Acceleration and collimation of Poynting dominated jets

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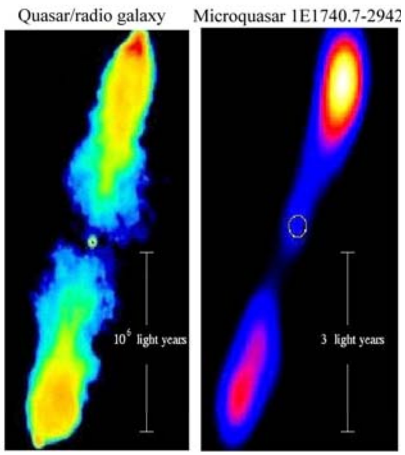
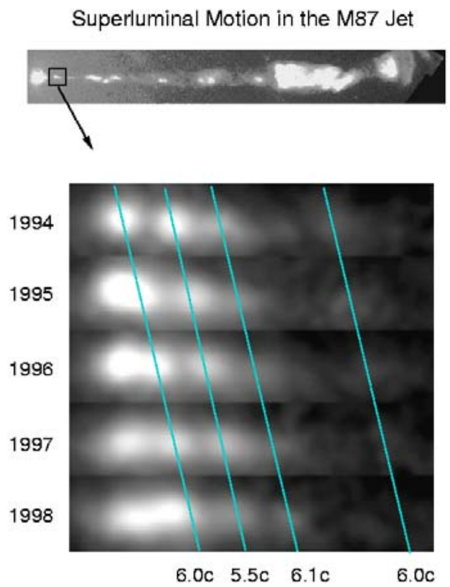
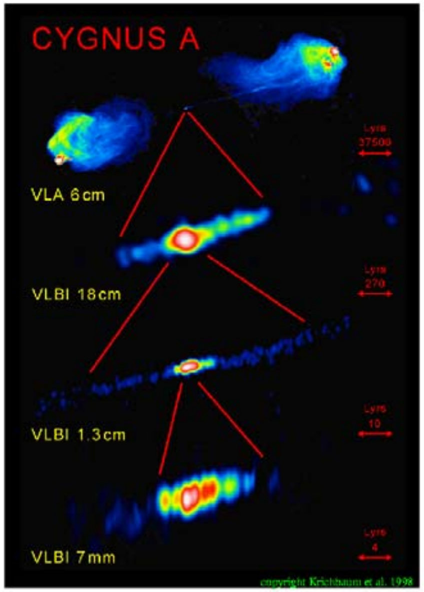
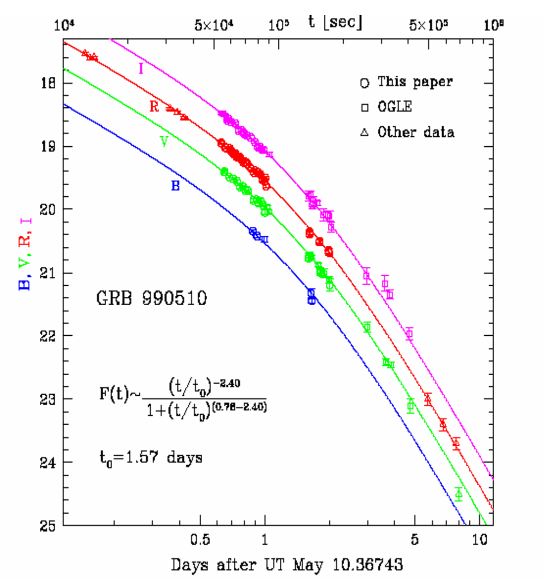
Universality of relativistic jet phenomenon

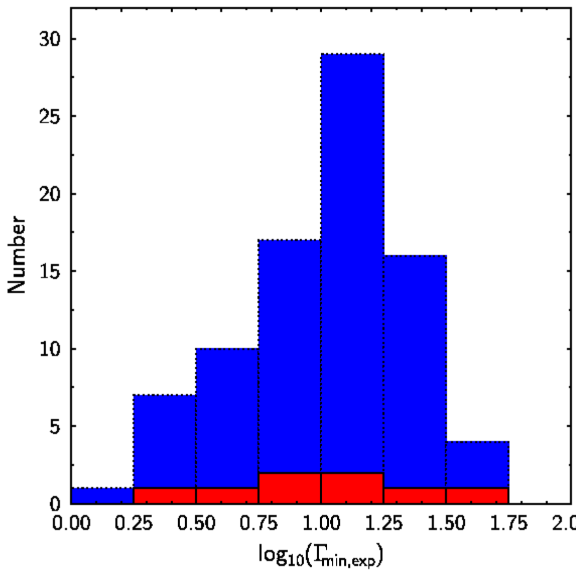






GRBs apparently involve ultrarelativistic (γ=100-1000), highly collimated (Θ=2°-5°) outflows. They likely arise during the collapse of star's core.





Distribution of Lorentz factors
Blue – AGNs; red – microquasars
(Fender 2005)

All relativistic cosmic jet sources may be connected by a common basic mechanism

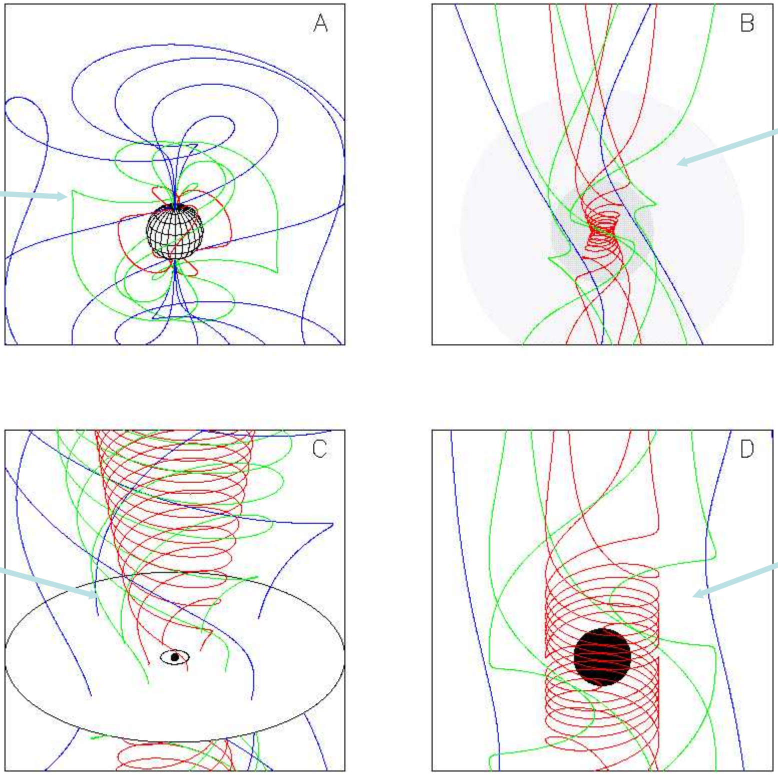
A promising model is magnetohydrodynamic acceleration by rotating, twisted magnetic fields

