The Structure of Relativistic Shock Precursors and Its Implications for Gamma-Ray Bursts

M. Milosavljevic, E. Nakar (2006), ApJ 651, 979

S. M. Couch, M. Milosavljevic, E. Nakar (2008), ApJ 688, 462

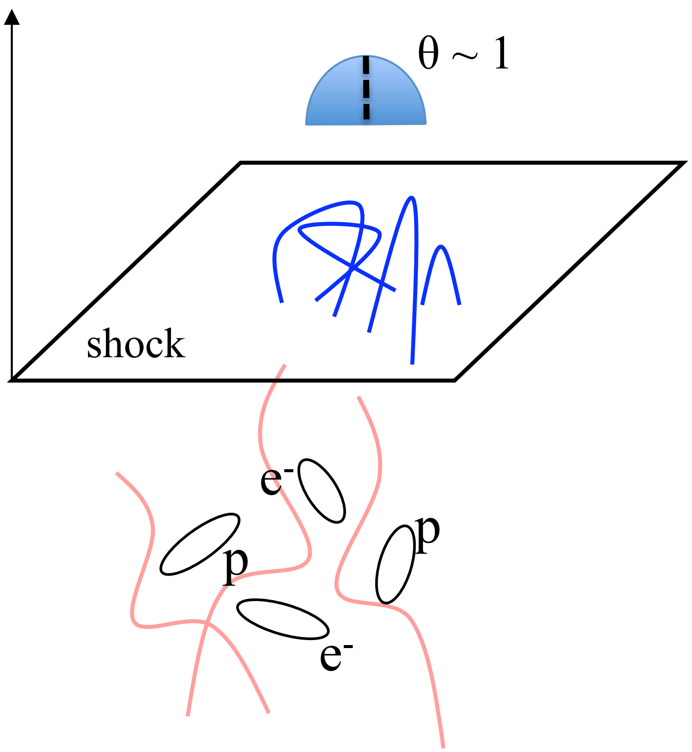
Gamma-Ray Burst Afterglows and the Problem of Magnetic Field Amplification

- A collisionless relativistic blast wave that has Lorentz factor $\Gamma \sim 1-10^3$ and isotropic equivalent energy $E_{tot} \sim 10^{52} - 10^{54}$ ergs.
- GRB afterglow spectra suggest that the synchrotron emitting region contains magnetic energy fraction $\varepsilon_B \sim 10^{-2} - 10^{-4}$.
- The pre-existing, unshocked plasma contains a weak initial magnetic field characteristic of the interstellar medium, e.g., magnetic/rest energy $\varepsilon_B \sim 10^{-9}$.
- A process amplifying the magnetic field in the upstream *or* the downstream by a factor of about $\times 10^5$ should operate.
- The amplified field should persist over $> 10^6$ proton skin depths in the shock downstream.
- Skin-depth scale fields are rapidly damped. Ideally, a process will produce a field on scale much larger than the skin depth.

