Self-Similar Evolution of Cosmic-Ray Modified Shocks

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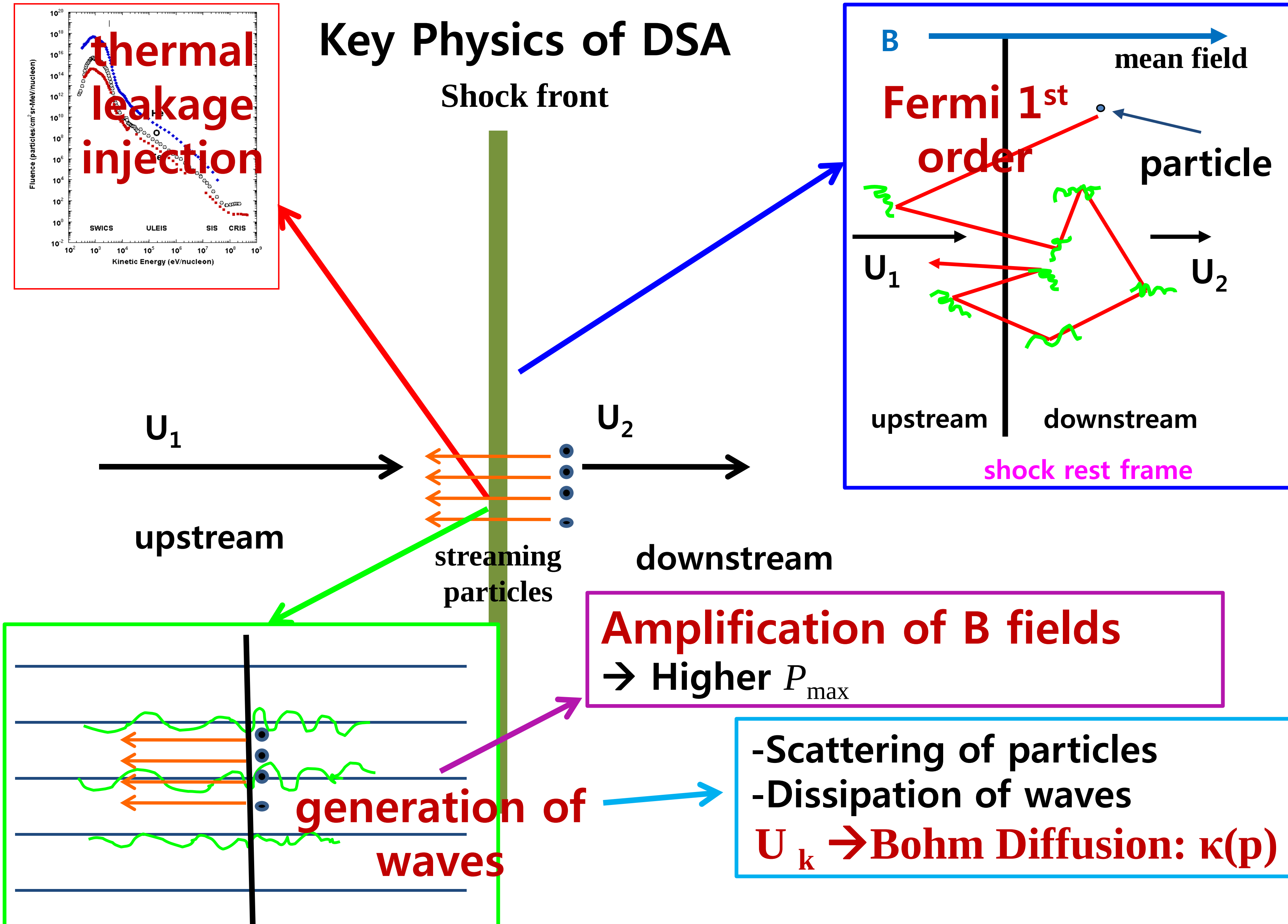
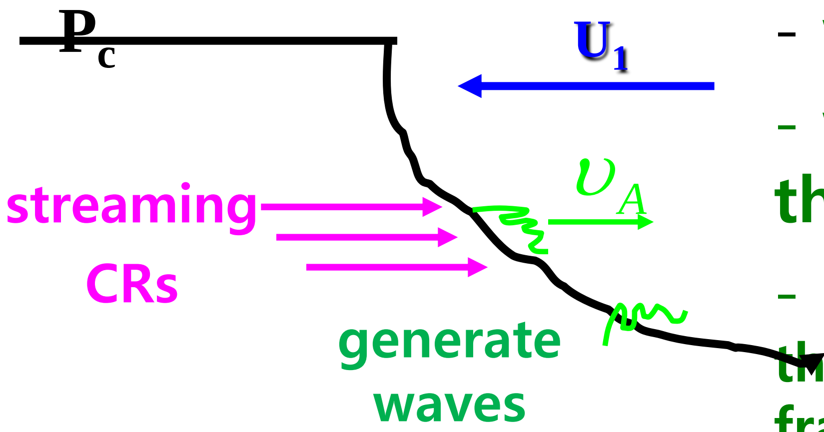
Outline

- **Key Physics of Diffusive Shock Acceleration (DSA)**
  - **Fermi 1st order acceleration process**
  - **wave-particle interactions:** CR injection, wave generation,..
- **Numerical Method**
  - **Time-dependent Kinetic simulations using**

    CRASH (Cosmic Ray Acceleration Shock) code
- **Self-Similar Evolution of CR modified shocks**
  - **Analytic form for time-dependent CR spectrum, *f(p,t)***
  - **comparison of time-dependent and steady-state solutions**
- **Summary**


Key Physics of DSA
Shock front
thermal leakage injection
SWICS
ULEIS
SIS
CRIS
Kinetic Energy (eV/nucleon)
Fluence (particles/cm2 sr-MeV/nucleon)
B
mean field
Fermi 1st order
particle
U1
U2
upstream
downstream
shock rest frame
U1
U2
upstream
streaming particles
downstream
Amplification of B fields
→ Higher P max
-Scattering of particles
-Dissipation of waves
U k →Bohm Diffusion: κ(p)
generation of waves

Particle injection, Wave generation, drift & dissipation



- waves drift upstream
- waves dissipate energy & heat the gas.
- CRs are scattered and isotropized in the wave frame rather than the fluid frame.

suprathermal particles

→leak upstream and become CRs (thermal leakage injection implemented in Kang et al. 2002)

→ generation of waves by wave-particle interactions & Amplification of B fields

(not implemented yet in CRASH: next task, see Vladimirov's and Blasi's talks later).

Basic Equations for Kinetic DSA Simulations

(1D plane-parallel)

$$\frac{\partial \rho}{\partial t} + \frac{\partial (u\rho)}{\partial x} = 0$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + P_g + \underline{P_c}) = 0$$

$$\frac{\partial (\rho e_g)}{\partial t} + \frac{\partial}{\partial x}(\rho e_g u + P_g u) = \underline{-u\frac{\partial P_c}{\partial x} + W - L}$$

ordinary gasdynamics EQs

+ P_c terms

Diffusion Convection Eq. for isotropic part of f(p)

$$\frac{\partial f}{\partial t} + (u + u_w)\frac{\partial f}{\partial r} = \frac{1}{3}\frac{\partial}{\partial x}(u + u_w) \cdot p\frac{\partial f}{\partial p} \quad + \quad \frac{\partial}{\partial x}[\kappa(x,p)\frac{\partial f}{\partial x}] \; + \; Q(x,p)$$

$$P_c = \frac{4}{3}\pi m_p c^2 \int_0^\infty f(p)\frac{p^4 dp}{\sqrt{p^2+1}}$$

W= wave dissipation heating, u_w = drift speed of waves

L= thermal energy loss due to injection, Q= CR injection

$$\frac{\partial f}{\partial t} + (u+u_w)\frac{\partial f}{\partial x} = \frac{1}{3}\frac{\partial}{\partial x}[(u+u_w)]\, p\frac{\partial f}{\partial p} + \frac{\partial}{\partial x}[\kappa(x,p)\frac{\partial f}{\partial x}] + Q(x,p)$$

Simple models for wave transport, diffusion, injection

$$u_w \approx -\upsilon_A = -\mathrm{B}_0/\sqrt{4\pi\rho(x)} \text{ in upstream, } u_w \approx 0 \text{ in downstream,}$$

$$\kappa(x,p) = \kappa^* p(\rho/\rho_0)^{-1} \text{ : Bohm-like power-law}$$

$$\delta B \sim B_0 \text{ \& compression of field}$$

$$Q(x,p) = \text{thermal leakage injection}$$ → **next slides**

$$\frac{\partial(\rho e_g)}{\partial t} + \frac{\partial}{\partial x}(\rho e_g u + P_g u) = -u\frac{\partial P_c}{\partial x} + W - L$$

$$W(x,t) \approx -\upsilon_A \frac{\partial P_c}{\partial x} \text{: wave disspation \& heating}$$

$$L = \text{thermal energy loss due to CR injection}$$

Numerical Model for Thermal Leakage Injection in CRASH

$$\tau_{esc}(\varepsilon_B, \frac{\upsilon}{u_d}) = H\left[\frac{\varepsilon_B \upsilon}{u_d} - (1.07+\varepsilon_B)\right]\left(1-\frac{u_d}{\upsilon}\right)^{-1}\left(1-\frac{u_d}{\varepsilon_B \upsilon}\right)\exp\left\{-\left[\frac{\varepsilon_B \upsilon}{u_d} - (1+\varepsilon_B)\right]^{-2}\right\}$$

"Transparency function": probability that particles at a given velocity can leak upstream. **(adopted from Malkov 1998)**

