Phenomenological approaches to the problem of electron acceleration in Supernova Remnants

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i. Some statistical properties of SNRs
(with Oleh Petruk – 1st paper submitted)

ii. SN 1006 3-dimensional structure
(work in progress)

Inferences on the electron acceleration in non-relativistic collisionless shocks

“Phenomenology” in physical sciences

(from Wikipedia)

There are cases in physics when it is not possible to derive a theory for describing observed results from the known first principles.

For example, the underlying theory is not yet discovered, or the mathematics to describe the observations is too complex.

In these cases sometimes simple algebraic expressions may be used to model the observations or experimental results.

… and to hopefully provide some hints to the theory.

Old-fashioned topics ?

- Electrons

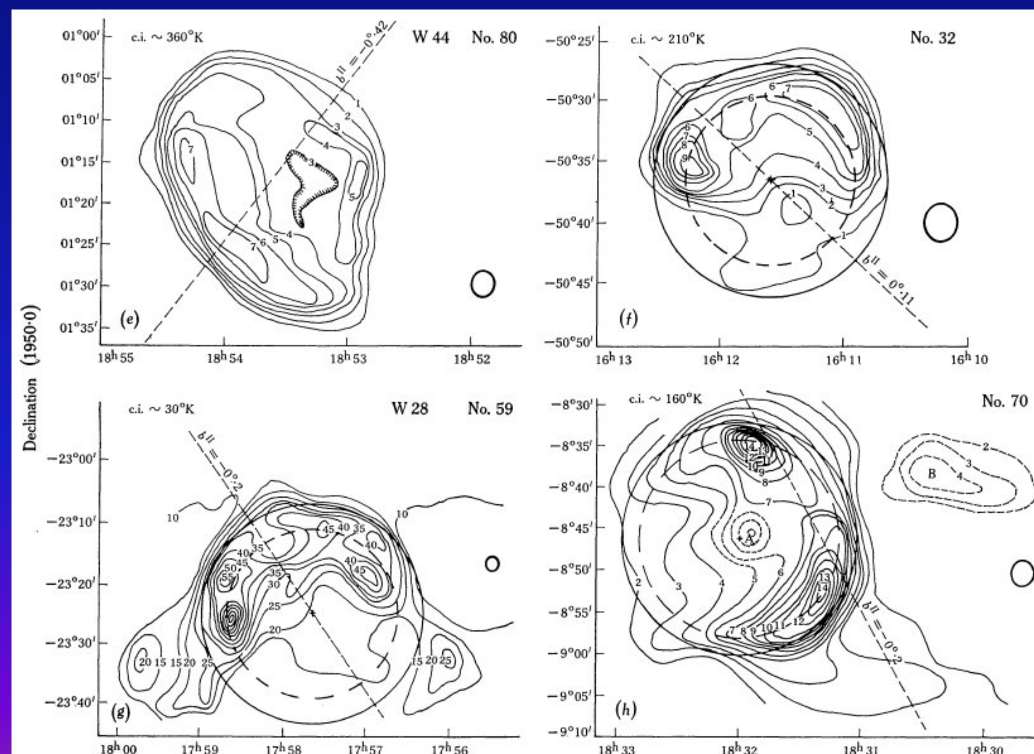
  They emit the non-thermal radiation that we detect

- Radio observations

  Most SNRs are bright radio synchrotron sources

~80 Galactic SNRs already known in the late '60s

(Kesteven 1968)



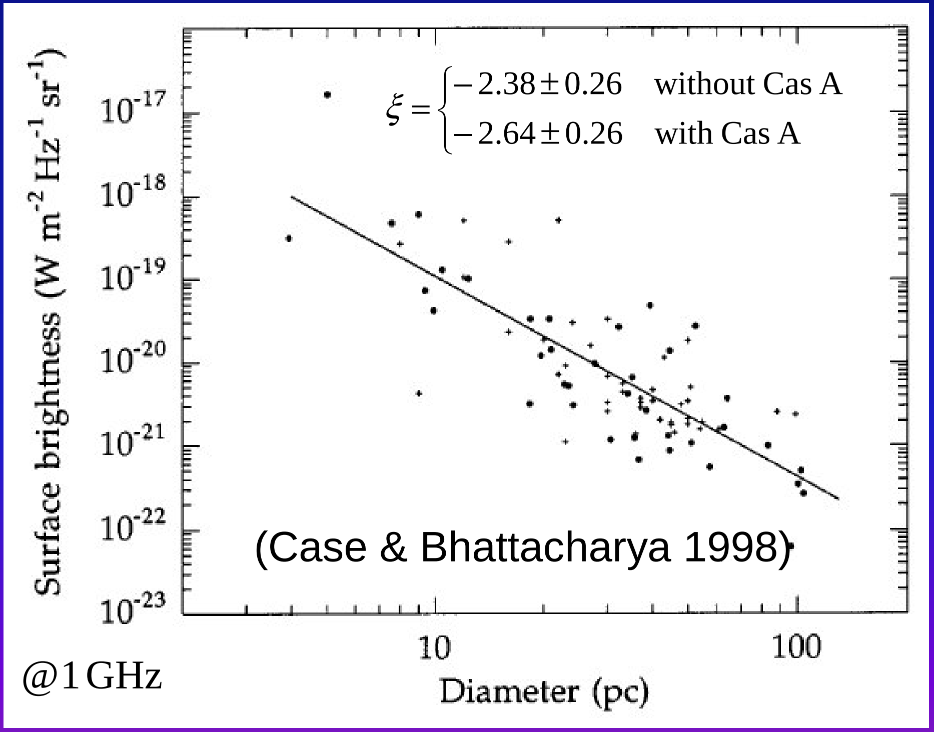
The "infamous" Σ-D relation

- Sample of shell-type SNRs at known distances
- Empirical relation, between the average radio surface brightness (Σ) and diameter (D)
- Known since the '70s. Slope estimates:

$$\Sigma \propto D^{\xi}$$

$$\xi \approx -3.0 \pm 1.0$$

- Used to get distance estimates (from Σ, θ) but rather inaccurate



What to get from the Σ-D relation ?

Usually considered only as
a "tool" for estimating distances

Take it instead as a "diagnostic tool" to constrain electron acceleration and / or the SNR evolution

especially if combined
with correlations between other quantities

Requirement: samples of SNRs at known distances

Basics of synchrotron emission

$$\begin{cases} \nu \propto BE^2 \\ S(\nu) \propto BE\, f(E) \end{cases} \qquad f(E) = K E^{-\gamma} \quad (\gamma \geq 2)$$

$$S(\nu) \propto K B^{(\gamma+1)/2} \nu^{-(\gamma-1)/2} \propto K B^{1+p} \nu^{-p}$$

- In terms of the energy densities (and p=1/2)

$$\left.\begin{aligned} K &\propto \varepsilon_{CR} \\ B &\propto \varepsilon_B^{1/2} \end{aligned}\right\} \Rightarrow S(\nu) \propto \varepsilon_{CR} \varepsilon_B^{3/4} \nu^{-1/2} \Rightarrow \begin{cases} F(\nu) \propto \varepsilon_{CR} \varepsilon_B^{3/4} R^3 \nu^{-1/2} \\ \Sigma(\nu) \propto \varepsilon_{CR} \varepsilon_B^{3/4} R \nu^{-1/2} \end{cases}$$

(degeneracy)

Which expansion phase ?

Most radio SNRs with sizes ~ 10 - 50 pc

- Not too young (M$_{swept-up}$>>M$_{ejecta}$)
- Not too old (E_{SNR}~E_{SN})

- Sedov phase $E_{SN} \approx \rho_o R^3 V_{sh}^2$

$$\begin{cases} R \approx \left(E_{SN}/\rho_o\right)^{1/5} t^{2/5} \\ t \approx \left(\rho_o/E_{SN}\right)^{1/2} R^{5/2} \end{cases}$$

$$V_{sh} = \frac{2}{5}\frac{R}{t} \approx \left(E_{SN}/\rho_o\right)^{1/5} t^{-3/5} \approx \left(E_{SN}/\rho_o\right)^{1/2} R^{-3/2}$$

The many "flavours" of the predictions

- Given $\Sigma \propto \varepsilon_{CR} \varepsilon_B^{3/4} R$
- Bubble of magnetic fields and electrons:

  $\varepsilon_{CR} \propto \varepsilon_B \propto R^{-4}$ $\Sigma \propto R^{-6}$ (Shklovsky 1960)

  + similar (van der Laan 1962; Lequeux 1962; Poveda & Woltjer 1962; Kesteven 1968)
- If continuous injection of electrons & fresh field. THEN emission decrease more slowly
- Magnetic field efficiency

  compression: $\varepsilon_B \propto \varepsilon_{B,ISM}$

  turbulent amplification: $\varepsilon_B \propto \rho_o V_{sh}^2$

- Particle efficiency

  in energy: $\varepsilon_{CR} \propto \rho_o V_{sh}^2$

  in number: $f(E) \approx \begin{cases} n E_o^{(\gamma-1)/2} E^{-(\gamma+1)/2} & E < mc^2 \\ n\left(mc^2 E_o\right)^{(\gamma-1)/2} E^{-\gamma} = K E^{-\gamma} & E > mc^2 \end{cases}$

$$E_o \propto m V_{sh}^2 \Rightarrow \varepsilon_{CR} \propto n\left(mc^2\right)^{(3-\gamma)/2}\left(m V_{sh}^2\right)^{(\gamma-1)/2} \propto \rho_o V_{sh}^{2p} \quad \Rightarrow \rho_o V_{sh}$$

(Bell 1978)

- Fitting the behaviour of B: (Duric & Seaquist 1986)
  - constant number efficiency for particles
  - Sedov expansion for the SNR
  - From $\Sigma \propto D^{-3.5}$ then $B \propto D^{-2}$

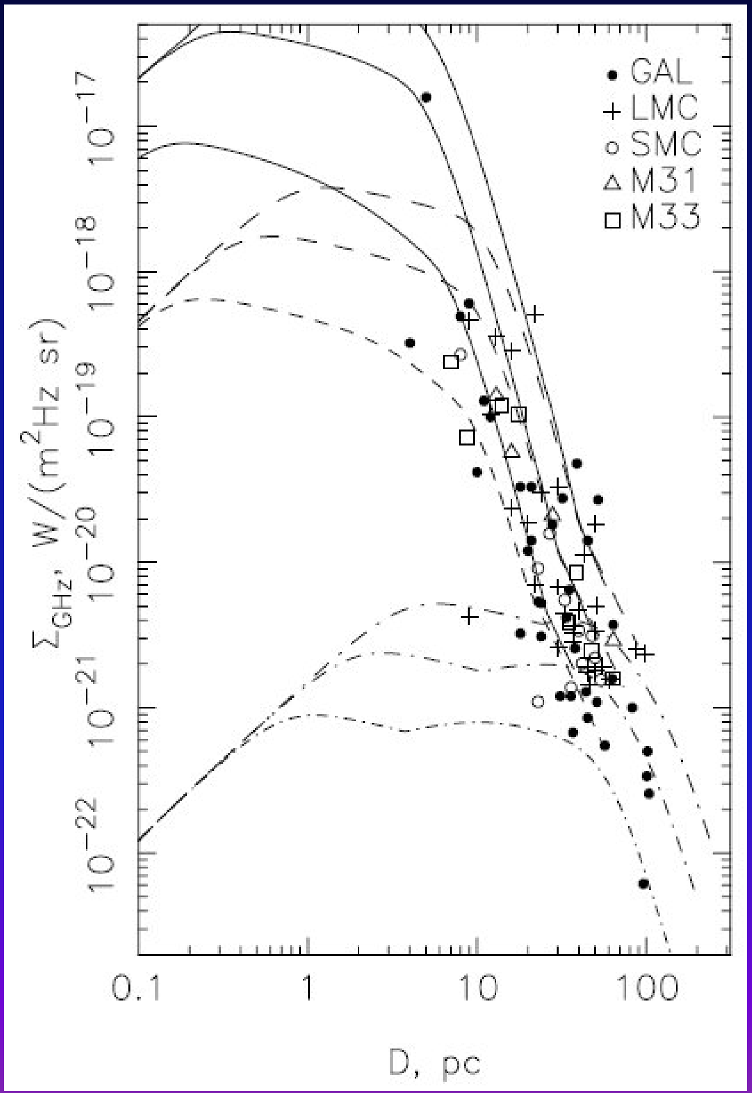
The "Standard Model"

