On the plasma temperature in SNRs

Drury, Aharonian, Malishev, Gabici

Accepted by A&A, 14 Jan 2009

arXiv:0811.3566

X-ray diagnostics

- Claimed to set stringent limits on density in some cases (eg RXJ1713-3945)
- Depend crucially on strong shock heating
- Most analyses neglect particle acceleration

But...

- If strong particle acceleration (>50%)
  - less energy available for heating
  - shock structure changes quite dramatically
- How does this affect the SNR evolution and diagnostics?

A SNR, once it has ended the initial ballistic expansion phase, is just a bubble of high energy-density material expanding into the ambient medium.

$$\mathcal{E} \approx \frac{E_{\rm SN}}{\frac{4\pi}{3} R^3}$$

Interior pressure is of order $\frac{1}{3}\mathcal{E}$ to $\frac{2}{3}\mathcal{E}$

and must balance the ram pressure of the flow into the outer shock, thus

$$\rho_0 \dot{R}^2 \approx \frac{E_{\rm SN}}{R^3}$$

$$\Longrightarrow \dot{R} \approx \sqrt{\frac{E_{\rm SN}}{\rho_0} R^{-3/2}}$$

from which follows the well know Sedov relation

$$R \approx \left(\frac{E_{\rm SN}}{\rho_0}\right)^{1/5} t^{2/5}$$

This is thus a very robust relation and is not expected to change significantly even with very strong particle acceleration

However the shock does become much more compressive for at least four reasons:

- Softer equation of state
- Escape of high-energy particles
- Geometrical dilution
- Diffusion to the interior

Can capture much of this with an additional energy flux of high-energy particles

$$\begin{aligned}
\rho_0 U_0 &= \rho_1 U_1 = A \\
\rho_0 U_0^2 &= \rho_1 U_1^2 + P \\
\frac{1}{2}\rho_0 U_0^3 &= \frac{1}{2}\rho_1 U_1^3 + U_1\left(\mathcal{E} + P\right) + \Phi
\end{aligned}$$

Additional term

(NB not exact) which give shock compression ratio of

$$s = \frac{U_0}{U_1} = 1 + \frac{2\mathcal{E}}{P} + \frac{2\Phi}{U_1 P}.$$

Estimate of energy flux term.

$$\frac{4\pi p^3}{3} f(p)\,(U_0 - U_1)$$

is acceleration number flux, thus associated energy flux at the top end of the spectrum is

$$\frac{4\pi p_{max}^3}{3} f(p_{max})\,(U_0 - U_1)\,c p_{max}$$

But accelerated particle pressure is

$$P_C = \int \frac{pv}{3} 4\pi p^2 f(p)\,dp = \int \frac{4\pi p^3}{3} pv f(p) d\ln(p)$$

and thus we can estimate

$$\Phi \approx \frac{P_C(U_0 - U_1)}{\lambda},$$

where lambda measures the dominance of particles near the upper cut-off in the total pressure

$$\begin{aligned} \lambda &\approx \ln(p_{\max}/mc) \qquad f(p) \propto p^{-4} \\ &\approx 2 \qquad f(p) \propto p^{-3.5} \end{aligned}$$

This gives

$$s = 1 + \frac{2\mathcal{E}}{P - 2P_C/\lambda}$$

or equivalently

$$s = 1 + \frac{3P_G + 6P_C}{P_G + P_C\,(1 - 2/\lambda)}$$

Note that

$s \to \infty$ for $P_C >> P_G$ and $\lambda \to 2$

Thus expect generally quite compressive shocks,

$$s \approx 10...20$$

This allows use of a thin shell approximation due originally to Chernyi

Mass of swept-up shell $M = \frac{4\pi}{3} R^3 \rho_0$

and Newton's law of motion gives

$$\frac{d}{dt} \left[M (U_0 - U_1) \right] = 4\pi R^2 P_{\mathrm{int}}$$

writing

$$P_{\text{int}} = \alpha A(U_0 - U_1)$$

$$\frac{d}{dt}\left[M(U_0 - U_1)\right] = 4\pi R^2 \rho_0 U_0 (U_0 - U_1)\alpha$$

and it is not hard to show that consistency requires

$$\alpha = 1/2$$

Interior pressure is just half the
immediate post-shock pressure

So fluid element is swept up by the remnant, compressed in the shock, and then gradually relaxes into equilibrium with the interior.