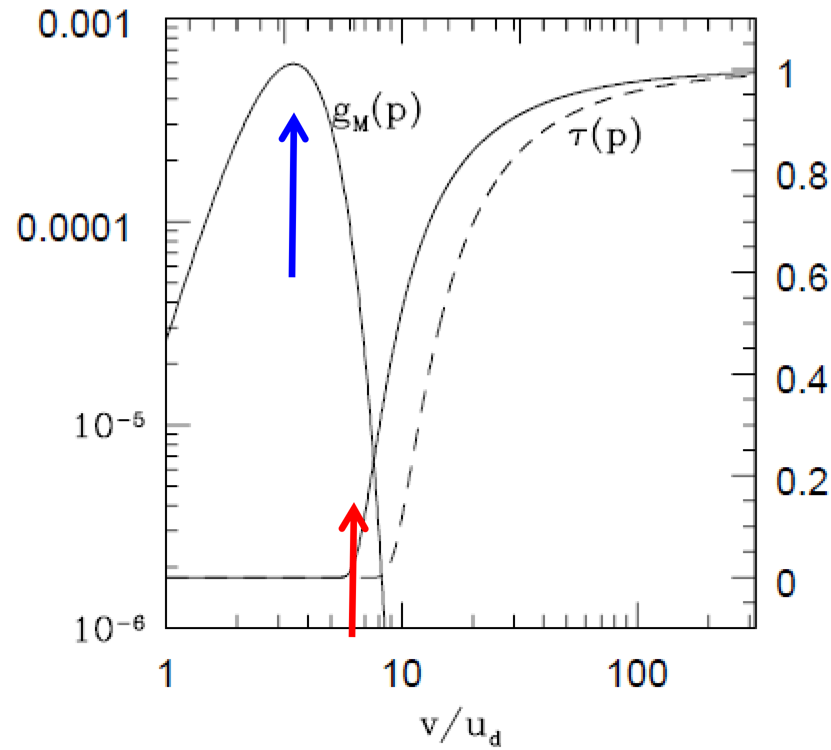
e.g. $\tau_{esc} = 1$ for CRs, $\tau_{esc} = 0$ for thermal ptls

$$\upsilon_{inj}/u_d \approx (1+1.07\varepsilon_B^{-1})$$

$$\upsilon_{inj}/\upsilon_{th} \approx \sqrt{\frac{5}{12}\frac{(M_s^2+3)}{(5M_s^2-1)}}(1+1.07\varepsilon_B^{-1})$$

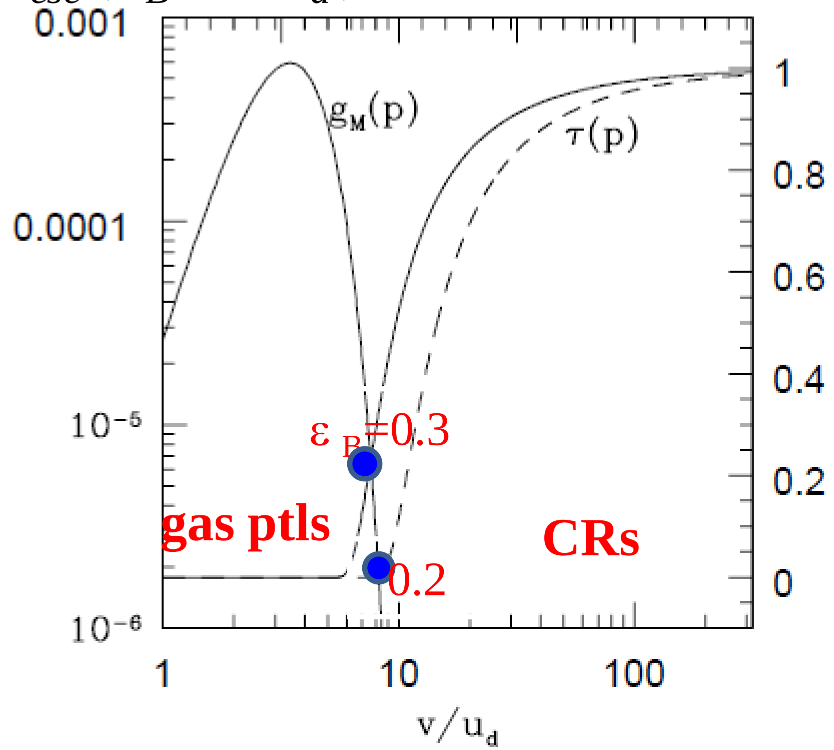
u_d = downstream flow speed

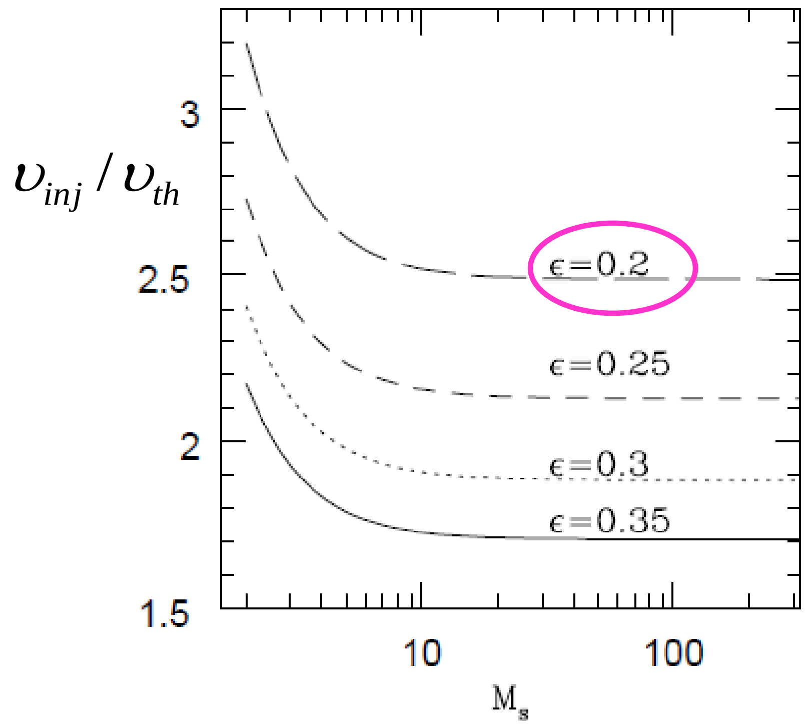
$$\varepsilon_B = \frac{B_0}{B_\perp} = \frac{\text{mean field}}{\text{turbulent field}} = \varepsilon_B(M)$$



Numerical Model for Thermal Leakage Injection in CRASH

$\tau_{esc}(\varepsilon_B, \upsilon / u_d)$ **: transparency function**





$$\upsilon_{inj} / \upsilon_{th} \approx fcn(M_s)\left[(1+1.07\varepsilon_B (M)^{-1})\right]$$

larger $\varepsilon_B \rightarrow$ smaller p_{inj}

$\rightarrow$ higher injection rate

$$\upsilon_{th} = 2\sqrt{k_B T_2 / m_p}$$

$$R_{inj} = \upsilon_{inj} / \upsilon_{th} \approx 2.5 - 2.7$$

for $\varepsilon_B = 0.2$

$$f(p_{inj}) = n_2 (2\pi T_2)^{-1.5} \exp(-2R_{inj}^2)$$

Numerical Tool: CRASH Code (Kang et al. 2001)

Bohm type diffusion: $\kappa(p) \propto p$

- **wide range of diffusion length scales to be resolved:** $l_{diff} = \kappa(p)/u_s$

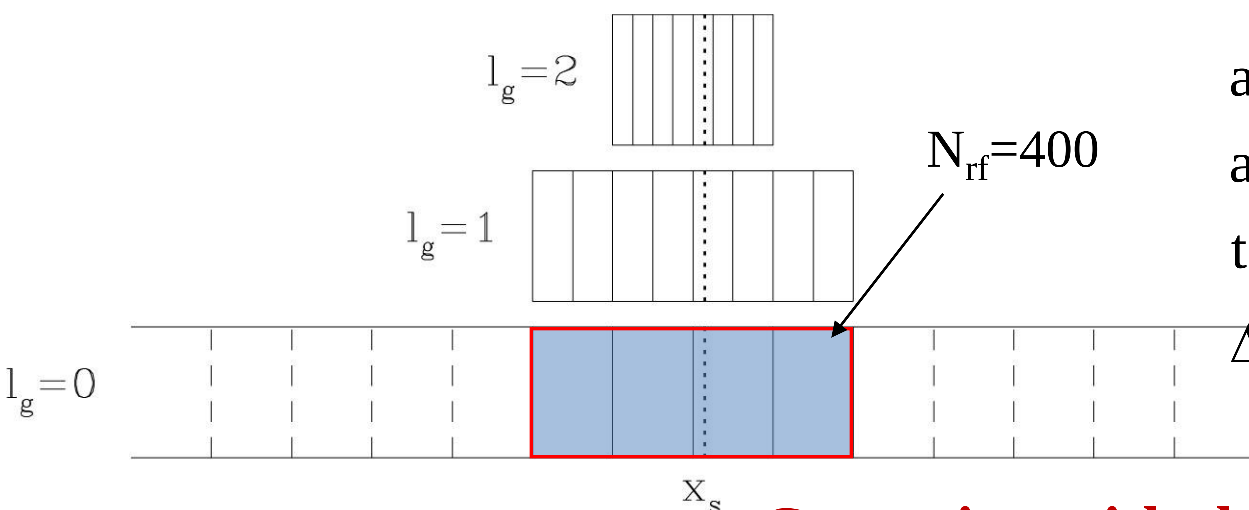
from p_{inj}/mc(~10^{-2}) to outer scales for the highest p_{max}/mc (~10^{6})

1) Shock Tracking Method (Le Veque & Shyue 1995)

- **tracks the subshock as an exact discontinuity**

2) Adaptive Mesh Refinement (Berger & Le Veque 1997)

- **refines region around the subshock with multi-level grids**



a factor of two refinement at each grid level, typically $l_{g,\max} = 8-10$

$\Delta x_{10} = \Delta x_0 / 1024$

Comoving with the shock

Prediction of DSA theory in test particle limit

(when non-linear feedback due to CR pressure is insignificant)

$$\frac{\Delta p}{p} \sim \frac{u_1 - u_2}{v}, \quad p_{esc} = \frac{u_2}{v} \text{(escape prob.)} \Rightarrow f(p) \propto p^{-q}$$

where the slope, *q* = 3σ/(σ-1) = $3u_1/(u_1-u_2)$

(**$\sigma = \rho_2/\rho_1 = u_1/u_2$** determined by the shock Mach No.)

