Non-thermal X-ray Filaments in Cas A: Testing Plasma Turbulence

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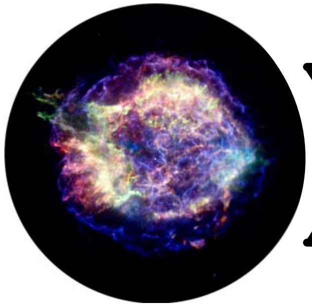
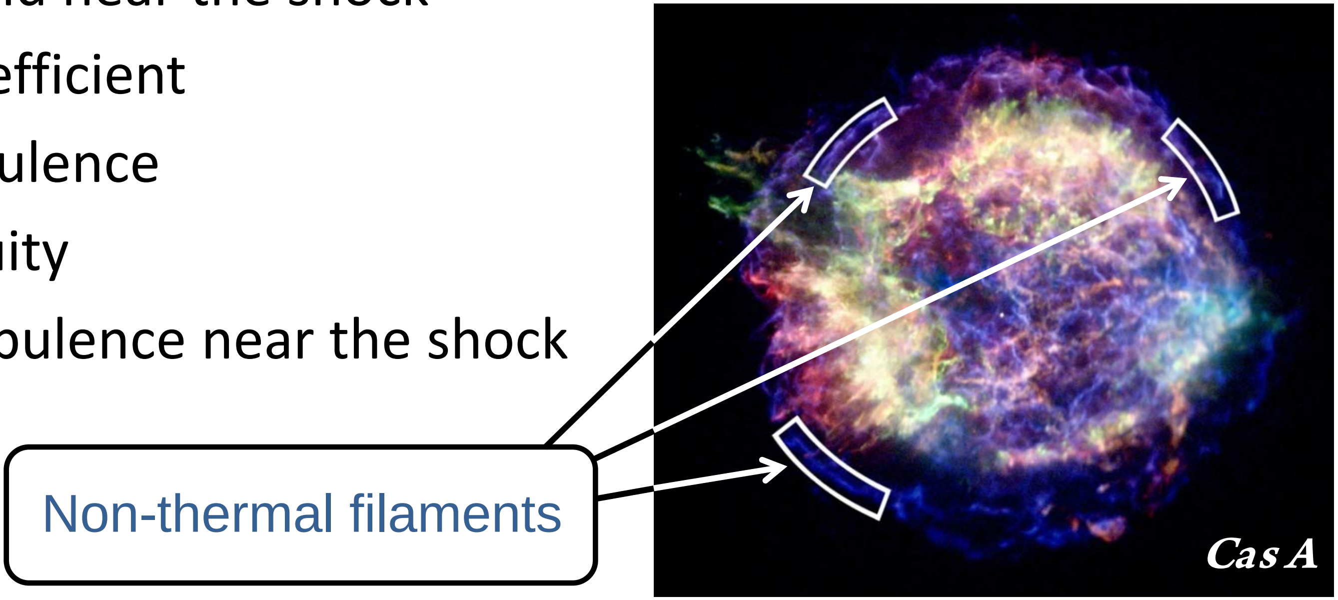
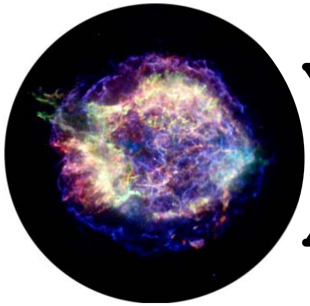


