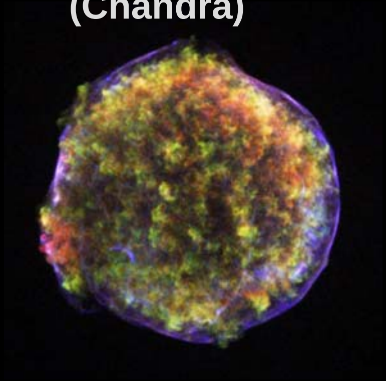
Modeling nonlinear shocks: a Monte Carlo method

Andrey Vladimirov, Don Ellison,
Andrei Bykov*

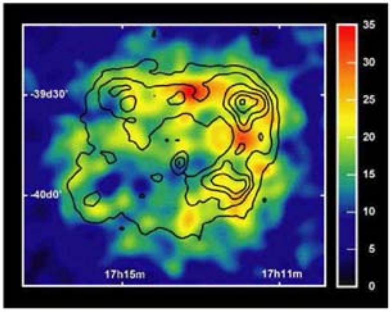
* Names ordered according to the distance from the projector screen

Goals

Tycho's SNR
(Chandra)



RX J 1713-3946
(HESS)



G1.9+0.3
(composite)

- Simulate full-scale nonlinear shocks (efficient acceleration, strong B-fields, high E_{max}), aiming to model SNR shocks
- Eliminate some approximations of kinetic models (e.g., isotropy of f(p) → model injection, oblique and relativistic shocks, large gradients of B-field)
- Self-consistently include important processes (various possible MFA mechanisms, Fermi-II, precursor heating, multiple particle species)

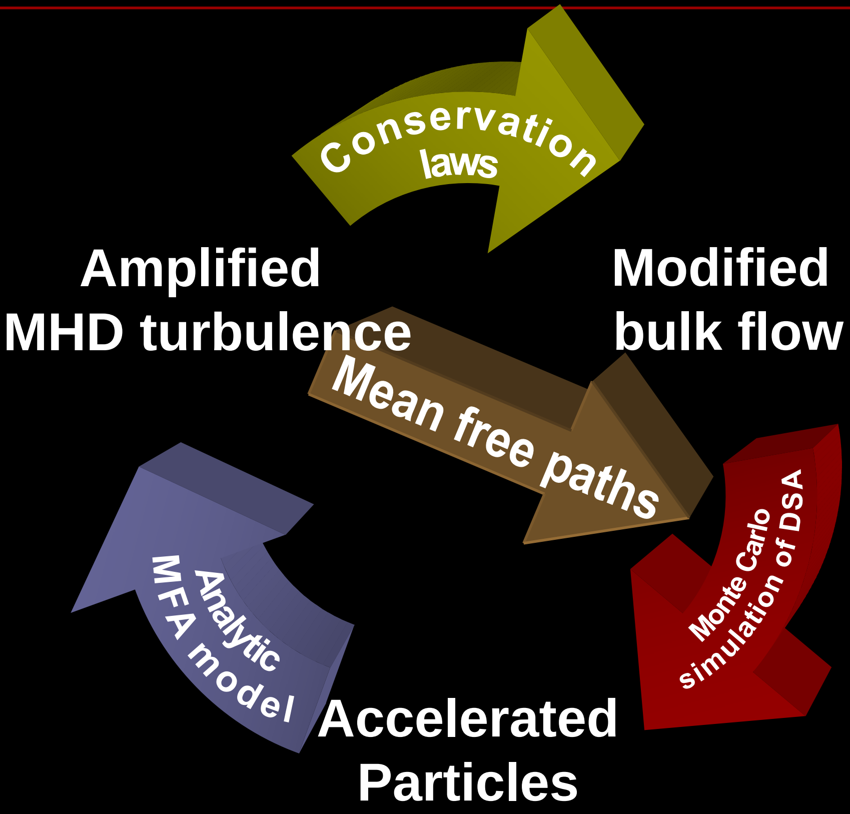
Model

- Monte Carlo simulation was originally developed by D. Ellison and others (M. Baring, D. Eichler, F. Jones and others)
- Shown to be in an excellent agreement with spacecraft observations (Earth’s bow shock, heliospheric shocks)
- Re-written for magnetic field amplification by Vladimirov, Ellison and Bykov; also adopted for parallel processing.

