A semi-analytic approach to non-linear shock acceleration

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The SNR paradigm for galactic CRs

- SNe may account for galactic CR energetics
- Diffusive Shock Acceleration provides power law spectra with the *correct* index

BUT

- Are CRs passive spectators of the shock dynamics?
- What is the maximum energy SNRs can accelerate CRs at?
- How do particles escape from SNRs?

Need for a Non-Linear theory of DSA

Kinetic approaches to NLDSA

Semi-analytic	• Flexible, computationally *extremely* cheap
Monte Carlo	• Relativistic shocks
Fully Numerical	• Time dependent solution

All methods require an *a priori* description of:

- Particle transport
- Magnetic field amplification
- Injection
- Particle escape

A semi-analytic approach

- Solution of the stationary diffusion-convection equation (Amato & Blasi 05-06)
  - Injection via thermal leakage (Malkow 1998, Kang et al. 2002, Blasi et al. 2005)

$$u\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p)$$

$$Q(x,p) = \eta\frac{n_1 u_1}{4\pi p_{inj}^2}\delta(p-p_{inj})\delta(x)$$

Transport eq. + injection recipe

$$f(x,p) = f_1(p)\exp\left[-\frac{q(p)}{3}\frac{R_{sub}}{R_{sub}-1}\int_x^0 \mathrm{d}x'\frac{u(x')}{D(x',p)}\right]$$

$$f_1(p) = \left[\frac{3R_{tot}}{R_{tot}U(p)-1}\right]\frac{\eta n_0}{4\pi p_{inj}^3}\exp\left[-\int_{p_{inj}}^{p}\frac{\mathrm{d}p'}{p'}\frac{3R_{tot}U(p')}{R_{tot}U(p')-1}\right]$$

$$u_p \equiv u_1 - \frac{1}{f_1(p)}\int_{-\infty}^{0}\mathrm{d}x\frac{\mathrm{d}u}{\mathrm{d}x}f(x,p)$$

CR Distribution function

$$\xi_c(x) = 1 + \frac{1}{\gamma_g M_0^2} - U(x) - \frac{1}{\gamma_g M_0^2}U(x)^{-\gamma_g}$$

Momentum conservation eq.

+

$$\xi(x) = \frac{1}{3\rho_0 u_0^2}\int_{p_{inj}}^{p_{max}}\mathrm{d}p\, 4\pi\, p^3 v(p) f(x,p)$$

CR pressure

- Very fast iterative method based on integral equation solution
- Allows arbitrary diffusion coefficient D(x,p)
- May be easily generalized in order to account for oblique shocks, perpendicular transport, Fermi II, nuclei acceleration,…

The maximum momentum

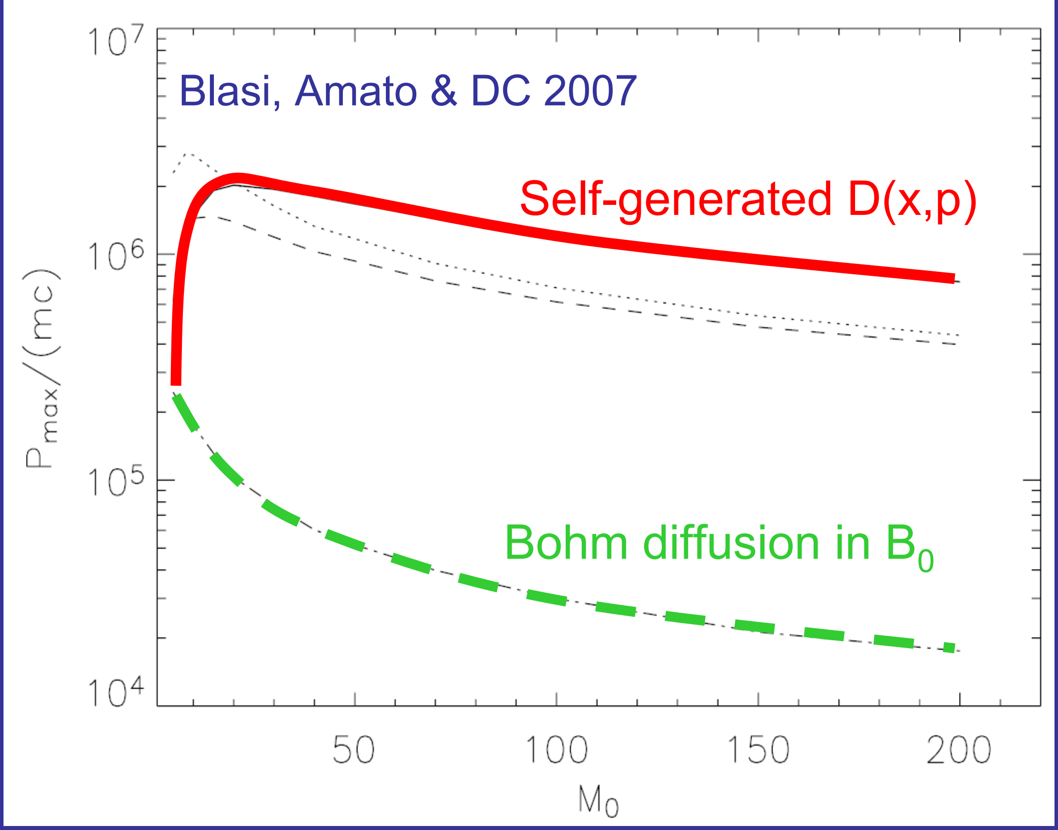
- Following Drury 1983, we calculated the acceleration time from the time-dependent transport equation (Blasi, Amato & DC 2007)
  - For modified shocks and arbitrary D(x,p)

$$\langle t \rangle = \frac{3R_{\rm tot}}{u_0^2} \int_{p_{\rm inj}}^{p} \frac{{\rm d}p'}{p'} \left[\frac{R_{\rm tot} D_2(p')}{R_{\rm tot} U_p(p') - 1} + \frac{u_0 \Lambda(p')}{R_{\rm tot} U_p(p') - 1} \right]$$

$$\Lambda(p) = \int_{-\infty}^{0} {\rm d}x \exp\left[\frac{q(p)}{3} \left(1 - \frac{u_2}{u_1}\right) \int_0^x {\rm d}x' \frac{u(x')}{D(x')} \right]$$

