Precise Manipulations of the Particle Phase Space with Nonadiabatic Ponderomotive Barriers

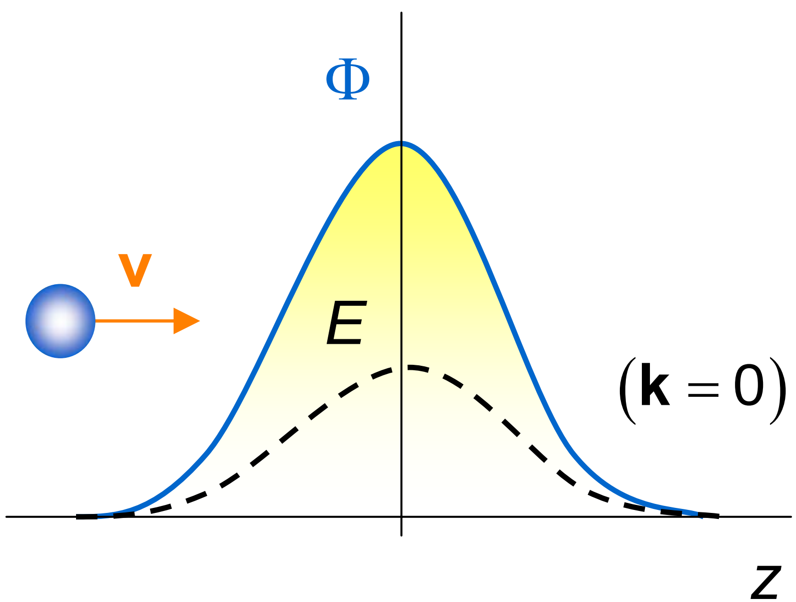
KITP, University of California, Santa Barbara

ILYA DODIN

Princeton University

A ponderomotive potential is an effective potential seen by a particle in ac field on average over the fast oscillations. It is not a true potential though, and hence can be used for particle manipulations more advanced compared to those via static potentials.

Classical Ponderomotive Force



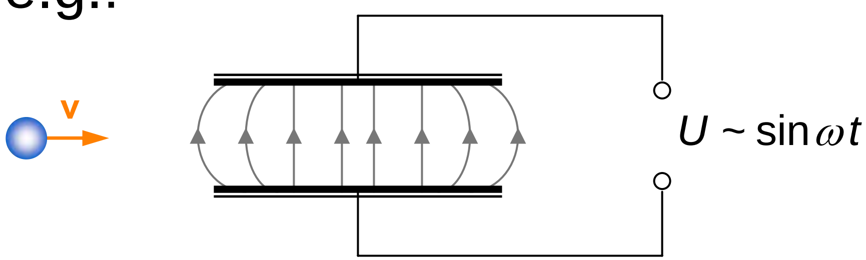
$$\mathbf{E}_{\sim}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\sin\omega t$$

$$\epsilon \sim \frac{\langle v \rangle T}{L} \ll 1$$

$$\Phi = \frac{e^2 E^2}{4m\omega^2}$$

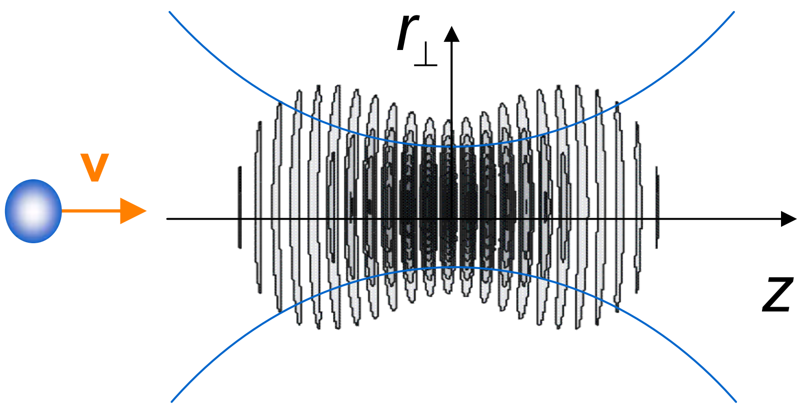
$$\mathfrak{E} = m\langle \mathbf{v} \rangle^2 / 2 + \Phi$$

e.g.:



Gaponov and Miller (1958);
Motz and Watson (1967) …

Relativistic Ponderomotive Force



$$\mathbf{E}_\sim(\mathbf{r},t) \approx \mathbf{E}(t - z/c)\sin\left[\omega(t - z/c)\right]$$

$$\epsilon \sim \frac{\langle v \rangle T}{L} \ll 1$$

$$m_{\text{eff}} = m\sqrt{1 + \overline{a^2}}$$

$$a = eE_\sim / m\omega c$$

Kibble (1966); Eberly and Sleeper (1968); Sarachik and Shappert (1970); Rax and Fisch (1992); Bauer *et al* (1995); Mora and Antonsen Jr. (1997); Tokman (1999); Dodin *et al* (2003)…

$$\mathfrak{E} = \sqrt{m_{\text{eff}}^2 c^4 + p_{\text{d}}^2 c^2}$$

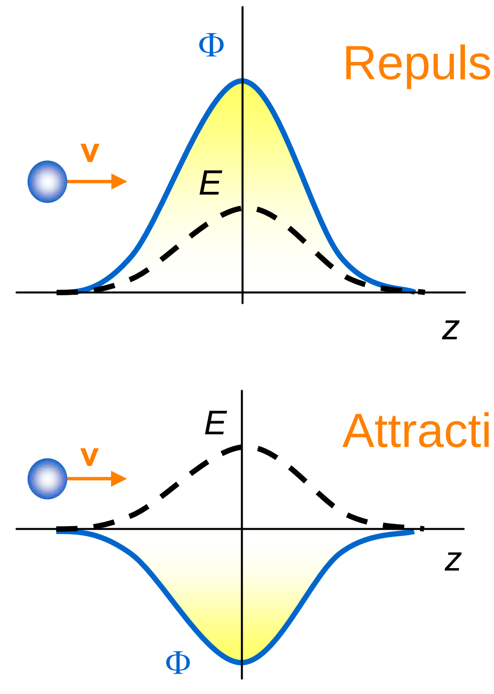
Question

Does it have anything to do with the attosecond science?