Basic assumptions

- Steady state
- Statistical description of particle transport
- Analytical models for turbulence generation
- Acceleration limited by size, time or E_{max}

Summary of the model



Amplified MHD turbulence

Modified bulk flow

Accelerated Particles

Our approximations: magnetic fields

- We do not follow the evolution of B-field structure, $\boldsymbol{B}(\mathbf{r},t)$
- Instead, we evolve the spectrum of stochastic B-fields (Fourier transform averaged over a small volume), $W(x, k)$
- Justification: size of system ~ $L_D=(c/u)\cdot\lambda(p_{\max})$, largest turbulent harmonics ~ $\lambda(p_{\max}) << L_D$
- Saves computational time at the cost of relying on analytical models for magnetic turbulence amplification.

Our approximations: particle transport

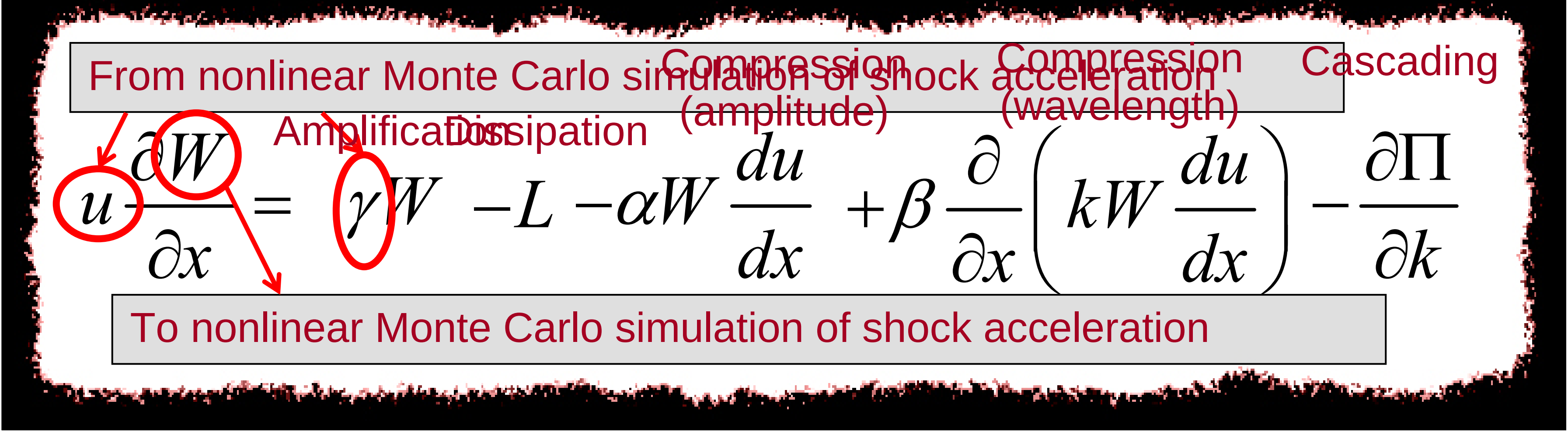
- In the absence of $\boldsymbol{B}(\mathbf{r},t)$, a statistical description is employed, based on $W(x,k)$
- Justification: length scale of parameter variability L_D, mean free path $\lambda = (u/c)\,L_D \ll L_D$ – OK
- Monte Carlo particle transport accurately describes anisotropic $f(\mathbf{p})$ (i.e., goes beyond the diffusion approximation).
- Slower than the kinetic approach, but makes it possible to:
  - Model thermal leakage injection, given a prescription for low energy particle transport;
  - Simulate relativistic shocks;
  - Include uniform $\boldsymbol{B}_0$ to study oblique shocks;
  - Deal with large gradients of $W(x,k)$

Our method: iterative procedure

- Takes computational time
- But! Conserves paper, pencils and mental energy: easy to implement any mean free path prescription, any MFA model, multiple ion species, synchrotron losses for electrons, Fermi-II, particle escape/age limit, shock transparency, etc.

Magnetic field amplification (MFA)

- Assume a phenomenological model for evolution of turbulence spectrum $W(x, k)$
- This model describes $\Delta B >> B_0$, but must also agree with the theoretical models for $\Delta B << B_0$ regime



From nonlinear Monte Carlo simulation of shock acceleration

Amplification

Dissipation

Compression (amplitude)

Compression (wavelength)

Cascading

$$u\frac{\partial W}{\partial x} = \gamma W - L - \alpha W \frac{du}{dx} + \beta \frac{\partial}{\partial x}\left(kW\frac{du}{dx}\right) - \frac{\partial \Pi}{\partial k}$$

To nonlinear Monte Carlo simulation of shock acceleration

Mechanisms of MFA

- Plasma instabilities in presence of accelerated particles: resonant and non-resonant.

