

Parquet RG approach to Moire heterostructures: a chiral phase diagram

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Alfred P. Sloan
FOUNDATION

Moire heterostructures

- 2d materials with Moire superlattices: Flat bands and correlation dominated physics
- Experiment: many correlated phases (superconductors, insulators, nematics...)
- Theory: How to understand?

Theoretical approaches

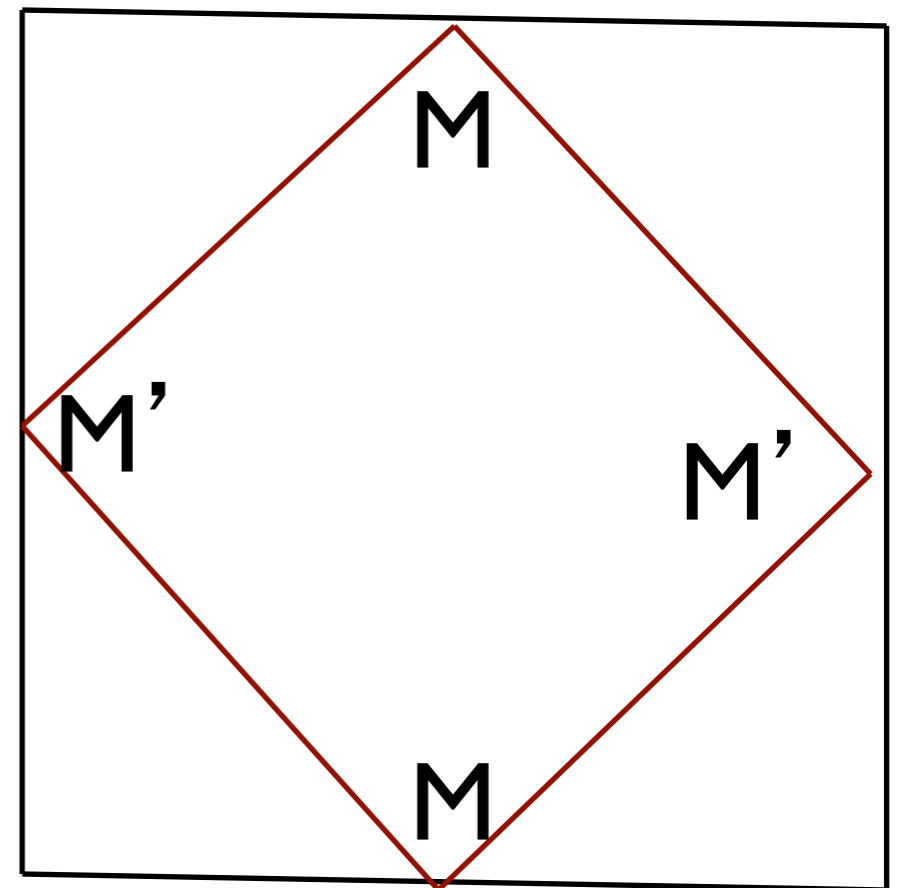
- One possibility: start from *strong coupling*, treat systems as doped Mott insulators
- Second possibility: start from *weak coupling*, examine instabilities of Fermi surface
- Empirically: intermediate coupling. Expect weak coupling appropriate for at least some Moire systems.

Weak coupling near Van Hove filling

- In 2d, Van Hove singularities (VHS) with log divergent density of states
- Near VHS, even weak coupling gives enhanced energy scales for correlated states (log square)
- Tuning to VHS in Moire subbands should be 'easy' (gating)
- If Fermi surface near VHS is *nested*, rich physics emerges

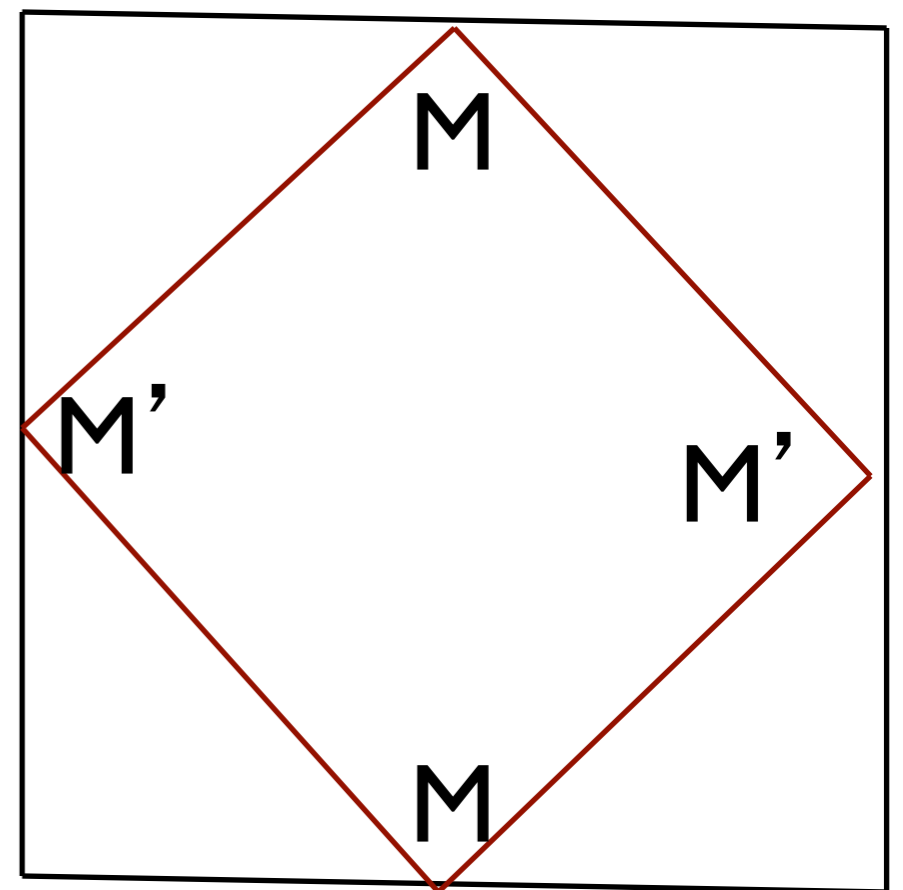
History: square lattice.

- Square lattice system near half filling
- Nearest neighbor tight binding model
- Van Hove singularity and nested Fermi surface



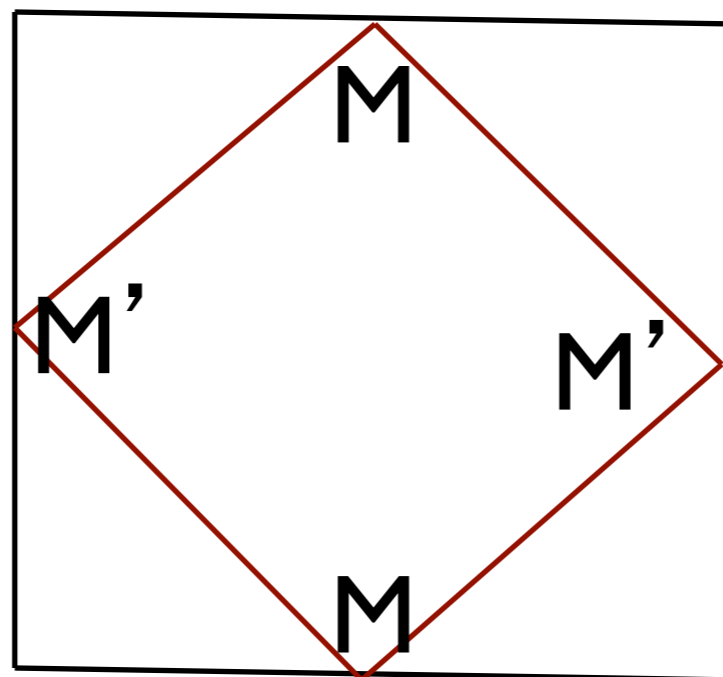
Square lattice review

- Instabilities in multiple channels, both particle particle and particle hole
- Log squared instabilities (enhanced energy scale)



Analyse competition between different orders using RG

- Schulz 1987, Dzyaloshinskii 1987, Furukawa, Rice 1998
- Progressively integrate out high energy states and examine flow of couplings. Marginal with log corrections
- Three sources of log divergence: BCS, nesting, DOS
- leading divergence is $\log^2(\Lambda/T)$
- One-loop RG for leading logs



Square lattice: bottom line

- At leading (log square) order, AF and d-SC are *degenerate*
- Incorporate subleading single logs: AF `wins' at half filling, gives way to d-SC upon doping
- Incorporation of subleading single logs is messy...

