Moiré is Different:

Wigner crystallization in TBLG

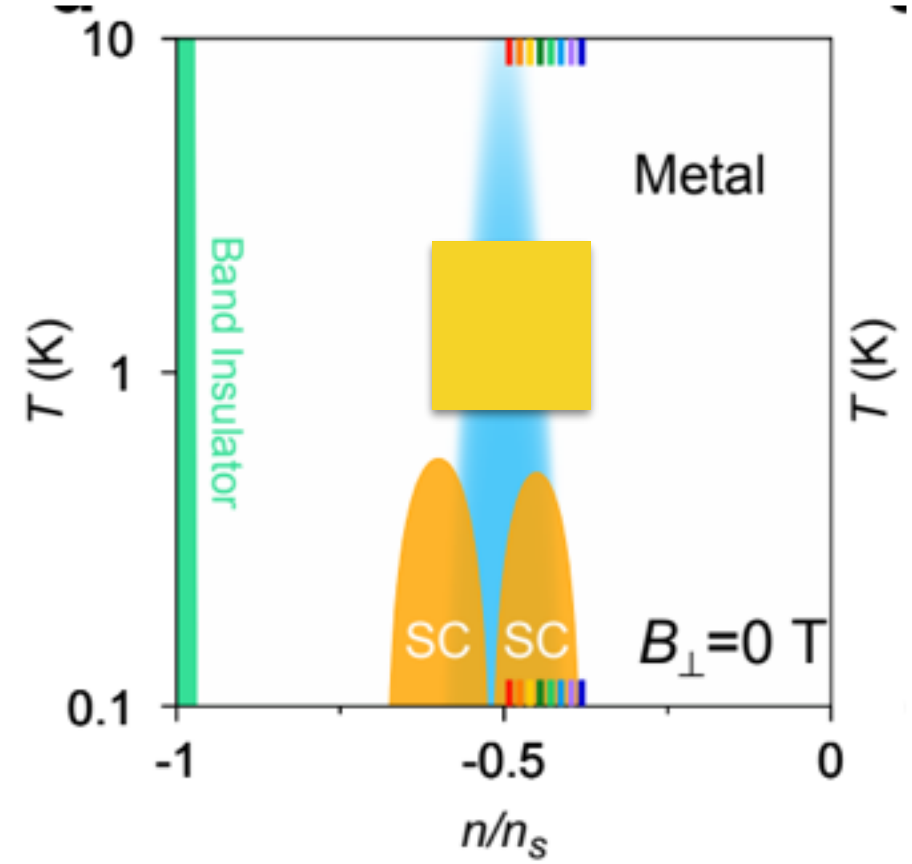
Bikash Padhi

Chandan Setty

Nano. Lett. 2018+
arXiv:1810.00884

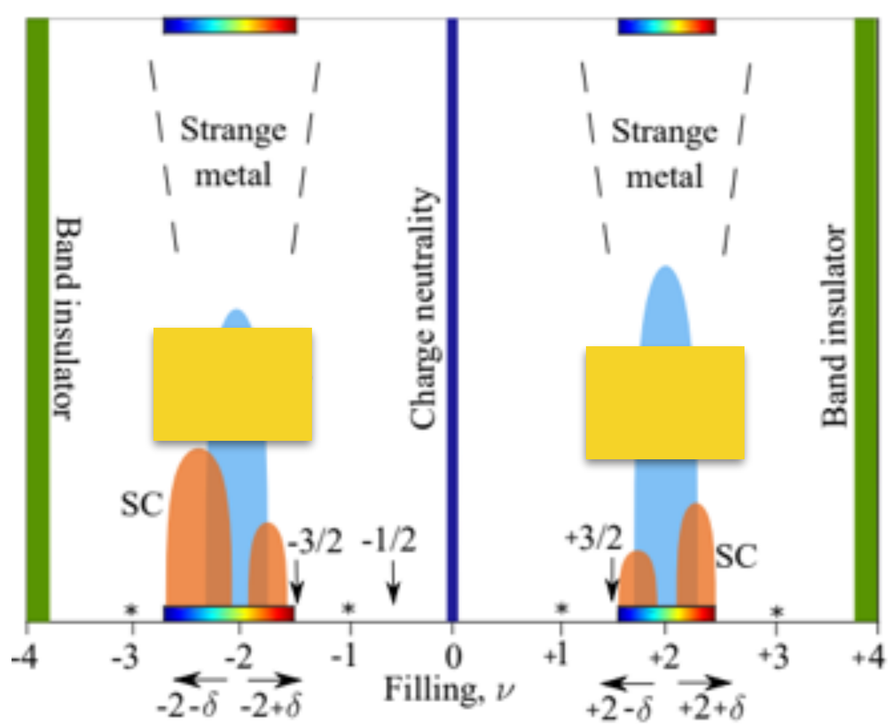
EFRC,NSF

2018



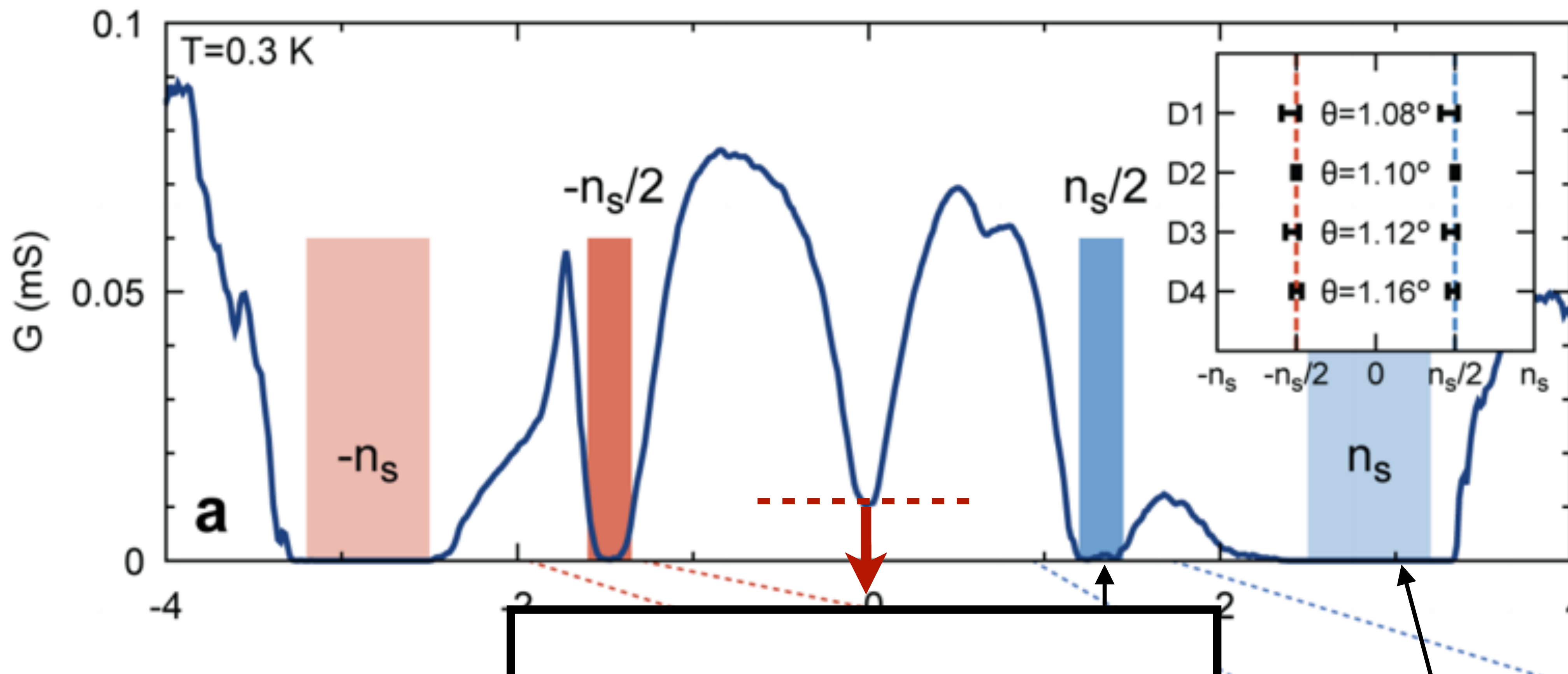
8months

2019



filling

$$\nu = n_e A_s$$



$$\sigma_{\min} = \frac{2e^2}{\pi^2 \hbar}\left(1 + \frac{1}{W}\right)$$

Koshino & Ando, PRB (2006)

Mott Insulator

$\nu = 4$

band insulator

Mottness in TBLG?

$$\nu = n_e A_s \qquad \nu_{\max} = 4$$

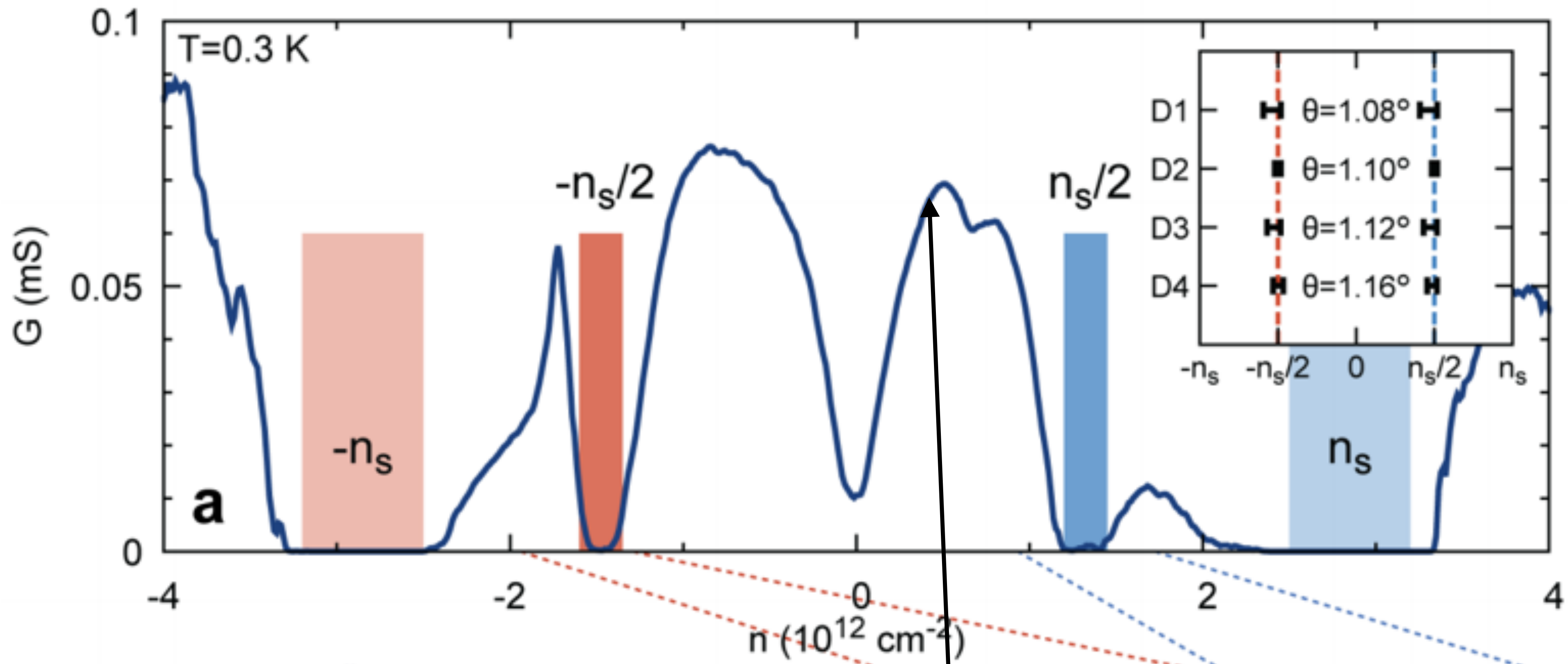
$$U = E^{\nu+1} + E^{\nu-1} - 2E^{\nu}$$

If Mott

$$\forall \, \nu < 4$$

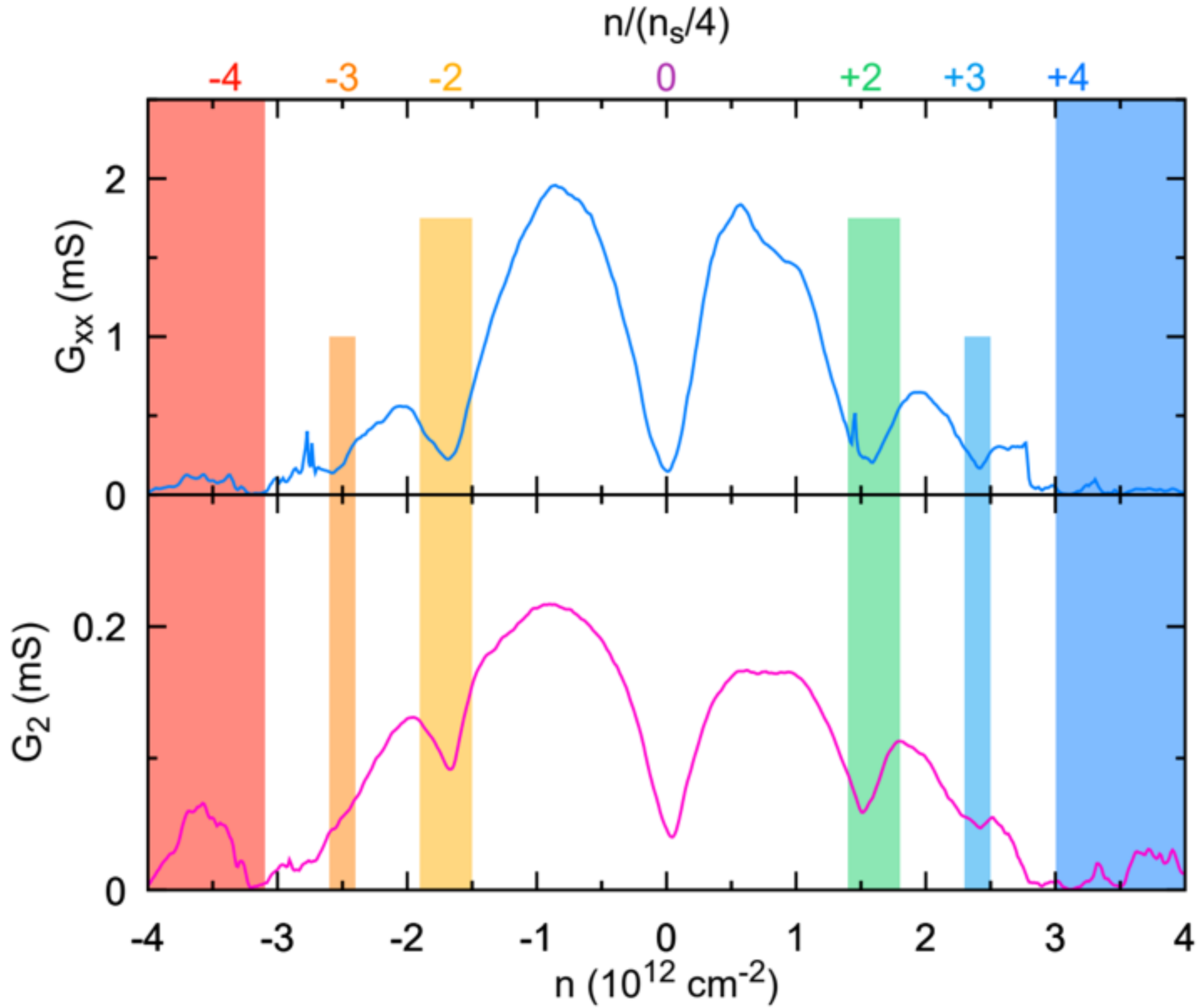
insulator

Where's the insulator at $\nu = 1$?