Determined by the Monte Carlo simulation

$$\gamma_{\text{res}}(k) = v_A \frac{dP_{\text{cr}}(x, p_{\text{res}})}{dx} \left| \frac{dp_{\text{res}}}{dk} \right| \frac{1}{W(x,k)}$$

where $p_{\text{res}} = \dfrac{eB_0}{ck}$

Resonant CR streaming instability

$$\gamma_{\text{nr}}(k) = v_A k \sqrt{\frac{k_c}{k} - 1},$$

Nonresonant Bell's instability

where $k_c = \dfrac{B_0 j_d}{c \rho v_A}$, and $\dfrac{1}{r_{g1}} < k < k_c$

Mechanisms of MFA

- Other instabilities are currently considered in literature
- Long-wavelength nonresonant kinetic instability (Bykov et al. 2009 – ask me for a copy)
- Magnetosonic instability (Malkov 2009)
- Firehose instability (Blandford, Funk, …)

Mean free path (MFP)

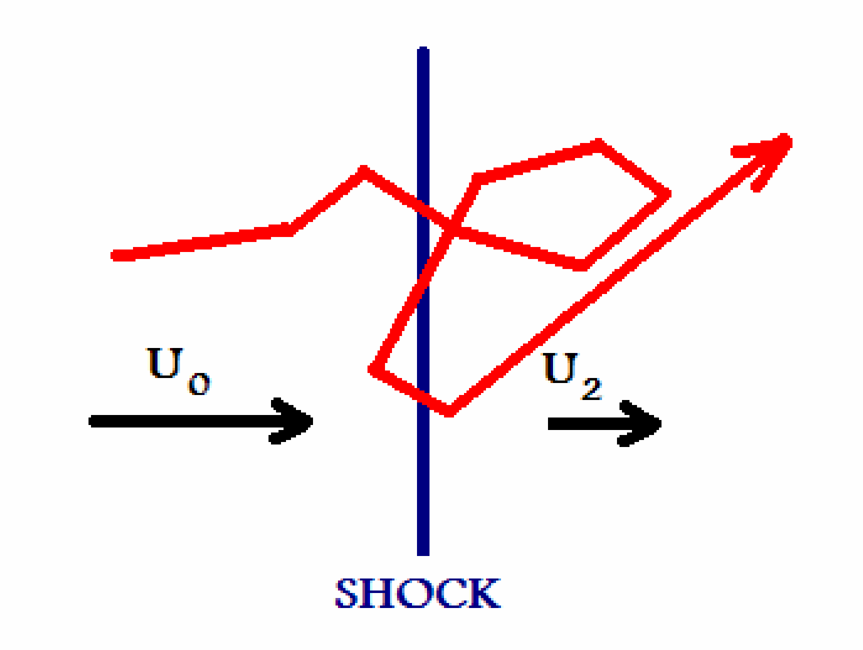
- Monte Carlo uses MFPs of particles in MHD turbulence to describe transport on scales larger than MFP
- MFP determines efficiency of particle acceleration and, subsequently, of MFA

The lowest energy particles get trapped in the large-scale turbulent structures and experience convective diffusion

For intermediate energy particles, either resonant scattering, or Bohm diffusion may be assumed

How Monte Carlo transport works

- Thermal particles are propagated assuming elastic momentum scattering in a small fraction of MFP λ(x, p)
- If a thermal particle crosses the shock against the flow and returns, it gains energy – thermal leakage injection
- Particles are traced in this way from thermal to the highest energies until they escape



How Monte Carlo transport works

- Tracing each particle allows the model to calculate all the necessary moments of the distribution function
- These data are then used as the input for the MFA model
- Output of MFA model plus superthermal particle distribution are used to iterate the solution to satisfy fundamental conservation laws

Achieving self-consistency

Pressure (momentum flux) of Alfven waves

$$\ldots\mathrm{nst},$$

$$\rho u^2 + P + P_w = \mathrm{const},$$

$$P_w(x) = \frac{1}{2}\int W(x,k)\,dk,$$

$$\frac{1}{2}\rho u^3 + \frac{\bar{\gamma}}{\bar{\gamma}-1}(P+\varepsilon)u + F_w = \mathrm{const},$$