Adiabatic Invariant and Particle Manipulation

- $\mathfrak{E}$ is an adiabatic invariant $(\partial_t \equiv 0)$
  - $\mathfrak{E} = m\langle \mathbf{v} \rangle^2 / 2 + \Phi = \text{inv}$
  - $\mathfrak{E} = \sqrt{m_{\text{eff}}^2 c^4 + p_{\text{d}}^2 c^2} = \text{inv}$

$$\epsilon \sim \frac{\langle v \rangle T}{L} \ll 1$$

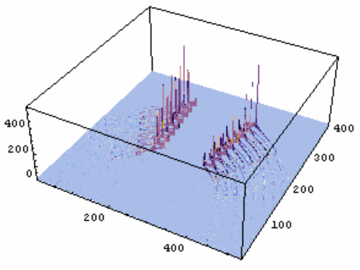


Conservation law is a constraint!

Main Idea

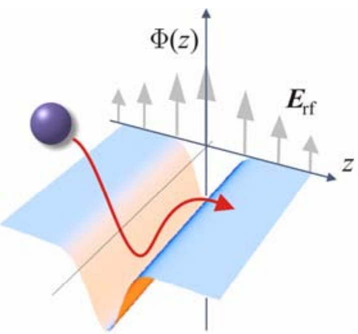
Removing the constraint of adiabaticity might give us additional <u>flexibility</u> for manipulating particle motion.

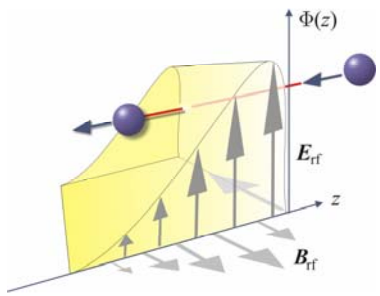
Examples of Advanced Barriers



- Attosecond bunches

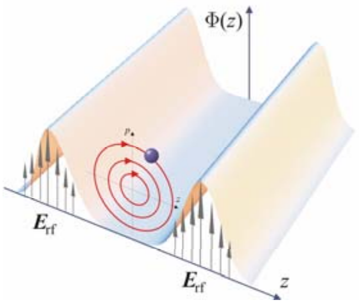
- Ponderomotive cooling

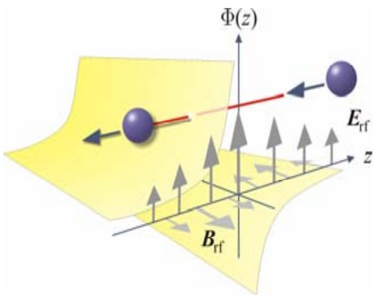




- Asymmetric barriers

- Quantumlike effects





- Singular quasi-potentials

How do we break adiabaticity?

average velocity of the particle

longest period of particle oscillations

$$\epsilon \equiv \frac{\langle v \rangle T}{L} \geq 1$$

spatial scale of the field

$$\frac{\langle v \rangle}{\omega L} \geq 1$$

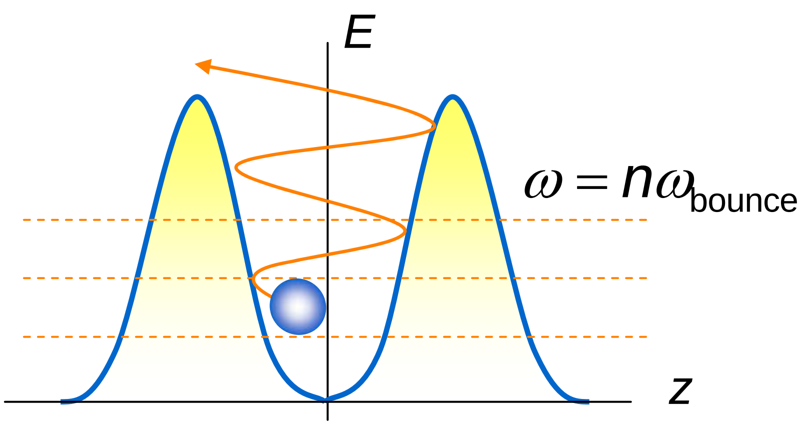
fast particle

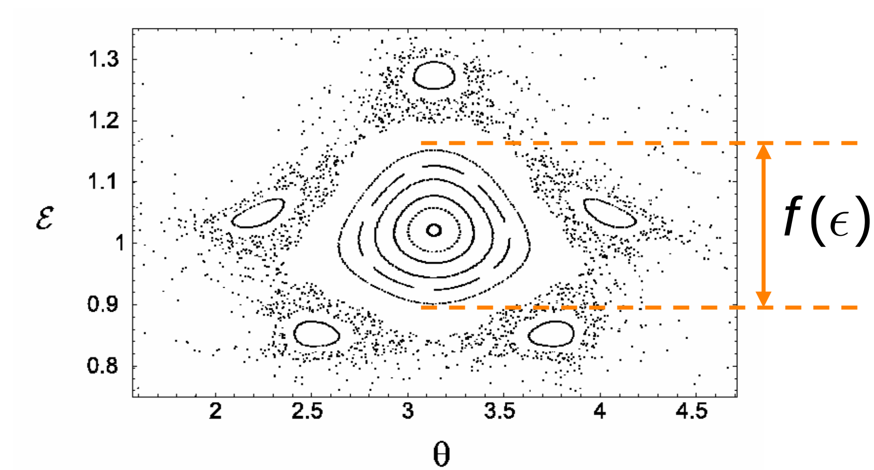
...