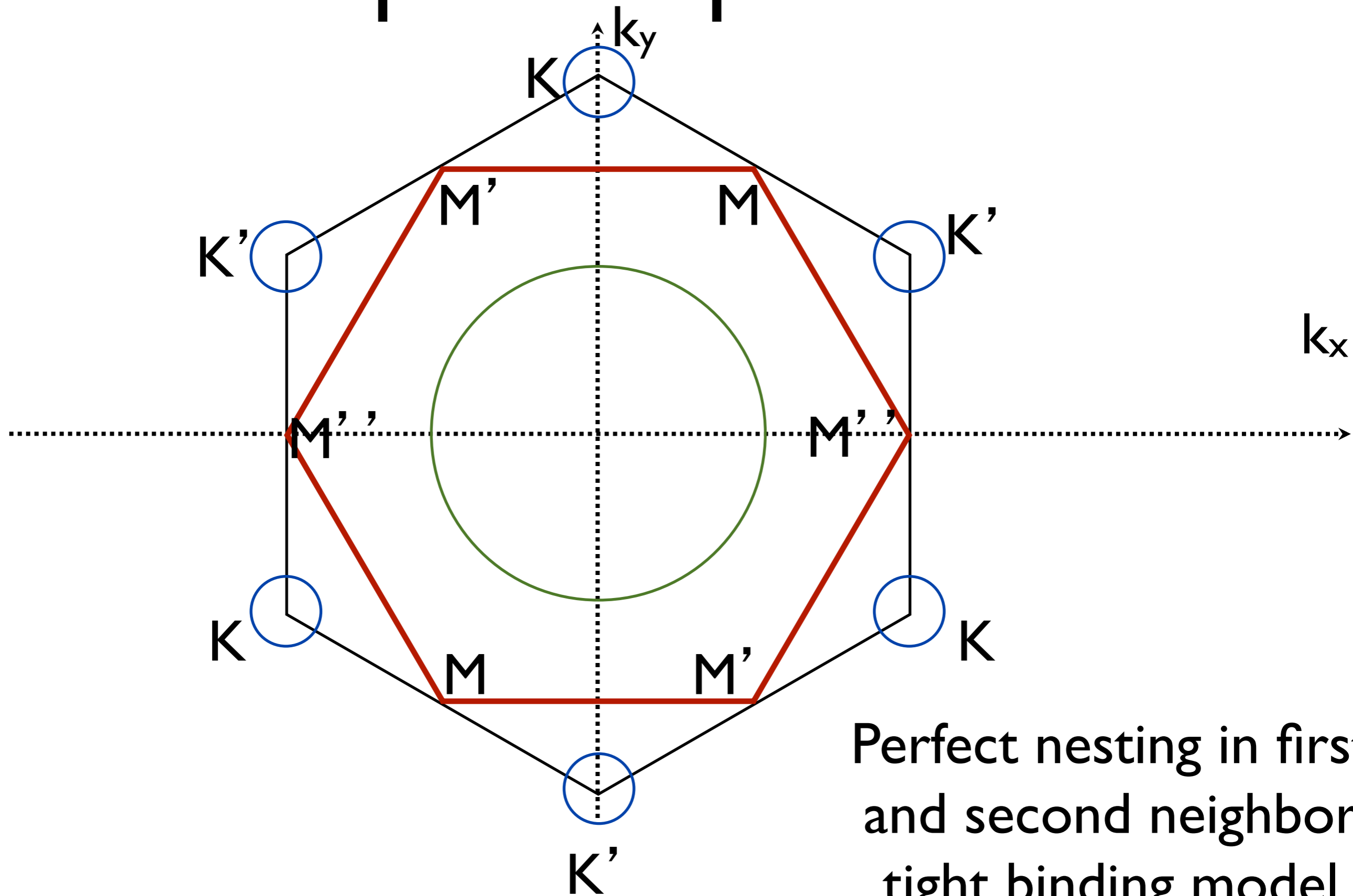
Honeycomb lattice is cleaner!



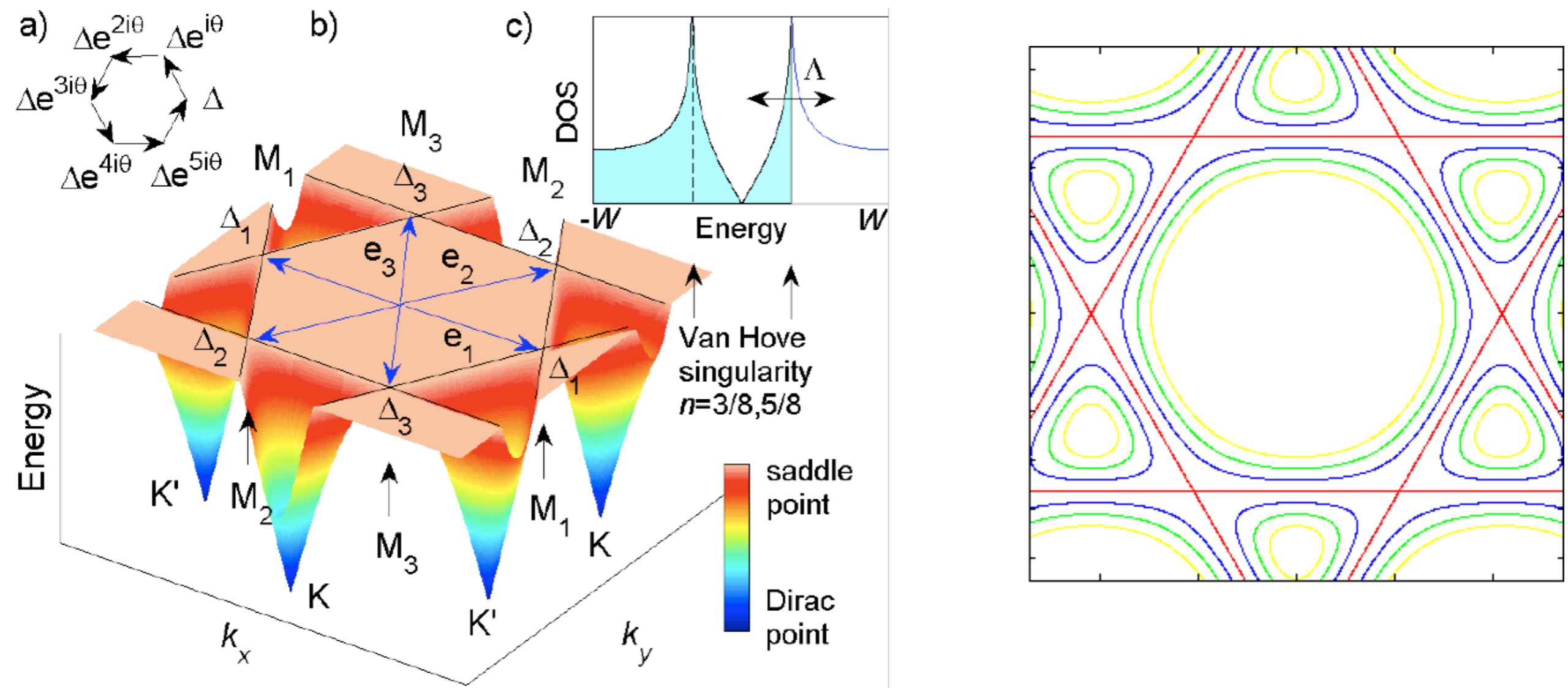
Rahul Nandkishore,
Leonid Levitov and
Andrey Chubukov

Nature Physics, 8,
158–163 (2012)

