Cosmic-ray Acceleration and Current-Driven Instabilities

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Outline

Analysis of yesterday's football match

Particle acceleration at non-relativistic shocks

Field amplification - streaming instabilities

Transport properties of test particles in amplified field

Cosmic ray modified shocks & Free escape boundaries

DSA - The diffusion approximation

- Transport equation

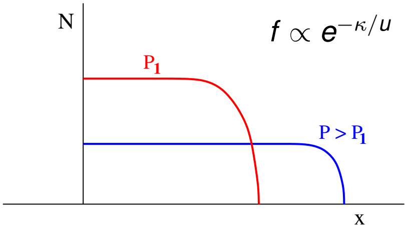
$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}\left(uf - \kappa(x,p)\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial p^3}\left(p^3 f \frac{\partial u}{\partial x}\right)$$

- Steady-state test particle solution:

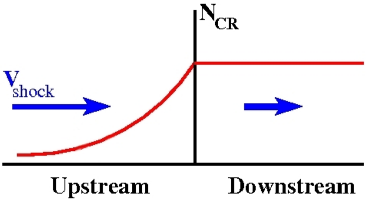
$$f_+(p) = ap^{-q} + q\int_0^p \frac{\mathrm{d}p'}{p}\left(\frac{p'}{p}\right)^q f_-(p')$$

where $q = 3r/r - 1$

- $N(E) \sim E^{-2}$ for strong shocks - consistent with GCR spectrum allowing for energy-dependent propagation
- Shape of the spectrum is independent of κ.



Wave modification supernova shock precursors



- CRs stream at $\approx$ shock speed
- Streaming instability (resonant Alfvén waves neglecting dissipation)

$$\frac{\partial U_a}{\partial t} + u\frac{\partial U_a}{\partial x} = v_a \frac{\partial P_{cr}}{\partial x}$$

- steady-state solution:

$$\left(\frac{\delta B}{B}\right)^2 = M_a \frac{P_{cr0}}{\rho u^2}$$

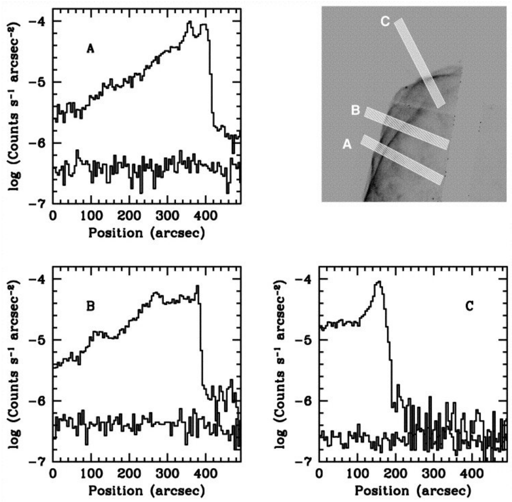
(Lucek & Bell '00, Bell & Lucek '01)

- non-linear McKenzie & Völk '81

$$\left(\frac{\delta B}{B}\right)^2 = \frac{M_a(1-y^2)}{2y(\sqrt{y} - 1/2M_a)}$$

see also Drury '83

Observational evidence of MFA?? (Long et al. 2003)



$$L_{min} \equiv \kappa/u \sim 10^{19} \gamma_{keV}^{1/2} B_{3\mu G}^{-3/2} u_{10^8}^{-1} \mathrm{cm}$$

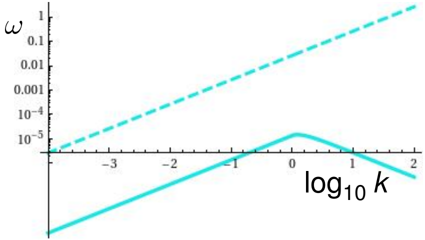
Chandra resolution at 2 kpc $\sim 3 \times 10^{16}$ cm

Cosmic-ray current driven instability (Bell 2004)

- 3 component plasma - thermal electrons, protons + non-thermal protons (CRs)
- non-thermal protons provide current - $J_{cr} \approx e n_{cr} v_{sh}$
- zero net current and charge

$$\sum q_s n_s \mathbf{v}_s = 0, \qquad \sum q_s n_s = 0$$

$$\nabla \times \mathbf{B} = \sum_{\alpha} J_{\alpha} = J_{\mathrm{MHD}} + J_{cr}$$