Quantumlike Effects: Discrete Energy Levels

- "De Broglie wavelength" $\lambda \equiv \frac{2\pi}{\omega}\langle v \rangle$

$$\frac{\langle v \rangle}{\omega L} \geq 1$$



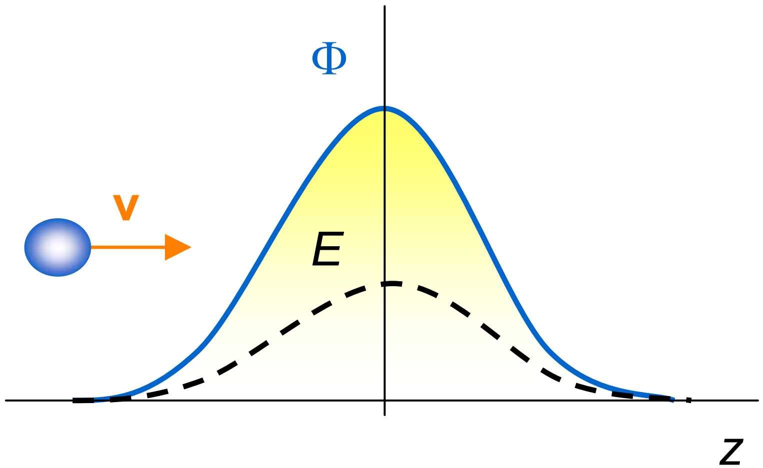


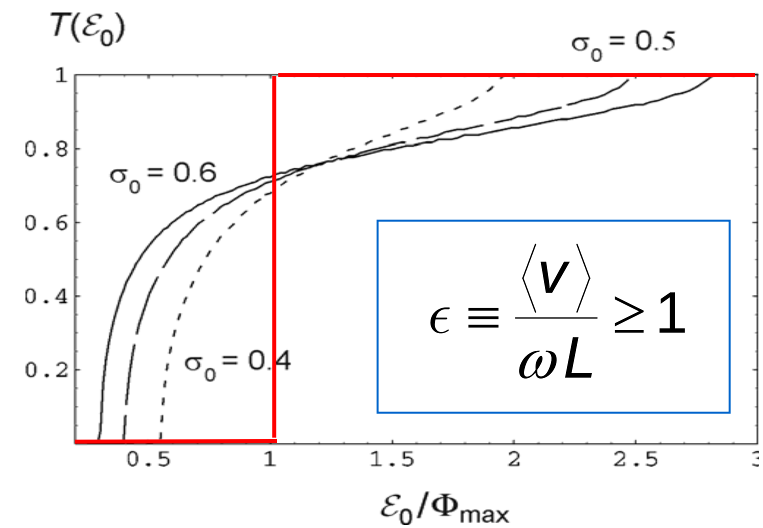
$$\oint k\,dz \approx 2\pi n, \qquad k = \frac{2\pi}{\lambda}$$

Dodin and Fisch, *PRL* (2006)

Quantumlike Effects: Nonadiabatic Tunneling

- Particle transmission depends on the initial phase ωt_0

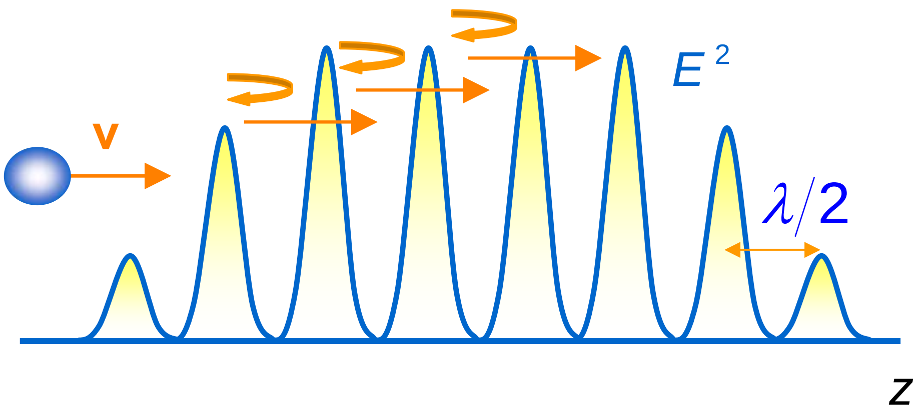




$$T \approx \frac{1}{2} - \frac{1}{\pi} \arcsin \frac{\Phi_{max} - \mathcal{E}_0}{|\delta \mathfrak{E}|}$$

Dodin and Fisch, submitted to *PRE*

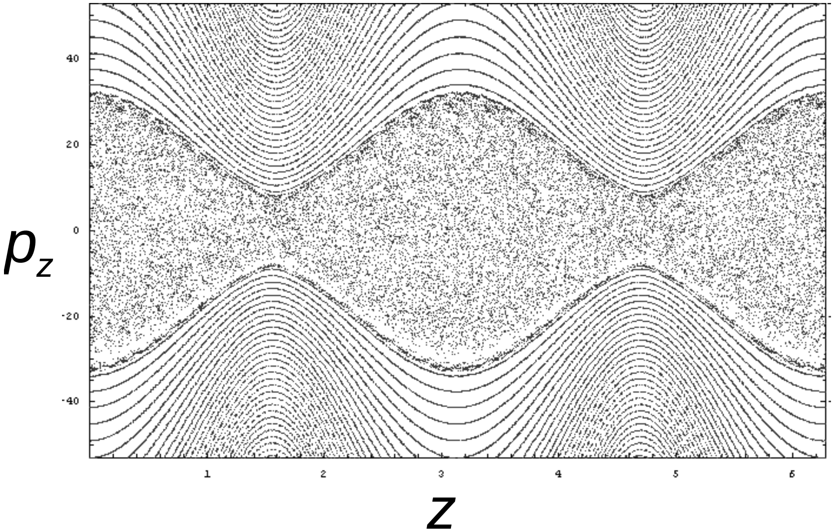
Attosecond Electron Bunches: $\partial_x = \partial_y = 0$