Doped Graphene



Perfect nesting in first
and second neighbor
tight binding model

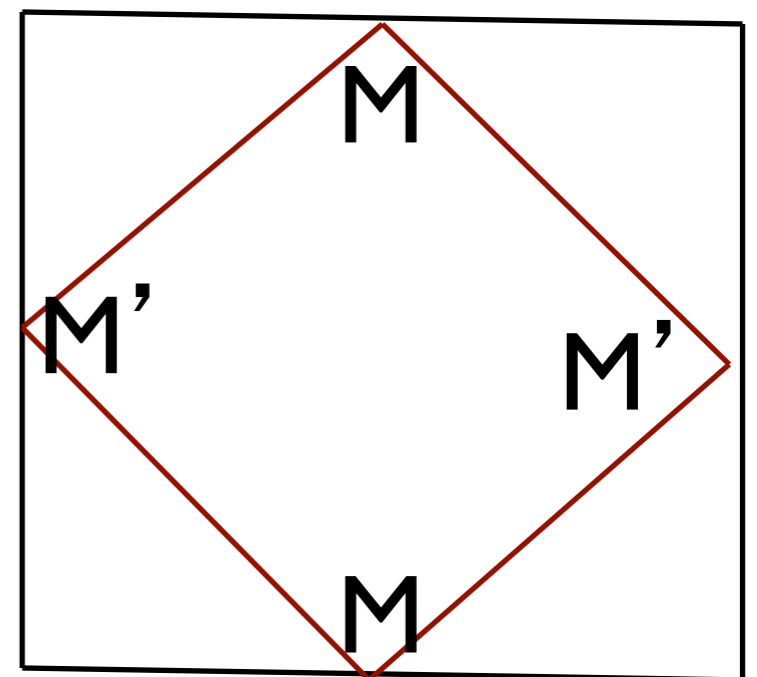
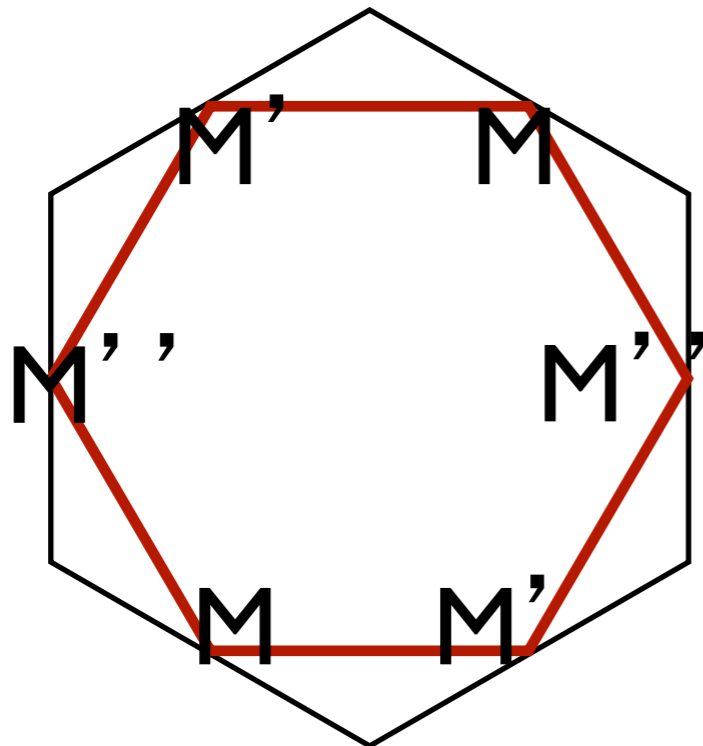


At $3/8$ or $5/8$ filling, the tight binding bandstructure displays a) Van Hove singularity (strong interactions) b) Nested FS (strong density wave fluctuations)

What happens?

Analyse competition between different orders using RG

- Progressively integrate out high energy states and examine flow of couplings. Marginal with log corrections
- Three sources of log divergence: BCS, nesting, DOS
- leading divergence is $\log^2(\Lambda/T)$
- One-loop RG for leading logs
- Similar to square lattice at half filling: Schulz 1987, Dzyaloshinskii 1987, Furukawa, Rice 1998



Weak coupling RG flow

- RG flows to strong coupling (instability)
- Unique and universal instability.

$$T_c \sim \Lambda \exp(-A/\sqrt{g_0\nu_0}).$$

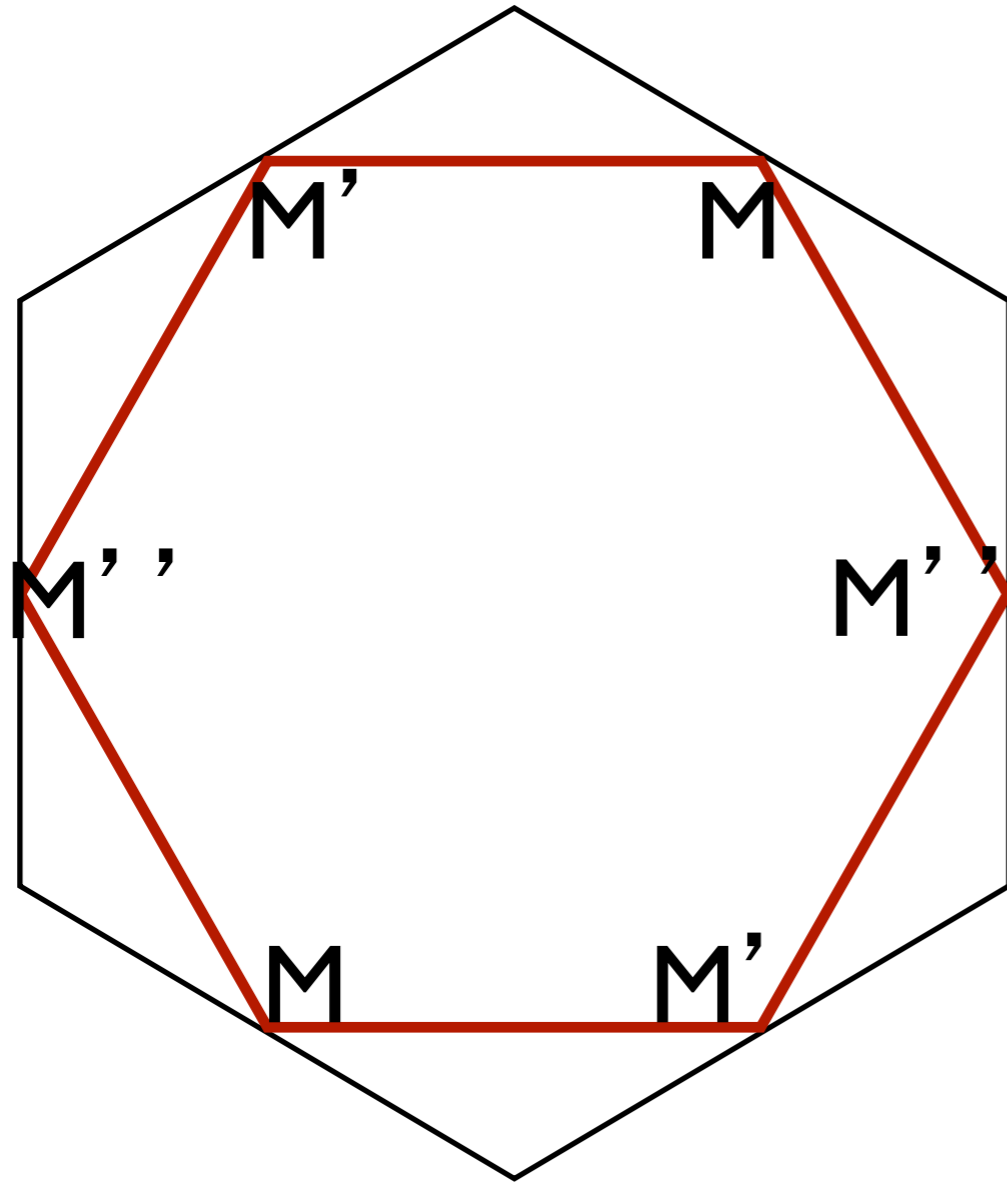
Contrast with standard weak coupling result