$\nu = 1$

$\nu = 1 \neq$ insulator



Where's the insulator at $\nu = 1$?

Tuning superconductivity in twisted bilayer graphene

Matthew Yankowitz[1*], Shaowen Chen[1,2*], Hryhoriy Polshyn[3*], K. Watanabe[4], T. Taniguchi[4], David Graf[5], Andrea F. Young[3†], and Cory R. Dean[1†]
[1]Department of Physics, Columbia University, New York, NY, USA
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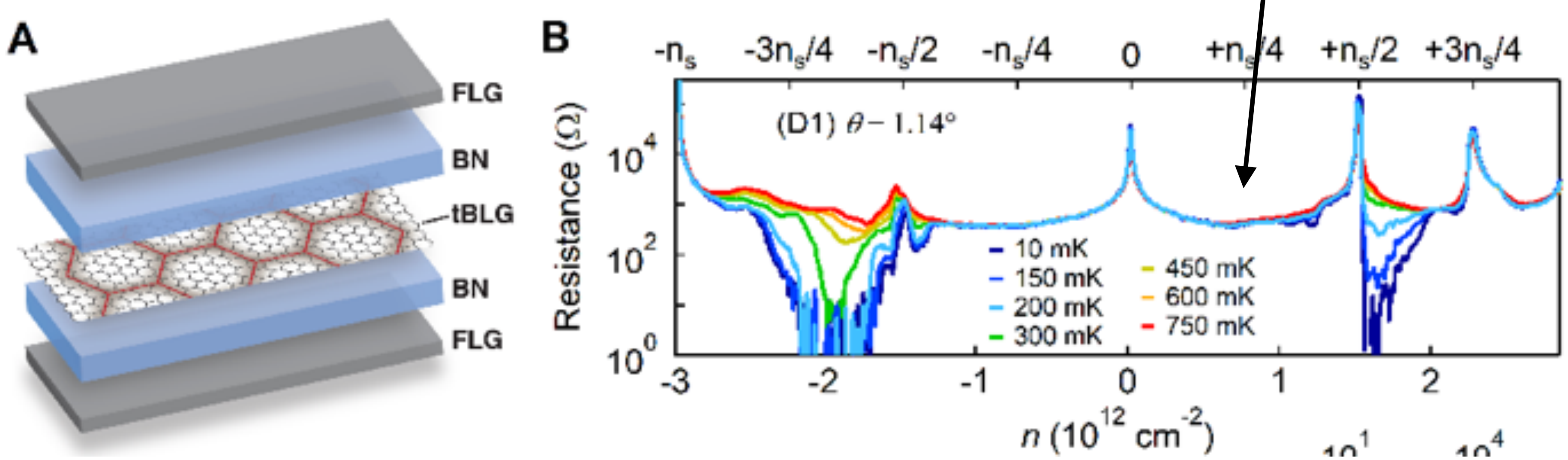
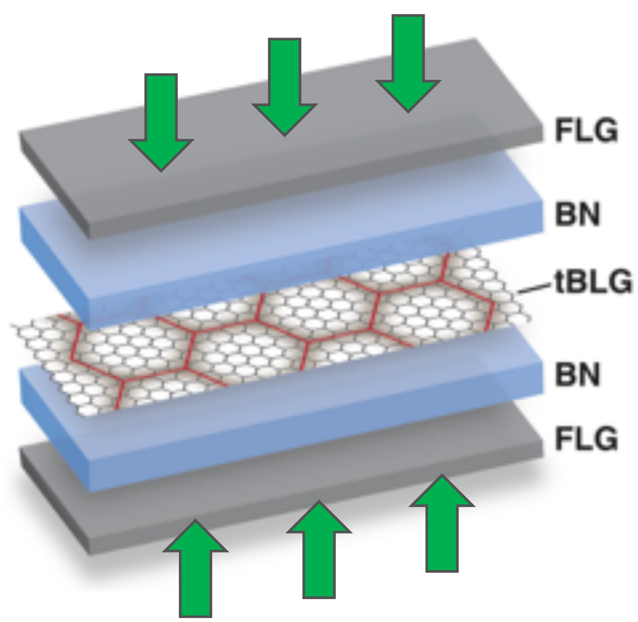
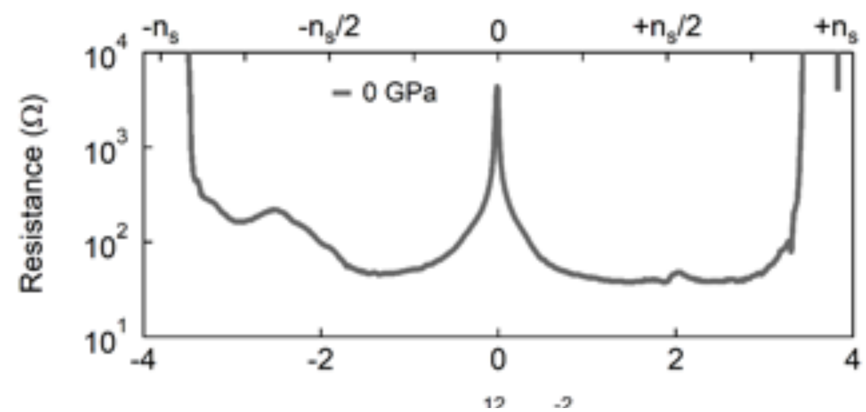
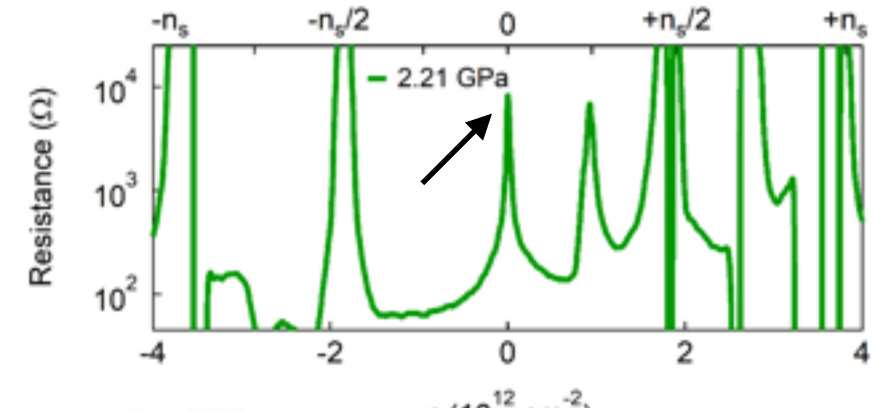


metallic

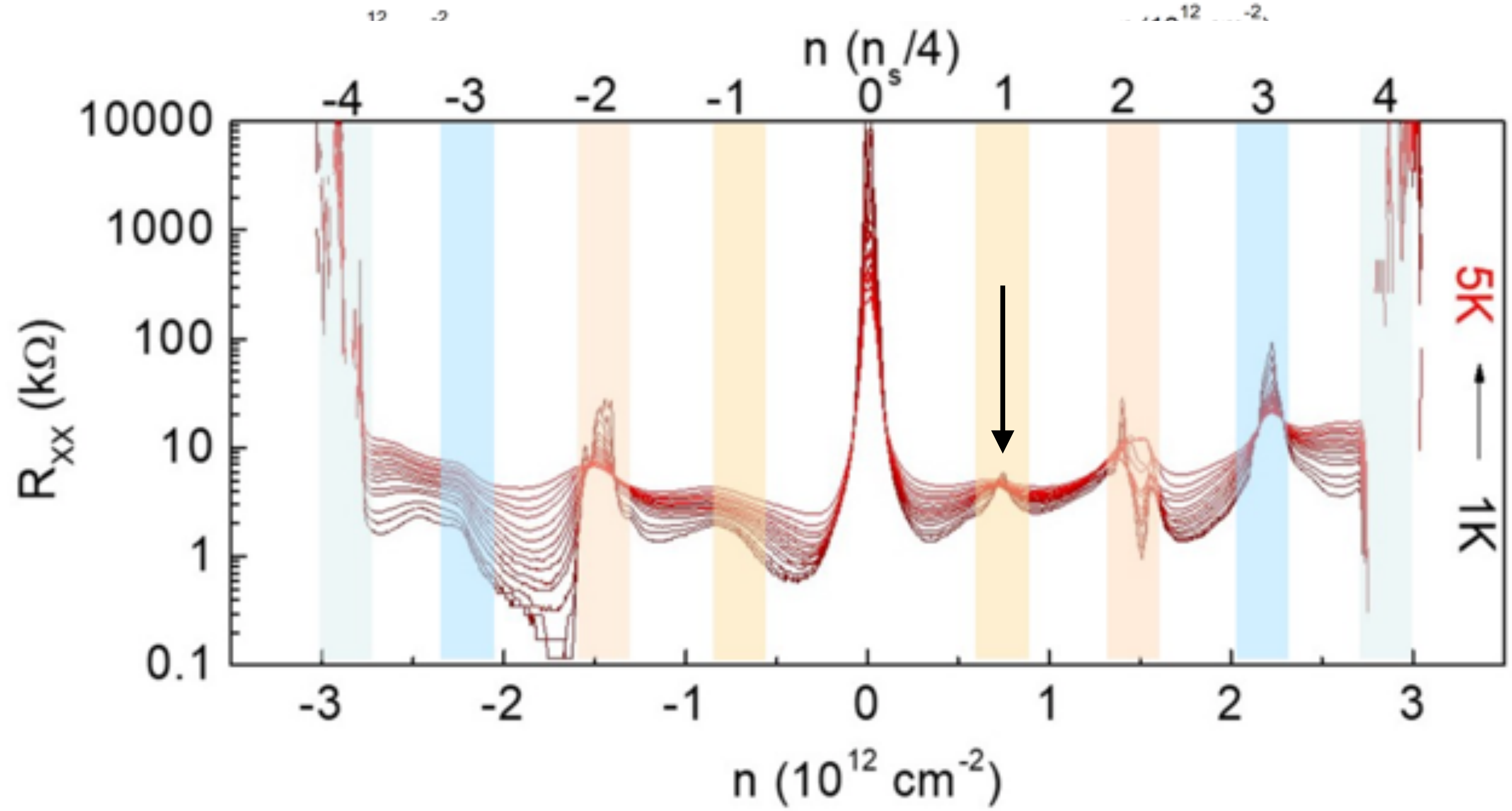
Modifying metallic-state resitivity with pressure