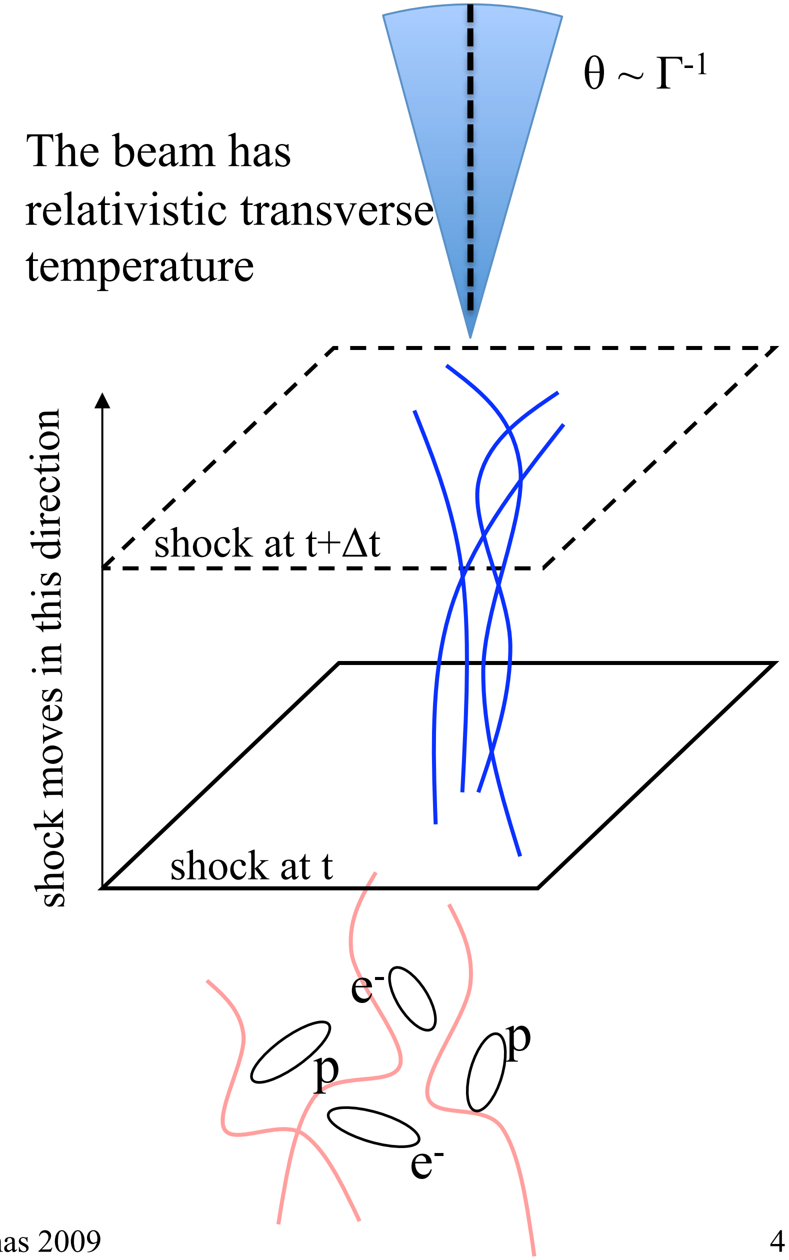
Assumption

- DSA operates in the collisionless relativistic blast wave with Lorentz factor $\Gamma >> 1$ and accelerates protons and electrons, such that at least one of the following is true:
  - Electron acceleration is less efficient than ion acceleration, e.g., $E_{p,max} >> E_{e,max}$ or $N_p(E_{max}) >> N_e(E_{max})$
  - Electron acceleration is as efficient as ion acceleration, but a radiation field, with energetics comparable to that of the blastwave, is present.

