Quantum mechanics of the electronic fluid in chiral solids

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KITP, Correlated bands in TBLG
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Often, chiral solids are twisted in some way, *e.g.* twisted bilayer graphene.

Reduced symmetry yields new responses:

$$\boldsymbol{j}_{\mathrm{AHE}} \propto \boldsymbol{E} \times \boldsymbol{M}, \boldsymbol{j}_{\mathrm{CME}} \propto \boldsymbol{B}$$

- electrodynamic responses

$$\nabla \cdot \Pi^{2d} \propto \eta^{H} \Delta \boldsymbol{u} \times \boldsymbol{z}, \;\; \boldsymbol{j}^{3d}_{\mathrm{CVE}} \propto \boldsymbol{\omega},$$

- hydrodynamic responses

Effects of "band geometry"

Ultrapure electronic systems, where hydrodynamic flow was claimed:

GaAs – Wurzburg, ETH
Graphene -- Manchester, Weizmann
$PdCoO_2$ and other "delafossites") -- MPI CPS Dresden
WP_2 – IBM Zurich+Dresden

Classical mechanics of electrons in solids

Promote the band energy to the classical Hamiltonian:

$$H(\boldsymbol{r}, \boldsymbol{k}) = E_{\boldsymbol{k}-e\boldsymbol{A}} + e\phi(\boldsymbol{r})$$

"Peierls substitution"

Write down the equations of motion:

$$\dot{\boldsymbol{r}} = \partial_{\boldsymbol{k}} E_{\boldsymbol{k}},$$

$$\dot{\boldsymbol{k}} = -\partial_{\boldsymbol{r}} E_{\boldsymbol{k}} + e\partial_{\boldsymbol{k}} E_{\boldsymbol{k}} \times \boldsymbol{B}.$$

Then perhaps solve the Boltzmann equation:

$$\underbrace{\partial_t f + \dot{\boldsymbol{r}} \boldsymbol{\nabla} f + \dot{\boldsymbol{k}} \partial_{\boldsymbol{k}} f}_{} = \underbrace{\hat{I}_{st}}_{}$$

$+O(\hbar)$corrections $\rightarrow$ "unusual" E&M

$+O(\hbar)$correction $\rightarrow$ "unusual" hydrodynamics

Semiclassical motional in external fields

Proceed by comparison:

$$\hat{\mathrm{p}} = \frac{1}{i}\nabla_{\mathbf{r}} - e\boldsymbol{A}_{\mathbf{r}}$$

$$\hat{\mathrm{r}} = i\nabla_{\mathbf{p}} + \mathbf{A}_{\mathbf{p}},\ \mathbf{A}_{\mathbf{p}} = i\langle u_{\mathbf{p}}|\nabla_{\mathbf{p}}|u_{\mathbf{p}}\rangle$$

\- looks like a "vector potential" in the momentum space

Motion in ***external*** fields is semiclassical:

$$\dot{\mathrm{p}} \quad = \quad -e\frac{\partial\phi}{\partial\mathbf{r}} + e\dot{\mathbf{r}}\times\mathbf{B},\ \mathbf{B} = \nabla_{\mathbf{r}}\times\boldsymbol{A}_{\mathbf{r}}$$

$$\dot{\mathrm{r}} \quad = \quad \frac{\partial\epsilon_{n\mathbf{p}}}{\partial\mathbf{p}} - \dot{\mathrm{p}}\times\boldsymbol{\Omega}_{n\mathbf{p}},\ \boldsymbol{\Omega}_{n\mathbf{p}} = \nabla_{\mathbf{p}}\times\mathbf{A}_{\mathbf{p}}$$

$$\boldsymbol{\Omega}_{n\mathbf{p}} = i\langle\partial_{\mathbf{p}}u_{n\mathbf{p}}|\times|\partial_{\mathbf{p}}u_{n\mathbf{p}}\rangle$$

Motion in magnetic field

$$\dot{\mathbf{p}} = e\boldsymbol{E} + e\dot{\mathbf{r}} \times \mathbf{B}$$

$$\dot{\mathbf{r}} = \mathbf{v}_{\mathbf{p}} - \dot{\mathbf{p}} \times \boldsymbol{\Omega}_{n\mathbf{p}}$$

