High Harmonics and Attosecond Pulses in Relativistic Regime

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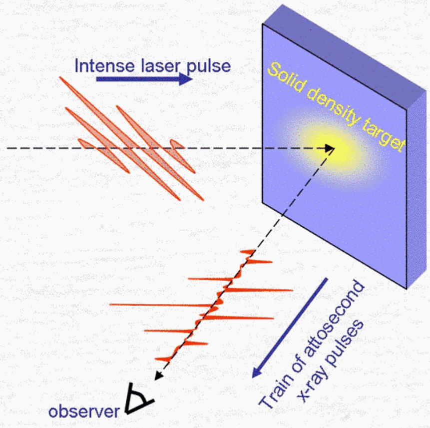

When laser plasma is relativistic?

Dimensionless laser amplitude $a = \frac{eA}{mc^2}$

Electron quiver energy (plane wave) $\mathrm{E} = \frac{a^2}{2} \cdot mc^2$

Electron motion becomes relativistic when $a \approx 1$

$$a = 1 \;\leftrightarrow\; I\lambda^2 = 1.37\times 10^{18}\ \mathrm{W}\ \mu\mathrm{m}^2/\mathrm{cm}^2$$

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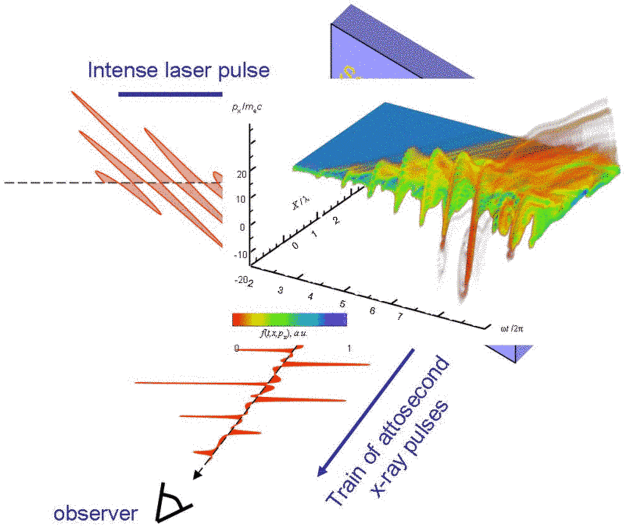
The apparent reflecting point oscillates at relativistic velocities together with the plasma surface

A. Pukhov, **NATURE** Physics, Vol. **2**, p. 439 (2006).



Gordienko, et al., Phys. Rev. Lett. 2004

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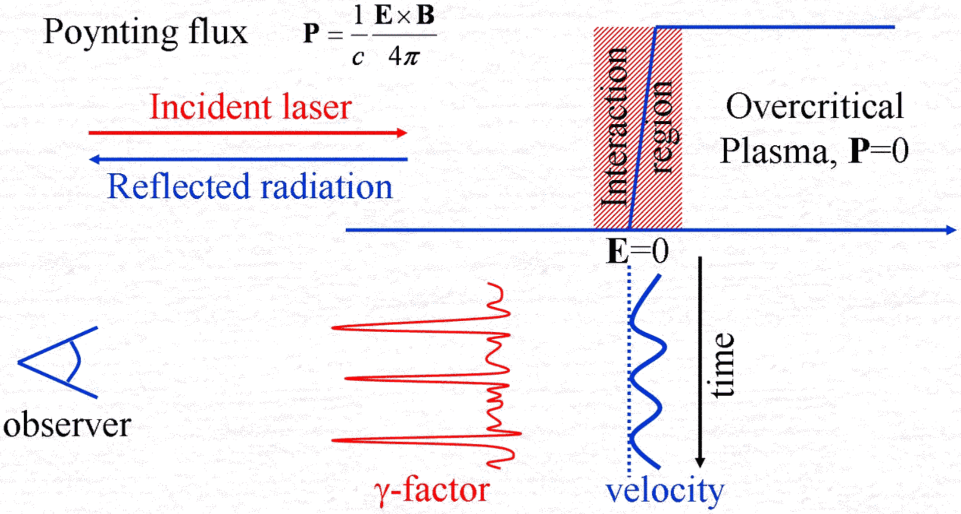
Boundary Condition: **E**=0

S. Gordienko, A. Pukhov, O. Shorokhov, T. Baeva, *Phys. Rev. Letters*, 93, 115002 (2004)

Poynting flux $\mathbf{P} = \frac{1}{c}\frac{\mathbf{E}\times\mathbf{B}}{4\pi}$



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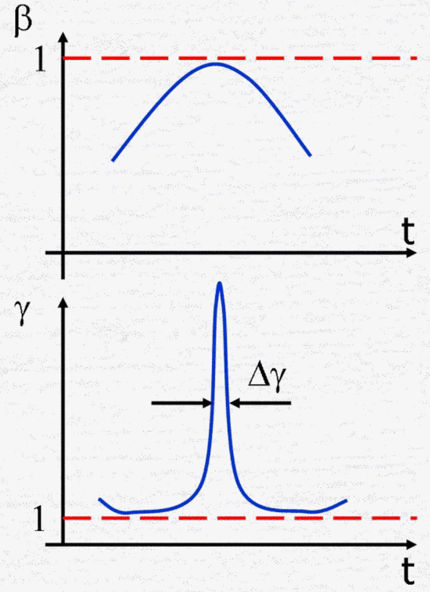
Universal Surface Dynamics