The Shock Frame



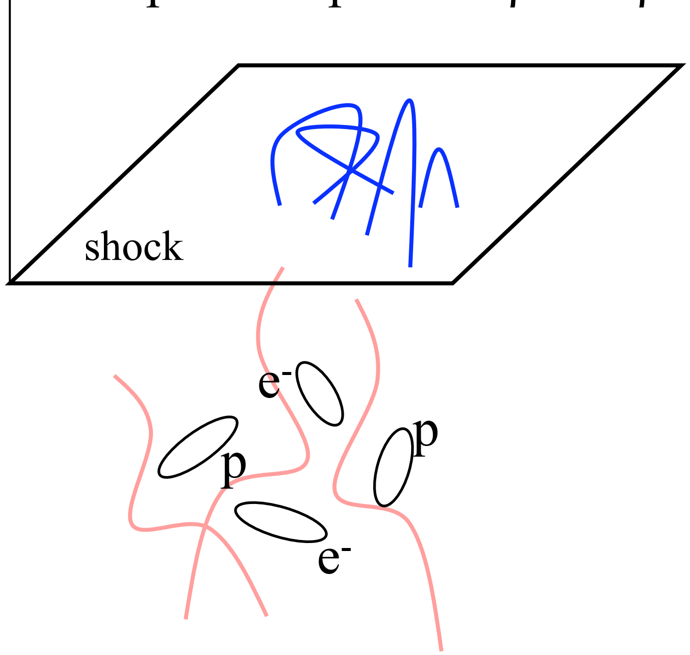
The Upstream Frame



The Shock Frame

High energy protons: ρ^{+}

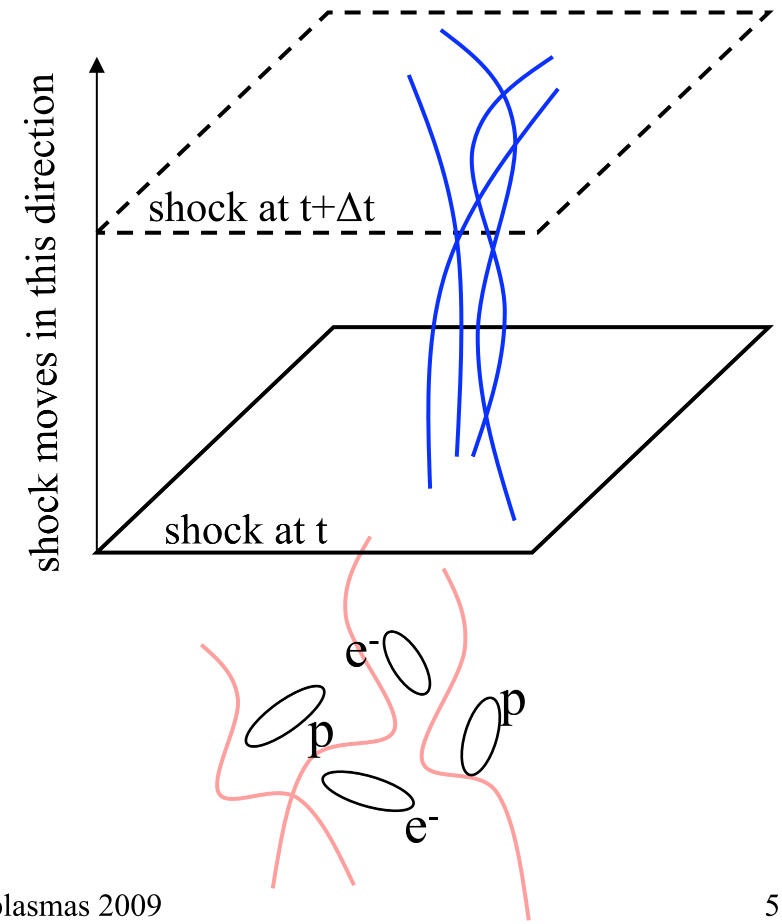
Upstream plasma: $\rho^{-} = -\rho^{+}$

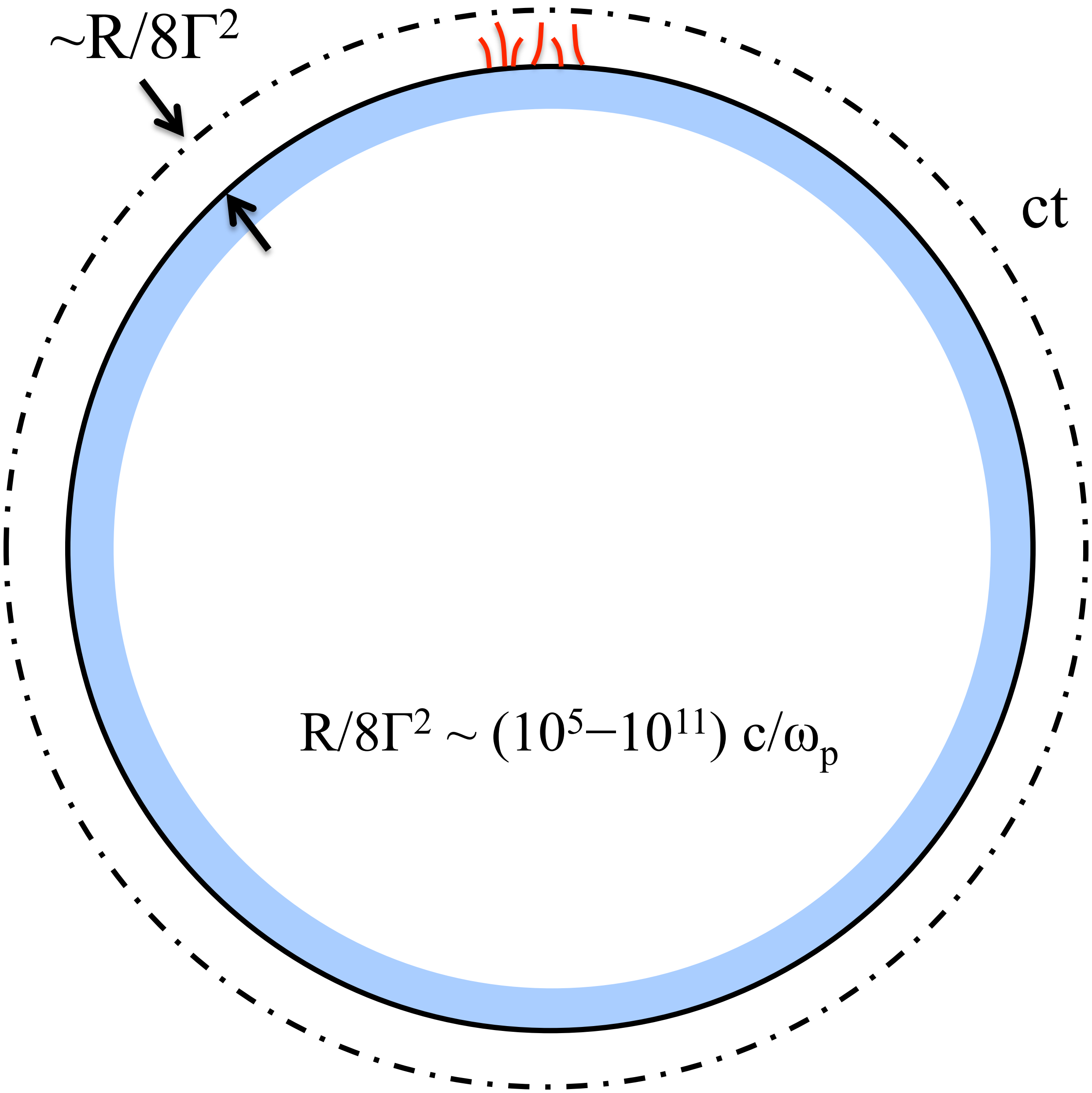


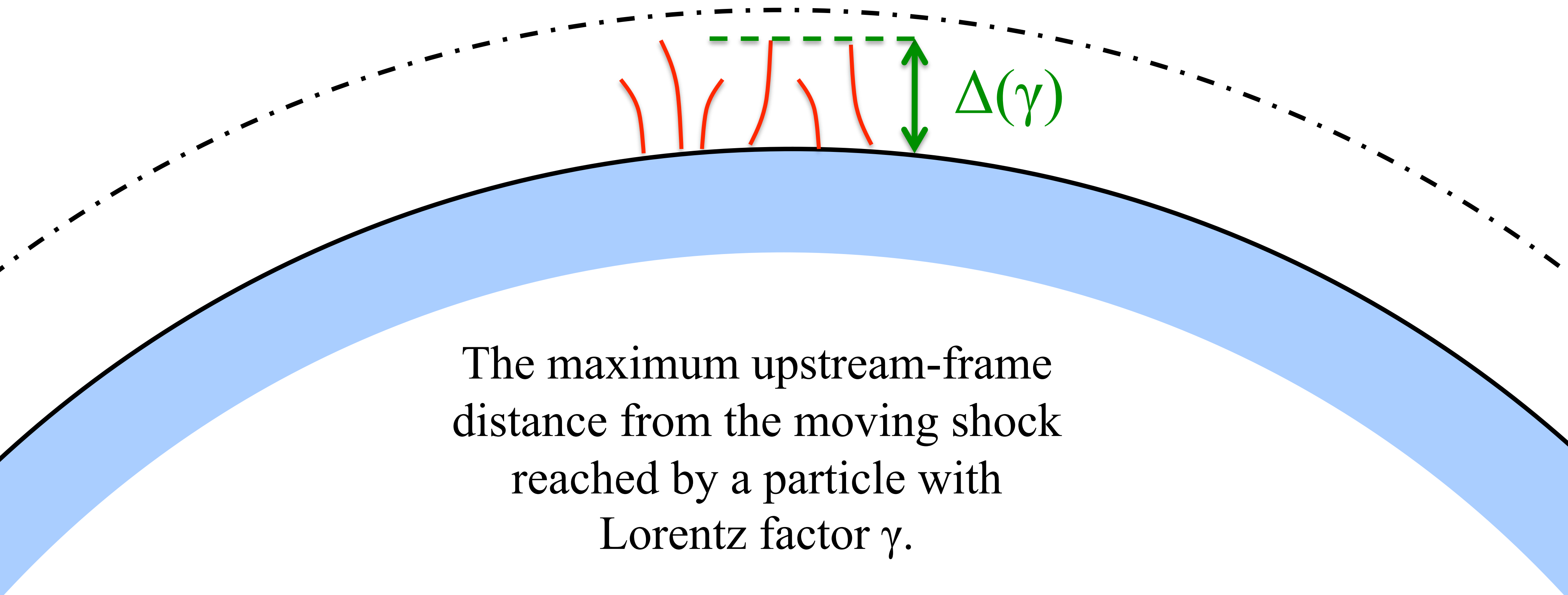
The Upstream Frame

High energy protons: j^{+}

Upstream plasma: $j^{-} = -j^{+}$ (return current)




~R/8Γ²
ct
R/8Γ² ~ (10⁵–10¹¹) c/ω_p


Δ(γ)
The maximum upstream-frame
distance from the moving shock
reached by a particle with
Lorentz factor γ.

Confinement of Particles to the Shock