Chiral anomaly

$$\dot{\mathbf{p}} = \frac{1}{D_B}\left(e\boldsymbol{E} + e\mathbf{v}_{\mathbf{p}} \times \mathbf{B} - e^2(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}_{\mathbf{p}}\right)$$

$$\dot{\mathbf{r}} = \frac{1}{D_B}\left(\mathbf{v}_{\mathbf{p}} - e\boldsymbol{E} \times \boldsymbol{\Omega}_{n\mathbf{p}} - e(\mathbf{v}_{\mathbf{p}}\cdot\boldsymbol{\Omega}_{\mathbf{p}})\boldsymbol{B}\right)$$

AHE

Chiral magnetic effect

$$D_{\boldsymbol{B}} = 1 - e\boldsymbol{B}\boldsymbol{\Omega}_{\boldsymbol{p}}$$

So far mostly electromagnetic phenomena

What about hydrodynamics?

$$\boldsymbol{j}_{\mathrm{AHE}} \propto (E - \tfrac{1}{e}\boldsymbol{\nabla}\mu) \times \boldsymbol{M},$$

$$\boldsymbol{j}^{3d}_{\mathrm{CVE}} \propto \boldsymbol{\omega},$$

$$\nabla \cdot \Pi^{2d} \propto \eta^H \Delta \boldsymbol{u} \times \boldsymbol{z},$$

responses to "statistical" forces

$$\partial_t f + \dot{\boldsymbol{r}}\boldsymbol{\nabla} f + \dot{\boldsymbol{k}}\partial_{\boldsymbol{k}} f = \hat{I}_{st}$$

$+O(\hbar)$corrections $\rightarrow$ "unusual" E&M

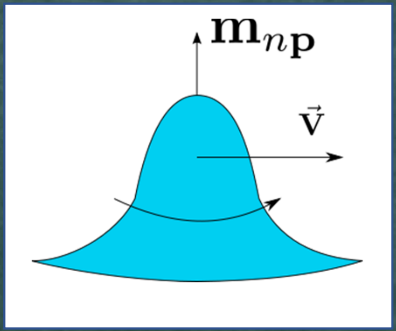
$+O(\hbar)$correction $\rightarrow$ "unusual" hydrodynamics

Band geometry effects in collisions

Case study: Weyl fermions

$$H_{\boldsymbol{p}} = v\boldsymbol{\sigma}\cdot\boldsymbol{p},\ \ \varepsilon_{\boldsymbol{p}} = \pm vp$$

Intrinsic magnetic moment (conduction band):



$$\mathbf{m}_{\boldsymbol{p}} = \frac{ie}{2}\langle\partial_{\boldsymbol{p}}u_{\boldsymbol{p}}|\times(h_{\boldsymbol{p}}-\epsilon_{\boldsymbol{p}})|\partial_{\boldsymbol{p}}u_{\boldsymbol{p}}\rangle = \frac{ev\hbar}{2p}\boldsymbol{e}_{\boldsymbol{p}}$$

Intrinsic angular momentum:

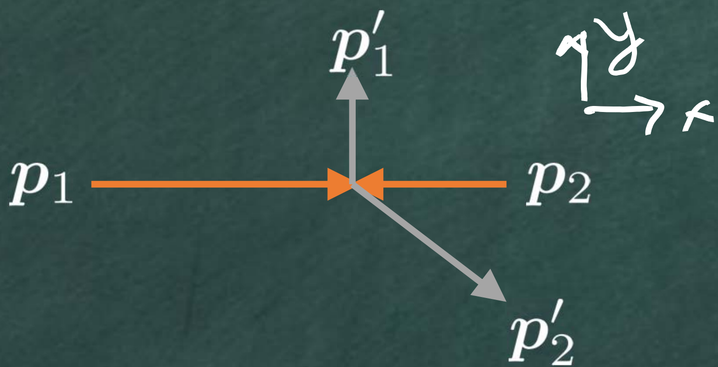
$$\boldsymbol{M}_p = \langle(\boldsymbol{r}-\boldsymbol{r}_c)\times(\boldsymbol{p}-\boldsymbol{p}_c)\rangle + \frac{\hbar}{2}\langle\boldsymbol{\sigma}\rangle = \frac{\hbar}{2}\boldsymbol{e}_{\boldsymbol{p}} \equiv \boldsymbol{s}_{\boldsymbol{p}}$$