Plasma surface velocity $\beta = v_n/c$ is a smooth function.
At the maximum it can be approximated by a parabola:

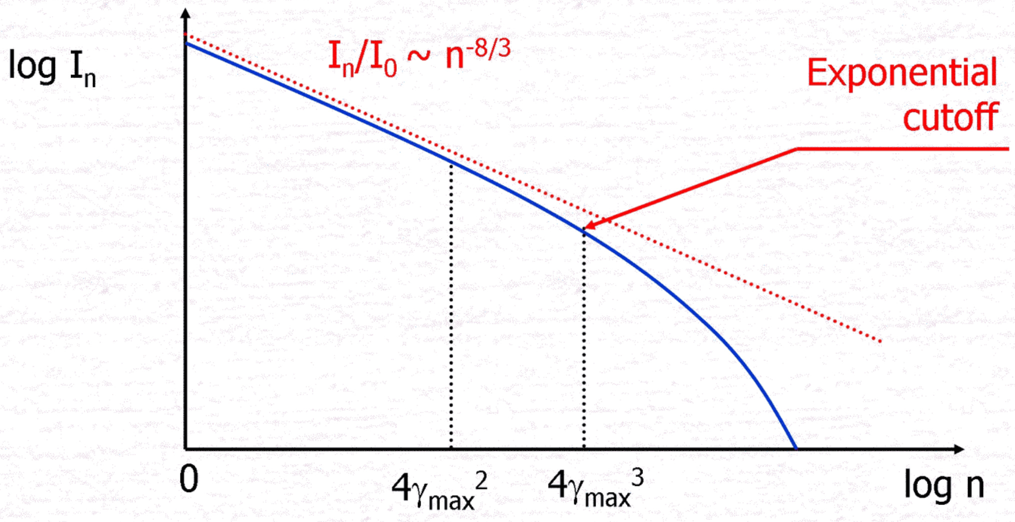
$\beta(t) \approx \beta_{max}(1-\alpha^2 t^2)$,
$\alpha \approx \omega_0$.

Its γ-factor $\gamma = 1/\sqrt{1-\beta^2}$
has a sharp spike of the width

$\Delta\gamma \approx 1/\alpha\gamma_{max}$

Universal Spectra

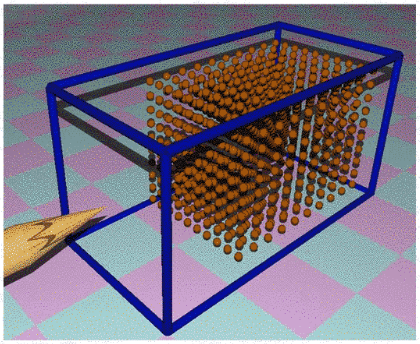
S. Gordienko, A. Pukhov, O. Shorokhov, T. Baeva, *Phys. Rev. Letters*, 93, 115002 (2004)



Virtual Laser Plasma Lab
3D PIC code

VLPL runs on parallel clusters

We use up to 10^9 particles and 10^8 grid cells



$$d_t p = eE + \frac{e}{c}\mathbf{v}\times B$$
$$\partial_t E = c\nabla\times B - 4\pi j$$
$$\partial_t B = -c\nabla\times E$$

Reference:A.Pukhov, J.Plasma Phys, 1999

Motion of the electron fluid boundary:
PIC simulations

S. Gordienko, A. Pukhov, O. Shorokhov, T. Baeva, *Phys. Rev. Letters*, 93, 115002 (2004)

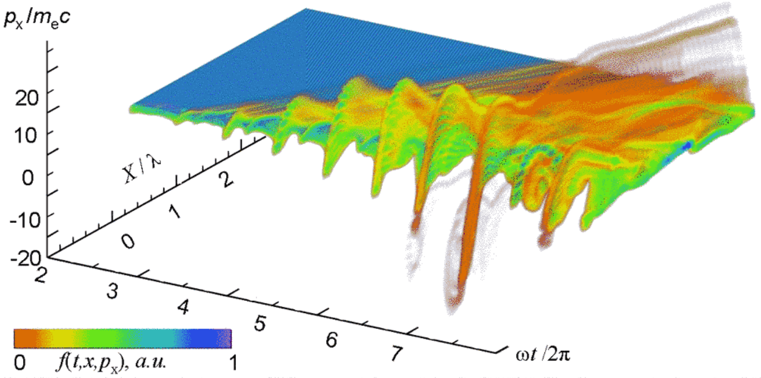
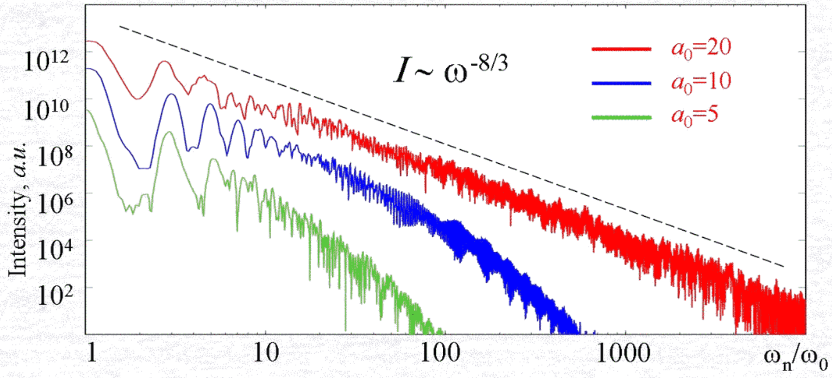


