Strong-field approximation (SFA) vs Coulomb effects

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Direct-electron SFA in brief

$$M_{\mathbf{p}} = -i \int_{-\infty}^{\infty} dt \langle \psi_{\mathbf{p}}^{(Volkov)}(t) | V(\mathbf{r}) | \psi_0(t) \rangle$$

$$\psi_{\mathbf{p}}^{(Volkov)}(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2}} \exp[i(\mathbf{p} - e\mathbf{A}(t)) \cdot \mathbf{r}]$$

$$\times \exp[-\frac{i}{2m} \int^{t} d\tau (\mathbf{p} - e\mathbf{A}(\tau))^2]$$

Evaluation by stationary phase:

$$\frac{1}{2m}(\mathbf{p} - e\mathbf{A}(t))^2 + I_P = 0$$

the (complex) solutions $t = t_s$ determine the tunneling times

Saddle-point approximation to the SFA

$$M_{\mathbf{p}} \propto \sum_s \sqrt{\frac{2\pi i}{S_{\mathbf{p}}''(t_s)}} \exp[i(I_P t_s + S_{\mathbf{p}}(t_s)] \langle \mathbf{p} - e\mathbf{A}(t_s) | V | \psi_0 \rangle$$

form factor

$$S_{\mathbf{p}} = \frac{1}{2m} \int^{t} d\tau [\mathbf{p} - e\mathbf{A}(\tau)]^2$$

$t = t_s$ (complex) saddle-point solutions

approximately $p - eA(t_s) = 0$

Quantum-orbit expansion of the transition amplitude

$$M(\mathbf{p}) = \sum_{\text{orbits } s} a_s(\mathbf{p}) \exp[iS_s(\mathbf{p})]$$

cf. Feynman's path integral

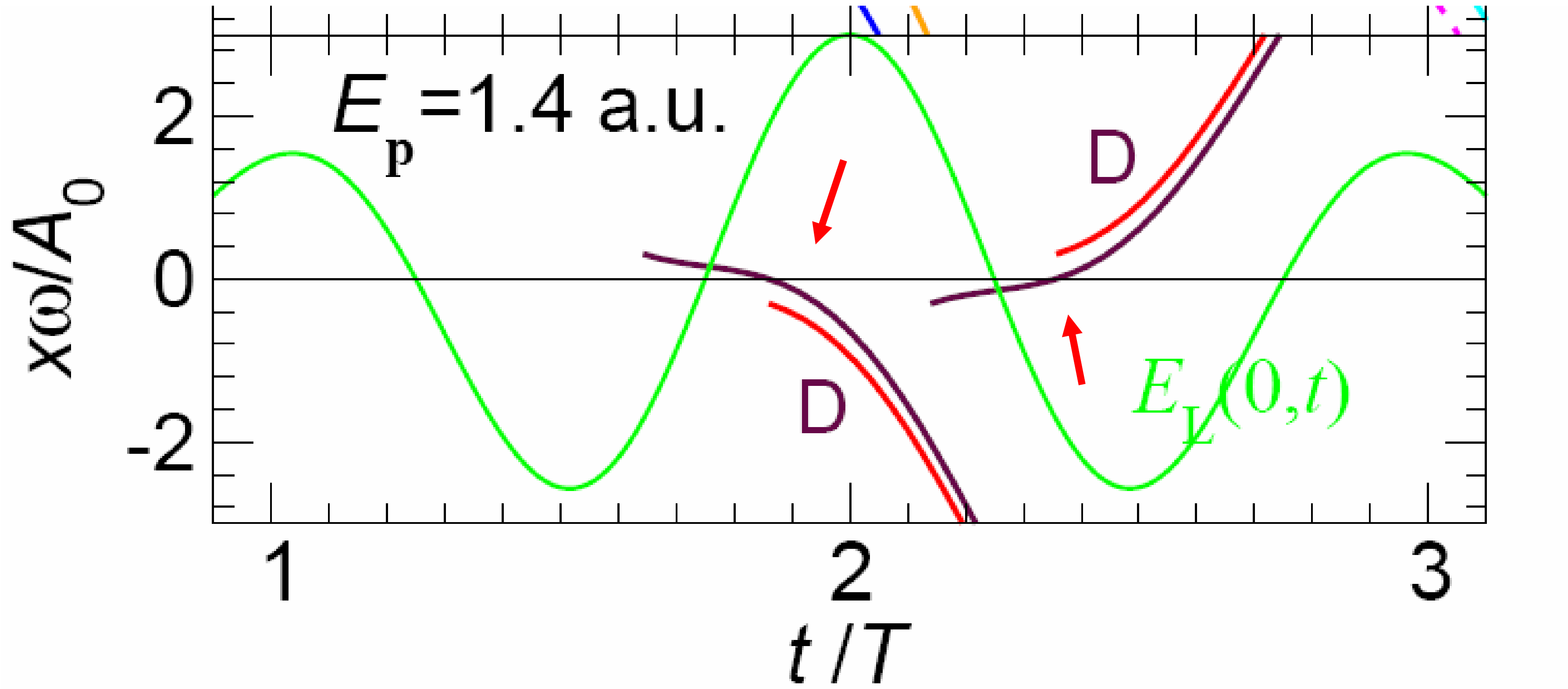
P. Salières et al., Science 292, 902 (2001)

The quantum orbits are defined by the solutions $(t_s, t'_s, \mathbf{k}_s)$ $(s = 1, 2, \ldots)$ of the saddle-point equations:

$$m\mathbf{x}(t) = \begin{cases} (t - t'_s)\mathbf{k}_s - \int_{t'_s}^{t} d\tau e\mathbf{A}(\tau), & (\mathrm{Re}\, t'_s \leq t \leq \mathrm{Re}\, t_s) \\ (t - t_s)\mathbf{p} - \int_{t_s}^{t} d\tau e\mathbf{A}(\tau). & (t \geq \mathrm{Re}\, t_s) \end{cases}$$