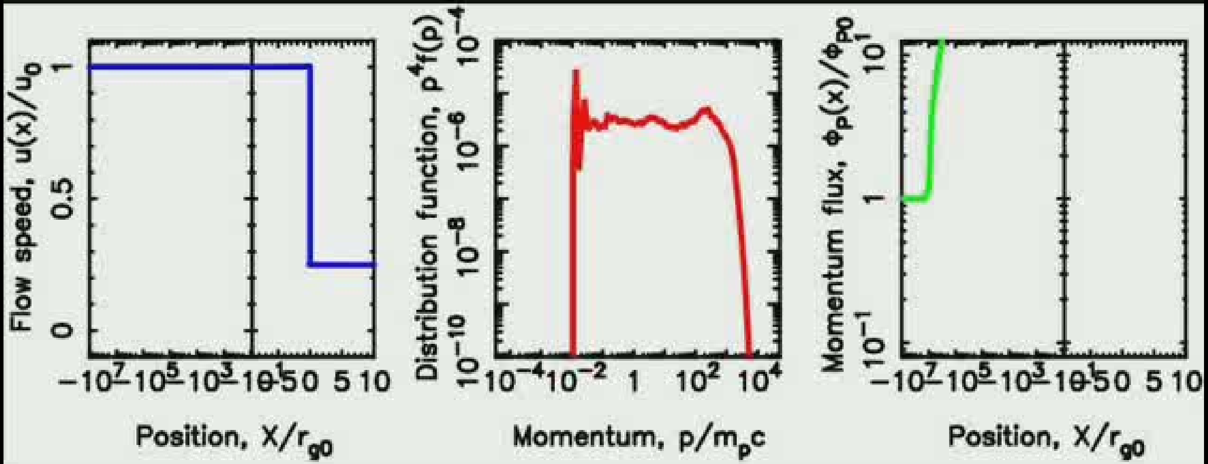
$$F_w(x) = \frac{3}{2}u(x)\int W(x,k)\,dk$$

$$\frac{u\rho^{\gamma}}{}\frac{d}{}\left(P_{th}\rho^{-\gamma}\right) = L.$$

Energy flux of Alfven waves

- Generated MHD waves contribute to momentum and energy balance in the nonlinear shock structure;
- Dissipation of waves in the precursor heats the thermal plasma and affects particle injection.
- These effects depend on turbulence properties.

Iterative procedure in action



Output of the model

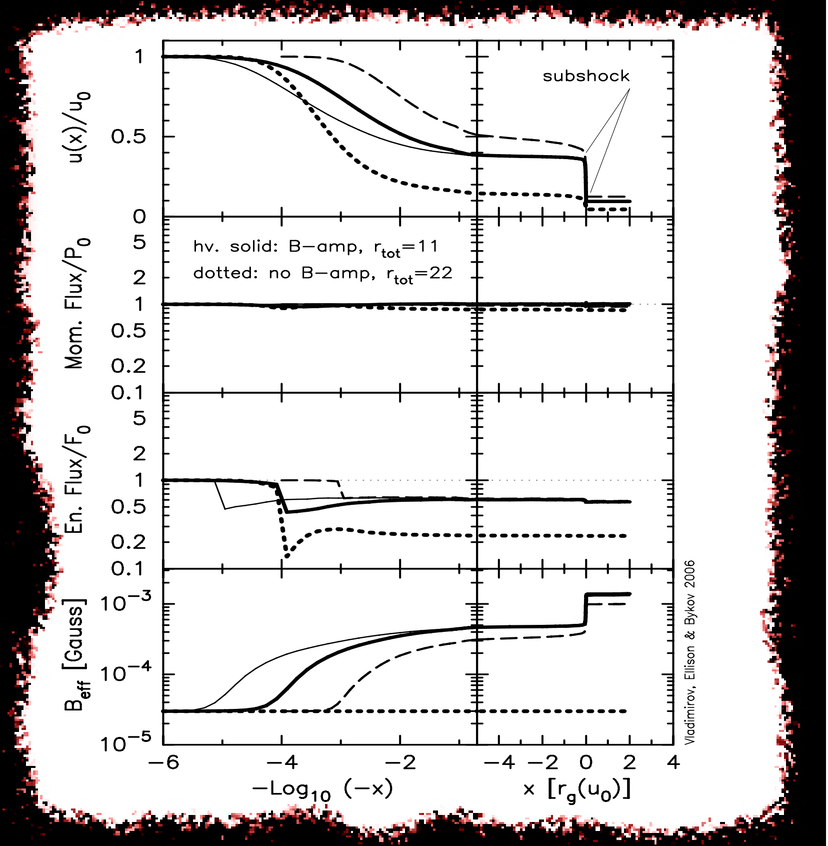
- Fraction of energy in superthermal particles
- Total plasma compression and heating
- Spectra of escaping CR particles
- Spectra of MHD turbulence, values of amplified effective B-fields
- Maximum particle energy for given shock parameters
- Photon emission can be calculated