(from Berezhko & Völk 2004 "The theory of synchrotron emission from SNRs")

- Efficient magnetic field amplification $\varepsilon_B \propto \rho_o V_{sh}^2$
- Efficient acceleration of electrons $\varepsilon_{CR} \propto \rho_o V_{sh}^2$
- Sedov phase $\rho_o V_{sh}^2 \approx E_{SN} / D^3$
- Resulting emission

$$\Sigma(D, n_0) \propto D \varepsilon_{CR} \varepsilon_B^{3/4} \propto D\left(\rho_o V_{sh}^2\right)^{7/4} \propto D(E_{SN} / D^3)^{7/4} \propto E_{SN}^{7/4} D^{-17/4}$$

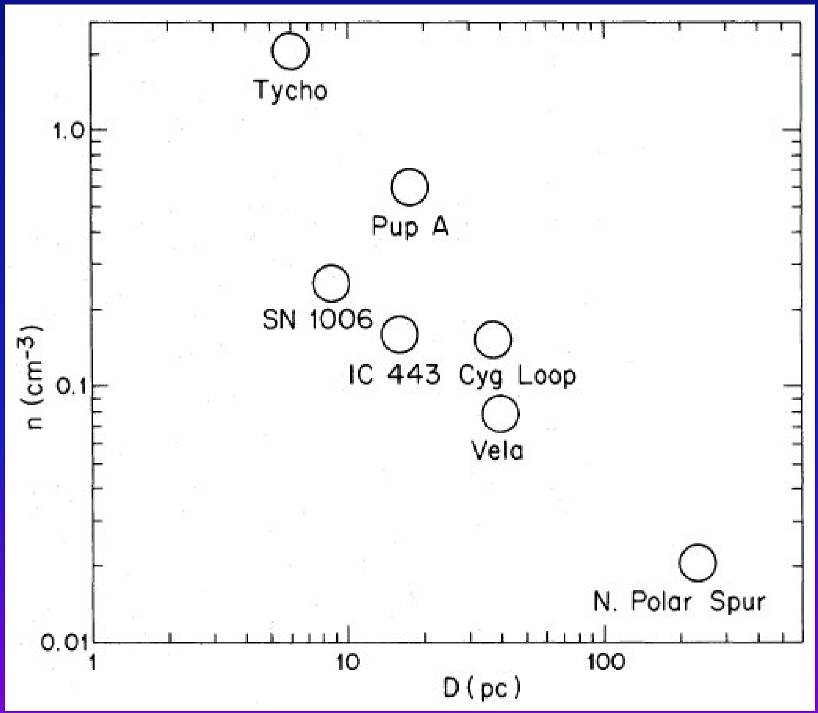
NO DEPENDENCE ON ρ_o !



Changing perspective

- So far, Σ-D relation taken as the evolutionary track of an "average" supernova remnant
- Evidence that is could, instead, be a combined result of SNRs evolving under very different ambient conditions
- Correlation between the SNR size and the ambient density (from thermal X-rays)

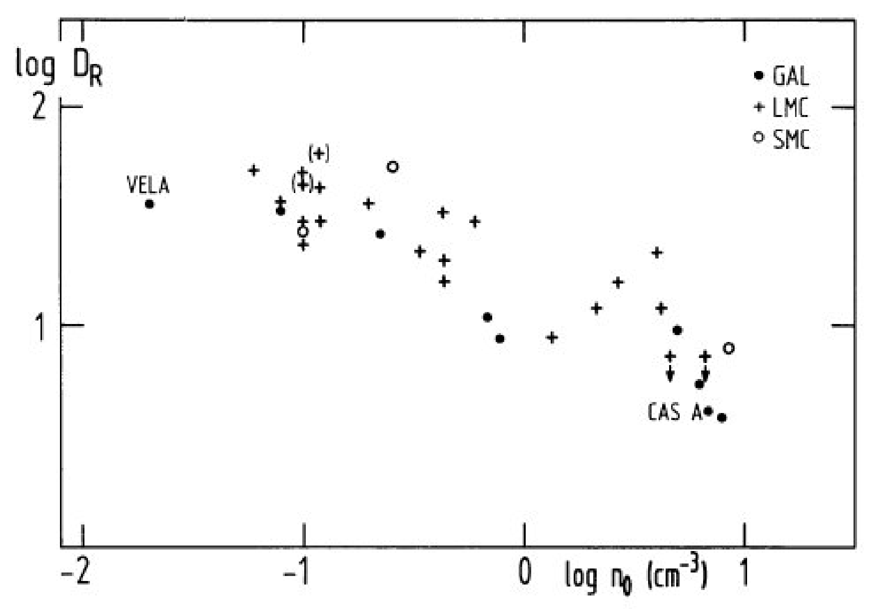
$$L_X \propto n_o^2 R^3 f(T_X)$$



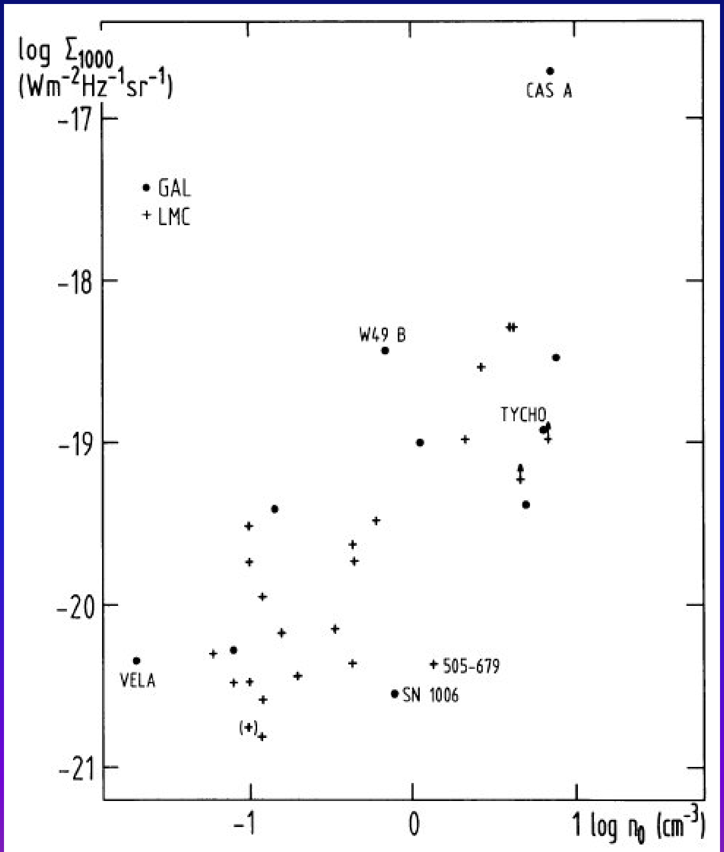
"The trend of lower densities observed for larger SNR is unmistakable"

(McKee & Ostriker 1977)

Σ-D as a secondary relation



$$D_R \propto n_o^{-0.39\pm0.04}$$
$$\Sigma_R \propto n_o^{1.37\pm0.21}$$
$$\Rightarrow \Sigma_R \propto D_R^{-3.5}$$



"It may be concluded that variations in the apparent ambient density may indeed be responsible for the slope of the observed Σ-D diagram"

(Berkhuijsen 1986)

Basic assumptions & considerations

ABOUT THE SOURCES:

- We observe a combined effect of evolutionary tracks under different ambient conditions
- The slopes of individual tracks may even be different from the best fit Σ-D slope
- A SNR in decelerated expansion spends most of its time near its maximum D (as a radio source)
- The lifetime of a SNR as an efficient radio source may be shorter than its “dynamical” life

ABOUT THE DATA:

- Σ-D-n_0 are a reasonable set of parameters for describing this sample
- The information contained in the data is more than "just a slope"
- The density of points in the parameter space (selection effects allowing) also depends on the residence time of individual SNRs