$\mathbf{x}(t=t_s') = 0$, but Re $[\mathbf{x}(\mathrm{Re}\ t_s')]$ different from 0

Examples of direct quantum orbits



One member of a pair of orbits experiences the Coulomb potential more than the other (see later)

Generalized Keldysh theory: Rescattering (cont.)

an alternative expression:

$$M_{\mathbf{p},E_0} = -i\int_{-\infty}^{\infty} dt \langle \psi_{\mathbf{p}}^{(\mathrm{Vv})(t)} | H_I(t) | \psi_0(t') \rangle$$

direct electrons

$$-i\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \langle \psi_{\mathbf{p}}^{(\mathrm{Vv})}(t) | V U^{(\mathrm{Vv})}(t,t') H_I(t') | \psi_0(t') \rangle$$

rescattered electrons

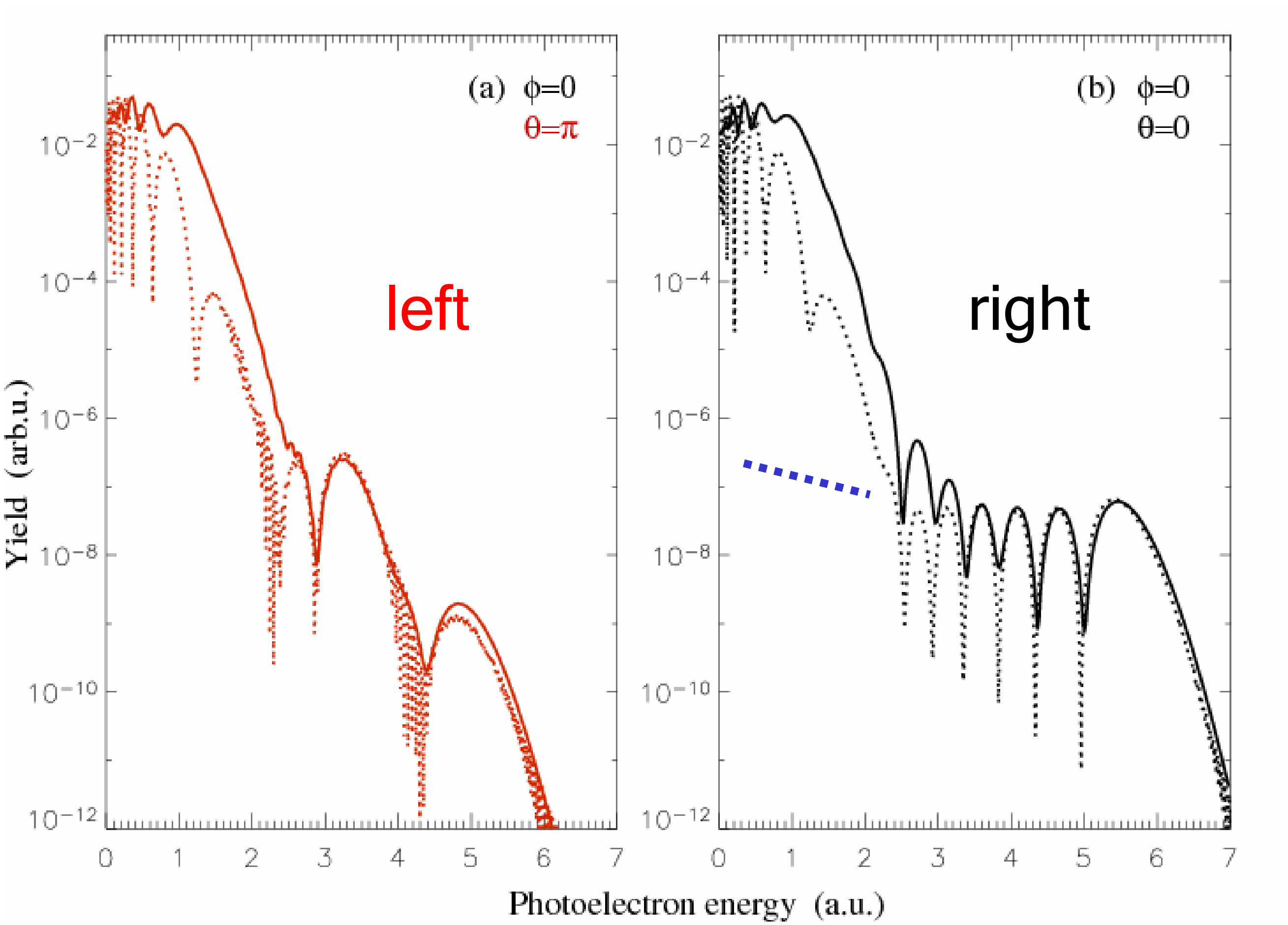
in the last line, may replace

$$V(\mathbf{r}) \to V_{\mathrm{scatt}}(\mathbf{r})$$