- Linearizing the MHD/kinetic equations results:

$$\omega^2 = v_{\mathrm{A}}^2 k^2 \pm \frac{B_0 J_{cr}}{\rho_0 c} k[1 - S(k)]$$

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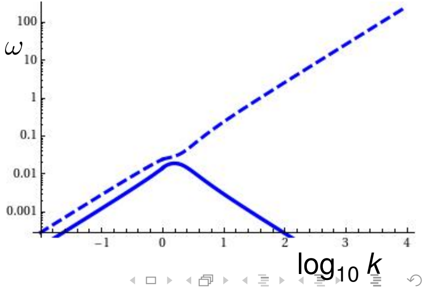

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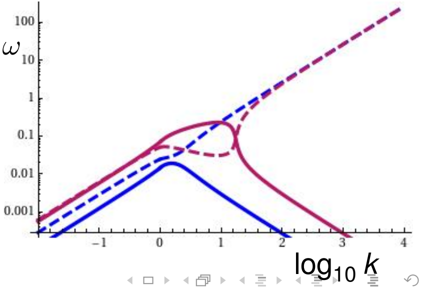

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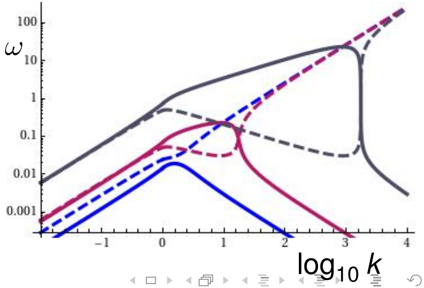
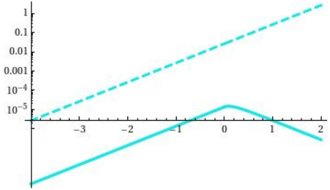


Resonant Mode $kr_{g0} < 1$- linear theory



$$\frac{P_{cr}}{\rho v_s^2}\frac{v_s}{c}M_A^2 \ll 1$$

- waves are unmodified, i.e. $\omega_r = v_A k$
- growth rate given by ion-cyclotron resonance

$$s(k) \approx 1 + \mathrm{i}\frac{3\pi}{16}kr_{g0}$$

- $$\Gamma = \frac{\sqrt{9\pi^3}}{16}\frac{e}{\rho_0^{1/2}c}n_{cr}(p > p_{res})\left[< v > -\frac{1}{3}\sigma A\right]$$

where $p_{res} = eB/ck$, $\sigma = -\frac{\partial \ln f}{\partial \ln p}$
Kulsrud & Pearce, Melrose & Wentzel, Cesarsky etc.

Non-resonant mode - Growth and saturation

- Condition for non-res instability $\zeta M_A^2 \gg 1$
- Maximum growth rate

$$\Gamma_{max} = \frac{1}{2}\zeta M_A \frac{v_s}{r_g} \quad \text{where} \quad \zeta = \frac{n_{cr} p_{res}}{n_0 m_p v_s}$$

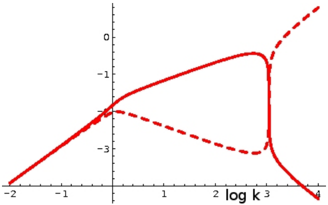
- saturation when currents associated with waves:

$$|\mathbf{k} \times \delta\mathbf{B}| \approx 4\pi n_{cr} e \beta_{sh}$$

- $k \sim 1/r_g$ - saturated field energy

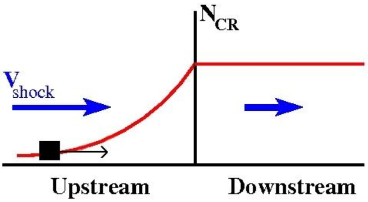
$$\frac{B_w^2}{8\pi} \sim U_{cr}\beta_{sh} = \eta\rho u_{sh}^2 \beta_{sh}$$

- fast, efficiently accelerating shocks ideal for magnetic field amplification
- What do the numerical simulations tell us?