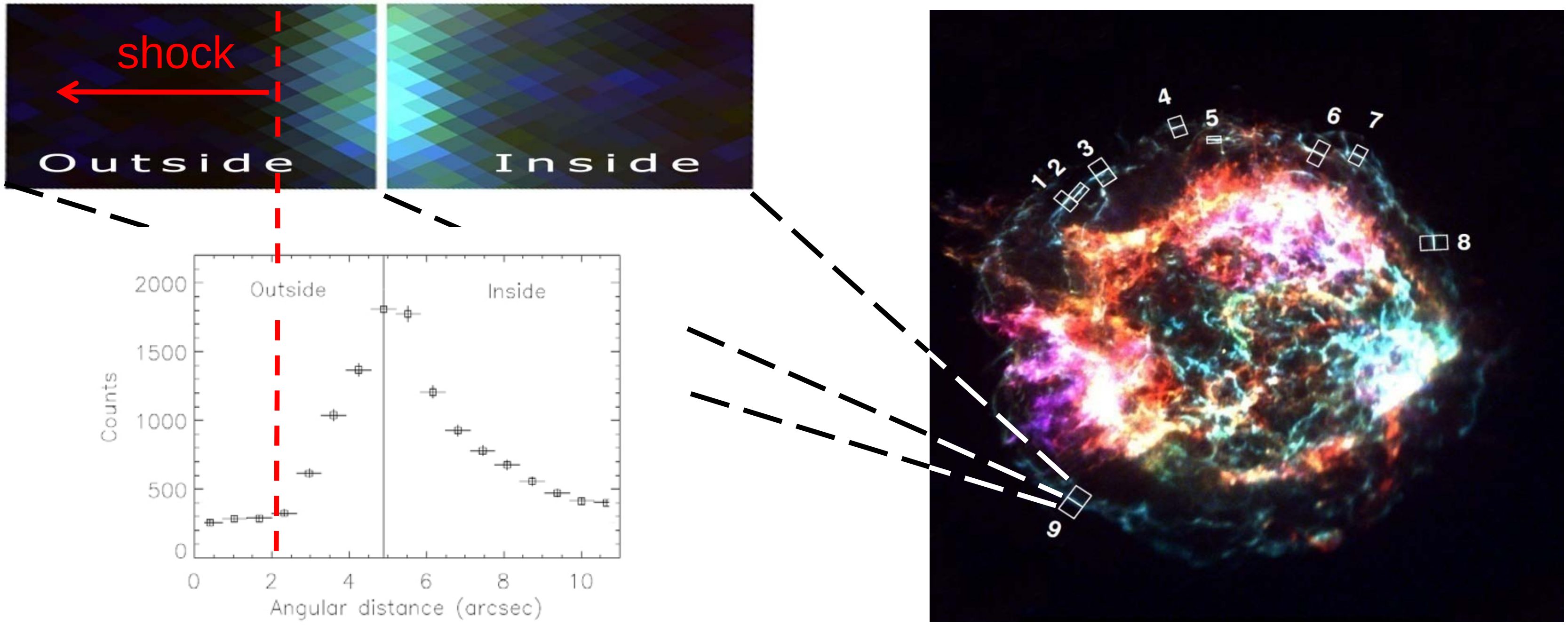
Motivation

- Cosmic-Ray acceleration requires a strong turbulence, particle diffusion near astrophysical shocks
- Non-thermal X-ray filaments in shell type SNRs
- Estimate :
  - magnetic field near the shock
  - diffusion coefficient
  - level of turbulence
  - shock obliquity
- Test plasma turbulence near the shock


Non-thermal filaments



Observational Data

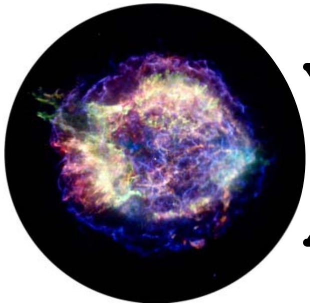


1 Ms *Chandra* observation of Cas A (*Hwang et al. 2004*)

- Obtain:
  - spatially resolved spectra
  - radial profiles at different energies

Araya et al. (in preparation)

- Expect :
  - spectrum softer inside
  - larger width at lower photon energies



Model

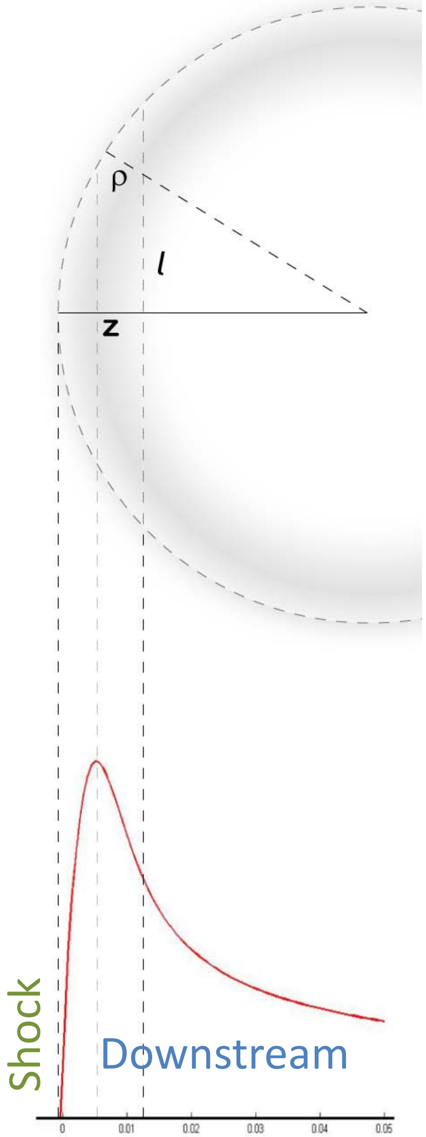
- Non-thermal filaments modeled as thin spherical regions associated with forward shock
- Particles injected at the shock have a power-law distribution
- Advected downstream
- Diffusion
- Two different scenarios are considered:
  - Advection + Synchrotron losses– “Advection-Loss”
  - Advection + Synchrotron losses + Diffusion - “Advection - Diffusion” (with different types of energy dependence of diffusion coefficient)
- kinetic equation :

