Strong Correlations, Electronic Ordered States, and Potts-Nematicity in Twisted Bilayer Graphene

Rafael M. Fernandes

University of Minnesota

U.S. DEPARTMENT OF ENERGY | Office of Science

KITP, Jan 23rd, 2019

In collaboration with:

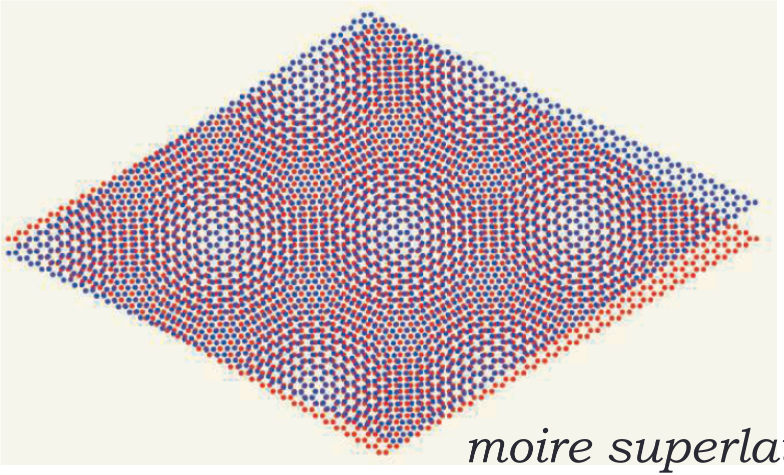
Jörn Venderbos, University of Pennsylvania

Venderbos and RMF, Phys. Rev. B **98**, 245103 (2018)

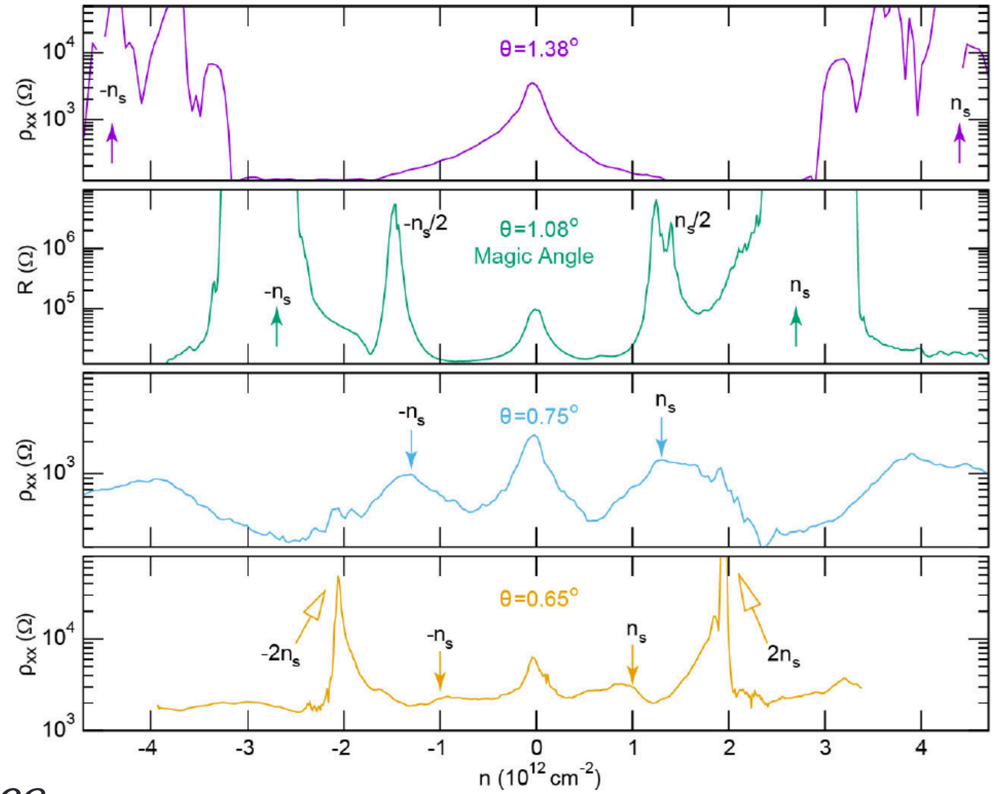
Acknowledgments:
O. Vafek and J. Kang (FSU);
P. Jarillo-Herrero (MIT); L. Fu (MIT)

Surprises in twisted bilayer graphene (TBG)

- emergence of correlated insulator at "half filling" for magic twist angle



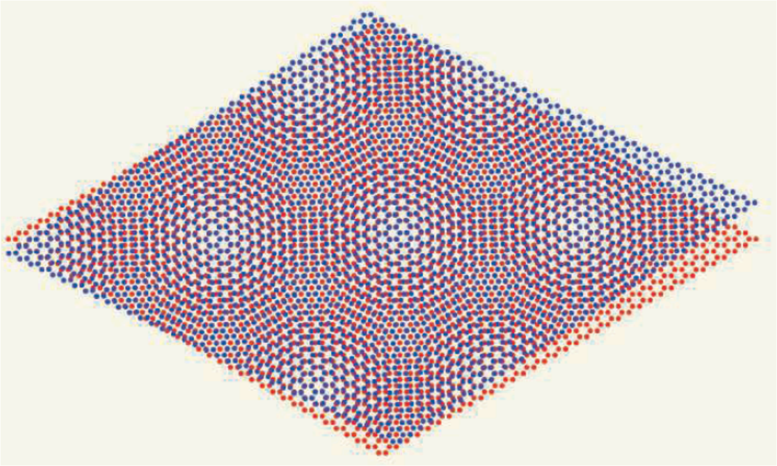
moire superlattice

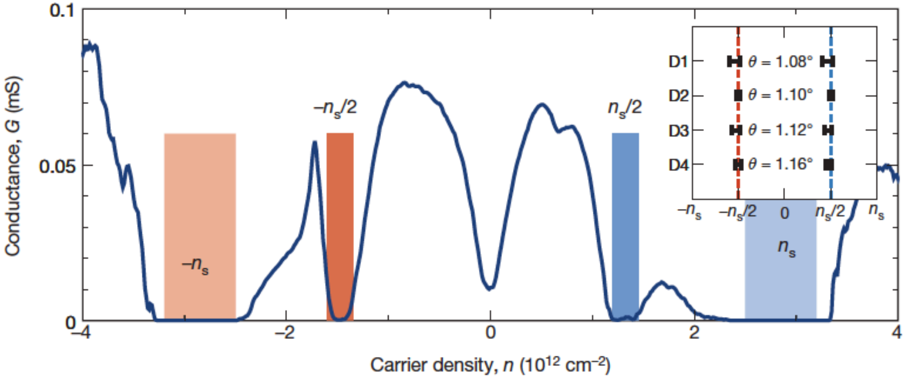


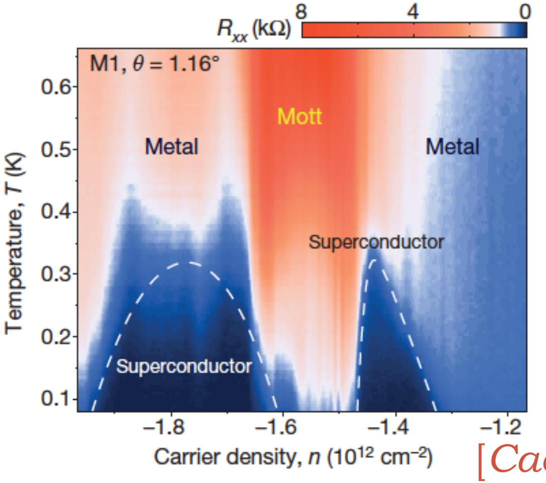
[Cao et al, Nature (2018)][2]

Surprises in twisted bilayer graphene (TBG)

- emergence of correlated insulator at “half filling” for magic twist angle
- superconductivity near an insulating state



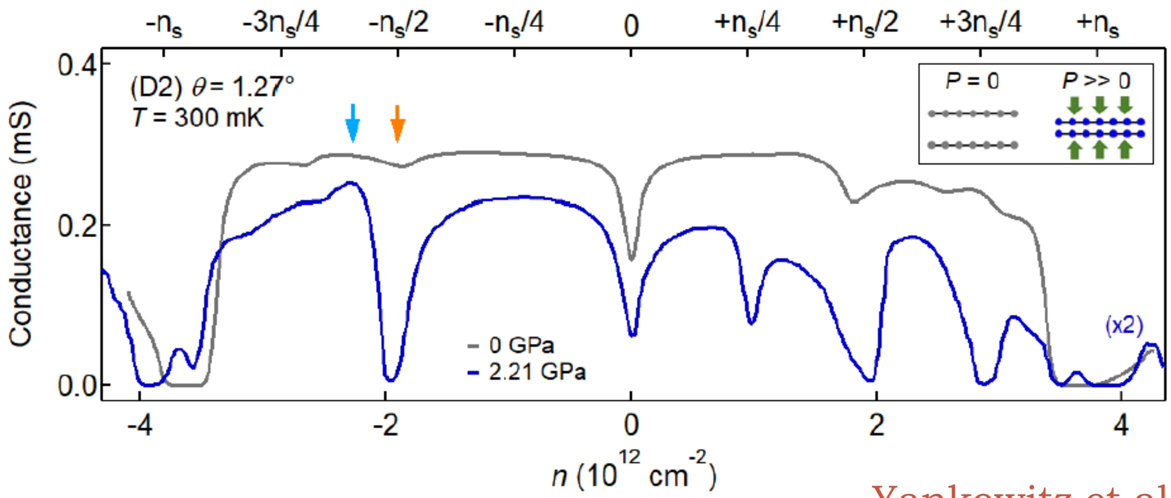




[Cao et al, Nature (2018)][2]

Surprises in twisted bilayer graphene (TBG)

- Electronic states can also be controlled with pressure



Yankowitz et al, arXiv (2018)

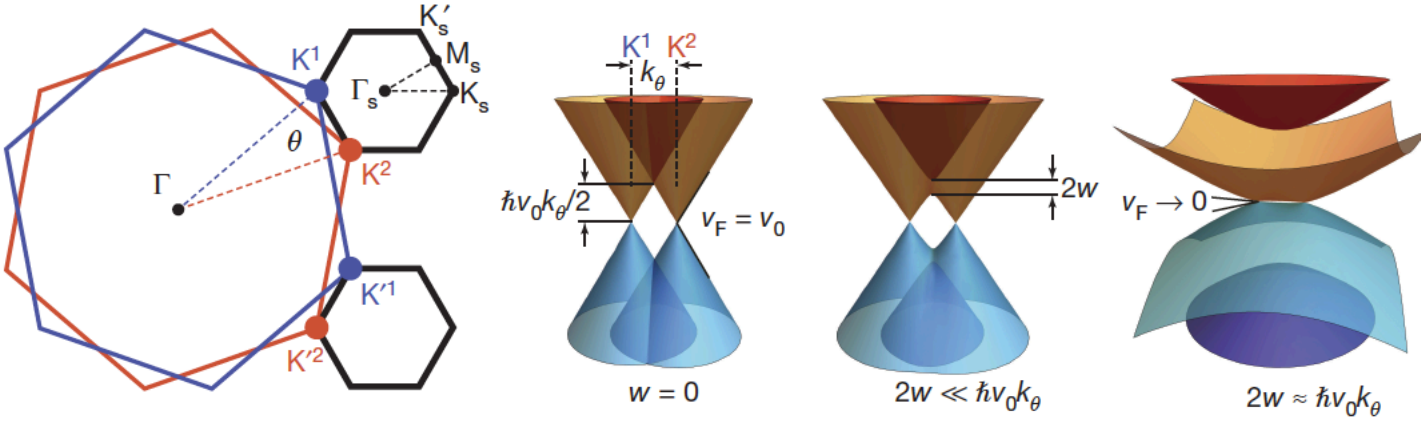
- Similar phenomena seen in other twisted systems (TB^{2}G, trilayer graphene, …) P. Kim group, F. Wang group

Surprises in twisted bilayer graphene (TBG)

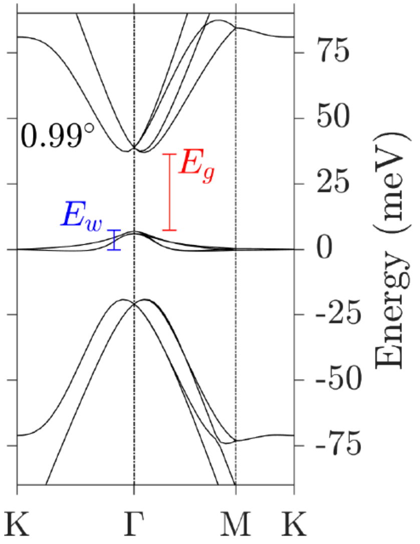
- Magic angle: isolated nearly-flat bands in the moiré Brillouin zone.