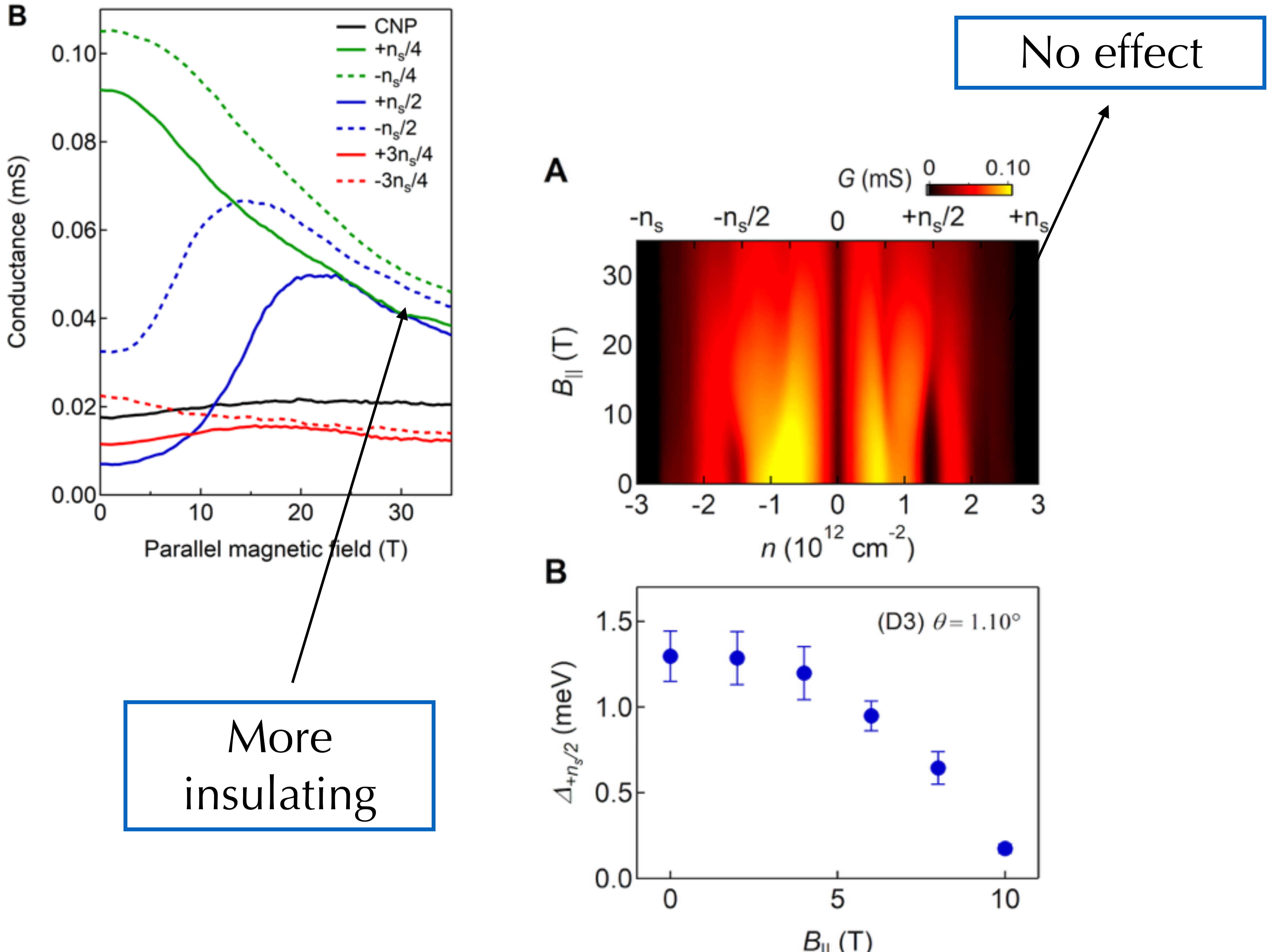


D. Efetov



M. Yankowitz *et al.*, *Science* 10.1126/science.aav1910 (2018)

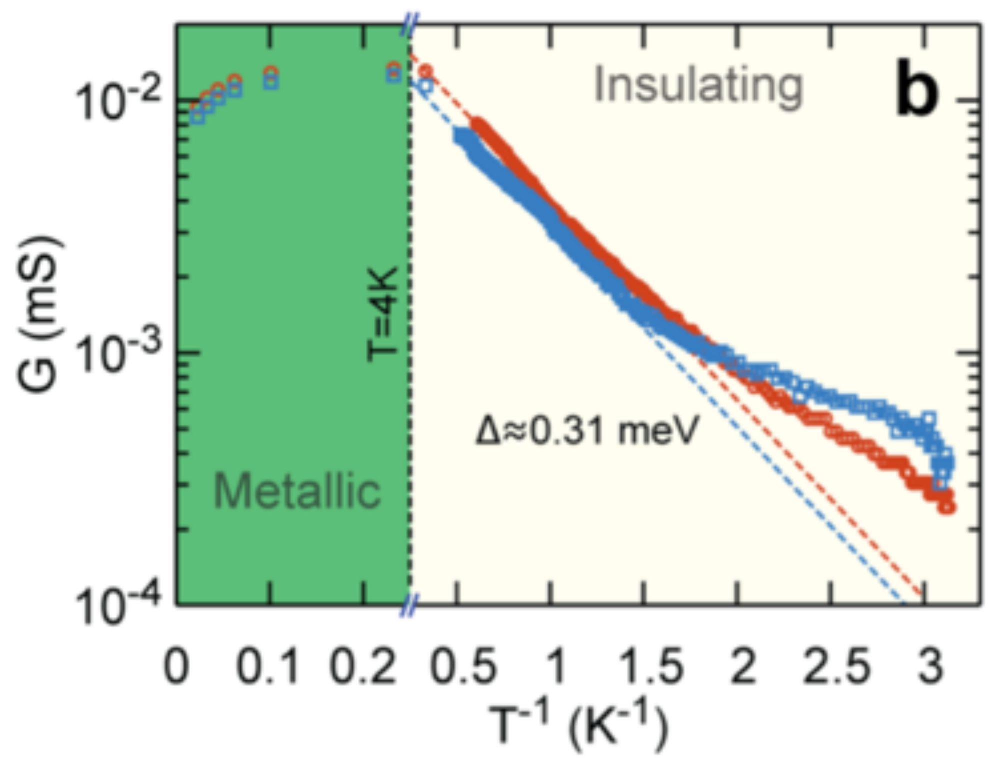
H. Polshyn*, M. Yankowitz* *et al.*, *in prep* (2019)

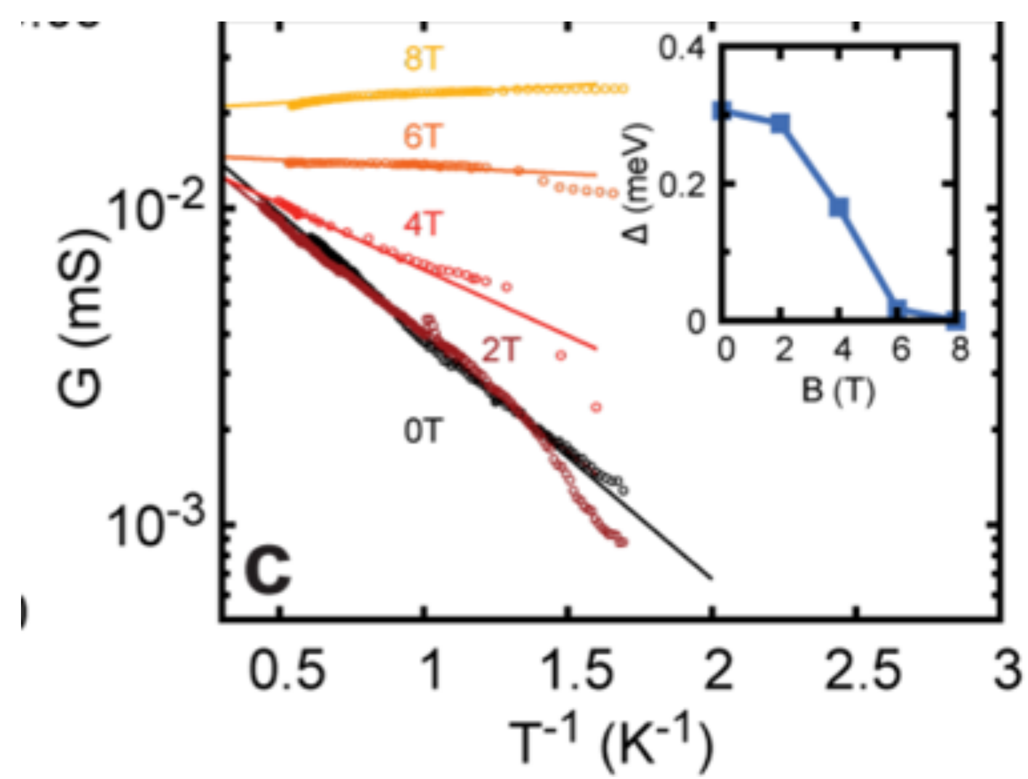


B
Conductance (mS)
0.00
0.02
0.04
0.06
0.08
0.10
0
10
20
30
Parallel magnetic field (T)
CNP
+n_s/4
-n_s/4
+n_s/2
-n_s/2
+3n_s/4
-3n_s/4
More insulating
No effect
A
G (mS)
0
0.10
-n_s
-n_s/2
0
+n_s/2
+n_s
B_|| (T)
n (10^12 cm^-2)
B
(D3) θ= 1.10°
Δ_{+n_s/2} (meV)
B_|| (T)