- Particles are deflected by:
  - The pre-existing field of the shock upstream
  - A field amplified in the shock precursor
- The distance in the upstream frame that particles reach from the shock depends on the field geometry
  - If the field is uniform on scales r_g/Γ, where $r_g = \gamma mc^2/eB$, the distance is

    $\Delta(\gamma) \sim r_g/\Gamma^3$

  - If the field is tangled on scales r_g/Γ, the deflections are diffusive, and the distance is

    $\Delta(\gamma) \sim r_g^2/\lambda_{def}\Gamma^4$

- The maximum distance that any particle can reach is the distance between the blastwave and the associated light pulse is

  $\Delta(\gamma_{max}) \sim R/8\Gamma^2$

  - This condition limits γ_{max} for protons, e.g., for a coherent field:

$$\gamma_{p,\max} \sim 7\times 10^5 \left(\frac{\bar{B}_{\rm def}}{10^{-6}}\right)\left(\frac{E_{\rm tot}}{10^{53}\ {\rm erg}}\right)^{1/3}\left(\frac{\Gamma}{100}\right)^{1/3}\left(\frac{n}{1\ {\rm cm}^{-3}}\right)^{1/3}$$

The Compton Cooling of Electrons

- Assuming that fraction ε_{rad} of the blastwave energy in radiation, the radiation energy density equals, in the shock frame,