Published results

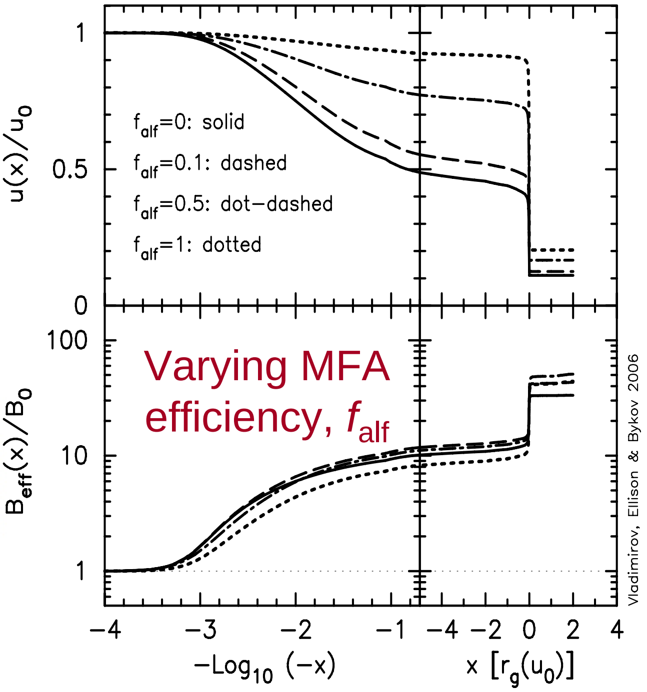
- Vladimirov, Ellison & Bykov, 2006, ApJ : introduced the MC simulation with MFA
- Ellison & Vladimirov, 2008, ApJL: analyzed the effect of MFA on Emax
- V, B & E, 2008, ApJ: studied the importance of precursor heating and its connection to particle injection
- V, B & E, 2009, ApJL: self-consistent solutions with Bell’s instability

Results: nonlinear shocks with MFA

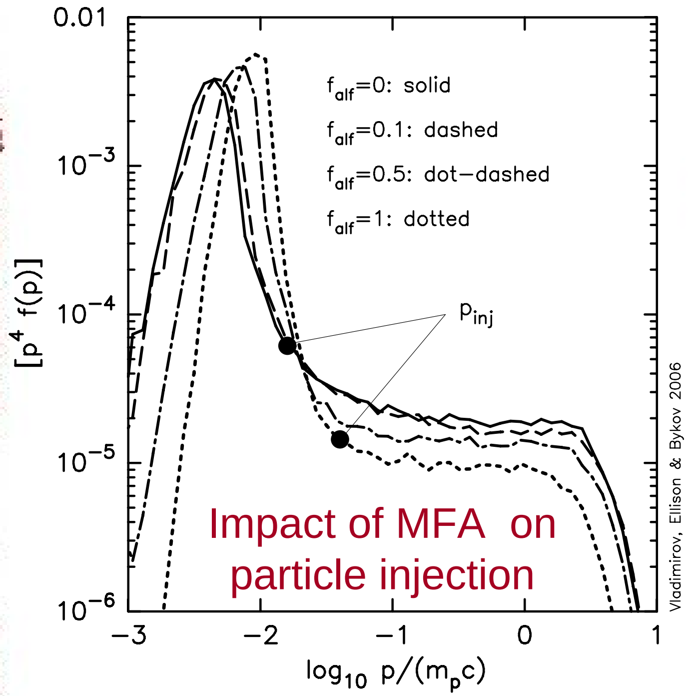
- VEB 2006
- Shown: self-consistent structure of shocks with resonant MFA.
- *Principal result: MFA in NL-DSA is possible to the high levels observed in some SNRs*
- Shocks remain efficient particle accelerators in the presence of MFA