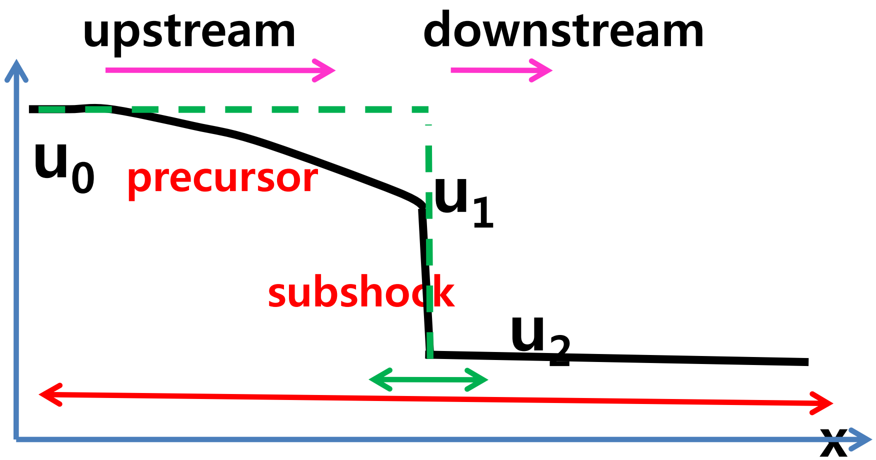
for strong gas shock : σ → 4, q → 4,

(for γ = 5/3 adiabatic index) **independent of *M***

But DSA is quite efficient → shock structure is modified by CR pressure.

$$q(p) = \frac{3U(p)}{U(p) - u_2} + \frac{d\ln(U(p) - u_2)}{d\ln p}$$

U(p) is the precursor velocity that particles with *p* feel on average.



$$\sigma_s = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \text{subshock comp.}$$

$$\sigma_t = \frac{u_0}{u_2} = \frac{\rho_2}{\rho_0} = \text{total shock comp.}$$

for $\kappa(p) = \kappa^* p^{\alpha}$

momentum dependent

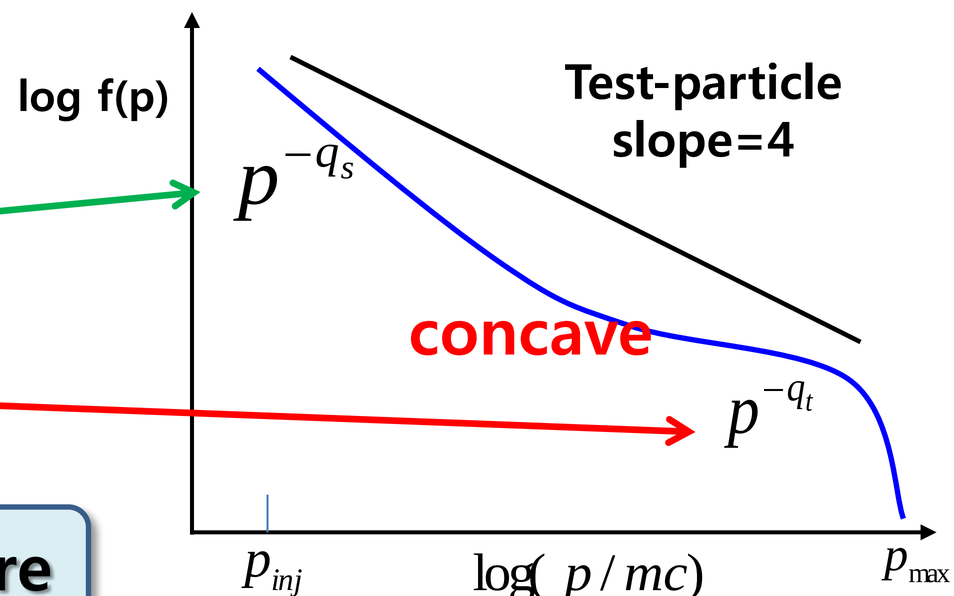
$$l_{diff}(p_{\min}) = \frac{\kappa(p_{\min})}{u_s}$$ → **feel only σ_s**

Particles with different *p* experience different Δu.

$$l_{diff}(p_{\max}) = \frac{\kappa(p_{\max})}{u_s}$$ → **feel σ_t**

$$q_s = \frac{3 \cdot \sigma_s}{\sigma_s - 1} > 4 : \text{subshock}$$

$$q_t = \frac{3 \cdot \sigma_t}{\sigma_t - 1} < 4 : \text{total shock}$$



CR modified shock structure

Wave drift steepens CR spectrum & reduces acceleration efficiency

$$q(p) = \frac{3[u(p)+u_w(p)]}{[u(p)+u_w(p)-u_2]} + \frac{d\ln[u(p)+u_w(p)]}{d\ln p}$$

$$\approx \frac{3[u(p)+u_w(p)]}{[u(p)+u_w(p)-u_2]}$$ for mildly modified shocks

$$q_s = \frac{3(u_1-\upsilon_A)}{(u_1-\upsilon_A-u_2)} = \frac{3(1-M_A^{-1})}{(1-M_A^{-1}-\sigma_s^{-1})}$$

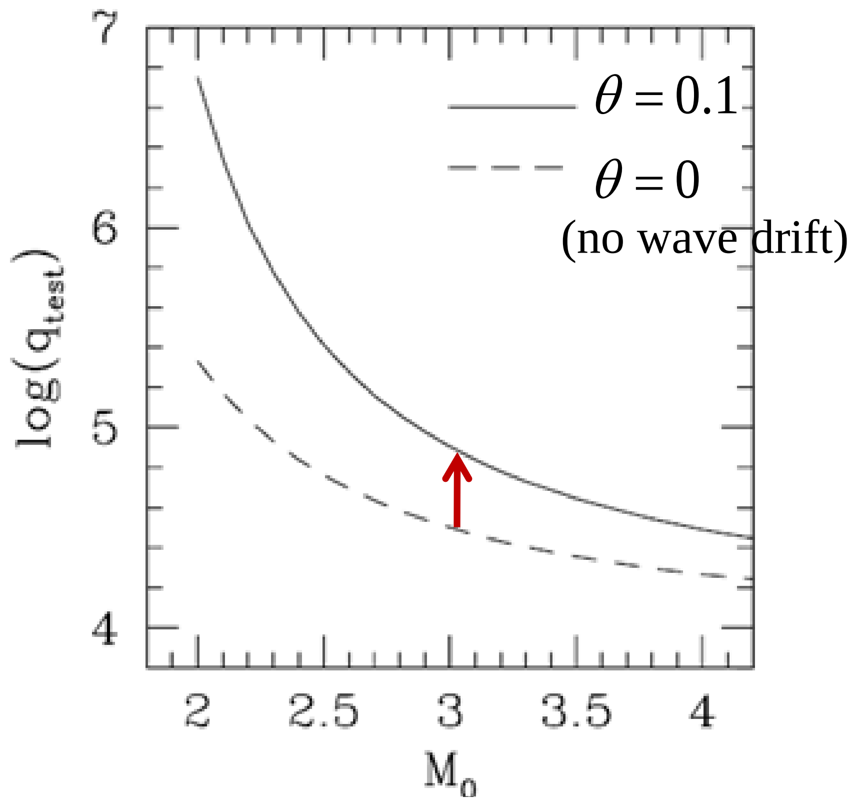
$$M_A = u_1/\upsilon_A\,,\ \sigma_s = u_1/u_2$$

$$\theta = \frac{E_B}{E_{th}} = \frac{8\pi/B^2}{1.5(P/\rho)}$$

for $\theta = 0.1,\ M_A = 2.36 M_0$

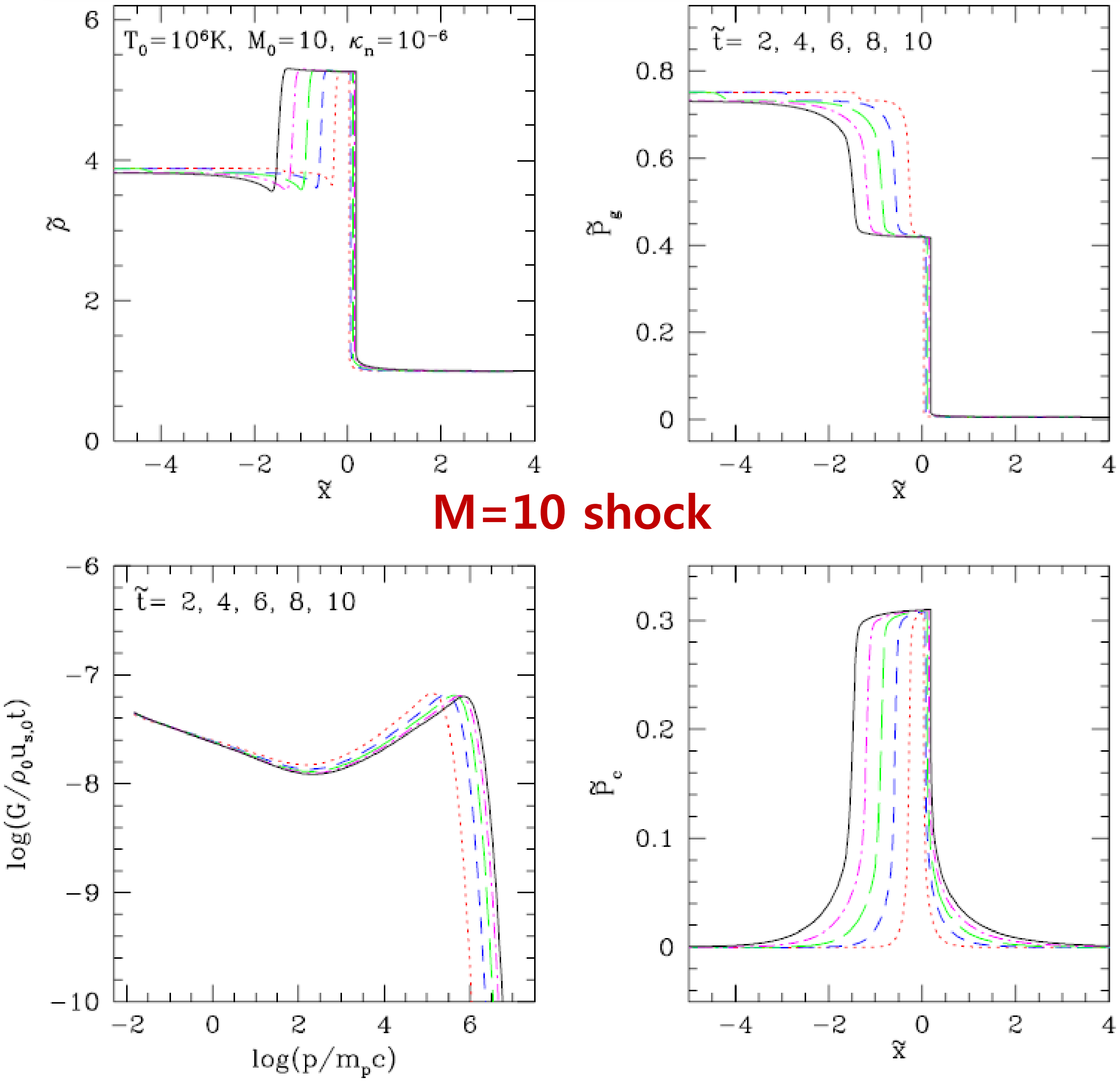
important for weak shocks

(e.g. shocks in the ICM)



Slope of test-particle spectrum

DSA Kinetic Simulation Results: Self-similar stage



M=10 shock

After $P_{c,2}$ reaches to an asymptotic value,

→ the shock flow becomes self-similar.