Zeroth order approximation

Neglecting the SNR evolution: $D = D_2$

For Sedov expansion: $\langle \lg D \rangle = \lg D_2 - \frac{2}{5} \lg e$

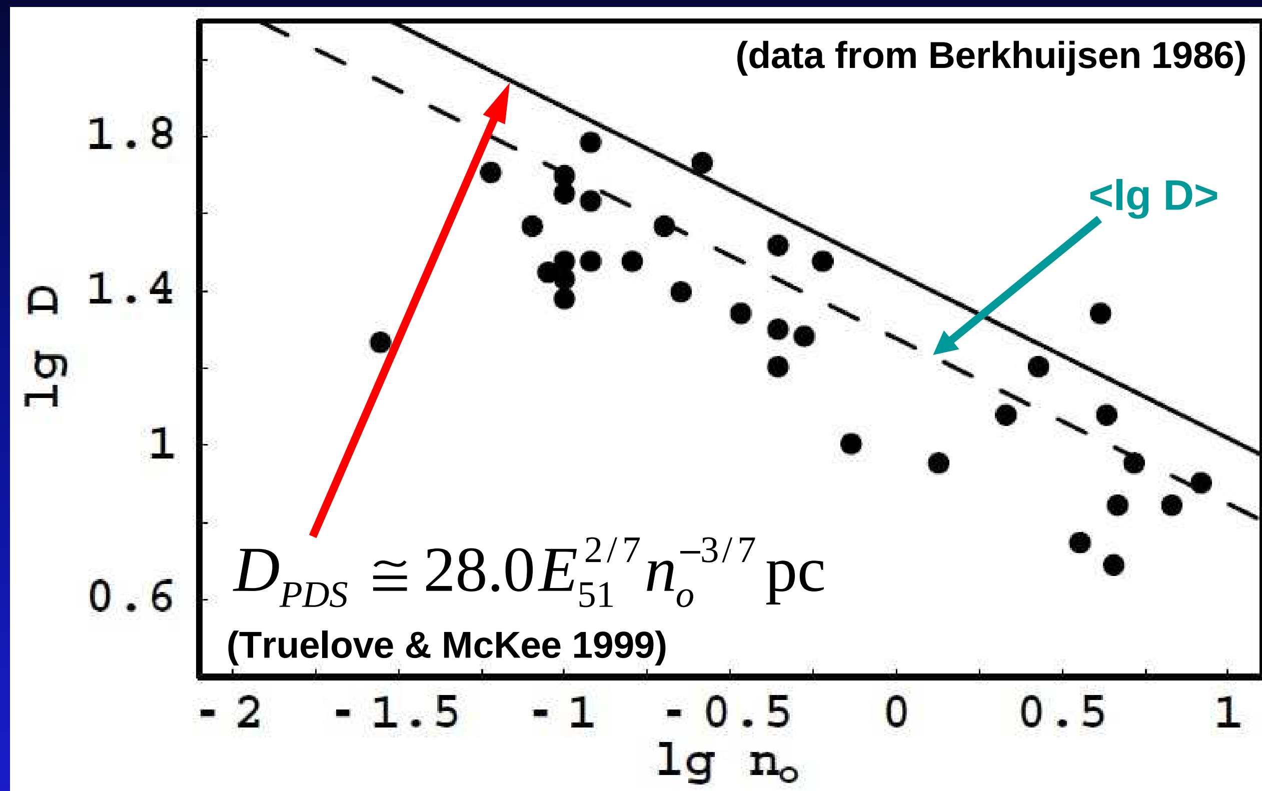
- D-n relation: $D_2 \propto n_o^m$
- Σ-n relation: $\Sigma_2 \propto n_o^n$
- DERIVED Σ-D relation: $\Sigma_2 \propto D_2^{\xi}; \quad \xi = n/m$
- Probability of finding a SNR in a region with given density: $\widetilde{P}(n_o) \propto n_o^w$
- Cumulative distribution with size:

$$N(D) \propto \widetilde{P}(n_o) n_o \propto D^{(w+1)/m}$$

No dependence on the expansion law !!

Endpoint of the radio emitting phase

Nice match with the endpoint of the Sedov phase



- Safe to use Sedov expansion in modelling
- Consistent with $E_{SN} \leq 10^{51}$ erg !!!
- What (if any) physical relation ?

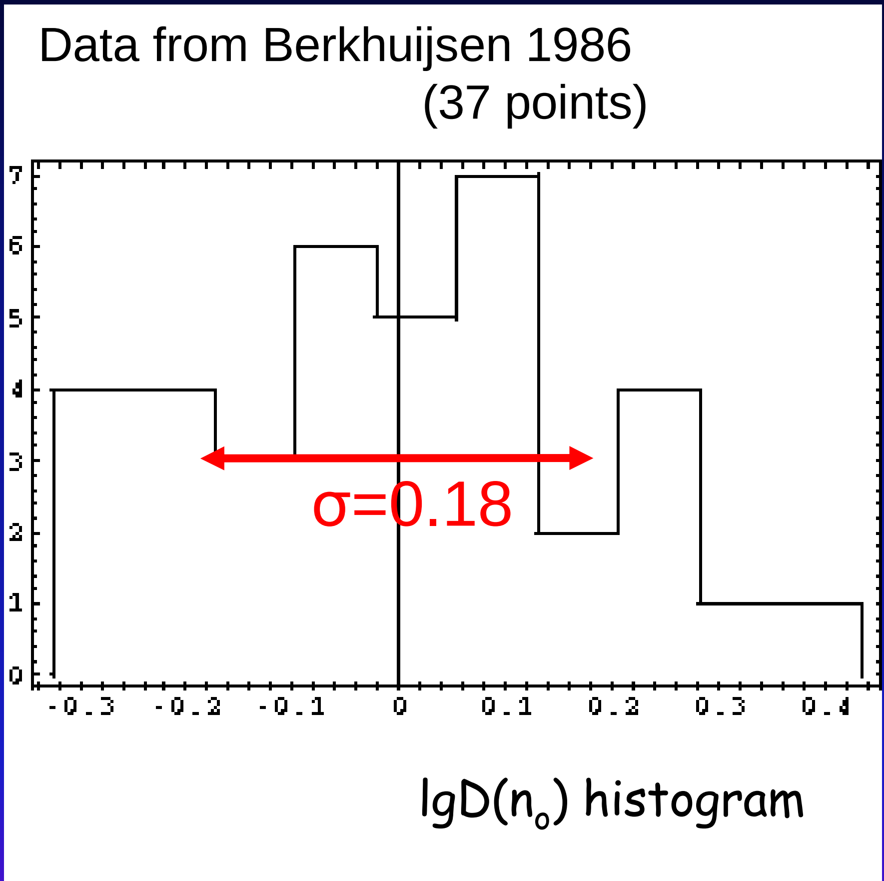
Individual SNR evolution tracks

- Residuals of the $D(n_o)$ fit
- Fixed n_o, i.e. single SNR evolution
- Dispersion in the distribution (LOW)

If $D \propto t^a$ with $D < D_2$ then

$$\sigma(\lg D) = 0.434 / a$$

i.e. $\qquad = 0.174$ for $a = 5/2$



Dispersion due to measurement uncertainties?
If so, even lower intrinsic dispersion.

A physical meaning for the endpoint ?

- Decelerated expansion
  $\Rightarrow$ most time spent when D near D_2
- Why should the efficiency in emitting radio synchrotron (= in injecting electrons?) turn down at the end of the Sedov phase?

- "Physical" arguments:
  - Increase of the compression ratio (even better?)
  - Radiative $\Rightarrow$ lower temperature (against?)
- Alternative reasons?

A parametric statistical model

- Physics is introduced implicitly
- Self-similarity is assumed !!!

$$D_2 \propto n_o^m$$
$$\Sigma_2 \propto n_o^n$$
$$\tilde{P}(n_o) \propto n_o^w$$

Basic assumptions:

- SNR dynamical evolution $t \propto D^a n_o^b$
- SNR radio emission $\Sigma = \Sigma_2 f\left(D / D_2\right)$

or, more simply, $\Sigma \propto D^p n_o^q$

$$\Sigma_2 \propto D_2^{\xi}; \quad \xi = n / m = p + q / m$$

p is the slope of an individual evolutionary track

Analysis of "ideal" data

Results of linear regressions:

- *p* and *q* from fitting $\Sigma \propto D^p n_o^q$ to the data (2-D regression)
- *m* from fitting $D \propto n_o^m$ to the data
- *w* from the cumulative distribution

$$N(D) \propto \tilde{P}(n_o) n_o \propto D^{(w+1)/m}$$

- The expansion law could be only extracted from the distribution across the $D(n_o)$ correlation

Some results from data analysis

SNR samples with information on Σ-D-n_o

(using extragalactic SNRs)

$$\Sigma \propto D^p n_o^q$$
$$D_2 \propto n_o^m$$
$$\tilde{P}(n_o) \propto n_o^w$$

- Berkhuijsen (37 radio + X-ray obj)

$$\xi = -2.01 \pm 0.34;\ p = -0.88 \pm 0.55;\ q = 0.62 \pm 0.25$$
$$\Rightarrow \quad m = -0.37 \pm 0.04$$

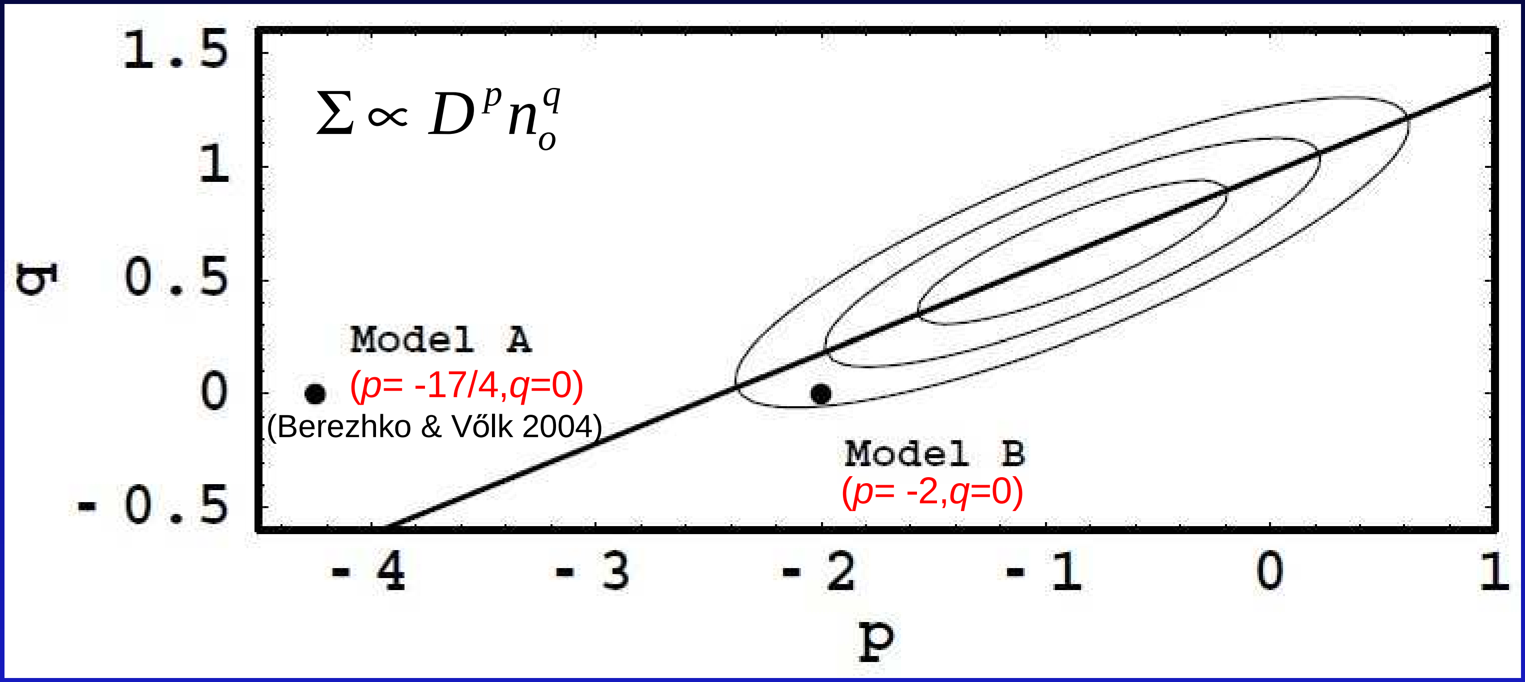
- LMC (28 radio + X-ray obj)

$$\xi = -2.57 \pm 0.38;\ p = -0.82 \pm 0.89;\ q = 0.71 \pm 0.31$$
$$\Rightarrow \quad m = -0.32 \pm 0.03$$

- M33 (22 radio + X-ray obj)

$$\xi = -2.20 \pm 0.46;\ p = -1.37 \pm 0.64;\ q = 0.52 \pm 0.30$$
$$\Rightarrow \quad m = -0.34 \pm 0.07$$