  Bistritzer and MacDonald, PNAS (2011)

- Small bandwidth: interactions become important



Cao et al, Nature (2018)



Kaxiras et al, arXiv (2019)

Surprises in twisted bilayer graphene (TBG)

- New platform to understand strong interactions, superconductivity, and their interplay with topology

- Bringing two communities together

Surprises in twisted bilayer graphene (TBG)

- New platform to understand strong interactions, superconductivity, and their interplay with topology

- Bringing two communities together

The New York Times

At Rio's Beaches, Kicks Go Over Net

Surprises in twisted bilayer graphene (TBG)

- New platform to understand strong interactions, superconductivity, and their interplay with topology

- Bringing two communities together

Outline

1. Low-energy model and interactions

2. Strong-coupling phase diagram: spin, orbital, and superconducting degrees of freedom

3. Potts-nematicity and nematic superconductivity

Outline

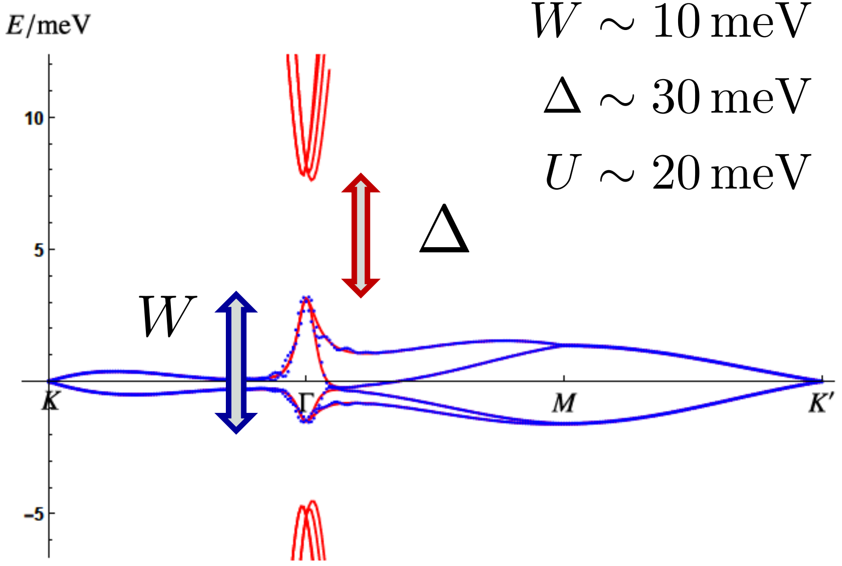
1. Low-energy model and interactions

2. Strong-coupling phase diagram: spin, orbital, and superconducting degrees of freedom

3. Potts-nematicity and nematic superconductivity

Low-energy model

- Energy scales (estimates):



$W \sim 10\,\text{meV}$

$\Delta \sim 30\,\text{meV}$

$U \sim 20\,\text{meV}$

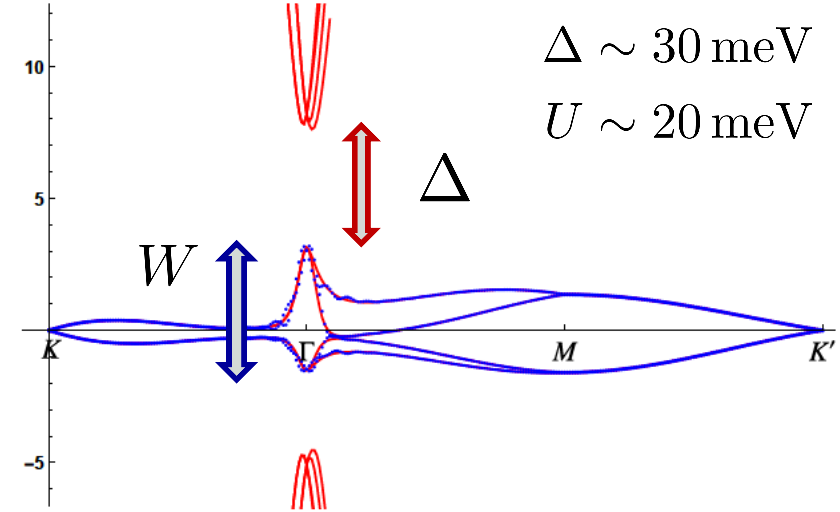
Kang & Vafek, PRX (2018)

- ➢ focus only on narrow bands as long as $\Delta > U$
- ➢ most likely, the narrow-band subsystem is in the intermediate coupling regime $U \sim W$
- ➢ starting point: weak-coupling or strong-coupling?

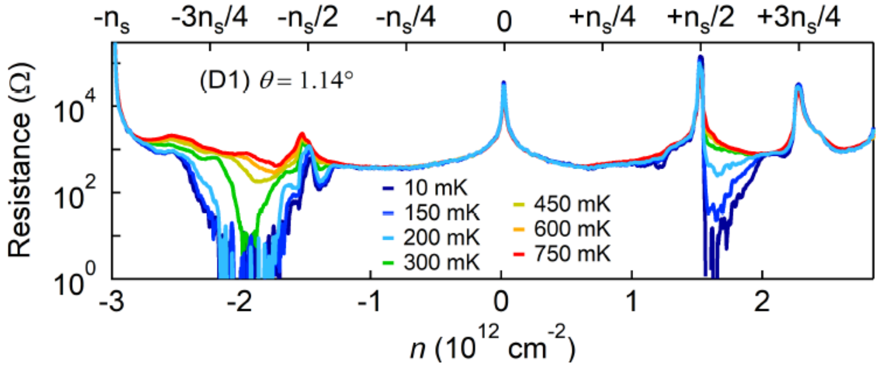
Low-energy model

- Energy scales (estimates):

$$W \sim 10\,\mathrm{meV}$$
$$\Delta \sim 30\,\mathrm{meV}$$
$$U \sim 20\,\mathrm{meV}$$



Kang & Vafek, PRX (2018)

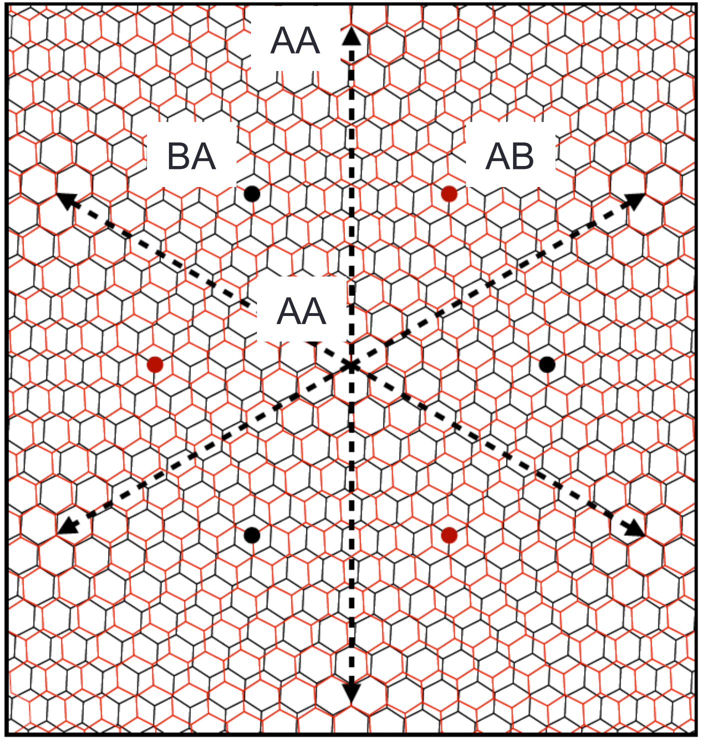


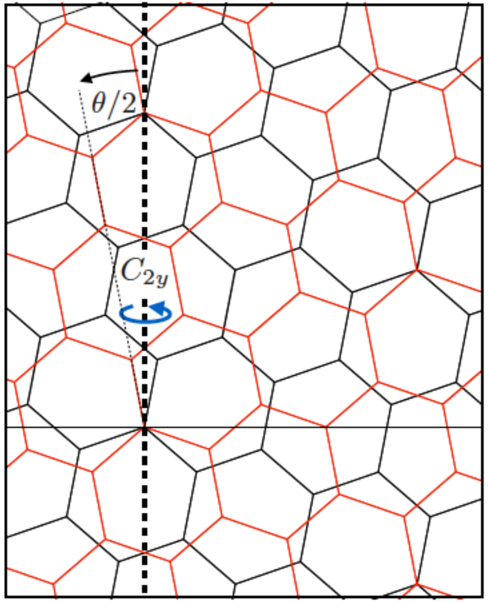
Yankowitz et al, arXiv (2018)

observation of insulating behavior at commensurate filling motivates us to start with a strong-coupling approach

Low-energy model: non-interacting part

- Commensurate twist from AA center: moiré superlattice



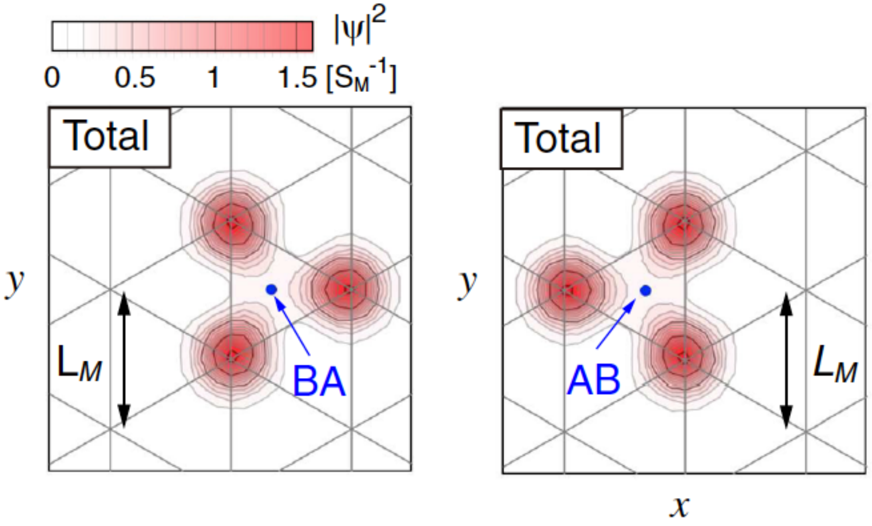


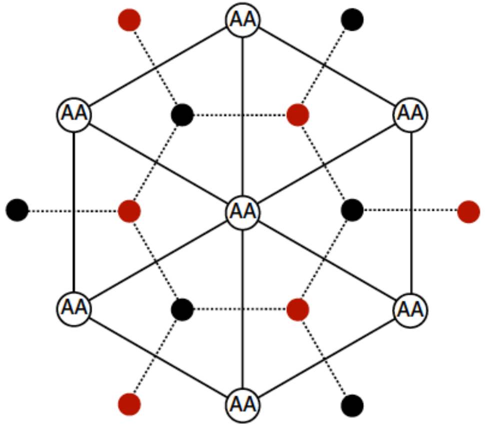
absence of C_{2x} symmetry (D_3 space group) removes any subtleties related to Wannier obstruction

Yuan & Fu, PRB (2018)
Kang & Vafek, PRX (2018)
Zou, Po, Vishwanath, Senthil, PRB (2018)

Low-energy model: non-interacting part

- Wannier states are peaked near AA points, but centered at AB/BA points: emergent honeycomb lattice