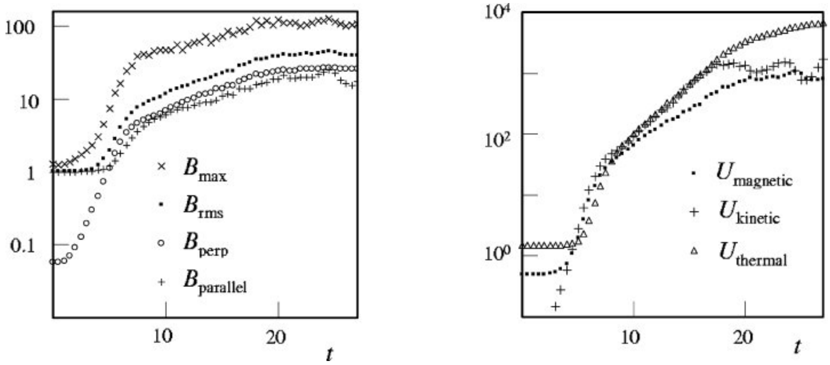
Numerical simulations

- quasi-linear steady-state solution
- CRs stream at $\approx$ shock speed
- simulation box $L_{box} \ll \kappa(p_{min})/u$
- Box is periodic
- $j_{cr}(\mathbf{x}, t) = \text{const.}$
  although see
  Lucek & Bell 2001,
  Zirakashvili et al 2008,
  Niemiec et al. 2008,
  Riquelme & Spitkovsky 2009



- $\mathbf{j}_{cr} \| \mathbf{B}_0$
- $\mathbf{j}_{cr} \cdot \mathbf{E} = \mathbf{u} \cdot (\mathbf{j}_{cr} \times \delta\mathbf{B})$
- once fluid is set in motion, energy pumped into system increases

Numerical simulations-MHD (Bell 2004)



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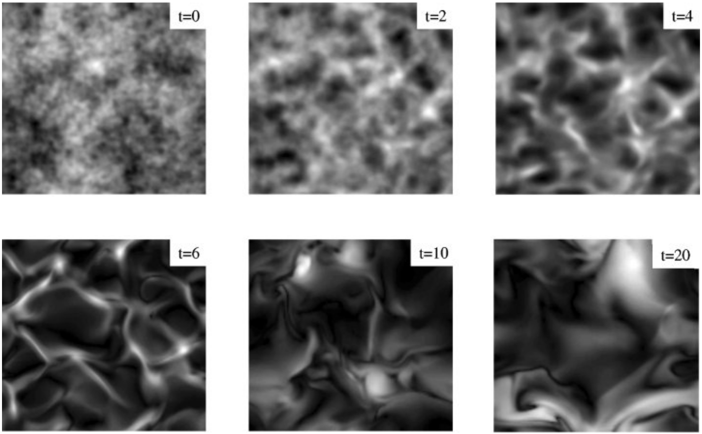
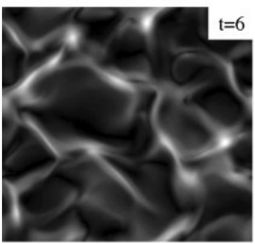
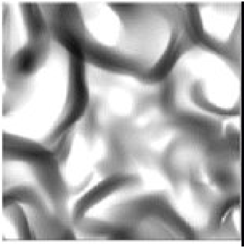
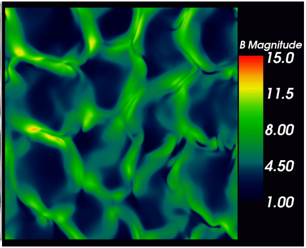


Numerical simulations - Non-linear structure





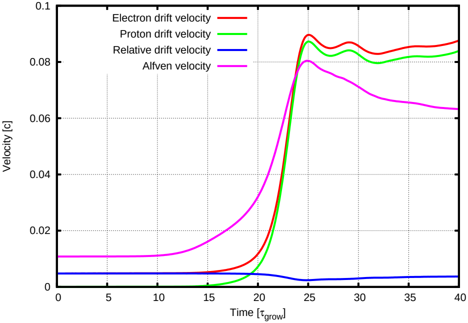


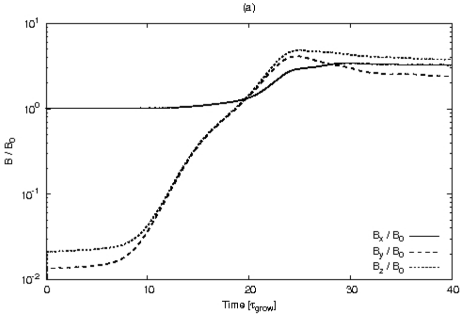
Bell 2004 Zirakashvili et al. 2008 BR et al. 2008