If shocked at time t, takes about the same time again to exit from the compressed shell.

After that expands in pressure equilibrium with interior where

$$P_{\rm int} \propto R^{-3} \propto t^{-6/5}$$

If pressure dominated by cosmic rays,

$$P_C \quad \propto \quad \rho^{4/3}$$

Thus density has to scale as

$$\rho \propto P^{3/4} \propto t^{-9/10}$$

and the ion temperature as

$$T_{\rm i} \propto \frac{P_G}{\rho} \propto t^{-3/5}$$

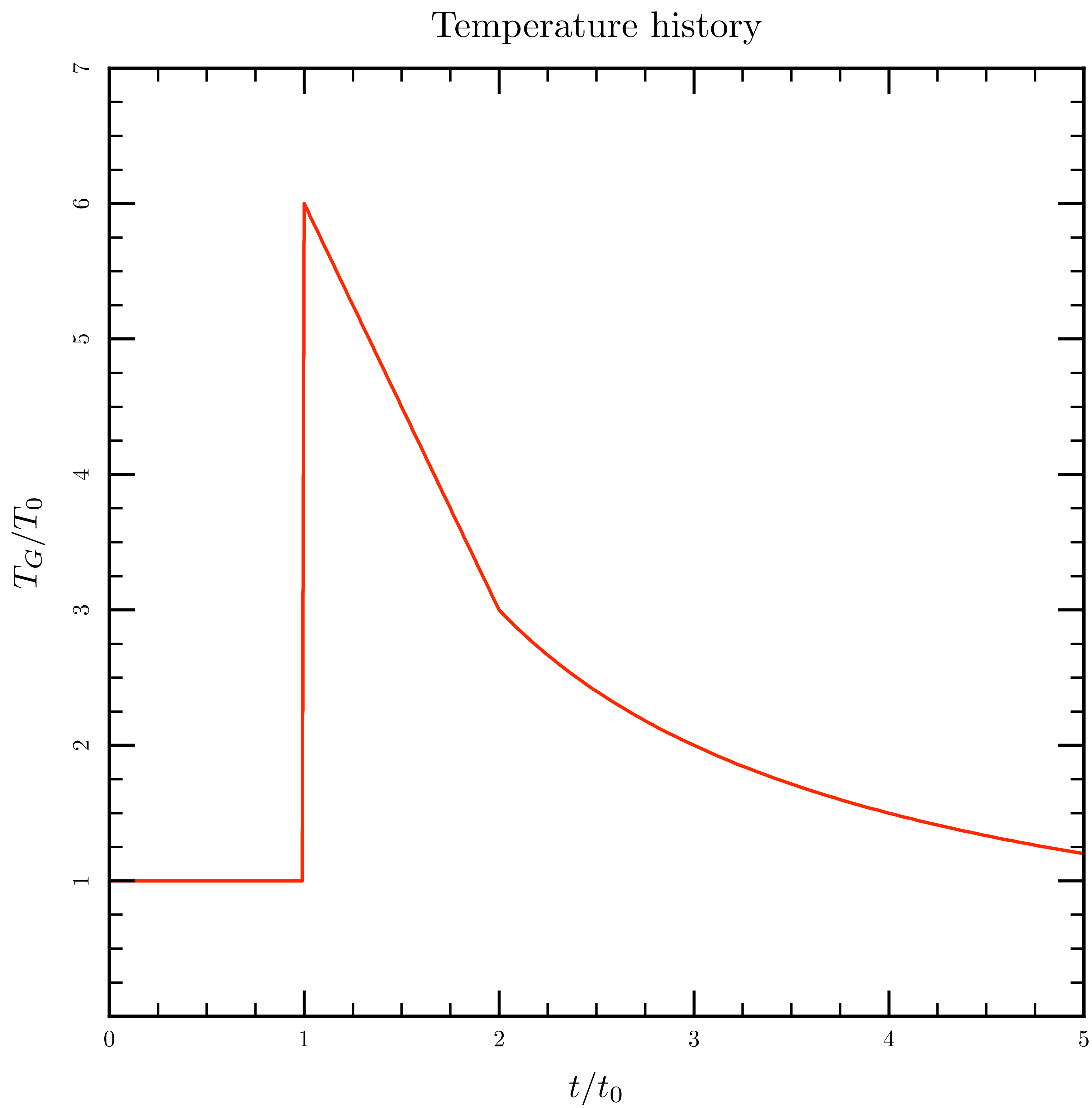
Thermal history of a fluid element

Initially shocked at time t_0

Expands and cools by a factor 2 over next t_0

Then slow cooling as $t^{-3/5}$

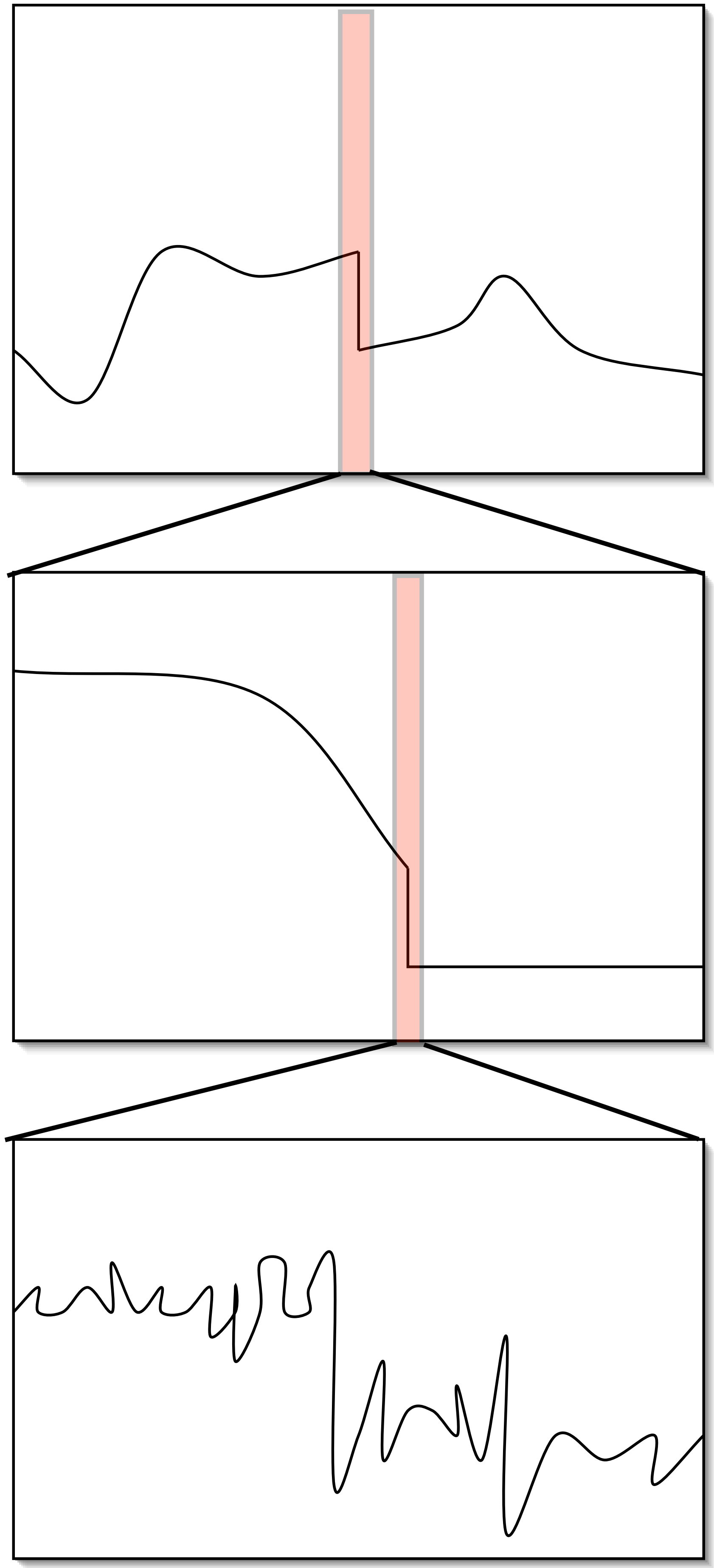
But ... how much heating in the shock?