experiment

$$\Delta = .31meV \approx 4K \approx \mu_B B_c$$

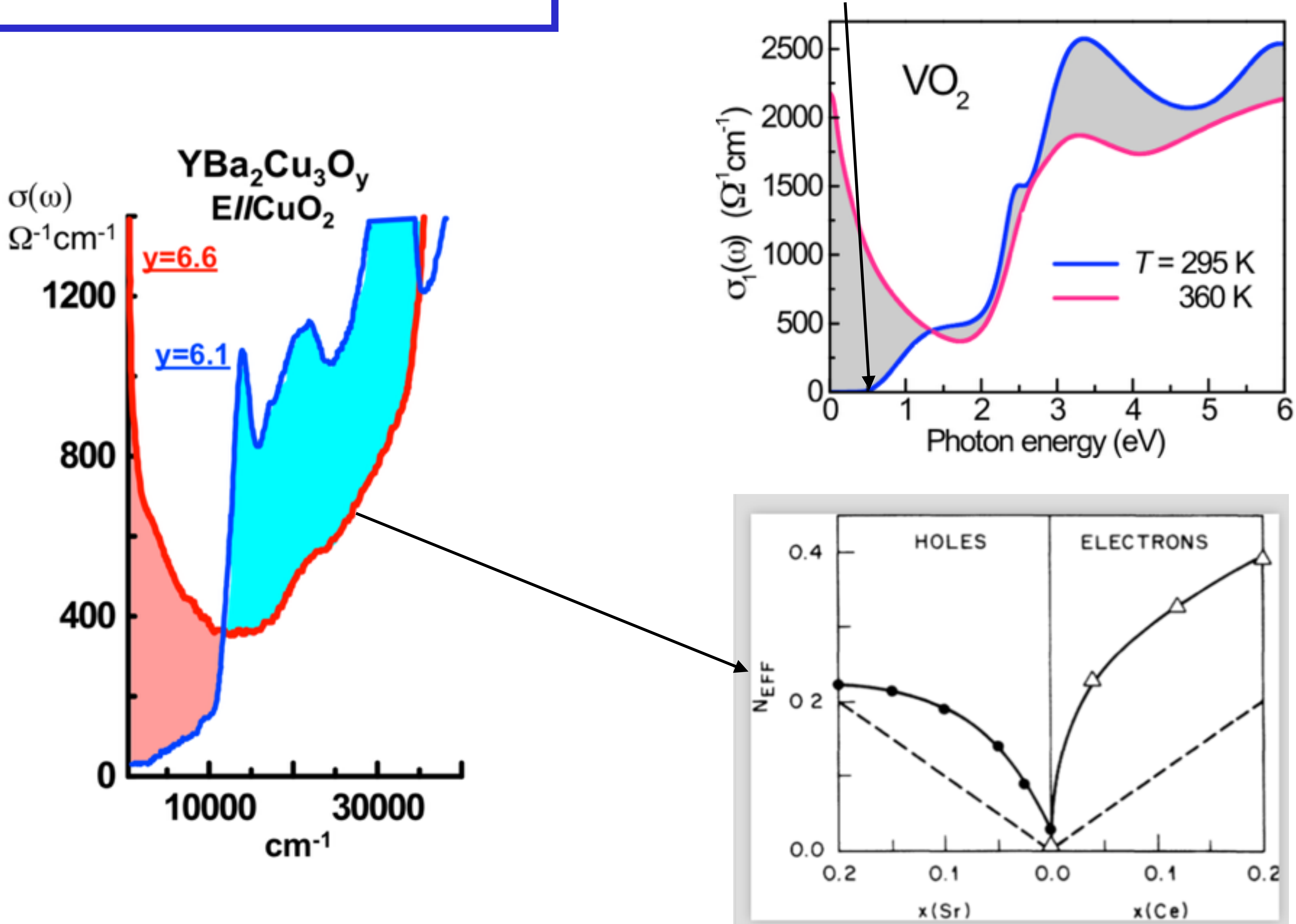




one energy scale

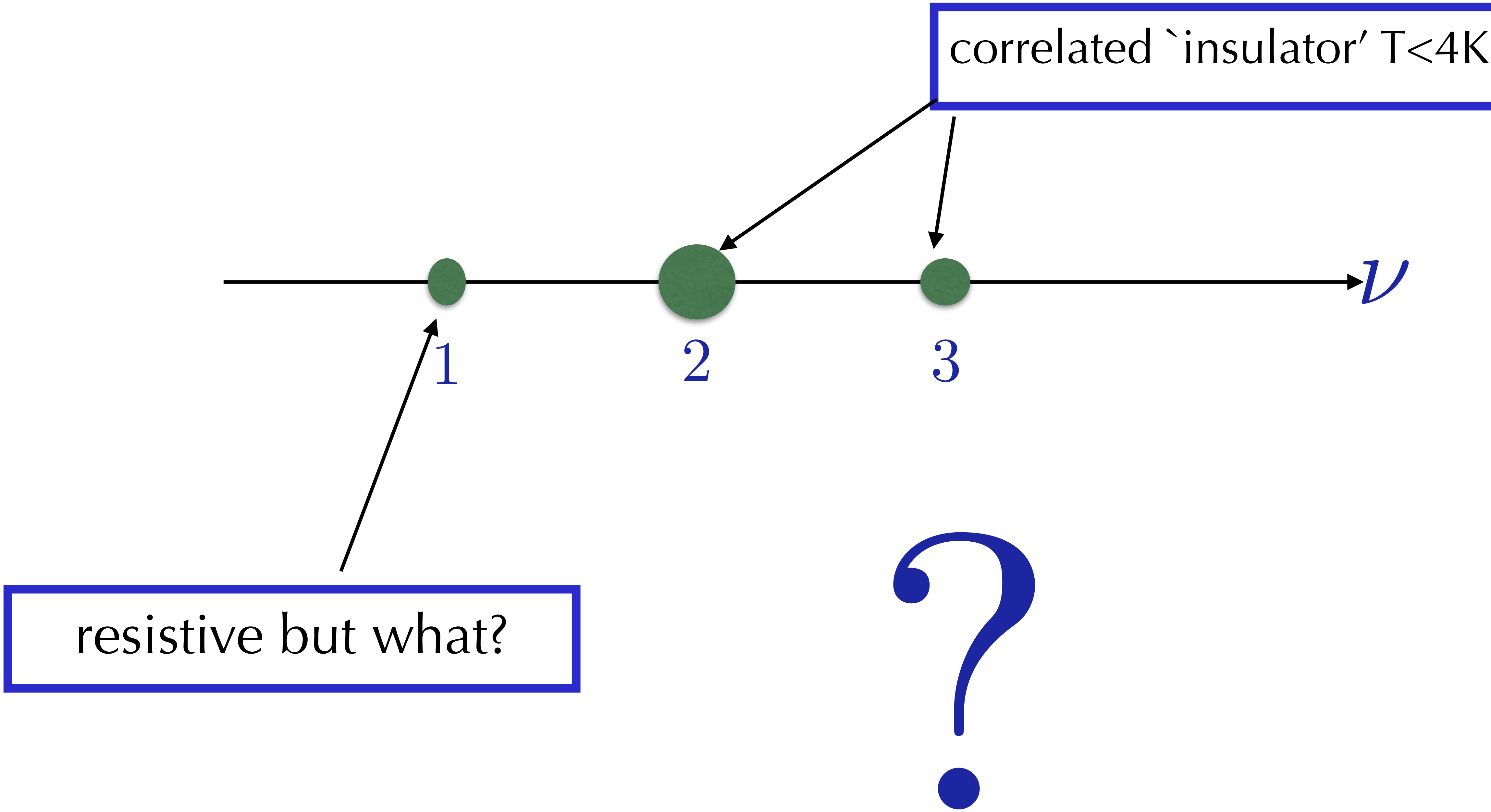
Mottness: 2-energy scales

$\Delta = .5eV \gg 295K$



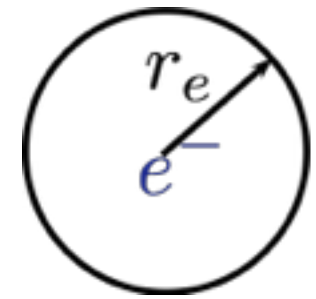
Not Mott
Insulation

Experimental puzzle: hierarchy of insulators



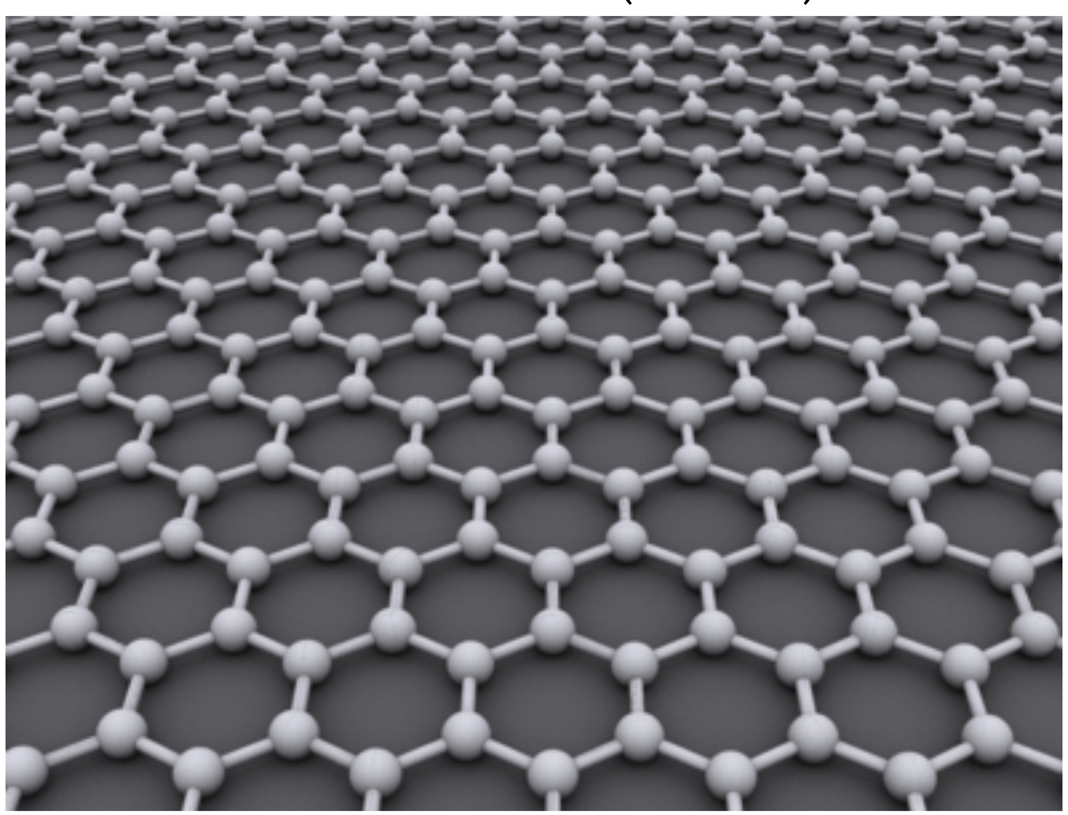
when are interactions important?

$$r_s = \frac{V_{ee}}{E_{kin}} = \frac{e^2}{\epsilon c_n} r_e^{n-1}$$



$c_n k^n$

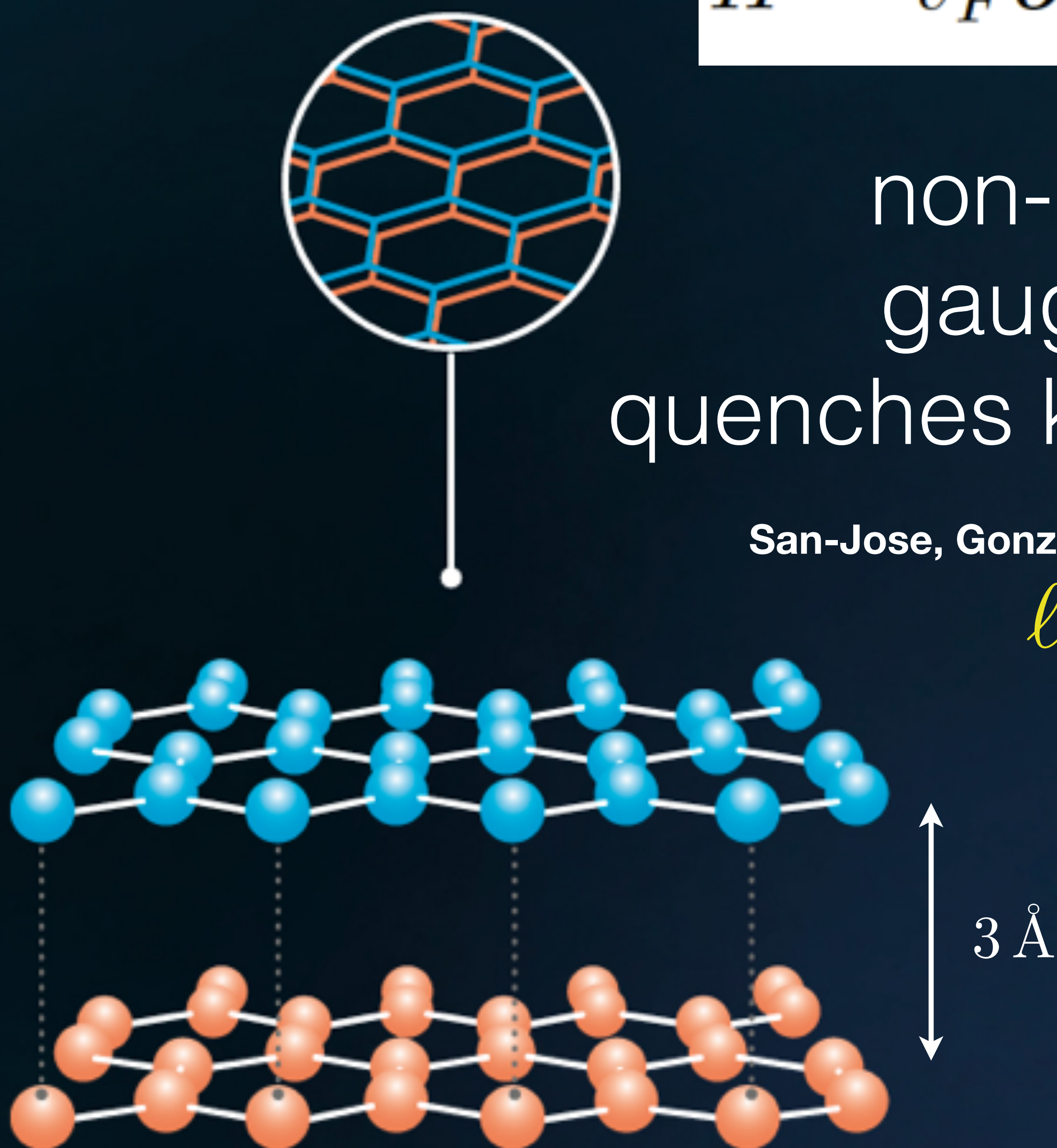
Dirac spectrum
(n=1)



density independent

interactions are irrelevant

$$H = v_F \boldsymbol{\sigma} \cdot (-i\boldsymbol{\partial} - \hat{\mathbf{A}})$$



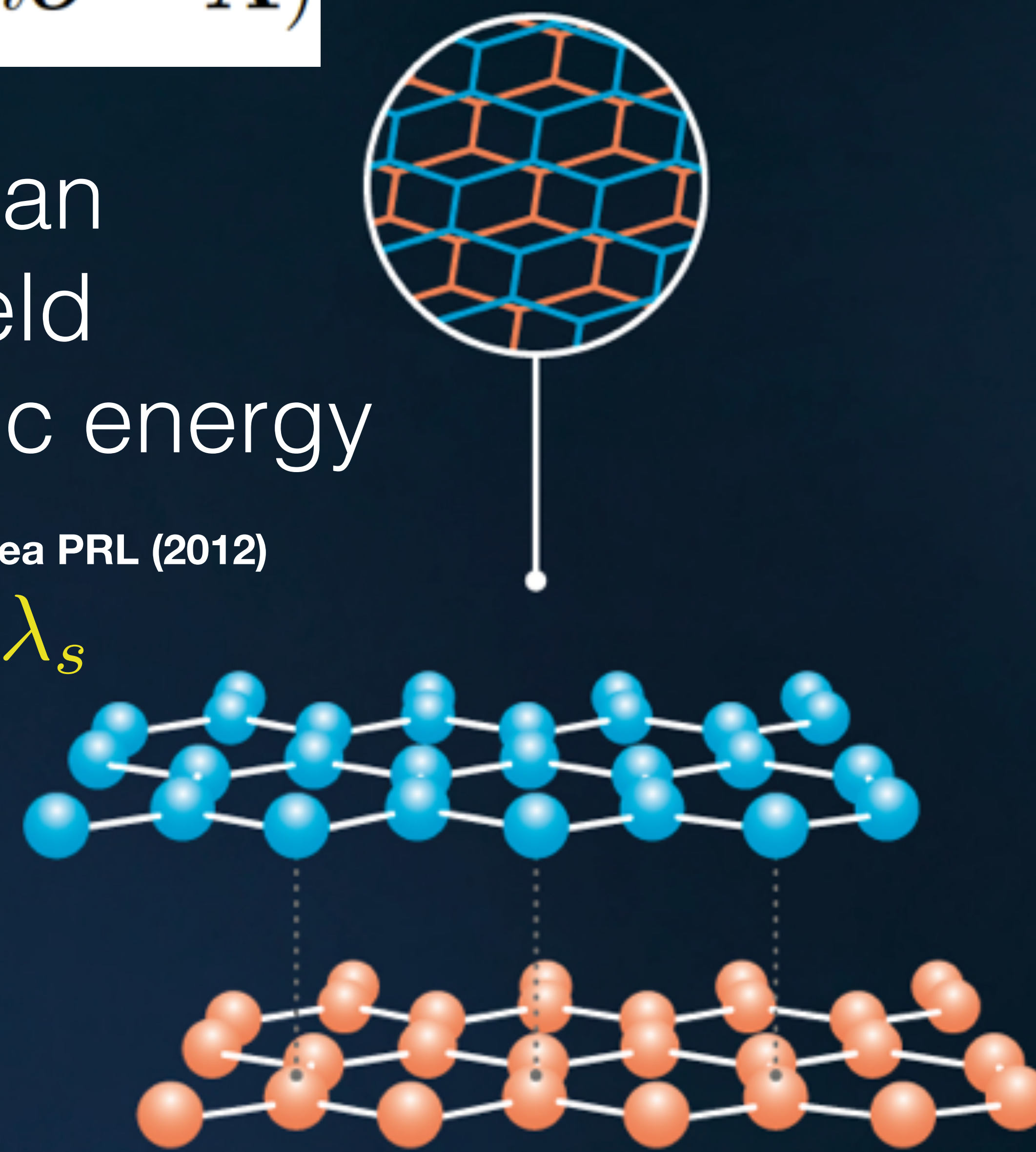
AA-stacking

non-abelian
gauge field
quenches kinetic energy

San-Jose, Gonzalez, Guinea PRL (2012)

$\ell_B \approx \lambda_s$

Wigner
crystallization
(Kindermann,…)