Fig.: Electron distribution function. The helix represents the electron surface motion in the laser field. The reddish downward spikes stand for the surface relativistic motion towards the laser. These spikes are responsible for the zeptosecond pulse generation.

Reflected radiation spectra in 1D PIC simulations

S. Gordienko, A. Pukhov, O. Shorokhov, T. Baeva, *Phys. Rev. Letters*, 93, 115002 (2004)



The Gaussian laser pulse $a=a_0\exp[-(t/\tau)^2]\cos\omega_0 t$ is incident onto an overdense plasma layer with $n=30n_c$.
The color lines correspond to laser amplitudes a_0=5,10,20.
The broken line marks the analytical scaling $I \sim \omega^{-8/3}$.

VULCAN Experiment: Harmonics down to "Water Window"

B. DROMEY, M. ZEPF, et al., **NATURE** Physics, Vol. **2**, p. 456 (2006).

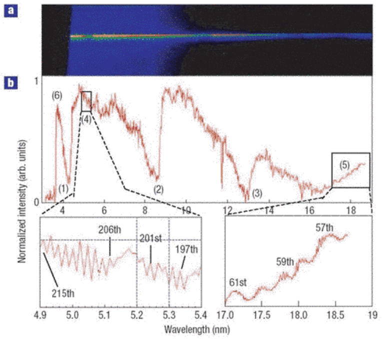
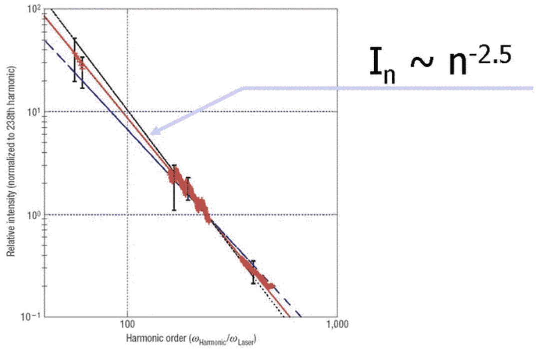
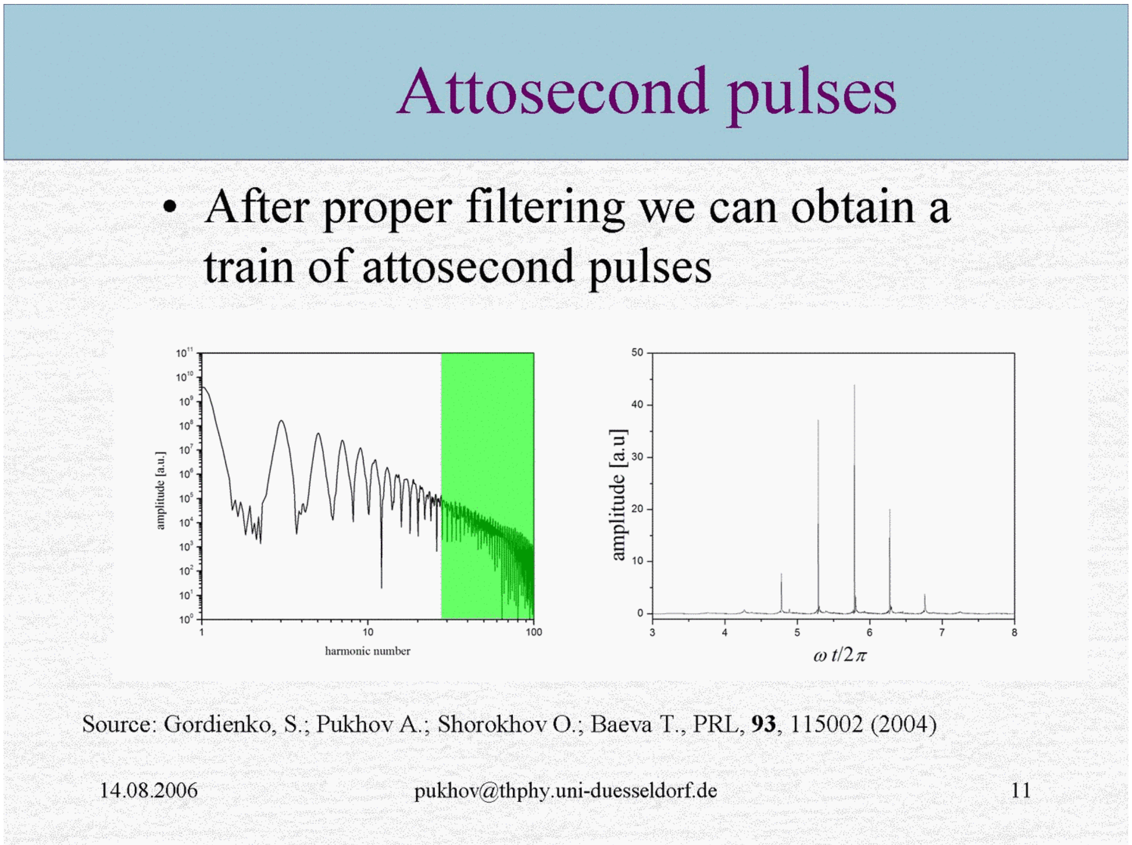