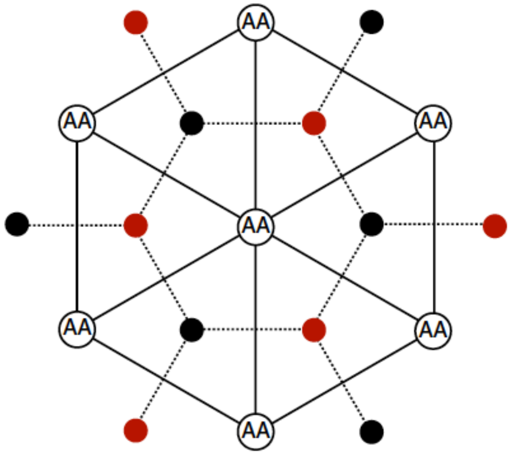
Koshino et al, PRX (2018)

Kang & Vafek, PRX (2018)

8 states per moiré unite cell:
2 (spin) x 2 (sublattice) x 2 (“orbitals”)

Low-energy model: non-interacting part

- Two "p-orbital" honeycomb lattice model:



Yuan & Fu, PRB (2018)

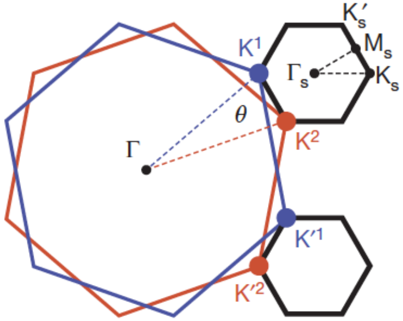
Kang & Vafek, PRX (2018)

- the 2 "orbitals" in each honeycomb sublattice site are eigenstates of C_{3z}
- $L_z = \pm 1$ angular momentum eigenstates (nearly valley-polarized): equivalent to $p_x \pm ip_y$ "orbitals"
- unitary transformation: p_x and p_y "orbitals" on a honeycomb lattice
- hopping amplitudes **not** given by Slater-Koster rules

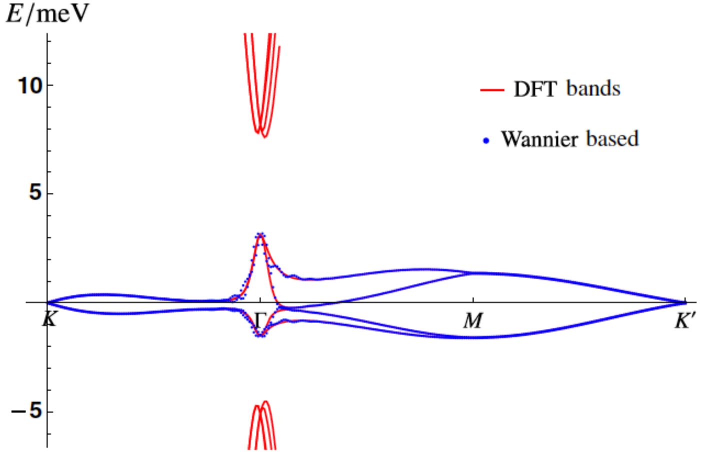
Low-energy model: non-interacting part

- Two-orbital honeycomb lattice model: tight-binding band dispersions

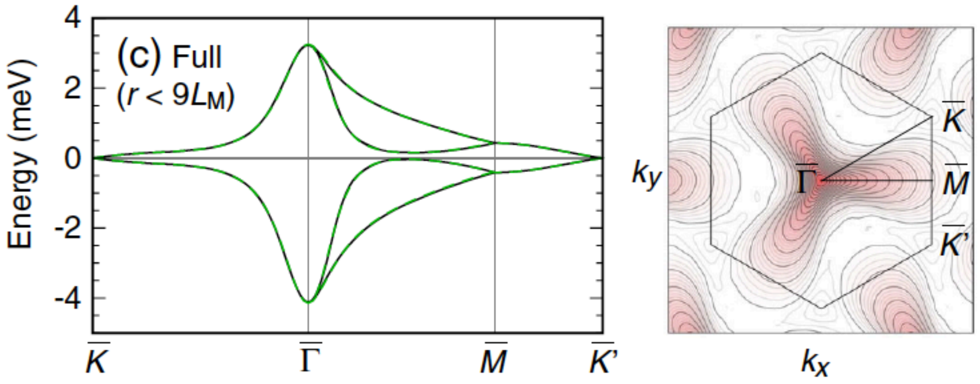
$$H_K = \sum_{ij} c_i^\dagger \hat{T}(\mathbf{r}_{ij}) c_j + \text{H.c.}$$



Brillouin zone is folded due to moiré superlattice



Kang & Vafek, PRX (2018)



Koshino et al, PRX (2018)

Low-energy model: interaction terms

- Extended Hubbard-Kanamori model: density-density and exchange-like interactions

$$H_I = \frac{1}{2}\sum_{ij} V_{ij}^{\alpha\beta} n_{i\alpha} n_{j\beta} + \frac{1}{2}\sum_{ij,\alpha\beta} J_{1,ij}^{\alpha\beta} c_{i\alpha\sigma}^{\dagger} c_{j\beta\sigma'}^{\dagger} c_{i\beta\sigma'} c_{j\alpha\sigma}$$
$$+ \frac{1}{2}\sum_{ij,\alpha\neq\beta} J_{2,ij}^{\alpha\beta} c_{i\alpha\sigma}^{\dagger} c_{j\beta\sigma'}^{\dagger} c_{i\alpha\sigma'} c_{j\beta\sigma}$$
$$+ \frac{1}{2}\sum_{ij,\alpha\neq\beta} J_{3,ij}^{\alpha\beta} c_{i\alpha\sigma}^{\dagger} c_{j\alpha\sigma'}^{\dagger} c_{i\beta\sigma'} c_{j\beta\sigma},$$

- Main challenges: determine the most important interaction terms and solve the problem in the intermediate coupling regime.

Outline

1. Low-energy model and interactions
2. Strong-coupling phase diagram: spin, orbital, and superconducting degrees of freedom
3. Potts-nematicity and nematic superconductivity

Strong-coupling expansion

- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

$\mathbf{S}_i$	$\boldsymbol{\tau}_i$
SU(2) spin	SU(2) orbital
↓	↓
preserved if spin-orbit coupling is absent	*broken by the symmetries of the system*

see also: Xu & Balents, PRL (2018); Dodaro *et al*, PRB (2018)

Strong-coupling expansion

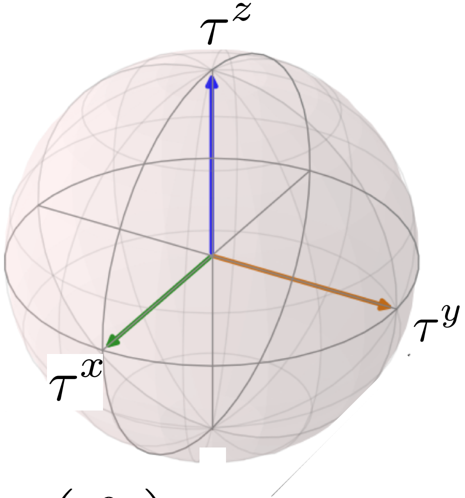
- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

$\boldsymbol{\tau}_i$

SUU(2) orbital

broken by the symmetries of the system

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow p_x$$



$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow p_y$$

Strong-coupling expansion

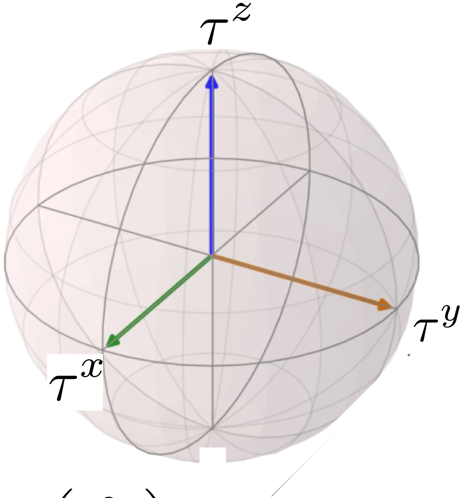
- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

$\boldsymbol{\tau}_i$

SU(2) orbital

broken by the symmetries of the system

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow p_x$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow p_y$

τ^z: *orbital order (C_{3z} symmetry breaking)*

$$n_{p_x} \neq n_{p_y}$$

Strong-coupling expansion

- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

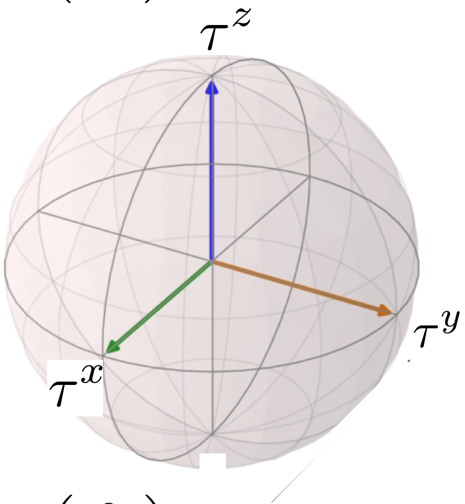
$\boldsymbol{\tau}_i$

SU(2) orbital

↓

broken by the symmetries of the system

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow p_x$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow p_y$

τ^z: *orbital order (C_{3z} symmetry breaking)*

$n_{p_x} \neq n_{p_y}$

τ^x: *orbital order (C_{3z} symmetry breaking)*

$n_{p_x+p_y} \neq n_{p_x-p_y}$

Strong-coupling expansion

- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

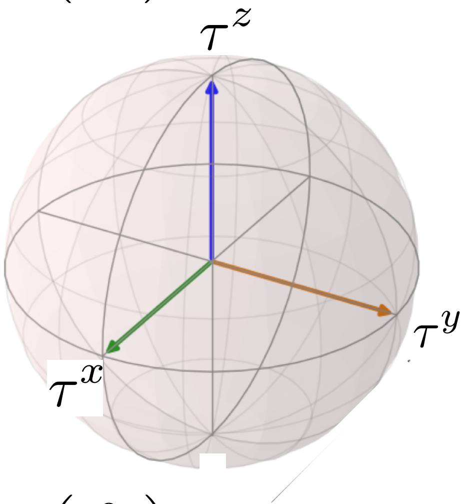
$\boldsymbol{\tau}_i$

SU(2) orbital

↓

broken by the symmetries of the system

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow p_x$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow p_y$

τ^z: *orbital order (C_{3z} symmetry breaking)*

$$n_{p_x} \neq n_{p_y}$$

τ^x: *orbital order (C_{3z} symmetry breaking)*

$$n_{p_x+p_y} \neq n_{p_x-p_y}$$

τ^y: *orbital magnetism (T symmetry breaking)*

$$n_{p_x+ip_y} \neq n_{p_x-ip_y}$$

Strong-coupling expansion

- What to expect? In the insulating regime, with one electron per site ("half-filling"), two unquenched degrees of freedom are left.

$$\boldsymbol{\tau}_i \quad \longrightarrow \quad \boldsymbol{\tau}_i^{\|} = (\tau_i^z, \tau_i^x) \quad \& \quad \tau_i^y$$

SU(2) orbital

↓

broken by the symmetries of the system

- ➢ *approximate U(1) symmetry (no intervalley scattering)*
- ➢ *C_{3z} symmetry-breaking: nematic*

- ➢ *orbital magnetism*

Strong-coupling expansion

- Strong-coupling expansion: Hamiltonian in terms of $\mathbf{S}_i$ and $\boldsymbol{\tau}_i$

  - Kugel-Khomskii Hamiltonian

Crystal structure and magnetic properties of substances with orbital degeneracy

K. I. Kugel' and D. I. Khomskiĭ
P. N. Lebedev Physics Institute
(Submitted November 13, 1972)
Zh. Eksp. Teor. Fiz. **64**, 1429-1439 (April 1973)

- First step: onsite interactions (Hubbard *U* and Hund's *J*) and nearest-neighbor hopping only.