The shock structure stretches linearly with *t*, independent of κ(p).

$$l_{\max} \approx \frac{\kappa(p_{\max})}{u_s} \approx \frac{1}{8} u_s t$$

$$t_{acc} \approx 8 \frac{\kappa(p_{\max})}{u_s^2}$$

for $\kappa(p) = \kappa^* p^\alpha (\rho/\rho_0)^{-1}$, $\quad p$ in units of $m_p c$, $\alpha = 1$

$$t_{acc} \approx \frac{3}{u_1 - u_2}\left(\frac{\kappa_1(p)}{u_1} + \frac{\kappa_2(p)}{u_2}\right) = \frac{8M_s^2}{M_s^2 - 1}\frac{\kappa(p)}{u_s^2} \quad \text{for } M >> 1$$

$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t \Rightarrow \kappa(p_{\max}) \approx (1/8) u_s^2 t \text{ at a given shock age}$$

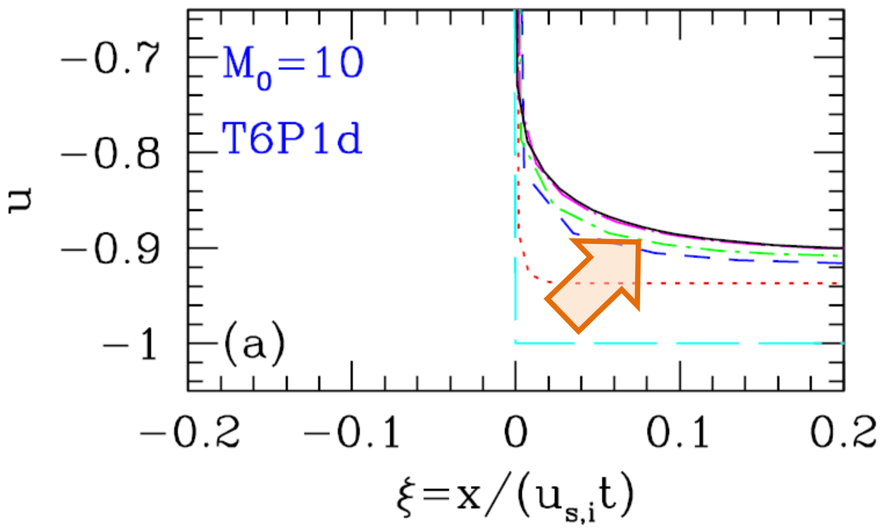
$$l_{\max} \approx \frac{\kappa(p_{\max})}{u_s} \approx (1/8)\, u_s t \quad \Rightarrow \xi \equiv \frac{x}{u_{s,i} t} \text{ similarity variable}$$

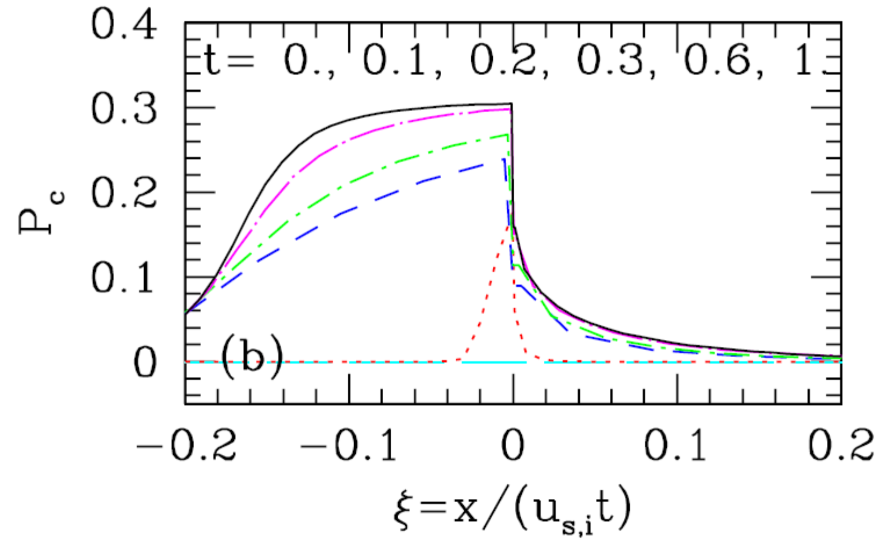
Shock structure broadens linearly with time independent of κ^*.

smaller $\kappa^* \rightarrow$ higher $p_{\max}$ at a given shock age.

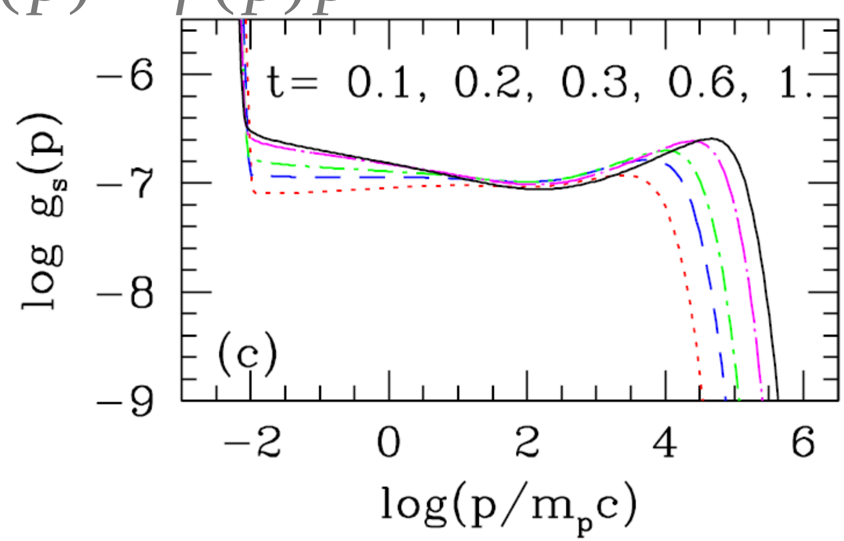
But hydrodynamic structure is independent of κ^*.

DSA kinetic simulation results: Early Evolution





$g(p) = f(p)p^4$



Initial conditions at t=0

M_0=10 gasdynamic shock

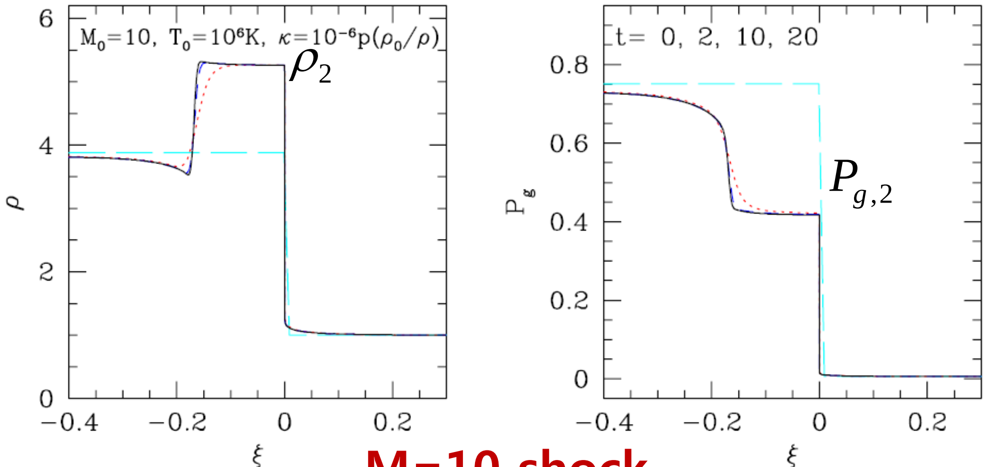
No pre-existing CRs

ε_B=0.2, θ=0.1

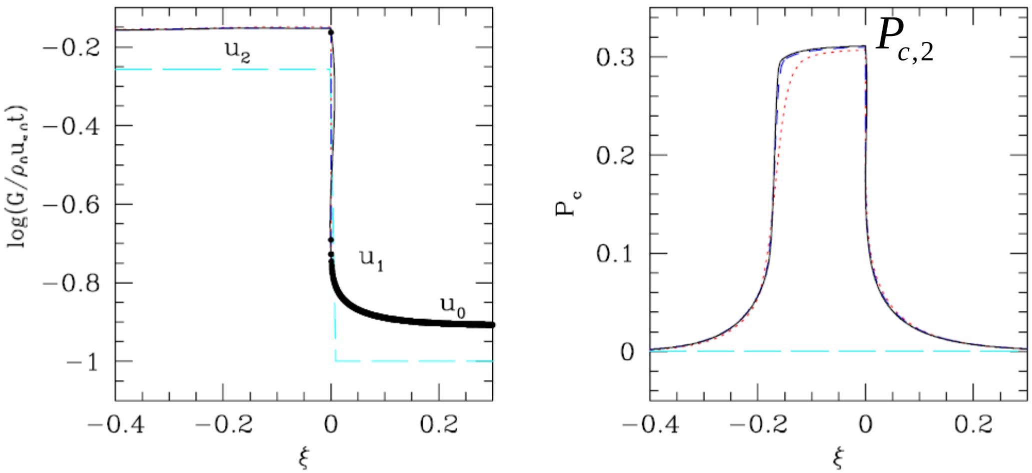
$\kappa(p)=10^{-6}p(\rho/\rho_0)^{-1}$

$$\theta = \frac{E_B}{E_{th}}, \quad \frac{\upsilon_A}{c_s} = \frac{M_0}{M_A} = \sqrt{\frac{9\theta}{5}}$$