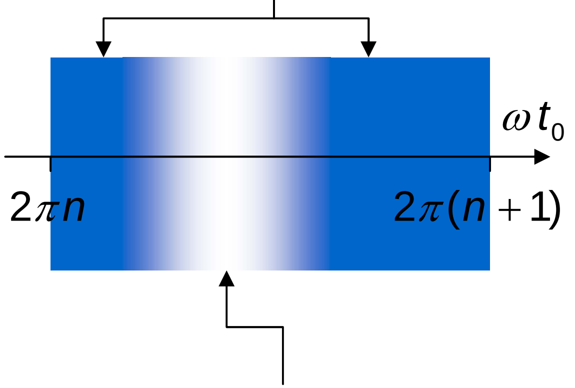


$$a_0 = eE/m\omega c$$

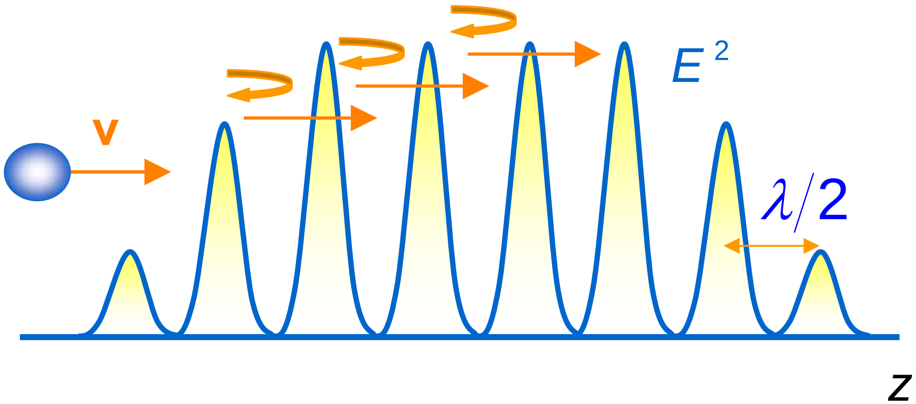
$$\frac{p_z}{mc} \sim a_0 \gg 1$$

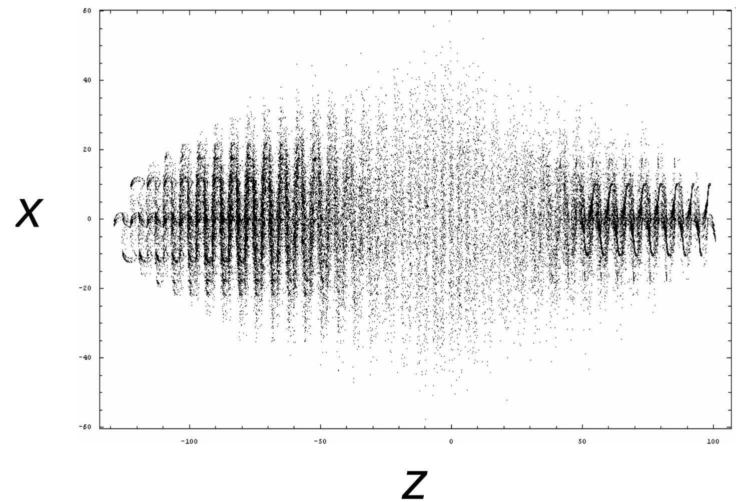
$$\epsilon \equiv \frac{\langle v \rangle}{\omega L} \sim 1$$