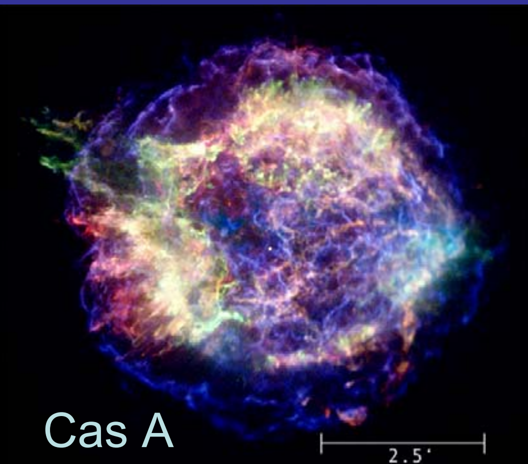
- Shock modification lead to:
  - P_{max} decrease due to fluid slowing down
  - P_{max} increase due to MFA

P_{max} as high as the knee within ~ 1000 yr



- Also confirmed by Monte Carlo calculation by Ellison & Vladimirov 2008

Magnetic Field Amplification



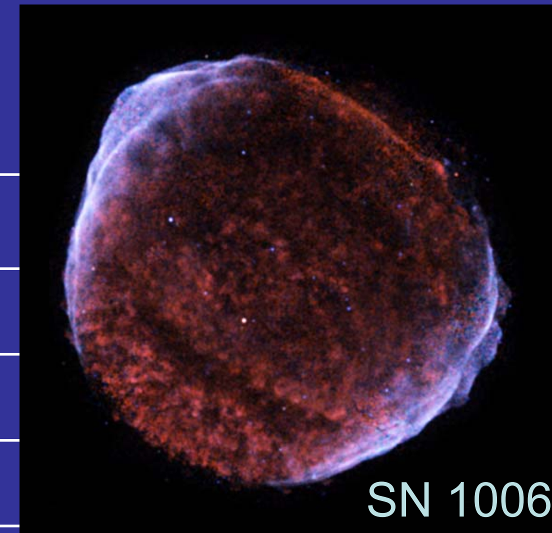
The width of the rims requires

- $B_{ds} \approx 70\text{-}500\ \mu G \gg B_0$

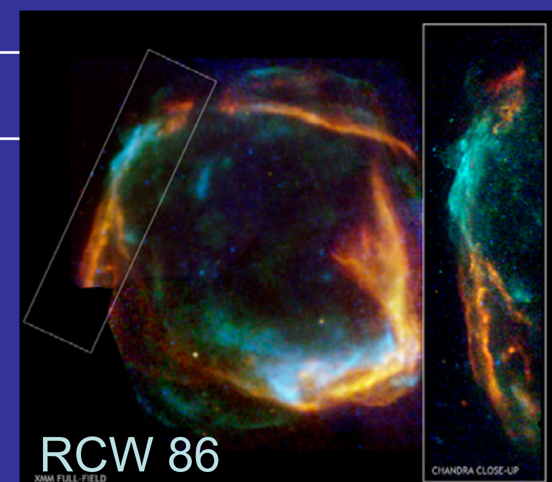
SNR	B_{ds} (μG)	$P_{w,ds}$(%)
Cas A	250-390	3.2-3.6
Kepler	210-340	2.3-2.5
Tycho	240-530	1.8-3.1
SN1006	90-110	4.0-4.2
RCW 86	75-145	1.5-3.8

Völk, Berezhko & Ksenofontov 2005

Parizot et al. 2006



Downstream magnetic pressure
Is at most 2 - 4% of bulk pressure



But upstream P_w very likely dominates over P_{gas}, since:

$$\frac{B^2}{8\pi} > nkT \Rightarrow B > 6\mu G\ \mathrm{n}^{1/2}\left(\frac{\mathrm{T}}{10^4 K}\right)$$

Shock dynamics with magnetic waves

- Three-fluid approach: gas, relativistic particles, magnetic turbulence
  - Vainio & Schlickeiser 1999

$$[\rho u]_1^2 = 0\,,$$
$$[\rho u^2 + p + p_w]_1^2 = 0\,,$$
$$\left[\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1} u p + F_w\right]_1^2 = 0$$

$$\boxed{\frac{p_2}{p_1} = \frac{(\gamma+1)R_{sub} - (\gamma-1) + (\gamma-1)(R_{sub}-1)\Delta}{\gamma+1-(\gamma-1)R_{sub}}}\,, \quad \frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{1}{R_{sub}}$$

$$\Delta = \frac{R_{sub}+1}{R_{sub}-1}\frac{[p_w]_1^2}{p_1} - \frac{2R_{sub}}{R_{sub}-1}\frac{[F_w]_1^2}{p_1 u_1}$$

- For Alfvèn waves:

$$F_w = \sum_\mu \frac{\delta \vec{B}_\mu^2}{4\pi}(u + H_{c,\mu} v_A) + \frac{\left(\sum_\mu \delta \vec{B}_\mu\right)^2}{8\pi} u\,, \quad p_w = \frac{1}{8\pi}\left(\sum_\mu \delta \vec{B}_\mu\right)^2 \quad \delta \vec{u}_\mu = -H_{c,\mu}\frac{\delta \vec{B}_\mu}{\sqrt{4\pi\rho}}$$

- At the subshock waves are partially reflected and transmitted in order to satisfy Maxwell and conservation equations (Scholer & Belcher 1971)

$$\mathcal{T} \simeq \frac{R_{sub} + \sqrt{R_{sub}}}{2} \quad \mathcal{R} \simeq \frac{R_{sub} - \sqrt{R_{sub}}}{2} \quad \Rightarrow \quad p_{w2} = p_{w1} R_{sub}^2 \quad \boxed{\Delta = (R_{sub}-1)^2 \frac{p_{w1}}{p_1}}$$

- If one naively assumed $F_w = 3u\, P_w$ everywhere, then

$$\Delta' = [(R_{sub}-1)^2 - 2R_{sub}]W < \Delta \qquad \boxed{W = p_{w1} / p_1}$$

thus $\Delta' < 0$ for $R_{sub} < \sim 3.7$ and opposite effect on the temperature jump!