Temperature history
T_G/T_0
7
6
5
4
3
2
1
0
1
2
3
4
5
t/t_0

Self-regulated injection

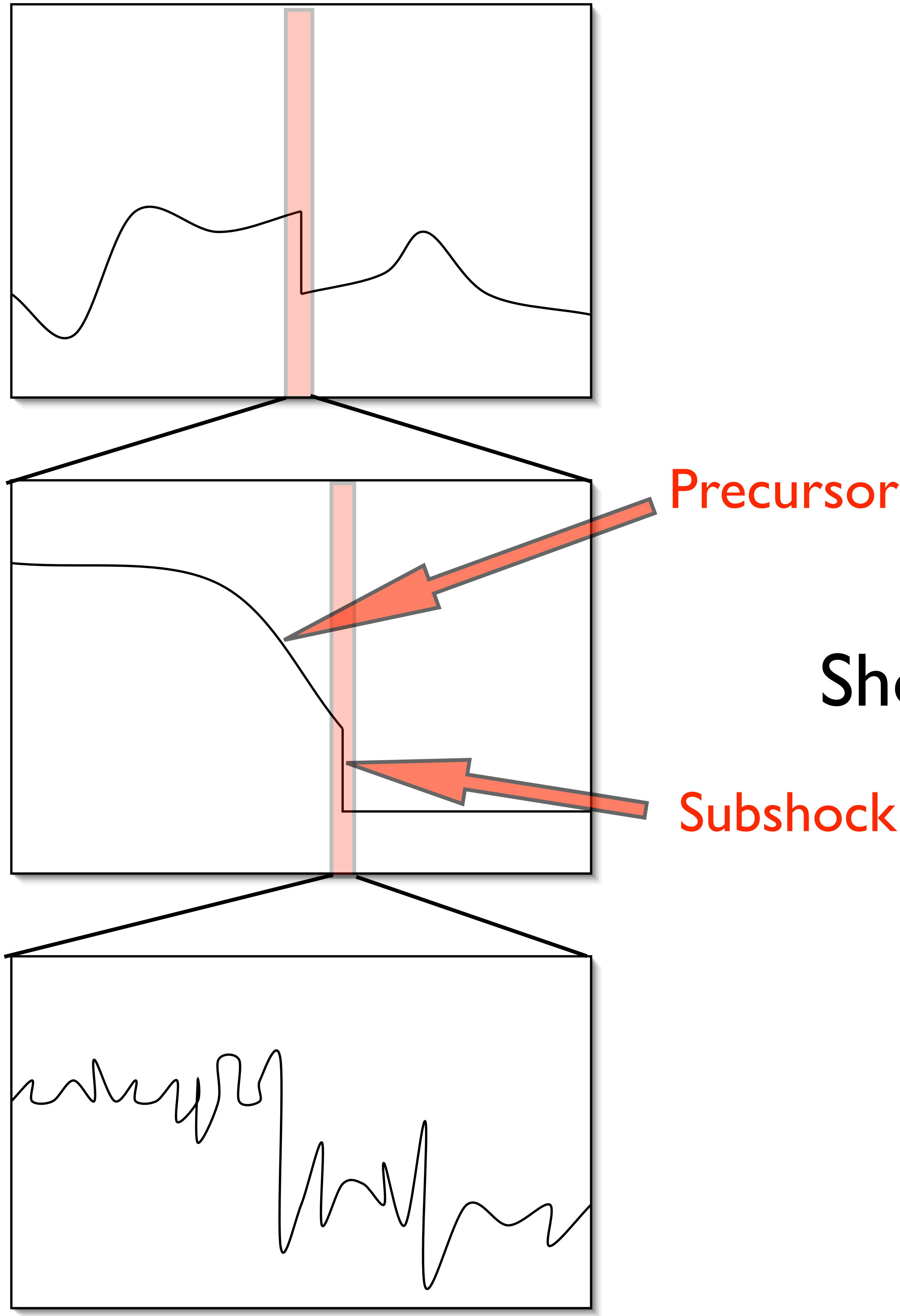
- Assume injection (of ions) is “easy”
- Shock has to “throttle back” injection to avoid excessive energy demands on the acceleration
- Does this by weakening the sub-shock