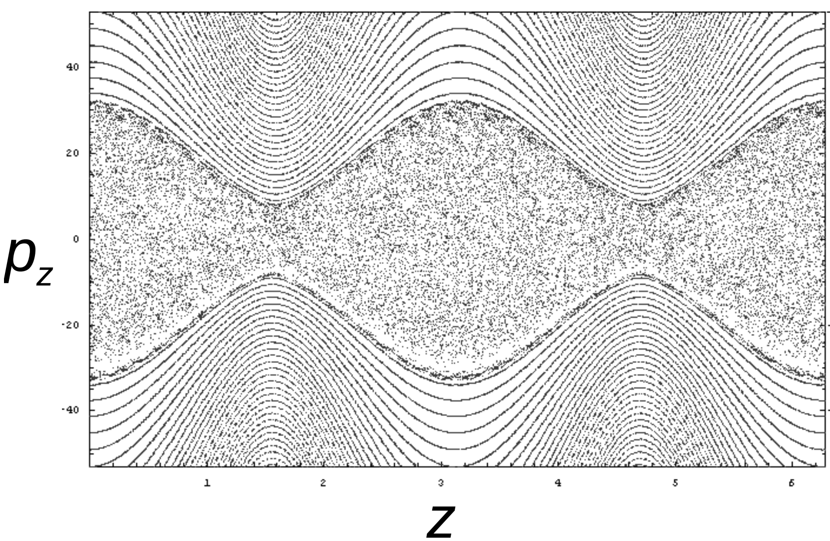


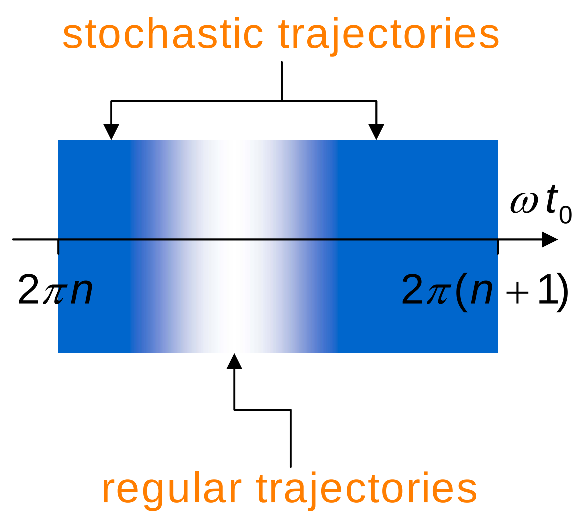


Attosecond Electron Bunches: $\partial_x = \partial_y = 0$

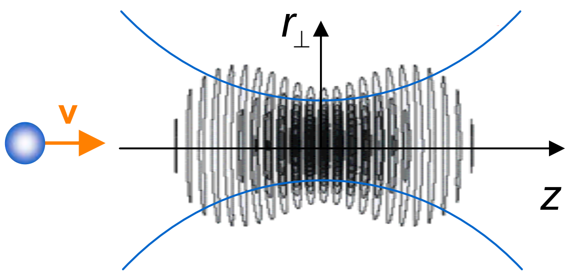




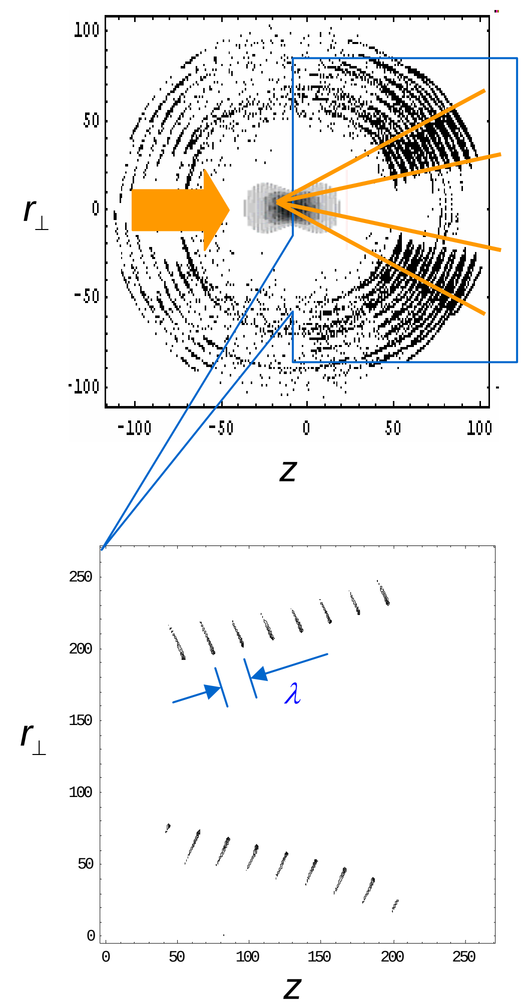




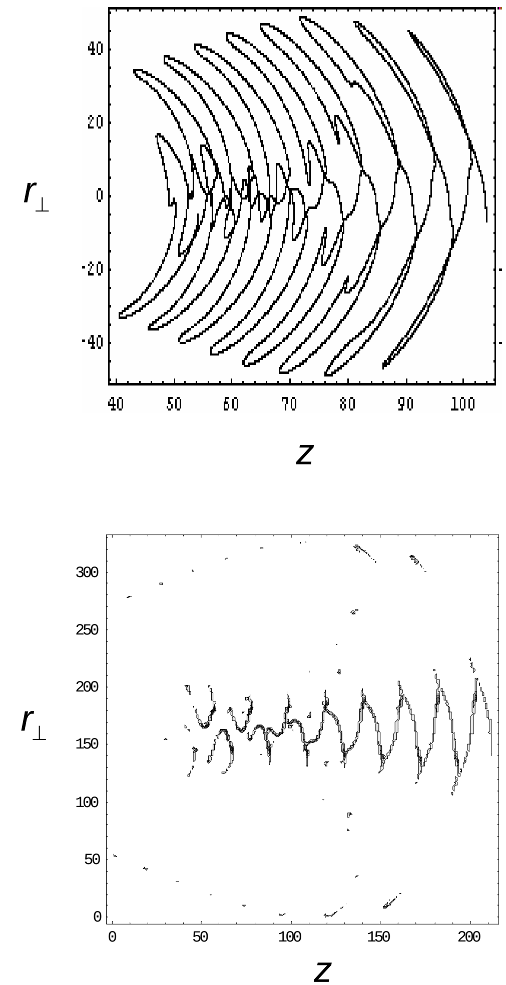
Focused Field: Standing Wave



standing wave



traveling wave



- Chirped energy distribution, like in

  Naumova *et al* (2004);
  Nees *et al* (2005)

How do we break adiabaticity?

average velocity of the particle

longest period of particle oscillations

$$\epsilon \equiv \frac{\langle v \rangle T}{L} \geq 1$$

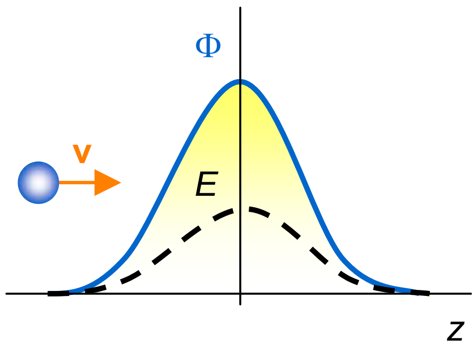
spatial scale of the field

$$\frac{\langle v \rangle}{\omega L} \geq 1$$

fast particle

?

Ponderomotive Potential ≡ Energy of Dipole Interaction



$$E(z,t) = \mathrm{Re}\left[\tilde{E}(z)\exp(-i\omega t)\right]$$

$$F_{\mathrm{PMF}}(z,t) = eE(Z + z_{\mathrm{osc}}, t)$$

$$F_{\mathrm{PMF}} \approx eE(Z,t) + ez_{\mathrm{osc}} \cdot \partial E(Z,t)/\partial Z$$

$$\tilde{z}_{\mathrm{osc}} \approx -e\tilde{E}(Z)/m\omega^2$$

$$\langle F_{\mathrm{PMF}} \rangle \approx -\Phi'$$

$$\Phi = e^2 \left|\tilde{E}\right|^2 / 4m\omega^2$$

Gaponov and Miller (1958)

$$F_{\mathrm{PMF}} \approx eE(Z,t) + \mathcal{P} \cdot \partial E(Z,t)/\partial Z$$

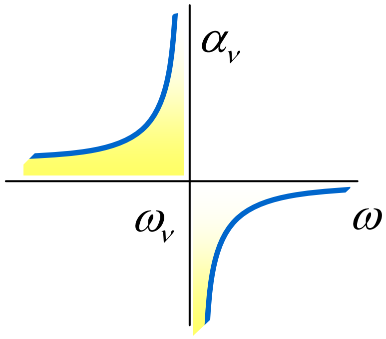
$$\tilde{\mathcal{P}} \equiv ez_{\mathrm{osc}} = \alpha(\omega)\tilde{E}(Z)$$

$$\alpha = -e^2/m\omega^2$$

$$\Phi = -\alpha\left|\tilde{E}\right|^2/4$$

Particles Exhibiting Natural Oscillations

$$\Phi = -\left(\tilde{\mathbf{E}}^* \cdot \boldsymbol{\alpha} \cdot \tilde{\mathbf{E}}\right)/4$$



$$\alpha_\Omega \propto -\frac{1}{\omega - \Omega}$$

- Atoms, molecules – quantum oscillators
- Atomic cluster (core + electron cloud)

$$\Phi = \frac{1}{4}|E|^2 a^3 \frac{\omega_p^2}{3\omega^2 - \omega_p^2} \propto \frac{1}{\omega - \omega_p/\sqrt{3}}$$

