Electrons in narrow-band moire graphene: two-dimensional Bloch oscillations

Leonid Levitov, MIT

Correlations in Moire Flat Bands, KITP
Jan 16, 2019

Dynamics of a Bloch electron in 1D solids

Bloch waves in a periodic lattice: $H_0 = \frac{p^2}{2m} + U(x)$:

$$\psi_n(x) = u_{n,k}(x)e^{ikx}, \quad \epsilon_n(k)\psi_n(x) = H_0\psi_n(x)$$

with band dispersion $\epsilon_n(k)$ periodic in k, and the envelop functions $u_{n,k}(x)$ periodic in x.

The k-space form of Newton's law for Bloch waves in the presence of an external field, $H = H_0 - eEx$:

$$\hbar\frac{d\mathbf{k}}{dt} = e\mathbf{E} \tag{1}$$

Follows from adiabatic apprx for one band, or rigorously for a multiband dynamics.

Oscillatory movement in a DC electric field (video)

- Electron wavepacket for nth band: $\tilde{\psi}_{n,k}(x) \sim \int dk' e^{-\alpha(k'-k)^2} \psi_{n,k'}(x)$. The velocity is the group velocity $v = \frac{1}{\hbar} \frac{\partial \epsilon(k)}{\partial k}$
- Sign-changing $v(t)$, current oscillations!
- Bloch oscillator DC→AC

Quasiclassical explanation: $\hbar \frac{dk}{dt} = eE$ gives $k(t) = k_0 + eEt/\hbar$ that sweeps the entire Brillouin zone. After reaching the right end point $k = \frac{\pi}{a}$ electron Bragg-reflects and continues from the left end point $k = -\frac{\pi}{a}$, since Brillouin zone is a circle (in 1D). Sign-changing velocity $v(t)$, time period $T = (2\pi/a)\hbar/eE = h/eEa$.
Bloch oscillation frequency $\nu = eEa/h$.

Experiments: an ongoing quest

- Requirements: narrow bandwidth $J < \omega_{\rm ph}$, and low disorder $\gamma_{\rm dis} < \nu$
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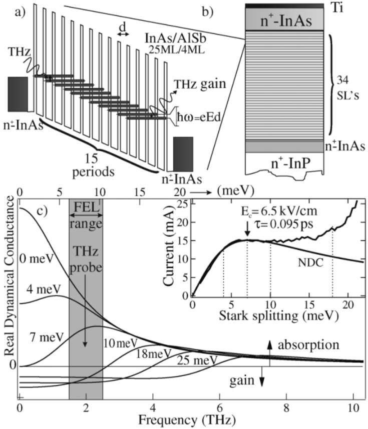

THz Loss and Gain in a Bloch Oscillating InAs/AlSb Superlattice [Savvidis et al, PRL 92, 196802 (2004)]

Experiments: an ongoing quest

"The THz part of the electromagnetic spectrum is marked by a lack of commercial technology. Bloch oscillating superlattices have the potential to provide broad band gain at THz frequencies and may be the basis of a technology that will help fill the gap in solid state THz fundamental oscillators." (2004)

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- Oscillation at a frequency controlled by the external resonator or circuit but only at frequencies below the Bloch frequency
- Promising, but plauged by charge instabilities due to negative differential conductivity
- Coherent THz emission?
- Proof-of-principle demonstration with cold atoms in optical lattices (Weld @ UCSB)

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Other promising systems?

Narrow bands in moire graphene

- High optical phonon frequency $\omega_{\rm ph} \sim 200$mV
- Condition $J < \omega_{\rm ph}$ easy to fulfill
- Large lattice constant $a \sim 10$nm
- Opportunity: demonstrate BO in moire bands at $eE < J/a$, avoiding Wannier-Stark ladder localization and charge instabilities

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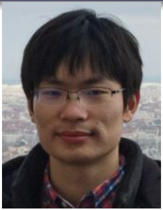
Feasible?