Total angular momentum is conserved in collisions:

$$[\boldsymbol{\ell}+\boldsymbol{s}]^{in} = [\boldsymbol{\ell}+\boldsymbol{s}]^{out}$$

The need for two-particle shifts in two-particle collisions

Naïve scattering diagram:

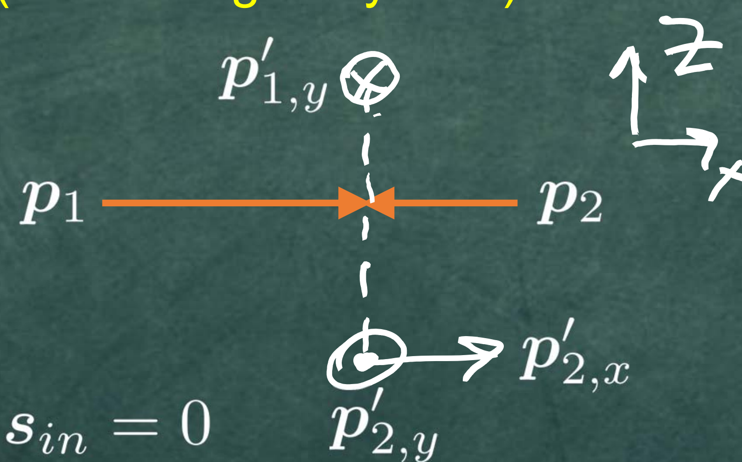


$$\boldsymbol{\ell}_{in} = 0, \ \boldsymbol{s}_{in} = 0$$

$$\boldsymbol{\ell}_{out} = 0, \ \boldsymbol{s}_{out} \neq 0$$

Something is wrong!

Reality (view along the y-axis)



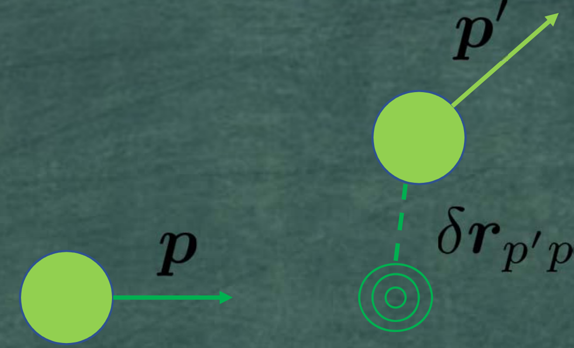
$$\boldsymbol{\ell}_{in} = 0, \ \boldsymbol{s}_{in} = 0$$

$$\boldsymbol{\ell}_{out} \neq 0, \ \boldsymbol{s}_{out} \neq 0, \ \boldsymbol{\ell}_{out} + \boldsymbol{s}_{out} = 0$$

Something is right!

Main observation: Shift happens

"Side jump" (Berger, 1970):



Qualitative picture for a smooth impurity potential: displacement in the impurity's electric field due to the anomalous velocity

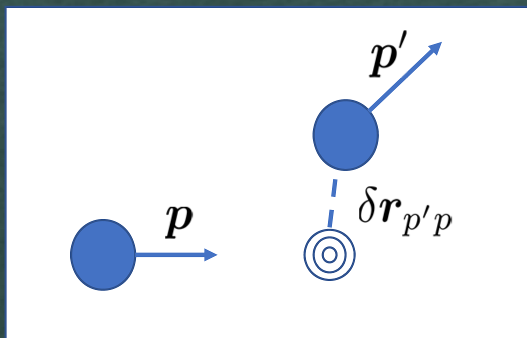
$$\delta \boldsymbol{r}_{\boldsymbol{p}\boldsymbol{p}'} = \int_{t_i}^{t_f} dt\, \boldsymbol{\Omega} \times \dot{\boldsymbol{p}} = \boldsymbol{\Omega} \times (\boldsymbol{p} - \boldsymbol{p}')$$