- Particle in a dc magnetic field

$$\Phi = \frac{e^2 |E_\parallel|^2}{4m\omega^2} + \frac{e^2 |E_\circlearrowright|^2}{4m\omega(\omega + \Omega)} + \frac{e^2 |E_\circlearrowleft|^2}{4m\omega(\omega - \Omega)}$$

Gaponov and Miller (1958); Motz and Watson (1967); Minogin and Letokhov (1987)...

$$\mathfrak{E} \neq \text{invariant}$$

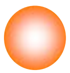
How do we break adiabaticity?

average velocity of the particle

longest period of particle oscillations

$$\epsilon \equiv \frac{\langle v \rangle T}{L} \geq 1$$

spatial scale of the field

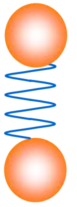


$$\frac{\langle v \rangle}{\omega L} \geq 1$$

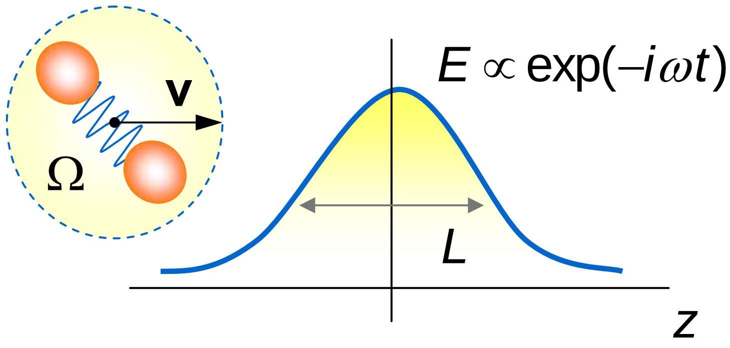
fast particle

$$\frac{\langle v \rangle}{(\omega - \Omega) L} \geq 1$$

resonant oscillator



Effective Ponderomotive Potential

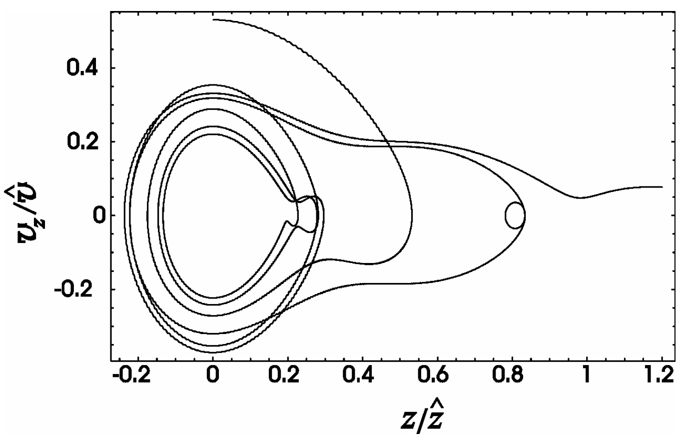


$$L = \frac{1}{2}mv^2 + L_p + \boldsymbol{\mathcal{P}} \cdot \mathbf{E}$$

$$L_p = \frac{1}{2}M_{ij}\dot{\mathcal{P}}_i\dot{\mathcal{P}}_j - R_{ij}\dot{\mathcal{P}}_i\mathcal{P}_j + \frac{1}{2}Q_{ij}\mathcal{P}_i\mathcal{P}_j$$

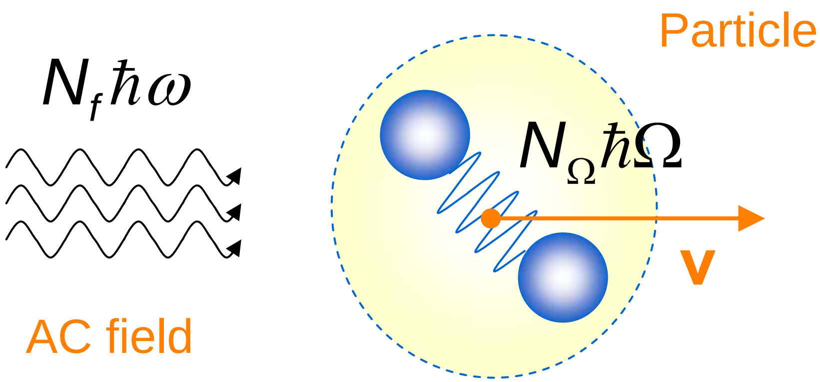
$$mv^2/2 + \underbrace{\Phi(\mathbf{r}) - \frac{\omega - \Omega}{\Omega}\mathcal{E}_\Omega}_{\Phi_{\text{eff}}} = \text{const}$$

e.g., $\Phi_{\text{eff}} = \Phi + \mu(B - B_{\text{resonant}})$



Dodin and Fisch, *PLA* (2006)

Quantum Interpretation



$$\mathcal{E} = mv^2/2 + \Phi$$

$$\Phi = -\langle \mathbf{p} \cdot \mathbf{E} \rangle / 2$$

$$\mathcal{E}_\Omega = N_\Omega \hbar\Omega$$

$$\mathcal{E} + \hbar N_\Omega \Omega + \hbar N_f \omega = \text{const}$$

$$N_\Omega + N_f = \text{const}$$

$$mv^2/2 + \underbrace{\Phi(\mathbf{r}) - \frac{\omega - \Omega}{\Omega}\mathcal{E}_\Omega}_{\Phi_{\text{eff}}} = \text{const}$$

Nonadiabatic Acceleration, Trapping, and Cooling

$$\frac{mv^2}{2} + \Phi - \frac{\omega - \Omega}{\Omega}\mathcal{E}_\Omega = \text{const}$$