Figure 3 Unprocessed high harmonic spectrum recorded with the extreme-ultraviolet spectrometer. a, Raw CCD image obtained with the double PM setup ($E = 70$ J on target, false colours). **b**, A lineout of **a**. Spectral features: (1) first-order carbon K-edge (4.36 nm), (2) second-order carbon K-edge, (3) third-order carbon K-edge, (4) region of resolved harmonics around 200th order



$I_n \sim n^{-2.5}$

Figure 4 Relative intensity of harmonics normalized to the 238th harmonic (at the carbon K-edge). The lines are fits to the data with the exponent p as a fitting parameter such that $I(n)/I(238) = n^{-p}/238^{-p}$. The best fit (red line) corresponds to a value of $p = 2.5$ confirming harmonic production in the relativistic limit. The error



Attosecond pulses

- After proper filtering we can obtain a train of attosecond pulses

Source: Gordienko, S.; Pukhov A.; Shorokhov O.; Baeva T., PRL, **93**, 115002 (2004)

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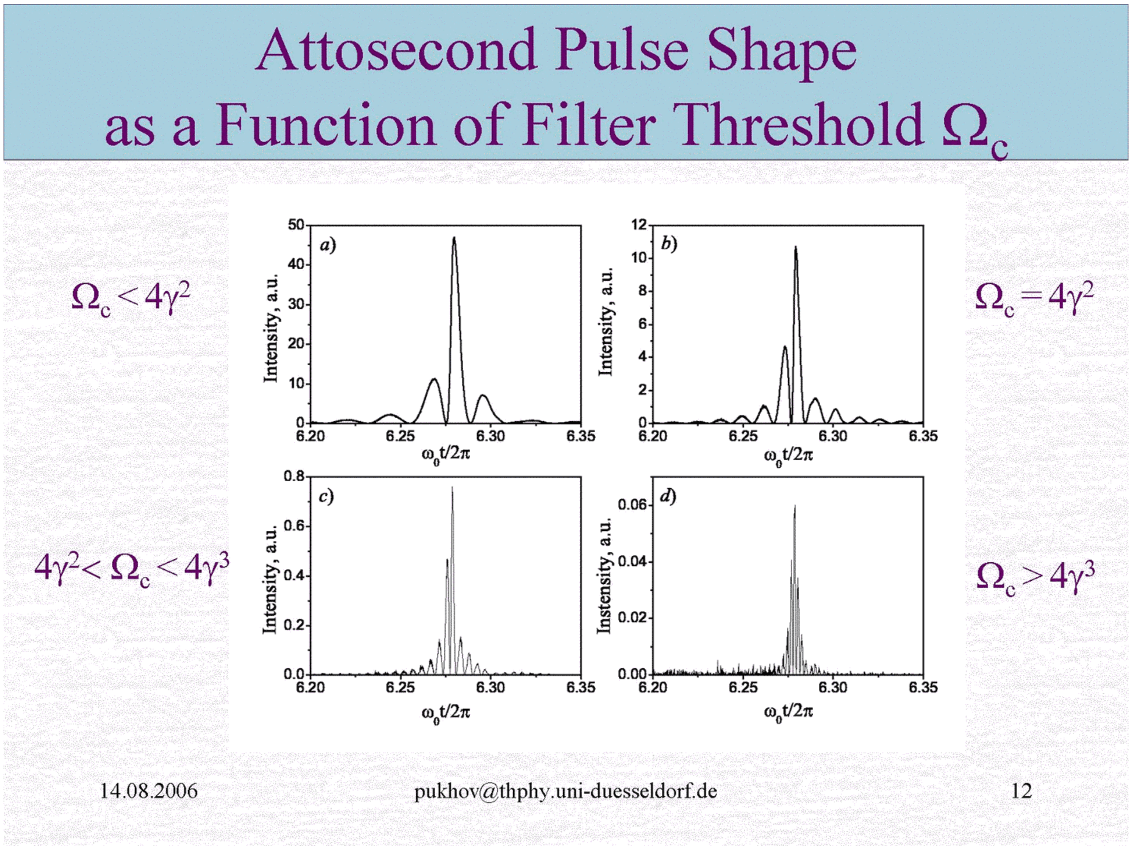

Attosecond Pulse Shape as a Function of Filter Threshold Ω_c

$\Omega_c < 4\gamma^2$

$\Omega_c = 4\gamma^2$

$4\gamma^2 < \Omega_c < 4\gamma^3$

$\Omega_c > 4\gamma^3$

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Shortest Pulse Duration

Gordienko, et al., Phys. Rev. Lett. 2004

Can be zeptosecond!