going beyond the SAEA

restored „hard" Coulomb effects in first-order Born approximation

Hydrogen H(1s) ATI spectra via TDSE and SFA



solid: TDSE;
dashed: SFA

ω = 0.056 a.u.
E_0 = 0.834 a.u.
4-cycle sine-square sine pulse

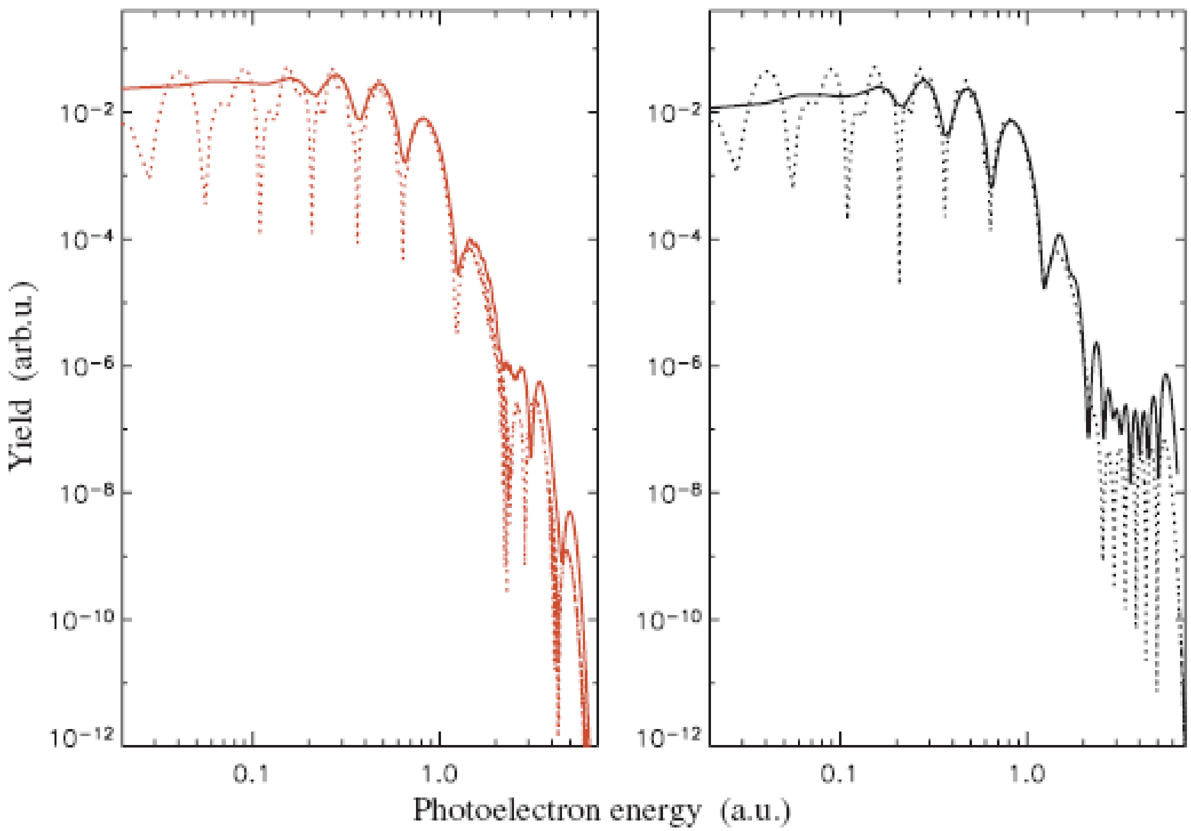
Problem areas:

very low energies

transition region between $2U_p$ and $5U_p$

D. Bauer, D.B. Milosevic, WB, JMO 53, 135 (2006)

Origin of interferences: short-range potential



solid: TDSE
dashed: SFA

TDSE:
Coulomb potential cut at r_c = 2 a.u.

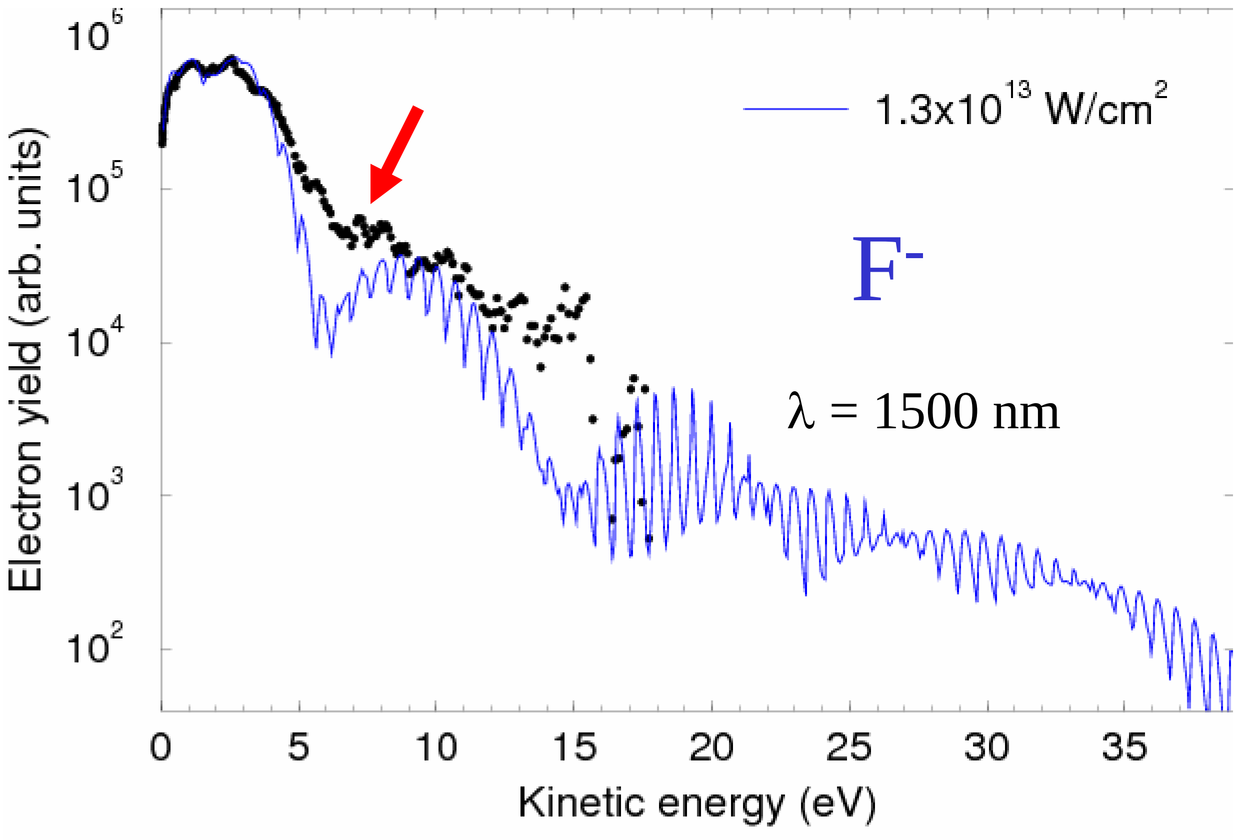
SFA:
Yukawa wave function

Interferences are not an artifact of the SFA

Interference of the two solutions from within one cycle

(includes focal averaging)