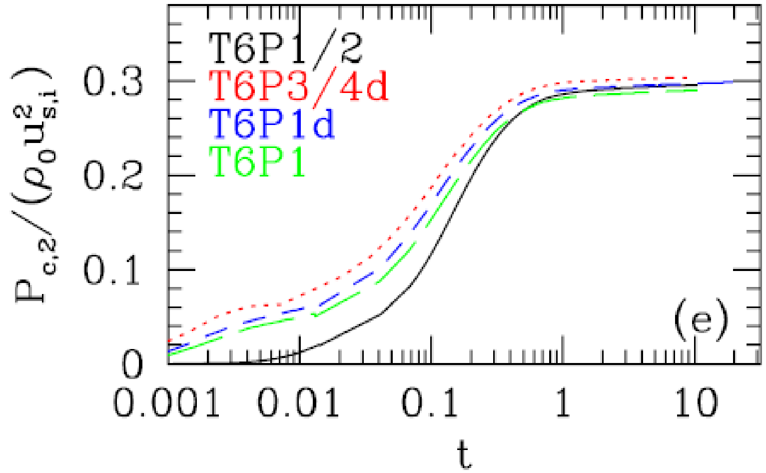
DSA Kinetic Simulation Results: Self-similar stage



M=10 shock

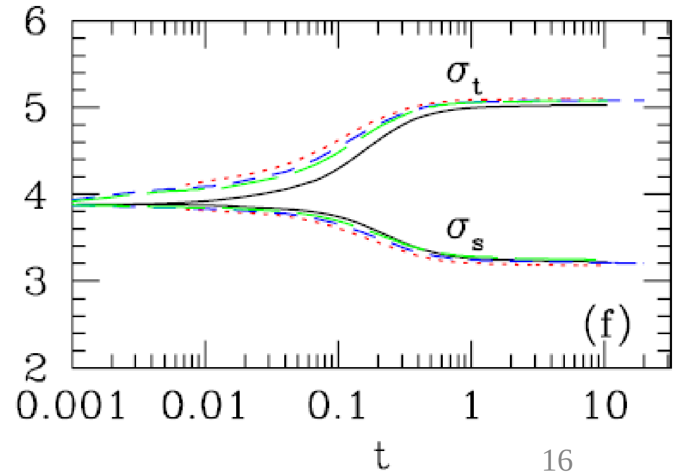


$P_{g,2}$, $P_{c,2}$ $\rightarrow$ constant in time



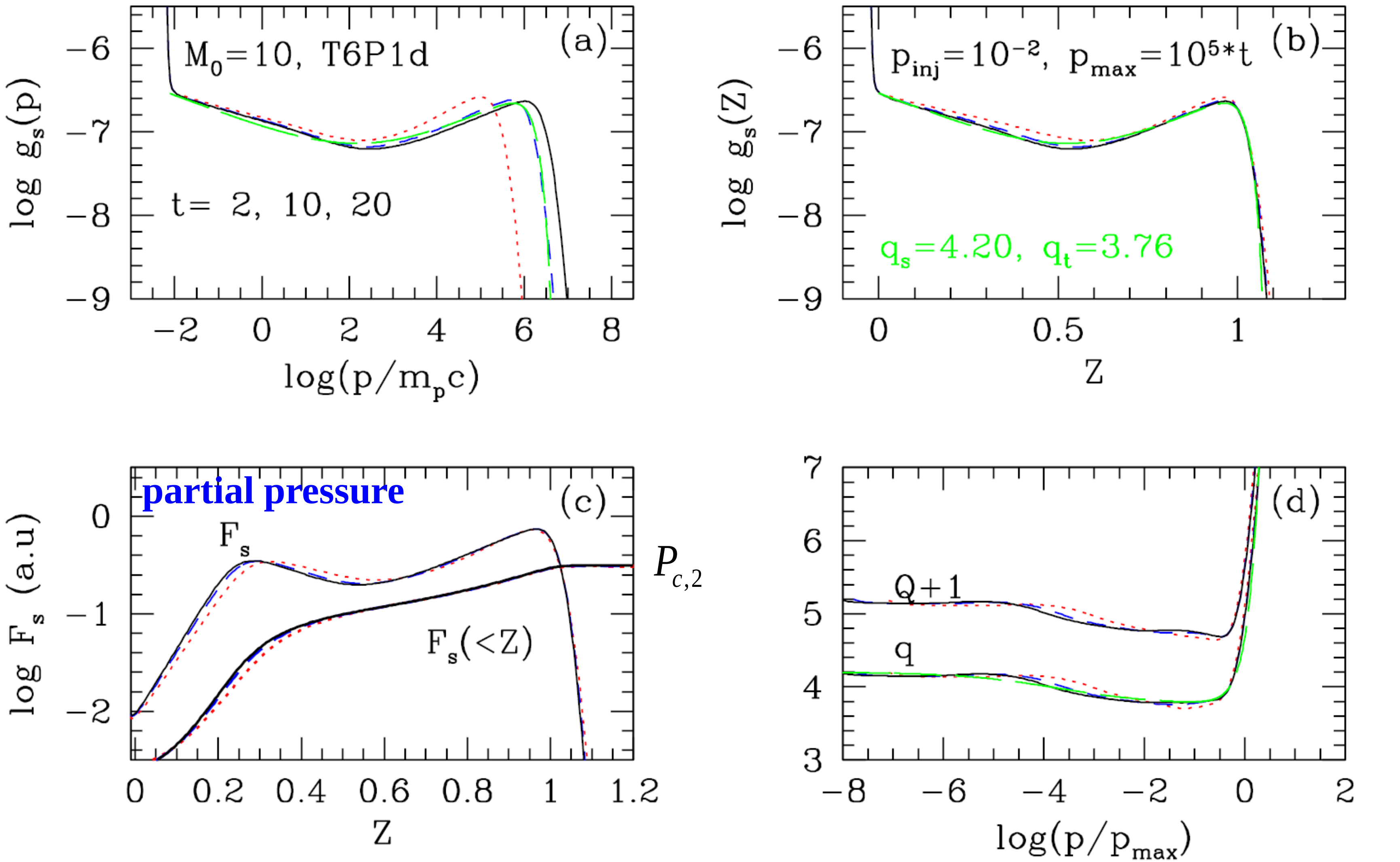
$\sigma_t = \rho_2 / \rho_0 \rightarrow$ constant in time

$\sigma_s = \rho_2 / \rho_1$



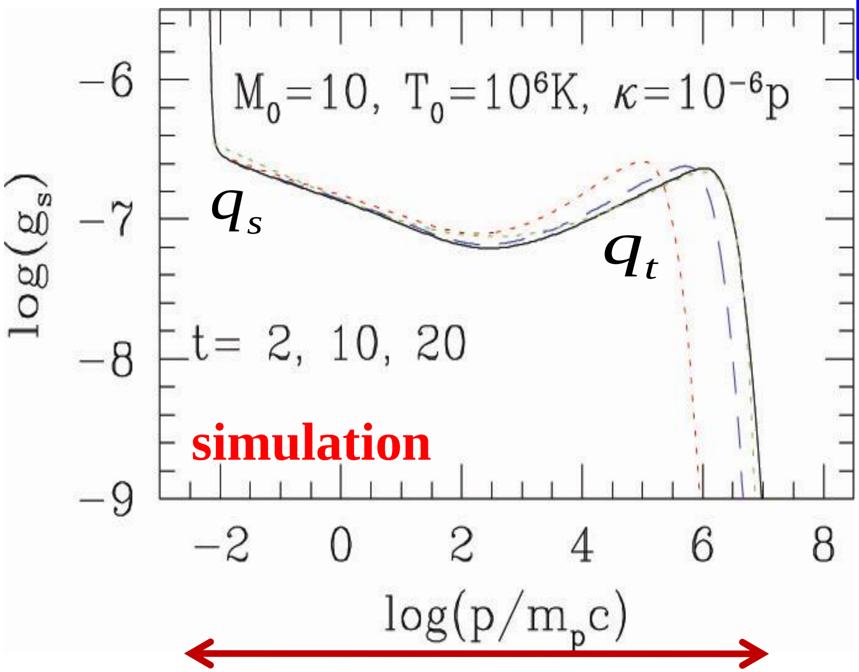
$$\xi = \frac{x}{l_{\max}(t)} \propto \frac{x}{u_s \cdot t}$$ = **similarity variable**

CR spectrum during the Self-similar stage



$$F(Z) \equiv g(Z)\frac{p}{\sqrt{p^2+1}}\ln\left(\frac{p_{\rm max}}{p_{\rm inj}}\right) \qquad Z \equiv \ln(p/p_{\rm inj})/\ln[p_{\rm max}(t)/p_{\rm inj}]$$

CR distribution function at shock

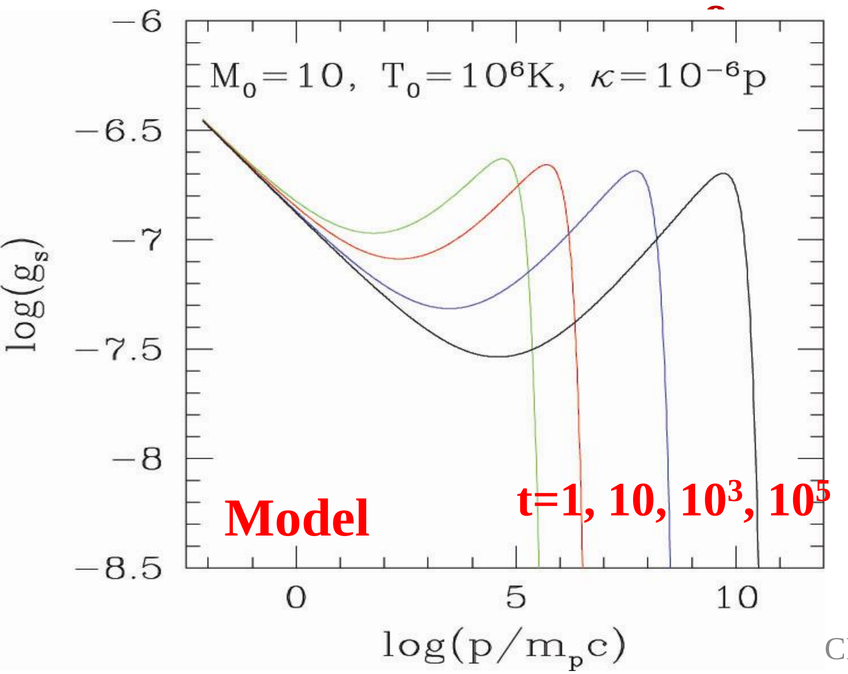


$$q_s = \frac{3(u_1+u_w)}{u_1+u_w-u_2} \quad \text{subshock jump at low } p$$

$$q_t = \frac{3(u_0+u_w)}{u_0+u_w-u_2} \quad \text{total shock jump at high } p$$

$$p_{\min} = p_{\text{inj}} \approx 2.5 p_{th}$$

$$p_{\max} \approx \frac{u_s^2}{8\kappa^*} \cdot t\,, \quad \text{where } \kappa(\mathrm{p}) = \kappa^* p\left(\frac{\rho}{\rho_o}\right)^{-1}$$



possible analytic form : **two power-laws**

$$f(x_s,p) = [\,f_1 \cdot (p/p_{\min})^{-q_s} + f_2 \cdot (p/p_{\max})^{-q_t}\,] \cdot \exp[-(p/p_{\max})^2]$$

where $f_1 = f_{th}(p_{\text{inj}})$ at thermal tail

f_2 : to be determined by $P_{c,2}$

Why CR modified shocks become self-similar ?