$$H_I^{(\text{onsite})} = U \sum_{i,\alpha} n_{i\alpha\uparrow} n_{i\alpha\downarrow} + (U-2J) \sum_i n_{ix} n_{iy} + J \sum_{i,\sigma,\sigma'} c^\dagger_{ix\sigma} c^\dagger_{iy\sigma'} c_{ix\sigma'} c_{iy\sigma} + J \sum_{i,\alpha\neq\beta} c^\dagger_{i\alpha\uparrow} c^\dagger_{i\alpha\downarrow} c_{i\beta\downarrow} c_{i\beta\uparrow}.$$

$$H_K = \sum_{ij} c_i^\dagger \hat{T}(\mathbf{r}_{ij}) c_j$$

$$\hat{T}_1^{(1)} = t_1 + t_1' \tau^z$$

t_1': *approximately zero (intervalley scattering)*

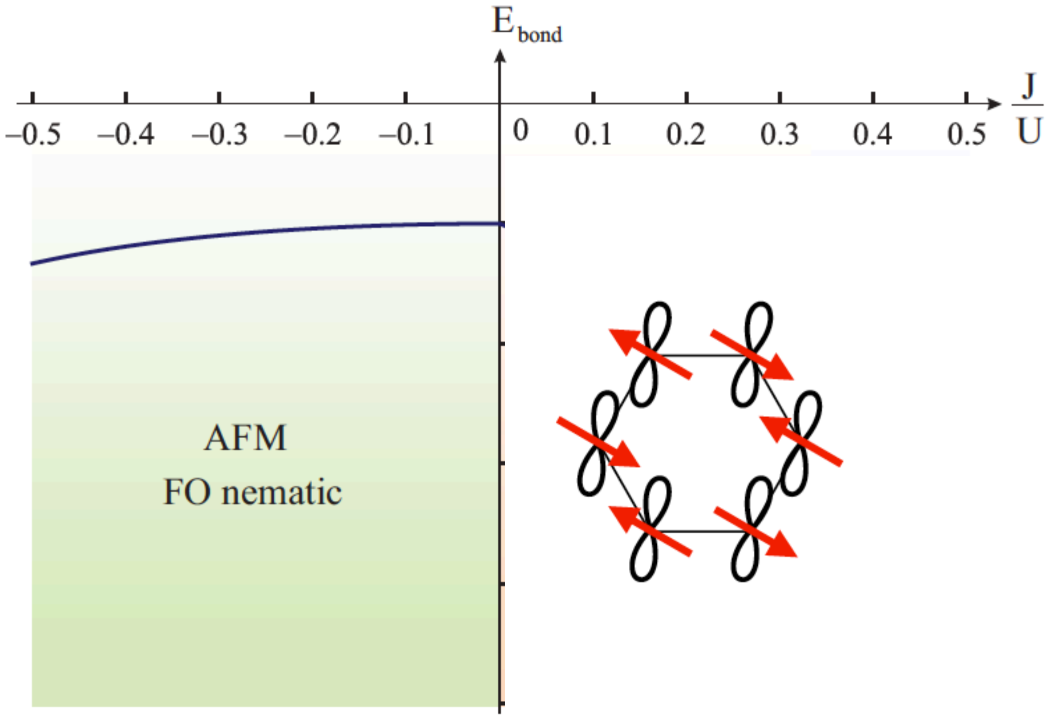
Strong-coupling Hamiltonian

- One electron per site (“half-filling”, but moiré unit cell is 1/4 filled)

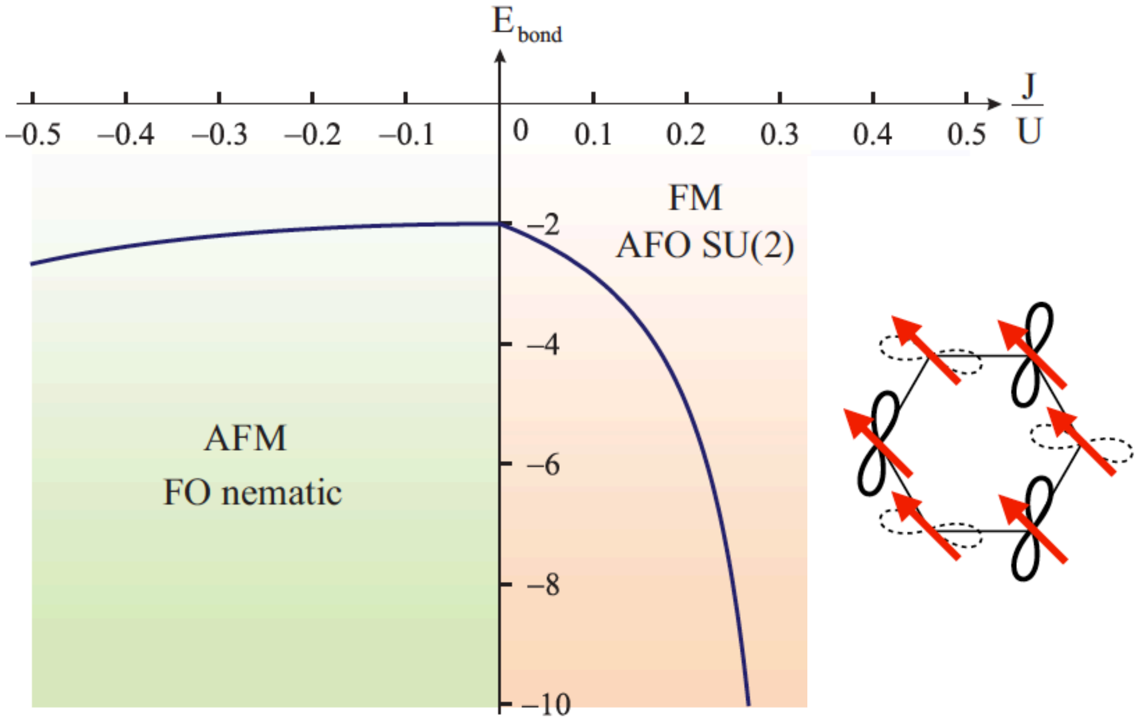
$$\mathcal{H} = \sum_{\langle ij\rangle} \left\{ \frac{t^2}{(U-3J)} \left(\frac{3}{4} + \mathbf{S}_i \cdot \mathbf{S}_j \right) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - 1) - \frac{t^2}{U+J} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j \right) (1 + \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - 2\tau_i^y \tau_j^y) - \frac{2t^2}{U-J} \left(\frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j \right) (\tau_i^y \tau_j^y + 1) \right\}.$$

several approaches to capture the effects of interactions: Balents, Xu, Fu, Vafek, Kang, Isibo, Sachdev, Kivelson, Rademaker, Mellado, Senthil, Vishwanath, Guinea, Bascones, Martin, MacDonald, Lee, Law, Ma, Koshino, Kuroki, Kennes, Thomson, Phillips, Betouras, Nandkishore, Bernevig, Thanos, …

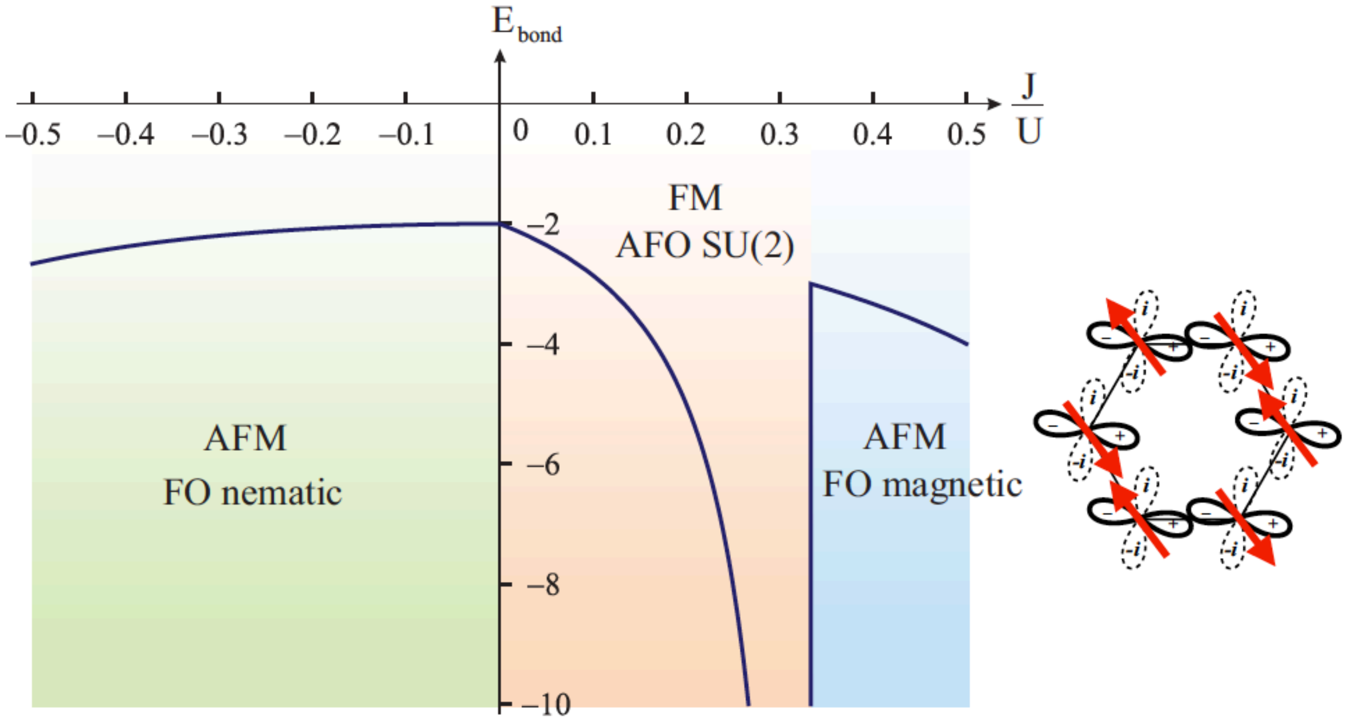
- Strong-coupling phase diagram: either $\mathbf{S}_i$ or $\boldsymbol{\tau}_i$ (but not both) are staggered (translational symmetry-breaking)



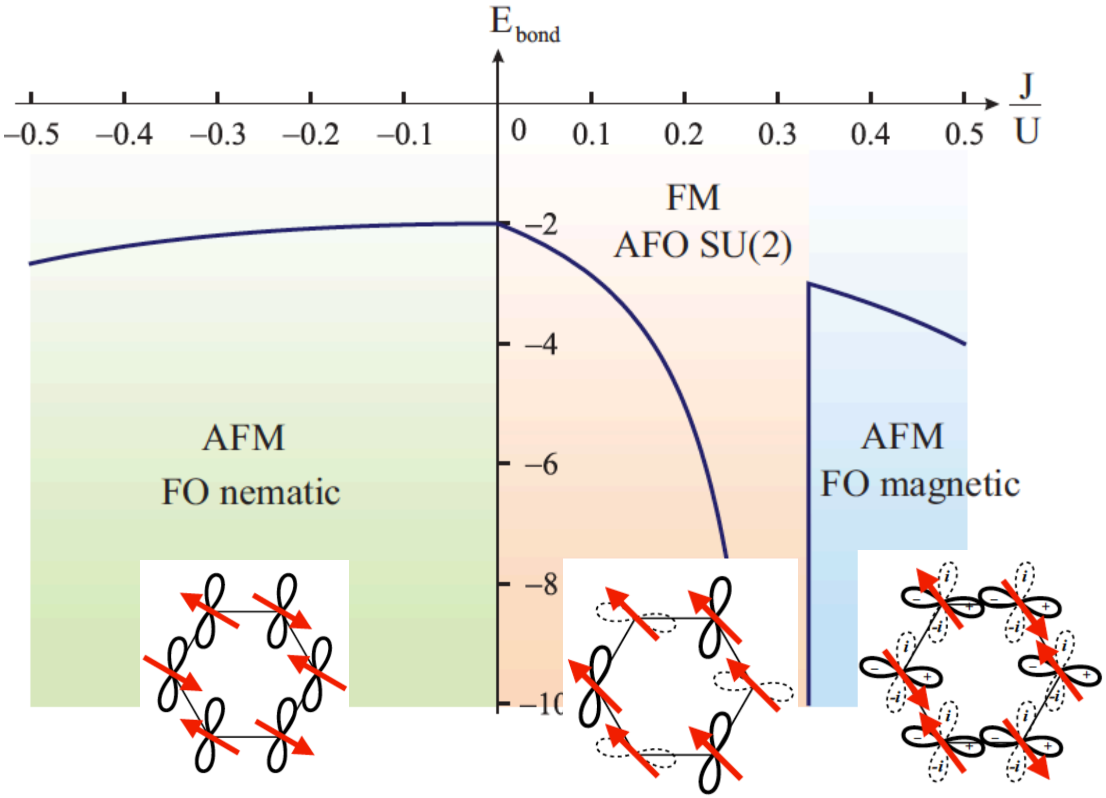
- Strong-coupling phase diagram: either $\mathbf{S}_i$ or $\boldsymbol{\tau}_i$ (but not both) are staggered (translational symmetry-breaking)



- Strong-coupling phase diagram: either $\mathbf{S}_i$ or $\boldsymbol{\tau}_i$ (but not both) are staggered (translational symmetry-breaking)



- Strong-coupling phase diagram: either $\mathbf{S}_i$ or $\boldsymbol{\tau}_i$ (but not both) are staggered (translational symmetry-breaking)



What about superconductivity?