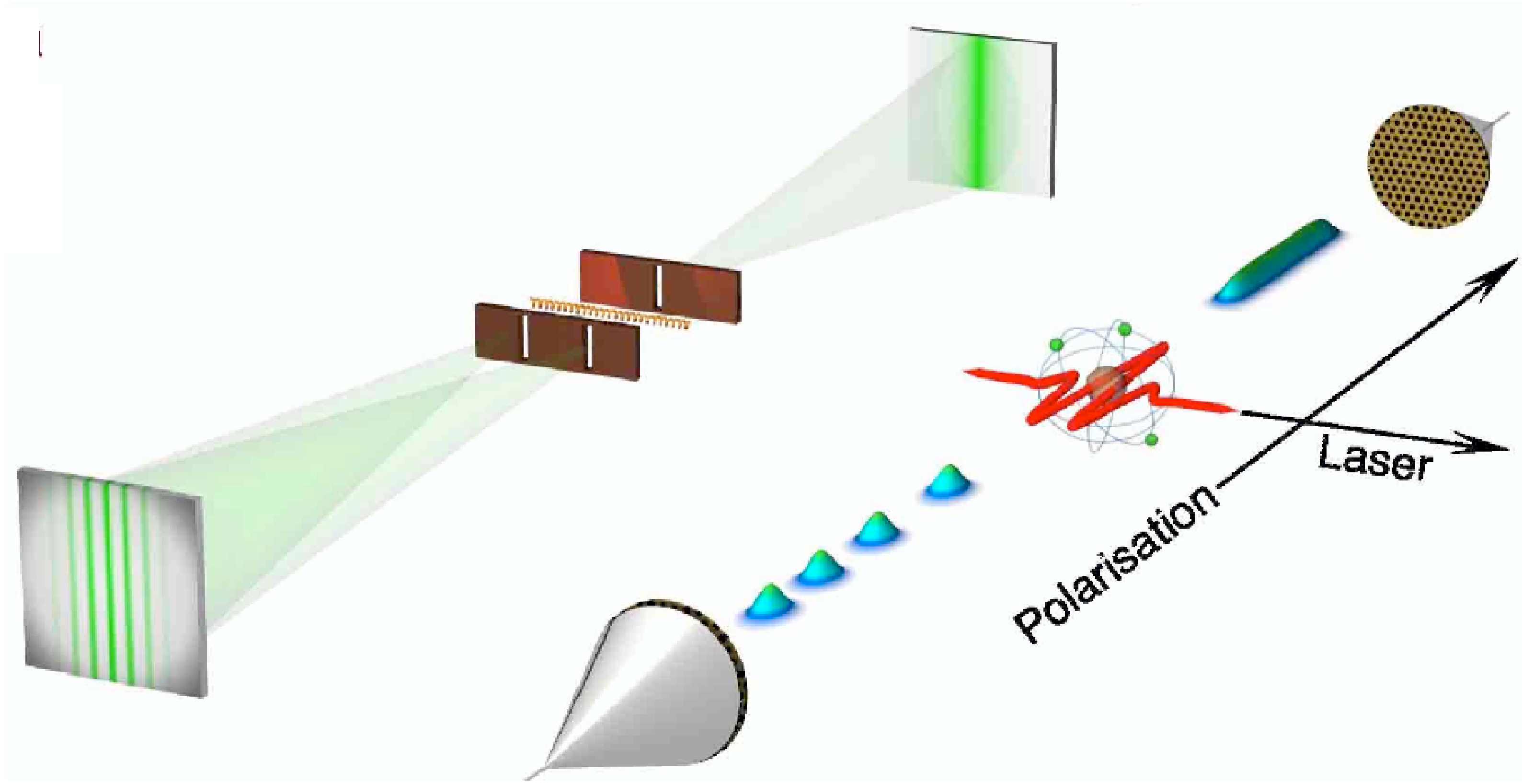
Detachment, no Coulomb potential!



Data: I. Yu Kiyan, H. Helm, PRL 90, 183001 (2003)
(1.1 x 10^{13} Wcm^{-2})
Theory: D.B. Milosevic et al., PRA 68, 070502(R) (2003)
(1.3 x 10^{13} Wcm^{-2})

cf. M.V. Frolov, N.M. Manakov, E.A. Pronin, A.F. Starace, JPB 36, L419 (2003)

The attosecond double slit

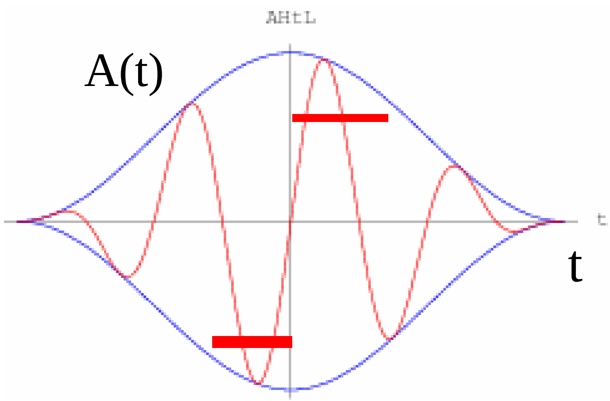


one and the same atom can realize the single slit and the double slit at the same time