Results: nonlinear shocks with MFA

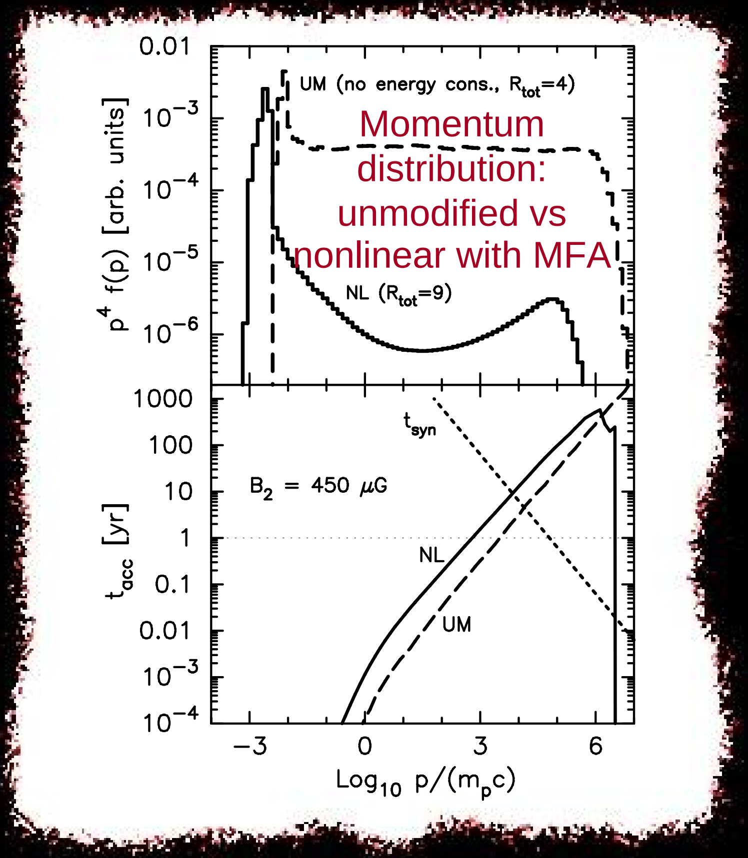


Varying MFA efficiency, f_{alf}



Impact of MFA on particle injection

Results: maximum energy of accelerated particles

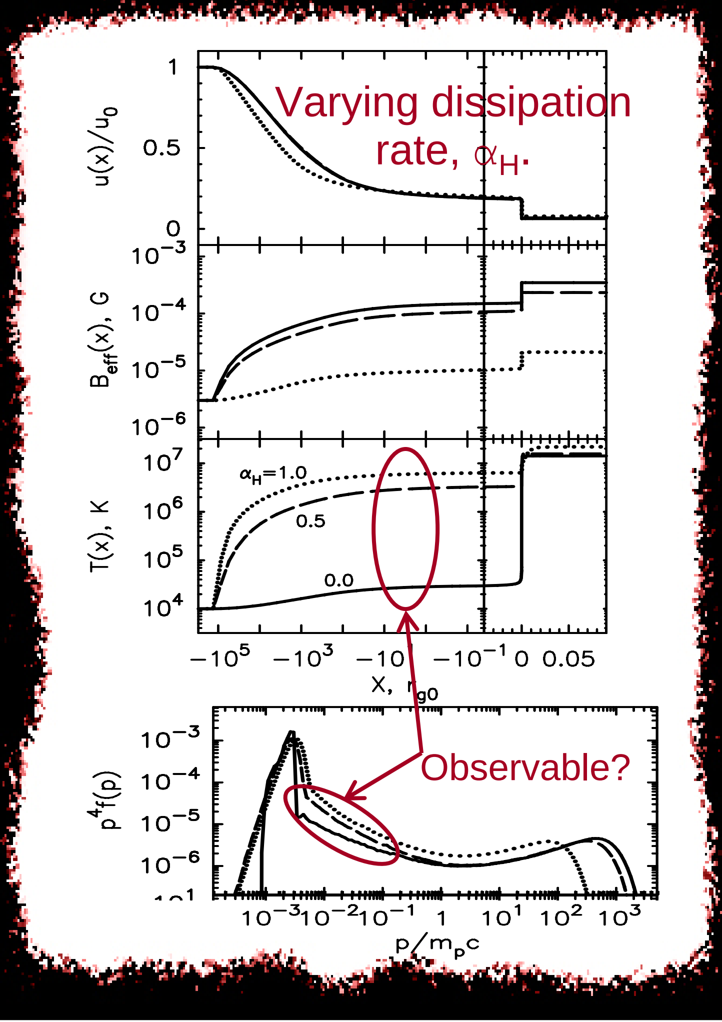


Momentum distribution: unmodified vs nonlinear with MFA

- EV 2008
- *Maximal particle energy,* E_{max}, *does not scale as the amplified field,* B_{eff}
- Important for the interpretation of some observations

Results: turbulence dissipation and particle injection

- VBE 2008
- *Dissipation of turbulence efficiently heats the precursor plasma, boosts particle injection, but does not affect shock compression*
- Dissipation effects may be observable in the X-ray range
- High energy particles not sensitive to injection



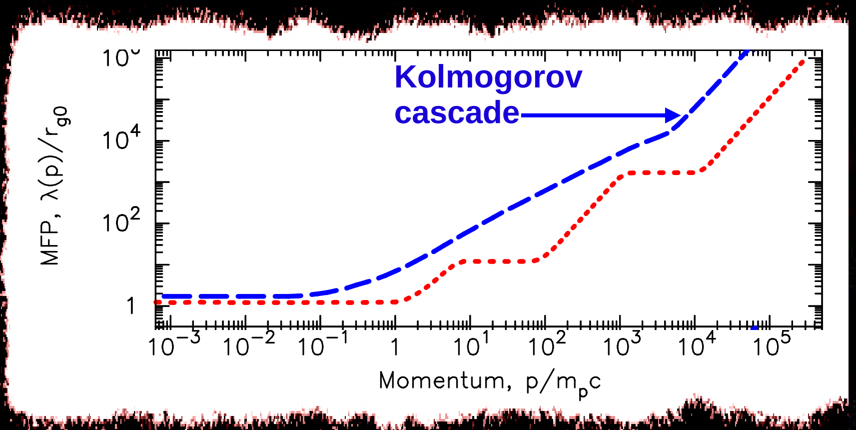
Varying dissipation rate, α_H.