$$T_c \sim \Lambda \exp\left(-\frac{A'}{g_0\nu_0}\right)$$

Tc enhanced by large DOS

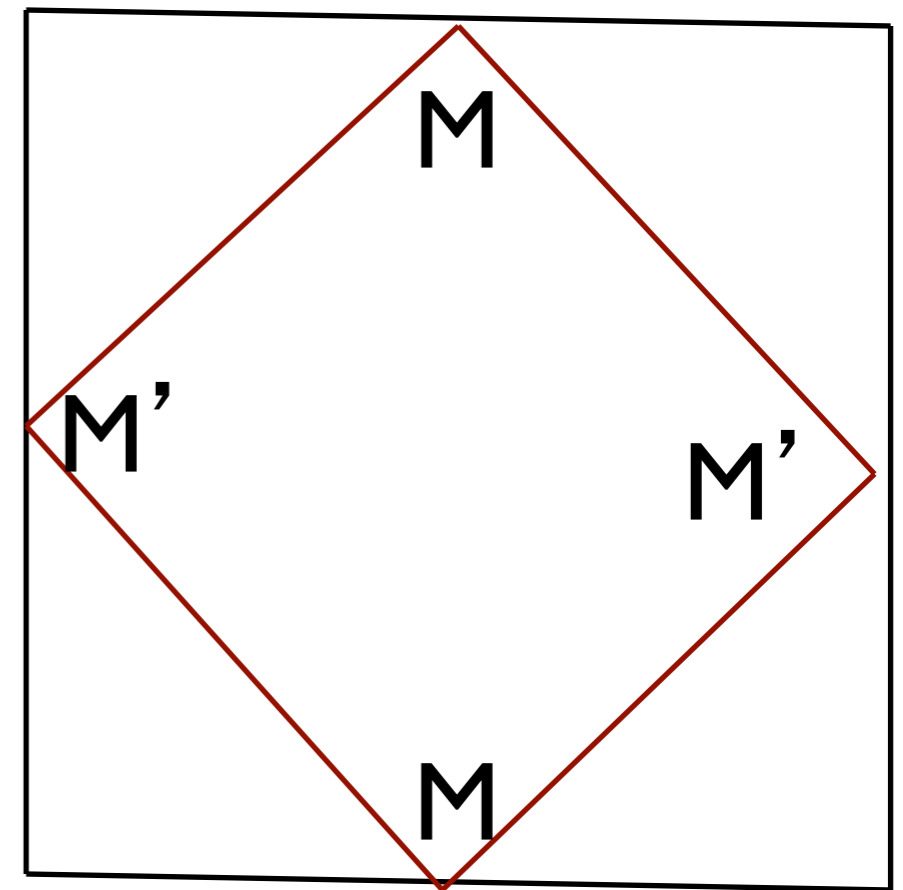
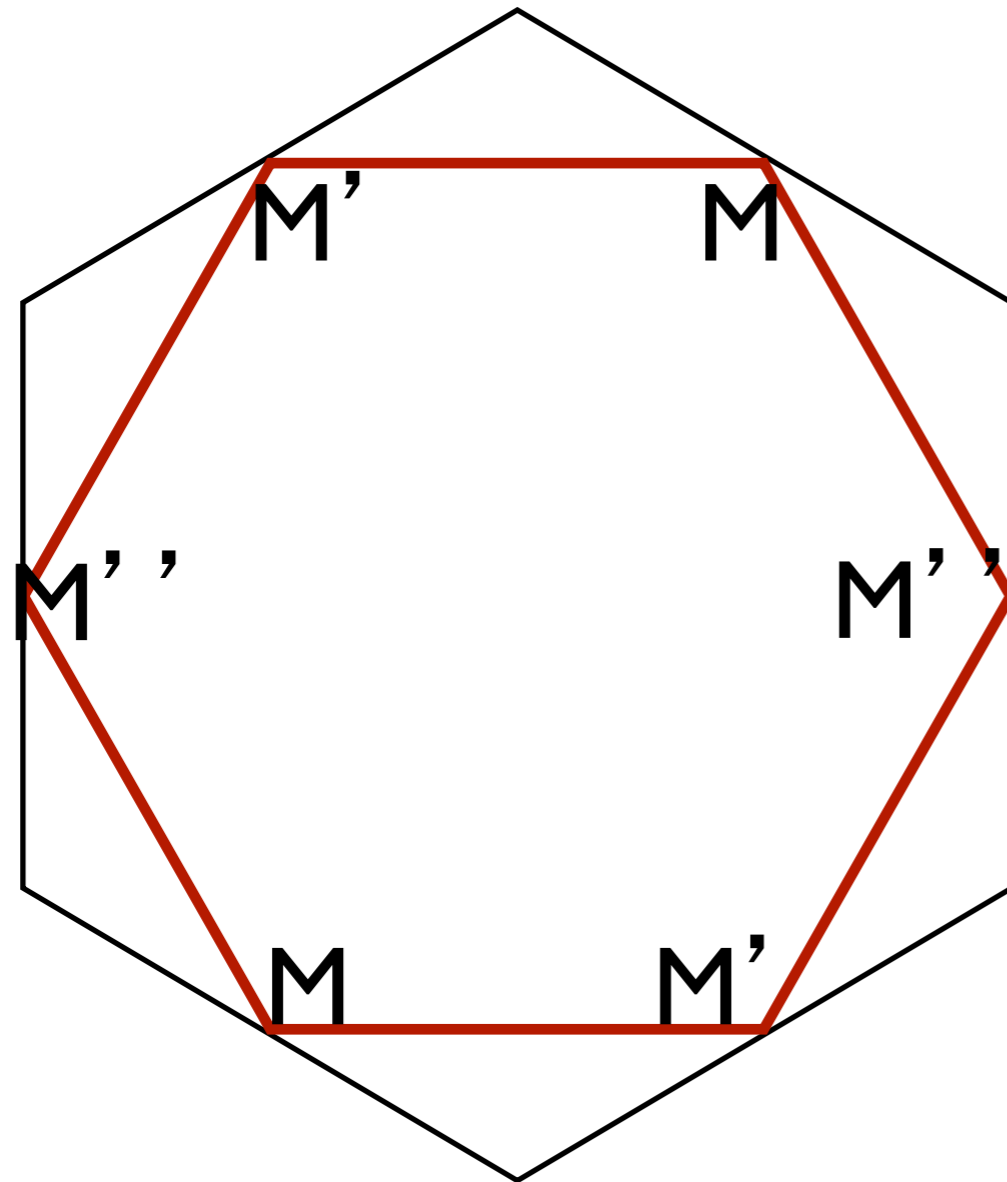
What kind of ordering?

Honeycomb lattice



- Instabilities to SDW and d-SC
- d-SC wins at leading log order

Why the honeycomb lattice is better for SC



How to optimize for (two dimensional) SC?

- Saddle points of dispersion (the more the better)
- Need nesting (generate attraction)
- Saddle points separated by half-reciprocal lattice vector
- Honeycomb lattice as ideal platform