The dynamical feedback of MFA

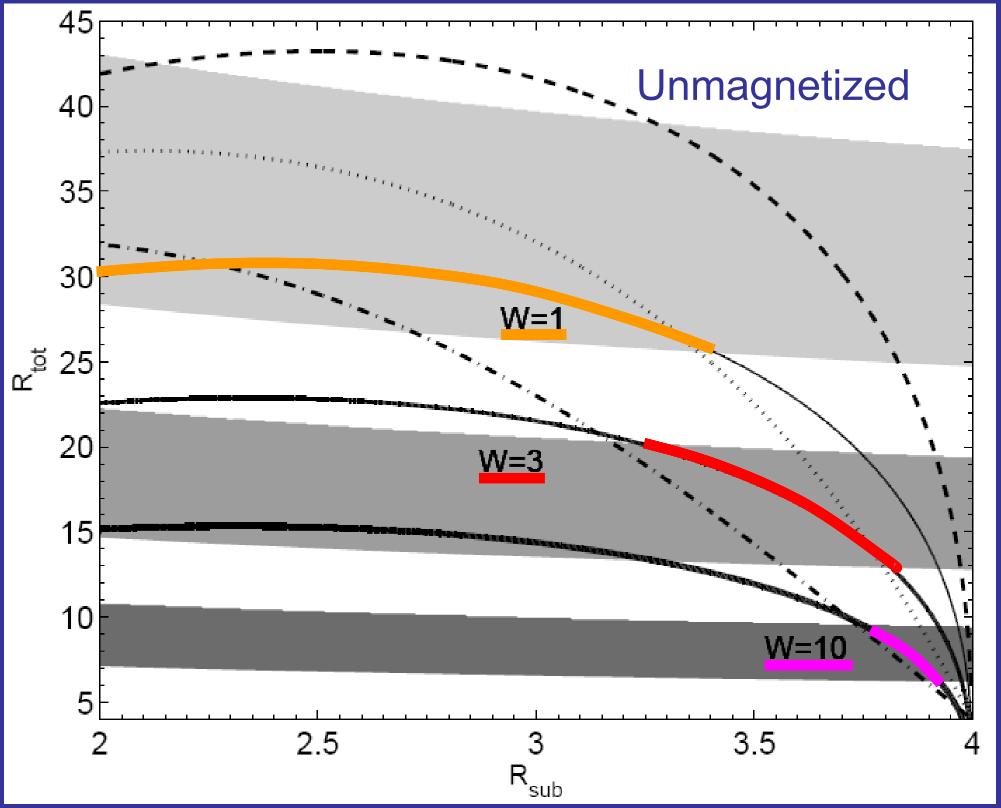
- Three-fluid model with Alfvén waves excited by streaming instability

$$R_{tot}^{\gamma+1} = \frac{M_0^2 R_{sub}^{\gamma}}{2}\left[\frac{\gamma + 1 - R_{sub}(\gamma - 1)}{1 + \Lambda_B}\right]$$

$$\Lambda_B = W\left[1 + R_{sub}\left(2/\gamma - 1\right)\right]$$

$W = P_w / P_{gas}$ Ratio between magnetic and plasma pressure upstream

- In $W = P_{w1}/P_1$ ≈ 1-100



DC, P. Blasi, E. Amato & M. Vietri 2008, ApJL

The magnetic turbulence feedback cannot be neglected and provides a smoothening of the precursor

MFA and damping in equations

- We start from the equation for magnetic turbulence transport

$$\frac{\partial \mathcal{F}_w(k,x)}{\partial x} = u(x)\frac{\partial \mathcal{P}_w(k,x)}{\partial x} + \sigma(k,x)\mathcal{P}_w(k,x) - \Gamma(k,x)\mathcal{P}_w(k,x)$$

  - with resonant growth $\sigma(k,x) = \frac{4\pi}{3}\frac{v_A(x)}{\mathcal{P}_w(k,x)}\left[p^4 v(p)\frac{\partial f(x,p)}{\partial x}\right]_{p=\bar{p}(k)}$
  - and damping $\Gamma(x) = \zeta\sigma(x)$

- By integrating over k one gets $\frac{dF_W(x)}{dx} = u(x)\frac{dp_W(x)}{dx} + (1-\zeta)v_A(x)\frac{dp_{CR}(x)}{dx}$

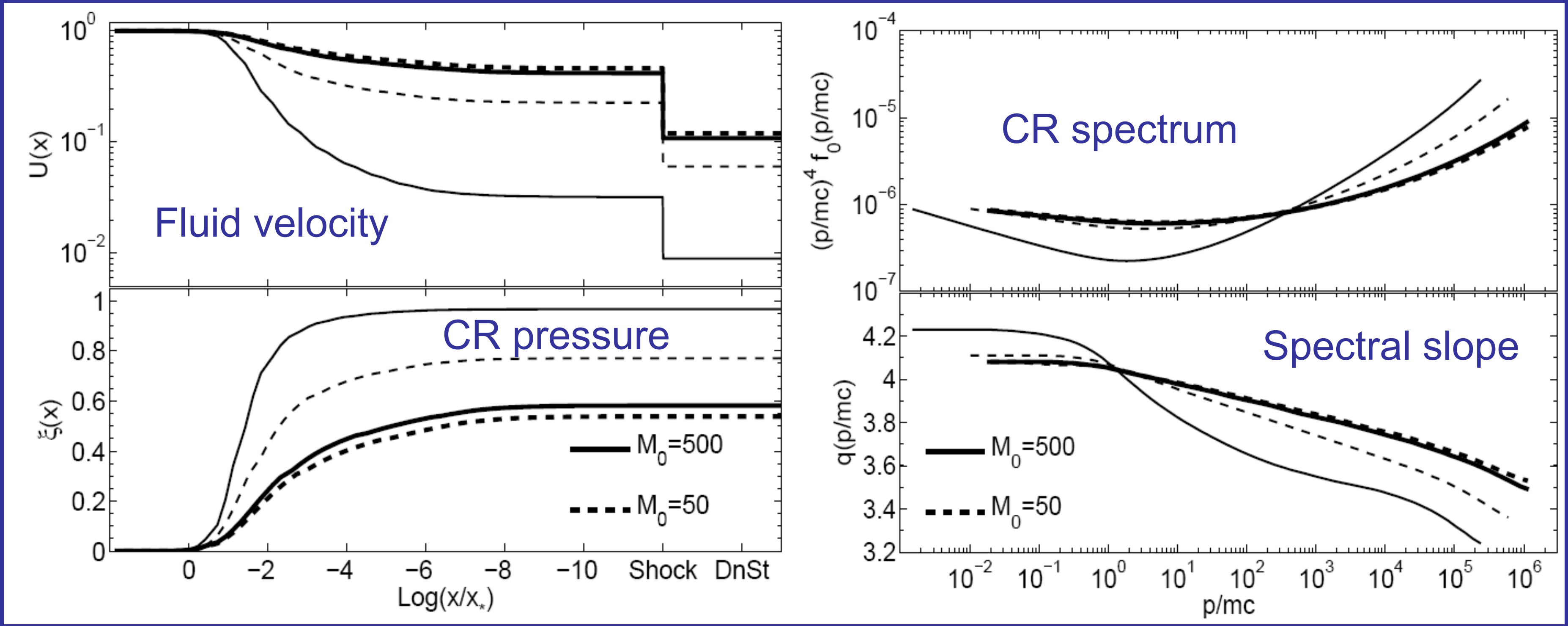
and in the limit $M_A \gg 1$, $M_0^2 \gg 1$ $\quad \frac{p_W(x)}{\rho_0 u_0^2} \approx (1-\zeta)\frac{1-U(x)^2}{4M_A(x)U(x)} \qquad U(x) = \frac{u(x)}{u_0}$