Full result, any weak impurity

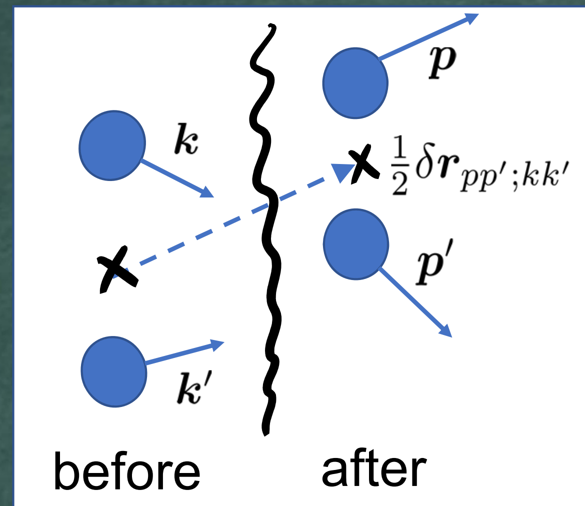
$$\delta \boldsymbol{r}_{pp'} = \langle u_{\boldsymbol{p}} | i\partial_{\boldsymbol{p}} u_{\boldsymbol{p}} \rangle - \langle u_{\boldsymbol{p}'} | i\partial_{\boldsymbol{p}'} u_{\boldsymbol{p}'} \rangle - (\partial_{\boldsymbol{p}} + \partial_{\boldsymbol{p}'}) \mathrm{Arg} \langle u_{\boldsymbol{p}} | u_{\boldsymbol{p}'} \rangle$$

(Belinicher, Ivchenko, Sturman, 1982; Sinitsyn, MacDonald, Niu, 2007)

Two-particle collisional coordinate shift



VS



- Individual shifts are ill-defined due to particle indistinguishability
- Symmetric combination – total shift – is well-defined:

$$\boldsymbol{R}^{(-\infty)} = \boldsymbol{v}_{\mathbf{k}} t + \boldsymbol{v}_{\mathbf{k}'} t + \delta \mathbf{r}^{(-\infty)}$$

$$\longrightarrow \delta \boldsymbol{r}_{\mathbf{pp}';\mathbf{kk}'} = \delta \mathbf{r}^{(+\infty)} - \delta \mathbf{r}^{(-\infty)}$$

$$\boldsymbol{R}^{(+\infty)} = \boldsymbol{v}_{\mathbf{p}} t + \boldsymbol{v}_{\mathbf{p}'} t + \delta \mathbf{r}^{(+\infty)}$$

Main results for 2p coordinate shifts

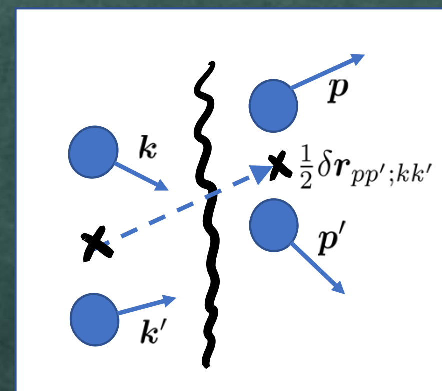
For $\langle \boldsymbol{p}\boldsymbol{p}'|\hat{V}_{e-e}|\boldsymbol{k}\boldsymbol{k}'\rangle \equiv V_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'}$:

$$\delta \boldsymbol{r}_{\mathbf{p}\mathbf{p}';\mathbf{k}\mathbf{k}'} = \langle u_{\boldsymbol{p}}|i\partial_{\boldsymbol{p}} u_{\boldsymbol{p}}\rangle + \langle u_{\boldsymbol{p}'}|i\partial_{\boldsymbol{p}'} u_{\boldsymbol{p}'}\rangle - \langle u_{\boldsymbol{k}}|i\partial_{\boldsymbol{k}} u_{\boldsymbol{p}}\rangle - \langle u_{\boldsymbol{k}'}|i\partial_{\boldsymbol{k}'} u_{\boldsymbol{k}'}\rangle$$
$$- (\partial_{\boldsymbol{p}} + \partial_{\boldsymbol{p}'} + \partial_{\boldsymbol{k}} + \partial_{\boldsymbol{k}'})\arg V_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'}$$