  $U_{sh} \sim \varepsilon_{rad} E_{tot} / R^3$

- Equating the IC power

  $\Delta E/\Delta t \sim - 4/3\ \sigma_T\, c\, (\gamma_e/\Gamma)^2\, U_{sh}$

  to the energy gain per DSA cycle

  $\Delta E \sim (\gamma_e/\Gamma)\, m_e\, c^2$

  of duration

  $\Delta t \sim \Gamma\, \Delta_e(\gamma_e)/c$

  one obtains

Magnetic field dependence is encapsulated in $\gamma_{p,max}$

$$\gamma_{e,\max} \sim \frac{4.4 c^{1/3} m_p^{1/2} \gamma_{p,\max}^{1/2} \Gamma^{1/3}}{\epsilon_{\rm rad}^{1/2} \bar{\rho}^{1/3} E_{\rm tot}^{1/6} \sigma_{\rm T}^{1/2}} \quad \text{(coherent field)},$$

$$\sim \frac{2.7 c^{2/9} m_p^{2/3} \gamma_{p,\max}^{2/3} \Gamma^{2/9}}{\epsilon_{\rm rad}^{1/3} \bar{\rho}^{2/9} E_{\rm tot}^{1/9} m_e^{1/3} \sigma_{\rm T}^{1/3}} \quad \text{(tangled field)},$$

$$\frac{\Delta_e(\gamma_{e,\max})}{\Delta_p(\gamma_{p,\max})} \sim \left(\frac{\gamma_{p,\max}}{460}\right)^{-1/2} \left(\frac{\epsilon_{\rm rad}}{0.1}\right)^{-1/2} \left(\frac{E_{\rm tot}}{10^{53}\ {\rm erg}}\right)^{-1/6} \times \left(\frac{\Gamma}{100}\right)^{1/3} \left(\frac{n}{1\ {\rm cm}^{-3}}\right)^{-1/3} \text{ (coherent field)},$$

$$\sim \left(\frac{\gamma_{p,\max}}{440}\right)^{-2/3} \left(\frac{\epsilon_{\rm rad}}{0.1}\right)^{-2/3} \left(\frac{E_{\rm tot}}{10^{53}\ {\rm erg}}\right)^{-2/9} \times \left(\frac{\Gamma}{100}\right)^{4/9} \left(\frac{n}{1\ {\rm cm}^{-3}}\right)^{-4/9} \text{ (tangled field)},$$

Thus for $\gamma_{p,max} >> 10^3$, electrons are confined closer to the shock than the protons. The nonthermal precursor is charged, and a return current must flow in the upstream frame.

Nonthermal Proton Density

- The spectrum of nonthermal particles in the upstream is harder than that in the downstream

  $N^{(up)}(\gamma) \sim (\gamma/\gamma_{max})^s N^{(down)}(\gamma)$

  where s = 1 for coherent field, s = 2 for tangled field.

- We have made the simplifying assumption that the character of the field (coherent vs. tangled, etc.) is uniform on all scales.

- For an $dN/dE \sim E^{-2}$ spectrum in the downstream, the density of nonthermal protons of any energy expressed in terms of the faction x of the distance than any particle can travel ahead of the shock [$x = \Delta/(R/8\Gamma^2)$] then equals

$$n_{\rm ntp}(x) \sim \frac{0.3}{x^{1/s}} \left(\frac{\gamma_{p,\rm max}}{10^6}\right)^{-1} \left(\frac{\Gamma}{100}\right)^4 \left(\frac{\bar{n}}{1\ {\rm cm}^{-3}}\right) \left(\frac{\epsilon_{\rm nt}}{0.1}\right)$$

Return Current and the Ampère Force

- Assuming that $n_{ntp}(\Delta) << n_{up}$, the return current is proportional to the nonthermal proton density