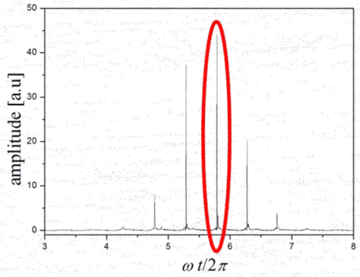
$$\tau_{\text{pulse}} \sim \frac{1}{\gamma_{\max}^3} \sim \frac{1}{a^6}\left(\frac{n_e}{n_c}\right)^2$$

$$\gamma_{\max} \sim a^2 \frac{n_c}{n_e}$$

High harmonics (applications)

Yet many applications...

- molecule tomography
- quantum control
- quantum computing
- etc.



... need ***single*** attosecond pulses!

Can we extract one pulse from the train?

Yes : Relativistic Plasma Control (RPC)

S–Similarity for Ultra-Relativistic Plasmas, $I\lambda^2 \gg 10^{18}$ Wμm²/cm²

Gordienko & Pukhov *Phys. Plasmas* **12**, 043109 (2005)

The similarity parameter (*S*-number) $S = \dfrac{n_e}{a_0 n_c}$

Dynamics of plasmas with ***S=const*** is similar.
Electrons move along the same trajectories,
their momenta scale as $\mathbf{p} = a_0 \hat{\mathbf{p}}(k_0 R, \omega_0 \tau, S)$

The parameter ***S*** has the role of relativistically corrected plasma density.

It separates	relativistically overdense plasmas,	***S***>>1,
from	relativistically underdense ones,	***S***<<1.

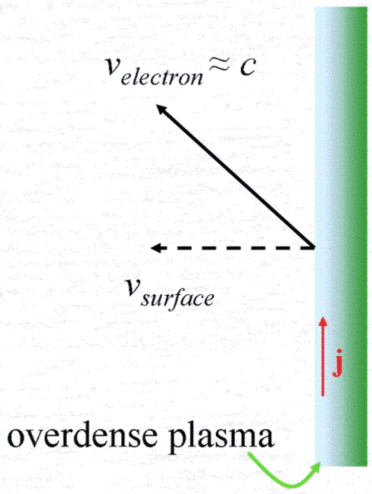
High harmonics (theory)

- Ultra-relativistic similarity theory shows that

$$S = \frac{N_e}{a_0 N_c} = const$$

$$a_0^2 >> 1$$

$$p_\tau \sim a_0 \quad p_n \sim a_0$$



$$v_{surface} \neq v_{electron}$$

High harmonics (theory)

Simple algebra shows...

$$\mathbf{p}_n = a_0 \mathbf{P}_n(S, \omega t)$$
$$\mathbf{p}_\tau = a_0 \mathbf{P}_\tau(S, \omega t)$$

$$\beta_s(t) = \frac{p_n(t)}{\sqrt{m_e^2 c^2 + p_n^2(t) + p_\tau^2(t)}} = \frac{P_n(t)}{\sqrt{P_n^2(t) + P_\tau^2(t)}} - O(a_0^{-2})$$

$$\gamma_s(t) = \frac{1}{\sqrt{1 - \beta_s^2(t)}} = \sqrt{1 + \frac{P_n^2(t)}{P_\tau^2(t)}} + O(a_0^{-2}),\ p_\tau \neq 0$$

$$\gamma_s(t) = \sqrt{\frac{p_n^2 + m_e^2 c^2}{m_e^2 c^2}} \propto a_0,\ \ p_\tau = 0$$

Apparent Reflection Point

- External observer sees the reflection at x(t), where

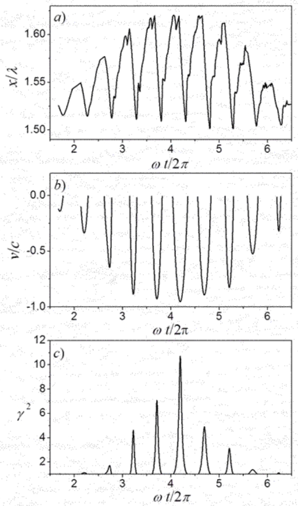
$$E_{||}(x(t))=0$$

- Equation for the apparent reflection point

$$E_{||}^{i}(x\text{-}ct)+E_{||}^{r}(x+ct)=0$$

PIC Plasma Modelling

- Results of PIC simulations



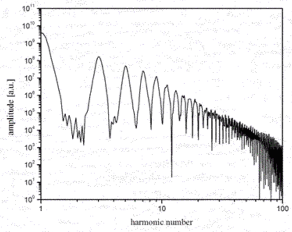
Plasma surface oscillations.
Initial position of the plasma slab: x_0=1.5
Displacement due to light pressure.