Two-dimensional Bloch oscillations

with Zhiyu Dong 

- Two independent frequencies (trajectories wind BZ in two different directions)
- Values tunable by the field polar angle $\mathbf{E} = E(\cos\theta, \sin\theta)$
- Sensitive to band dispersion, a probe of carrier dynamics
- Interesting dynamics in magnetic field (phase transition to one-frequency oscillation)

Illustrate for triangular lattice tight-binding band:

$$\epsilon(\mathbf{k}) = -J \sum_{j=0,1,2} \cos(\mathbf{k}\mathbf{a}_j), \ \ \mathbf{a}_j = a(\cos\theta_j, \sin\theta_j)$$

$\theta_j = \frac{2\pi}{3} j$. Combining with Newton's dynamics $\hbar \frac{d\mathbf{k}}{dt} = e\mathbf{E}$, gives trajectories $\mathbf{k}(t) = e\mathbf{E}t + k_0$. An oscillatory particle velocity time dependence

$$\mathbf{v}(t) = -J \sum_j \mathbf{a}_j \sin(\frac{e}{\hbar}\mathbf{E}\cdot\mathbf{a}_j t + \phi_0) \qquad (2)$$

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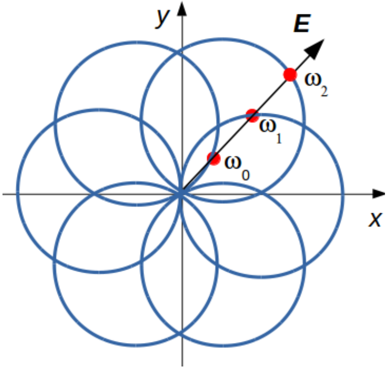
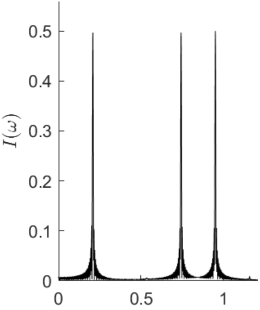
Three angle-dependent frequency values

$$\omega_j(\theta) = \frac{e}{\hbar}\mathbf{E}\cdot\mathbf{a}_j = \frac{e}{\hbar} Ea\cos(\theta - \theta_j) \qquad (3)$$

Only two frequencies are independent, $\omega_2 = \omega_0 + \omega_1$ as expected.

Three Bloch frequencies: angle dependence





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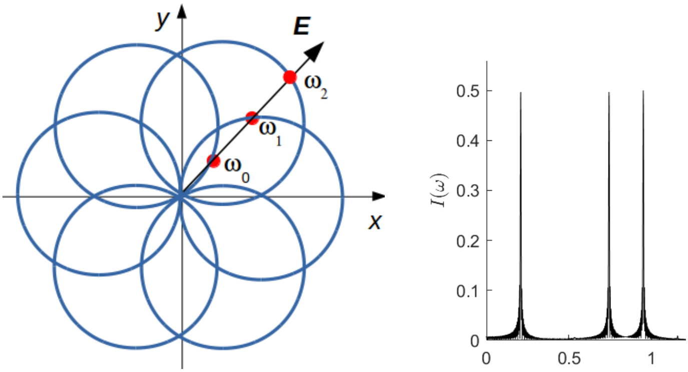

More frequencies for a more general dispersion.
Tomography of the bandstructure?

Adding magnetic field:

Newton's EOM for Bloch electron (use mks units):

$$\hbar\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \quad (4)$$

Lorentz force with electron band velocity
$\mathbf{v}(k) = \frac{1}{\hbar}\nabla_k \epsilon(\mathbf{k})$ (periodic in k)

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Two distinct types of orbits arise depending on the relative strength of $\mathbf{E}$ and B:
1) confined orbits (reside within BZ), B-dominated, one Bloch frequency
2) deconfined orbits (extended over infinitely many BZ's), E-dominated, two or more frequencies.

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2) deconfined orbits (extended over infinitely many BZ's), E-dominated, two or more frequencies.
Confined in k space means deconfined in r space, and vice versa.

Geometric visualization of dynamics

Try to put EOM in a Hamiltonian form (k only).

$$\dot{k}_1 = \partial_2 H(k_1, k_2), \quad \dot{k}_2 = -\partial_1 H(k_1, k_2) \qquad (5)$$