F. Lindner et al., PRL 95, 040401 (2005)

Single slit vs. double slit by variation of the absolute phase

$$A(t) = A_0\, e_x \cos^2(\pi\, t/nT)\, \sin(\omega t - \phi)$$

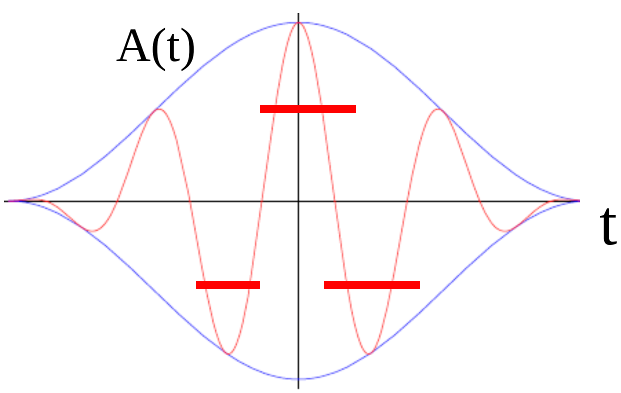


$\phi = 0$

„cosine“ pulse

one window in either direction

$p=eA(t)$



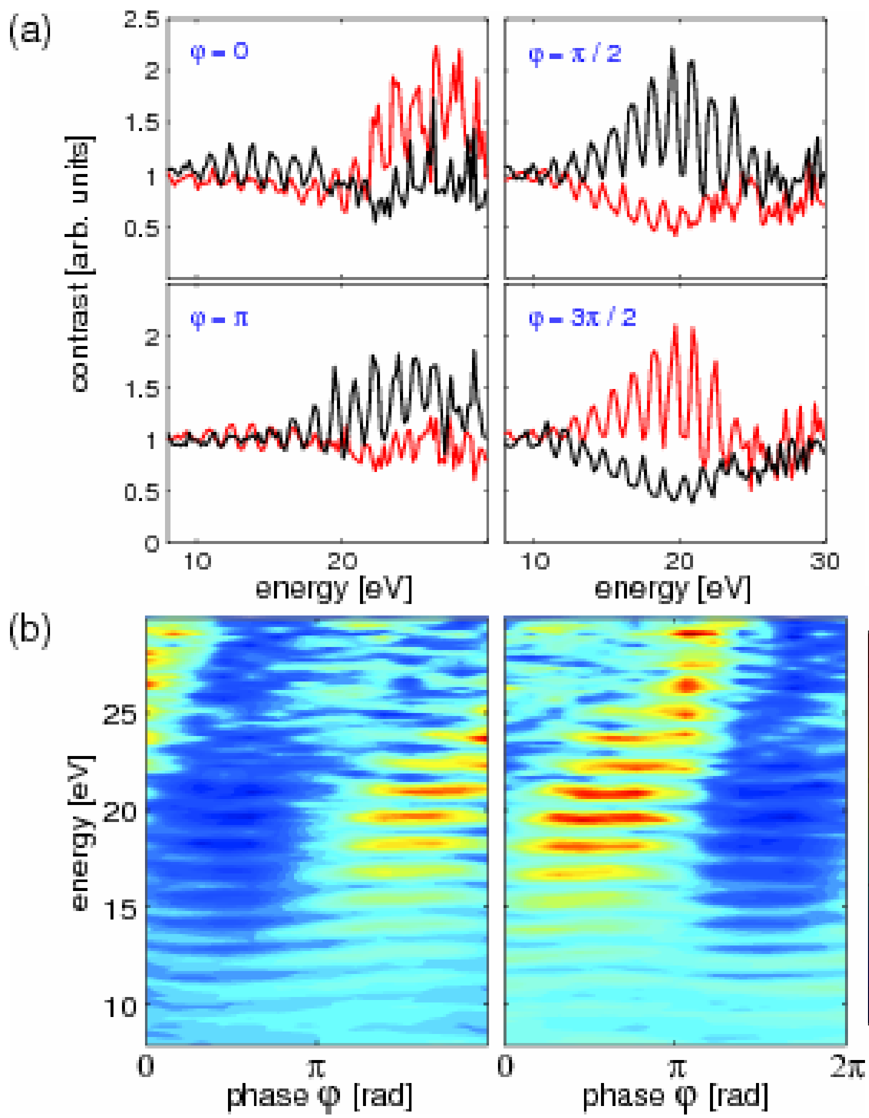
$\phi = \pi/2$

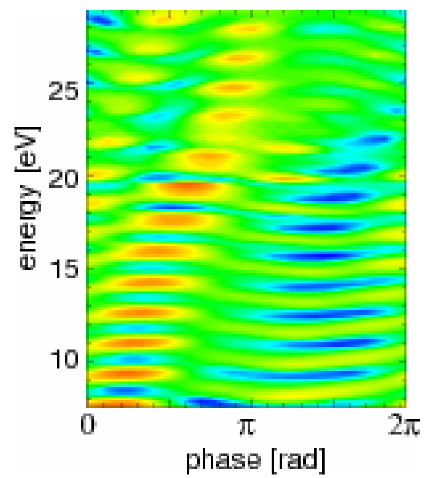
„sine“ pulse

one window in the positive direction,
two windows in the negative direction

Theory vs. experiment:

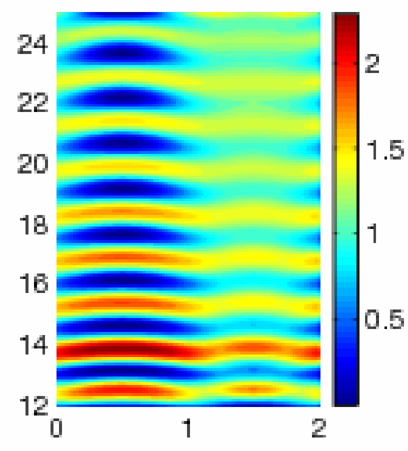
The Coulomb field **IS** important





solution of the TDSE including the Coulomb field

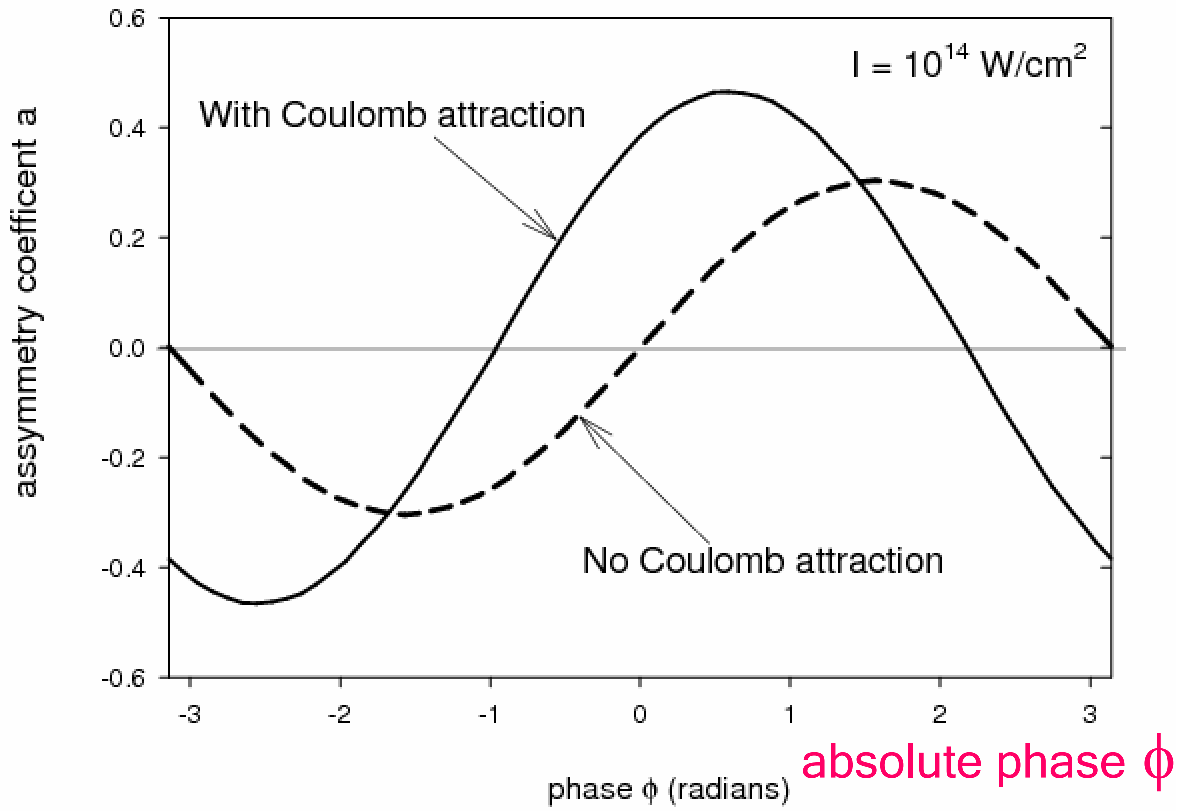
F. Lindner et al.
PRL 95, 040401 (2005)



„simple-man“ model ignoring the Coulomb field

Backward-forward asymmetry for a few-cycle field as a function of the absolute phase

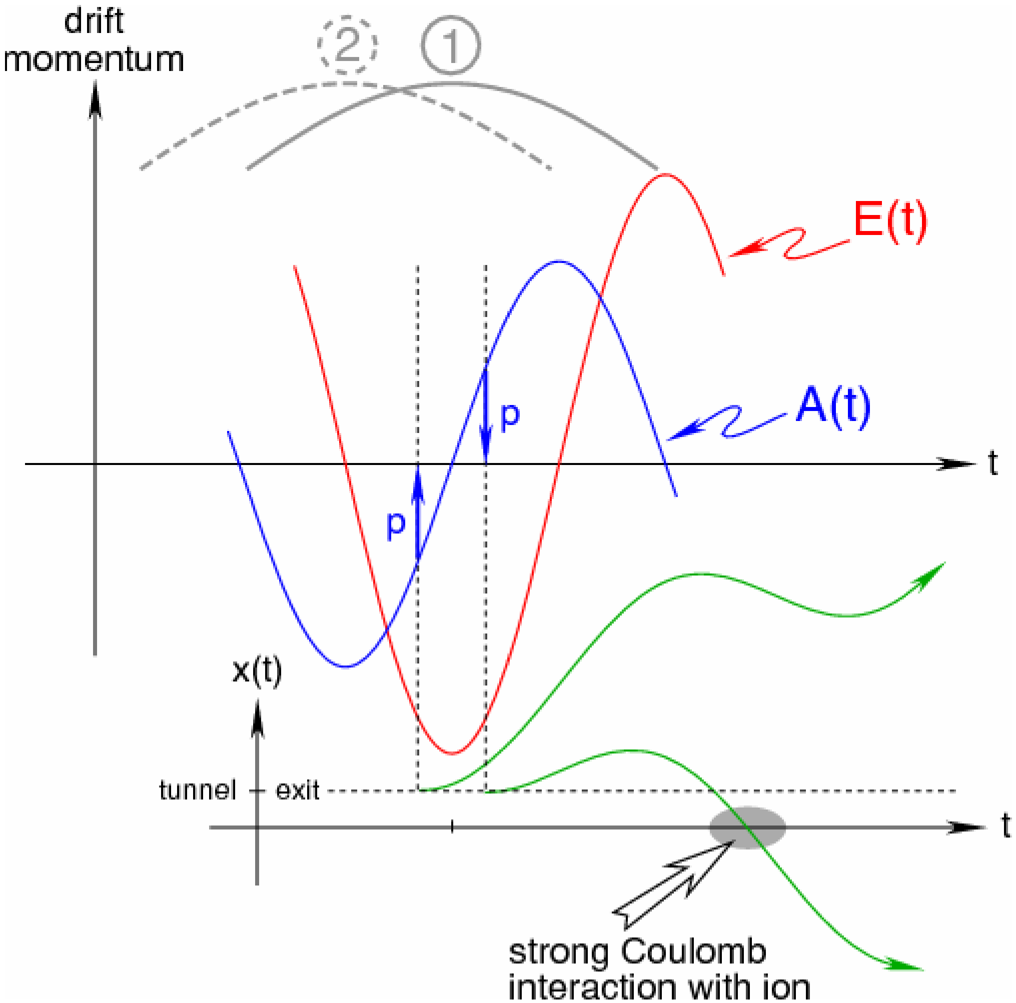
R = [W(left)-W(right)]/[W(left)+W(right)]



SFA predicts R = 0 for $\phi = 0$,
TDSE for $\phi = -0.3$

Chelkowski and Bandrauk
PRA 71, 053815 (2005)

Physical consequences of the Coulomb field



If the Coulomb field is ignored, envelope 1 yields backward-forward symmetry.