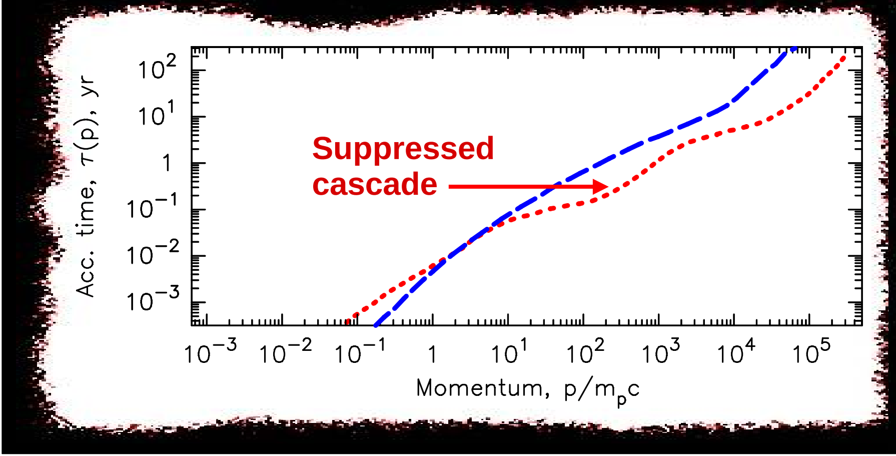
Observable?

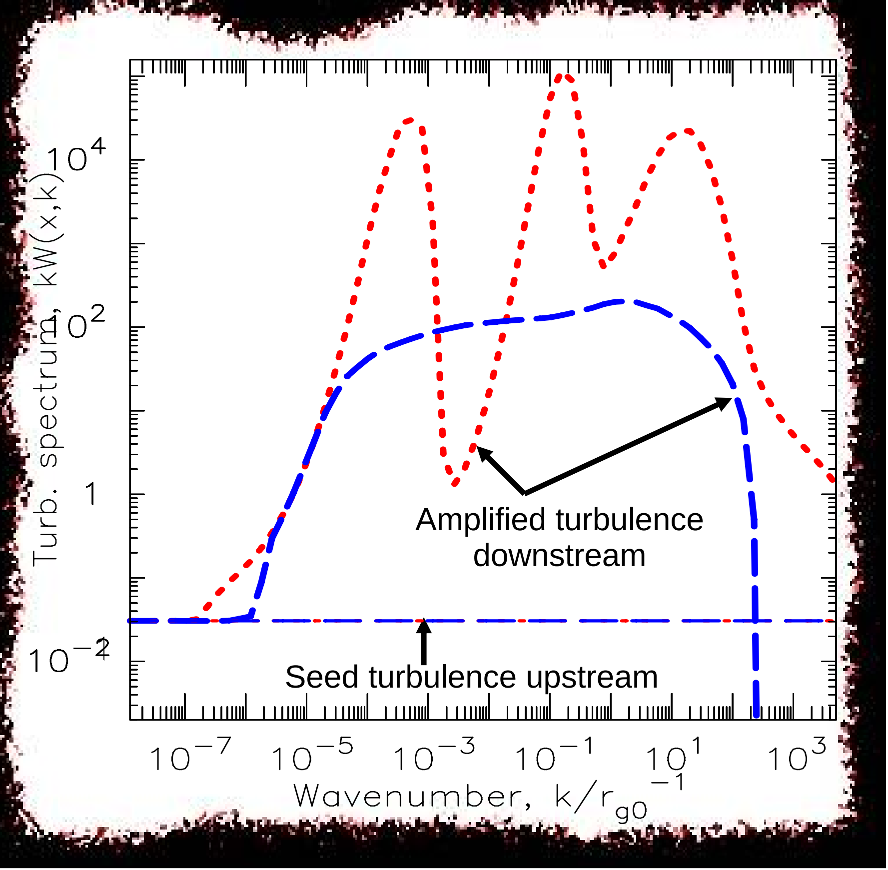
Results: nonresonant instability and precursor stratification

- Submitted to APJL (I’ll be happy to send or print you a copy)
- Spectral energy transfer (cascading) is uncertain
- The nonresonant streaming instability may lead to *peculiar turbulence spectra* with discrete peaks
- Observable features are discussed

Results: nonresonant instability and precursor stratification







This is not the end!