Poynting dominated outflows in astrophysics

Rotation twists up field into toroidal component, slowing rotation

Pulsar magnetosphere, beyond the light cylinder

Collapsing, magnetized supernova core



Magnetized accretion disks around neutron stars and black holes

Magnetospheres of Kerr black holes, with differentially-rotating metric

Picture by David Meier

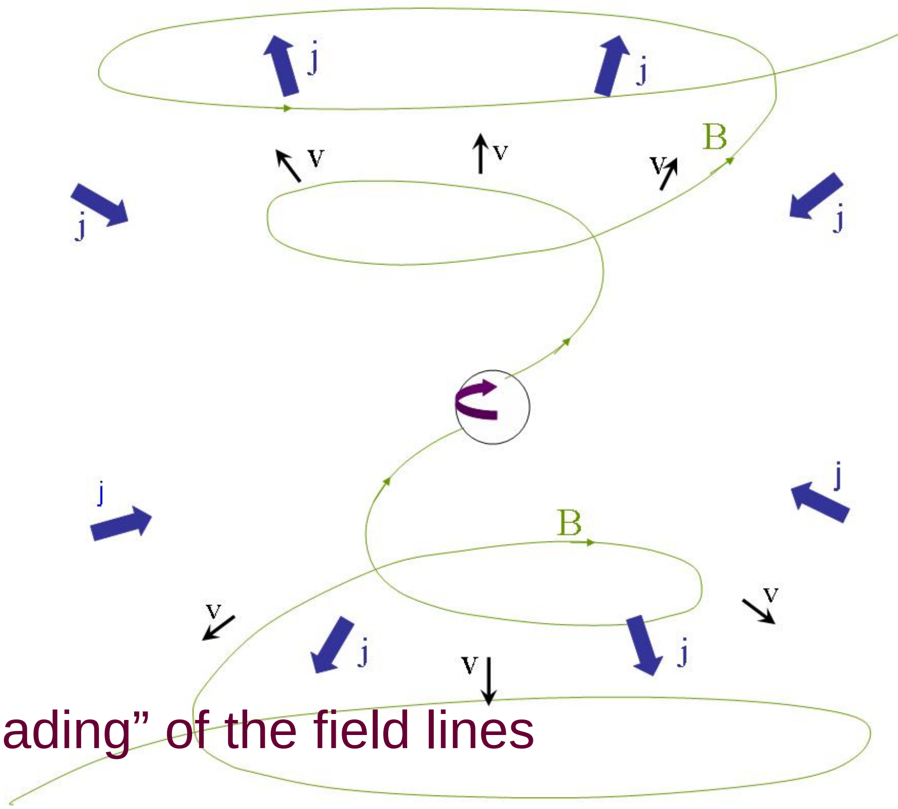
Energy balance in magnetized outflows

In the proper plasma frame $\mathbf{E'} = \mathbf{0}$

In the lab frame $\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} = \mathbf{0}$

Poynting flux $\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} \approx \frac{B^2}{4\pi}\mathrm{v}$

Rotational energy → Poynting → ?



Relativistic flow can be produced by having a very strong rotating magnetic field such that $B^2 \gg 4\pi\rho c^2$ $\Rightarrow$ low "mass loading" of the field lines

The flow starts as Poynting dominated, $\sigma \gg 1$. How could the electro-magnetic energy be transformed into the plasma energy?

$$\sigma = \frac{\text{Poynting flux}}{\text{plasma energy flux}}$$

Force balance in Poynting dominated flows

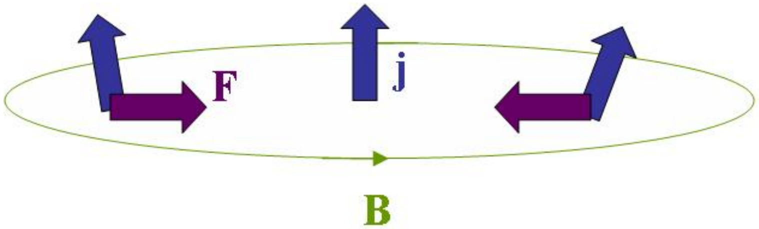
Plasma pressure and inertia<< Lorentz force.
Does it mean that huge tension of wound up magnetic field (hoop stress) compresses the flow towards the axis?

NO!

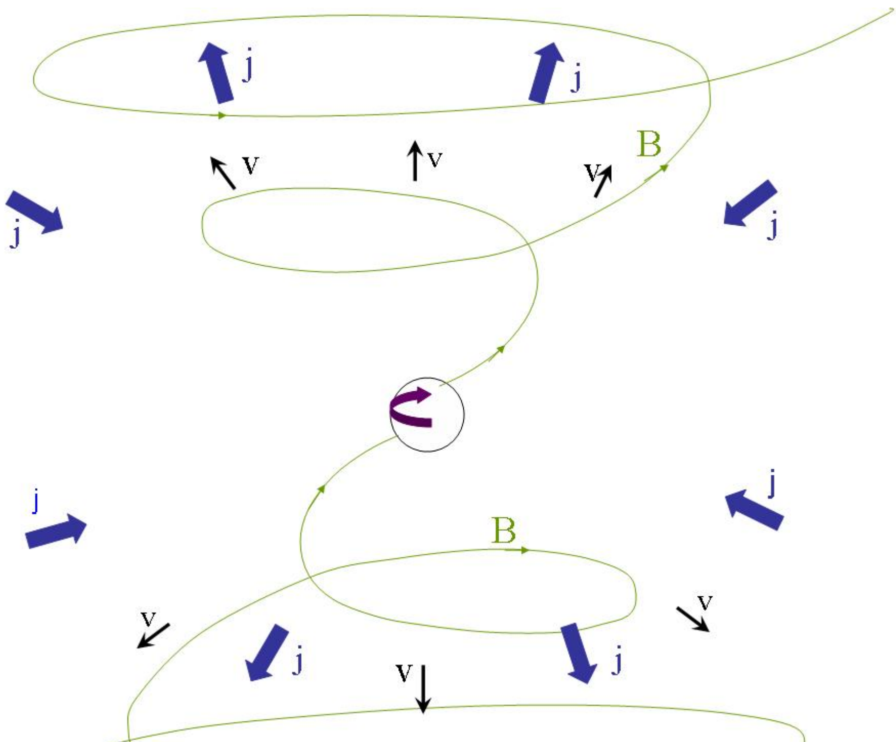
In the current closure region, the force is decollimating.

An external confinement is necessary!

Magnetic hoop stress

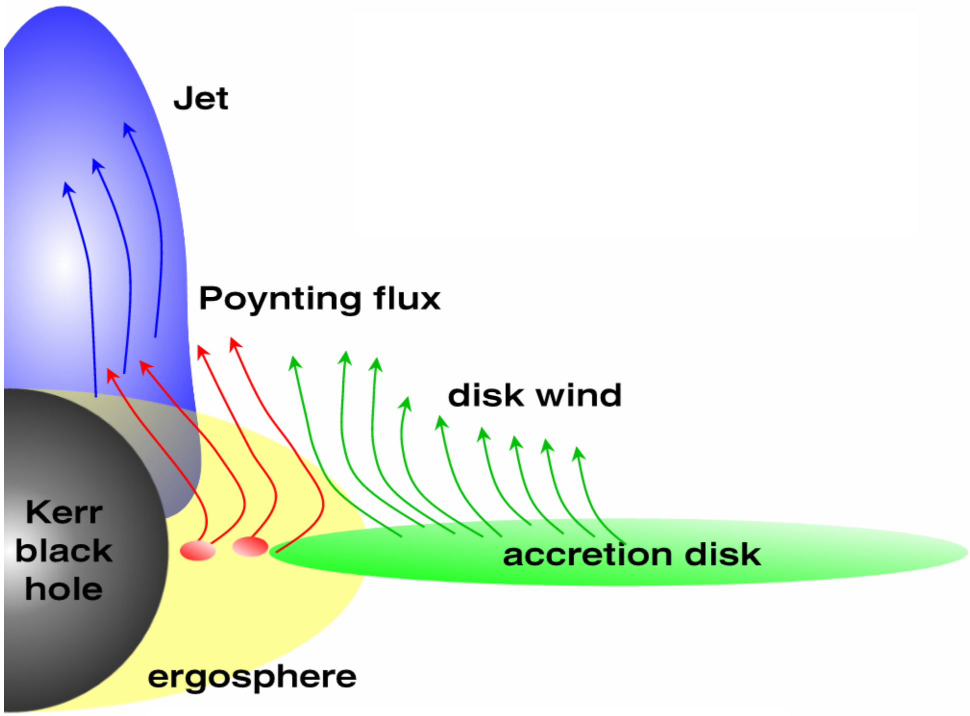


$$\mathbf{F} = \frac{1}{c}\,\mathbf{j}\times\mathbf{B}$$

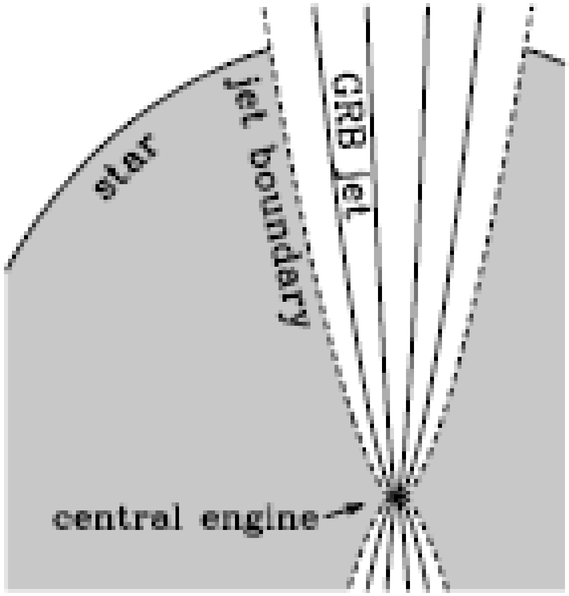


Externally confined jets

In accreting systems, the relativistic outflows from the black hole and the internal part of the accretion disc could be confined by the (generally magnetized) wind from the outer parts of the disk.