Pesin, PRL 2018

Distinguishable particles: $\boldsymbol{p} \to \boldsymbol{k},\ \boldsymbol{p}' \to \boldsymbol{k}'$

$\delta \boldsymbol{r}_{\boldsymbol{p}\boldsymbol{p}';\boldsymbol{k}\boldsymbol{k}'} = \delta \boldsymbol{r}_{\boldsymbol{p};\boldsymbol{k}} + \delta \boldsymbol{r}_{\boldsymbol{p}';\boldsymbol{k}'}$ sum of individual 1p shifts



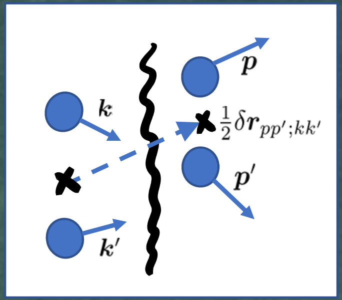
Weyl fermions with point interaction: $h_{\boldsymbol{p}} = v\boldsymbol{\sigma}\cdot\boldsymbol{p}$

$$\delta \mathbf{r}_{\mathbf{k}\mathbf{k}';\mathbf{p}\mathbf{p}'} = -\frac{v}{2}\left(\frac{1}{\varepsilon_k} - \frac{1}{\varepsilon_{k'}}\right)\frac{\mathbf{e}_{\mathbf{k}}\times\mathbf{e}_{\mathbf{k}'}}{1-\mathbf{e}_{\mathbf{k}}\cdot\mathbf{e}_{\mathbf{k}'}} + \frac{v}{2}\left(\frac{1}{\varepsilon_p} - \frac{1}{\varepsilon_{p'}}\right)\frac{\mathbf{e}_{\mathbf{p}}\times\mathbf{e}_{\mathbf{p}'}}{1-\mathbf{e}_{\mathbf{p}}\cdot\mathbf{e}_{\mathbf{p}'}}$$

Individual shifts can be defined for point interaction, Lorentz-inv case, and zero angular momentum: Chen, Son, Stephanov, PRL 2015.

Corrections to kinetics with 2p shifts

Physical considerations (1ps case review: Sinitsyn, J. Phys. CM 2008)



- Current due to shift accumulations:

$$\boldsymbol{j}^{sj} = \frac{1}{4}e\int_{\mathbf{pp'kk'}} W_{\mathbf{pp';kk'}}(1-f_{\mathbf{p}})(1-f_{\mathbf{p'}})f_{\mathbf{k}}f_{\mathbf{k'}}\delta\mathbf{r}_{\mathbf{pp';kk'}}$$

- Energy change during a collision:

$$\delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k'}}-\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p'}}) \to \delta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{k'}}-\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p'}}-e\mathbf{E}\delta\mathbf{r}_{\mathbf{kk';pp'}})$$

- The collision integral no longer nullified by the equilibrium d.f.:

$$I_{e-e}[\phi_{\mathbf{k}}] \to I_{e-e}[\phi_{\mathbf{k}}] - e\boldsymbol{E}\boldsymbol{g}_{\mathbf{k}},$$

$$\boldsymbol{g}_{\mathbf{k}} = \frac{1}{2T}\int_{\mathbf{pp'k'}} W_{\mathbf{pp';kk'}}(1-f_{\mathbf{p}})(1-f_{\mathbf{p'}})f_{\mathbf{k}}f_{\mathbf{k'}}\delta\mathbf{r}_{\mathbf{pp';kk'}}$$