Usual implementations of the turbulent heating assume ζ=1 (Mc Kenzie & Völk 1981), which is however incosistent with MFA!

- The heating of the plasma is given by $\frac{d}{dx}\left[p(x)u(x)^\gamma\right] = \zeta(\gamma-1)v_A(x)u(x)^{\gamma-1}\frac{dp_{CR}(x)}{dx}$

and has been easily implemented in the formalism above (DC et al. 2009)

Magnetic feedback on the spectra



DC, Blasi, Amato & Vietri 2009

U_0=5900 km/s; SNR age=1000yr

T_0(K)	Λ_B	ξ_1	$p_{max}(10^6 GeV)$	R_{sub}	R_{tot}	$T_2(10^6\,\mathrm{K})$
10^4	No	0.97	0.24	3.58	112.1	0.88
10^4	Yes	0.58	1.17	3.84	9.22	126.5
10^6	No	0.77	0.59	3.76	16.6	42.3
10^6	Yes	0.54	1.14	3.84	8.44	154.8

Scattering centre velocity

- The velocity of the scattering centers naturally enters the transport equation

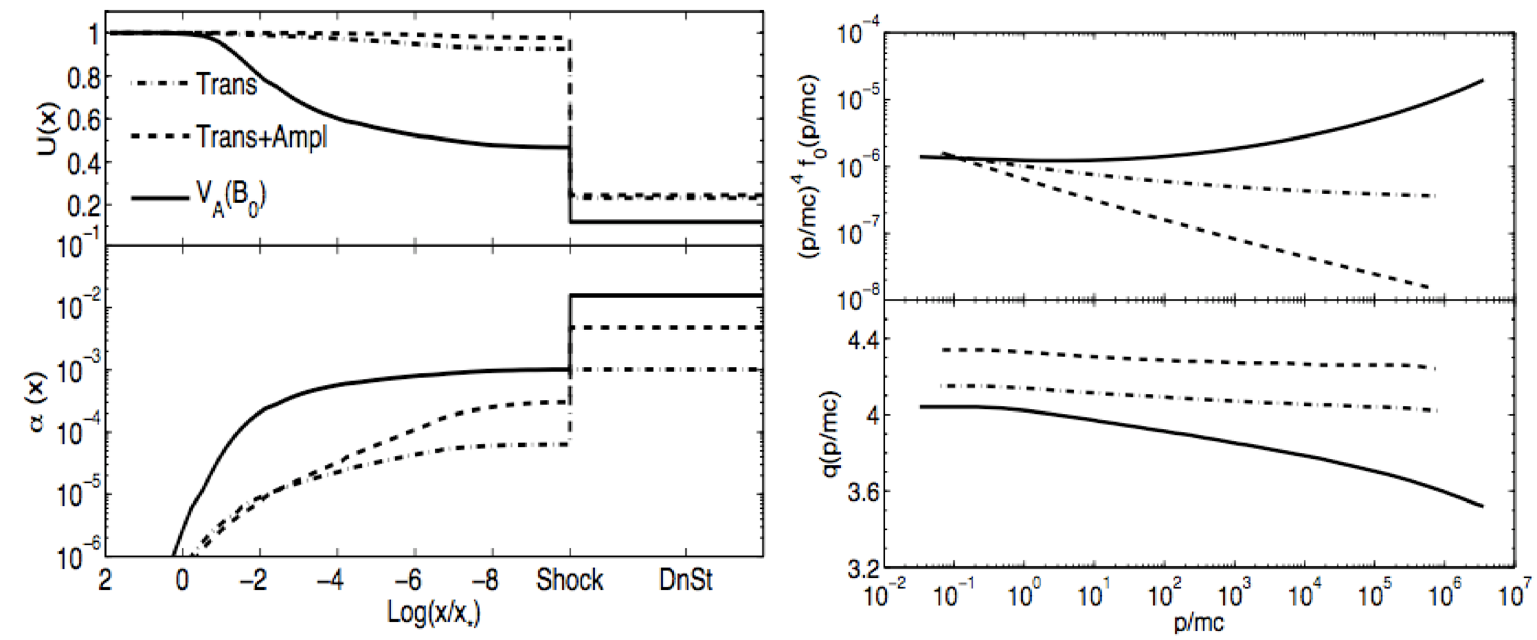
$$\tilde{u}(x)\frac{\partial f(x,p)}{\partial x}=\frac{\partial}{\partial x}\left[D(x,p)\frac{\partial}{\partial x}f(x,p)\right]+\frac{d\tilde{u}(x)}{dx}\frac{p}{3}\frac{\partial f(x,p)}{\partial p}+Q(x,p)$$

$$\tilde{u}(x)=u(x)-v_W$$

How does it depend on the nature of the turbulence?