- Structures are approaching scale of simulation box
- However, no bulk acceleration of fluid observed

Particle in Cell simulations

$$v_{d,cr} = 0.1c$$





Ohira, BR, Kirk, Takahara 2009

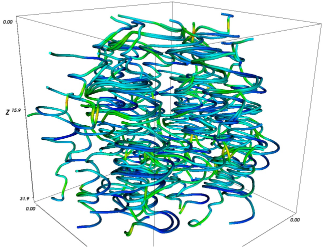
(see also: Niemiec et al. 2008, Riquelme & Spitkovsky 2009)

Test particle transport in amplified field

- Take a snapshot of field in early stages of non-linear development, $kL_{box} \approx 1$

- determine statistical properties of field lines and diffusion

- integrate particle trajectories



$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q\mathbf{v} \times \mathbf{B}$$

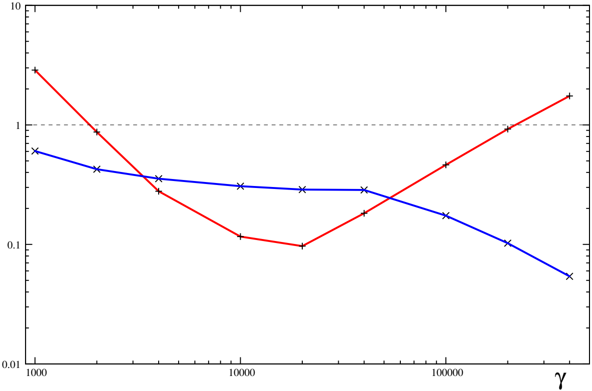
$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}$$

for many particles, random initial positions and directions

Diffusion coefficients

Comparison of diffusion coefficients to Bohm limit in the pre-amplified field $B_0 = 3\mu\text{G}$, $B_{rms} \sim 15\mu\text{G}$

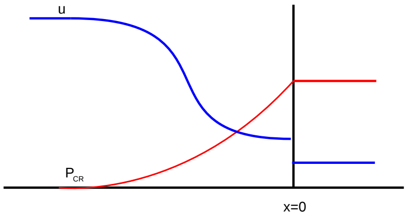
$\kappa_{\parallel}/\kappa_{Bohm}$, $\kappa_{\perp}/\kappa_{Bohm}$



BR, O'Sullivan, Duffy, Kirk (2008) MNRAS

Cosmic-ray modified shocks - Non-linear DSA

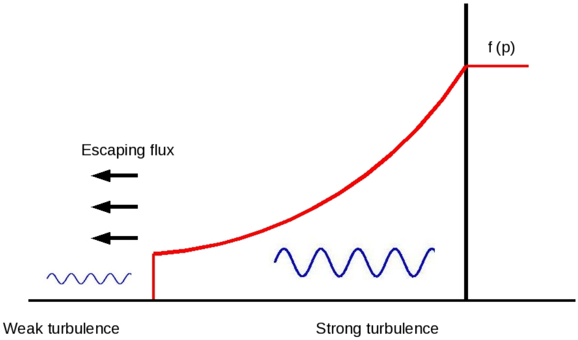
- When CR pressure becomes large we must go beyond test-particle limit



- Cosmic ray pressure gradient decelerates incoming plasma
- Total shock compression ratio $\rho_{ds}/\rho_0 > 4$
- sub-shock compression ration< 4
- larger *effective* compression ratio, hardens spectra at higher energies

Cosmic-ray modified shocks - Free escape boundary

- Scattering waves are self-generated



- At a certain distance upstream, growth of hydromagnetic waves too slow to confine particles - CRs decouple from plasma
- magnetic field growth (amplification) driven in transition region