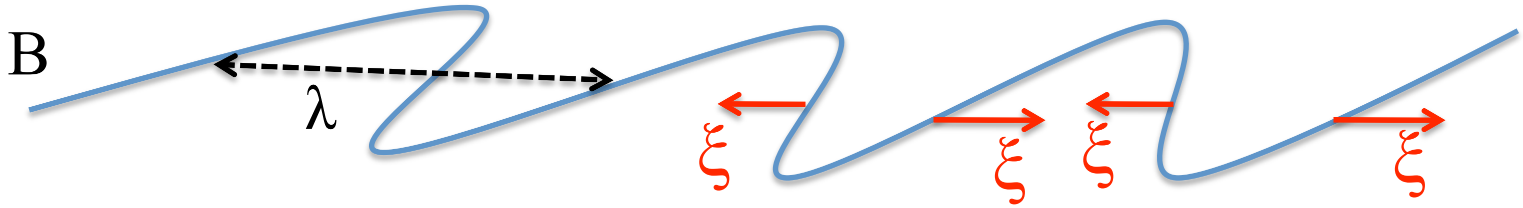
  $J_{ret}(\Delta) = -ecn_{ntp}(\Delta)$

- Consider a fluid element with magnetic field **B** on scales λ *with a perpendicular component*. This may not be the same field that is responsible for confining particles to the shock.

- The Newtonian equation of motion of the fluid element's displacement reads

  $$\frac{d^2\boldsymbol{\xi}}{d\tau^2} = \frac{\boldsymbol{J}_{\mathrm{ret}}(\tau) \times \boldsymbol{B}}{\rho c}$$

  where τ is the time since the arrival of the light shell



$$\xi \sim \frac{R}{8\Gamma^2}\left(\frac{\gamma_{p,\max}}{1.6\times 10^6}\right)^{-1}\left(\frac{\epsilon_B}{10^{-9}}\right)^{1/2}\left(\frac{E_{\rm tot}}{10^{53}\ {\rm erg}}\right)^{1/3}\left(\frac{\epsilon_{\rm nt}}{0.1}\right)\left(\frac{\Gamma}{100}\right)^{4/3}\left(\frac{\bar{n}}{1\ {\rm cm}^{-3}}\right)^{1/6} g(x)\sin\phi.$$

The displacement results in nonlinear fluid perturbation on scales λ if $\xi > \lambda$. Of course it should be checked that the transverse motion does not become relativistic. If the fluid motion comes close to becoming relativistic, an alternate expression can be derived.

For $\gamma_{p,max}$ that is neither too small nor too large, e.g., ~ 10^6, the maximum displacement $\xi_{max} \sim R/8\Gamma^2$ is many times, e.g., ~ 10^5 larger than the upstream proton skin depth. On these scales, the plasma dynamics should be in the MHD regime.

The Origin of the Reversing Field

- Consider a coherent field *with a parallel component*, such that the system is linearly unstable to the nonresonant mode.
- The fastest growing mode has wavelength (Bell 2004, 2005)

  $\lambda_{fast} \sim B_{par} / e\, n_{ntp}$

  which is microscopic.
- The growth rate of all longer wavelengths equals

  $\Upsilon(\lambda) \sim e\, n_{ntp} (n\, m_p)^{-1/2} (\lambda/\lambda_{fast})^{-1/2}$.
- A mode has time to grow if

  $\Upsilon(\lambda) >> (R/8\Gamma^2 c)^{-1}$.

Bell 2005

The Origin of the Reversing Field

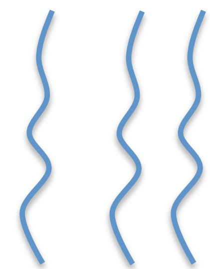
- The wavelengths that become nonlinear have

$$\lambda \ll \frac{R}{8\Gamma^2}\left(\frac{\gamma_{p,\max}}{4.6\times10^6}\right)^{-1}\left(\frac{\epsilon_{B_0}}{10^{-9}}\right)^{1/2}\left(\frac{\epsilon_{\rm nt}}{0.1}\right)\left(\frac{\Gamma}{100}\right)^{4/3} \times\left(\frac{n}{1\ {\rm cm}^{-3}}\right)^{1/6}\left(\frac{E_{\rm tot}}{10^{53}\ {\rm erg}}\right)^{1/3},$$