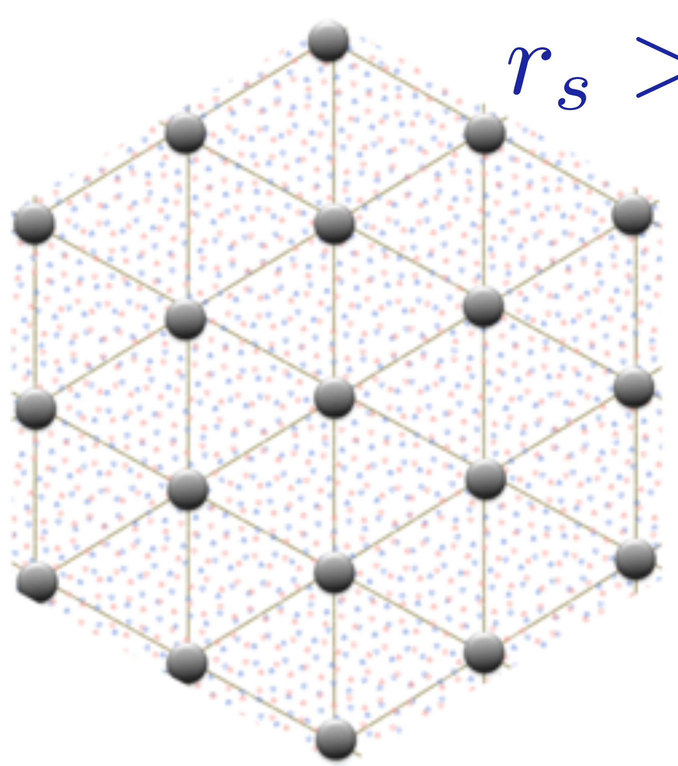


AB-stacking

Wigner crystallization

no underlying lattice

symmetry-broken state



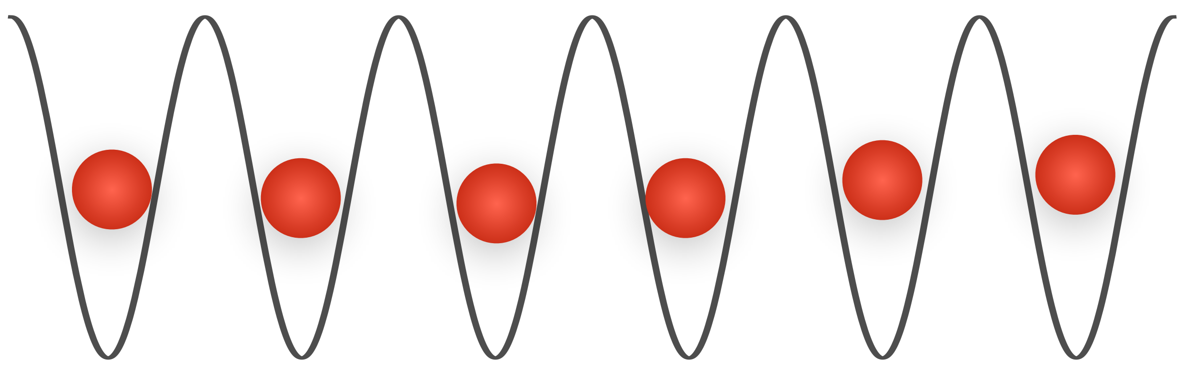
$r_s > 37$

Mott Insulation

lattice needed

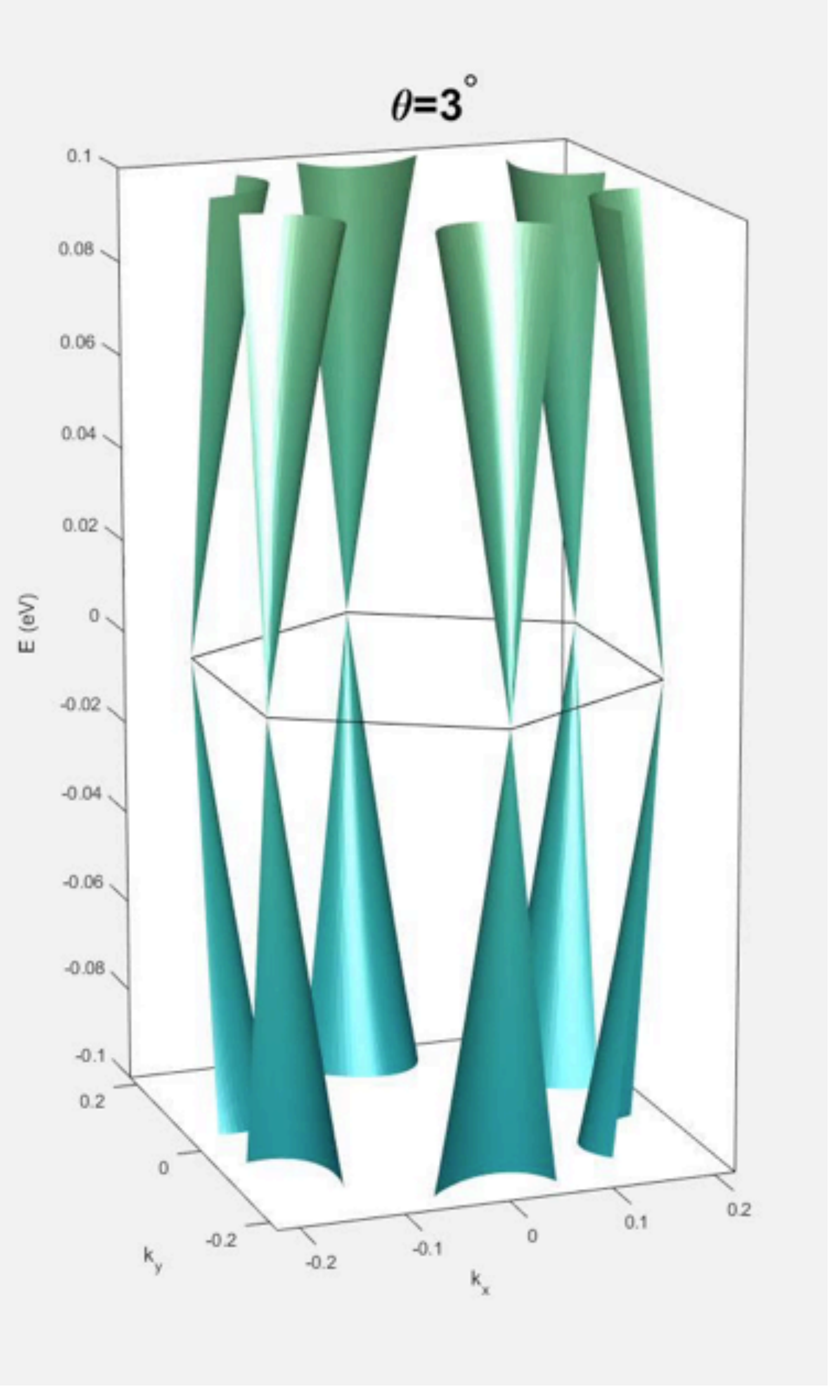
symmetry intact

$U \gg W$



$$n_e^{1/2} a_0^* = \frac{a_0^*}{r_e} = r_s^{-1} \approx O(1)$$

Evolution of Band Structure



recall

Dirac spectrum

$$r_s = \text{const}$$

TBLG: deviation from Dirac

$$H_\theta(\mathbf{k}) = \begin{pmatrix} 0 & v_F \mathbf{k}^\dagger + \frac{\mathbf{1}}{\mathbf{2m}} \mathbf{k}^\mathbf{2} \\ v_F \mathbf{k} + \frac{\mathbf{1}}{\mathbf{2m}} (\mathbf{k}^\dagger)^\mathbf{2} & 0 \end{pmatrix}$$

flat-band condition

$$\tilde{v} = \frac{v_0}{v_F} = \frac{\partial_\mathbf{k} E_\mathbf{k}|_{\mathbf{k}=\mathbf{\Gamma}}}{\partial_\mathbf{k} E_\mathbf{k}|_{\mathbf{k}=\mathbf{K},\mathbf{K}'}} = \frac{1+6\kappa^2}{1-3\kappa^2}$$

$$\kappa = 1/\sqrt{3}$$

compute r_s

$r_s^{(1)}(\theta)$

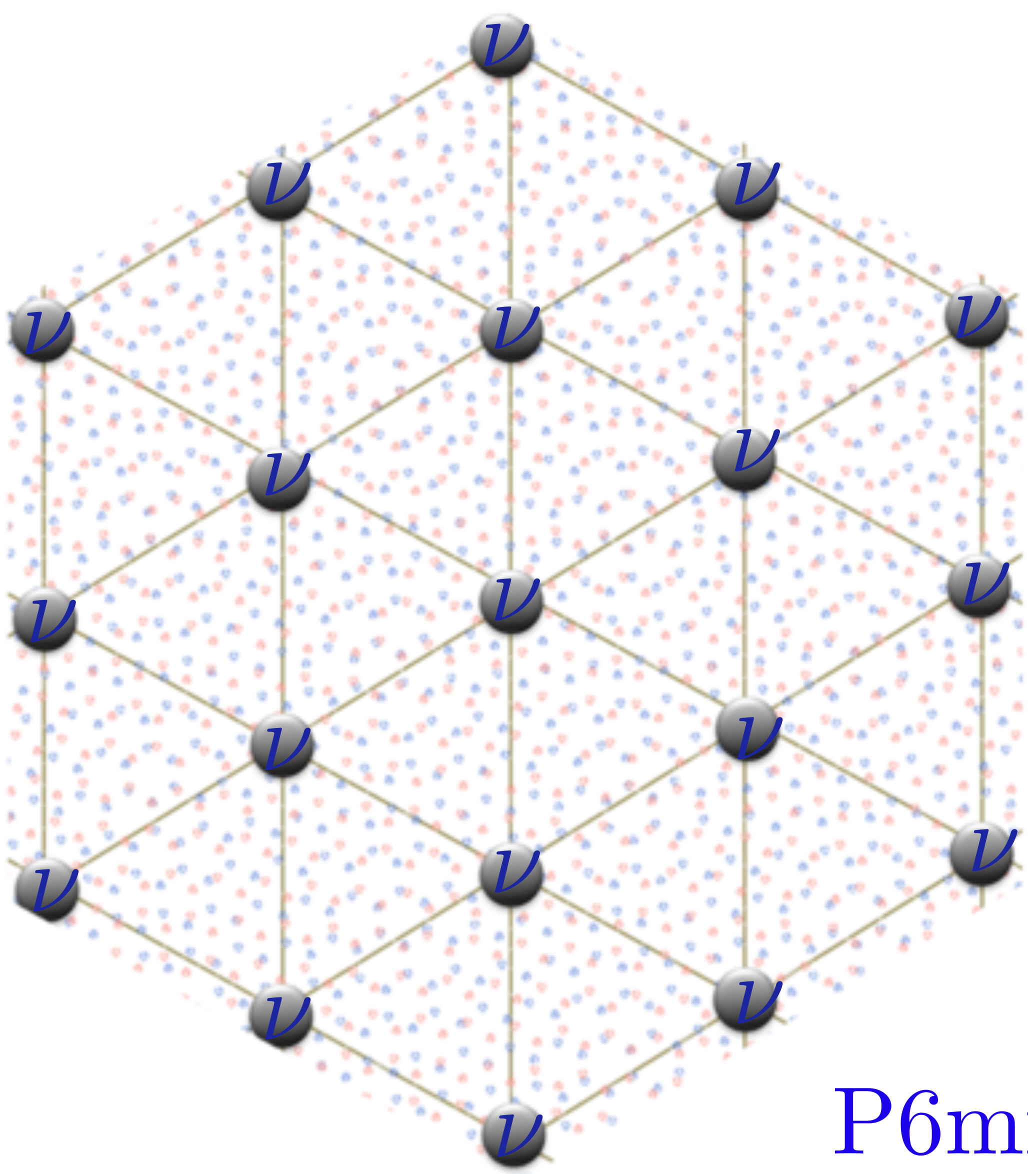
$$r_s(\theta,\nu) = \frac{\alpha}{\epsilon}\tilde{v}(\theta) \times \left(1-\gamma+\frac{1}{2}\gamma^2\right)^{-1/2}$$

$$\alpha = \frac{e^2}{\hbar v_0} \sim 2$$

$$\gamma = \frac{\sqrt{2}\hbar}{m_* v_F}\frac{1}{2r_e} = \frac{\theta}{a}\frac{\hbar}{m_* v_F}\sqrt{\frac{\pi\nu}{\sqrt{3}}} \sim \frac{\theta^\circ}{100}\frac{\tilde{v}(\theta)}{\tilde{m}(\theta,\nu)}\sqrt{\nu}$$

filling dependence

Wigner Crystals for $\nu \neq 1$



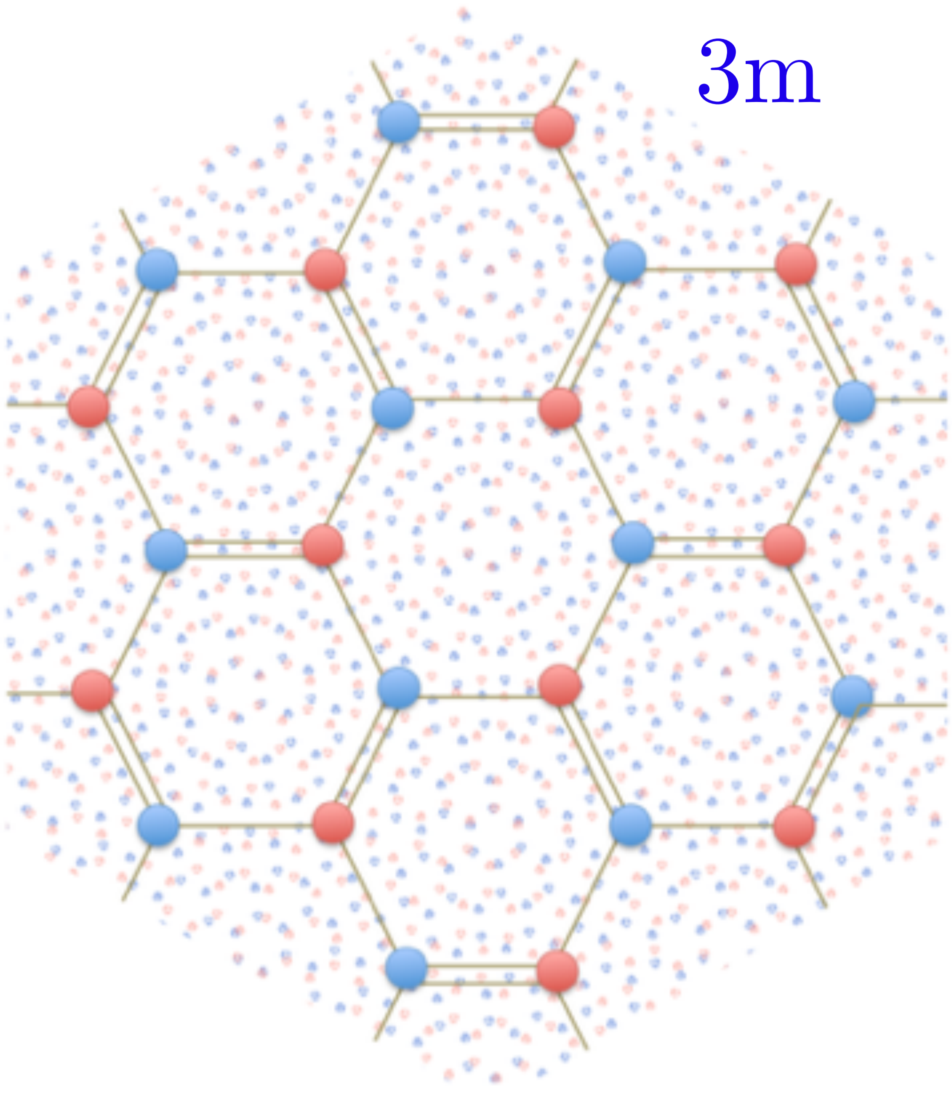
$$r_s^{\text{crit}} = \frac{37}{\nu^2}$$