Outer scale
Astrophysics

Intermediate scales
Shock acceleration theory

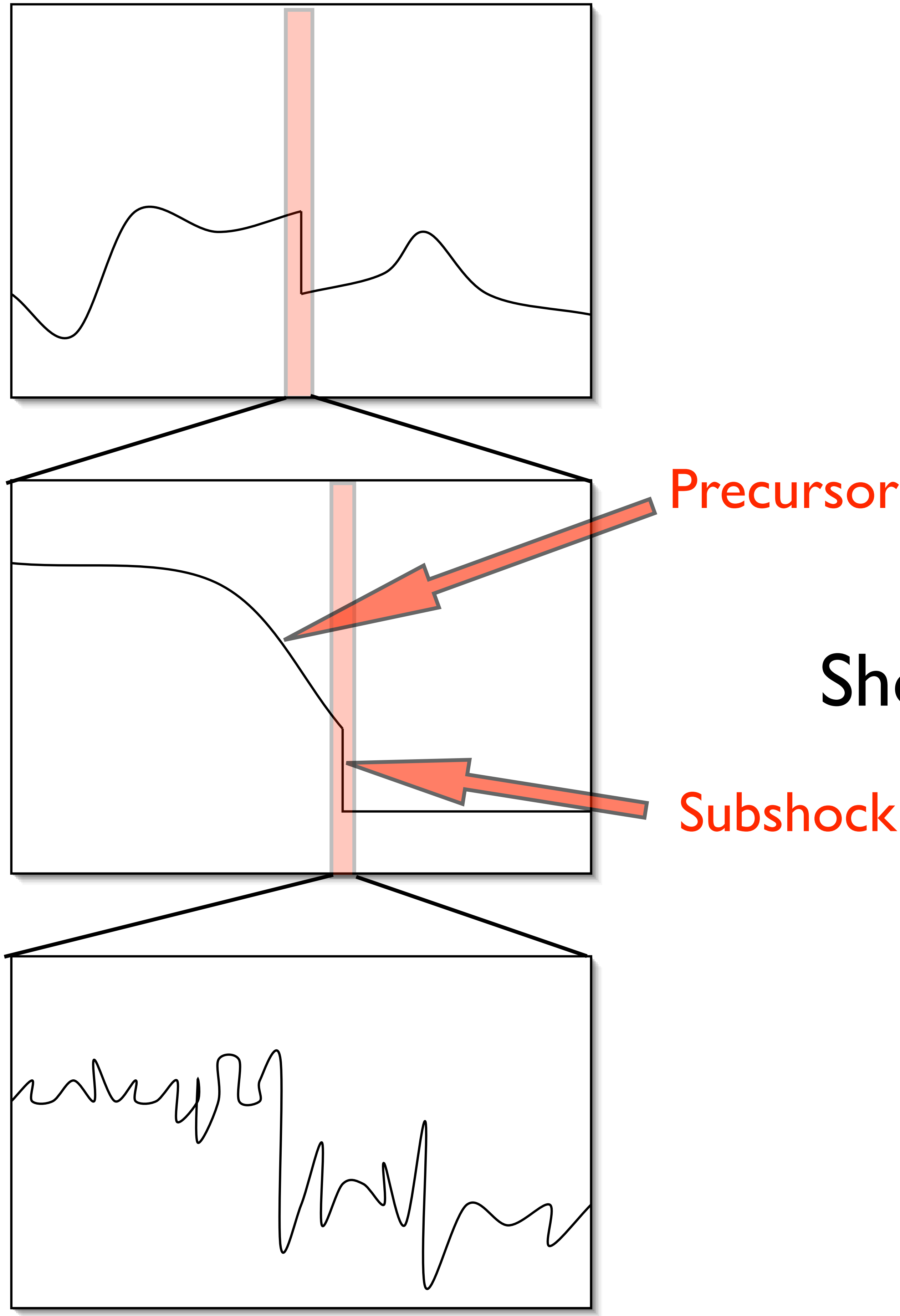
Inner scale
Plasma physics



Outer scale
Astrophysics

Intermediate scales