2-D analysis of the confidence levels



- Near degeneracy in the p-q plane

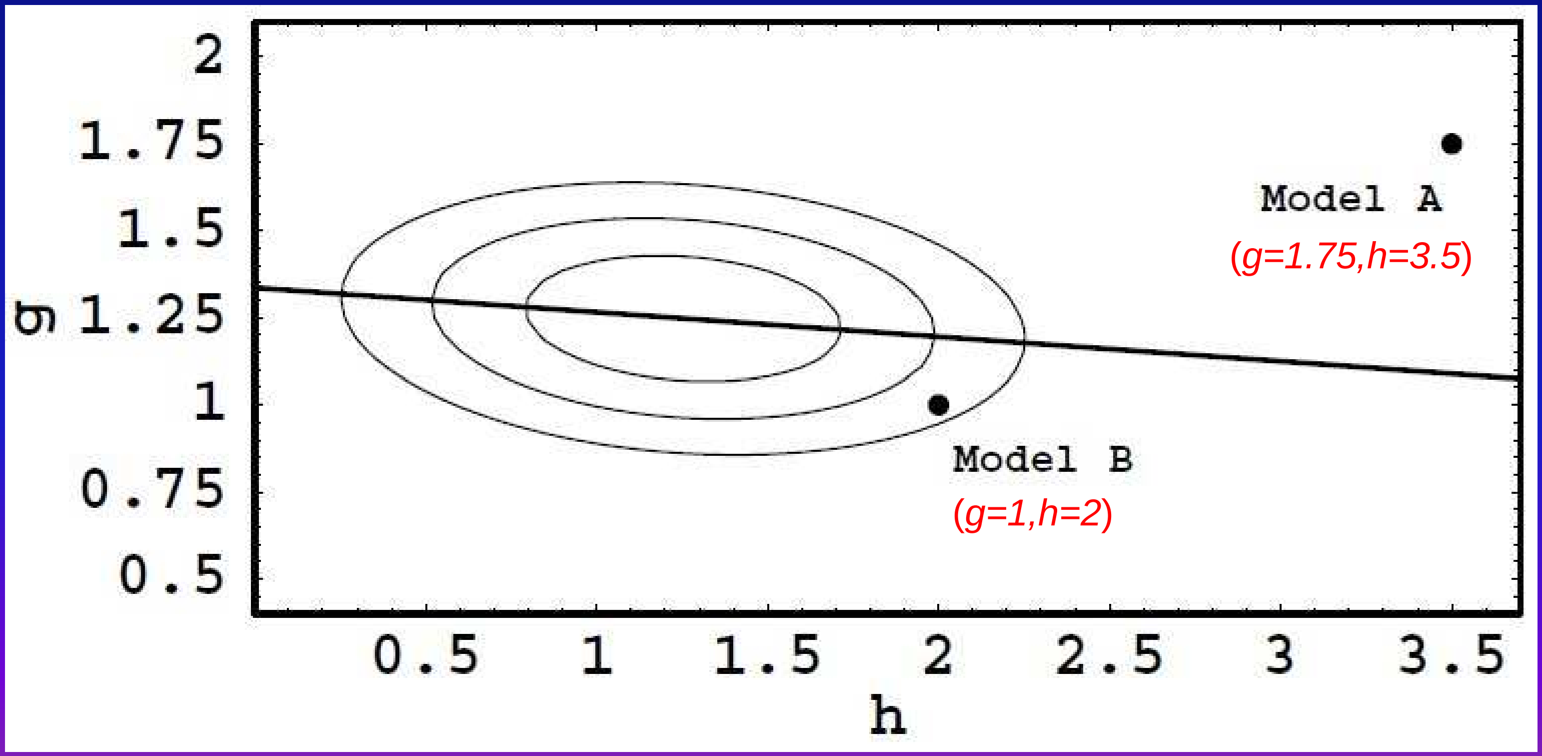
$$q - 0.40\,p = 0.97 \pm 0.14$$

…with a more "physical" flavour

- Assuming Sedov expansion

$$\varepsilon_{CR}\varepsilon_B^{3/4} \propto n_o^g V_{sh}^h \quad \Rightarrow \quad \Sigma \propto D^{1-3h/2} n_o^{g-h/2}$$

- Best fit: $g = 1.25 \pm 0.14; \quad h = 1.27 \pm 0.37$



- Best- fit solution:

$$g = 1.25 \pm 0.14; \quad h = 1.27 \pm 0.37$$

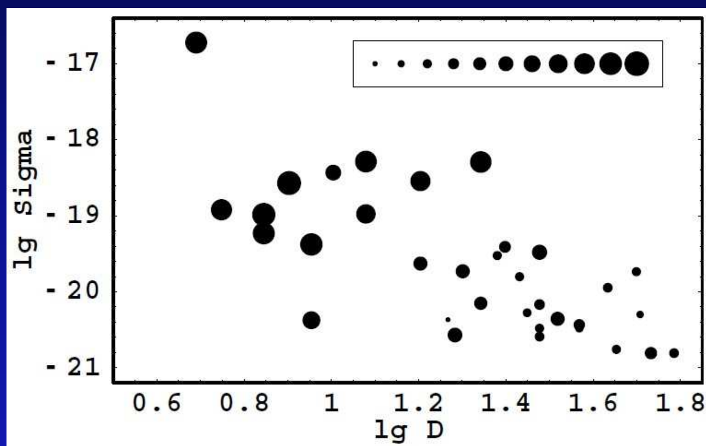
- Against field amplification

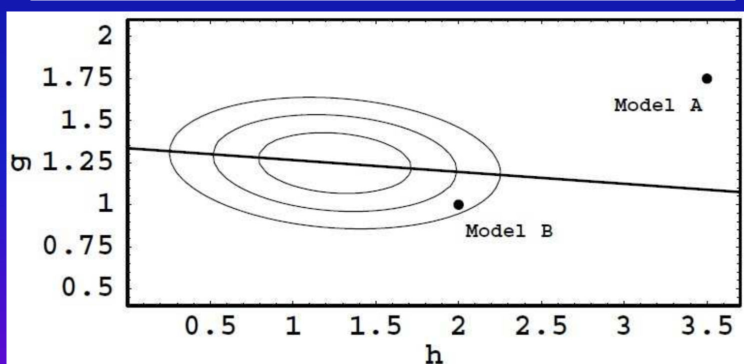
ε_{CR}	ε_B	$\varepsilon_{CR}\varepsilon_B^{3/4}$	g	h
ρV^2	ρV^2	$\rho^{7/4} V^{7/2}$	1.75	3.5
ρV	ρV^2	$\rho^{7/4} V^{5/2}$	1.75	2.5
ρV^2	1	ρV^2	1	2
ρV	1	ρV	1	1

(for middle-age SNRs !!!)

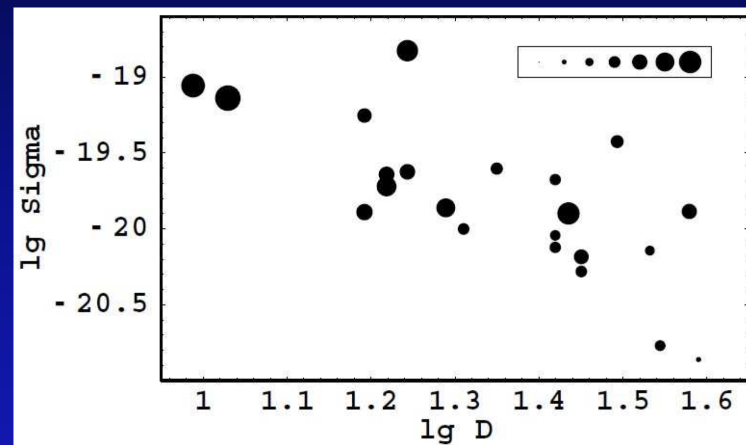
How robust is this method?