In GRBs, a relativistic jet from the collapsing core pushes its way through the stellar envelope.



Force balance in Poynting dominated flows (cont)

Total electromagnetic force $\mathbf{F} = \rho_e \mathbf{E} + \frac{1}{c} \mathbf{j} \times \mathbf{B}$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = 0 \qquad \rho_e = \frac{1}{4\pi} \nabla \cdot \mathbf{E}$$

In the far zone, v $\longrightarrow$ c and E $\longrightarrow$ B.
The Lorentz force is nearly compensated by the electric force.

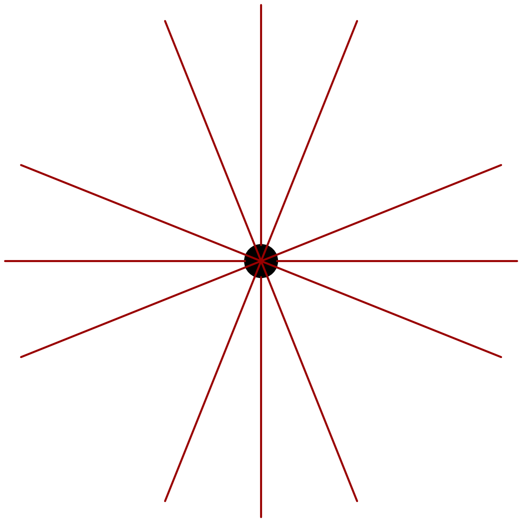
Examples of relativistic MHD outflows.

1. Pulsar wind – no external confinement

Without external confinement, the characteristic collimation/acceleration scale is exponentially large. The flow is practically radial and the acceleration practically stops (Tomimatsu 1994; Beskin, Kuznetsova & Rafikov 1998)

$$\gamma \approx \Omega r; \qquad \gamma < \gamma_{\max}^{1/3}$$

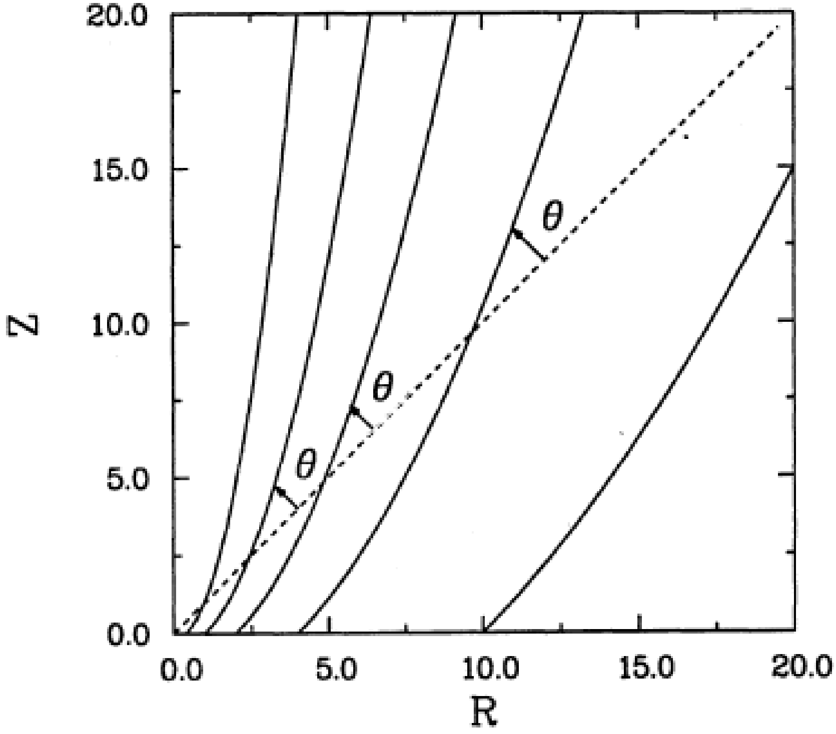
$$\gamma \approx \left[\gamma_{\max} \ln\left(\Omega r / \gamma_{\max}\right)\right]^{1/3}; \quad \gamma > \gamma_{\max}^{1/3}$$



γ_{max}>>1 - the Lorentz factor achieved when and if the Poynting flux is completely transformed into the kinetic energy

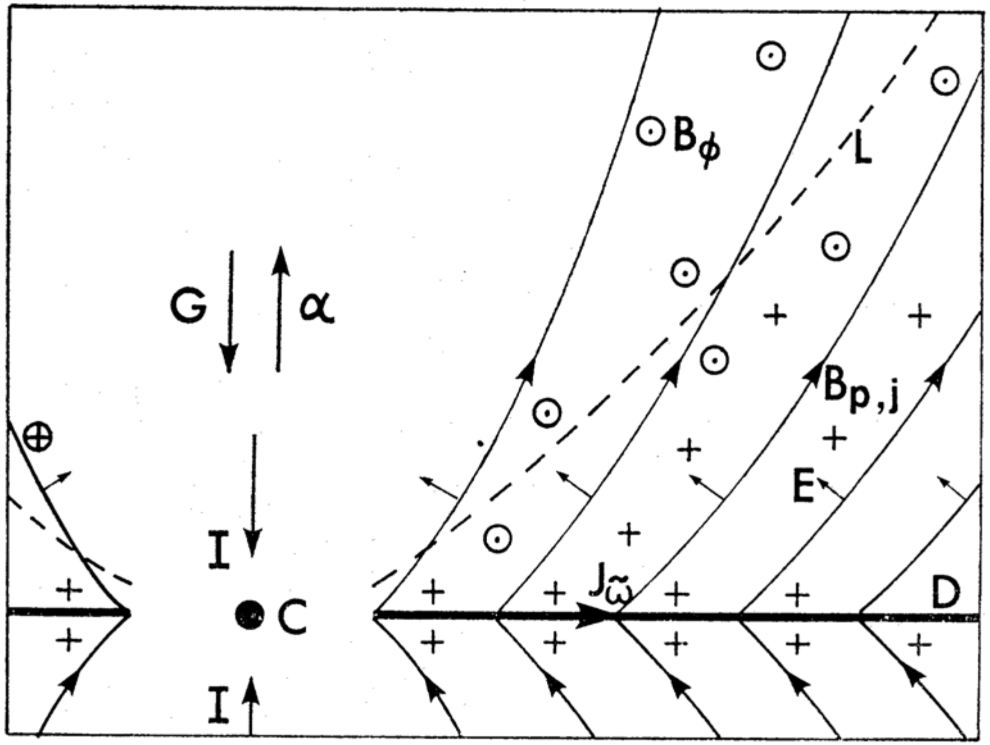
Examples of relativistic MHD outflows.
2. Self-similar solutions

Li, Chiueh, Begelman 1992;
Contopoulos1995;
Vlahakis, Konigl 2003;
Narayan et al 2007



Total $\Psi \rightarrow \infty$.
Collimation and acceleration could occur at a reasonable, even though large, scale.

Examples of relativistic MHD outflows.
3. Parabolic flux surfaces



Blandford (1976); Beskin, Nokhrina (2006): Parabolic collimation, $z \propto r^2$.
The flow is accelerated to equipartition level.