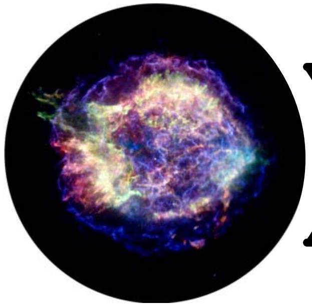
$$\frac{\partial n}{\partial t} - \beta B^2 \frac{\partial(\gamma^2 n)}{\partial \gamma} - D(\gamma)\Delta n = Q(\gamma_0)\delta(r - r_0)$$

Synchrotron losses

Diffusion

Source function





Diffusion

Magnetic turbulence → Particle diffusion due to the scattering on magnetic irregularities

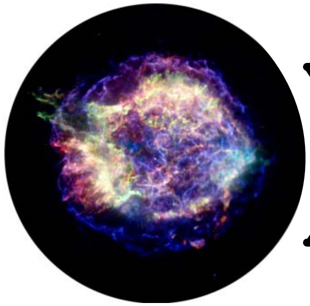
Spectrum of magnetic Turbulence – $W(k) \propto k^{-q}$ → Spatial diffusion coefficient $D \propto \gamma^{2-q} \quad (q - 1 \equiv \delta)$

We parameterize diffusion coefficient as :

$$D(\gamma) = \kappa D_0 \gamma^{1-\delta}$$

$$D_0 = (mc^3/3eB)$$

Type of Turbulence	***q***	***δ***
White noise	1	0 (*Bohm*)
Kraichnan	1/2	1/2
Kolmogorov	5/3	2/3



Model

- Solution of the kinetic equation (*added Advection*):

$$n(\gamma,r) = \frac{1}{\beta B^2\gamma^2}\int_{\gamma}^{\gamma_0} d\gamma_0\; Q(\gamma_0)\,\frac{exp\left[-\frac{(r - V_{adv}t(\gamma,\gamma_0))^2}{4\lambda(\gamma,\gamma_0)}\right]}{(4\pi\lambda(\gamma,\gamma_0))^{3/2}}$$

$$\lambda(\gamma,\gamma_0) = \int_{\gamma}^{\gamma_0} d\gamma' \frac{D(\gamma')}{\beta B^2\gamma'^2}$$ - *Syrovatskii* parameter

$$Q(\gamma_0) = K\gamma_0^{-\Gamma}$$

$\lambda(\gamma,\gamma_0)$ is equivalent to the squared distance traveled by the particle while "cooling" from γ_0 to γ

$$l_{adv} = V_{adv}t_{cool}$$

$$l_{dif} \sim \sqrt{\lambda(\gamma,\gamma_0)}$$

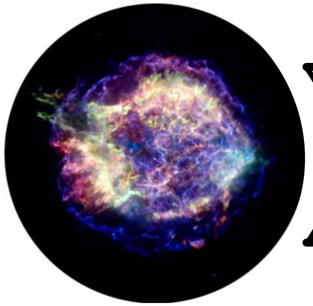
$$R$$

$$C_1 = \frac{l_{dif}(1keV)}{R}$$

$$C_2 = \frac{l_{adv}(1keV)}{l_{dif}(1keV)}$$

$$l_{dif}(1keV) = \left(\frac{\kappa D_0}{\beta B^2}\left(\frac{1.5h\nu_L}{1keV}\right)^{\delta/2}\right)^{1/2}$$

$$l_{adv}(1keV) = \frac{V_{adv}}{\beta B^2}\left(\frac{1.5h\nu_L}{1keV}\right)^{1/2}$$