Shock acceleration theory

Inner scale
Plasma physics



Outer scale
Astrophysics

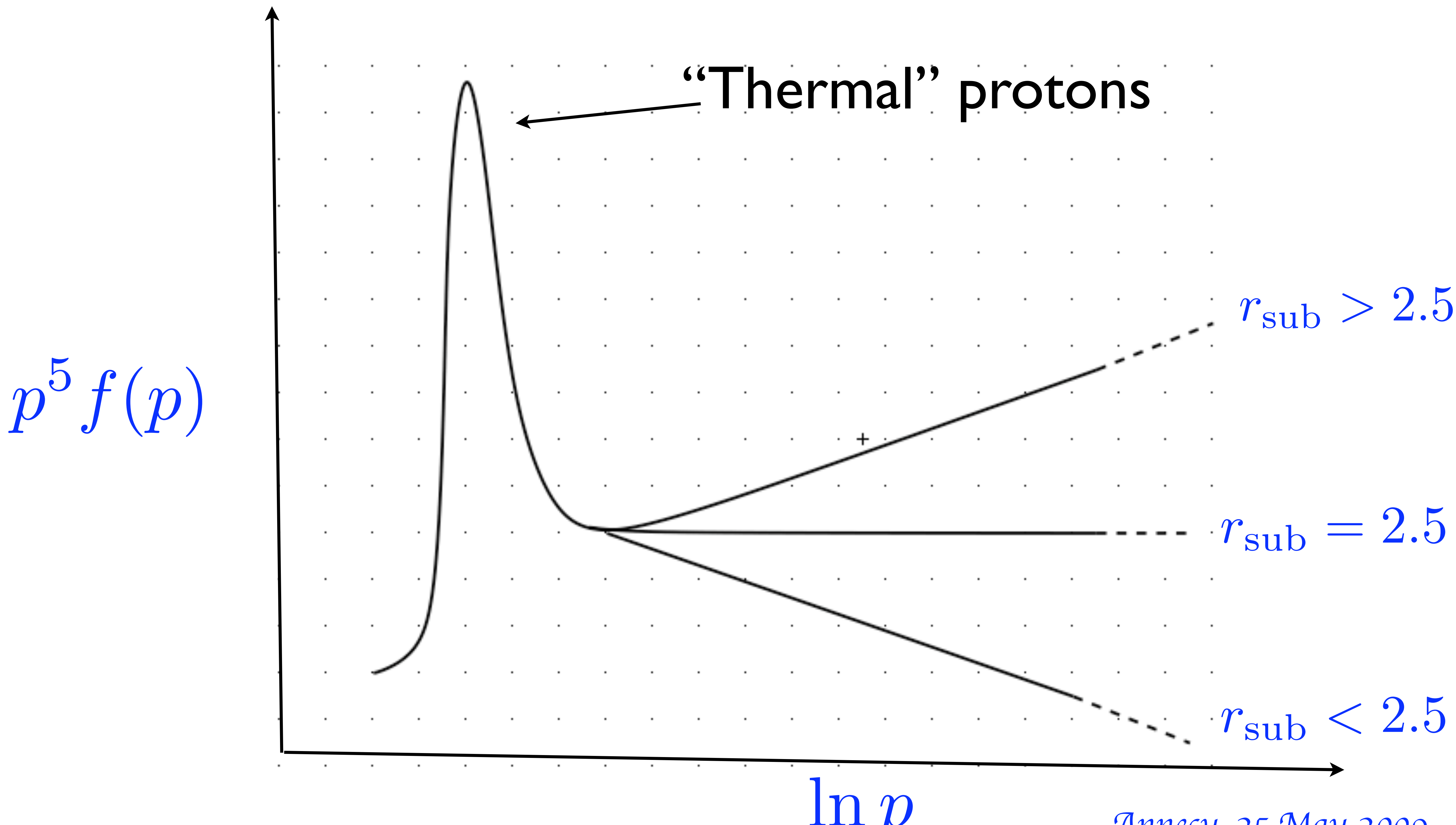
Intermediate scales
Shock acceleration theory