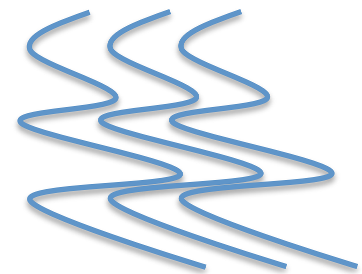
which suggests that nonlinear growth is broadly expected on scales $\lambda < R/8\Gamma^2$ when $\gamma_{p,max}$ is such that the fluid does not attain relativistic velocity when pushed by the Ampère force in the transverse direction. When $\gamma_{p,max}$ is smaller than the critical value for relativistic acceleration, the nonlinear growth is still expected on scales $c/\omega_p << \lambda << R/8\Gamma^2$.

The Scenario for Interaction of Accelerated Protons with the Shock Upstream

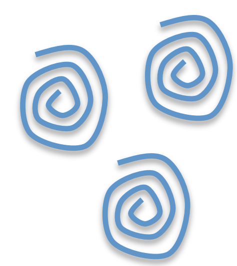
1. If the weak, seed field is tangled on small scales (much smaller than the light crossing time of the precursor, but still much larger than the skin depth), skip to step 3.



2. If the weak, seed field is uniform on small scales, the linear nonresonant mode grows until it produces a comparable field that is tangled on small scales.
3. As in Bell 2004, 2005, Niemiec et al. 2008, Riquelme & Spitkovsky 2009, Ohira et al. 2009, etc., the Ampère force pushes the small-scale field in the transverse direction and in the process generates compressible perturbations and vorticity in the shock upstream.



4. Provided that one is safely in the MHD regime, the upstream vorticity winds up the magnetic field further and perhaps leads to some cascading to even smaller scales.



Potential (?) Implications

- Some upstream heating and backreaction on the bulk upstream flow.
- Magnetic field amplification through the winding of a pre-existing field.
- Generation of vorticity in the shock upstream and a cascade to small scales.
- If the field is amplified, this allows acceleration of confinement of protons closer to the shock and acceleration to higher energies, but this weakens the return current far from the shock and so the process may be self-limiting.
- The transverse acceleration of the upstream plasma produces strongly nonlinear inhomogeneities, that, when being swept by the shock, can generate downstream vorticity (e.g., Goodman & MacFadyen 2007, Sironi & Goodman 2007) that may wind up the magnetic field further. The expected vortical energy density is $\varepsilon_{\rm vort} \sim \Gamma^{-1}$, consistent with the magnetic energy density inferred from the observations.

UHECR from GRB External Shocks?

- The maximum length scale of the amplified field allowed by causality is $\sim R/8\Gamma^2$.
- If the energy density in the amplified field could be as large as the rest energy density of the upstream, $\sim n_{up} m_p c^2$, then the particles would be accelerated to energy

  $$\gamma_{p,max} < 10^{10}\, n^{1/2}\, (R/10^{18}\mathrm{cm})$$

  just short of what is required.
- However, as $\gamma_{p,max}$ increases above 10^6, the maximum length scale on field amplification is possible through the current-driven mechanism decreases below $\sim R/8\Gamma^2$. Thus, in fact,

  $$\gamma_{p,max} <<< 10^{10}\, n^{1/2}\, (R/10^{18}\mathrm{cm})$$

  and UHECR cannot be produced in the external shock.

Skeletons in the Closet

- Because of its transverse thermal spread, the nonthermal precursor should be stable to filamentation as long as

  $n_{ntp,rest} / n_{up,rest} << \beta_{perp}^2 \sim \Gamma^{-2}$ (Silva et al. 2002)

- The precursor may be unstable to other kinetic modes, such as the "oblique mode". The relevance will depend on the timescales on which these modes grow and their saturation amplitude.
- If the streaming-driven field amplification and/or heating is particularly effective on intermediate scales $c/\omega_p << \lambda <<< R/8\Gamma^2$, this may produce quench filamentation and produce a magnetized shock.