Model

- Synchrotron emissivity integrated along the line of sight :

$$\boxed{F_\nu(z) = \int_0^{\bar{l}_{max}(z)} d\bar{l} I_\nu(\rho(\bar{l}, z))}$$

$$I_\nu(\rho) = \int_\gamma d\gamma P_\nu(\gamma) n(\gamma, \rho)$$

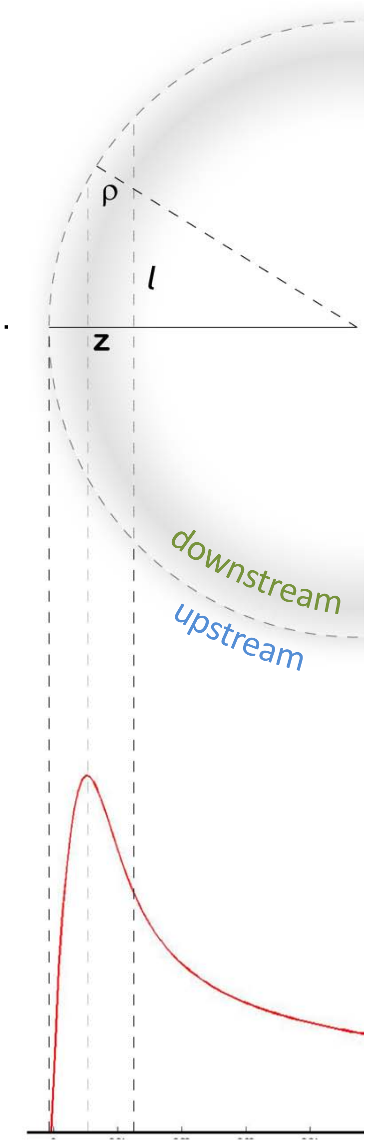
- Synchrotron emissivity

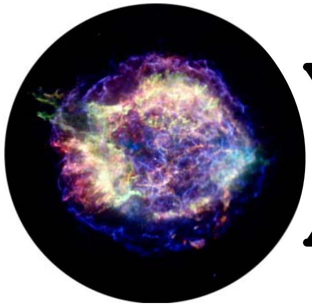
$$P_\nu(\gamma) = \frac{\sigma_T c B^2 \gamma^2}{6\pi} \delta(\nu - \nu_c)$$

- Specific power by single electron

$$\boxed{\begin{aligned} F_E(z) \quad &\propto \quad E_{keV}^{-\Gamma/2+3\delta/4} \int_{\bar{l}_{min}}^{\bar{l}_{max}} d\bar{l} \int_1^\infty d\epsilon \frac{\epsilon^{-\Gamma+3\delta/2}}{(\epsilon^\delta - 1)^{3/2}} \\ &\times \quad exp\left[-\frac{\delta \cdot E_{keV}^{\delta/2}}{4\,(1 - \epsilon^{-\delta})} \left(\frac{\rho(\bar{l}, z)}{C_1} - \frac{C_2}{E_{keV}^{1/2}} \left(1 - \frac{1}{\epsilon}\right) \right)^2 \right] \end{aligned}}$$

- Precursor (upstream) solution same as downstream except:
$(V_{adv})_{down} \leftrightarrow (- 4^* V_{adv})_{up}$, $(B)_{down} \leftrightarrow (B/4)_{up}$





Model

Radial profiles and spectra of the filament are fully defined by C_1, C_2, δ and Γ