- Assuming an effective Alfvén velocity in the amplified B leads to much steeper spectra!

$$v_A(x)=\frac{\delta B(x)}{\sqrt{4\pi\rho(x)}}$$

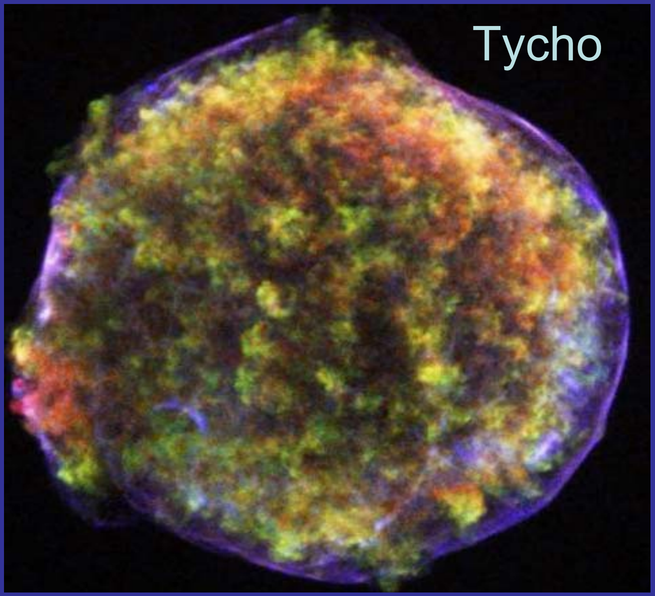


DC, Blasi, Amato & Vietri 2009

Why semi-analytical?

- The formalism outlined above for stationary, non-relativistic shocks is a very powerful tool since:
  - It is very fast (1 run takes from 10'' to 1')
  - It has virtually no dynamical range limitation on P_{max}, M_0, …
  - Allows to scan wide range of environmental parameters
  - Allows the inclusion of nuclei *(work in progress)*
  - Can implement new physical insights as soon as they are available

- Applications to SNR shocks:
  - Hydro + Multi-wavelength analysis of single SNRs *(see Giovanni's talk)*
  - Test the SNR paradigm for the origin of galactic CRs *(see this talk)*

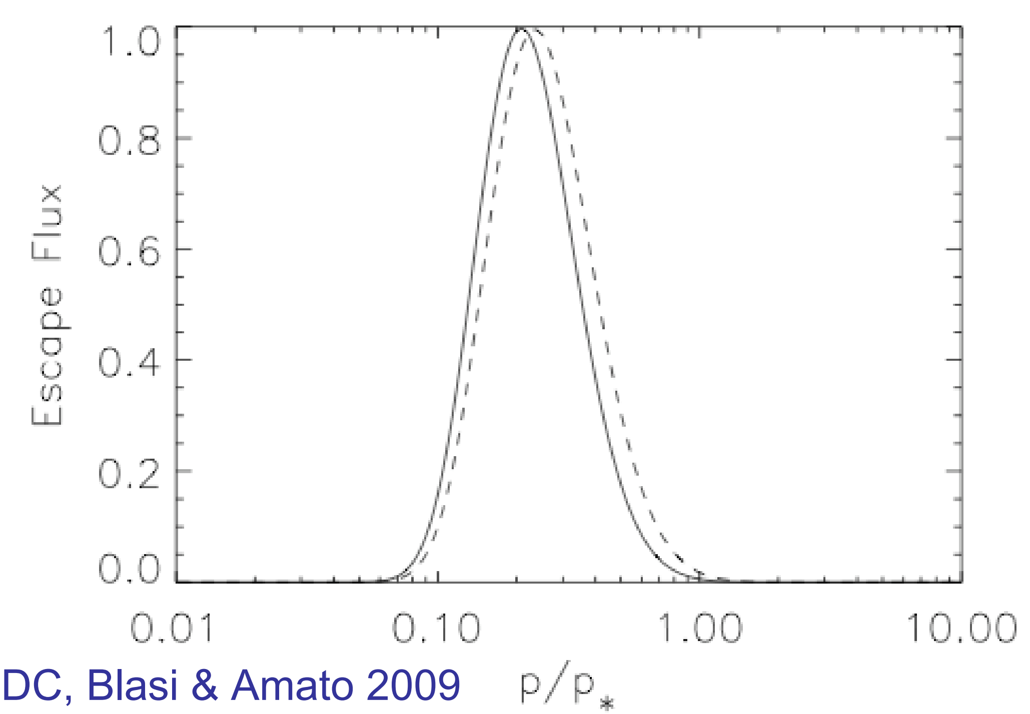


The escape flux

- Ejecta dominated stage
  - P_{max} and magnetic turbulence increase with time
- Sedov-Taylor stage
  - V_{sh}, P_{max} and δB decrease, and the SNR confining power too
  - Particles with momentum close to P_{max} escape the system from upstream
- An escape tern … as in the stationary regi…

Momentum … dary

$F_{esc} = \int_{-\infty}^{0^+} dx \frac{1}{3}\frac{du}{dx} 4\pi p_m^3$ … $\phi_{esc} = -D\left[\frac{\partial f}{\partial x}\right]_{x=x_0}$



DC, Blasi & Amato 2009

- And the *back o* … s?