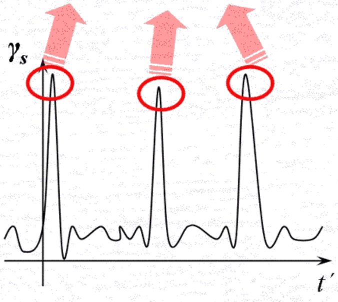
Velocity of the plasma surface (only the negative velocities are of physical interest).
The velocity is a **smooth** function of time!

The γ – factor of the plasma surface has sharp spikes!

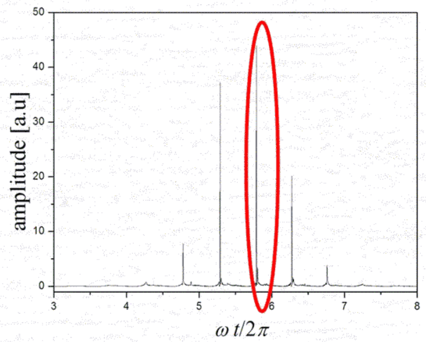
High harmonics (generation)

- At the times when
  $\mathbf{p}_\tau = 0$
  high harmonics
  are generated!
- $\mathbf{p}_\tau = \frac{e}{c}\mathbf{A}_\tau$
- The high harmonics
  are generated at those
  times when $\mathbf{A}_\tau = 0$





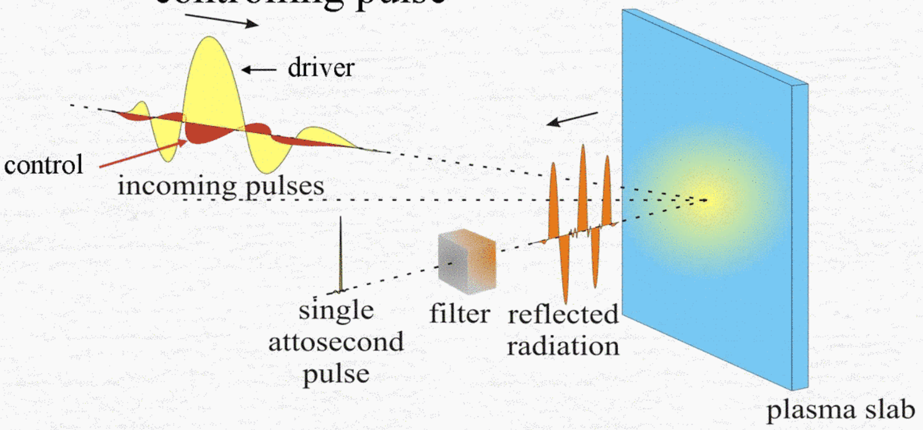
Single attosecond pulse



$\Leftrightarrow \mathbf{A}_\tau = 0$ once!

Relativistic plasma control

- Idea: manage the plasma dynamics by a controlling pulse



Source: T. Baeva, S. Gordienko, A. Pukhov, submitted to PRL, **arxiv: physics/0601044**

Relativistic plasma control

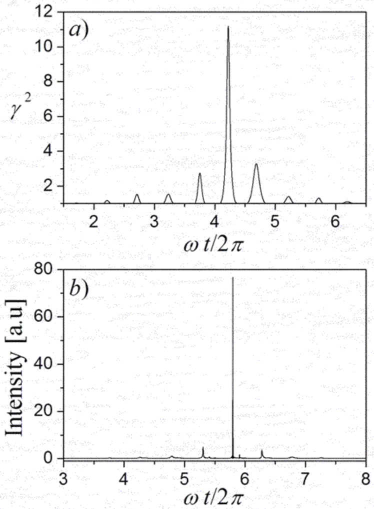
- PIC simulation results

Simulation parameters:

Driving pulse: ω_0=1, a_0=20

Controlling pulse: ω_d=1.25, a_d=6

Phase shift: $\Delta\varphi=\pi/8$



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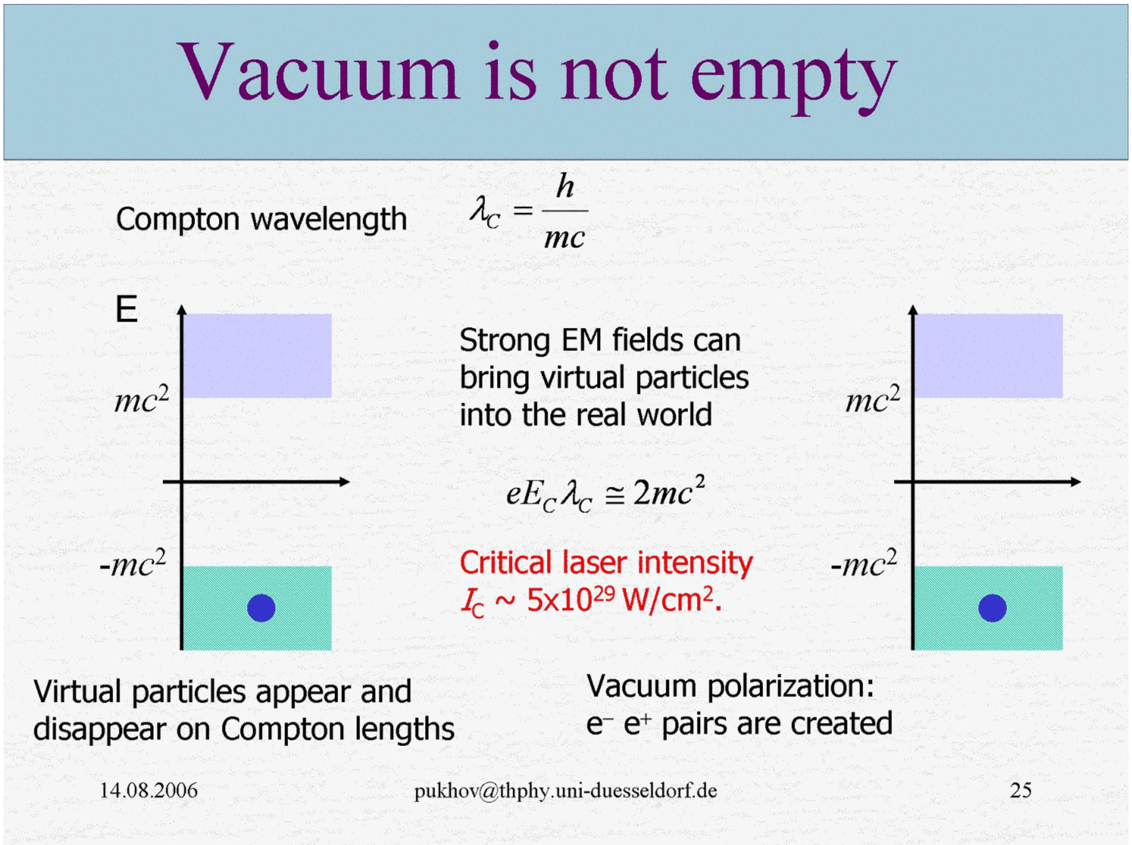
Coherent Harmonics Focusing

Gordienko, Pukhov, Shorokhov, Baeva, *Phys. Rev. Lett.* **94**, 103903 (2005)

Vacuum

Polarization

(Schwinger limit)



Vacuum is not empty

Compton wavelength $\lambda_C = \frac{h}{mc}$

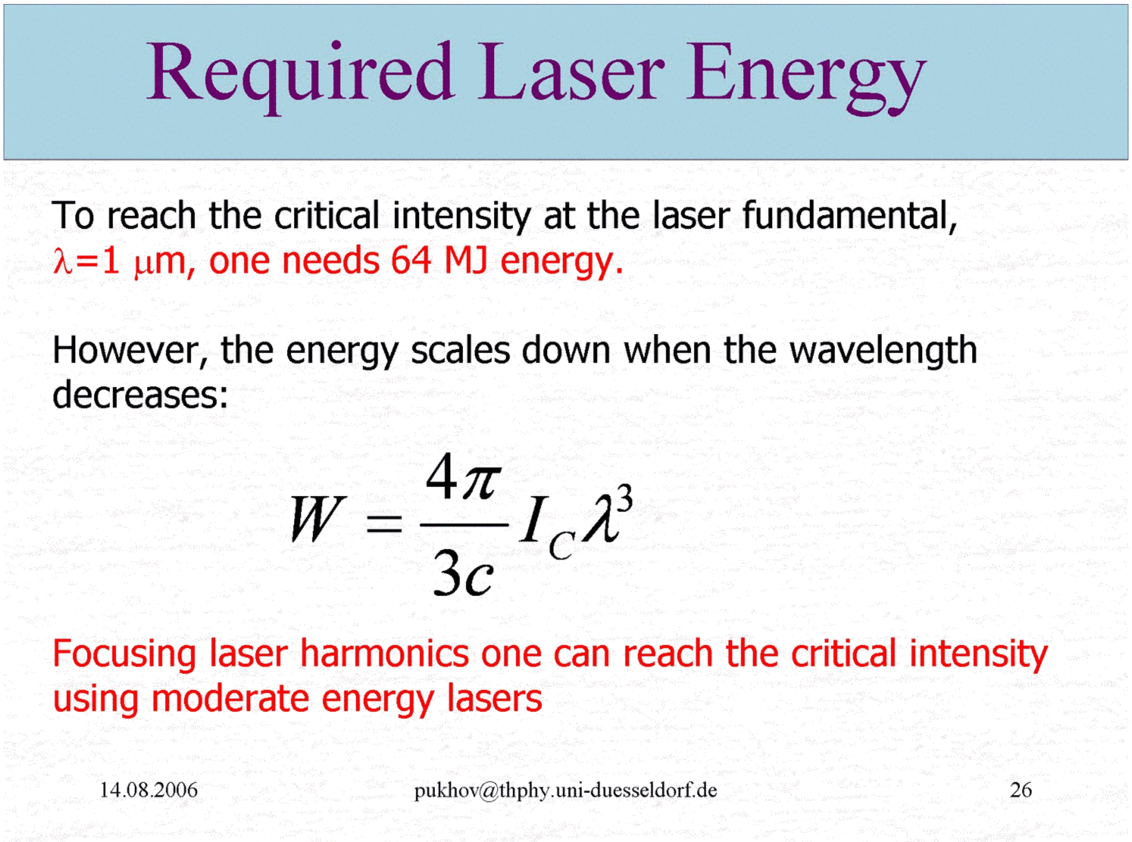
Strong EM fields can bring virtual particles into the real world

$$eE_C\lambda_C \cong 2mc^2$$

Critical laser intensity
I_C ~ 5x10^{29} W/cm^2.