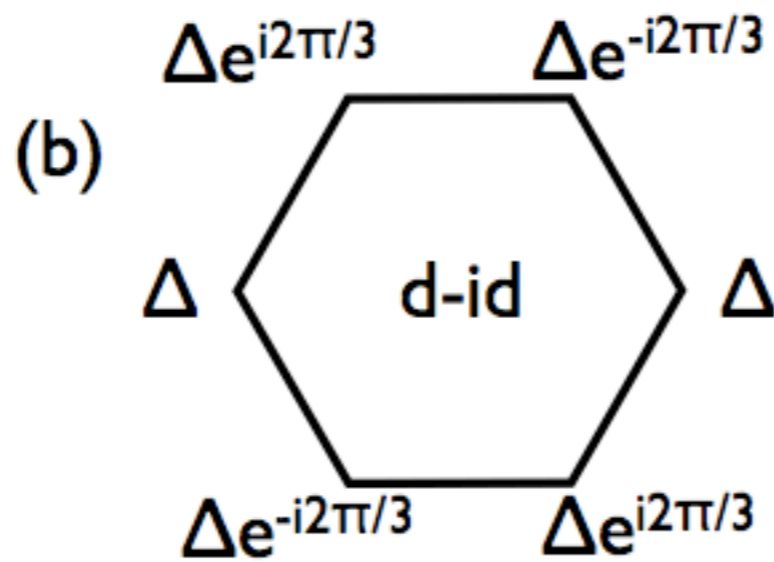
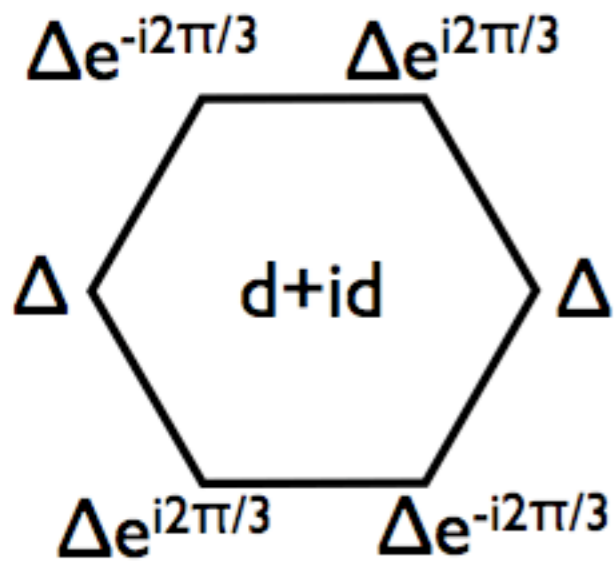
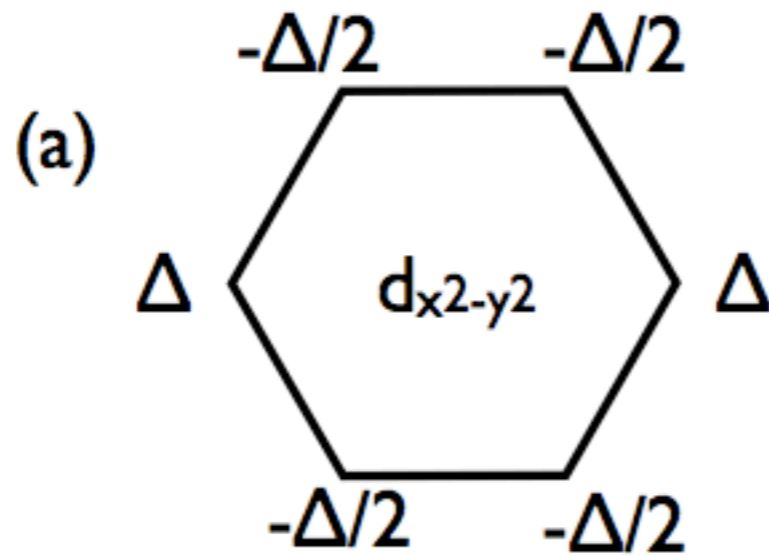
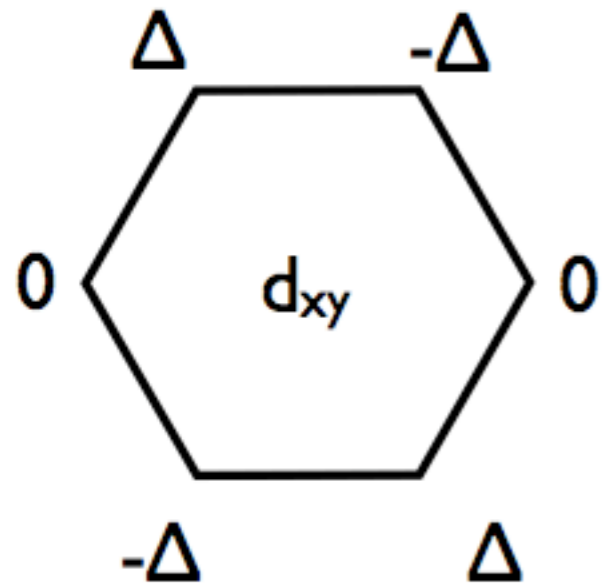
A side benefit...

Square lattice: d-wave

Honeycomb: doubly degenerate d-wave

Interplay of d-wave orders below T_c cannot be addressed through RG. Need a Ginzburg Landau theory.

$$F = \alpha(T - T_c)(|\Delta_{xy}|^2 + |\Delta_{x^2-y^2}|^2) + K_1(|\Delta_{xy}|^2 + |\Delta_{x^2-y^2}|^2)^2 + K_2|\Delta_{xy}^2 + \Delta_{x^2-y^2}^2|^2 + O(\Delta^6)$$



$$F = F(\Delta_1) + F(\Delta_2) + F(\Delta_3),$$

$$F(\Delta_i) = \alpha'(T - T_c)|\Delta_i|^2 + K|\Delta_i|^4, \quad K > 0$$

Expressing F in terms Δ_{xy} , $\Delta_{x^2-y^2}$, find $K_2 > 0$

Hence chiral SC

Evidence for superconductivity with broken time-reversal symmetry in locally noncentrosymmetric SrPtAs

P. K. Biswas,¹ H. Luetkens,^{1,*} T. Neupert,^{2,3} T. Stürzer,⁴ C. Baines,¹ G. Pascua,¹ A. P. Schnyder,⁵ M. H. Fischer,⁶ J. Goryo,^{3,7} M. R. Lees,⁸ H. Maeter,⁹ F. Brückner,⁹ H.-H. Klauss,⁹ M. Nicklas,¹⁰ P. J. Baker,¹¹ A. D. Hillier,¹¹ M. Sigrist,³ A. Amato,¹ and D. Johrendt⁴

- Honeycomb layers, near Van Hove doping
- Superconductor ($T_c=2\text{K}$).
- TRS breaking, nodeless...
- Most likely $d+id$.

PHYSICAL REVIEW B **89**, 020509(R) (2014)

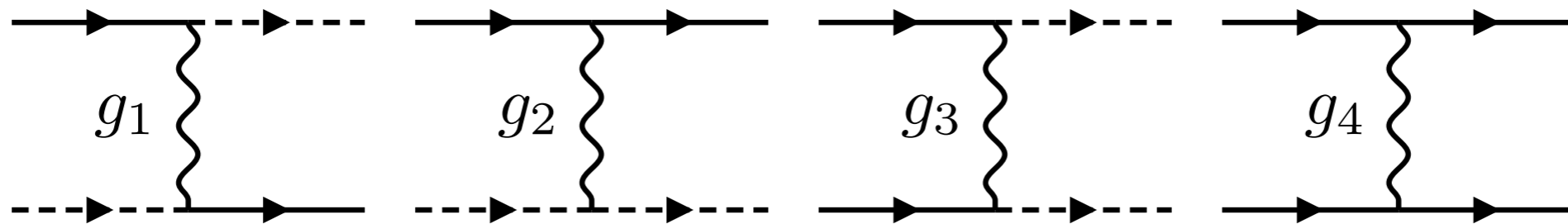
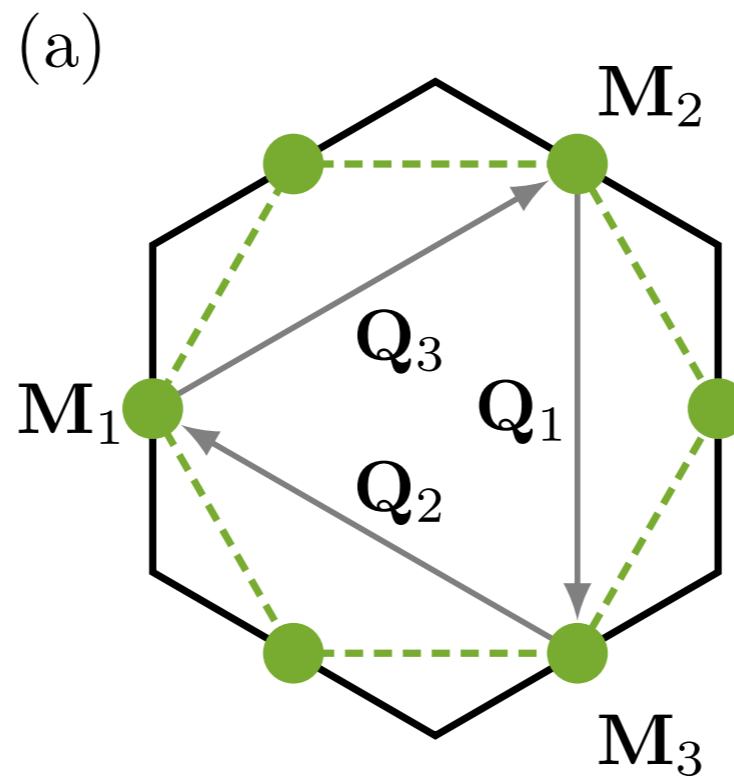