Due to Coulomb refocusing. the later orbit is preferred, violating b-f symmetry

The envelope 2 weakens the contribution of the later orbit and restores b-f symmetry.

argument explains the sign of the symmetry phase ϕ = -0.3

Electron-electron Coulomb interaction in the final state of nonsequential double ionization

collaborators:

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PRA **69**, 021402(R) (2004); 043405 (2004)

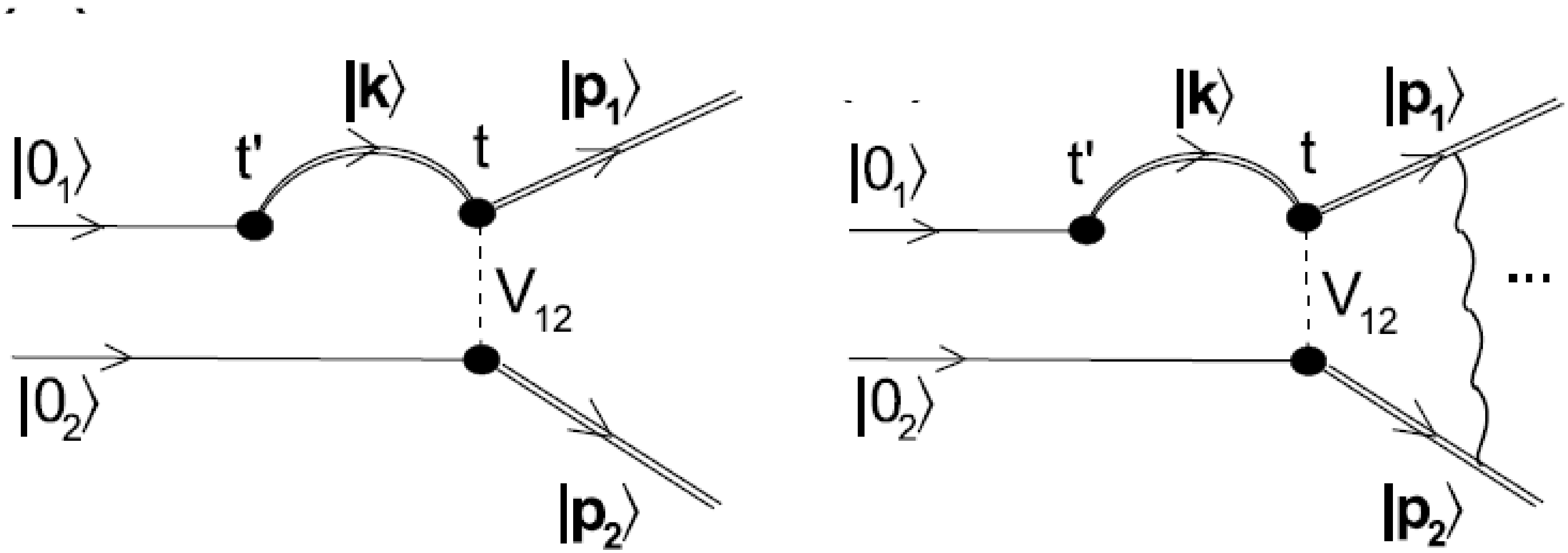
Two-electron Volkov state:

$$|\psi^{(\mathrm{Vv})}_{\mathbf{p}_1\mathbf{p}_2}(t)\rangle = |\psi^{(\mathrm{Vv})}_{\mathbf{p}_1}(t)\rangle \otimes |\psi^{(\mathrm{Vv})}_{\mathbf{p}_2}(t)\rangle$$
$$\times {}_1F_1(-i\gamma, 1; i(|\mathbf{p}||\mathbf{r}| - \mathbf{p}\cdot\mathbf{r}))e^{-\pi\gamma/2}\Gamma(1+i\gamma),$$

$\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\gamma = 1/(2|\mathbf{p}|)$

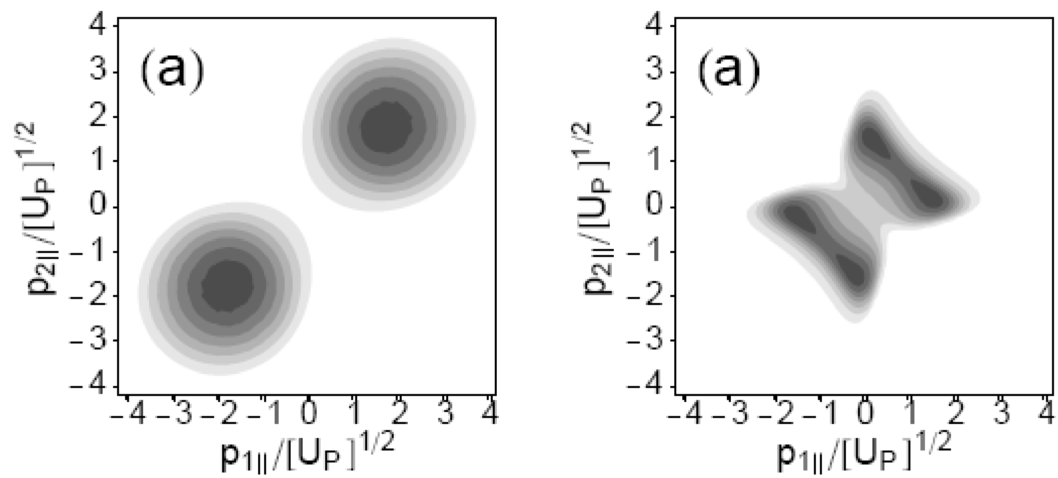
Note: Coulomb repulsion affects $\mathbf{r}_1 - \mathbf{r}_2$, laser fi eld couples to $\mathbf{r}_1 + \mathbf{r}_2$