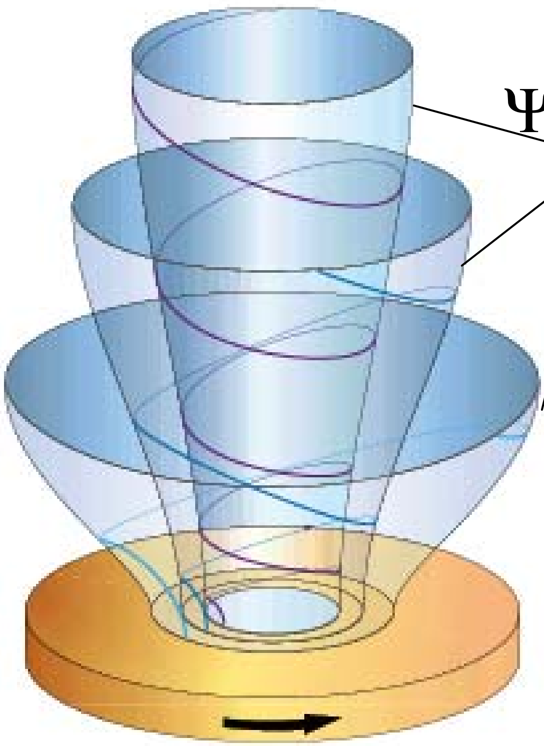
Externally confined jets.
What do we want to know?

What are the conditions for acceleration and collimation?
What is the terminal Lorentz factor, i.e. what fraction of the Poynting flux is transformed into the kinetic energy?
What is the final collimation angle?

One has to take into account that in the far zone, $v \to c$ and $E \to B$.
The Lorentz force is nearly compensated by the electric force.
Acceleration/collimation is spatially extended.

Asymptotic analysis is necessary.

Axisymmetric flows are nested magnetic surfaces of constant magnetic flux.
These surfaces are equipotential.
Plasma flows along these surfaces.



$$\Psi(r,z) = \mathrm{const}$$

$$\mathbf{B} = \mathbf{B}_p + B_\phi \mathbf{e}_\phi$$

$$\mathbf{B}_p = \frac{1}{r}\nabla\Psi \times \mathbf{e}_\phi$$

$$\mathbf{E} = -\Omega\nabla\Psi$$

At Ωr=c: $B_\phi \sim B_p \sim E$

$$\text{Poynting flux} = \frac{EB_\phi}{4\pi}c \sim \frac{B^2}{4\pi}c$$

At Ωr>>c: $B_\phi = \frac{2I}{cr} \propto \frac{1}{r}; \quad B_p \propto \frac{1}{r^2}; \quad E \propto \nabla\Psi \propto \frac{1}{r}$

In the far zone, E, $B_{\phi.}$>>B_p

Longitudinal dynamics of the flow

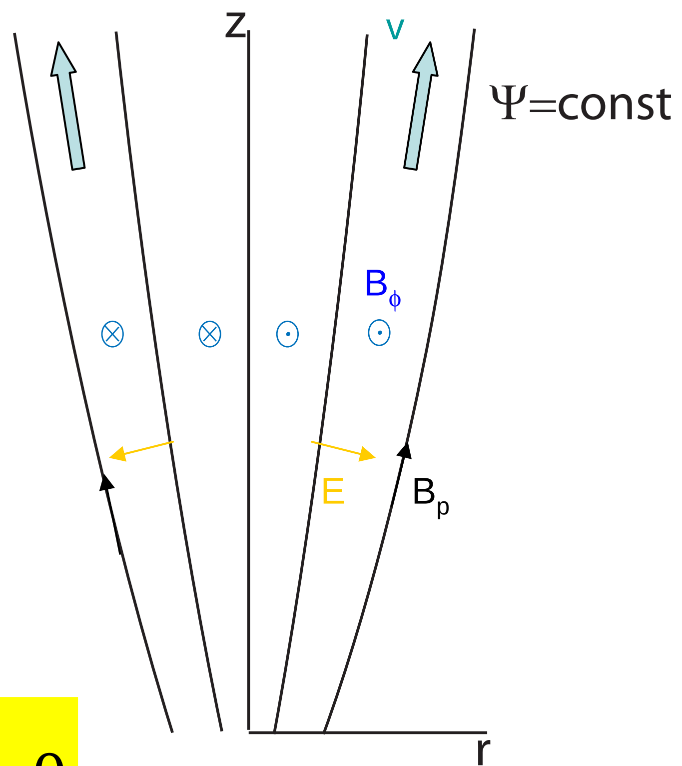
$$B_p = \frac{1}{r}|\nabla\Psi|$$

$$\gamma - \frac{r\Omega B_\phi}{\eta} = \mu(\Psi)$$ energy

$$\gamma r v_\phi - \frac{r B_\phi}{\eta} = l(\Psi)$$ angular momentum

$$4\pi\rho v_p \gamma = \eta(\Psi) B_p$$ continuity

$$B_p v_\phi - B_\phi v_p = r\Omega(\Psi) B_p$$ "bead on wire"



$$Y(\Omega r, \gamma, \Psi, \nabla\Psi) = 0$$

Bernoulli equation

Transverse force balance

$$\left[4\pi\rho\gamma(\mathbf{v}\cdot\nabla)\mathbf{v} - \mathbf{E}(\nabla\cdot\mathbf{E}) + \mathbf{B}\times(\nabla\times\mathbf{B})\right]\cdot\nabla\Psi = 0$$

$$G\left(\Omega r, \gamma, \Psi, \nabla\Psi, \nabla^2\Psi\right) = 0$$

transfield (Grad-Shafranov) equation

Boundary conditions and integrals of motion

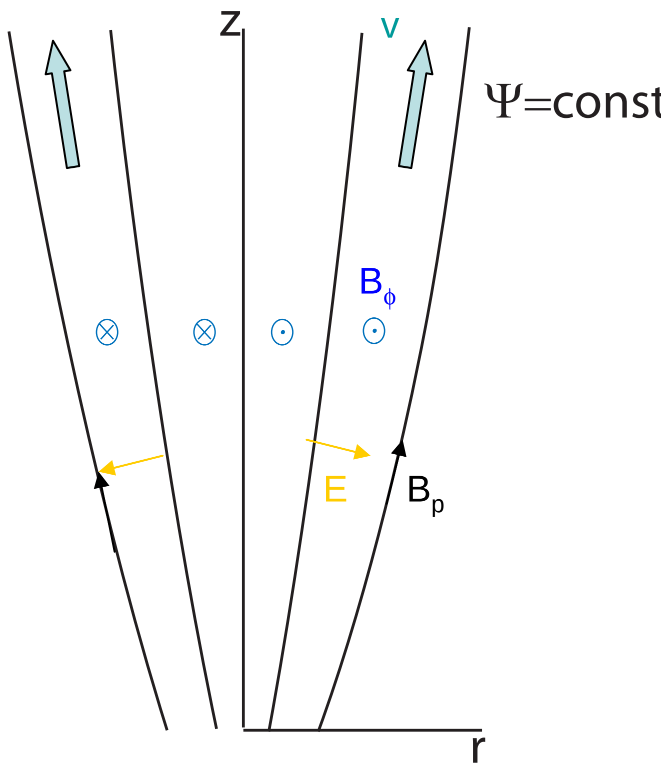
$$B_p = \frac{1}{r} |\nabla \Psi|$$

At the injection site, one prescribes
B_p, $\Omega(\Psi)$, $\eta(\Psi)$, $\gamma_{in}(\Psi)$

At the outer boundary $B'^2 \equiv B^2 - E^2 = 8\pi p_{ext}(z)$

$$\Omega l = \mu - \gamma_{in}$$

$\mu(\Psi)$ is determined from the condition of the smooth passage through the light cylinder