Chiral *d*-wave superconductivity in SrPtAs

Mark H. Fischer,^{1,2} Titus Neupert,^{3,4} Christian Platt,⁵ Andreas P. Schnyder,⁶ Werner Hanke,⁵ Jun Goryo,⁷
Ronny Thomale,⁵ and Manfred Sgrist⁴

Back to Moire

- Enhanced *orbital* degeneracy on top of spin degeneracy
- Consider $SU(N)$ symmetric extensions of previous models, $N > 2$
- Simple toy model, much less detail than fRG...but simplicity helps expose key physics



Evaluate β functions via one loop perturbative RG

Orbital degeneracy affects diagrams with loops

RG equations (general)

$$\frac{dg_1}{dy} = d_1 [g_1(2g_2 - N_f g_1) + (2 - N_f)g_3^2],$$

$$\frac{dg_2}{dy} = d_1 (g_2^2 + g_3^2),$$

$$\frac{dg_3}{dy} = 2d_1 g_3 [2g_2 - (N_f - 1)g_1] - g_3 [(N_p - 2)g_3 + 2g_4],$$

$$\frac{dg_4}{dy} = -(N_p - 1)g_3^2 - g_4^2.$$

Analysis of RG equations

$$\frac{dg_1}{dy} = d_1 [g_1(2g_2 - N_f g_1) + (2 - N_f)g_3^2],$$

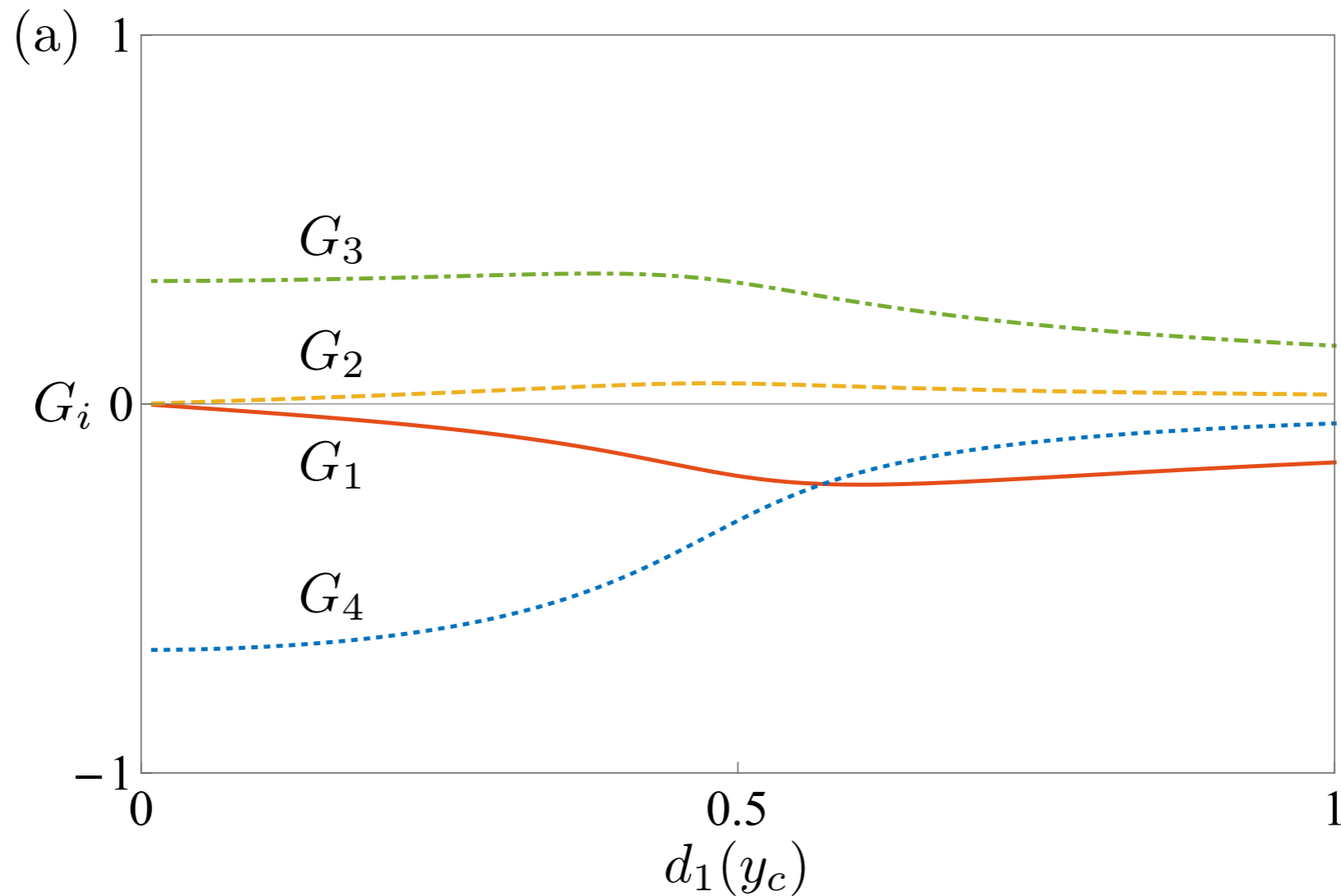
$$\frac{dg_2}{dy} = d_1 (g_2^2 + g_3^2),$$

$$\frac{dg_3}{dy} = 2d_1 g_3 [2g_2 - (N_f - 1)g_1] - g_3 [(N_p - 2)g_3 + 2g_4],$$