Inner scale
Plasma physics
Injection!

- Plausible arguments backed by simulations suggest that self-regulation requires the subshock to have an almost fixed compression ratio of between 2.5 and 3 if the injection is to be self-regulated

- In this case the temperature behind the subshock is a fixed (and small) multiple of the temperature upstream of the subshock.

$$P_C = \int 4\pi p^2 f(p) \frac{pv}{3} p \, d\ln p$$



- For compression 2.5 the temperature ratio is just 12/5
- For compression 3.0 the ratio rises to 11/3
- This is of course only the *subshock* heating, but in the limit of no wave dissipation in the precursor the *precursor* heating is just by adiabatic compression

- Heating in precursor is by factor of $s_{\rm pre}^{2/3}$
- Thus $T_1 = T_0 s_{\rm pre}^{2/3}$ $\qquad T_2 \approx 3T_1$
- And the postshock temperature is a multiple of the far upstream temperature

Big contrast to conventional shock heating,

$$\frac{3}{2}kT_G \approx \frac{1}{2}m_p\left(\frac{3}{4}U\right)^2,$$

Temperature is determined purely by the square of the shock speed

Extreme illustration, total shock compression 10 with factor 2.5 in subshock and 4 in precursor, then

$$T_2/T_0 \approx 4^{2/3}\frac{12}{5} \approx 6.05$$

Temperature can be as low as 6 times ambient!?

Clearly would not expect thermal X-rays in this case.

More generally

$$s_{\rm pre} \approx \left(\frac{M_0}{3}\right)^{3/4}$$

and

$$T_2 \approx \frac{11}{3}\left(\frac{M_0}{3}\right)^{1/2} T_0$$

so that even for

$$M_0 = 300 \qquad T_2 \approx 30 T_0$$

Conclusions

- No fundamental objection to *cold* SNRs if particle acceleration is very effective.
- Might even expect *anti-correlation* between strong TeV emission and *thermal* X-rays.
- Bulk dynamics remains Sedov like and is little affected.

A few caveats

- Have ignored all heating due to wave dissipation, magnetic reconnection etc.
- Additional complication is electron/ion thermalisation
- Purely theoretical discussion of ideal case, but aim was to establish the minimal ion heating allowed by theory.