Steady-state solutions

- coupled hydrodynamic - kinetic equations
- mass & momentum conservation

$$\rho(x)u(x) = \rho_0 u_0,$$
$$P_{\rm cr}(x) + \rho(x)u(x)^2 + P_g(x) = \rho_0 u_0^2 + P_{g,0}$$

- time dependent transport equation

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(\kappa\frac{\partial f}{\partial x}\right) = \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}x}p\frac{\partial f}{\partial p} + Q_0(x,p)$$

- Additional boundary condition $f(L_{\rm esc}) = 0$

See BR, Kirk & Duffy 2009 for details

Steady-state solutions - method of solution

- what is the location of escape boundary - transition zone from weak to strong turbulence

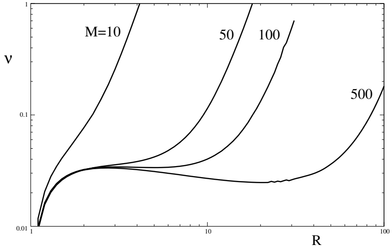
- diffusive current at escape boundary drives growth of waves

$$j_{\mathrm{cr}}(-L_{\mathrm{esc}}) = -4\pi e \int_{p_0}^{\infty} \kappa \frac{\partial f}{\partial x} p^2 \mathrm{d}p \;\; , \quad L_{\mathrm{esc}} = \frac{\kappa(p^*)}{u_0}$$

- nonresonant mode growth dominates provided:

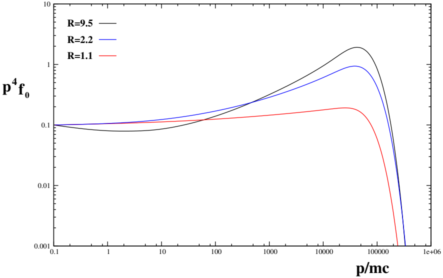
$$\zeta M_A^2 \equiv \frac{j_{cr} p^*}{e \rho_0 u_0^2} M_A^2 > 1$$

Injection efficiency $\nu - R$ diagram



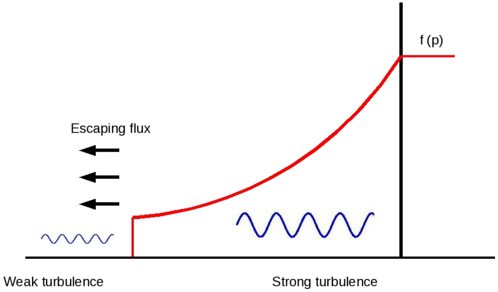
$$u_0 = 5000\text{km s}^{-1},$$
$$n_0 = 1\ \text{cm}^{-3},\ p^* = 10^3$$

$$\nu = \frac{4\pi}{3}\frac{mc^2}{\rho_0 u_0^2} p_0^4 f_0(p_0),$$



- $L_{\rm esc}$ determines max. energy
- How to determine $L_{\rm esc}$ in self-consistant manner?

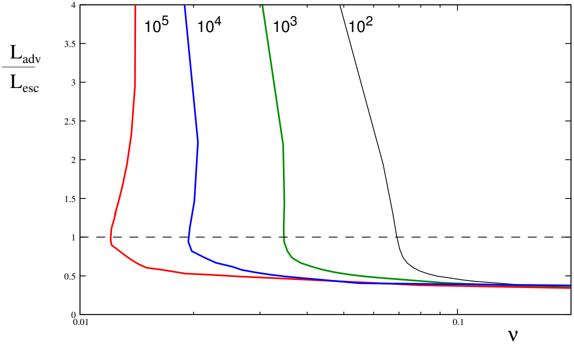
Maximum momentum



- Calculate CR flux (and p^{*}) from numerical SS solution
- Transition zone determined from condition

$$L_{\rm adv} \equiv u_{\rm sh}/\Gamma_{\rm max} \sim L_{\rm esc}$$

Maximum momentum



BR, Kirk & Duffy, ApJ

- Maximum energy can be calculated as a function of injection parameter

Summary

- Magnetic field amplification a natural consequence of efficient DSA
- Non-resonant instability first investigated by Bell likely mechanism
- Non-linear development and saturation still uncertain, bigger & better simulations required
- time asymptotic solutions to modified shock problem and boundary conditions investigated
- particle accelerating shocks appear to be self-organising /self-regulating systems