in the limit of $t \to \infty$, $p_{\max} \to \infty$

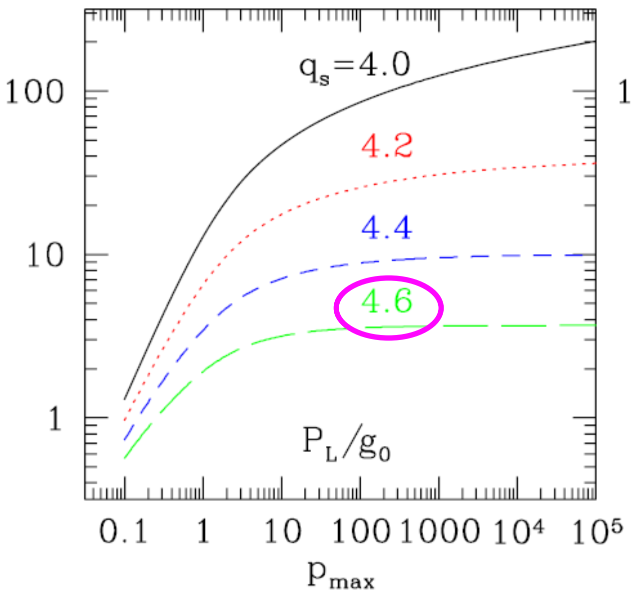
postshock $P_{c,2} \to$ constant

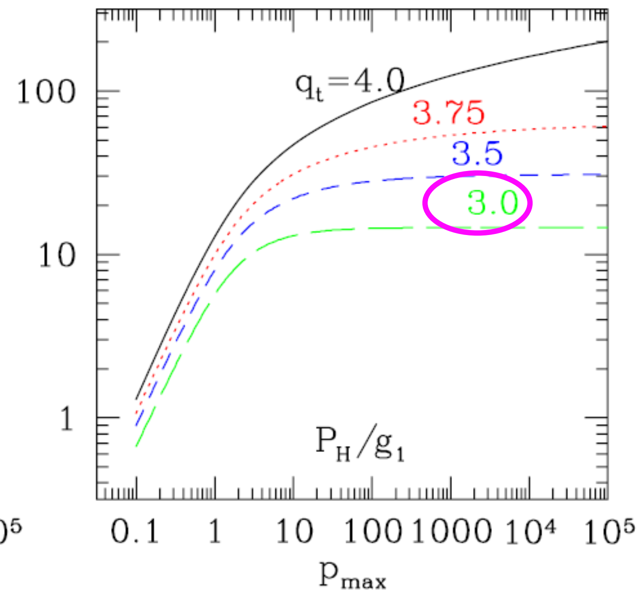
then shock structure $\to$ steady,

i.e. $P_{g,2}, \rho_1, \rho_2 \to$ constant

$$P_c = \frac{4\pi}{3} m_p c^2 \int_{p_{\text{inj}}}^{\infty} g(p) \frac{p}{\sqrt{p^2+1}} \frac{dp}{p}. = P_L + P_H$$

$$g_s(p) = \left[g_0 \cdot \left(\frac{p}{p_{\text{inj}}}\right)^{-q_s+4} + g_1 \cdot \left(\frac{p}{p_{\max}}\right)^{-q_t+4} \right] \exp\left[-\left(\frac{p}{1.5 p_{\max}}\right)^{2\alpha} \right]$$





if $q_s > 4.0$ (i.e. $\sigma_s < 4.0$),

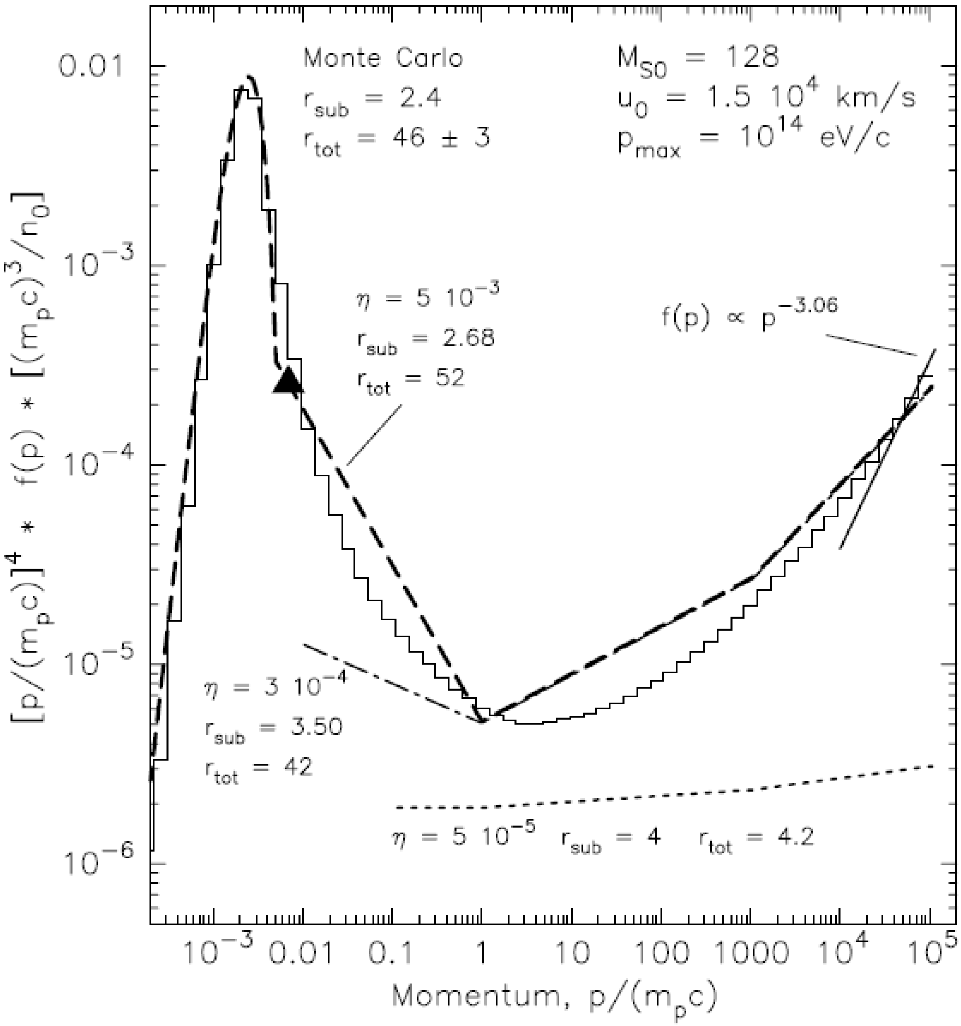
$P_L \to$ constant at large $p_{\max}$

if $q_t < 4.0$ (*i.e.* $\sigma_t > 4.0$),

$P_H \to$ constant at large $p_{\max}$

true for CR modified shocks.

semi-analytic model for f(p) for steady-state shocks by Berezhko & Ellison 1999



3 power-laws:

$$f(p) \propto p^{-q_{sub}} \text{ subshock}$$
$$\propto p^{-q_{int}} \text{ intermediate}$$
$$\propto p^{-q_{min}} \text{ minmum}$$

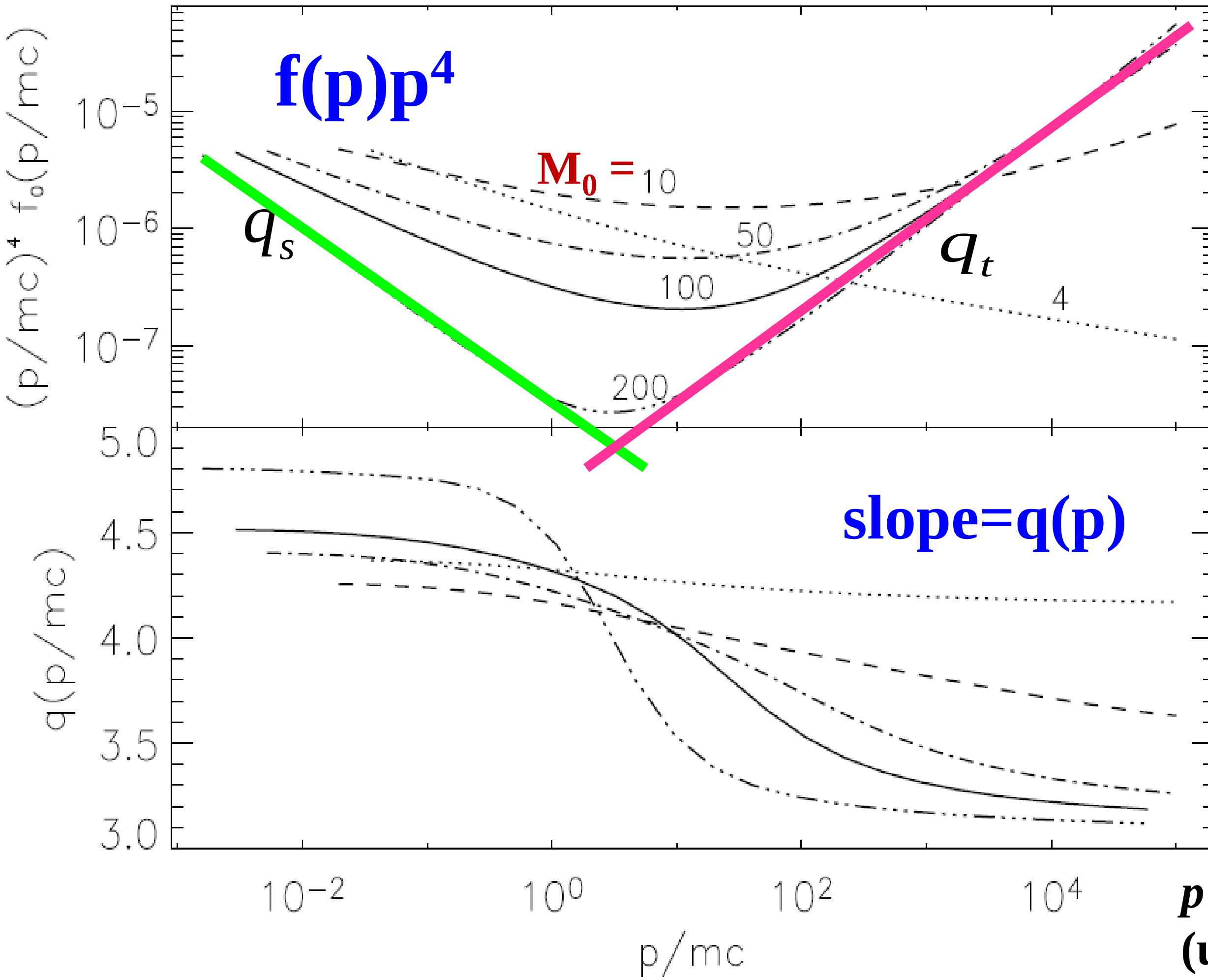
$$q_s = \frac{3\cdot\sigma_s}{\sigma_s - 1} : \text{subshock comp.}$$