$$\frac{dg_4}{dy} = -(N_p - 1)g_3^2 - g_4^2.$$

Unique instability

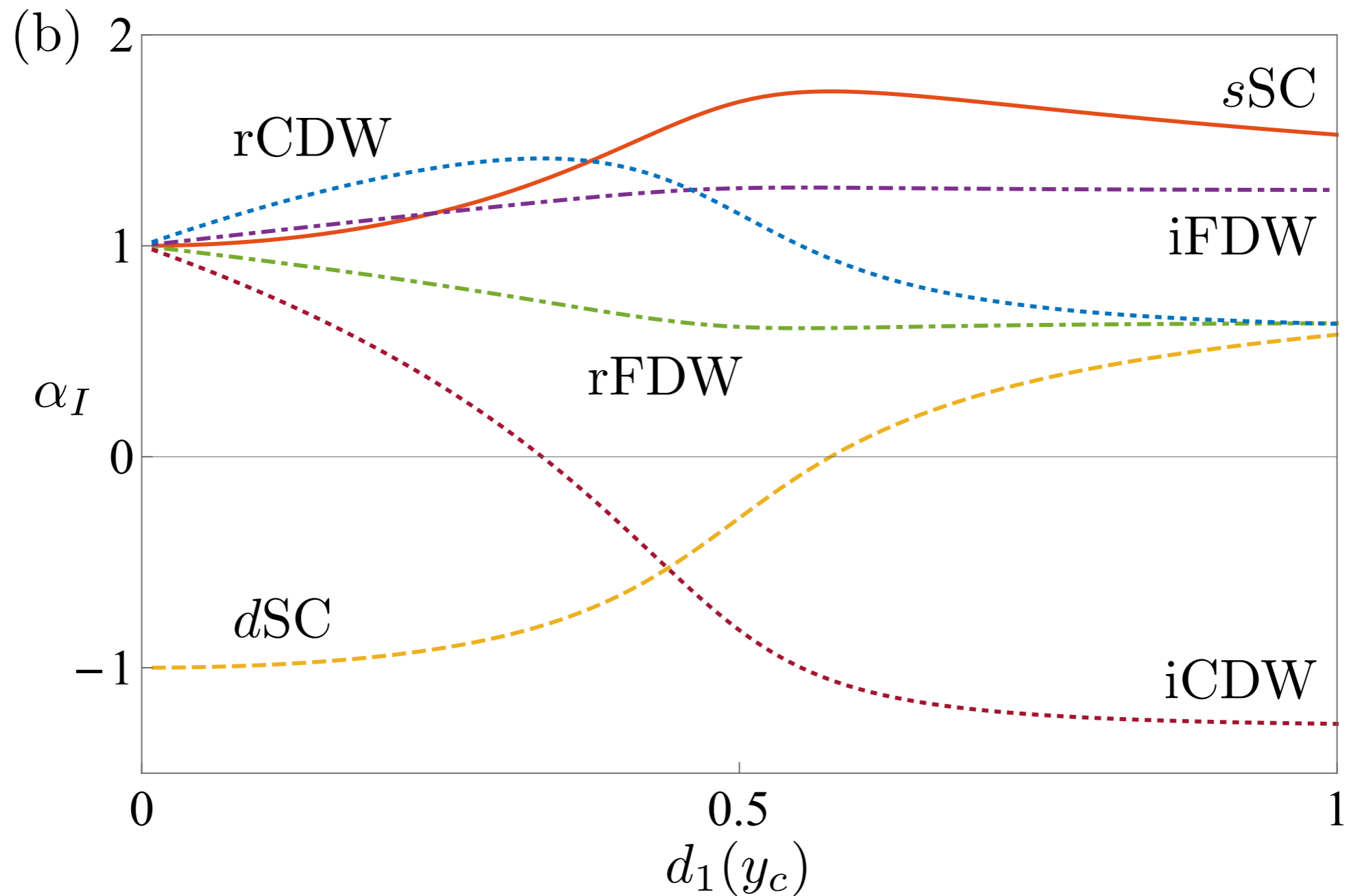
$$g_i = \frac{G_i}{y_c - y}.$$



Instability analysis

- Introduce test vertices corresponding to all possible fermion bilinears
- Calculate susceptibility towards ordering in every channel
- Most divergent susceptibility represents leading instability.

Leading instabilities



Leading instabilities

Leading instability is to an imaginary charge density wave (flux order), giving way upon doping to d-SC

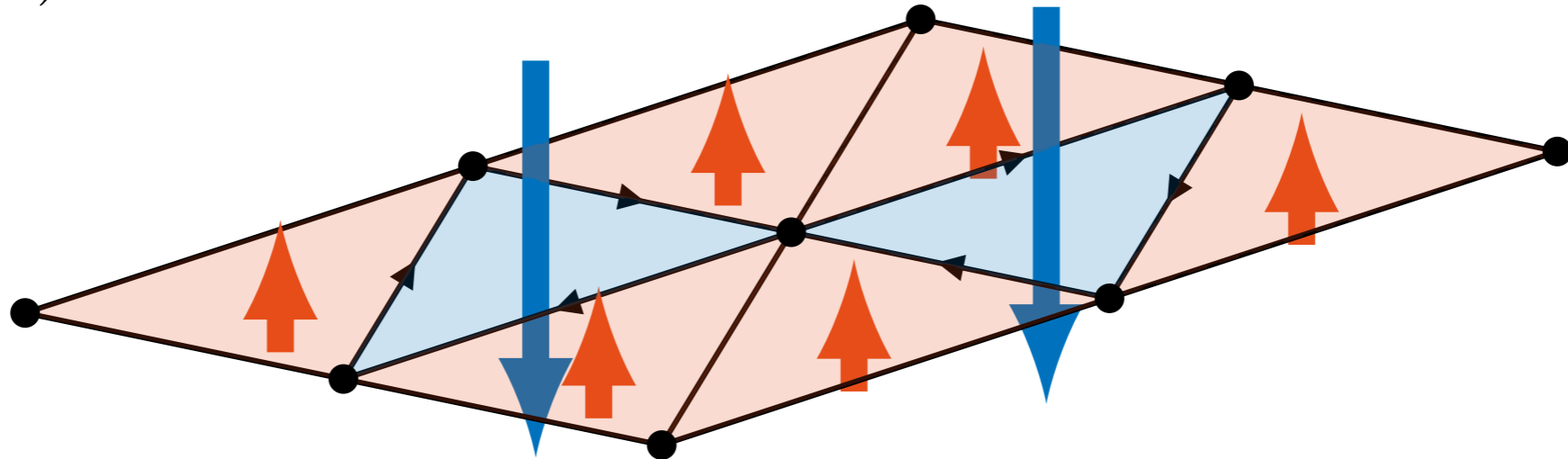
Flux order channel is *triply degenerate* (three nesting vectors). d-SC channel is doubly degenerate.

A chiral insulator

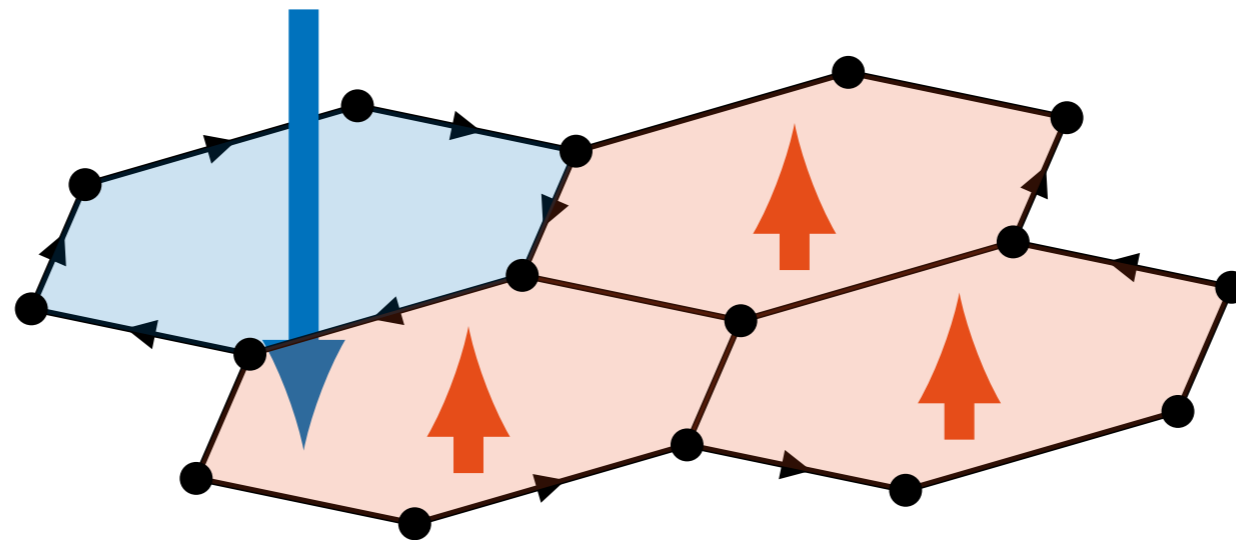
- Landau-Ginzburg analysis below T_c favors 3Q CDW order
- Fully gapped state with *Chern number* for occupied bands – a *Chern insulator*
- *Quantized anomalous Hall effect* – without magnetic order