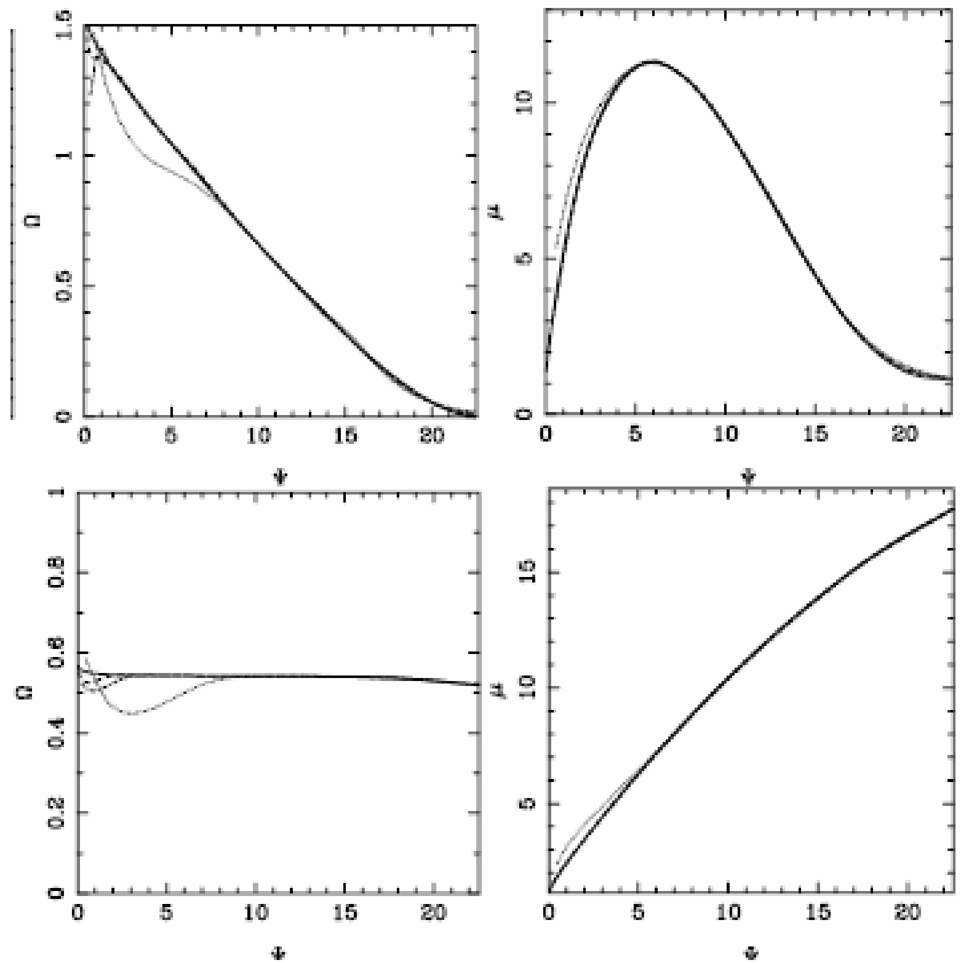


Poynting flux goes to 0 at the axis

$$P = \frac{EB_\varphi}{4\pi}c \approx \frac{B_\varphi^2}{4\pi}c;$$

$$B_\varphi = \frac{2I}{cr} = \frac{2}{cr}\pi\int jrdr \rightarrow \frac{2\pi j(0)}{c}r$$

$$\mu(\Psi) = \gamma_{in} + \frac{\Psi}{\tilde{\Psi}}; \quad \Psi \rightarrow 0$$



Komissarov et al. 2007

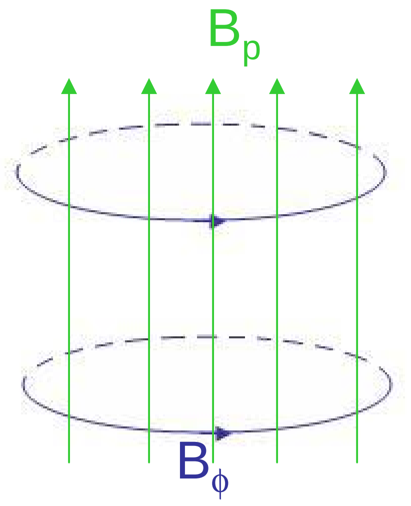
In the Poynting dominated outflows, the energy flux has a hollow cone distribution

Transverse force balance in cylindrical configuration.

1. v=0; E=0

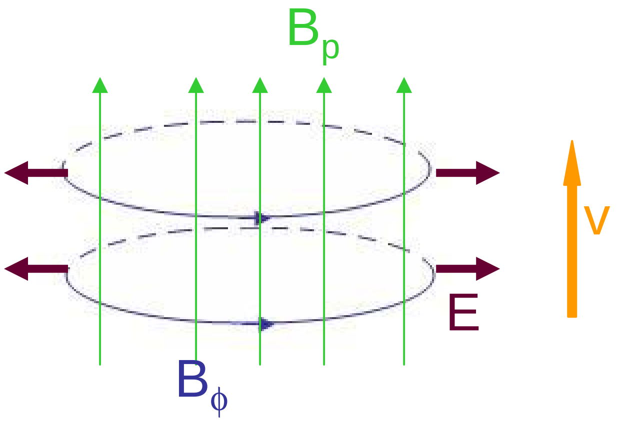
$$\frac{dB_p^2}{dr} + \frac{1}{r^2}\frac{d\left(rB_\varphi\right)^2}{dr} = 0$$

$$B_\phi \sim B_p$$



2. v →c; $B'_p = B_p$; $B'_\phi = \gamma B_\phi$; $E' = (v/c)\gamma B_\phi$

$$E' \approx B'_\phi >> B'_p$$



Relativistic MHD in the far zone, Ωr>>c

Vlahakis 2004; Tchekhovskoy, McKinney & Narayan 2008;
Komissarov, Vlahakis, Konigl & Barkov 2009; Lyubarsky 2009

At $\Omega r >> c, \quad E, B_\phi >> B_p$

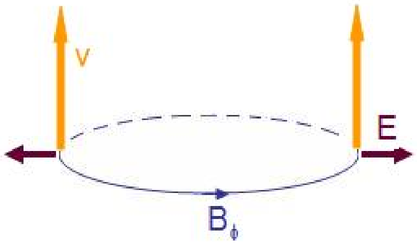
Transverse force balance, $G\left(\Omega r, \gamma, \Psi, \nabla\Psi, \nabla^2\Psi\right) = 0,$

implies $E^2 \approx B_\phi^2$

Bernoulli equation, $\Upsilon\left(\Omega r, \gamma, \Psi, \nabla\Psi\right) = 0,$

also implies $E^2 \approx B_\phi^2$

Drift $\quad \mathrm{v}_p = \frac{\mathrm{E}}{\mathrm{B}_\phi}\mathrm{c} \to \mathrm{c}$



The set of equations is nearly degenerate in the far zone.
One has to retain small terms.

Asymptotic equations for the flow in the far zone, Ωr>>c.

The procedure: expand Y=0 in r^{-1} and γ^{-1} to find $B^2-E^2=O(r^{-2},\gamma^{-2})$ and then to eliminate B^2-E^2 from G=0.