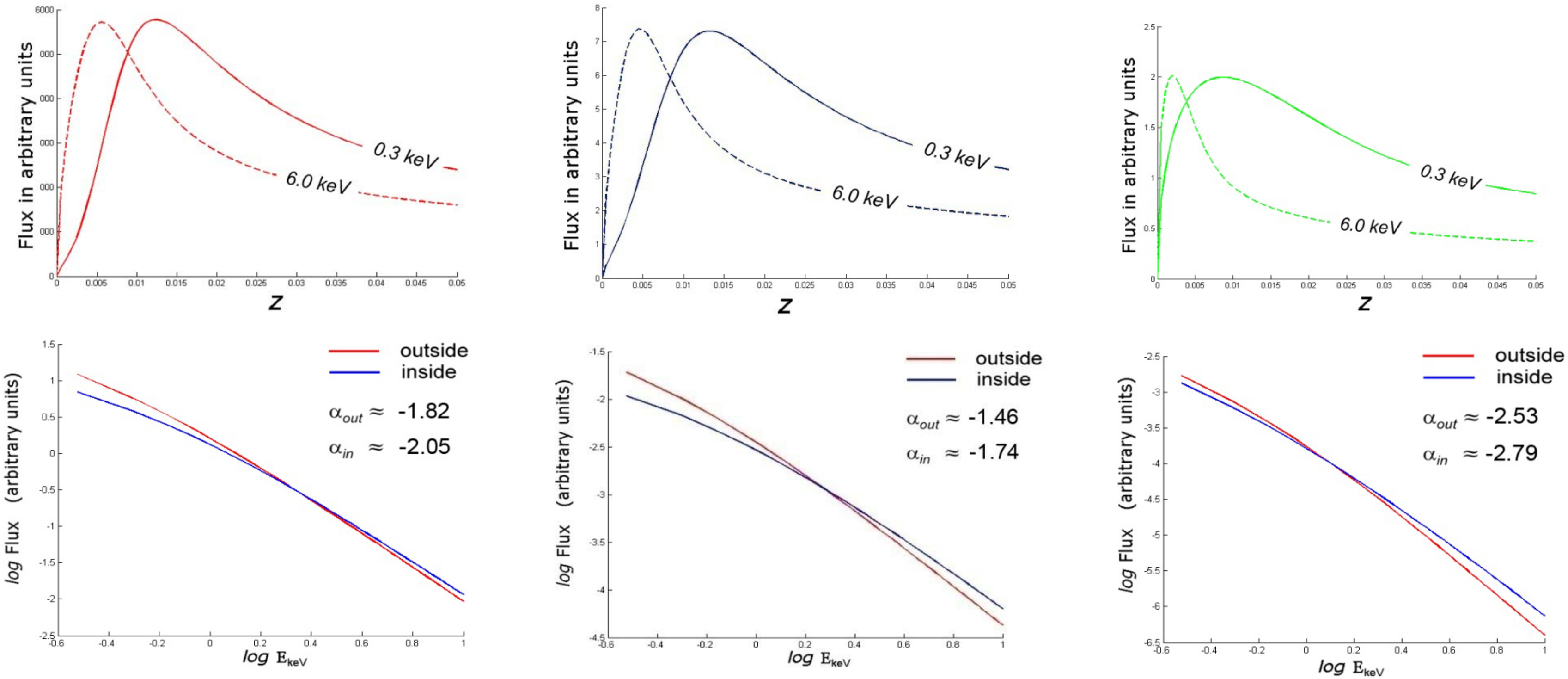
Bohm diffusion ($\delta = 0$)

Kolmogorov ($\delta = 0.66$)