Visualization – flux order

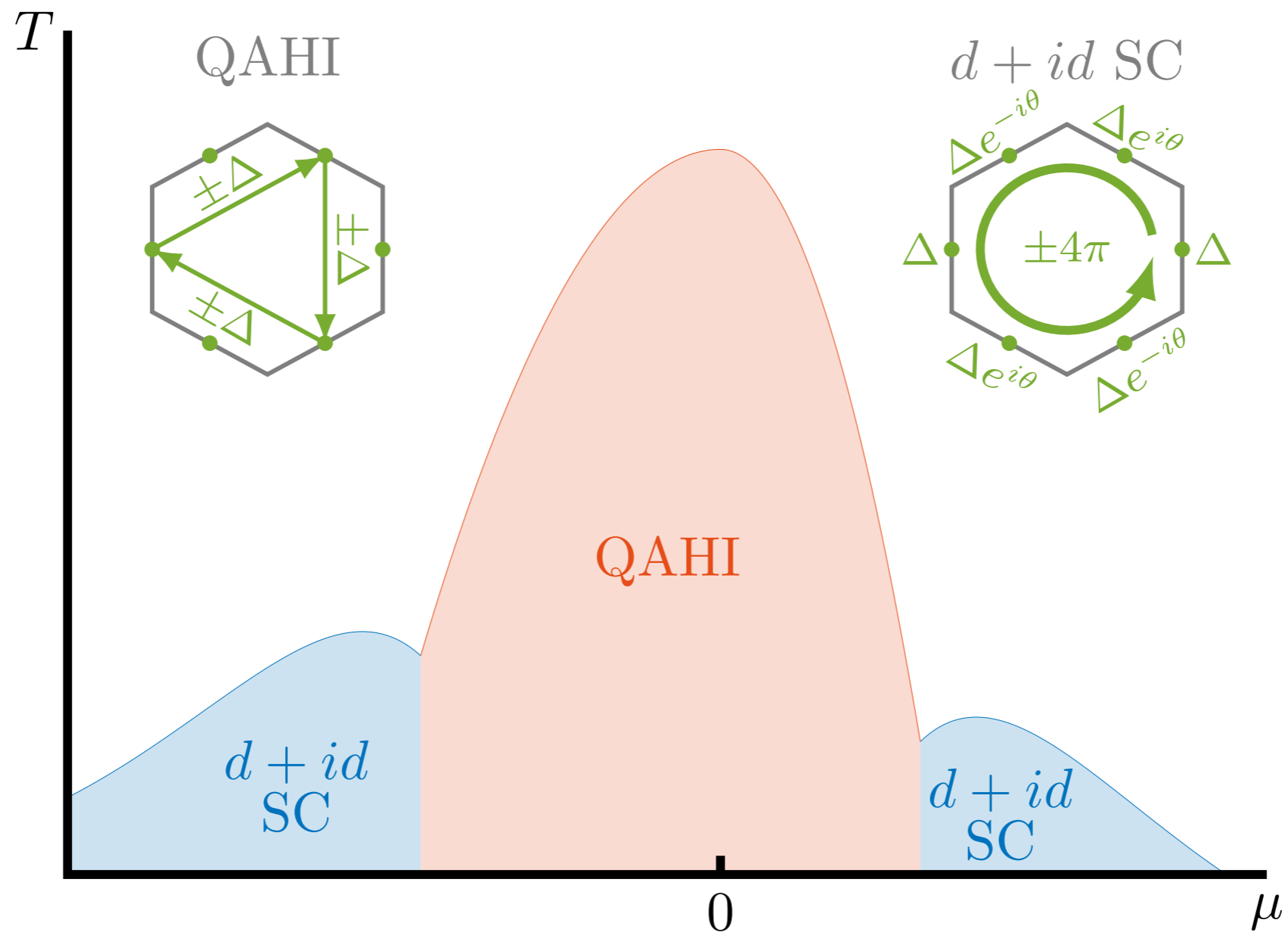
(a)



(b)



A chiral phase diagram



Experimental status?

- Sharpe...Goldhaber-Gordon (1901.03520)– signatures of Hall conductance at zero field (not quantized)
- Weckbecker...Shallicross (1901.04712)– signatures of flux order (current loops)

On square lattice

- Revisit square lattice with $SU(N)$ flavor
- Recall: $N=2$ AF and d-SC degenerate at leading log order
- $N>2$: Unique leading instability to (non-chiral) staggered flux state
- Gives way upon doping to nodal d-SC

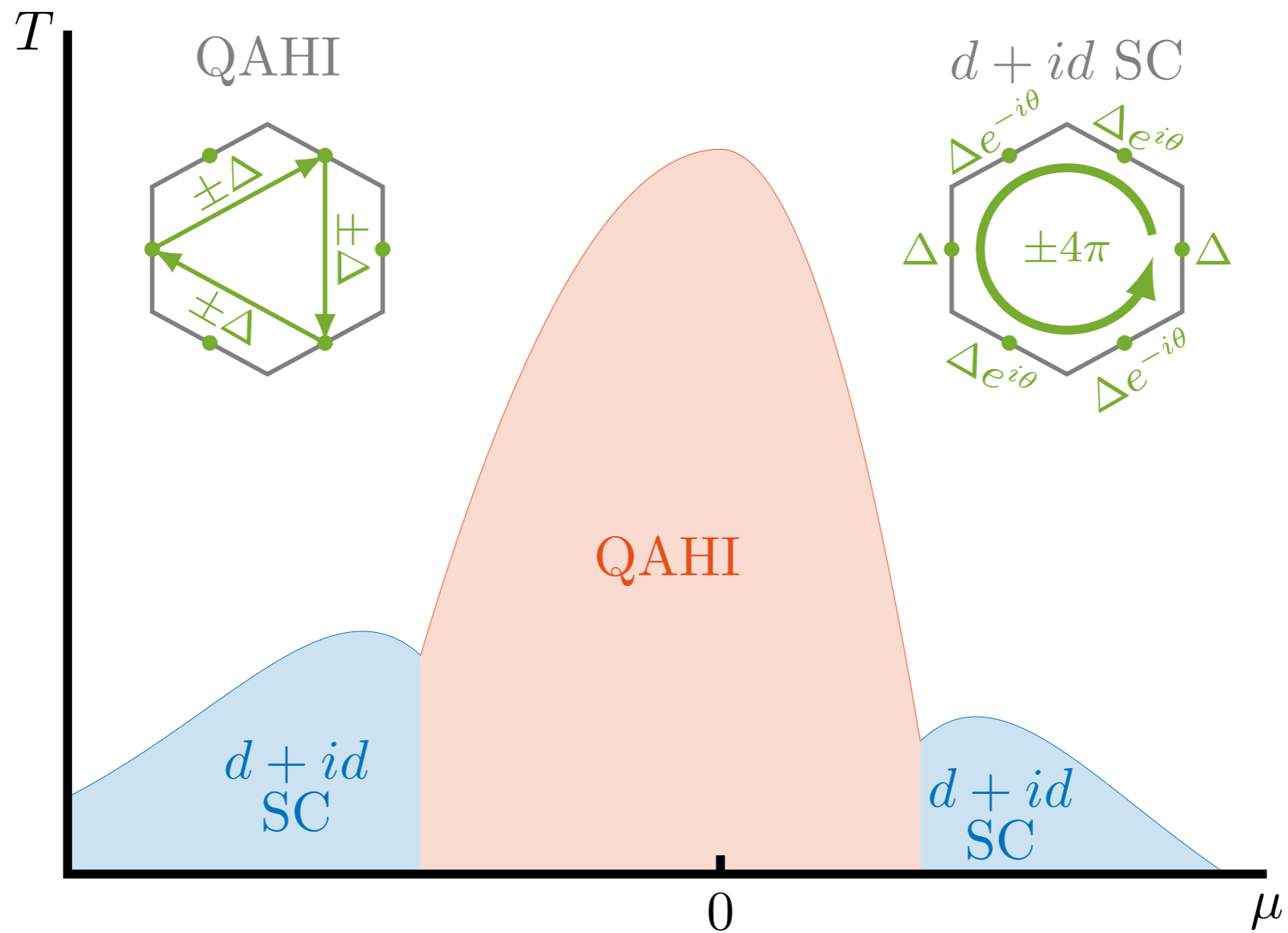
Conclusions

- Close to Van Hove filling, with nested FS, weak coupling can give access to rich phase diagram with enhanced energy scales
- Parquet RG is a good approach in this regime
- Flavor degeneracy in Moire systems makes an crucial difference – favors *flux ordered* states

Conclusions II

- On hexagonal lattices, leading weak coupling instability near VHS is to *Chern insulator* exhibiting QAH effect without magnetic order
- On doping, gives way to chiral superconductor

A chiral phase diagram



Thanks to collaborators

- Yu-Ping Lin (Boulder)
- L.S. Levitov (MIT)
- A. V. Chubukov (UW-Madison/Minnesota)