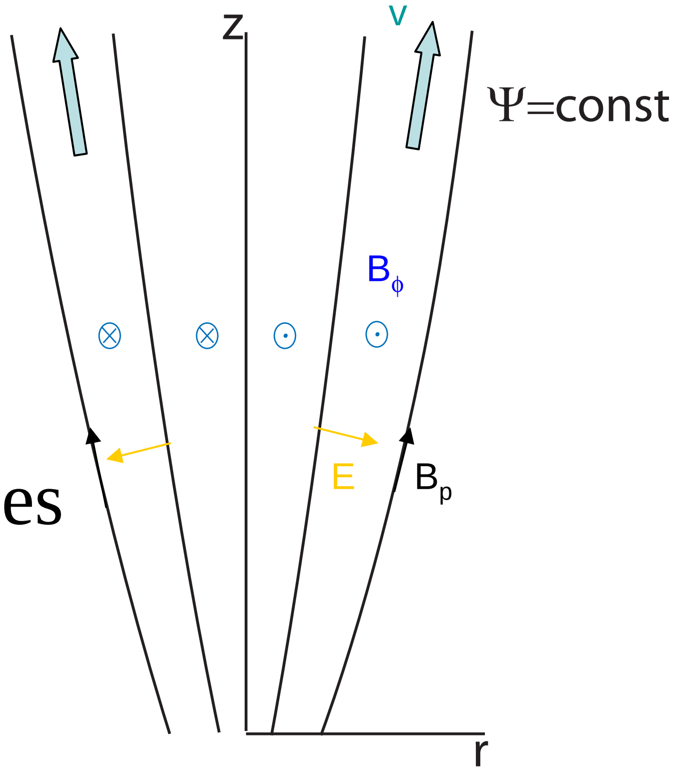
$$2\mu\eta r\left[-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\left(1-\frac{2\gamma_{in}}{\mu}+\frac{\gamma_{in}^2}{\gamma^2}\right)\right]=\left(1+\frac{\gamma_{in}^2}{\Omega^2 r^2}\right)\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

transfield

$$\eta(\mu-\gamma)=\Omega^2 r^2 B_p$$

Bernoulli

$r(\Psi,z)$ - shape of the flux surfaces



Transfield equation in different regimes

$$2\mu\eta r\left[-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\left(1-\frac{2\gamma_{in}}{\mu}+\frac{\gamma_{in}^2}{\gamma^2}\right)\right]=\left(1+\frac{\gamma_{in}^2}{\Omega^2 r^2}\right)\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

close to the axis

$$\frac{2\mu\eta r c^2}{\Omega^2 r^3}\left(1-\frac{2\gamma_{in}}{\mu}+\frac{\gamma_{in}^2}{\gamma^2}\right)=\left(1+\frac{\gamma_{in}^2}{\Omega^2 r^2}\right)\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

far from the axis

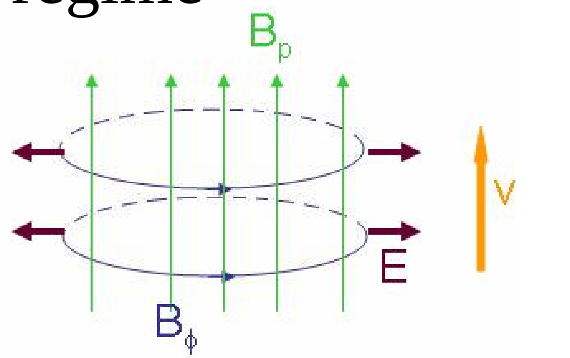
$$2\mu\eta r\left(-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\right)=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

Two collimation regimes

$$2\mu\eta r\left(-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\right)=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

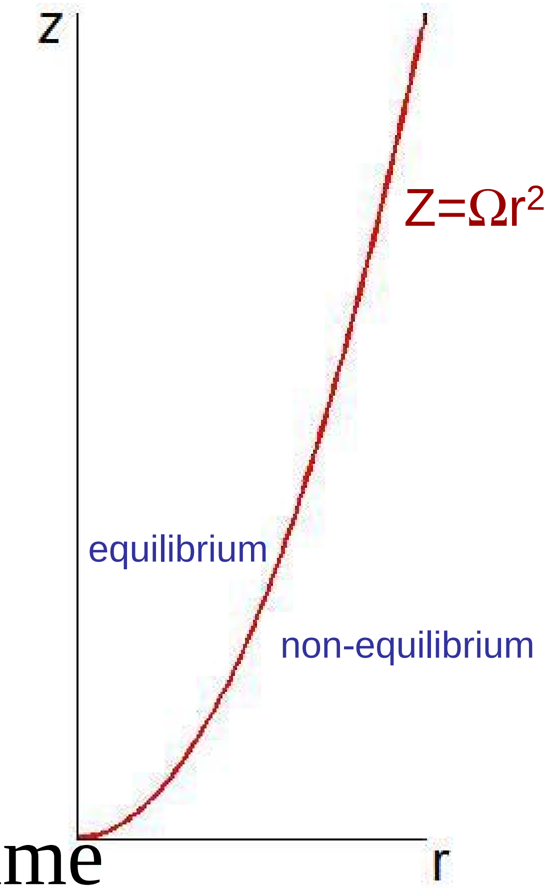
1. $\Omega r^2 << cz$ - equilibrium regime

$$\frac{2\mu\eta c^2}{\Omega^2 r^2}=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$



$$\gamma \sim \Omega r / c$$

cylindrical equilibrium at any z



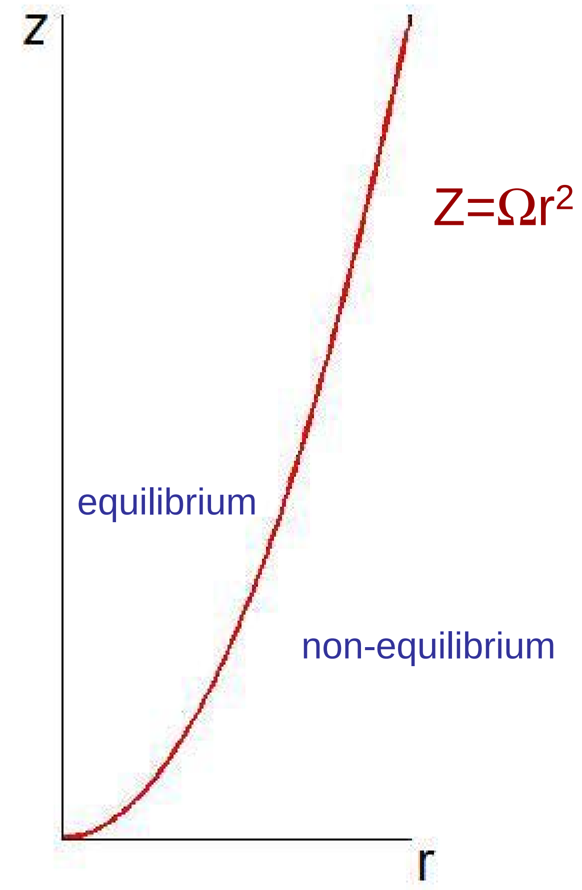
Causality: $t'=\frac{z}{c\gamma}$ - proper propagation time

$$\frac{ct'}{r}=\frac{z}{\gamma r}=\frac{zc}{\Omega r^2}<1$$

$\Theta\gamma<1$; $\Theta=\frac{\mathrm{dr}}{\mathrm{dz}}$ - opening angle

Two collimation regimes (cont.)

$$2\mu\eta r\left(-\frac{\partial^2 r}{\partial z^2}+\frac{c^2}{\Omega^2 r^3}\right)=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

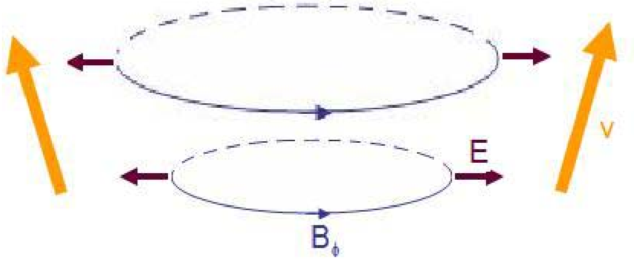


$$2.\ \Omega r^2 >> cz\ \text{- non - equilibrium regime}$$

$$-2\mu\eta r\frac{\partial^2 r}{\partial z^2}=\frac{\partial}{\partial\Psi}\left(\frac{\eta(\mu-\gamma)}{\Omega\gamma}\right)^2$$

$$\gamma\sim\left(r\frac{\partial^2 r}{\partial z^2}\right)^{-1/2}\sim\frac{z}{r}$$