$d\mathcal{E}(t) = \mathcal{F}_{esc}(t)\frac{1}{2}\rho V_{sh}^3($ …

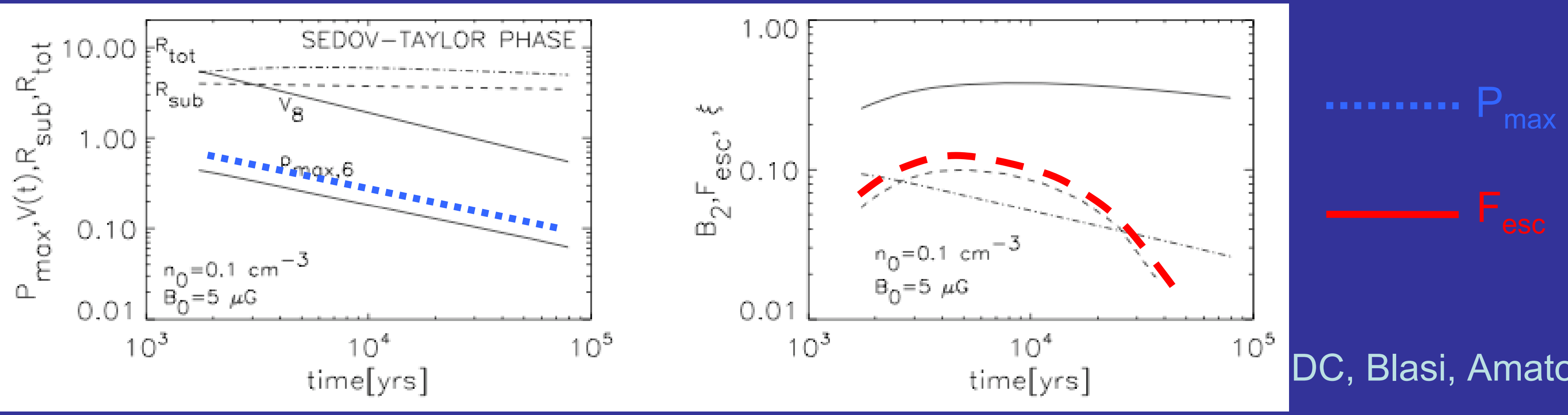
$d\mathcal{E}(p) = 4\pi p^2 N_e$ …

… $_{esc}(p) \propto p^{-4}$

Escape spectrum is no way related to E^{-2} Fermi's prediction for the acceleration one!

Step by step quasi-stationary SNR evolution

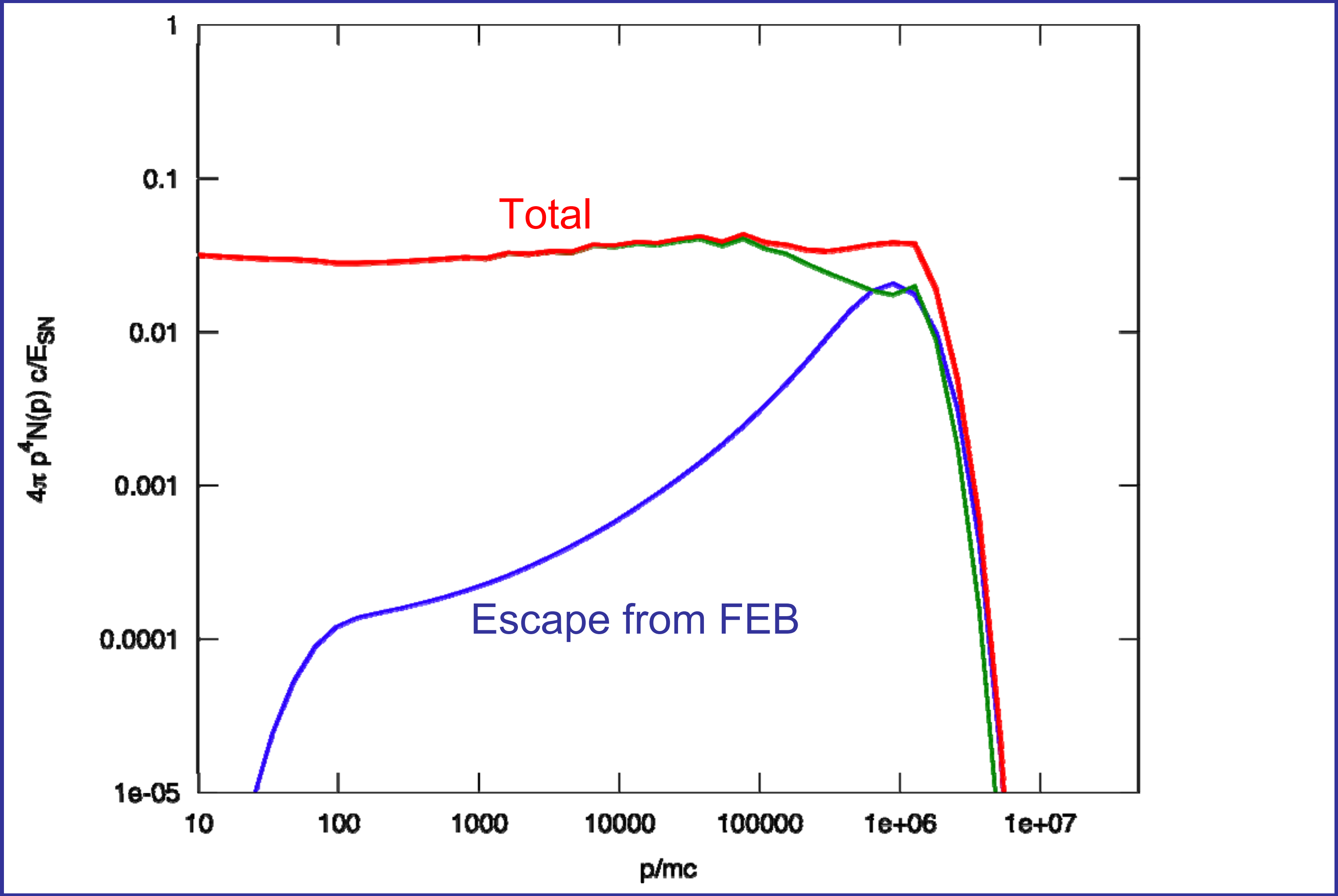
- Main ingredients:
  - Shock evolution according to self-similar solution (Truelove & McKee 1999)
  - Solution of NLDSA problem at each time step
  - Magnetic field amplification and damping (Ptuskin & Zirakashvili 2003/5)
  - Adiabatic losses in the downstream
  - Particle escape from *upstream*: free escape boundary at $x_0=0.15\ R_{sh}$
  - Leakage from *downstream* if the diffusion length is larger than $R_{sh}-R_{cd}$