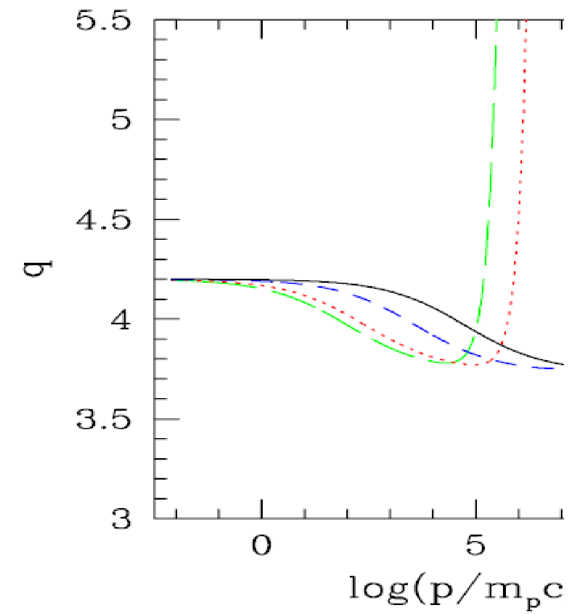
$$q_{min} = 3.5 + \frac{3.5 - 0.5\sigma_s}{2\sigma_t - \sigma_s - 1} : \text{minimum}$$

$$q_{int} = (q_{tp} + q_{min})/2$$

Semi-analytic model: Amato & Blasi 2005, 2006, Capriloi et al 2009
Non-linear amplification of B field driven by CR streaming

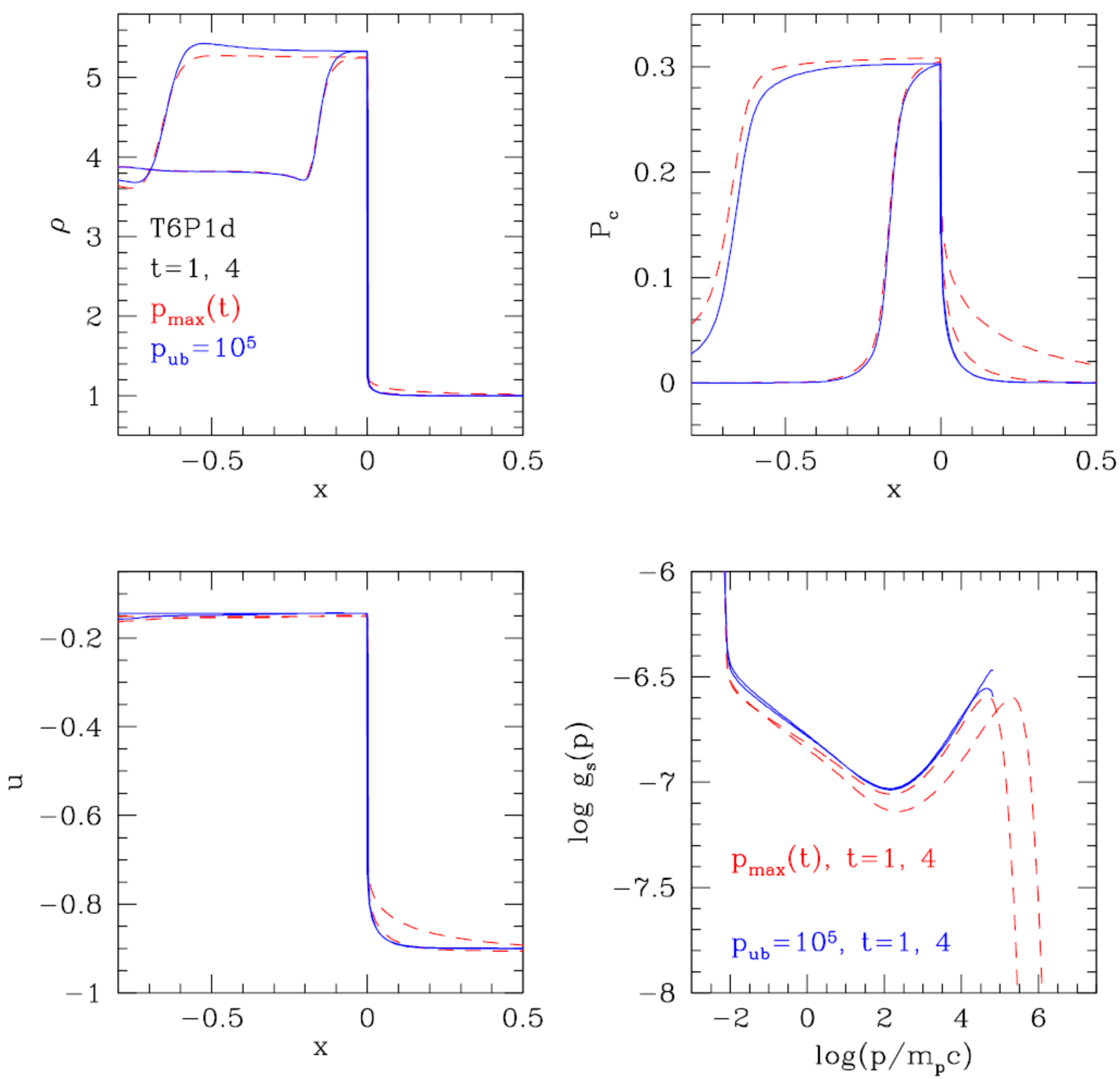
Solve DC equation for *f(x,p)* along with gasdynamic equations
in the steady state limit with a fixed p_{max}





$p_{max}=10^5$ mc
(upper momentum boundary)

time-dependent solution and steady-state solution with the same pmax are the same.



Steady state solution is achieved by setting a upper momentum boundary at $p_{ub} = 10^5$ (blue solid lines)

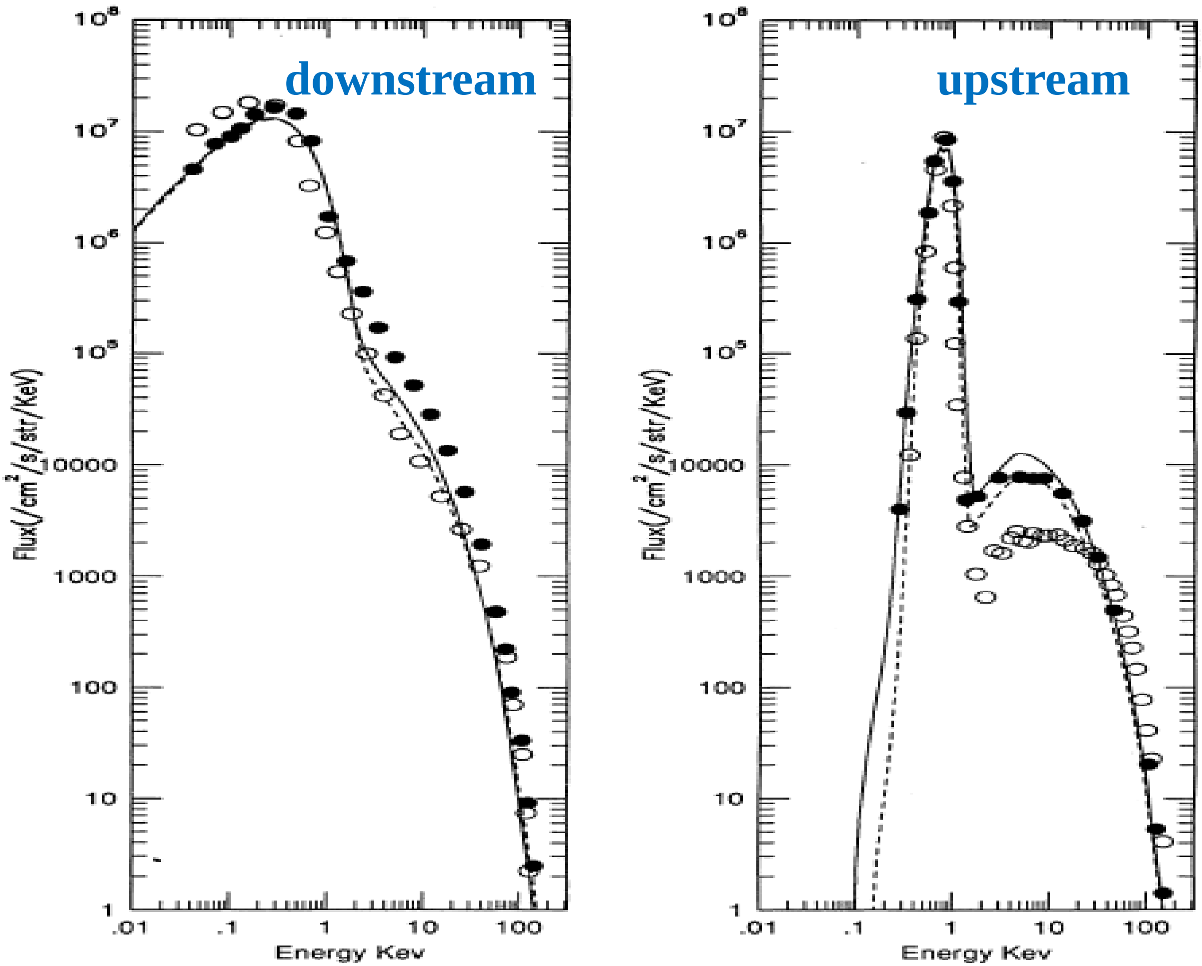
Precursor becomes steady.