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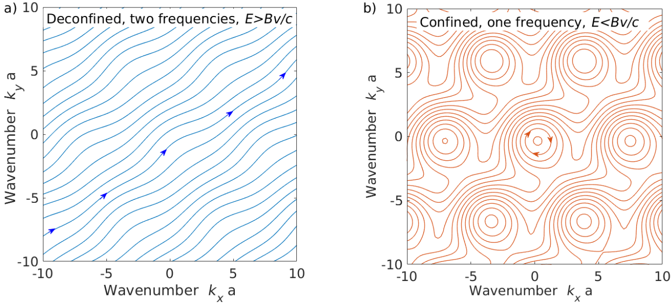
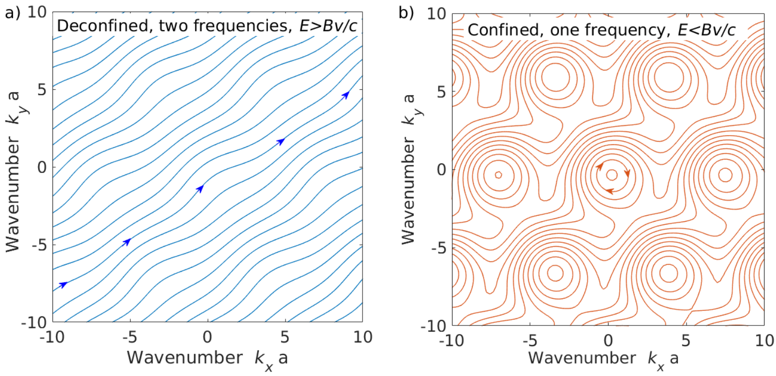
Trajectories are nothing but contours of $H(k_1, k_2)$!



Contours of a periodic function + a linear function:



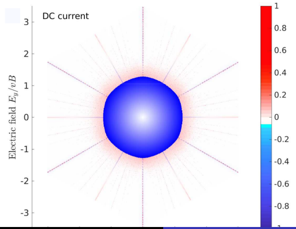
DC current at the transition

Find $\mathbf{R}(t) = \int_0^t \mathbf{v}(t')dt'$ by integrating EOM: $\hbar(\mathbf{k}(t) - \mathbf{k}(0)) = e\mathbf{E}t + e\mathbf{R}(t) \times \mathbf{B}$. Therefore,

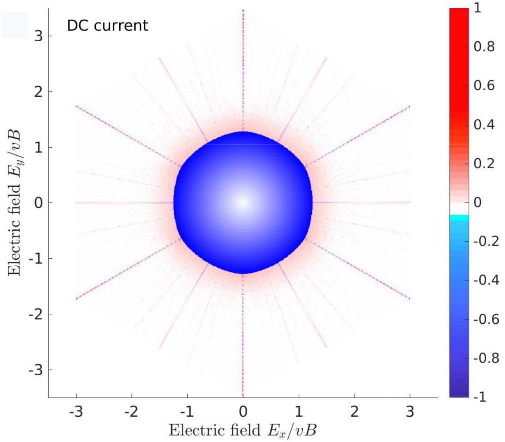
- In the confined (B-dominated) regime the drift velocity $\mathbf{u} = \mathbf{R}(t)/t|_{t=nT}$ equals

$$\mathbf{u} = \mathbf{E} \times \mathbf{B}/B^2$$

- At transition to the deconfined (E-dominated) regime drift velocity abruptly drops to zero.



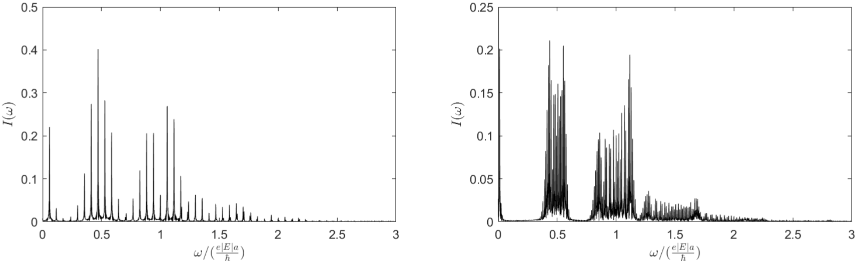
DC current component perpendicular to E



Abrupt drop to zero at transition; peculiar fine structure in the deconfined (E-dominated) state

Observations:

- Prominent for **E** near commensurability
- Turns into two-frequency spectrum at small B
- Indicates complex dynamics at the transition



2D Bloch oscillation summary

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- A new probe of carrier dynamics
- Dynamical transition driven by a B field
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Future:

- Coherent THz radiation?
- Finite carrier density? Charge instabilities?
- DC + AC driving. Negative differential conductivity? Gain vs. loss?
- Nonstationary Quantum Hall state in the B-dominated regime?