DC, Blasi, Amato 2009

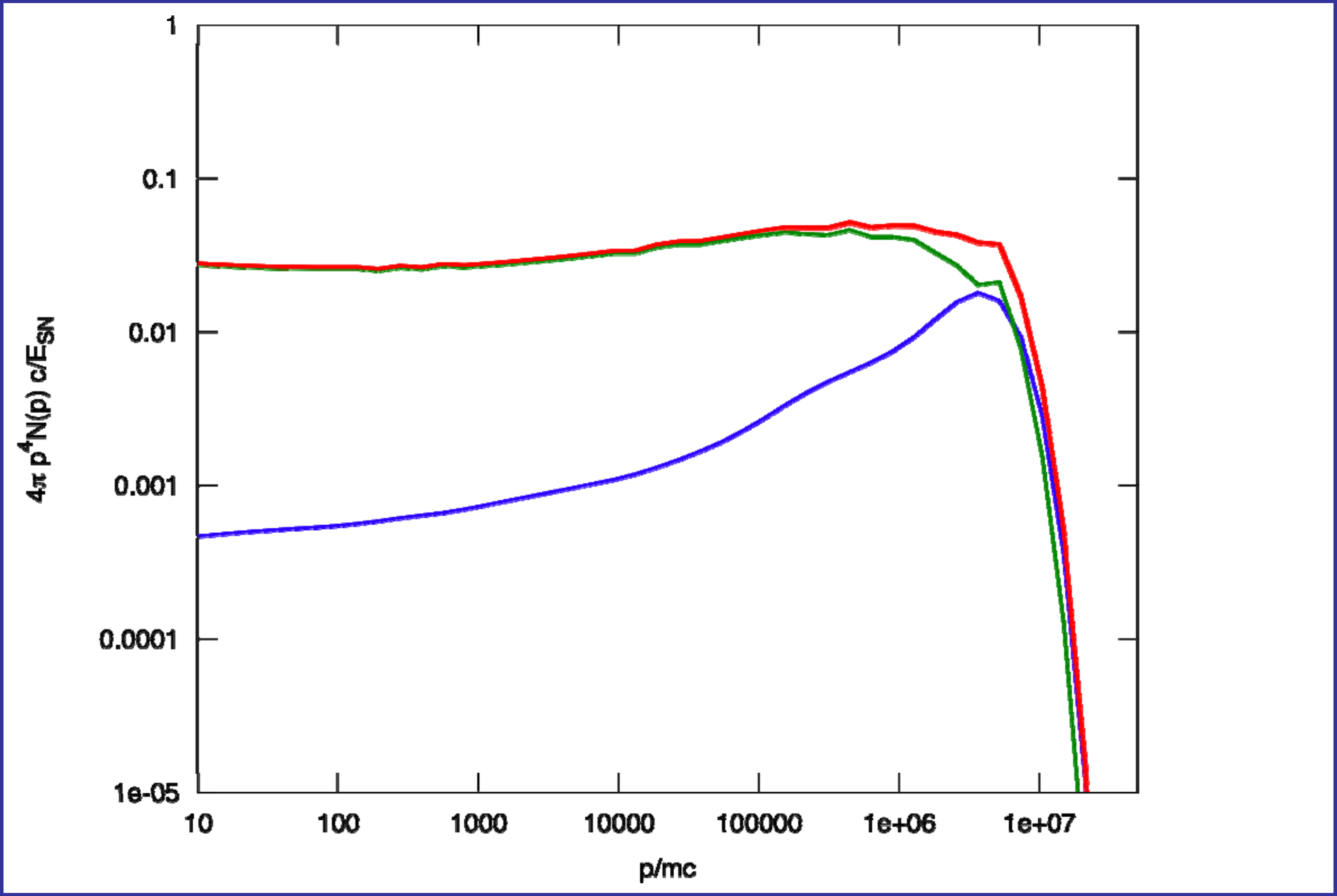
SNR lifetime contribution - 1

- $T_0=10^5$ K, $n_0=0.1$ cm^{-3}, $B_0=5$ μG, $x_0=0.15\ R_{sh}$, Bohm diffusion