$$\Theta\gamma>1$$



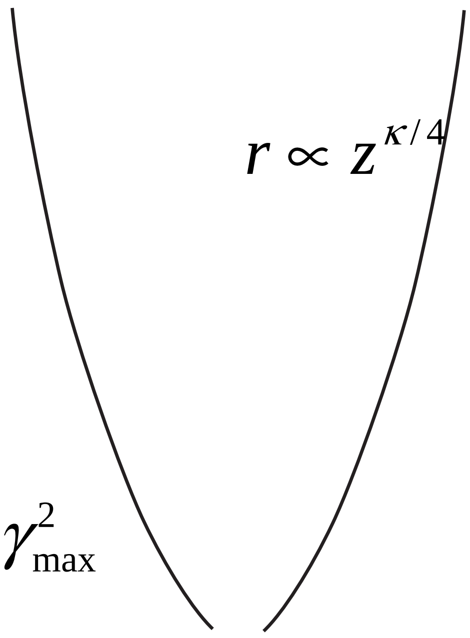
purely azimuthal field, $B_p=0$

Jet confined by the external pressure

$$1.\ p = p_0 (\Omega z / c)^{-\kappa}; \quad \kappa < 2$$

$r \propto z^{\kappa/4}$ equilibrium regime, $r < \sqrt{cz/\Omega}$

$$\gamma \approx \Omega r \propto z^{\kappa/4}$$



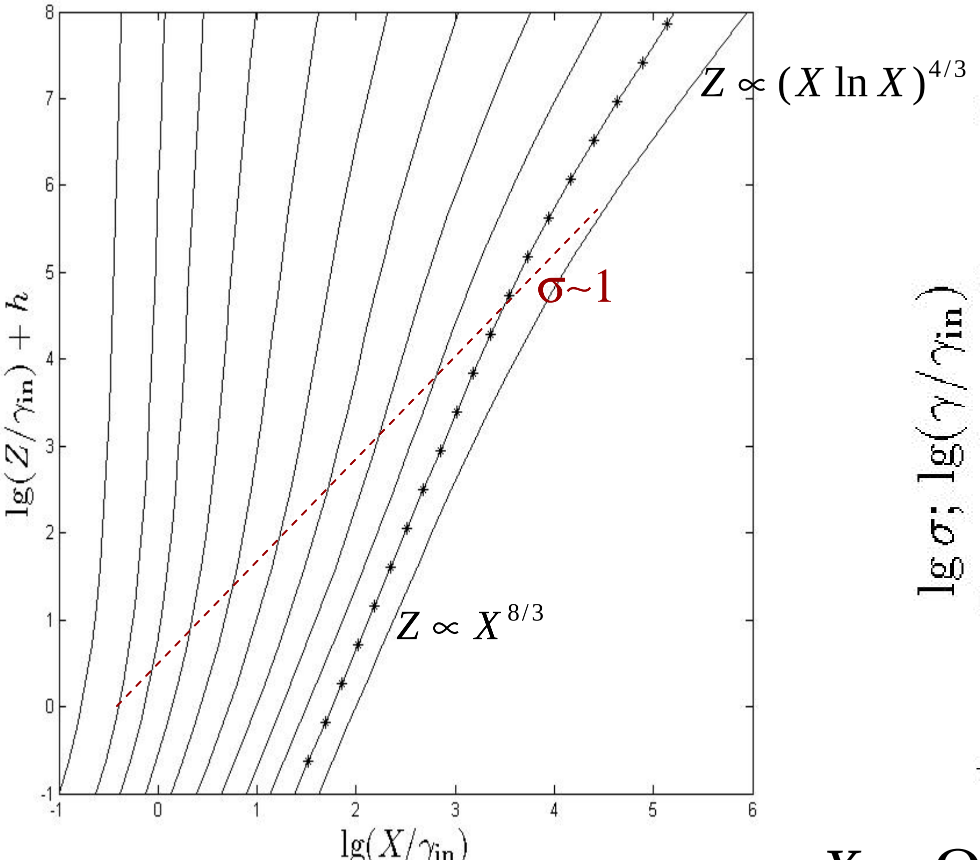
The Poynting flux is converted into the kinetic energy, $\gamma \sim \gamma_{max}$, at the distance

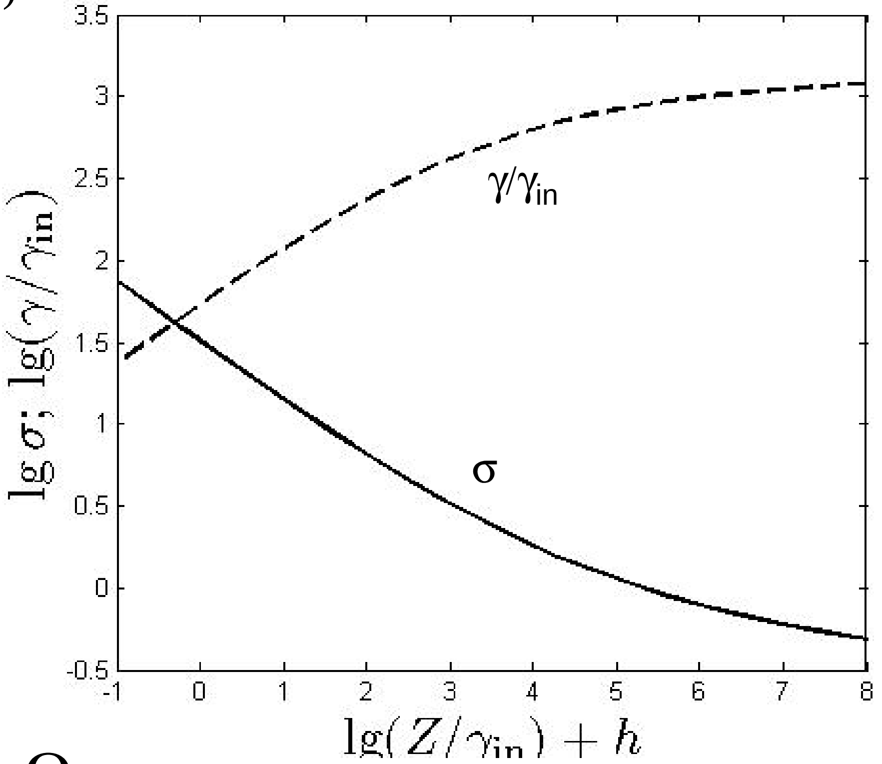
$$(z_0 \Omega / c) \sim \gamma_{max}^{4/\kappa}$$

The fastest acceleration at $\kappa \to 2$; then $(z_0 \Omega / c) \sim \gamma_{max}^2$

Example: in GRBs $\gamma \sim$ few 10^2; minimal $z_0 \sim 10^{11.5}$ cm – marginally OK.
But $\gamma\Theta < 1$, whereas there should be $\gamma\Theta >> 1$ in the prompt phase (achromatic breaks in the afterglow light curves, statistics)

Example: $\Omega = const; \;\; p \propto z^{-3/2}$





$X = \Omega r$

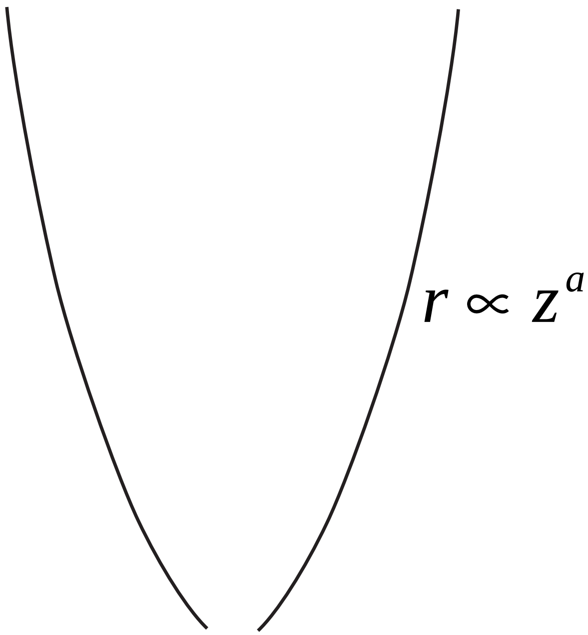
Jet confined by the external pressure (cont)

$$2.\ p = p_0 \left(\Omega z / c\right)^{-2}$$

$$r \propto z^a$$

$$a = 1/2; \quad \beta > 1/4; \qquad \beta = \frac{8\pi p_0}{B_0^2}$$

$$1/2 < a = \tfrac{1}{2}\left(1 + \sqrt{1 - 4\beta}\right) < 1; \quad \beta < 1/4$$