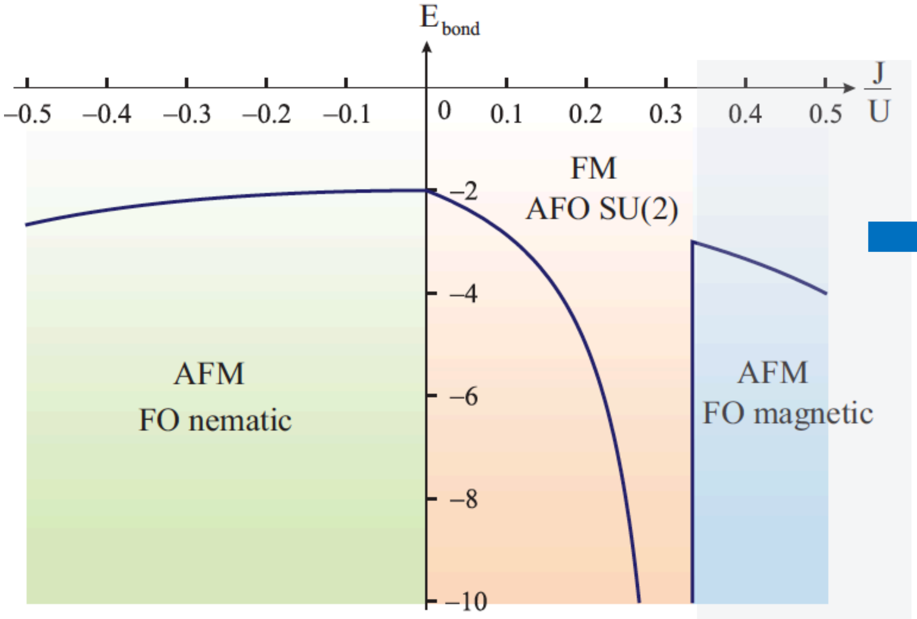
- Decomposition of the interaction terms in pair operator.
- Three types of **onsite** pairing: $\Delta^{\alpha\alpha'}_{\sigma\sigma'} c^\dagger_{i,\alpha\sigma} c^\dagger_{i,\alpha'\sigma'}$

$\Delta = \tau^0 \otimes (i\sigma^y)$	$\Delta = (\tau^z, \tau^x) \otimes (i\sigma^y)$	$\Delta = (i\tau^y) \otimes (\mathbf{d} \cdot \boldsymbol{\sigma}\, i\sigma^y)$
s-wave spin-singlet (A_1 symmetry)	*degenerate d-wave (E symmetry)*	*s-wave spin-triplet (A_2 symmetry)*

superconductivity proposals: Balents, Xu, Fu, Kivelson, Rademaker, Mellado, Guinea, Scalletar, Martin, MacDonald, Lee, Ma, Kennes, Nandkishore, Bernevig, …

What about superconductivity?

- s-wave spin-triplet channel is attractive already in the bare level in the regime of the phase diagram dominated by orbital ferromagnetism. $J > U/3$



speculation: s-wave spin-triplet SC near orbital FM?

c.f. P. Kim's talk and Goldhaber-Gordon's talk

Strong-coupling expansion

- Our onsite approximation neglects the impact of extended interactions, which are expected due to the extended nature of the Wannier functions.

Koshino et al, PRX (2018)

- Kang and Vafek: assisted-hopping interaction favors "ferro" alignment of *both* $\mathbf{S}_i$ and $\boldsymbol{\tau}_i$ already in the atomic limit.

Kang & Vafek, PRX (2018)

see also Senthil's talk

$$U \approx \frac{V_0}{2} \sum_{\mathbf{R}} \left(\sum_{j=\pm 1} \sum_{\sigma=\uparrow,\downarrow} O_{j,\sigma}(\mathbf{R}) \right)^2$$

$$O_{j,\sigma}(\boldsymbol{R}) = \frac{1}{3} Q_{j,\sigma}(\boldsymbol{R}) + \alpha_1 T_{j,\sigma}(\boldsymbol{R})$$

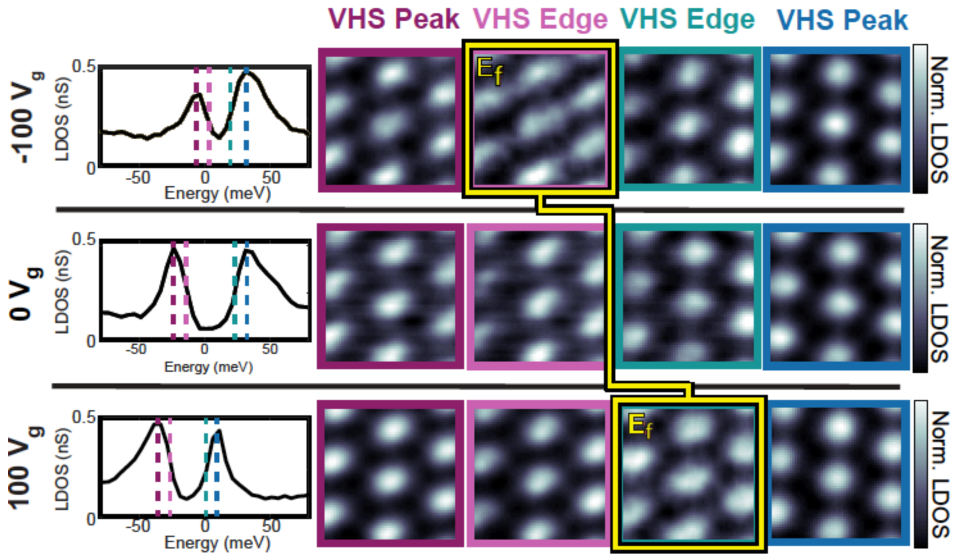
- Hopping beyond nearest neighbors are fundamental to correctly describe the narrow bands dispersions. Can they introduce frustration?

Outline

1. Low-energy model and interactions
2. Strong-coupling phase diagram: spin, orbital, and superconducting degrees of freedom
3. Potts-nematicity and nematic superconductivity

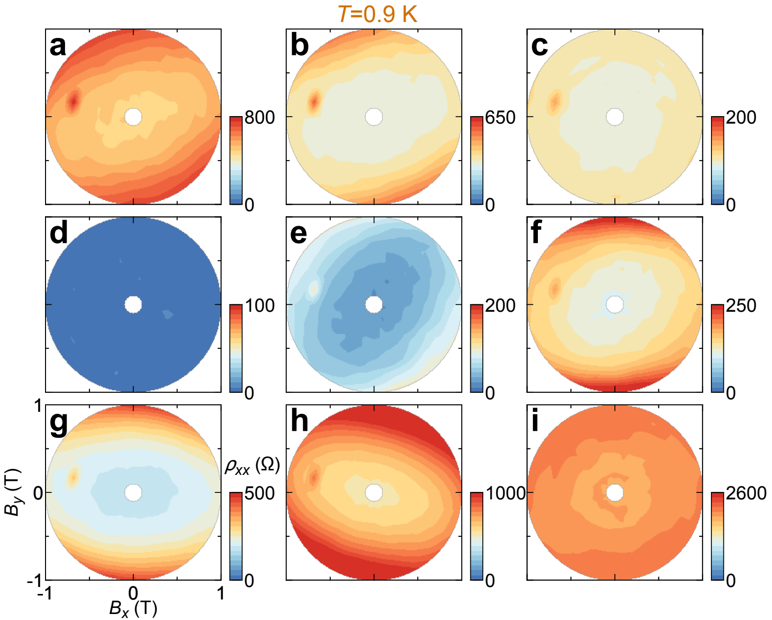
Electronic nematicity in TBG: experimental hints

- STM: spatial map of the local density of states is not three-fold symmetric



Kerelsky et al, arXiv (2018)

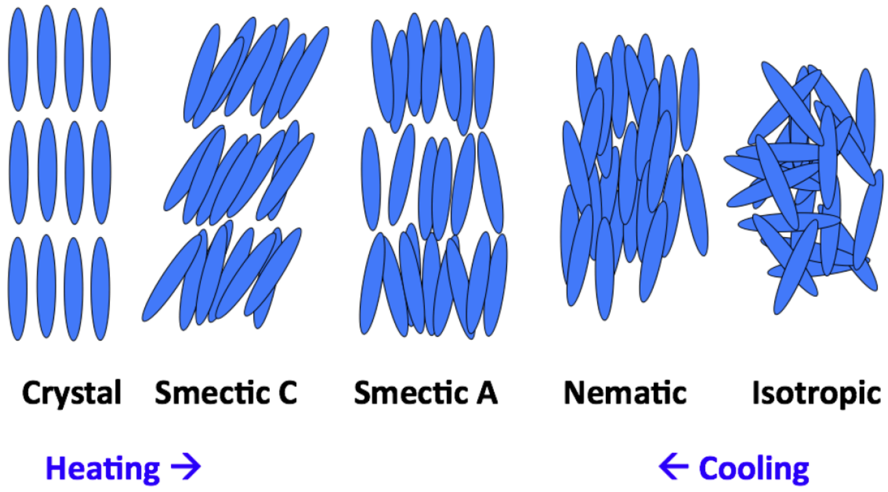
- superconducting upper critical field is two-fold symmetric



Jarillo-Herrero's talk

Electronic nematicity: phenomenology

- Nematic order in liquid crystals: orientational order without translational symmetry-breaking.



Mbanga, PhD thesis (2012)

- Order parameter (2D): $Q_{ij} = Q\left(2d_i d_j - \delta_{ij} d^2\right)$

director

Electronic nematicity: phenomenology

- Electronic nematicity: $\hat{Q}_{ij} = \psi^{\dagger}(\mathbf{r})\left(2\partial_i\partial_j - \delta_{ij}\nabla^2\right)\psi(\mathbf{r})$

Kivelson, Fradkin, Emery Nature (1998)

Electronic nematicity: phenomenology

- Electronic nematicity: $\hat{Q}_{ij} = \psi^\dagger(\mathbf{r})\left(2\partial_i\partial_j - \delta_{ij}\nabla^2\right)\psi(\mathbf{r})$

Kivelson, Fradkin, Emery Nature (1998)

➢ order parameter can be expressed in terms of quadrupolar charge densities:

$$\left\langle \hat{Q} \right\rangle = \begin{pmatrix} \rho_{x^2-y^2} & \rho_{xy} \\ \rho_{xy} & -\rho_{x^2-y^2} \end{pmatrix}$$

$$\begin{cases} \rho_{x^2-y^2} \equiv \left\langle (k_x^2 - k_y^2)\hat{\psi}^\dagger(\mathbf{k})\,\hat{\psi}(\mathbf{k}) \right\rangle \\ \rho_{xy} \equiv \left\langle (2k_x k_y)\hat{\psi}^\dagger(\mathbf{k})\,\hat{\psi}(\mathbf{k}) \right\rangle \end{cases}$$

Electronic nematicity: phenomenology

- Electronic nematicity: $\hat{Q}_{ij} = \psi^\dagger(\mathbf{r})\left(2\partial_i\partial_j - \delta_{ij}\nabla^2\right)\psi(\mathbf{r})$

Kivelson, Fradkin, Emery Nature (1998)

- order parameter can be expressed in terms of quadrupolar charge densities:

$$\left\langle \hat{Q} \right\rangle = \begin{pmatrix} \rho_{x^2-y^2} & \rho_{xy} \\ \rho_{xy} & -\rho_{x^2-y^2} \end{pmatrix} \qquad \begin{cases} \rho_{x^2-y^2} \equiv \left\langle (k_x^2 - k_y^2)\hat{\psi}^\dagger(\mathbf{k})\,\hat{\psi}\ (\mathbf{k}) \right\rangle \\ \rho_{xy} \equiv \left\langle (2k_x k_y)\hat{\psi}^\dagger(\mathbf{k})\,\hat{\psi}\ (\mathbf{k}) \right\rangle \end{cases}$$