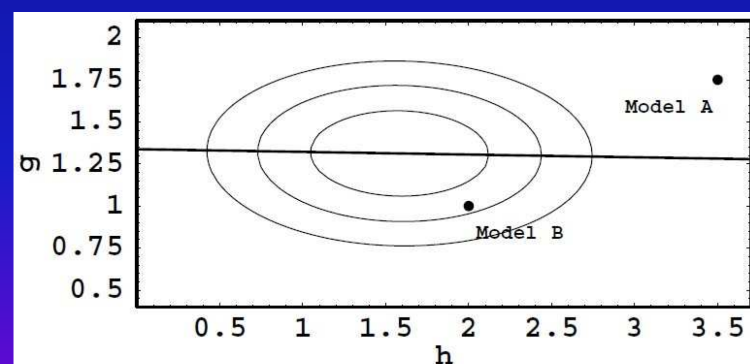
Berkhuijsen





M33

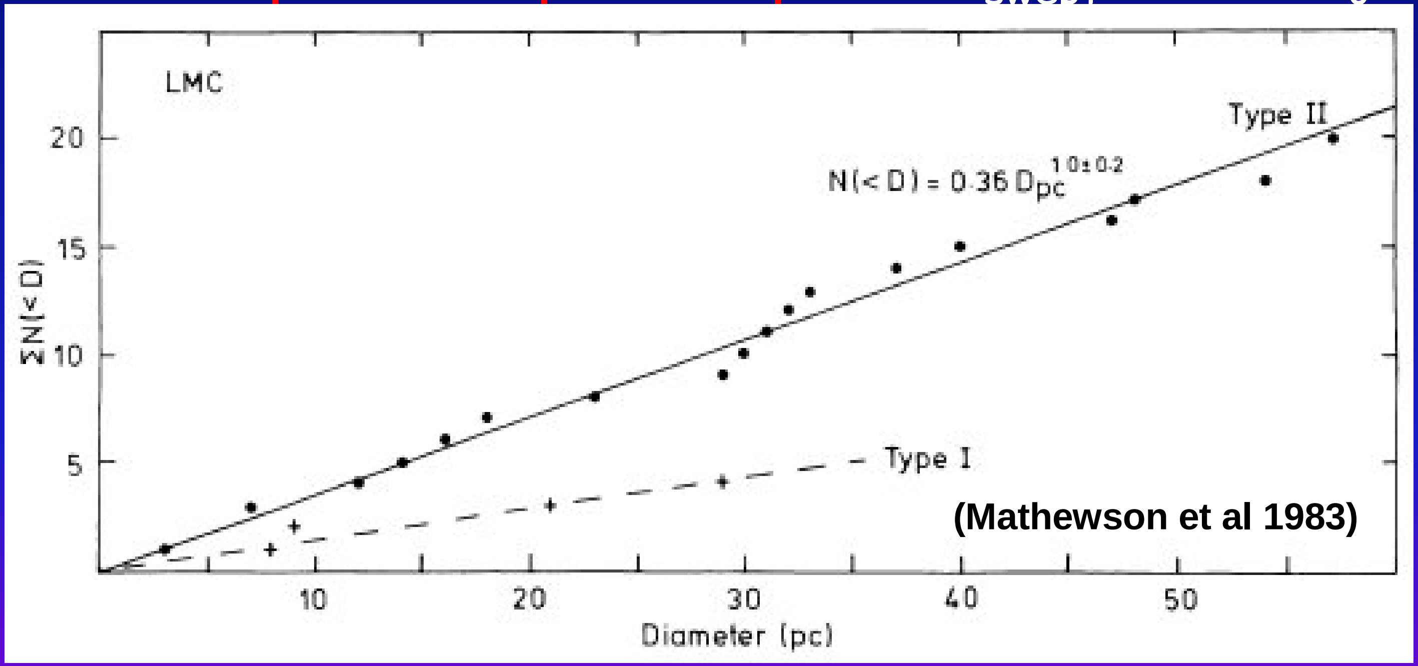




Different selection effects

Cumulative distributions

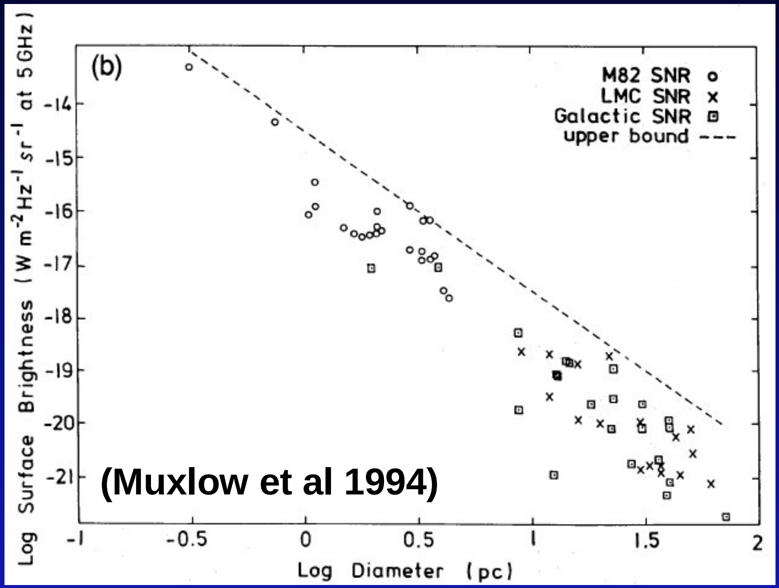
- Observed cumulative functions N(D) of SNRs in several galaxies show nearly linear behaviours
- Usual argument: if $D \propto t^{\alpha}$ then $N(D) \propto D^{1/\alpha}$
- Linear expansion up to 50 pc ? M_{swept}~2000 n_o M_{sun}

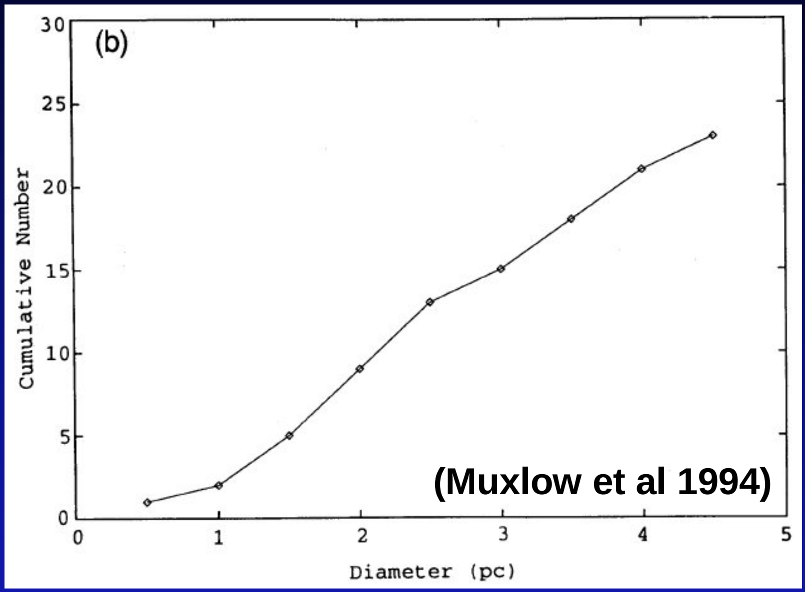


(Mathewson et al 1983)

- Independent of expansion law. $N(D) \propto \widetilde{P}(n_o) n_o \propto D^{(w+1)/m}$

The case of M82





- Linear N(D), then linear expansion (WRONG)
  then Vsh>5000 km/s and Ages<400 yr (for D=4 pc)
- Upper limit to the variability (< 0.1% per yr)

THEN cluster wind-driven bubbles, NOT SNRs

(Seaquist and Stankovic 2007)

Erroneous conclusion

What's next ?

- Extending the sample
  - SNRs in nearby galaxies
  - Well known distances
  - Rather homogeneous samples
- Investigate effects of:
  - Sample incompleteness
  - Detection thresholds
  - Barely resolved objects
  - False identifications
- Simulated data
  - On the basis of the above equations
  - To better check selections effects

Part 1 – Summary of results

- Σ-D (+n_o) relation may contain relevant information on:
  - the physics of electron injection
  - the SNR evolution phase
  - the distribution of ambient conditions
- Most radio SNRs close to the end of their "radio" life
  - Electron acceleration processes close to switch-off limit?
- Death of radio SNRs close to the end of Sedov phase.
- Measurement errors do not seem to play a major role.
- The "standard" model for synchrotron emission in SNRs does not fit the data
- Purely magnetic compression seems to be favoured (middle-age SNRs)

The case of SN 1006
(work in progress)

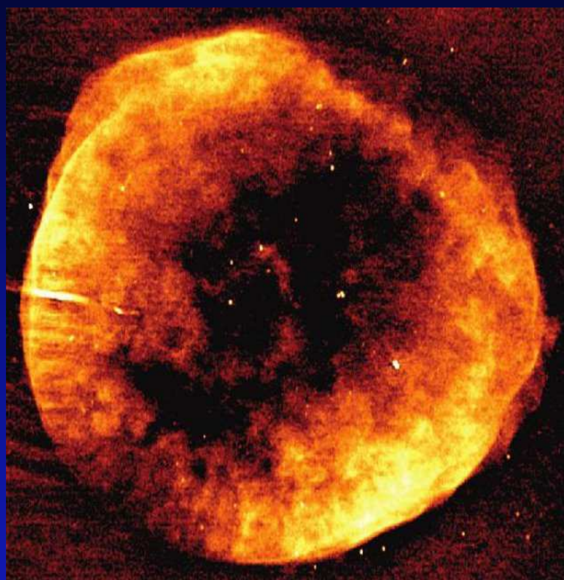
- Extremely well observed in radio and X-rays
  (+ Hα, UV, TeV, IR …)
- Taken as a "prototype" of shell-type SNRs
  (⇒ acceleration, modified shock, …)

BUT

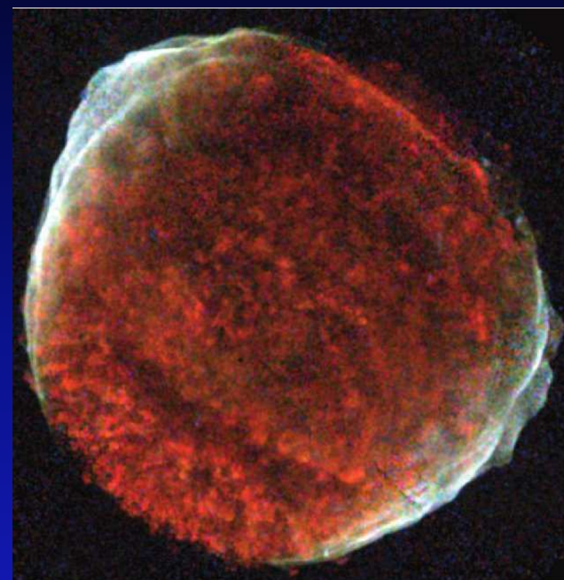
- Do we really understand its 3-D structure?
  (barrel-like vs polar caps)

Implications on acceleration models

Judging by eye…



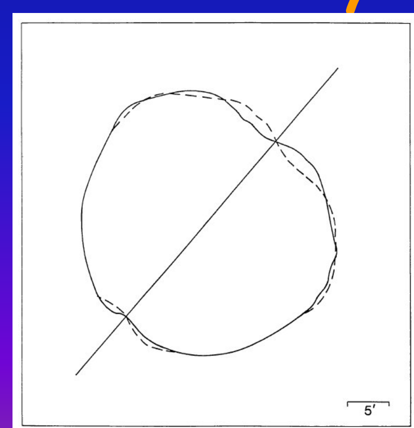
Radio



X-rays

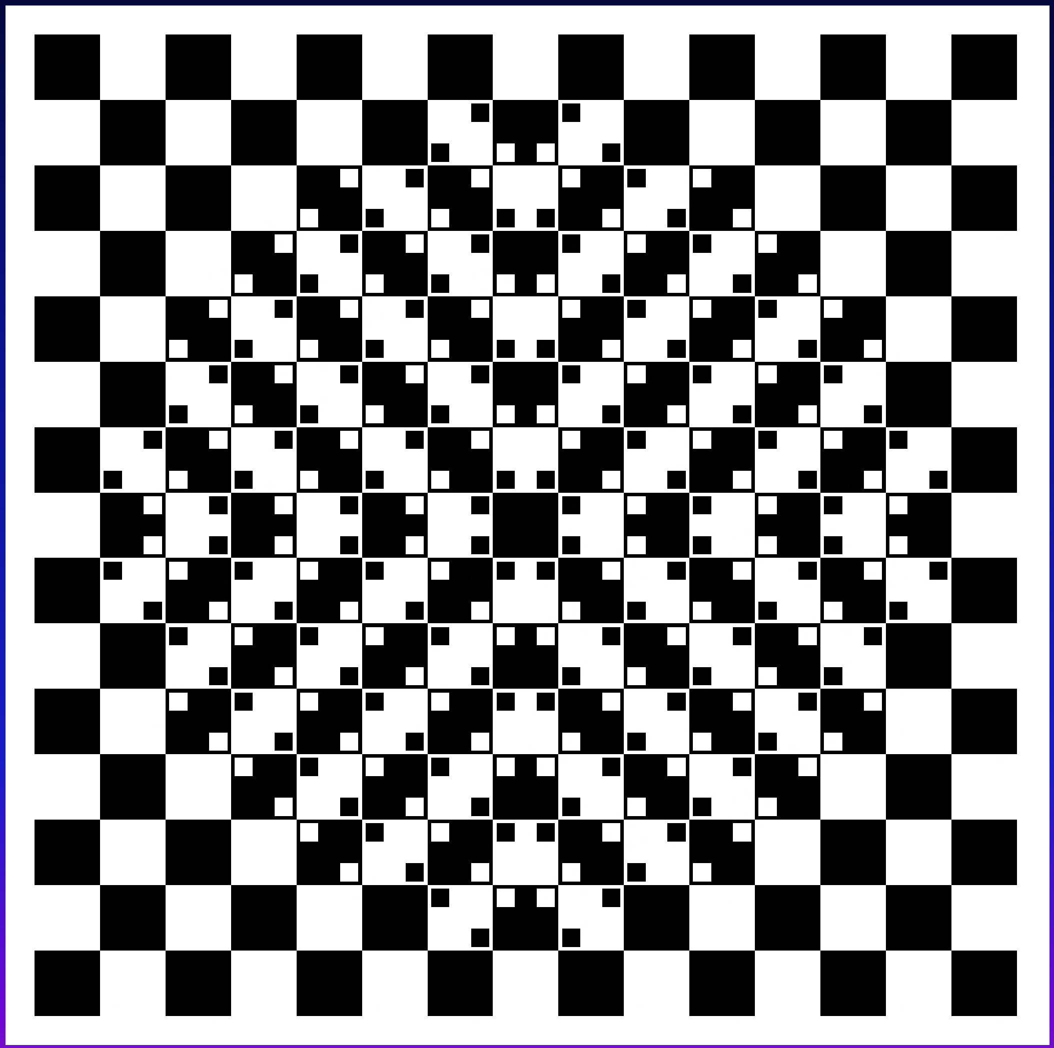
(from Cassam-Chenai 2008)

And even in
non-suspect times
(barrel-shaped)



(Kesteven & Caswell 1987)

but, can we trust in our eyes?