lower bound

P6mm

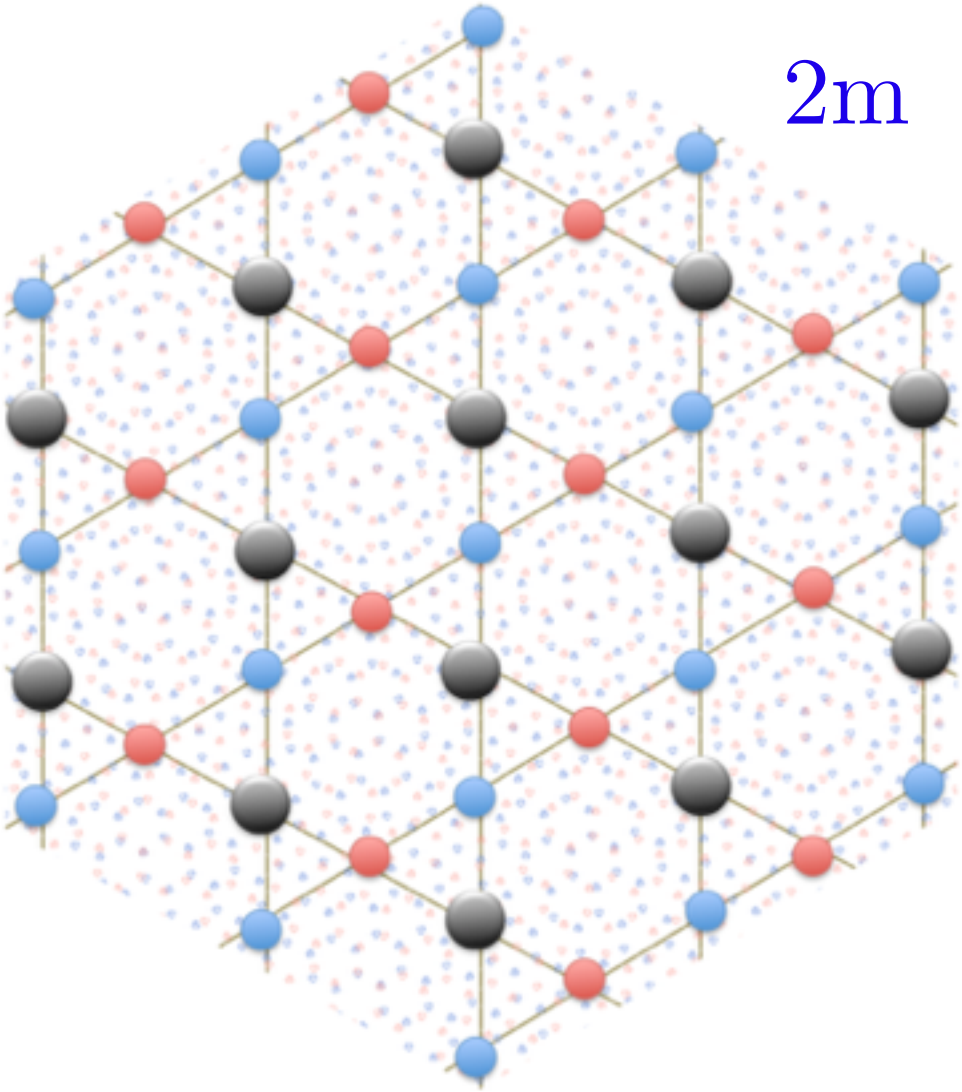
realistic lattices for $\nu > 1$

$\nu = 2$



$\lambda_s \to \lambda_s/\sqrt{3}$

$\nu = 3$



$\lambda_s \to \lambda_s/2$

$$d_2 = 1/\sqrt{3}$$

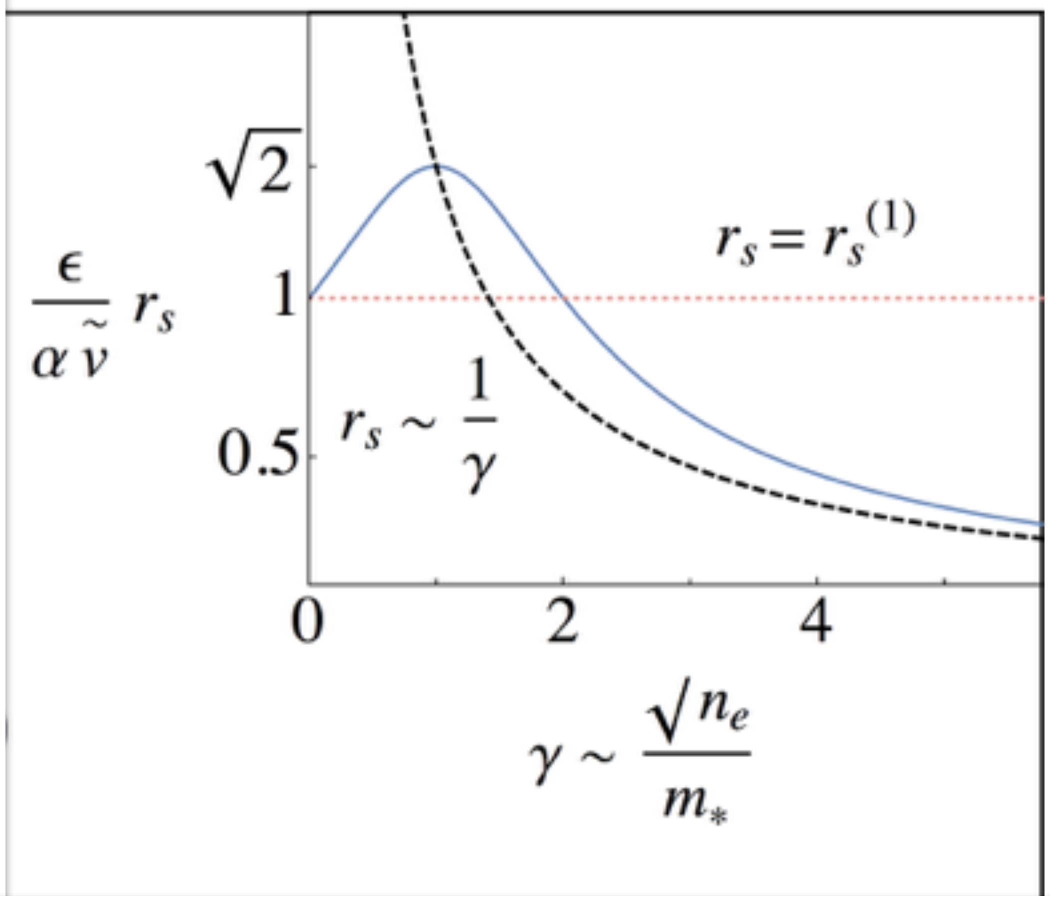
$$r_s = \frac{a}{a_0^*}$$

$$r_s = d_\nu 37$$

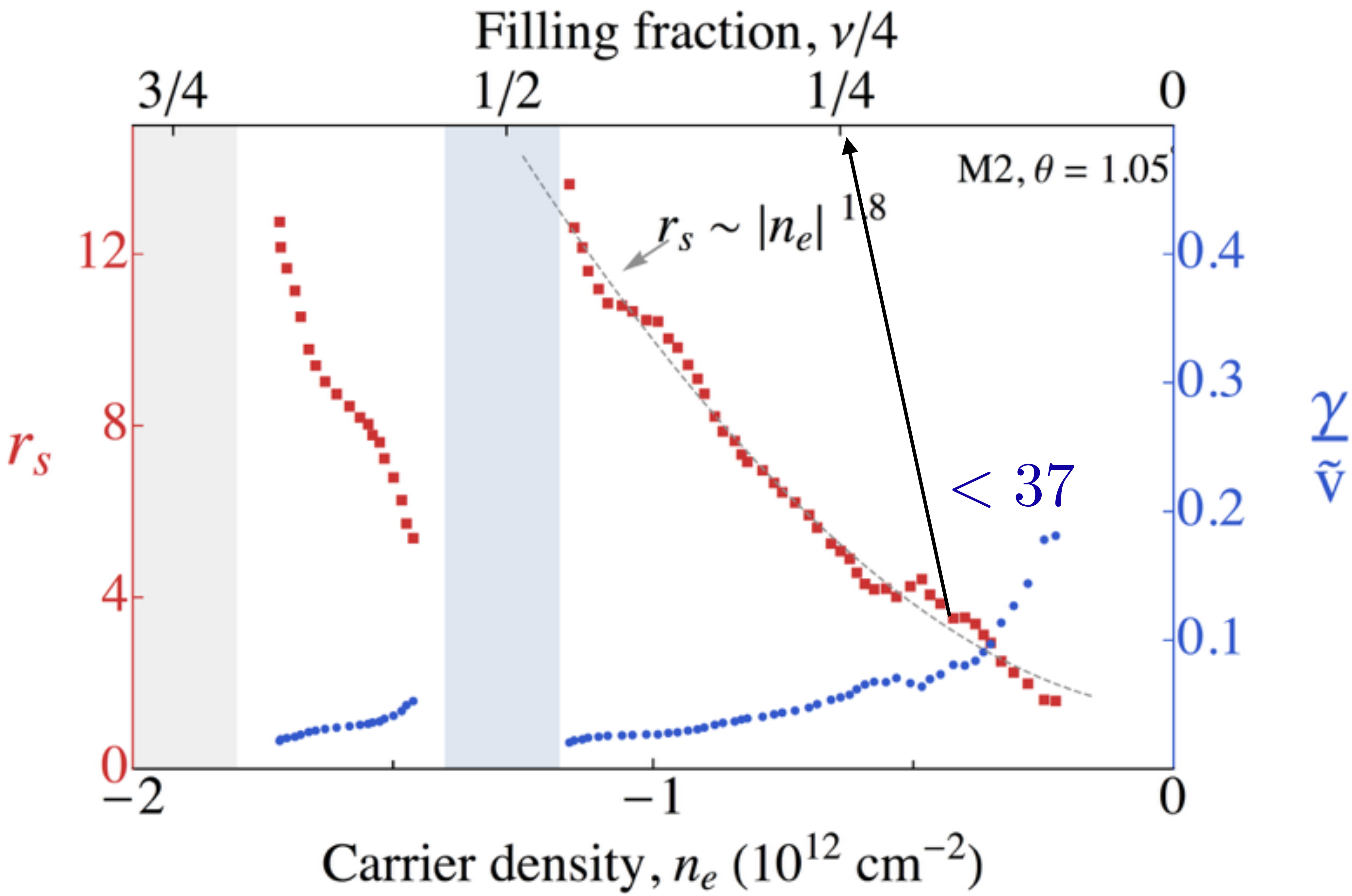
$$d_3 = 1/2$$

$$\frac{37}{\nu^2} < r_s^{\mathrm{crit}} < d_\nu 37$$

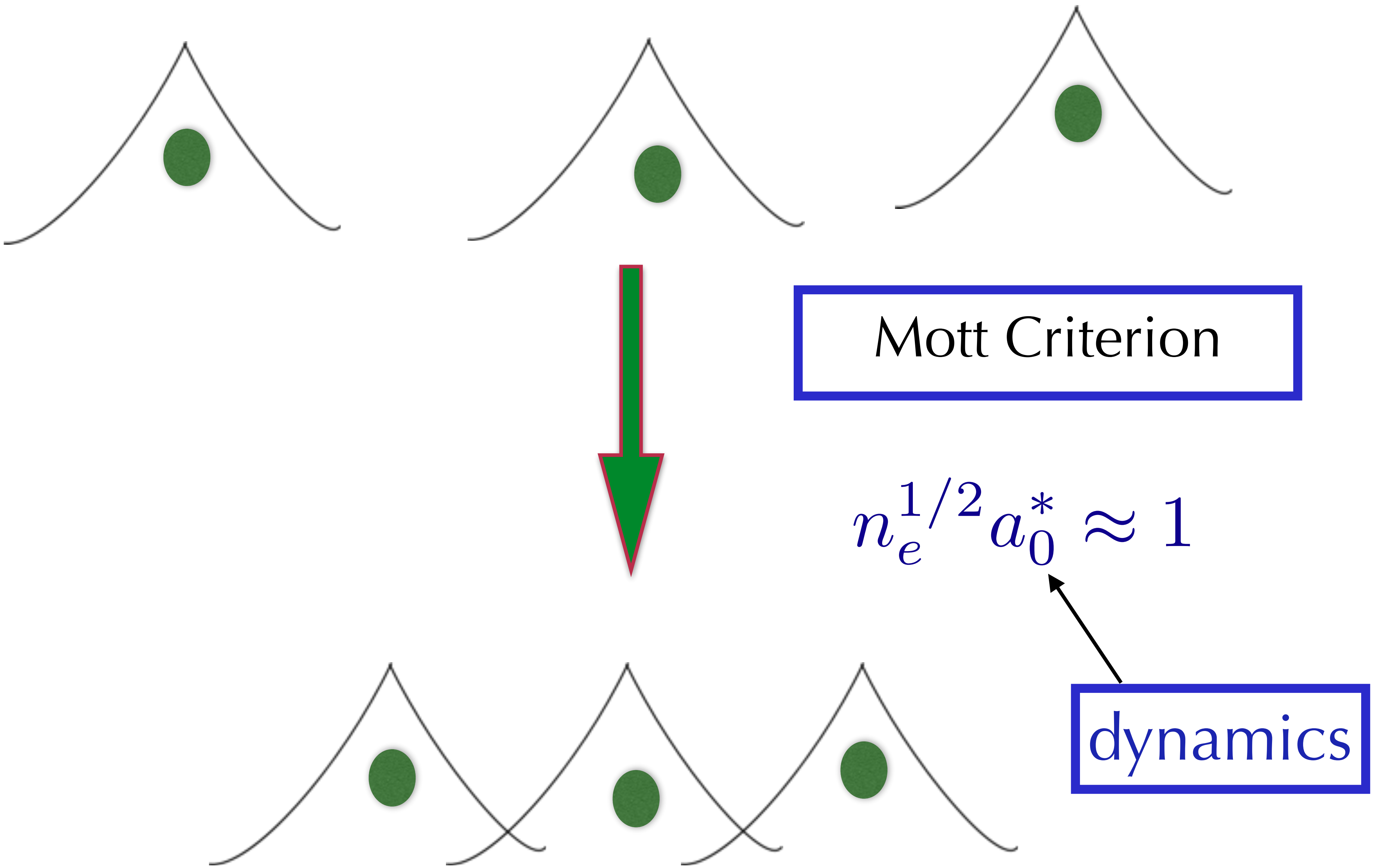
experiments



using experimental data for effective mass



What happened to Mottness?



$$m_*(\theta,\nu) = m_e \frac{\left(\gamma^2 - 2\gamma + 1\right)^{3/2}}{\gamma^3 - 3\gamma^2 + .45\gamma - 2}$$

Mott criterion

$$\frac{\lambda_s(\theta)}{a_0} \frac{m_*(\theta,\nu)}{m_e\sqrt{\nu}} \lesssim \epsilon$$

$\frac{\lambda_s(\theta)}{a_0} \to 150 - 200$

$\frac{m_*(\theta,\nu)}{m_e\sqrt{\nu}} \to \mathrm{const}$

never satisfied

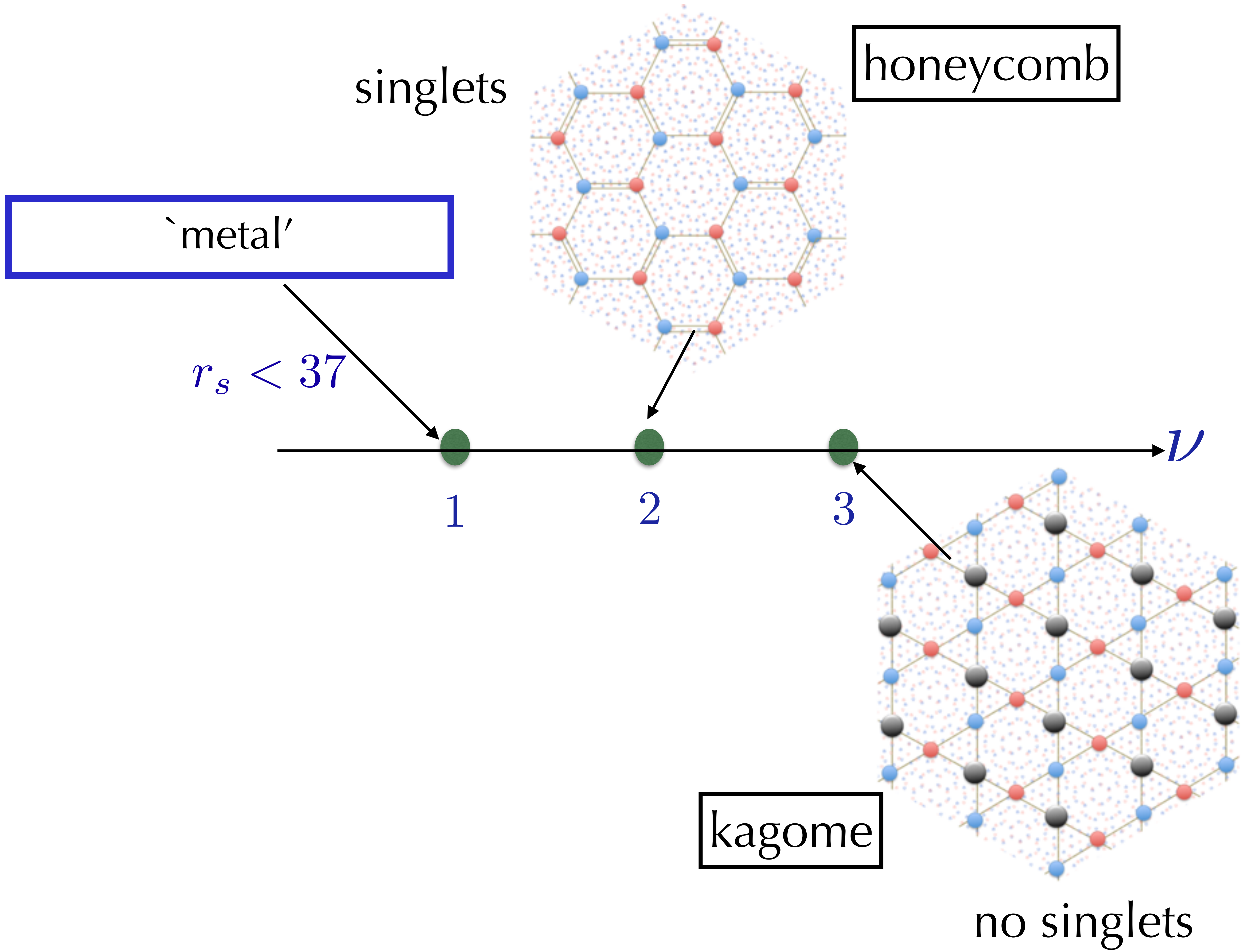
$$m_*/m_e = \tilde{v}\,\sqrt{h^2 n_e / 8\pi v_0^2 m_e^2}$$