- XY-nematic order parameter $\mathbf{\Phi}$

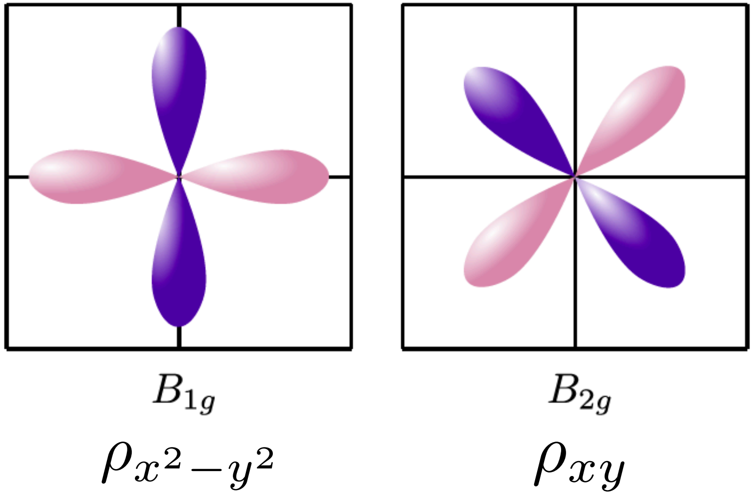
$$\left\langle \hat{Q} \right\rangle = \rho_{x^2-y^2}\sigma^z + \rho_{xy}\sigma^x \quad \Longrightarrow \quad \mathbf{\Phi} = \begin{pmatrix} \rho_{x^2-y^2} \\ \rho_{xy} \end{pmatrix}$$

Electronic nematicity: impact of the lattice

- Lattice generally breaks the continuous symmetry of the nematic OP:

  - **square lattice**: the two components of Φ transform as two different irreducible representations

$$\mathbf{\Phi} = \begin{pmatrix} \rho_{x^2-y^2} \\ \rho_{xy} \end{pmatrix}$$



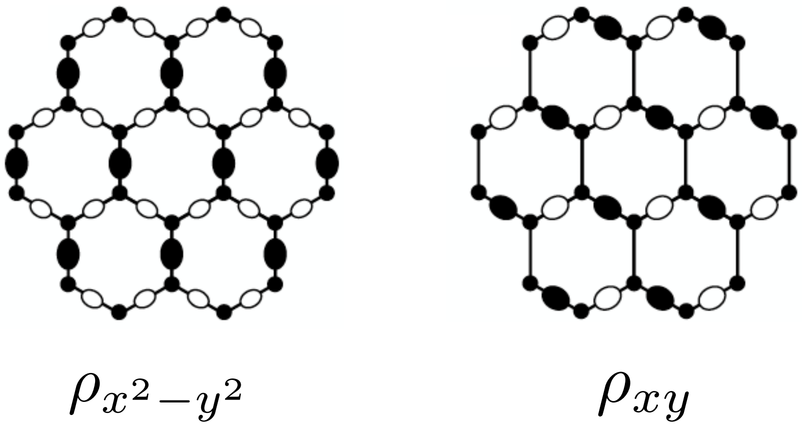
Ising-nematicity: cuprates, pnictides, ruthenates, heavy fermions, …

Electronic nematicity: impact of the lattice

- Lattice generally breaks the continuous symmetry of the nematic OP:

  - **honeycomb lattice**: the two components of $\mathbf{\Phi}$ transform as the same two-dimensional irrep *E*

$$\mathbf{\Phi} = \begin{pmatrix} \rho_{x^2-y^2} \\ \rho_{xy} \end{pmatrix}$$

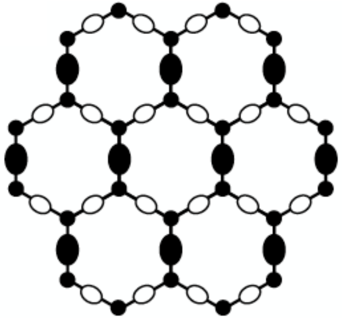


Electronic nematicity: impact of the lattice

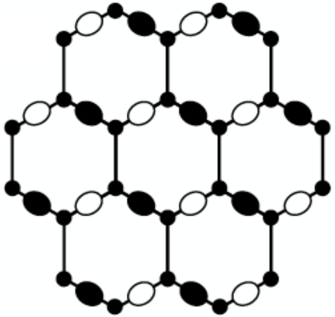
- Lattice generally breaks the continuous symmetry of the nematic OP:

  - **honeycomb lattice**: the two components of $\mathbf{\Phi}$ transform as the same two-dimensional irrep *E*

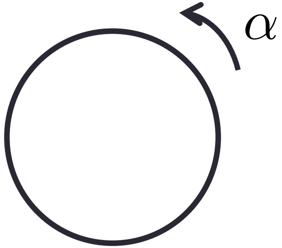
$$\mathbf{\Phi} = \begin{pmatrix} \rho_{x^2-y^2} \\ \rho_{xy} \end{pmatrix} \equiv \Phi_0 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$$



$\rho_{x^2-y^2}$



ρ_{xy}



XY-nematicity?

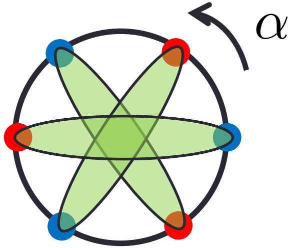
Electronic nematicity: impact of the lattice

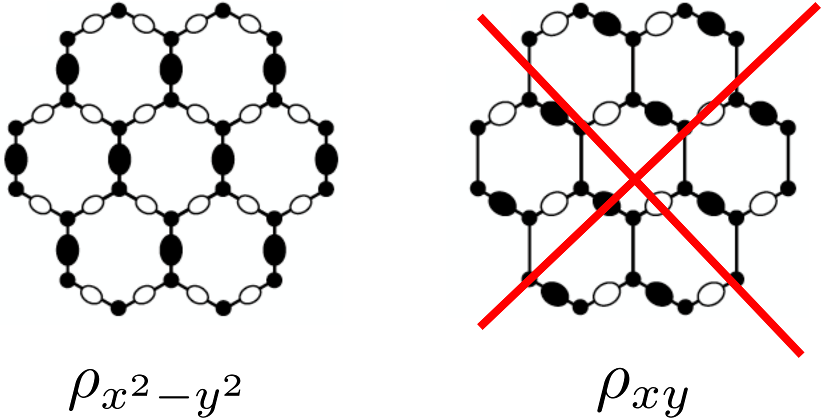
- Lattice generally breaks the continuous symmetry of the nematic OP:

  - **honeycomb lattice**: the two components of $\mathbf{\Phi}$ transform as the same two-dimensional irrep *E*

$$f = \frac{a}{2}\Phi_0^2 + \frac{\lambda}{3}\Phi_0^3\cos(6\alpha) + \frac{u}{4}\Phi_0^4$$

3-state Potts nematicity!





see also:

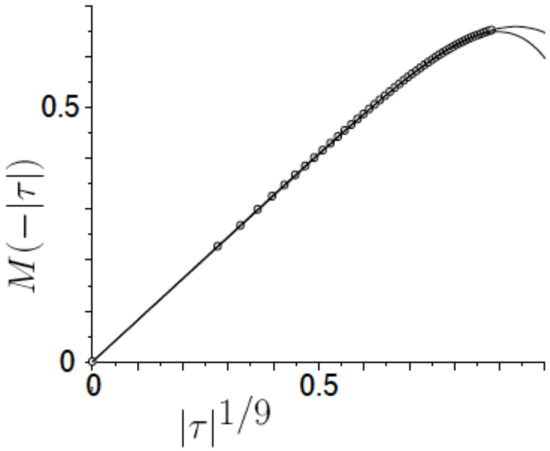
Hecker & Schmalian, npj Quant Mat (2017)

Fu et al, PRB (2016)

Electronic nematicity: impact of the lattice

- General properties of the two-dimensional 3-state Potts model:

➢ despite the cubic invariant, transition is second order



Shchur et al, PRB (2008)

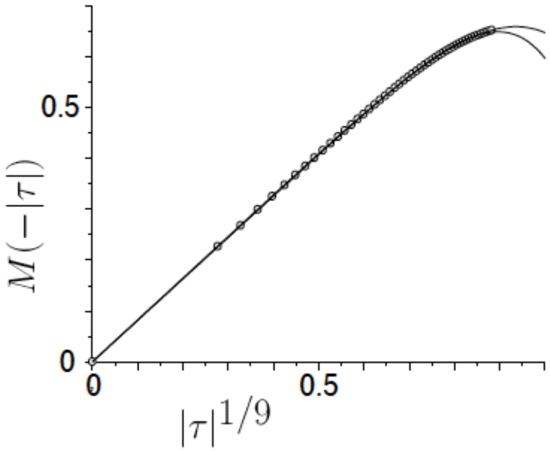
Electronic nematicity: impact of the lattice

- General properties of the two-dimensional 3-state Potts model:

➢ despite the cubic invariant, transition is second order

➢ Harris inequality is violated: $d\nu = \frac{5}{3} < 2$

importance of disorder, which is ubiquitous in TBG

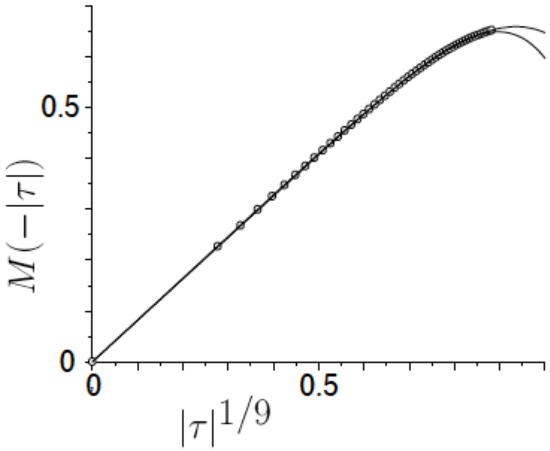


Shchur et al, PRB (2008)

Electronic nematicity: impact of the lattice

- General properties of the two-dimensional 3-state Potts model:

➢ despite the cubic invariant, transition is second order



Shchur et al, PRB (2008)

➢ Harris inequality is violated: $d\nu = \frac{5}{3} < 2$

importance of disorder, which is ubiquitous in TBG

➢ may explain the financial market $$$

Simulations of financial markets in a Potts-like model

Tetsuya Takaishi

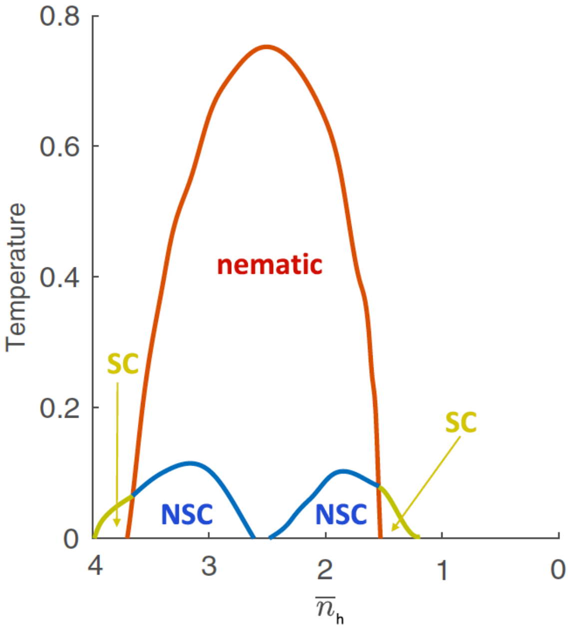
CERN, Physics Department, TH Unit, CH-1211 Genève 23, Switzerland

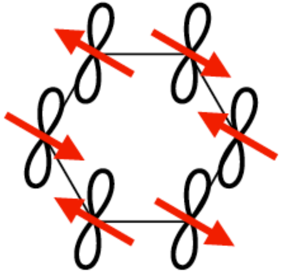
Hiroshima University of Economics, Hiroshima 731-0124, Japan

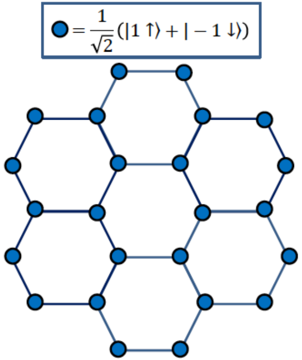
June 29, 2018

Electronic nematicity in TBG

- Possible microscopic origins:
  - spontaneous order of the emergent "orbital" degrees of freedom.