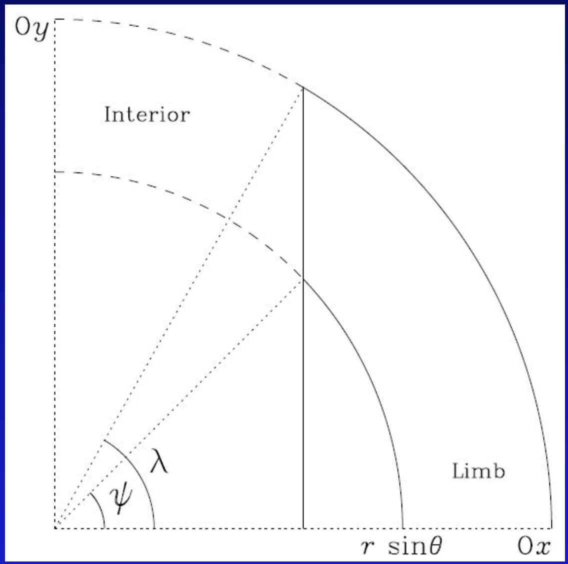
Not always !

Need for an objective test

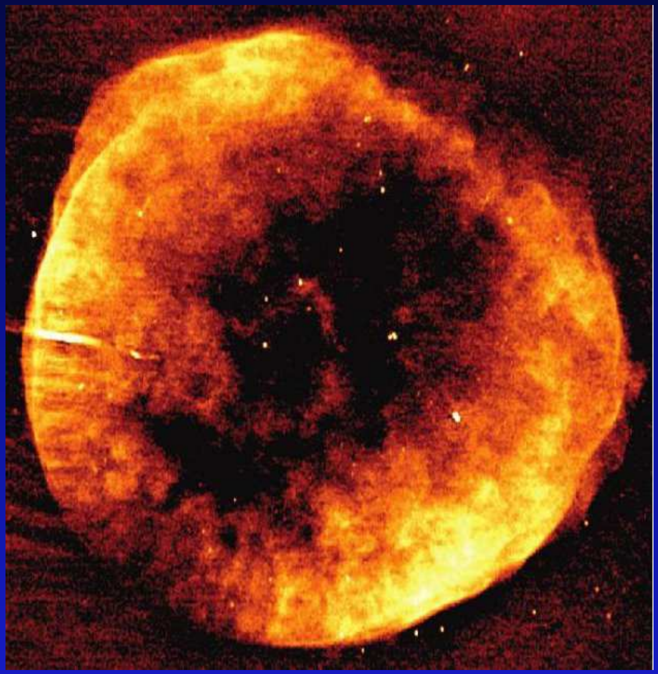
A mathematical theorem

(Rothenflug et al. 2004)

- Projection of an axisymmetric model
- Emission from inner strip
- Strip width = half of total size $\Rightarrow$ lower limit of 0.5 irrespective of orientation
- In SN 1006, 0.7 in radio, but 0.3 & 0.127 in X-ray bands !
- HYPOTHESIS: isotropic emissivity (reasonable, for a highly turbulent field)



Consequences



- Polar-cap 3-D structure for the emissivity
- Parallel shock scenario (result warmly welcomed by many theorists)
- Prediction of strong magnetic amplification seems also confirmed by the sharp X-ray limb

(Bamba et al. 2003)

Any room left for a barrel-shaped SNR?

Reviving the barrel-like model

- Let us try to release isotropy assumption for the emissivity
  - Ordered component of the magnetic field
  - Pitch angle anisotropy in the distribution (no)

FURTHER ASSUMPTIONS

- Axial symmetry, with axis on the sky plane
- Highly ordered magnetic field
- "Monochromatic" synchrotron emission

Directional effects

- On the emissivity $W = c_1 E^2 B^2 \sin^2 \alpha$
- On the emitted frequency $\nu = c_2 E^2 B \sin \alpha$
- Assumed energy distribution (Reynolds 1998)

$$f(E) = \tilde{K} E^{-2} \exp(-E/E_c)$$

- Derived local spectrum

$$S(\nu) = K\, B^{3/2} \sin^{3/2}\alpha\, \nu^{-1/2} \exp\left(-\sqrt{\nu/(\nu_c \sin\alpha)}\right)$$

(2 reasons for low emission at small $\sin\alpha$ values)

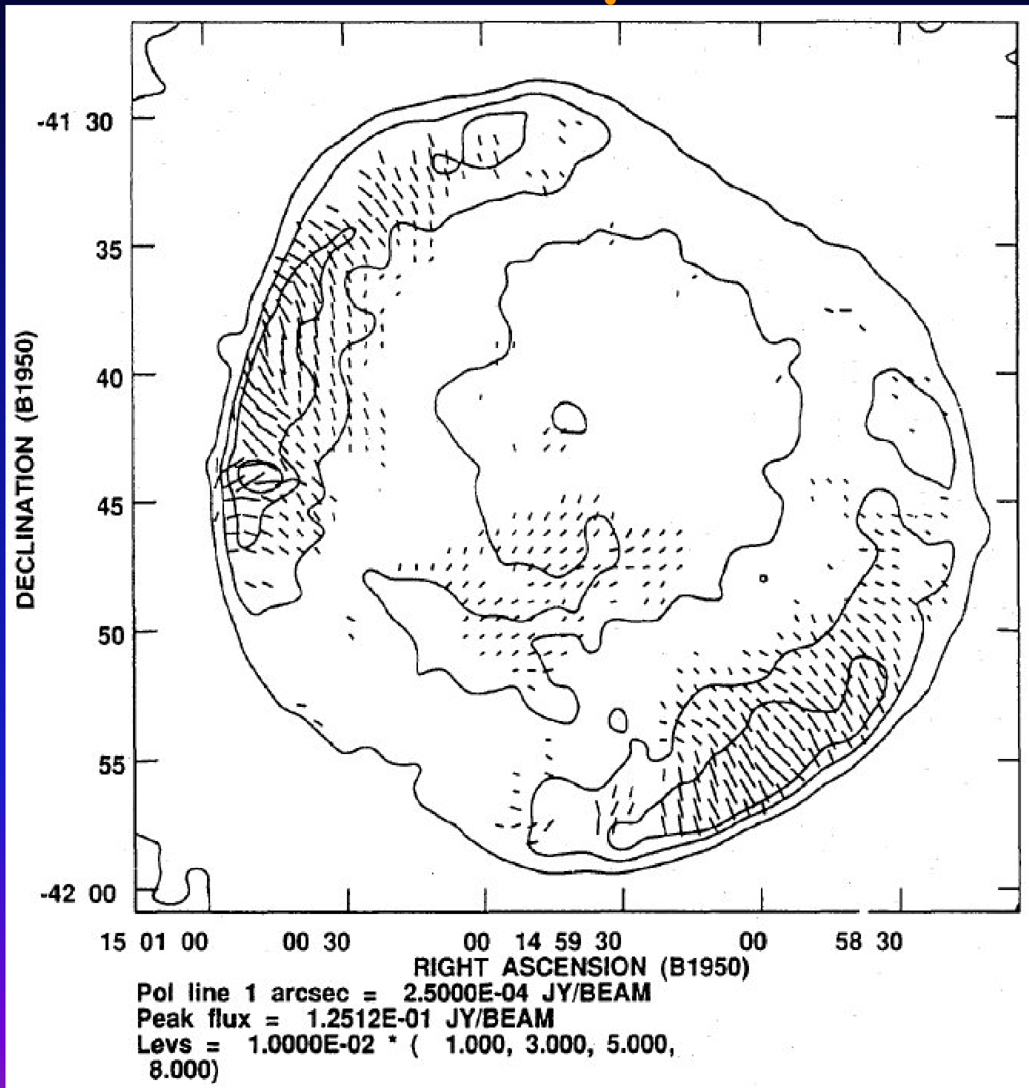
- "Phenomenological" latitude dependencies

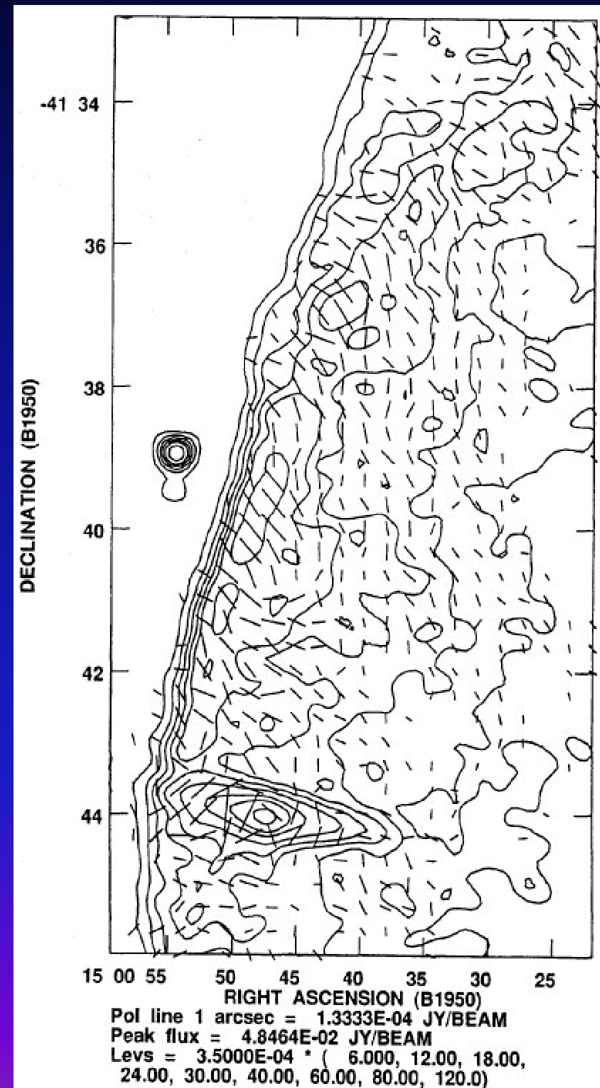
$$K = K_o |\sin\theta|^k; \quad B = B_o |\sin\theta|^b$$ & maybe $$\eta = \eta_o |\sin\theta|^e$$