“Hydrodynamic” anomalous Hall effect

- Kinetic equation:

$$e\mathbf{E}\left(\boldsymbol{v}_{\mathbf{k}}\frac{\partial f_0(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}}+\boldsymbol{g}_{\mathbf{k}}\right)=I_{st}(\phi_{\mathbf{k}}),\qquad \phi^{\boldsymbol{v}}=I_{st}^{-1}\left(e\mathbf{E}\boldsymbol{v}_{\ell}\frac{\partial f_0(\epsilon_{\ell})}{\partial \epsilon_{\ell}}\right),\quad \phi^{\boldsymbol{g}}=I_{st}^{-1}\left(e\mathbf{E}\boldsymbol{g}_{\ell}\right)$$

- Hall current: $\boldsymbol{j}^{\mathrm{AHE}}=\boldsymbol{j}^{\mathrm{shift}}+\boldsymbol{j}^{\mathrm{ballistic}},$

$$\boldsymbol{j}^{\mathrm{ballistic}}=-e\int_{\mathbf{k}}\boldsymbol{v}_{\mathbf{k}}\frac{\partial f_0(\epsilon_{\mathbf{k}})}{\partial \epsilon_{\mathbf{k}}}\phi_{\mathbf{k}}^{\boldsymbol{g}},\qquad \boldsymbol{j}^{\mathrm{shift}}=e\int_{\mathbf{k}}\boldsymbol{g}_{\mathbf{k}}\phi_{\mathbf{k}}^{\boldsymbol{v}}.$$

These expressions lead to an antisymmetric conductivity tensor

Picture thus far:

2p coordinate shifts carry particle number with them: hydro AHE

$$\delta \boldsymbol{r} = \langle W_{2p}^{out}| \int \Psi^\dagger \boldsymbol{r} \Psi | W_{2p}^{out}\rangle - \langle W_{2p}^{in}| \int \Psi^\dagger \boldsymbol{r} \Psi | W_{2p}^{in}\rangle$$

(drop "vt" terms)

$$|W_{2p}^{out}\rangle = \hat{T} |W_{2p}^{in}\rangle$$

Actually, 2p shifts carry *all* additive integrals of motion with them, for instance

$$\delta \pi_{ab} = \langle W_{2p}^{out}| \int \Psi^\dagger p_a r_b \Psi | W_{2p}^{out}\rangle - \langle W_{2p}^{in}| \int \Psi^\dagger p_a r_b \Psi | W_{2p}^{in}\rangle$$

(drop "vt" terms)

$$\delta \pi_{ab}^{(a)} = -\tfrac{1}{2} \epsilon_{abc} \delta \ell_c$$

The problem of rotating distribution

Additive conservation laws affect the local-equilibrium distribution:

$\sum_{in} 1, \varepsilon = \sum_{out} 1, \varepsilon :$ $\quad f_{\text{l.e.}} = f_{\text{eq}}(\beta(\boldsymbol{r})[\varepsilon_p - \mu(\boldsymbol{r})])$

$\sum_{in} 1, \varepsilon, \boldsymbol{p} = \sum_{out} 1, \varepsilon, \boldsymbol{p} :$ $\quad f_{\text{l.e.}} = f_{\text{eq}}(\beta(\boldsymbol{r})[\varepsilon_p - \mu(\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{p}])$

$\sum_{in} 1, \varepsilon, \boldsymbol{p}, \boldsymbol{\ell} + \boldsymbol{s} = \sum_{out} 1, \varepsilon, \boldsymbol{p}, \boldsymbol{\ell} + \boldsymbol{s} :$ $\quad f_{\text{l.e.}} = ???$

$$f_{\text{l.e.}} = f_{\text{eq}}(\beta(\boldsymbol{r})[\varepsilon_p - \mu(\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{p} - \tfrac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{s}_p])$$

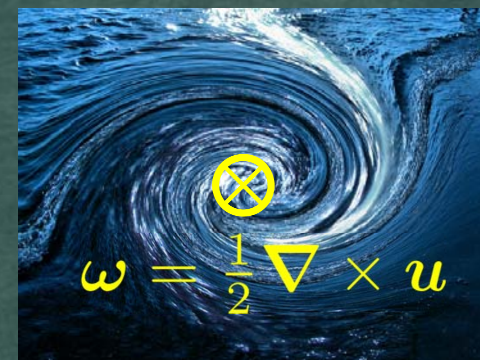
$$\delta\pi_{ab}^{(a)} = -\tfrac{1}{2}\epsilon_{abc}\delta\ell_c = \tfrac{1}{2}\epsilon_{abc}\delta s_c$$