Virtual particles appear and disappear on Compton lengths

Vacuum polarization:
e^- e^+ pairs are created



Required Laser Energy

To reach the critical intensity at the laser fundamental, λ=1 μm, one needs 64 MJ energy.

However, the energy scales down when the wavelength decreases:

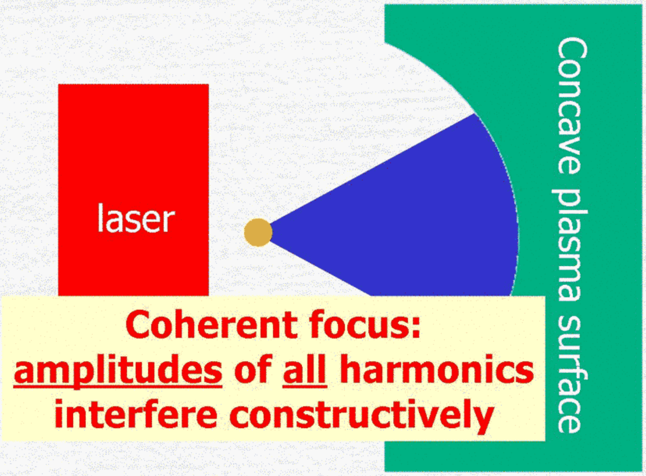
$$W = \frac{4\pi}{3c} I_C \lambda^3$$

Focusing laser harmonics one can reach the critical intensity using moderate energy lasers

Coherent Harmonics Focusing: plasma harmonics are phase-locked!

VLPL

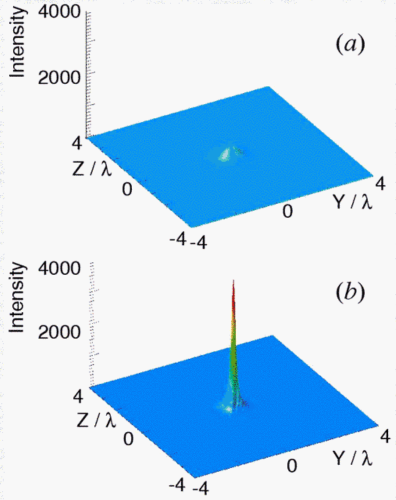
Gordienko, Pukhov, Shorokhov, Baeva, *Phys. Rev. Lett.* **94**, 103903 (2005)



laser

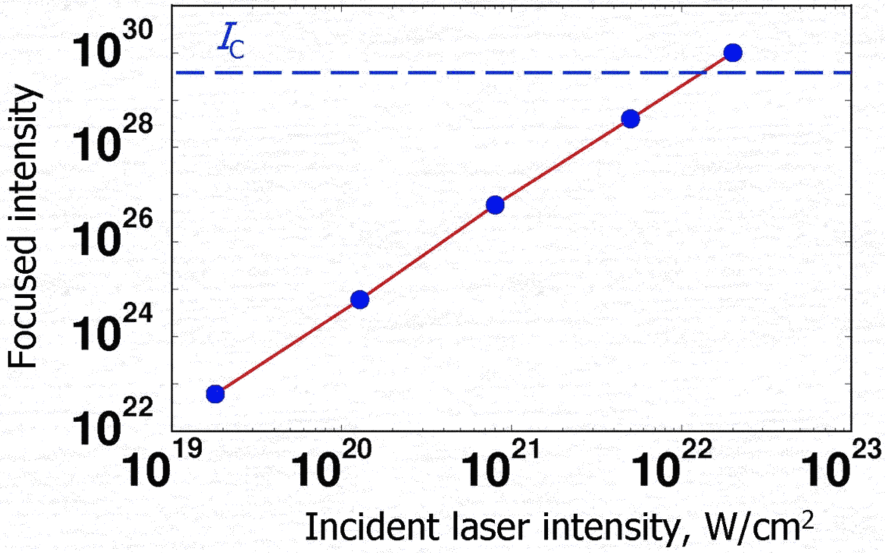
Concave plasma surface

Coherent focus:
<u>amplitudes</u> of <u>all</u> harmonics interfere constructively



Scaling of the CHF intensity

Gordienko, Pukhov, Shorokhov, Baeva, *Phys. Rev. Lett.* **94**, 103903 (2005)



Conclusions

1. Theory of high harmonis generation at plasma surfaces is developed.
2. The harmonics spectra are the universal power law $I_n{\sim}n^{-8/3}$
3. Harmonics appear as a train of attosecond pulses
4. Relativistic plasma control allows to seect a single attosecond pulse
5. Plasma harmonics are phase-locked, coherent, and can be focused to give huge intensities