Mott criterion

$$r_s^{(1)}(\theta) \lesssim \frac{1}{\sqrt{\pi}}$$

$$\theta - \theta_{\mathrm{magic}} = .7$$

$$n_e \approx 10^4 n_s$$



singlets
honeycomb
`metal'
$r_s < 37$
ν
1
2
3
kagome
no singlets

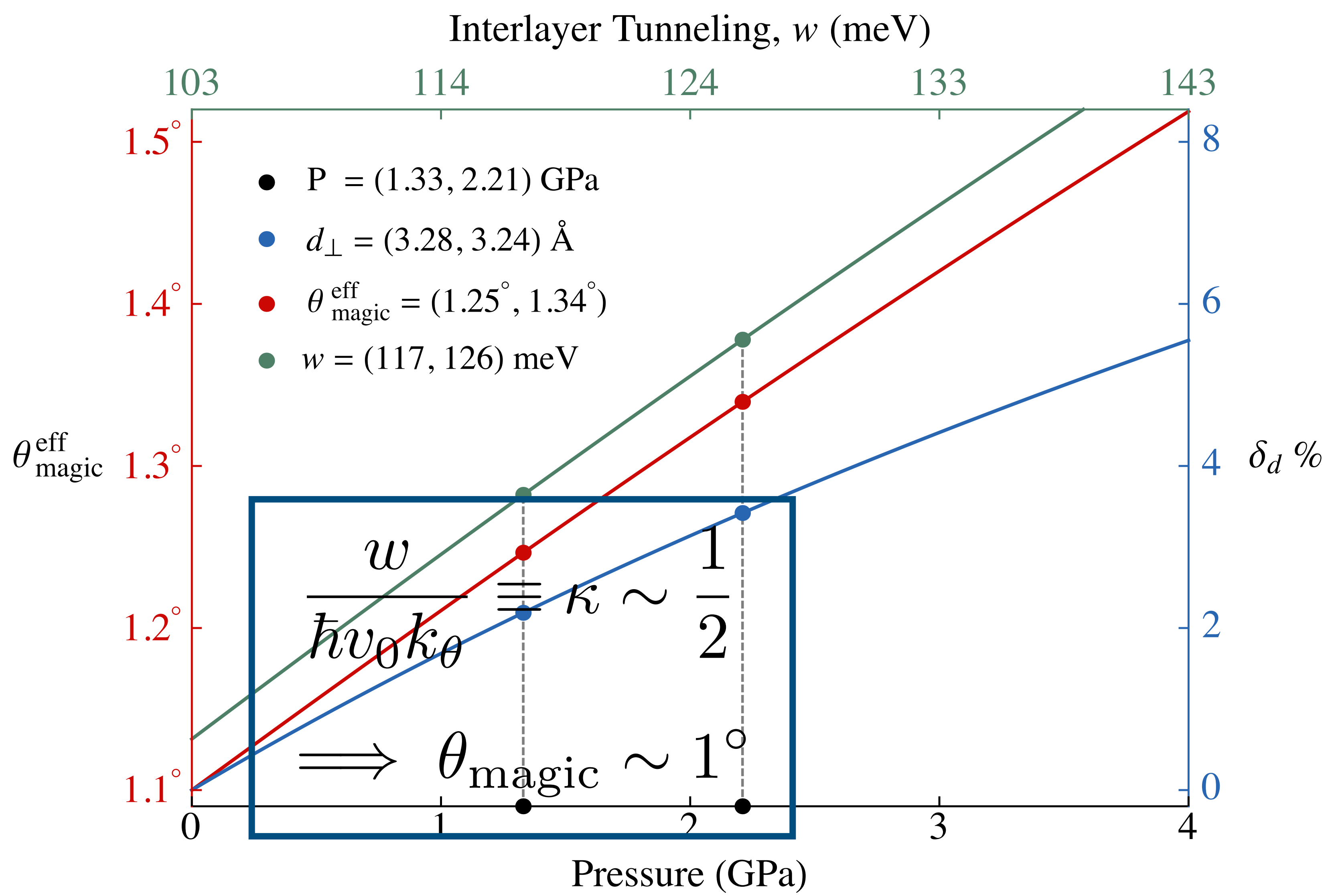
Effects of Pressure

Inter-particle Distance

$$1 - \frac{d_\perp}{\ldots} \equiv \delta_d = 10.48 \log\left(1 + \frac{\mathrm{P}}{\ldots}\right) \%$$

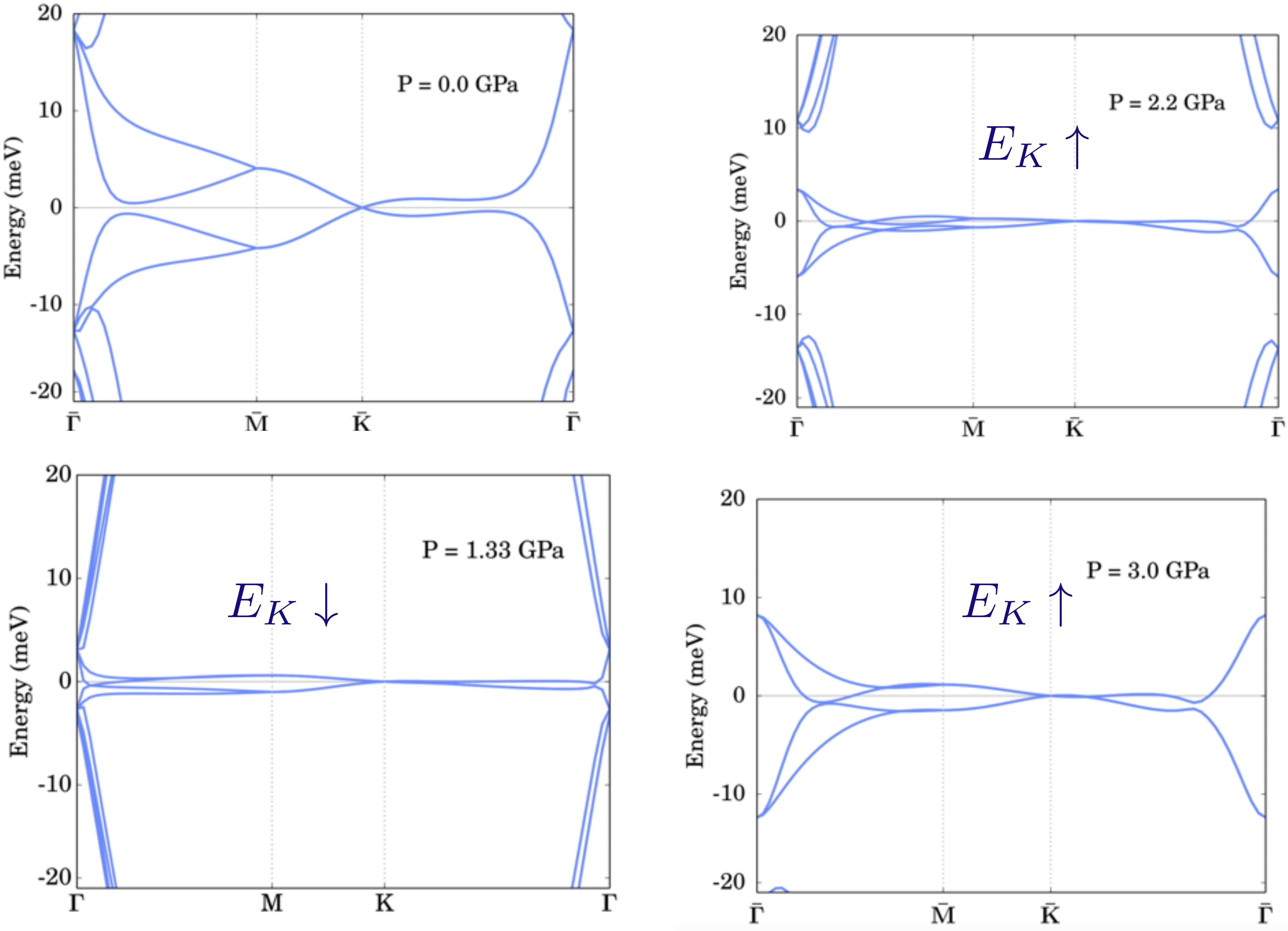
Inter-laye Tunnelin

Effective Magic-Ang



$$\frac{w}{\hbar v_0 k_\theta} \equiv \kappa \sim \frac{1}{2}$$

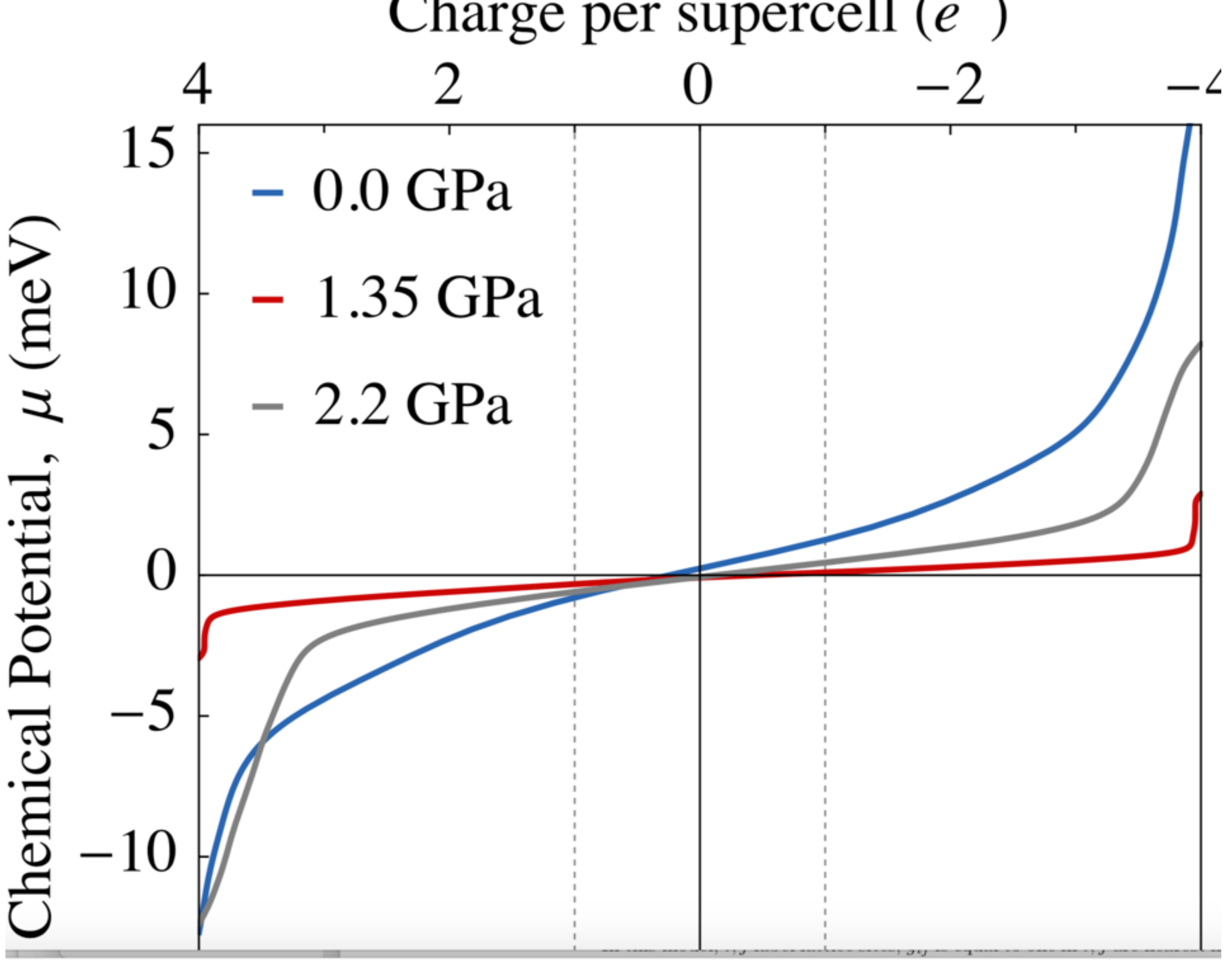
$$\implies \theta_{\text{magic}} \sim 1^\circ$$