F.H.M. Faisal, Phys. Lett. A **187**, 180 (1994); A. Becker, F.H.M. Faisal, PRA **50**, 3256 (1994)

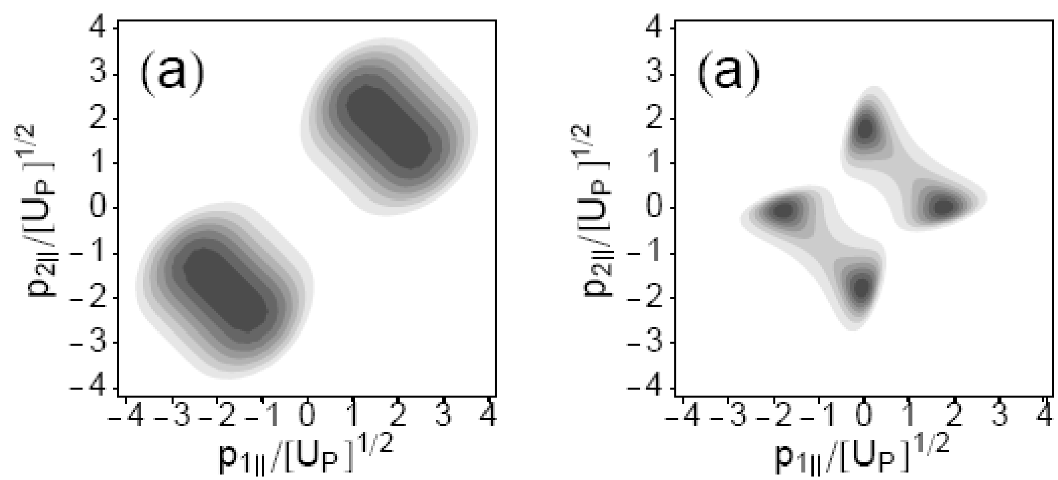


without with

final-state Coulomb repulsion between the two electrons



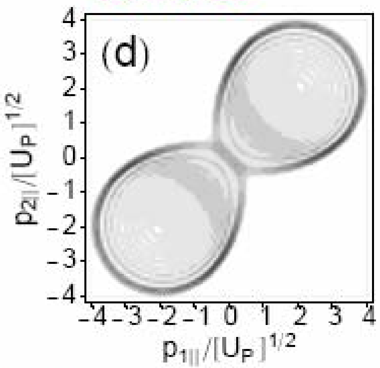
Including Coulomb repulsion in the final state



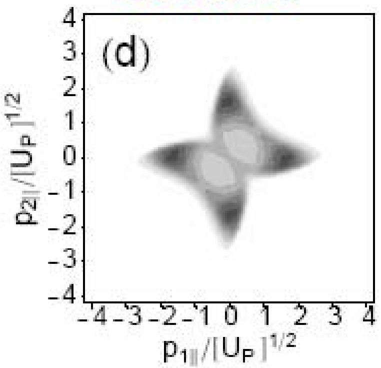
Small transverse momenta

$0 \leq p_{1\perp}, p_{2\perp} \leq 0.1\sqrt{U_p}$

contact



Coulomb



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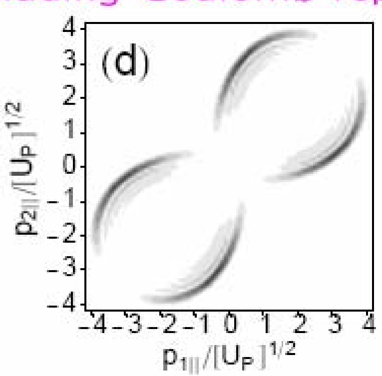

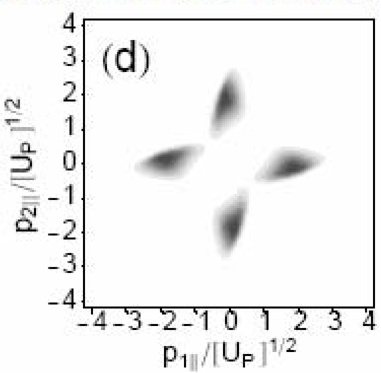

Large transverse momenta

$\sqrt{U_p} \leq p_{1\perp}, p_{2\perp} \leq 1.5\sqrt{U_p}$

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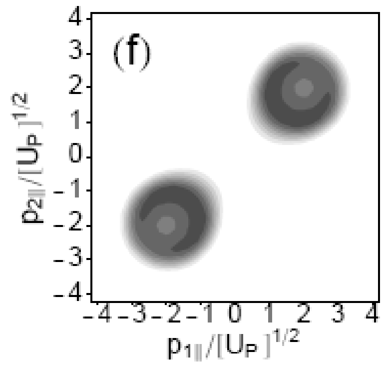
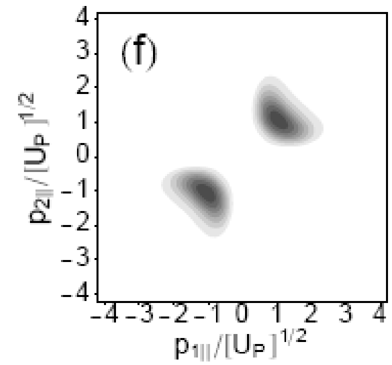


Coulomb



Including Coulomb repulsion in the final state