Highest energy particles escape through the upper momentum boundary.

Time-dependent solution at t=1, 4 → $p_{\max}(t) \approx 10^5$ (red dashed lines)

Precursor broadens as pmax increases with time.

Comparison of different methods: Kang & Jones 1995



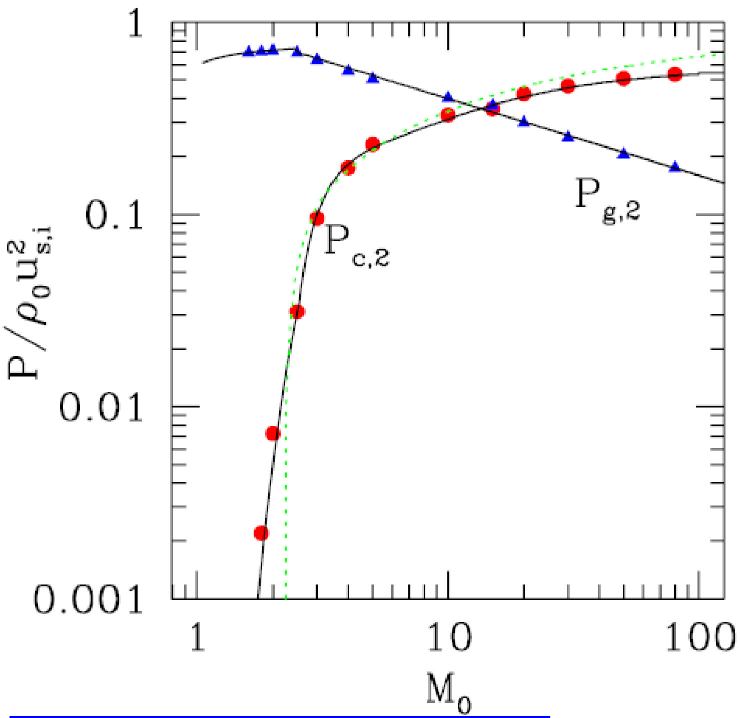
Solid lines: our kinetic simulation data

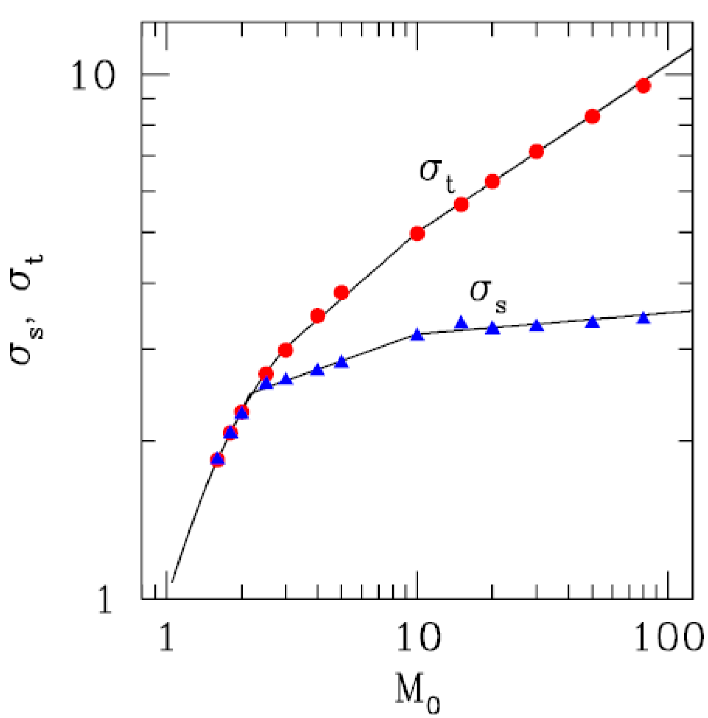
Filled circles: Monte Carlo simulation data from Ellison et al. 1993

Open circles: Hybrid simulation data from Ellison et al. 1993

Thus, we find that all of these computational methods (diffusion-advection, two-fluid, Monte Carlo, and hybrid) are in substantial agreement on the issues they can simultaneously address, so that the essential physics of diffusive particle acceleration is adequately contained within each. This is despite the fact that each makes what appear to be very different assumptions or approximations.

Time asymptotic solutions from DSA simulations





total compression

$$\sigma_t = \rho_2 / \rho_0$$

subshock compression

$$\sigma_s = \rho_2 / \rho_1 \sim 3$$

$$\frac{P_{c,2}}{\rho_0 u_s^2} \approx 0.5 \ \text{for } M_0 \geq 20$$

The solutions depend on the details of injection, wave generation, drift, and dissipation models, especially for weak shocks. Nonlinear feedback is important for strong shocks.

SUMMARY

In CR modified shocks, the precursor & subshock transition approach the time-asymptotic state.

$$P_{g,2}\,, P_{c,2}, \sigma_t = \rho_2 / \rho_0,\ \sigma_s = \rho_2 / \rho_1 \rightarrow \text{constant}$$

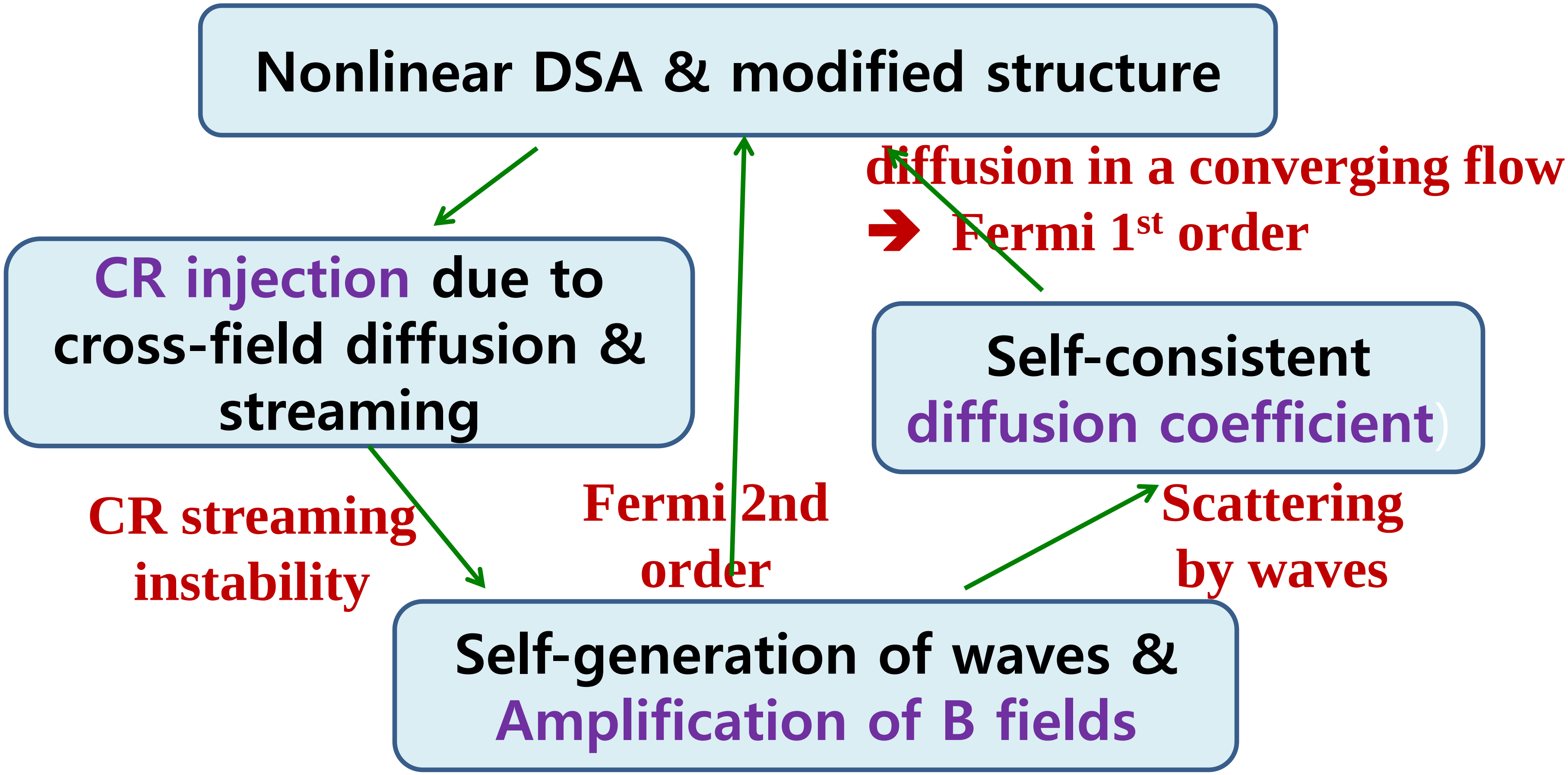
(need numerical simulations)

Then shock precursor structure evolves in a self-similar fashion, depending only on similarity variable, $\xi = x/(u_s\ t)$. During this self-similar stage, the CR distribution at the subshock maintains a characteristic form: two power-laws

$$f_s(p,t) = [\, f_1 \cdot (p / p_{\min})^{-q_s} + f_2 \cdot (p / p_{\max}(t))^{-q_t}\,] \cdot \exp[-(\frac{p}{1.5\, p_{\max}(t)})^{2\alpha}]$$

where $f_1 = f_{th}(p_{\text{inj}})$ at thermal tail, f_2 : to be determined by $P_{c,2}$

$$q_s(M) = \frac{3(u_1 + u_w)}{u_1 + u_w - u_2}\ ,\quad q_t(M) = \frac{3(u_0 + u_w)}{u_0 + u_w - u_2}\ , p_{\text{inj}} / p_{th} \approx 2.5,\ p_{\max}(t) \approx \frac{u_s^2}{8\kappa_n} \cdot t$$



Feedback from Plasma Physics on wave-particle interactions

Growth & Damping of waves ➔ **CR Injection & Scattering**

➔

Predict *f(p)*, p_{max}
non-thermal radiation (observation)