Chiral vortical effect of Weyl fermions

(broken I)

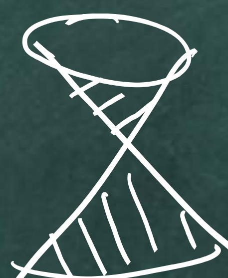
$$\boldsymbol{j} = en\boldsymbol{u} + \frac{1}{2}\lambda_{\text{cve}}\boldsymbol{\nabla} \times \boldsymbol{u} \equiv en\boldsymbol{u} + \lambda_{\text{cve}}\boldsymbol{\omega}$$

$$\lambda_{\text{cve}} = ?$$



Model of choice: single species of Weyl fermions

$$H = v\boldsymbol{\sigma} \cdot \boldsymbol{p},$$



-- “Conduction band” only

The CVE current does not appear in more conventional models, e.g. chiral suspension in a non-chiral fluid (Andreev, Son, Spivak 2009)

CVE: two currents

(Stephanov, Yin, PRL 2012)

$$\boldsymbol{j}_{\rm cve} = \boldsymbol{j}_{\rm bal} + \boldsymbol{j}_{\rm mag},$$ (ballistic+magnetization)

$$f_{\rm l.e.} = f_{\rm eq}(\beta(\boldsymbol{r})[\varepsilon_p - \boldsymbol{u}(\boldsymbol{r})\cdot\boldsymbol{p} - \tfrac{1}{2}\boldsymbol{\nabla}\times\boldsymbol{u}(\boldsymbol{r})\cdot\boldsymbol{s}_p])$$

$$\boldsymbol{j}_{\rm bal} = e\int_{\boldsymbol{p}} \boldsymbol{v}_{\boldsymbol{p}} f_{l.e.}(\boldsymbol{r}) = \tfrac{1}{3}e\frac{p_F^2}{4\pi^2}\boldsymbol{\omega}$$

$$\boldsymbol{j}_{\rm mag} = \boldsymbol{\nabla}\times\int_{\boldsymbol{p}} \boldsymbol{m}_{\boldsymbol{p}} f_{l.e.}(\boldsymbol{r}) = \tfrac{2}{3}e\frac{p_F^2}{4\pi^2}\boldsymbol{\omega}$$

$$\boldsymbol{j}_{\rm cve} = e\frac{p_F^2}{4\pi^2}\boldsymbol{\omega}$$

Microscopic treatment based on $\delta\pi_{ab}$ opens up a way toward “CVE in crystals”

“Anomalous” Hall viscosity in metals

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla P + \eta_{xx} \nabla^2 \boldsymbol{u} + \eta_{xy} \nabla^2 \boldsymbol{u} \times \boldsymbol{z}$$

Basic derivation ingredients:

$$\partial_t \int_p \boldsymbol{p} f_{\boldsymbol{p}} + \int_p \boldsymbol{p} [\boldsymbol{v} \boldsymbol{\nabla} f_{\text{l.e.}}] = 0$$

$$f_{\text{l.e.}} = f_{\text{eq}}(\beta(\boldsymbol{r})[\varepsilon_p - \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{p} - \tfrac{1}{2} \boldsymbol{\nabla} \times \boldsymbol{u}(\boldsymbol{r}) \cdot \boldsymbol{s}_p])$$

$$\nabla \cdot \boldsymbol{u} = 0$$

Result for the Hall viscosity:

$\eta_{xy}^{metal} = -\frac{1}{2} s_z(p_F) n$ compare to $\eta_{xy}^{(F)QHE} = -\frac{1}{2} s_z n$

(unpublished) (Avron et al. 1995, N. Read, 2009)

Conclusions:

- Band geometry manifests itself in two-particle collisions

- Collisional coordinate shifts are responsible for hydrodynamic
  - anomalous Hall effect,
  - chiral vortical effect,
  - anomalous Hall viscosity,
  - thermal Hall effect

- “Chiral hydrodynamics in solids” is a rich subject