The Poynting flux is converted into
the kinetic energy, $\gamma \sim \gamma_{max}$, at the distance

$$Z \sim \begin{cases} \gamma_{max}^2; & a = 1/2 \\ \gamma_{max}^{1/(1-a)}; & a > 1/2 \end{cases}$$

$$\gamma\Theta \sim 1$$

Jet confined by the external pressure (cont.)

$$3.\ p = p_0 \left(\Omega z / c\right)^{-\kappa}; \quad \kappa > 2$$

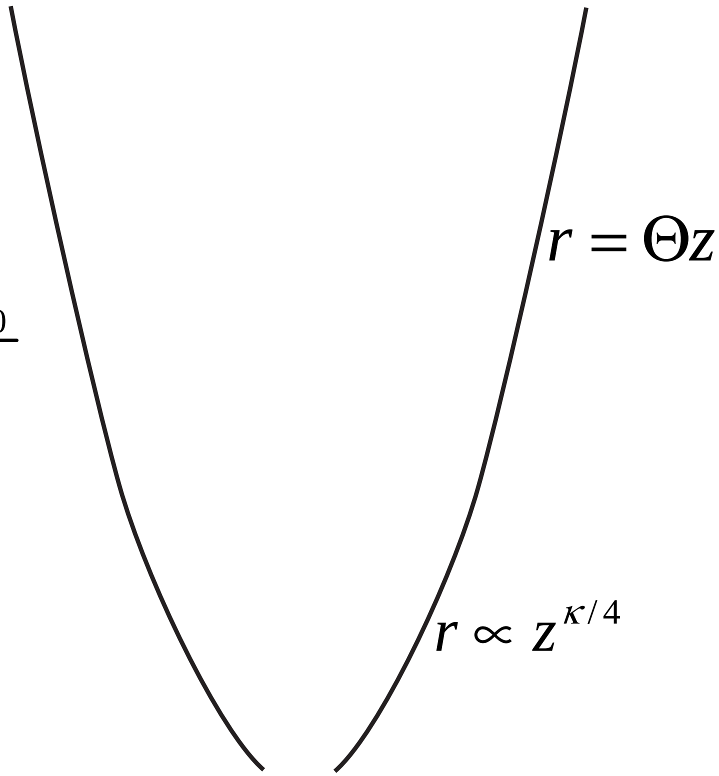
$$r = \Theta z \text{ - conical shape}$$

$$\Theta = \frac{1}{\sqrt{\pi}}\,\Gamma\!\left(\frac{\kappa-1}{\kappa-2}\right)\left(\frac{(\kappa-2)^{\kappa}}{\beta}\right)^{1/[2(\kappa\text{-}2)]}; \quad \beta = \frac{8\pi p_0}{B_0^2}$$

$$\Theta = 0.01/\beta^{2.5} \ \text{ at } \kappa = 2.2$$

$$\Theta = 0.2/\beta \ \text{ at } \kappa = 2.5$$

$$\Theta = 0.56/\sqrt{\beta} \ \text{ at } \kappa = 3$$



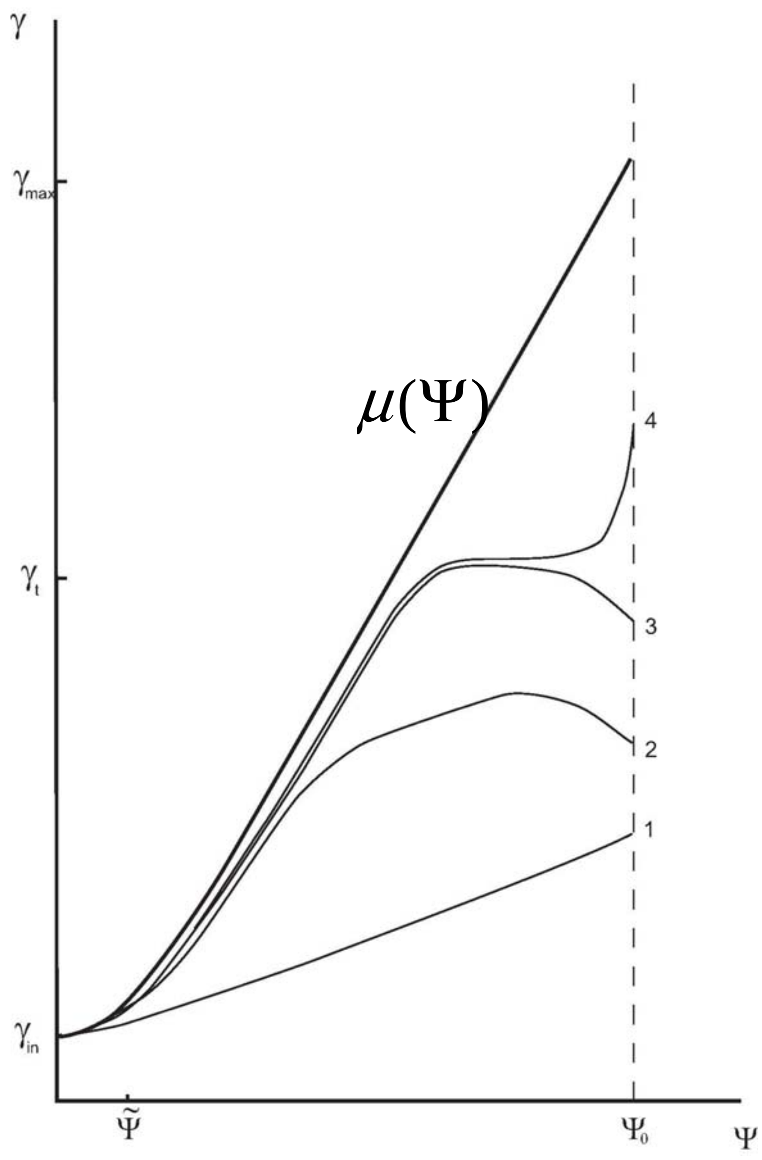
$$3.\ p = p_0(\Omega z/c)^{-\kappa}; \quad \kappa > 2; \text{ continued}$$

$$\gamma \text{ grows until } \gamma_{\rm t} \sim \left(\frac{\gamma_{\max}}{\Theta^2}\right)^{1/3}; \quad \gamma_{\max}\Theta > 1$$ non-equilibrium

$$\gamma_{\rm t} \sim \gamma_{\max}; \quad \gamma_{\max}\Theta < 1$$ equilibrium

$$\gamma_t \Theta \sim (\gamma_{\max}\Theta)^{1/3}$$

In GRBs, $\gamma\Theta>1$ (achromatic breaks etc). This condition is fulfilled only if the flow remains Poynting dominated. Magnetic dissipation is necessary.



$$\Omega = \text{const}$$

Conclusions I (formal)

1. External confinement is crucial for collimation and acceleration of Poynting dominated outflows.

2. For $p_{ext} \propto z^{-\kappa}$

$\kappa < 2$: the flow becomes asymptotically cylindrical;

$$\gamma_t \sim \gamma_{\max}$$

$2 < \kappa < 3$: the flow asymptotically conical

$$\gamma_t \sim \gamma_{\max}^{1/3} \Theta^{-2/3}; \quad \Theta = \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{\kappa - 1}{\kappa - 2}\right) \left(\frac{(\kappa - 2)^{\kappa}}{\beta}\right)^{1/[2(\kappa-2)]}$$

$\kappa > 3$: wide outflow

$$\gamma_t \sim \gamma_{\max}^{1/3}$$

3. Efficient acceleration (up to $\gamma \sim \gamma_{max}$) is possible only in causally connected flows ($\gamma\Theta < 1$).

4. The acceleration zone spans a large range of scales.

Conclusions II (informal)

Even though efficient transformation of the Poynting into the kinetic energy is possible in principle, the conditions are rather restrictive. It seems that in real systems, some sort of dissipation (reconnection) is necessary in order to utilize fully the electromagnetic energy.