Large scale field geometry

- Three natural geometries
  - Along meridians
  - Along parallels
  - Radial (only this choice gives small sin α values near the projected center)
    - Observed radio polarization
    - Some Models (Zirakashvili & Ptuskin 2008)
- Fit azimuthal distribution about the edge (sin $\alpha \cong 1$). Annuli with ranges: 400-950 arcsec in radio; 675-950 arcsec in X-rays

Radially oriented fields

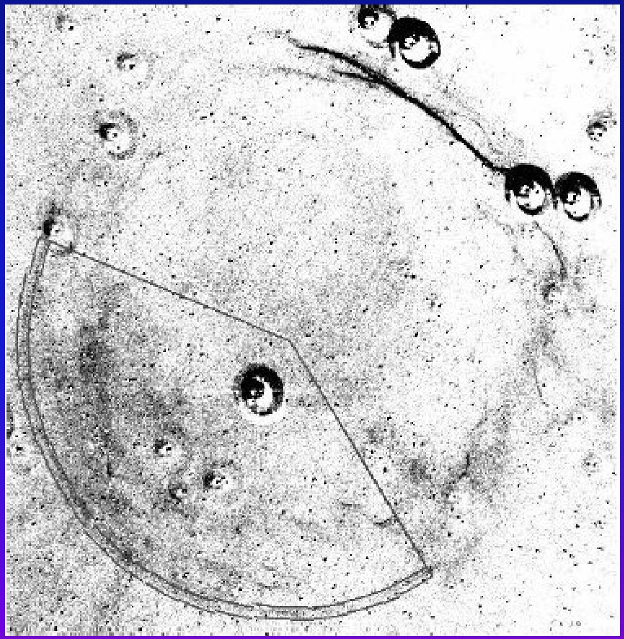




(Reynolds & Gilmore 1993)

The "right" side

- Circular Hα limb along the SE side
- Use only this half-remnant
- Use circle to accurately fix the center:



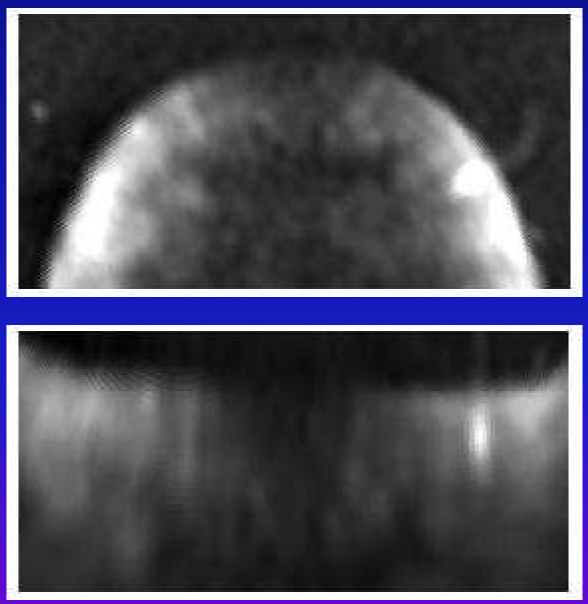
RA: $15^h 02^m 56.8^s$

Dec: -40°56'56.6"

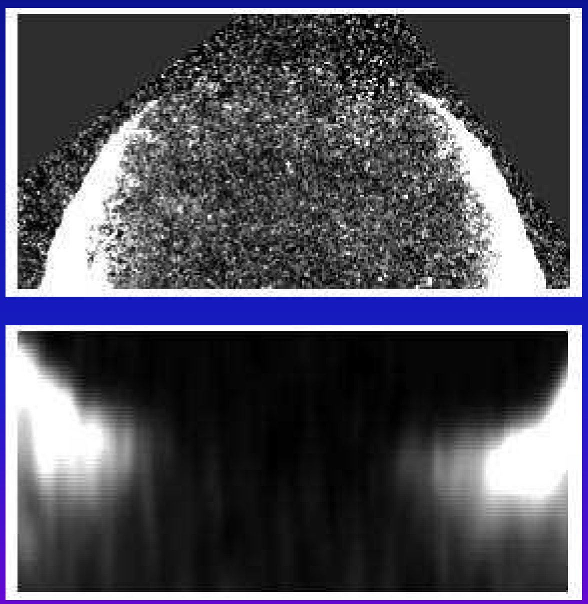
(Cassam-Chenai et al. 2008)

The available data

Radio @ 843 MHz (SUMSS)
X-rays @ 0.5-0.8, 0.8-2.0, 2.0-4.5 keV (Chandra)
(courtesy of Gamil Cassam-Chenai)

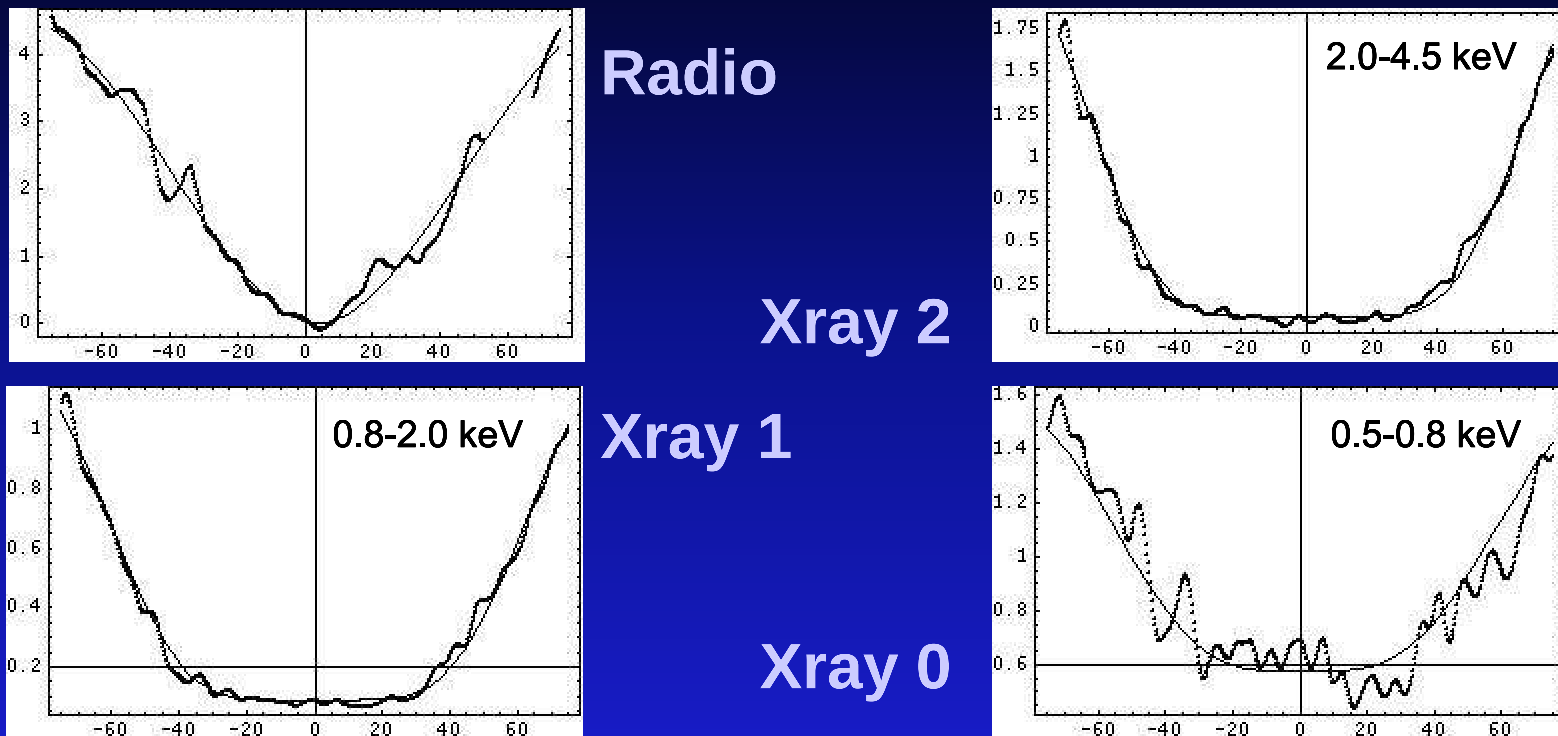


Radio



2.0-4.5 keV

A combined fit



Radio: no exponential cutoff

X-rays: cutoff more evident in the harder bands
offset related to the thermal component

Best-fit parameters

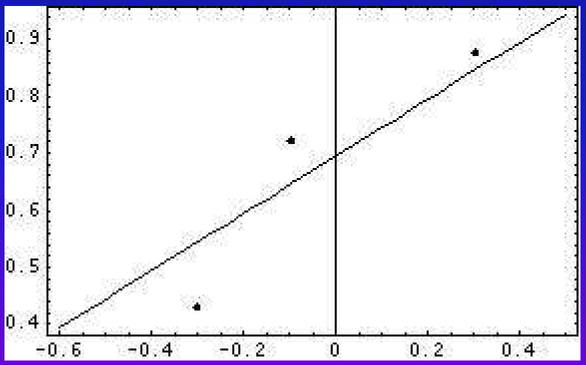
- Fit functions:

$$S_i(\theta) = Q_i \left|\sin(\theta-\theta_1)\right|^q \exp\left(-A_i \left|\sin(\theta-\theta_2)\right|^{-a}\right) + B_i; \quad i = r,0,1,2$$

- No absolute photometry (will constrain Q_i, B_i)
- Small asymmetries: $\theta_1 = 3.8^\circ; \quad \theta_1 = -0.8^\circ$
- Well-behaved A_i like $\nu^{1/2}$ ($A=(\nu/\nu_o)^{1/2}$)
- Relevant parameters;

$$Q \propto \left|\sin\theta\right|^q; \quad \nu_c = \nu_o \left|\sin\theta\right|^n$$

$$q = 1.85; \nu_o = 0.04\,\text{keV}; n = 0.55$$



Synchrotron vs age-dominated

- The nature of the cutoff
  - Age
    $$\tau_{age} = 3.2\,10^{10}\,\text{s}; \quad \text{V}_{\text{sh}} \cong 5000\ \text{km/s}$$
  - Accel. Time
    $$\tau_{acc} = 6.7\,10^{3}\eta\, E/B\ \text{s} = 2.6\,10^{3}\eta\, \nu_{keV}^{1/2} B^{-3/2}\ \text{s}$$
    $$\lambda_{par} = \eta\, r_g$$
  - Synch. time
    $$\tau_{syn} = 1.2\,10^{3}/E/B^{2}\ \text{s} = 3.1\,10^{3}\nu_{keV}^{-1/2} B^{-3/2}\ \text{s}$$
- The right balance
  - Accel = Age
    $$\nu_c = 1.5\,10^{14}\ B^{3}/\eta^{2}\ \text{keV}$$
  - Accel = Synchr
    $$\nu_c = 1.2/\eta\ \text{keV}$$

    Required large variations of η
- Transition field
  $$B_* = 20\,\eta^{1/3}\ \mu\text{G}$$

Age-dominated case ?