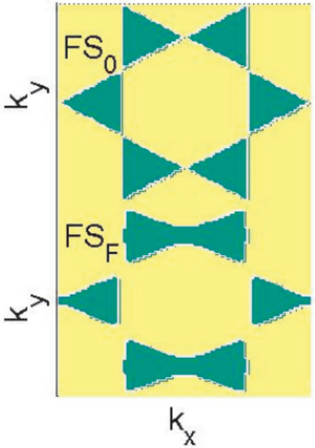




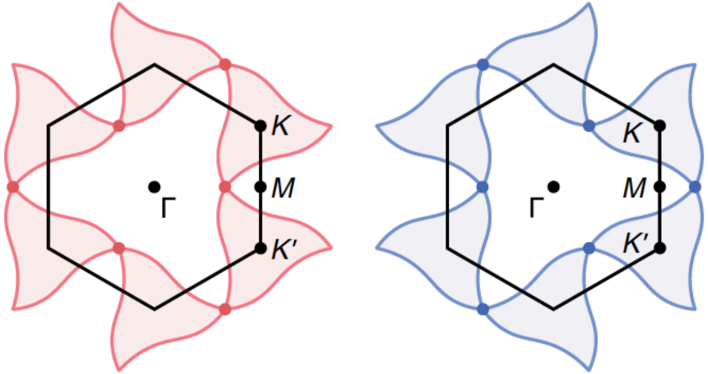
Dodaro et al, PRB (2018)
Venderbos & RMF, PRB (2018)
Kang & Vafek, arxiv (2018)

Electronic nematicity in TBG

- Possible microscopic origins:
  - spontaneous order of the emergent “orbital” degrees of freedom.
  - weak-coupling instability related to van Hove singularities? *But no gap is opened by nematic order.*



in the context of graphene doped to the vHS:
Valenzuela & Vozmediano, NJP (2008)



Isobe *et al*, PRX (2018)

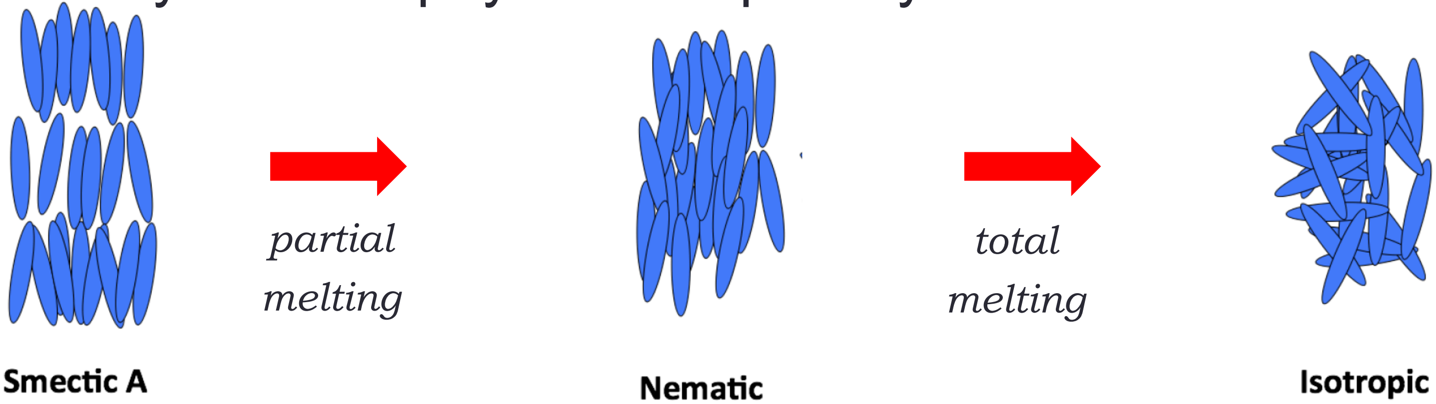
Electronic nematicity in TBG

- Possible microscopic origins:
  - spontaneous order of the emergent “orbital” degrees of freedom.
  - weak-coupling instability related to van Hove singularities? *But no gap is opened by nematic order.*
  - vestigial phase of a nematic superconducting state.

Venderbos & RMF, PRB (2018)

Vestigial order: partial melting of an ordered state

- Similarity with the physics of liquid crystals



Smectic A		Nematic		Isotropic	
translational	✗	translational	✓	translational	✓
rotational	✗	rotational	✗	rotational	✓

- In quantum matter: composite order $\langle \eta_\alpha \rangle = 0$ but $\langle \eta_\alpha \eta_\beta \rangle \neq 0$

RMF, Orth, Schmalian, Ann. Rev. Cond. Matt. Phys, *in press*

Vestigial nematic order in TBG

- One of the candidates for the superconducting state:

Balents, Xu, Fu, Rademaker, Mellado, Ma, Yang, Lin, Kennes, Nandkishore, ...

$\Delta = \tau^0 \otimes (i\sigma^y)$	$\Delta = (\tau^z, \tau^x) \otimes (i\sigma^y)$	$\Delta = (i\tau^y) \otimes (\mathbf{d} \cdot \boldsymbol{\sigma}\, i\sigma^y)$
s-wave spin-singlet *(A_1 symmetry)*	*degenerate d-wave* *(E symmetry)*	*s-wave spin-triplet* *(A_2 symmetry)*

$$\Delta = \eta_1 \tau^z (i\sigma^y) + \eta_2 \tau^x (i\sigma^y)$$

$\eta_1 \tau^z (i\sigma^y)$ ↓ $d_{x^2-y^2}$ -wave

$\eta_2 \tau^x (i\sigma^y)$ ↓ d_{xy} -wave

Vestigial nematic order in TBG

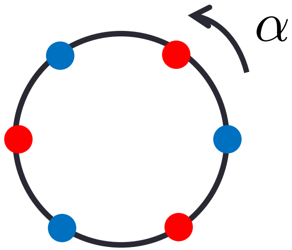
- One of the candidates for the superconducting state:

Balents, Xu, Fu, Rademaker, Mellado, Ma, Yang, Lin, Kennes, Nandkishore, …

$\Delta = \tau^0 \otimes (i\sigma^y)$	$\Delta = (\tau^z, \tau^x) \otimes (i\sigma^y)$	$\Delta = (i\tau^y) \otimes (\mathbf{d} \cdot \boldsymbol{\sigma}\, i\sigma^y)$
s-wave spin-singlet (A$_1$ *symmetry)*	*degenerate d-wave (E symmetry)*	*s-wave spin-triplet (A*$_2$ *symmetry)*

$$\Delta = \eta_1 \tau^z (i\sigma^y) + \eta_2 \tau^x (i\sigma^y)$$

- *d+id* chiral superconductivity $\boldsymbol{\eta} = \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$
- *d+d* nematic superconductivity $\boldsymbol{\eta} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$



Vestigial nematic order in TBG

- What are the possible composite operators? $\boldsymbol{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$$E \otimes E = A_1 \oplus A_2 \oplus E$$

Vestigial nematic order in TBG

- What are the possible composite operators? $\boldsymbol{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$$E \otimes E = A_1 \oplus A_2 \oplus E$$

$$\phi_0 = |\eta_1|^2 + |\eta_2|^2$$

A_1 amplitude fluctuations (no broken symmetries)

Vestigial nematic order in TBG

- What are the possible composite operators? $\boldsymbol{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$$E \otimes E = A_1 \oplus A_2 \oplus E$$

$$\phi_0 = |\eta_1|^2 + |\eta_2|^2$$

A_1 amplitude fluctuations (no broken symmetries)

$$\phi_2 = i\left(\eta_1^* \eta_2 - \eta_1 \eta_2^*\right)$$

A_2 chiral order, breaks time-reversal symmetry

Vestigial nematic order in TBG

- What are the possible composite operators? $\boldsymbol{\eta} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$$E \otimes E = A_1 \oplus A_2 \oplus E$$

$$\phi_0 = |\eta_1|^2 + |\eta_2|^2$$

A_1 amplitude fluctuations (no broken symmetries)

$$\phi_2 = i\left(\eta_1^* \eta_2 - \eta_1 \eta_2^*\right)$$

A_2 chiral order, breaks time-reversal symmetry

$$\boldsymbol{\Phi} = \begin{pmatrix} |\eta_1|^2 - |\eta_2|^2 \\ \eta_1^* \eta_2 + \eta_1 \eta_2^* \end{pmatrix}$$

E nematic order, breaks three-fold rotational symmetry

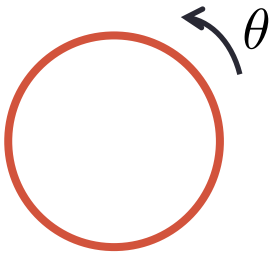
none of them break U(1) symmetry

Vestigial nematic order in TBG

- Can the composite nematic order parameter condense even in the absence of long-range superconductivity?
- Two phase variables: global phase and relative angle.

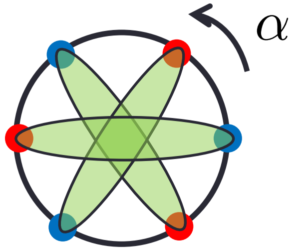
$$\boldsymbol{\eta} = |\eta| e^{i\theta} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \qquad \boldsymbol{\Phi} = \begin{pmatrix} |\eta_1|^2 - |\eta_2|^2 \\ \eta_1^* \eta_2 + \eta_1 \eta_2^* \end{pmatrix} \equiv |\eta|^2 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$$

$$\eta = |\eta| e^{i\theta}$$



$U(1)$ symmetry

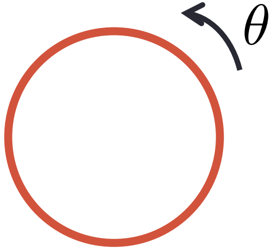
$$\boldsymbol{\Phi} = |\eta|^2 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$$



Z_3 symmetry

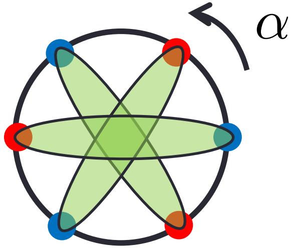
Vestigial nematic order in TBG

$\eta = |\eta| e^{i\theta}$



$U(1)$ symmetry

$\mathbf{\Phi} = |\eta|^2 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$



Z_3 symmetry

- Mean-field: both order parameters condense simultaneously

$\langle \mathbf{\Phi} \rangle \neq 0$

$\langle \eta \rangle \neq 0$

$\langle \mathbf{\Phi} \rangle = 0$

$\langle \eta \rangle = 0$

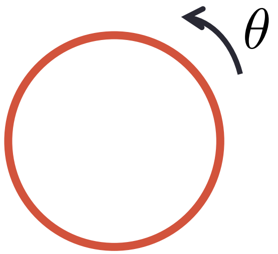
$T = 0$

T_c^{MF}

temperature

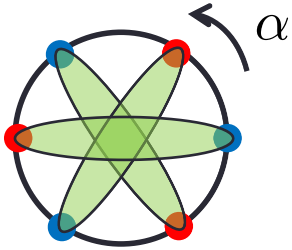
Vestigial nematic order in TBG

$\eta = |\eta| e^{i\theta}$



$U(1)$ symmetry

$\mathbf{\Phi} = |\eta|^2 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$



Z_3 symmetry

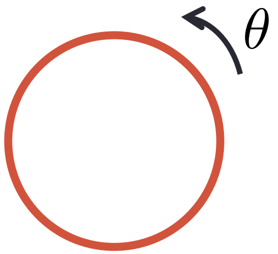
- Fluctuations kill superconductivity in 2D (Mermin-Wagner), but enhance the Potts transition. $\chi_{\mathrm{nem}}^{-1} \propto C - \xi_{\mathrm{SC}}^{-2}$