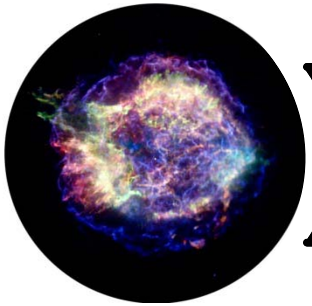
No diffusion

Radial Profiles

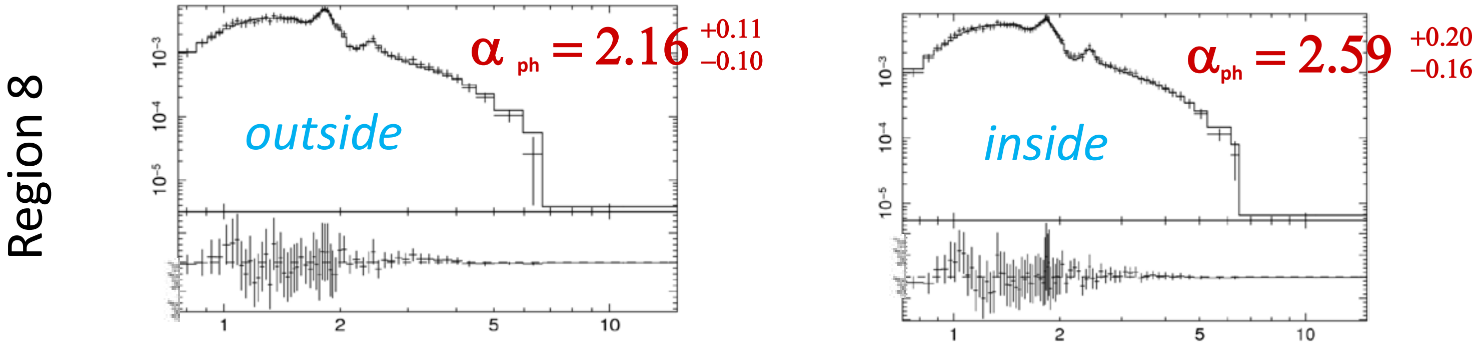
Spectra



$C_1 = 0.007$ $C_2 = 5$ $\Gamma = 2.5$ $B = 112\ \mu G$



Spatially Resolved Spectra



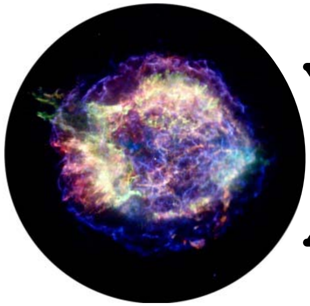
Region 8

$\alpha_{ph} = 2.16^{+0.11}_{-0.10}$

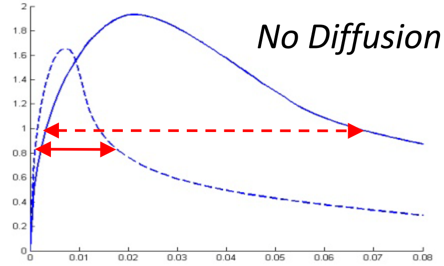
$\alpha_{ph} = 2.59^{+0.20}_{-0.16}$

- In ***all*** 9 filaments spectrum is harder *outside* than *inside* by about *1σ* (both *inside* and *outside* in a downstream region)
- This supports the assumption about *particle cooling*

Filament widths and spectral data from *Araya et al. (in preparation)*

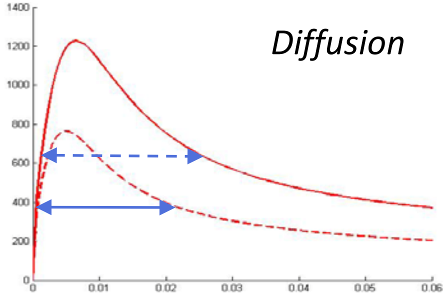


Weak Energy Dependence of Filament Widths



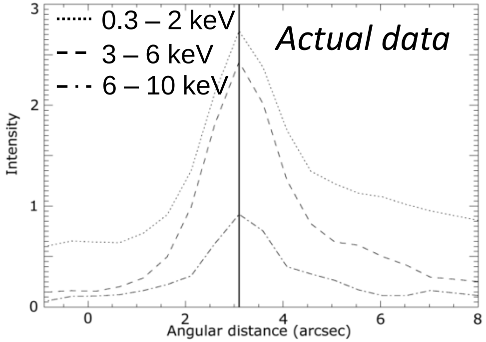
Ratio of the filament widths at different energies *(Model)* :

Advection -Loss - - - $\boxed{\frac{W_{0.3-2.0keV}}{W_{6.0-10.0keV}} > 4}$



Advection - Diffusion - - $\boxed{\frac{W_{0.3-2.0keV}}{W_{6.0-10.0keV}} > 1}$

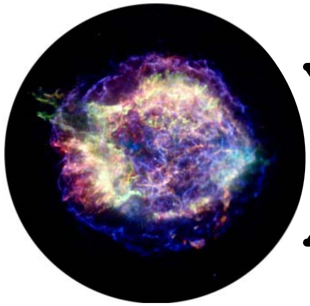
Ratio of the filament widths at different energies *(Data)* :



Region	1	2	3	4	5	6	7	8
$\frac{W_{0.3-2.0keV}}{W_{6.0-10.0keV}}$	$1.27^{+0.38}_{-0.35}$	$1.19^{+0.46}_{-0.34}$	$1.34^{+0.59}_{-0.41}$	$1.42^{+1.18}_{-1.02}$	$1.23^{+0.27}_{-0.23}$	$1.02^{+0.63}_{-0.86}$	$1.07^{+0.48}_{-0.49}$	$1.19^{+0.57}_{-0.37}$

"Advection-Loss" *(no diffusion)* – predicts strong energy dependence of filament widths

Inclusion of the diffusion in the model is necessary !!!