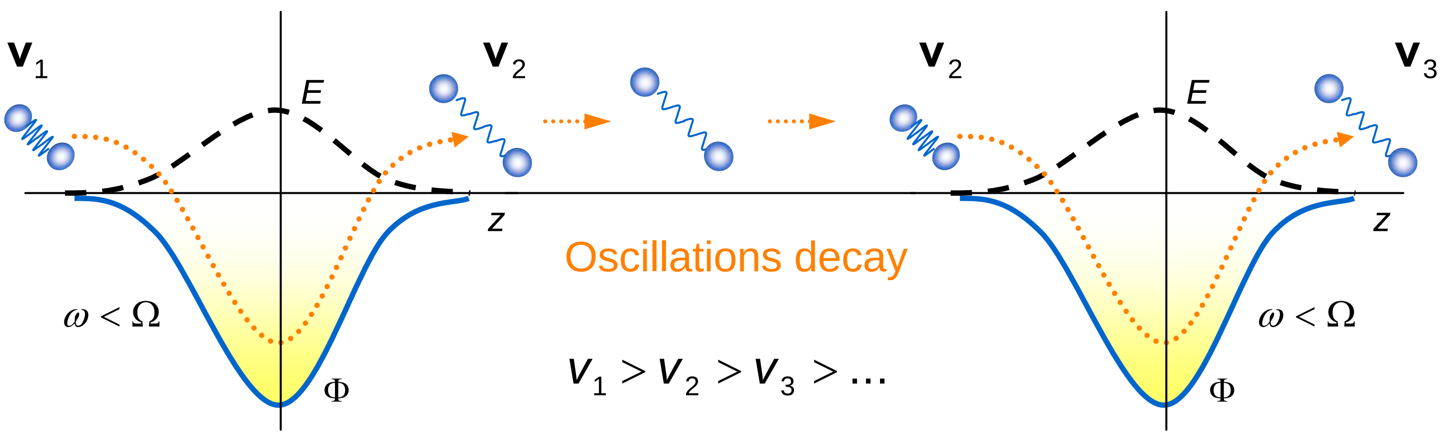
$$\Delta\mathcal{E}_\Omega \sim \left(\frac{\Omega}{\omega - \Omega}\right)\Phi \gg T \geq \Phi$$

$$t \to \infty: \quad \Delta\left(\frac{mv^2}{2}\right) = \frac{\omega - \Omega}{\Omega}\Delta\mathcal{E}_\Omega$$

$$0 < \Delta\mathcal{E}_\Omega \gg \mathcal{E}_{\Omega,0}$$

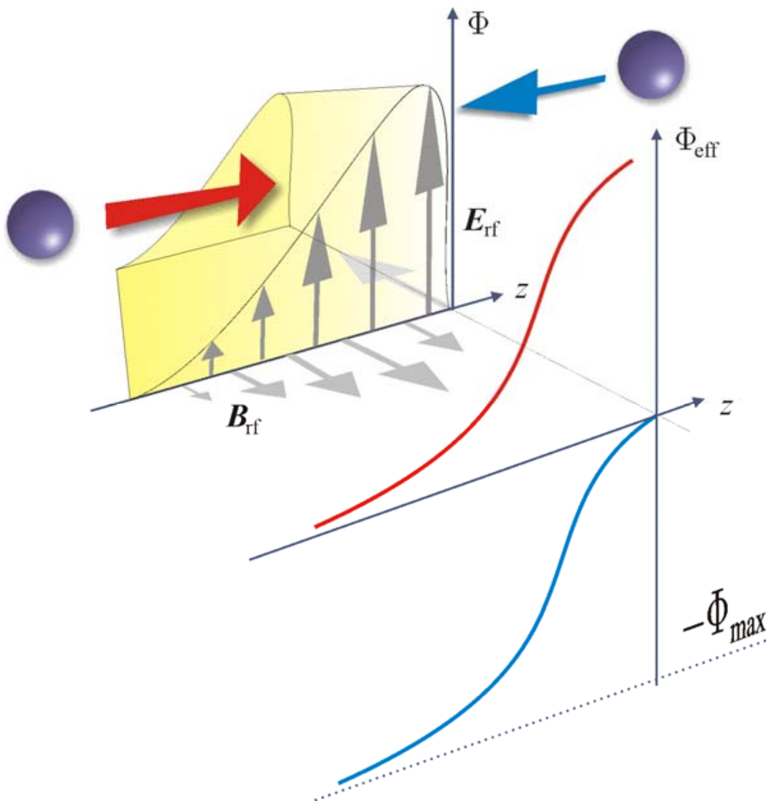
Heating and deceleration

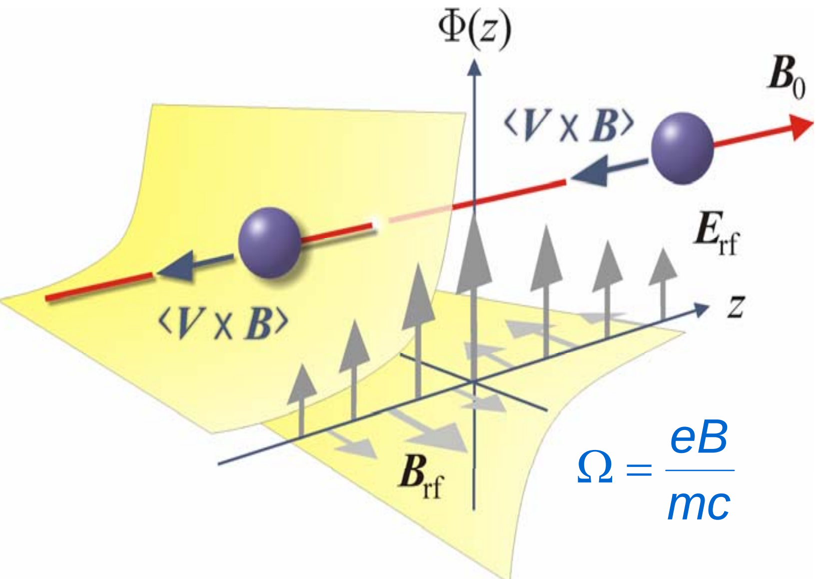
Heating and deceleration



Asymmetric Ponderomotive Potential

$$\langle \mathbf{F} \rangle = -\nabla \Phi_{\text{eff}}(\mathbf{r}, \mathcal{E}_\Omega), \qquad \Phi_{\text{eff}} = \Phi(\mathbf{r}) - \frac{\omega - \Omega}{\Omega} \mathcal{E}_\Omega$$