- Kinetic and MC models have a long way to go until a realistic model of a SNR shock is obtained with few parameters
- Major uncertainties are: mechanisms of MFA, structure of the self-generated MHD turbulence, particle transport in such turbulence, plasma physics of particle injection.

- PIC simulations may shed light on the particle injection process.
- Is it possible to describe the scattering rate of a thermal / low energy particle as a function of its energy in the self-generated fields?
- Study parameter dependence (on temperature, obliquity of the seed magnetic field, etc.) of the injection rate?
- Collisionless dissipation of turbulence is crucial for determining the precursor heating – PIC area.

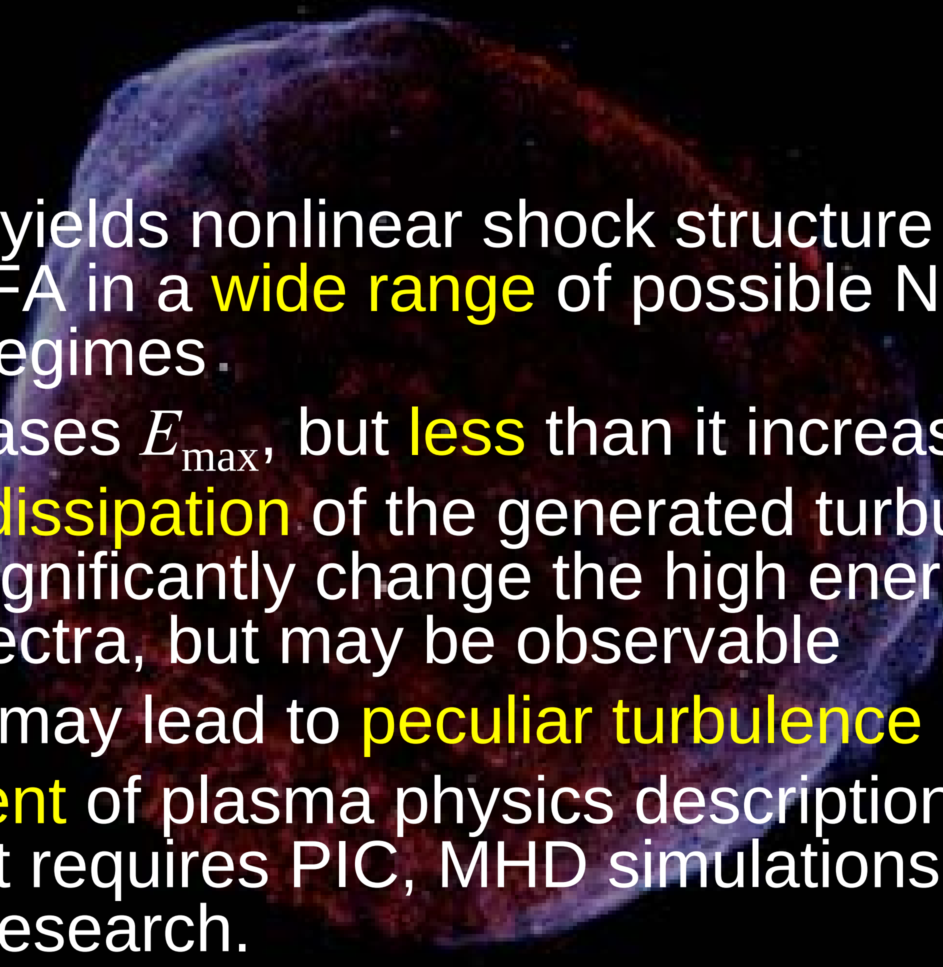
- MHD simulations of wave instabilities may be an efficient way to address problems like instability saturation, spectral energy transfer (e.g., Bell 2004, Zirakashvili, Ptuskin & Voelk 2008).
- Back-reaction of particles to the amplified B-field may be important (e.g., Lucek & Bell 2000).
- Particle transport in simulated stochastic fields can be studied numerically (e.g., Bell 2005)

Similar work* in progress

- Kinetic theory (e.g., Berezhko & Voelk, Blasi et al., Jones & Kang)
- Alternative models (e.g., Diamond & Malkov)
- Various other groups and individuals employing semi-analytical and combined methods

* nonlinear models with realistic dynamic range in particle energies

Summary



- MC model yields nonlinear shock structure with efficient MFA in a wide range of possible NL instability regimes.
- MFA increases E_{max}, but less than it increases B_{eff}
- Moderate dissipation of the generated turbulence does not significantly change the high energy particle spectra, but may be observable
- NL effects may lead to peculiar turbulence spectra
- Improvement of plasma physics description is essential; it requires PIC, MHD simulations and analytical research.