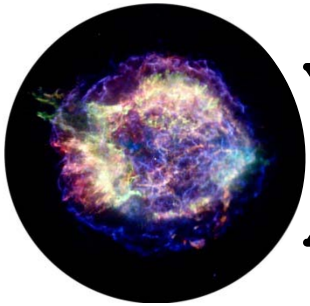
Results

- “Advection + Synchrotron” model ruled out
- “Advection + Synchrotron + Diffusion” model (*Bohm* type diffusion):
  - Determine *B*-field
  - Determine κ ($D = \kappa D_{Bohm}$)

- Average magnetic field $B \sim$ **60 μG** (30 – 160 μG)
- Diffusion scaling factor ($D = \kappa D_{Bohm}$) $\kappa \sim$ **0.1** (*most around 0.05*)
- Power-law index of electrons $\Gamma \sim$ **2.5** (2.0 – 4.0)

$$B \approx 56\mu G \left(\frac{C_1}{0.02}\right)^{-2/3} \left(\frac{C_2}{5.0}\right)^{-2/3} \left(\frac{V_{adv}}{1.3 \times 10^8 cm\ s^{-1}}\right)^{2/3} \left(\frac{R}{10^{19} cm}\right)^{-2/3}$$

$$\kappa \approx 0.05 \left(\frac{C_2}{5.0}\right)^{-2} \left(\frac{V_{adv}}{1.3 \times 10^8 cm\ s^{-1}}\right)^{2}$$



Level of Turbulence and Shock Obliquity

For the scalar diffusion coefficient:

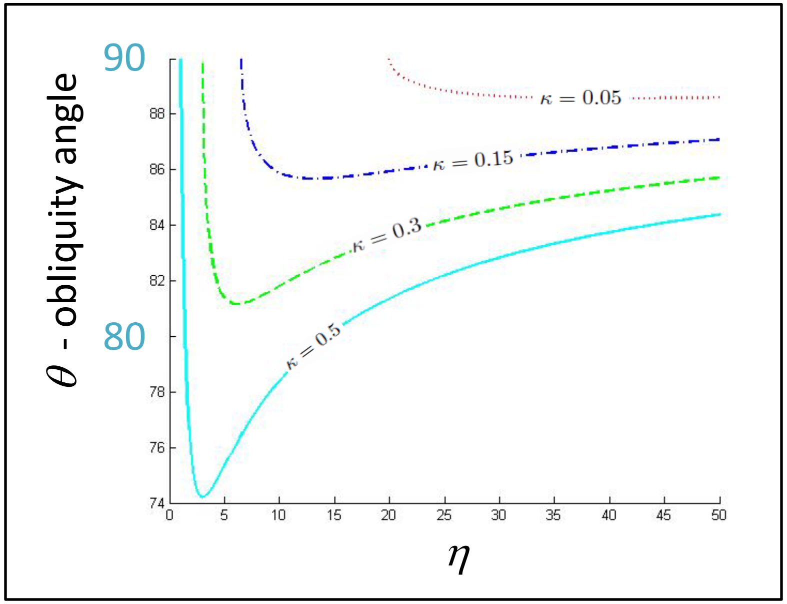
$$\boxed{D = D_{\|} \cos^2\theta + D_{\perp} \sin^2\theta}$$

$$\left.\begin{aligned} D &= D_{\|}\left[\left(\cos^2\theta + \frac{\sin^2\theta}{1+\eta^2}\right)\right] \\ D_{\|} &= \eta D_0 \gamma^{1-\delta} \\ D &= \kappa D_0 \gamma^{1-\delta} \end{aligned}\right\} \quad \boxed{\kappa = \eta\left(\cos^2\theta + \frac{\sin^2\theta}{1+\eta^2}\right)}$$

(Jokipii 1987)

$$\eta \equiv \left(\frac{\lambda_{mfp}}{r_g}\right) = (\delta B/B)^{-2}_{res}$$

$1 \leq \eta$ - gyrofactor



$$\kappa = 0.05 \;\Rightarrow\; \boxed{88.5^\circ \leq \theta \leq 90^\circ} \;\Rightarrow\; \eta \geq 20 \;\Rightarrow\; \boxed{\frac{\delta B}{B} \leq 0.22}$$

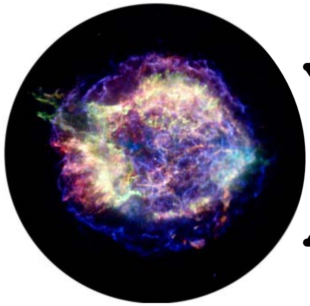
$$\kappa = 0.15 \;\Rightarrow\; \boxed{86^\circ \leq \theta \leq 90^\circ} \;\Rightarrow\; \eta \geq 6.5 \;\Rightarrow\; \boxed{\frac{\delta B}{B} \leq 0.4}$$

$$\kappa = 0.3 \;\Rightarrow\; \boxed{81^\circ \leq \theta \leq 90^\circ} \;\Rightarrow\; \eta \geq 3 \;\Rightarrow\; \boxed{\frac{\delta B}{B} \leq 0.6}$$