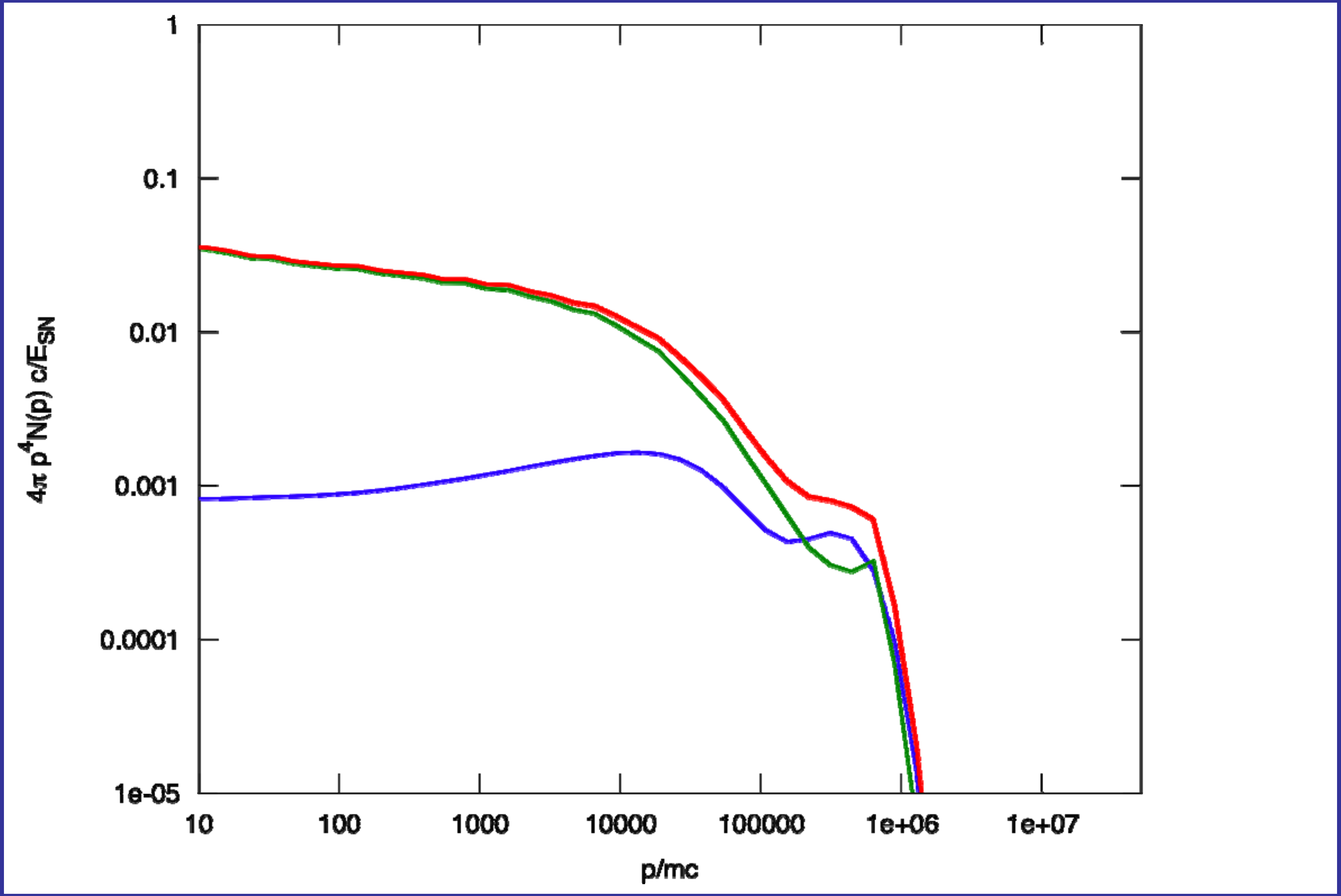
SNR lifetime contribution - 2

- $T_0=10^5$ K, $n_0=0.1$ cm^{-3}, $B_0=5$ μG, $x_0=0.15\ R_{sh}$, Self-generated diffusion



SNR lifetime contribution - 3

- $T_0=10^5$ K, $n_0=0.1$ cm^{-3}, $B_0=5$ μG, $x_0=0.15\ R_{sh}$, Bohm diffusion
- Alfvén velocity in the amplified magnetic field



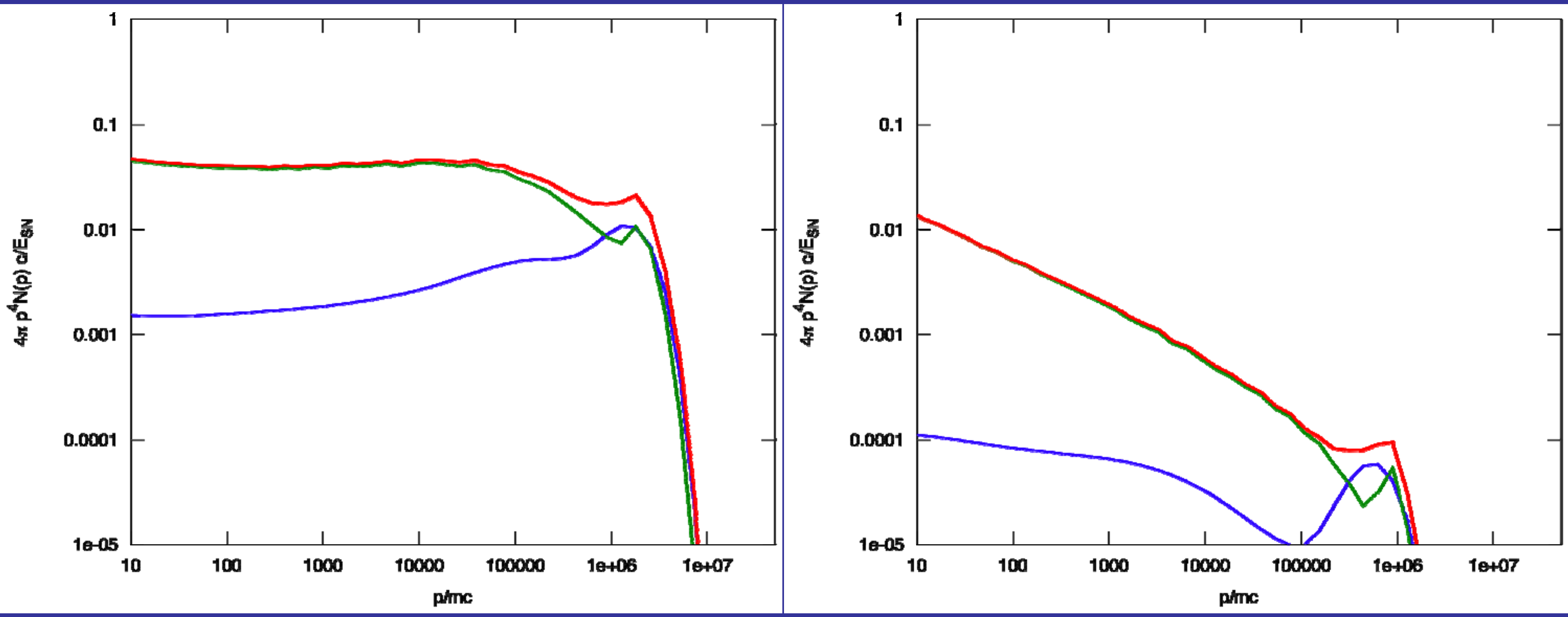
SNR lifetime contribution - 4

- Type I-like SNR

  $T_0=10^4$ K, $n_0=1$ cm^{-3}

- Type II-like SNR (no winds)

  $T_0=10^6$ K, $n_0=0.01$ cm^{-3}



The total diffuse spectrum of galactic CRs may be the superposition of very different contributions in terms of slope, normalization and composition

Open issues about galactic CR spectrum

- Which is the contribution by Type I/II SNe?
  - Role of pre-SN stages (winds, hot bubbles…)
- Which is the contribution of heavy nuclei to the SNR dynamics?
  - Abundances relative to H in the sources
- How does injection work?
- Is our knowledge of MF amplification and damping suitable to address this problem?
  - Magnetic feedback
  - Non-resonant modes properties
- Is secondary reacceleration relevant in order to explain positron/secondary nuclei fluxes at Earth?