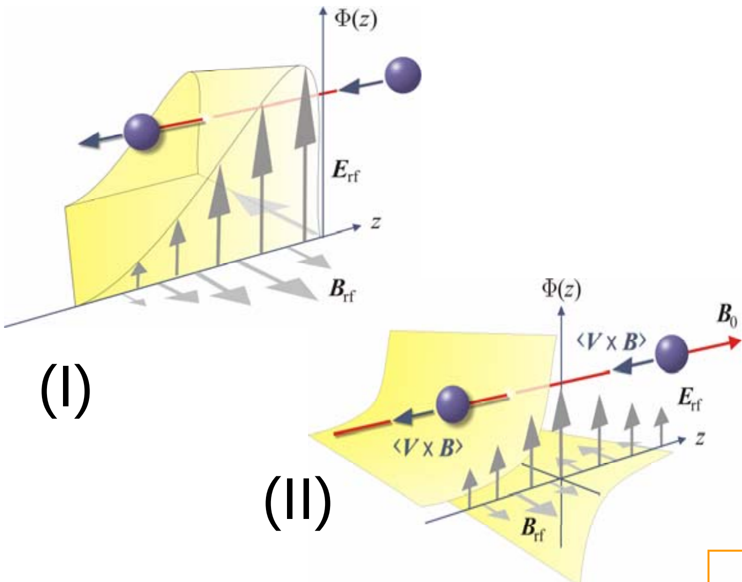


$$\Omega = \frac{eB}{mc}$$

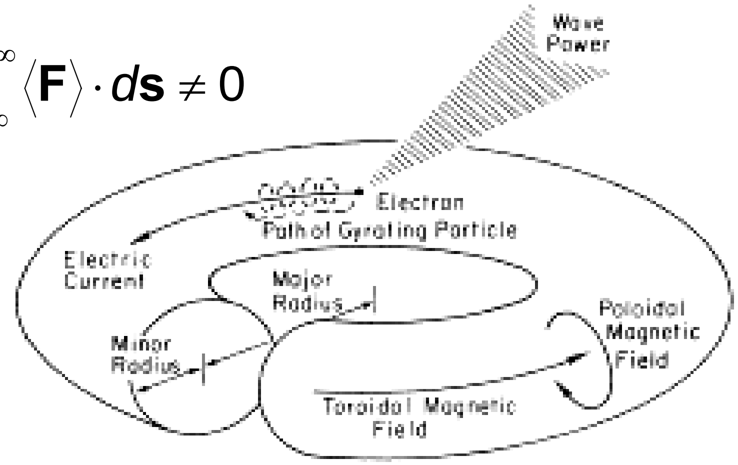
$$\Phi = -\frac{1}{4}\left(\mathbf{E}^* \cdot \boldsymbol{\alpha} \cdot \mathbf{E}\right) \propto \frac{|E_\nu|^2}{\omega - \Omega(z)}$$

Dodin and Fisch, *PRE* (2005); Raizen *et al* (2005); Ruschhaupt and Muga (2004)...

Current Drive



$$\int_{+\infty}^{-\infty} \langle \mathbf{F} \rangle \cdot d\mathbf{s} \neq 0$$



From [Fisch, *RMP* (1987)]

$$\text{Efficiency} = \frac{\text{Current}}{\text{Power}}$$

- Efficiency can be larger than that of conventional schemes
- Phase space conservation limits the amount of minimum heating

Fisch *et al*, *PRL* (2003); Suvorov and Tokman (1988)...

Main Message

As compared to truly conservative forces, the ponderomotive force is a more advanced tool for manipulating both charged and neutral particles.

Summary

- **Ponderomotive force is an advanced tool for particle manipulation**

- **Manipulating elementary particles:**
  - Quantum analogy, quantized energy levels in ponderomotive wells
  - Nonadiabatic tunneling
  - Attosecond electron bunches

- **Manipulating natural oscillators:**
  - Nonadiabatic ponderomotive potential, approximate integral
  - Ponderomotive cooling
  - One-way walls, current drive, phase-space limitations

Related Publications

- I. Y. Dodin and N. J. Fisch, *Nonadiabatic tunneling in ponderomotive barriers*, submitted to Phys. Rev. E (2006).
- I. Y. Dodin and N. J. Fisch, *Nonadiabatic ponderomotive potentials*, Phys. Lett. A **349**, 356 (2006).
- I. Y. Dodin and N. J. Fisch, *Ponderomotive ratchet in a uniform magnetic field*, Phys. Rev. E. **72**, 046602 (2005).
- I. Y. Dodin and N. J. Fisch, *Quantumlike dynamics of classical particles in ponderomotive potentials*, Phys. Rev. Lett. **95**, 115001 (2005).
- I. Y. Dodin and N. J. Fisch, *Variational formulation of the Gardner's restacking algorithm,* Phys. Lett. A. **341**, 187 (2005).
- I. Y. Dodin and N. J. Fisch, *Approximate integrals of radiofrequency-driven particle motion in a magnetic field*, J. Plasma Phys. **71**, 289 (2005).
- I. Y. Dodin, N. J. Fisch, and J.M. Rax, *Ponderomotive barrier as a Maxwell demon*, Phys. Plasmas **11**, 5046 (2004).
- N. J. Fisch, J. M. Rax, and I. Y. Dodin, *Current drive in a ponderomotive potential with sign reversal*, Phys. Rev. Lett. **91**, 205004 (2003); Erratum: Phys. Rev. Lett. **93**, 059902(E) (2004).
- I. Y. Dodin and N. J. Fisch, *Relativistic electron acceleration in focused laser fields after above-threshold ionization*, Phys. Rev. E 68, 056402 (2003).
- I. Y. Dodin, N. J. Fisch, and G. M. Fraiman, *Drift Lagrangian for relativistic particle in intense laser field*, JETP Lett. 78, 202 (2003).