Shocks are quasi-perpendicular !
consistent with wind environment

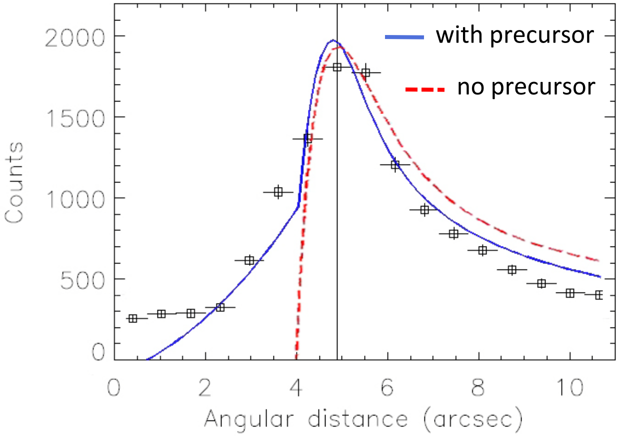
Maximum energy of accelerated electrons :

$$E_{max} \approx 330 \left(\frac{\kappa}{0.05}\right)^{-1/2} \left(\frac{B}{30\mu G}\right)^{-1/2} \left(\frac{V_{sh}}{5.2\times 10^8 cm\ s^{-1}}\right) TeV$$



Caveats

- We did δ-function profile for specific synchrotron power of a single electron.
- **More detailed calculations may be needed near the cut-off**

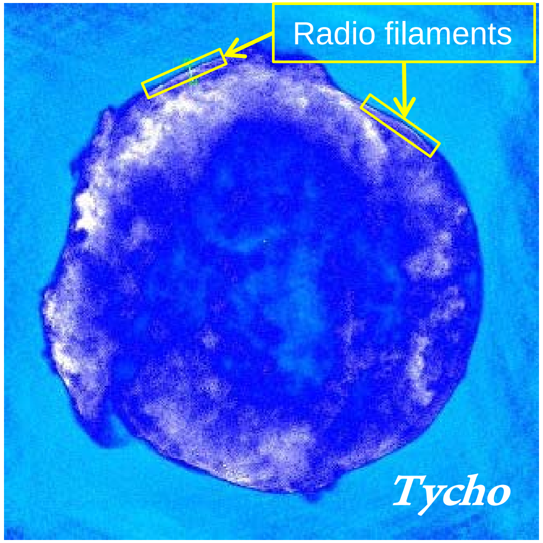


- Detailed shapes of profiles are not well reproduced

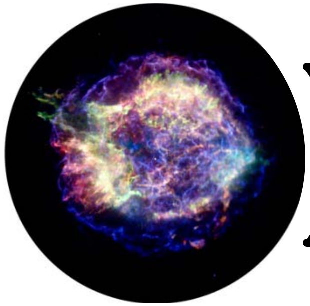
  Half width @ half-maximum still relevant
- **Possible solution – precursor (*upstream*) emission**

- Thin radio filaments found in some SNRs can't be explained by loss-limited approach
- **Possible solution – spatial structure of magnetic field**

  (Pohl, Yan, & Lazarian 2005)



- Can't yet distinguish different types of turbulence (White noise, *Kolmogorov*, *Kraichnan*)
- **Possible solution – Fit the data for different values of δ**



Summary

- Important observational test for the shock acceleration mechanisms
- Diffusion is important in the downstream region:
  - Various levels of *spatial evolution of spectra*
  - *Weak* energy dependence of filament widths
- Magnetic field in filaments : $\boldsymbol{B} \approx 30 - 160\ \mu G$
- Diffusion coefficient as a fraction of *Bohm* : $\boldsymbol{\kappa} \sim 0.05 - 0.35$
- Turbulence is moderately strong for all regions : $\delta \boldsymbol{B}/\boldsymbol{B} \sim 0.2\text{-}0.4$
- For all studied filaments shocks are *quasi-perpendicular*
- *Future work* : i) Fit the data for different values of δ to test the turbulence spectrum
  ii) Understand the implications of our results on *DSA* mechanism
  iii) Study other shell type SNRs