$\langle \mathbf{\Phi} \rangle \neq 0$, $\langle \eta \rangle = 0$

$\langle \mathbf{\Phi} \rangle = 0$, $\langle \eta \rangle = 0$

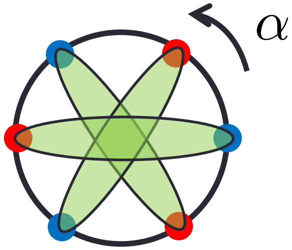
Vestigial nematic order in TBG

$\eta = |\eta| e^{i\theta}$



$U(1)$ symmetry

$\mathbf{\Phi} = |\eta|^2 \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \end{pmatrix}$



Z_3 symmetry

- Kosterlitz-Thouless transition: quasi-long-range SC order (microscopic calculation needed).

similar results for $Cu_xBi_2Se_3$, but in 3D

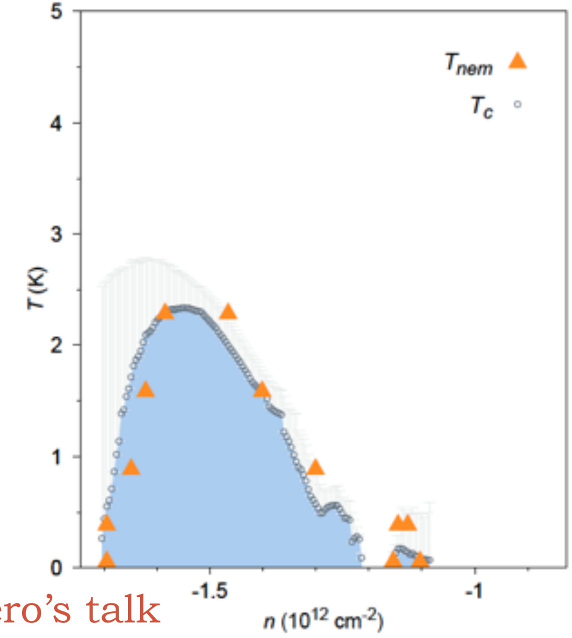
Hecker and Schmalian, npj Quant. Mat. (2018)

$\langle \mathbf{\Phi} \rangle \neq 0$ $\langle \eta \rangle \neq 0$ | $\langle \mathbf{\Phi} \rangle \neq 0$ $\langle \eta \rangle = 0$ | $\langle \mathbf{\Phi} \rangle = 0$ $\langle \eta \rangle = 0$

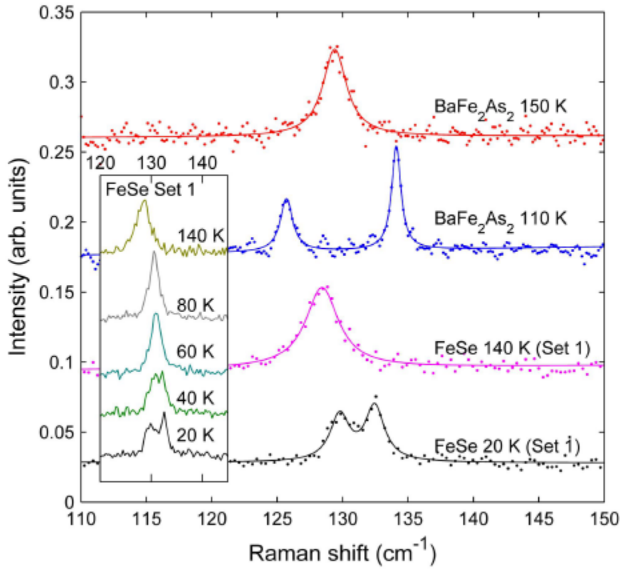
$T = 0$ $T_{\mathrm{BKT}}^{(\mathrm{SC})}$ $T_{Z_3}^{(\mathrm{nem})}$ temperature

Vestigial nematic order in TBG

- Experiments indicate that the entire superconducting dome is nematic.
- Possible nematicity in the normal state could be probed, for instance, by the Raman splitting of the E_g phonon mode (like in pnictides).



Jarillo-Herrero's talk



Hu et al, PRB (2016)

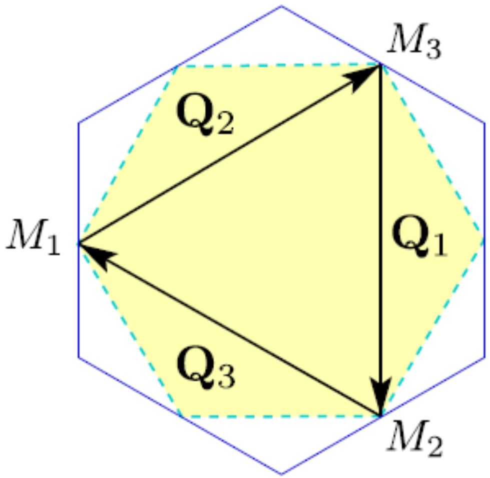
Electronic nematicity in TBG

- Possible microscopic origins:
  - spontaneous order of the emergent “orbital” degrees of freedom.
  - weak-coupling instability related to van Hove singularities? *But no gap is opened by nematic order.*
  - vestigial phase of a nematic superconducting state.
  - vestigial phase of a spin-density wave (SDW).

Vestigial nematic order in TBG: SDW case

- Similarly to monolayer graphene doped to the van Hove singularity, TBG may be unstable towards a SDW.

Nandkishore et al, Nat Phys (2013)
Platt et al, Adv Phys (2013)
Wang et al, PRB (2012)
Chern et al, PRB (2011)

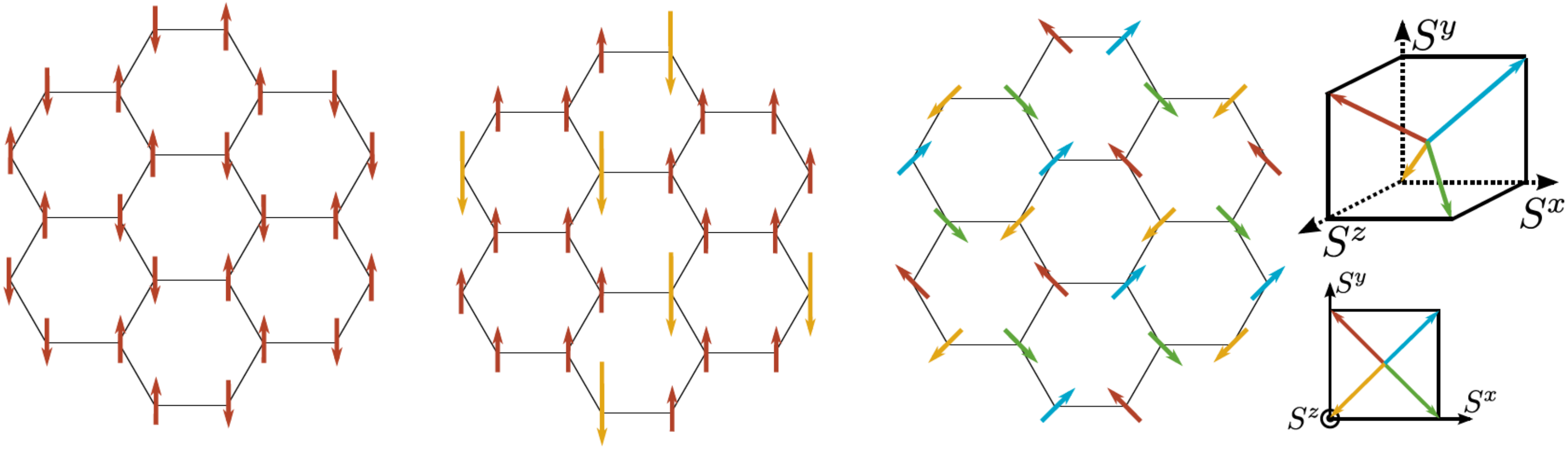


$$\mathbf{S}\left(\mathbf{r}\right) = \sum_{a=1,2,3} \mathbf{m}_a \cos\left(\mathbf{Q}_a \cdot \mathbf{r}\right)$$

three-fold degenerate SDWs

Vestigial nematic order in TBG: SDW case

- Three possible magnetic ground states



single-Q

collinear triple-Q

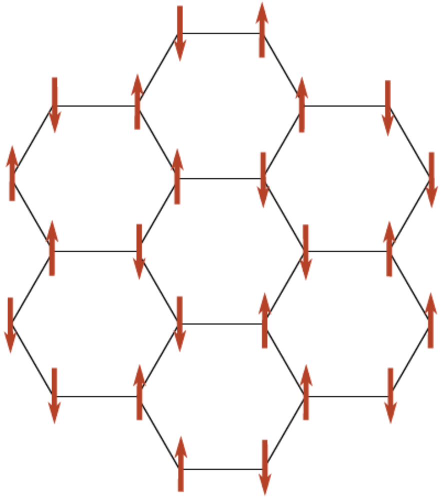
non-coplanar triple-Q

Vestigial nematic order in TBG: SDW case

- Composite 3-state Potts (Z_3) nematic order parameter can condense already in the paramagnetic phase.
  - long-range SDW not allowed in 2D

$$\phi^{\nu}_{(1,0)} = \left\{ m_1^2 - m_2^2,\ \frac{1}{\sqrt{3}} \left(m_1^2 + m_2^2 - 2m_3^2 \right) \right\}$$

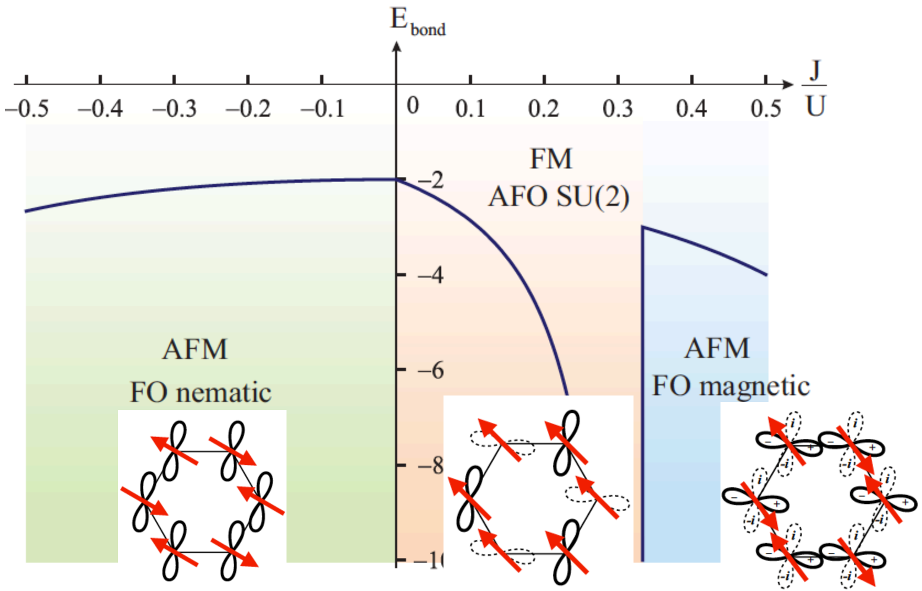
bond order

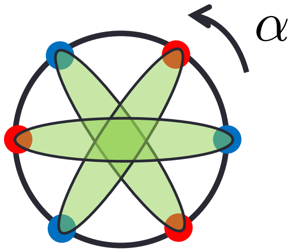


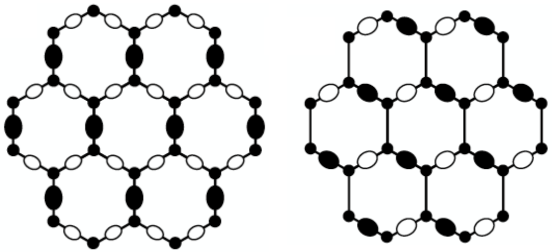
single-Q

RMF, Orth, Schmalian,
Ann. Rev. Cond. Matt. Phys, *in press*

Conclusions







- Rich interplay between spin and emergent orbital degrees of freedom in the strong-coupling insulating state of TBG.
- Potts-nematicity offers a new window to explore electronic nematic order.
- Nematic order expected to survive in the normal state as a vestigial order.

Venderbos and RMF, Phys. Rev. B **98**, 245103 (2018)