- Large azimuthal variations of ν_c . If synch. dominated, large variations of η
- Age-dominated case + nearly constant η
- IF SO (very speculative)

$$B = 6.5\,\eta^{2/3} \left|\sin\theta\right|^{0.18}\ \mu\mathrm{G}; \quad K \propto \left|\sin\theta\right|^{1.6}$$

VERY PRELIMINARY,
TO BE VERIFIED AND REFINED

Simulated radial profiles

MORE ROBUST

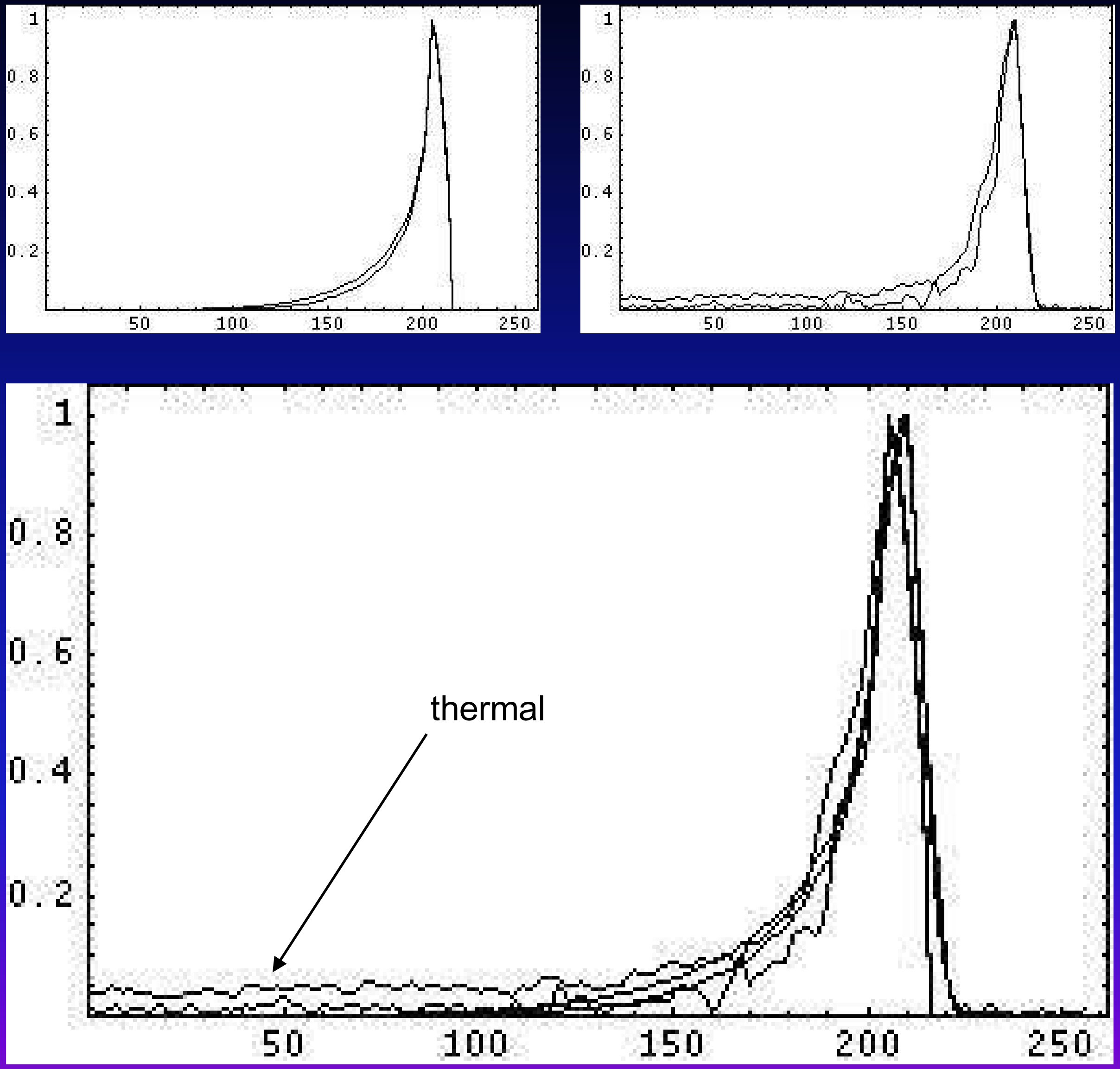
- The analysis about the edge determines uniquely the required parameters

$$S(\nu) = Q(\theta)\sin^{3/2}\alpha\, \nu^{-1/2} \exp\left(-\sqrt{\nu/(\nu_c(\theta)\sin\alpha)}\right)$$

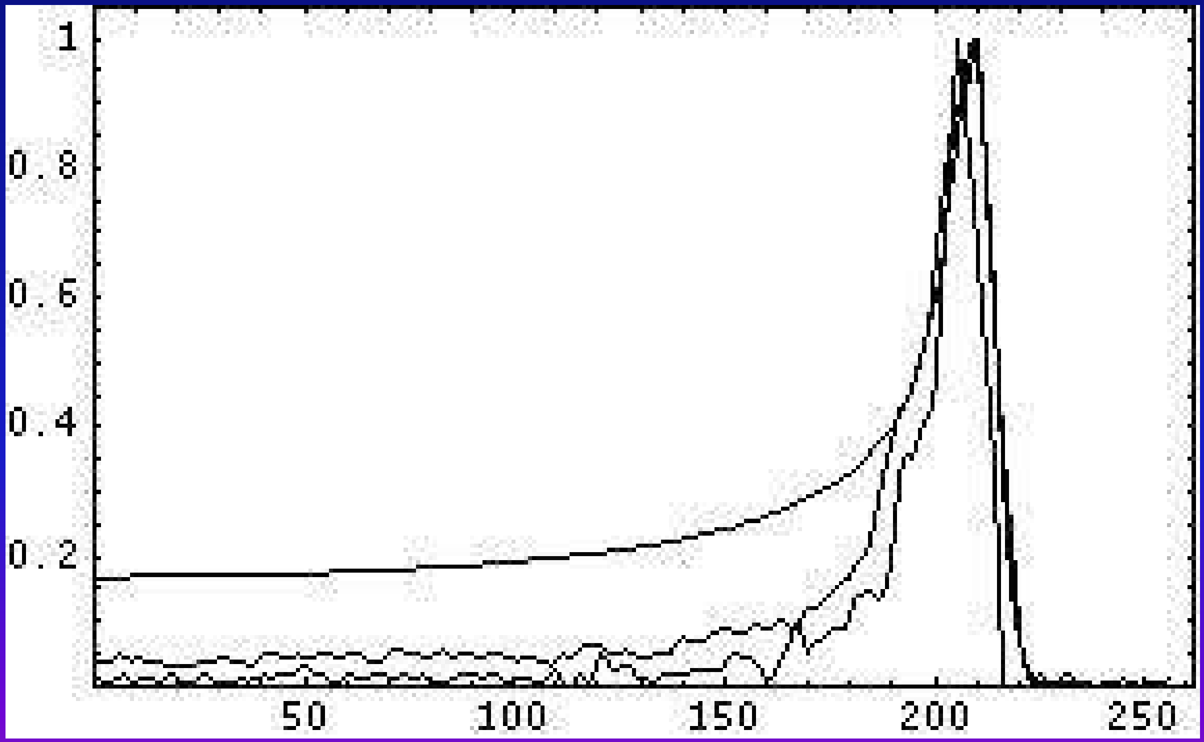
- Just one free parameter: shell thickness

Taken average value 5% **(Cassam-Chenai et al. 2008)**

- X-ray limbs reproduced reasonably well
- Radio probably too broad ?

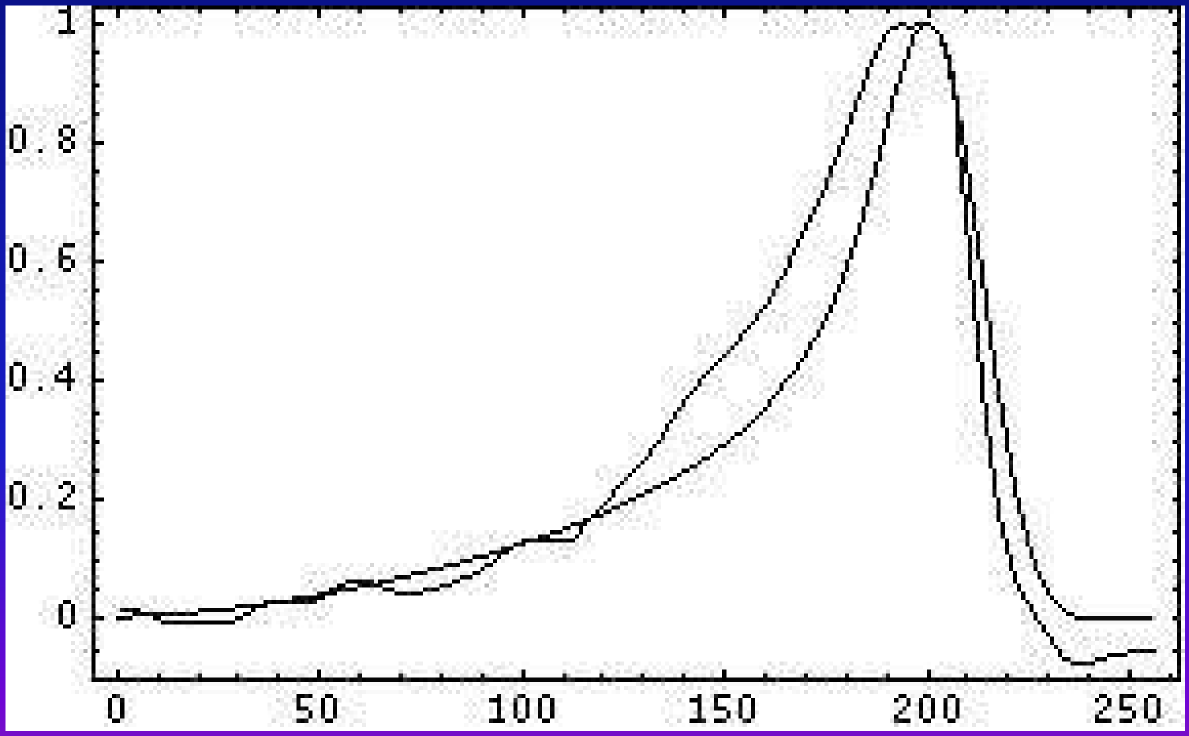

1
0.8
0.6
0.4
0.2
50
100
150
200
250
thermal

The isotropic case (for comparison)



Radio profile

- Lower resolution map (43 arcsec)
- Low sensitivity to large spatial scales



Part 2 - Conclusions

- SN 1006 3-D structure is not necessarily a polar-caps one.
- A barrel-like geometry is acceptable, provided that the field has an ordered component, directed radially.
- Anisotropic emissivity. In X-rays also important the effect on the cut-off frequency.
- A (preliminary) combined fit to radio and X-rays along the limb has been done.
- Radial profiles, computed using the parameters of the fit, agree with the observations.
- Origin of the radial component of the field?