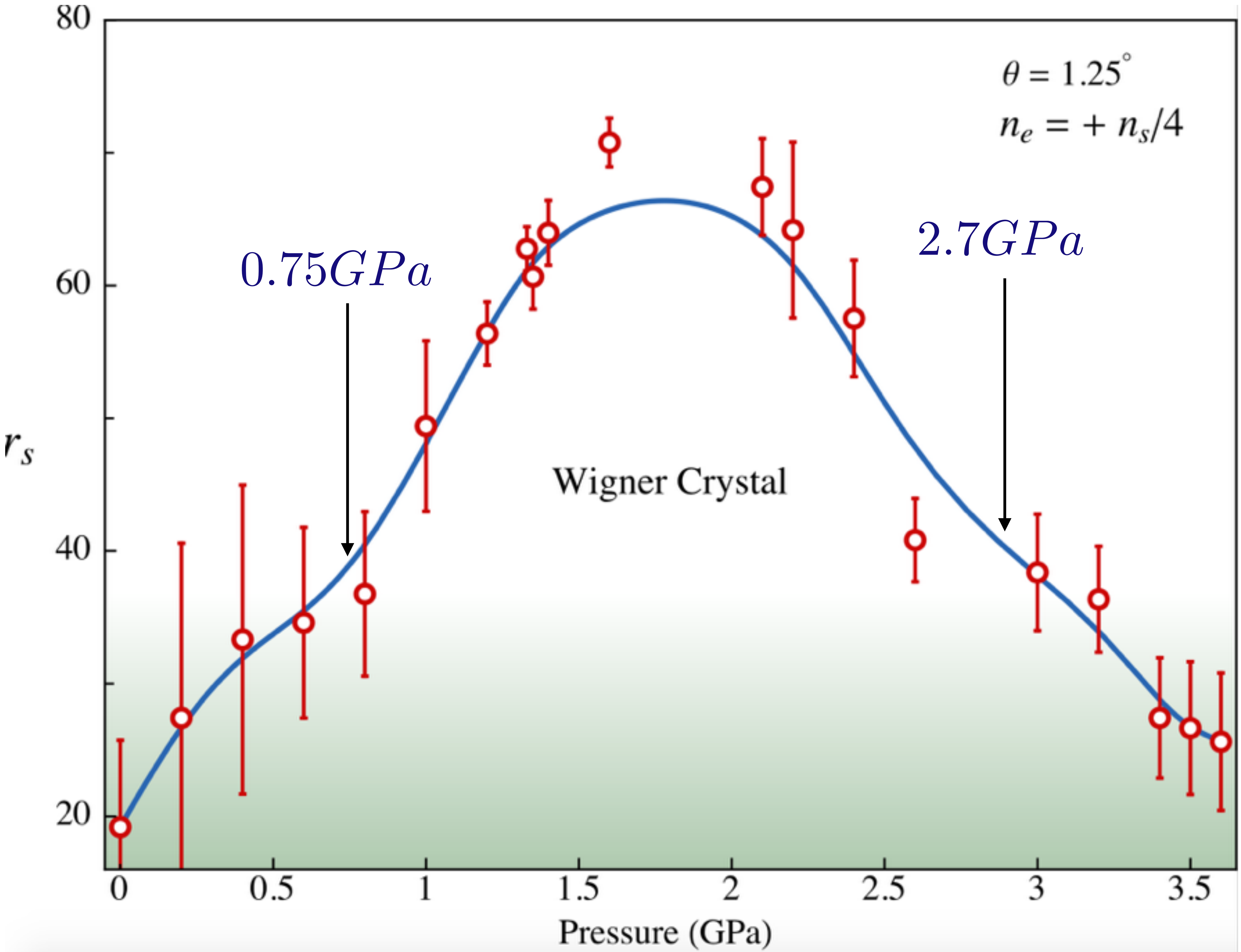
P = 0.0 GPa
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄
P = 2.2 GPa
$E_K \uparrow$
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄
P = 1.33 GPa
$E_K \downarrow$
Energy (meV)
20
10
0
-10
-20
Γ
M
K
Γ
P = 3.0 GPa
$E_K \uparrow$
Energy (meV)
20
10
0
-10
-20
Γ̄
M̄
K̄
Γ̄

estimated
r_s

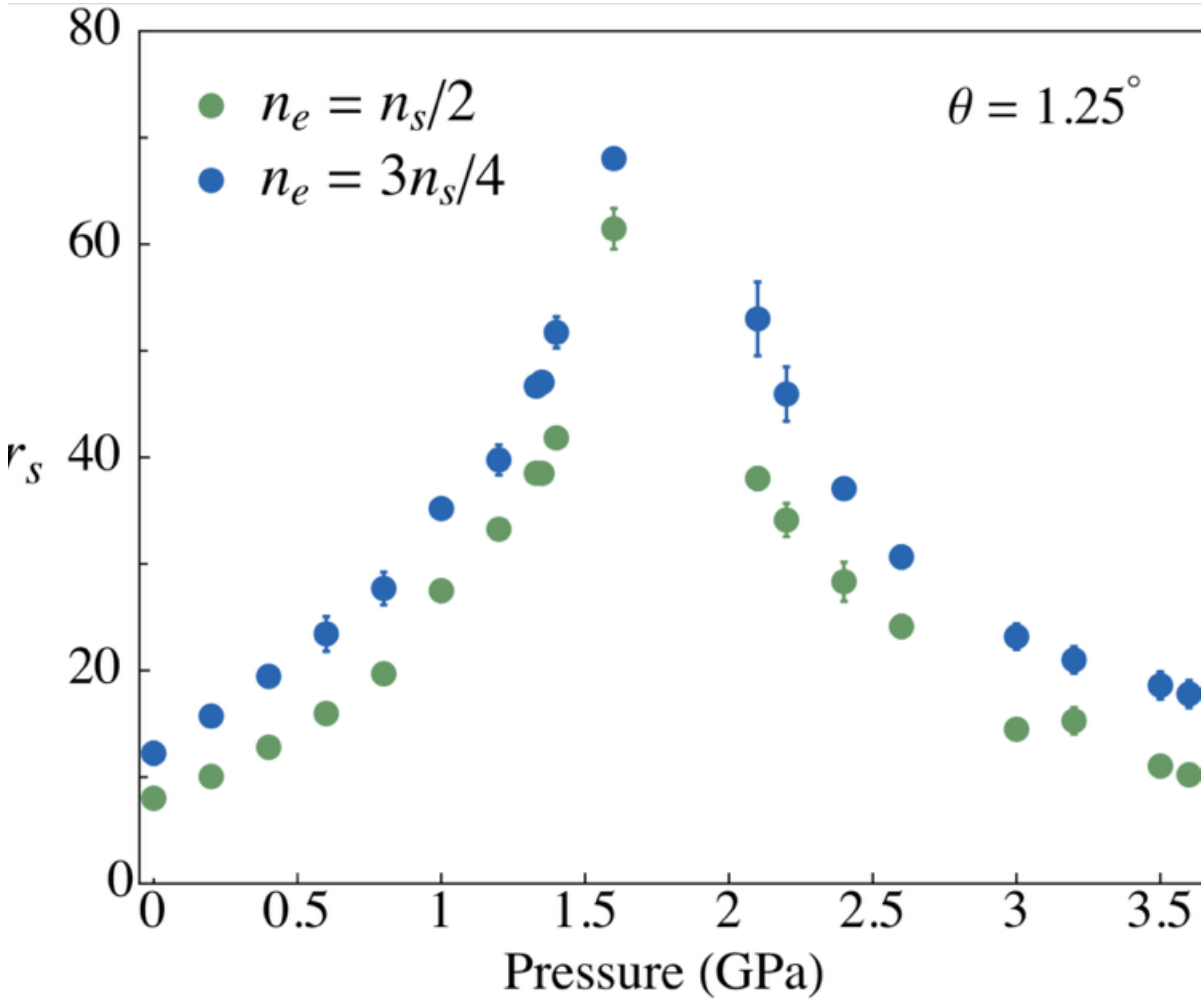
$$r_s \approx 15\,\mathrm{meV}\,\frac{\theta^\circ}{E_K}\sqrt{\nu} \xrightarrow[\mathrm{D2}]{\mathrm{Device}} \frac{20\,\mathrm{meV}}{E_K\,\mathrm{meV}}\sqrt{\nu}$$



Charge per supercell (e^-)
4
2
0
−2
−4
15
10
5
0
−5
−10
Chemical Potential, μ (meV)
0.0 GPa
1.35 GPa
2.2 GPa



80
60
40
20
r_s
$\theta = 1.25^\circ$
$n_e = + n_s/4$
$0.75GPa$
$2.7GPa$
Wigner Crystal
0
0.5
1
1.5
2
2.5
3
3.5
Pressure (GPa)



$n_e = n_s/2$
$n_e = 3n_s/4$
$\theta = 1.25^\circ$
r_s
80
60
40
20
0
0
0.5
1
1.5
2
2.5
3
3.5
Pressure (GPa)

Prediction

$$0.75GPa < P < 2.75GPa$$

dome-shaped
phase diagram

Melting temperature

$$\frac{e^2}{4\pi\epsilon\lambda_s} \approx 30meV$$

↓

$$T_{\rm melt} \propto .01U_{\rm coul}$$

↓

$$4K$$

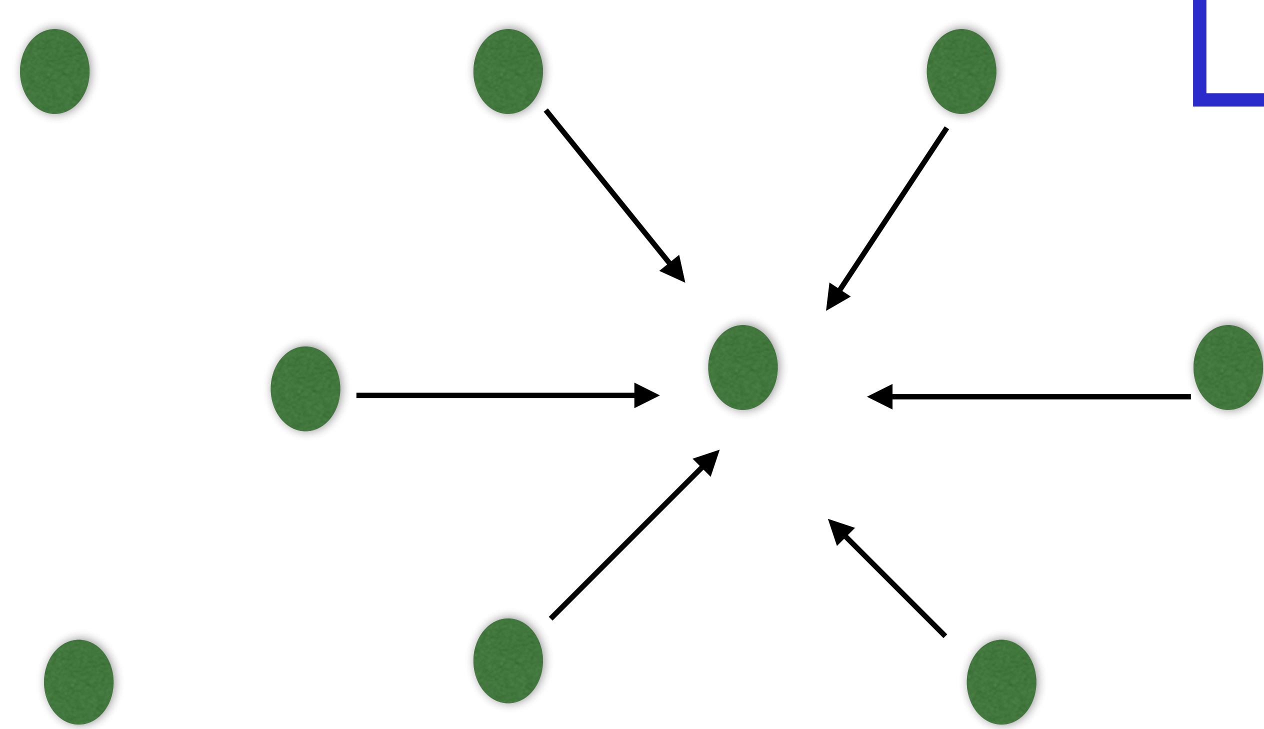
Letter

Superconductivity in a two-dimensional electron gas

Philip Phillips, Yi Wan, Ivar Martin, Sergey Knysh & Denis Dalidovich

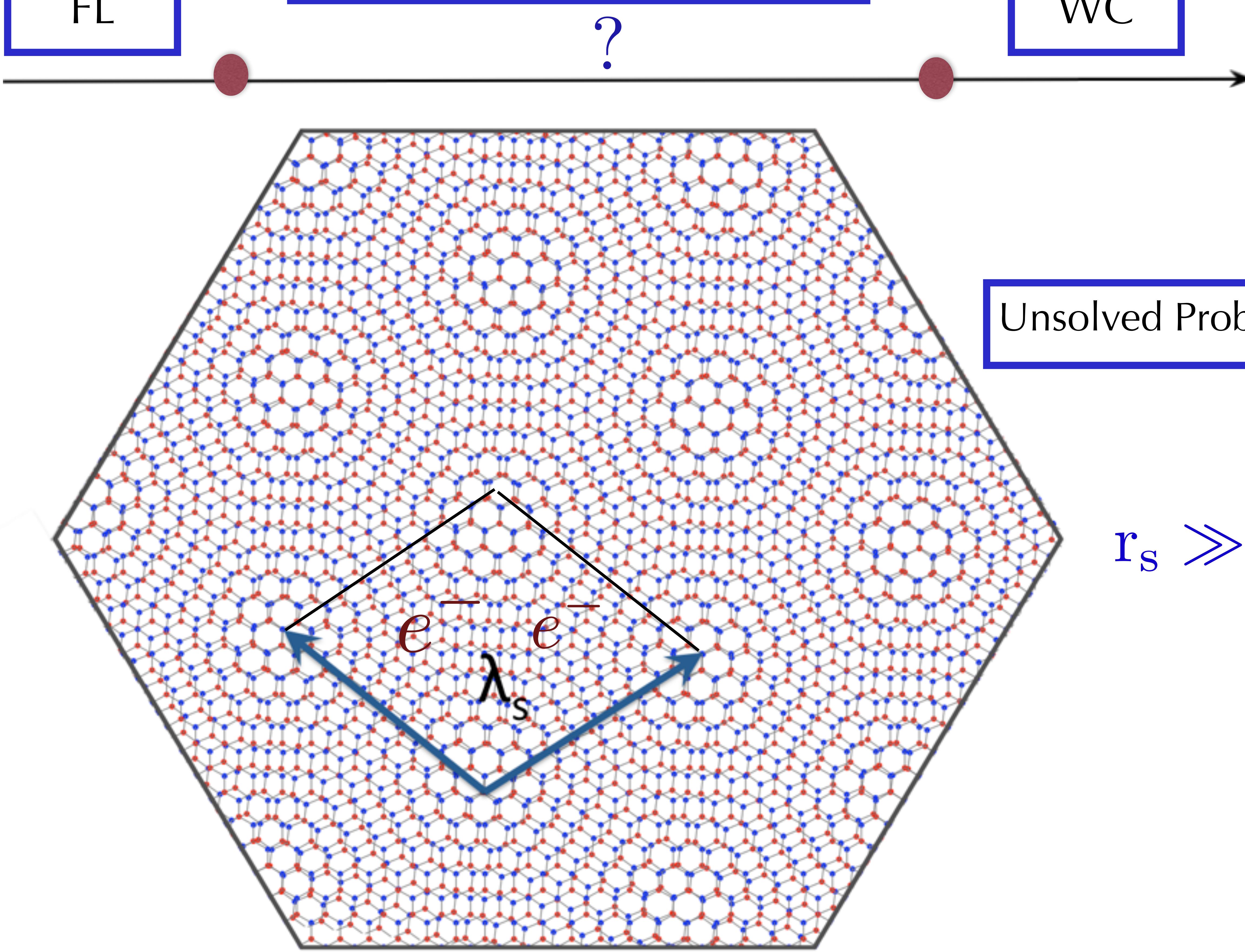
Nature **395**, 253–257 (17 September 1998) | Received: 19 May 1998

Wigner Crystal



dynamically generated attraction

Moiré is Different



Unsolved Problem

$r_s \gg 1$