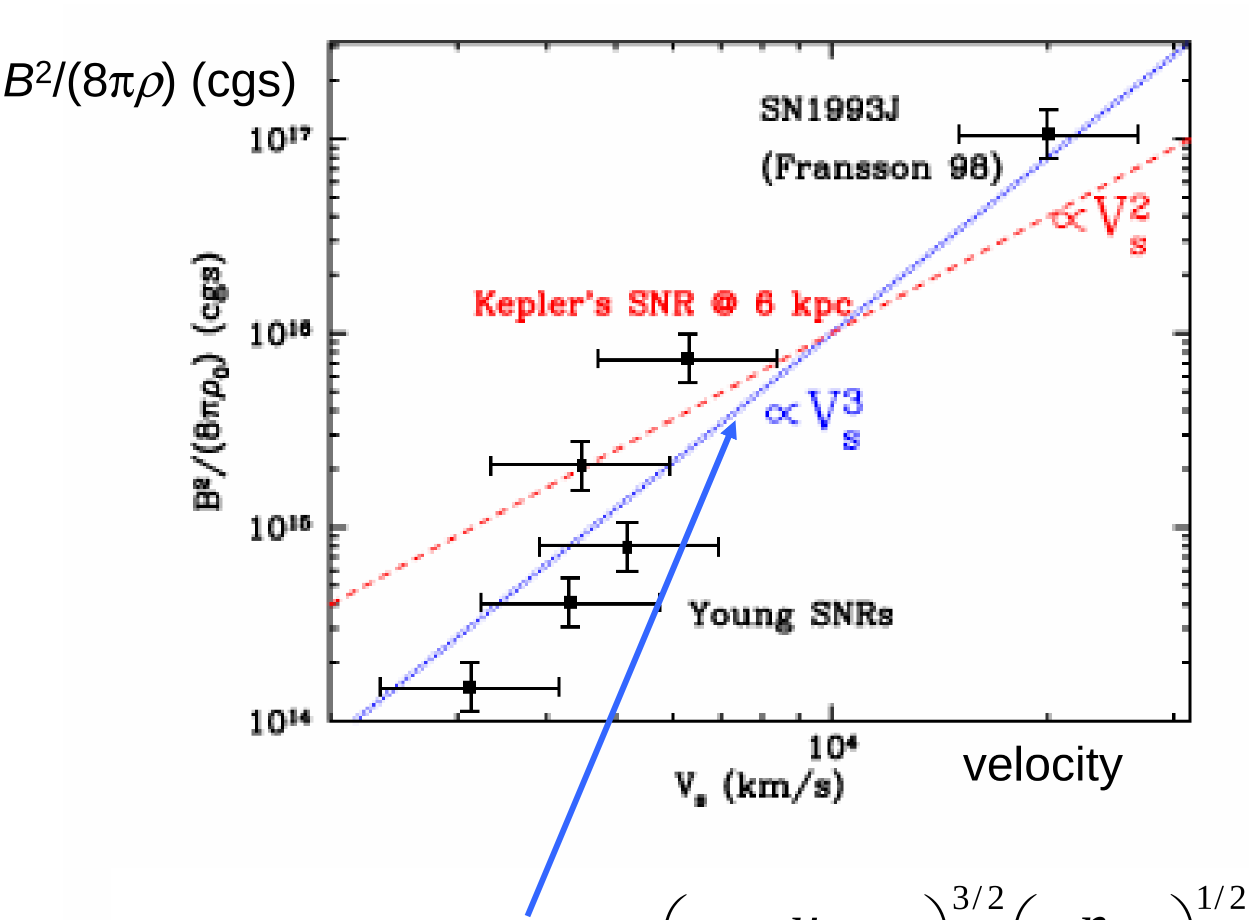
Allow 2x compression of *B* at shock

$$\frac{B^2_{downstream}}{2\mu_0} \approx \eta \frac{u}{c} \rho u^2$$

In real numbers

$$B_{downstream} \approx 400 \left(\frac{u}{10^4 \, \mathrm{kms}^{-1}} \right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{\eta}{0.1} \right)^{1/2} \mu \mathrm{G}$$

Inferred downstream magnetic field (Vink 2008)



Data for
RCW86, SN1006, Tycho,
Kepler, Cas A, SN1993J

Fit to obs (Vink):

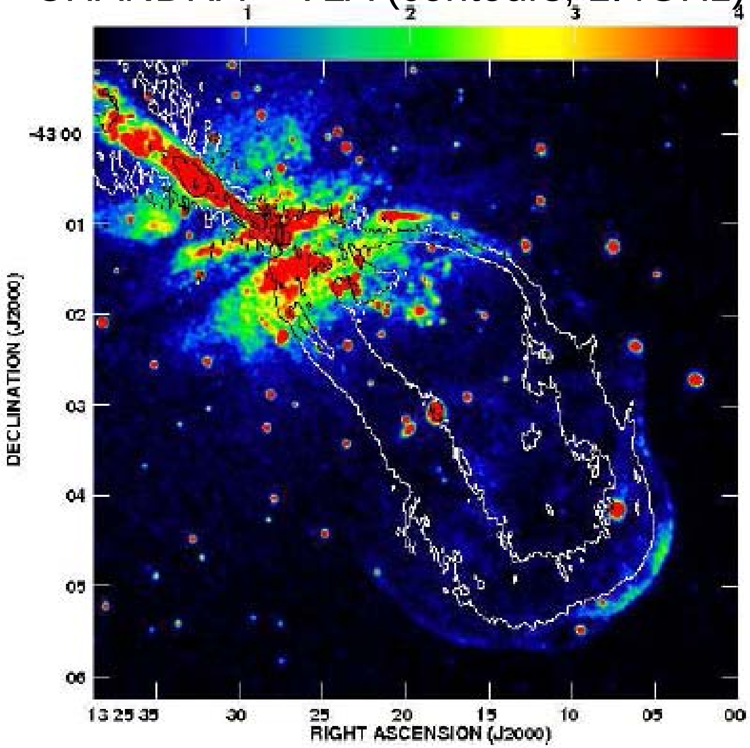
$$B \approx 700 \left(\frac{u}{10^4\,\mathrm{kms}^{-1}} \right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \mu\mathrm{G}$$

Theory:
B limited by tension

$$B \approx 400 \left(\frac{u}{10^4\,\mathrm{kms}^{-1}} \right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{\eta}{0.1} \right)^{1/2} \mu\mathrm{G}$$

Shocks in radio jets
Centaurus A (Croston et al 2008)

CHANDRA + VLA (contours, 1.4GHz)



Values taken by Croston et al:
$n_e = 10^{-3}\ \mathrm{cm}^{-3}$
$u = 2600\ \mathrm{kms}^{-1}$

Shell thickness ΔR = 300pc
Shell radius R = 2000pc

Shock thickness: $B \sim 1\ \mu\mathrm{G}$

Theory:

$$B \approx 400 \left(\frac{u}{10^4\,\mathrm{kms}^{-1}}\right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{\eta}{0.1}\right)^{1/2} \mu\mathrm{G} \quad \Longrightarrow \quad B \sim 1.7\ \mu\mathrm{G}$$

Summary

Dispersion relation derived from fluid model

Ions must be magnetised (Hall current)

Differs from Weibel by degree of magnetisation

Walls/cavities: diffusive shock acceleration is not diffusive

Non-diffusion can change spectral index

Observe field structure directly

Encouraged by success of saturation estimate

Application to extragalactic shocks

Diffusive shock acceleration may not be diffusive