Quantum matter without quasiparticles

Blackboard talk
Kavli Institute for Theoretical Physics,
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Talk online: sachdev.physics.harvard.edu

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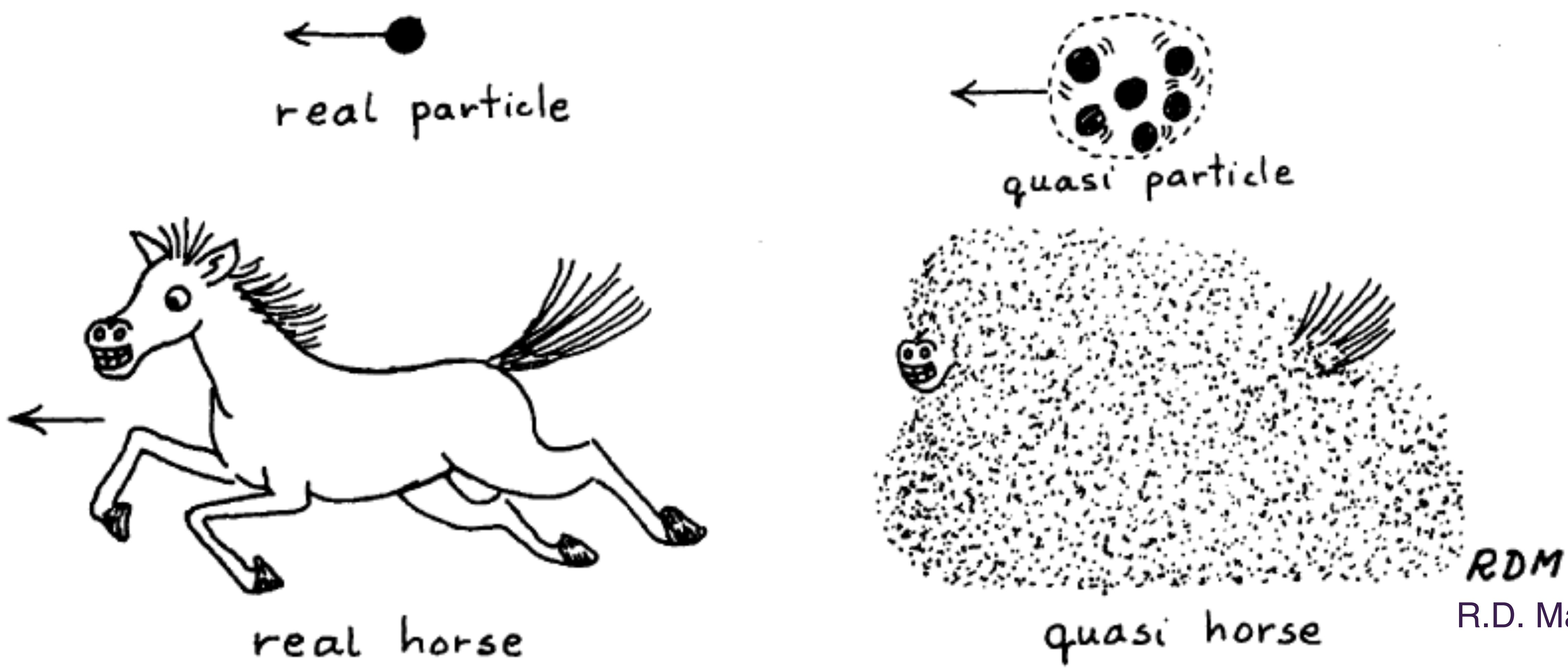
Quantum matter with quasiparticles:

A quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

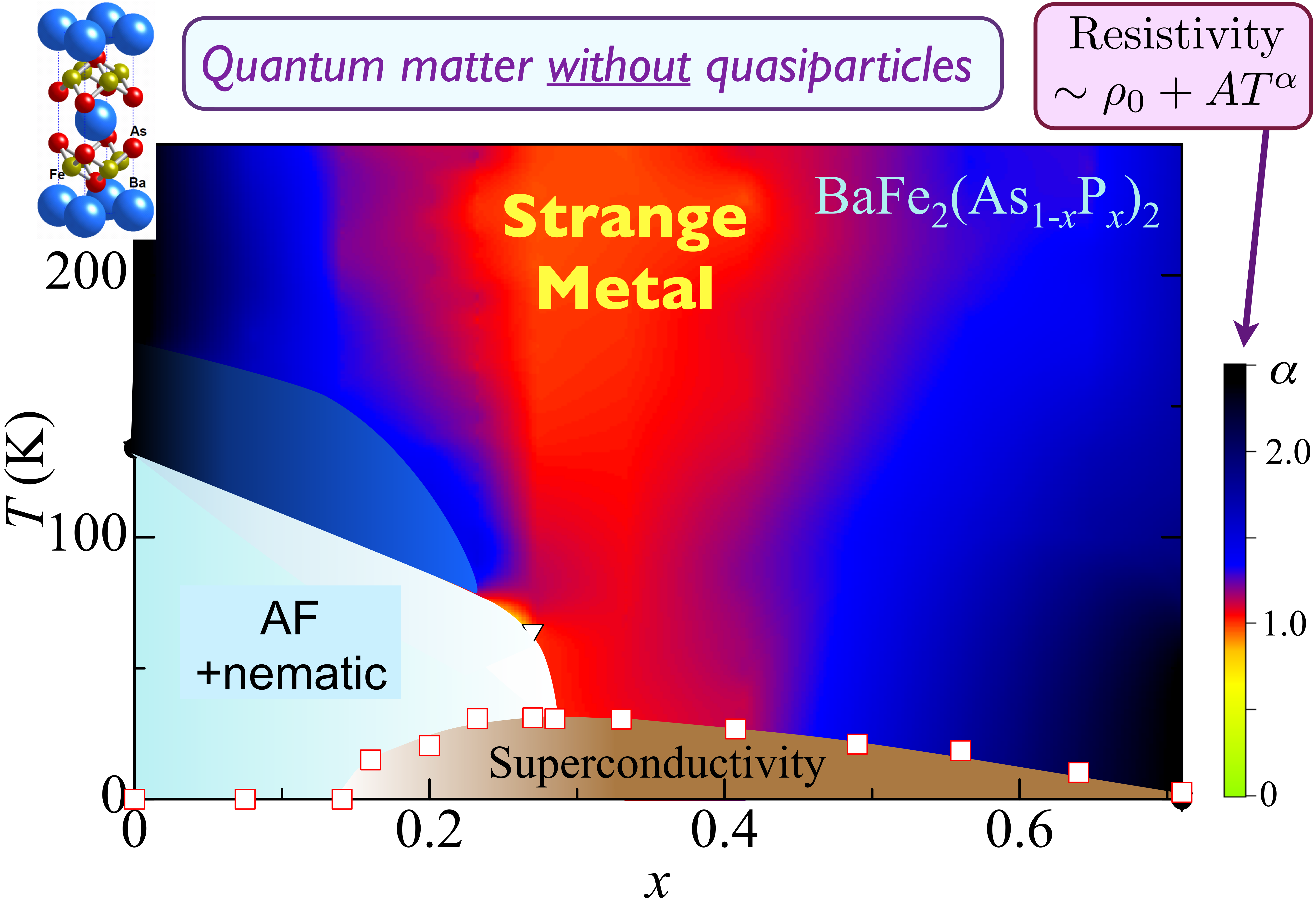
Quantum matter with quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are 'fractions' of an electron)

Quantum matter without quasiparticles

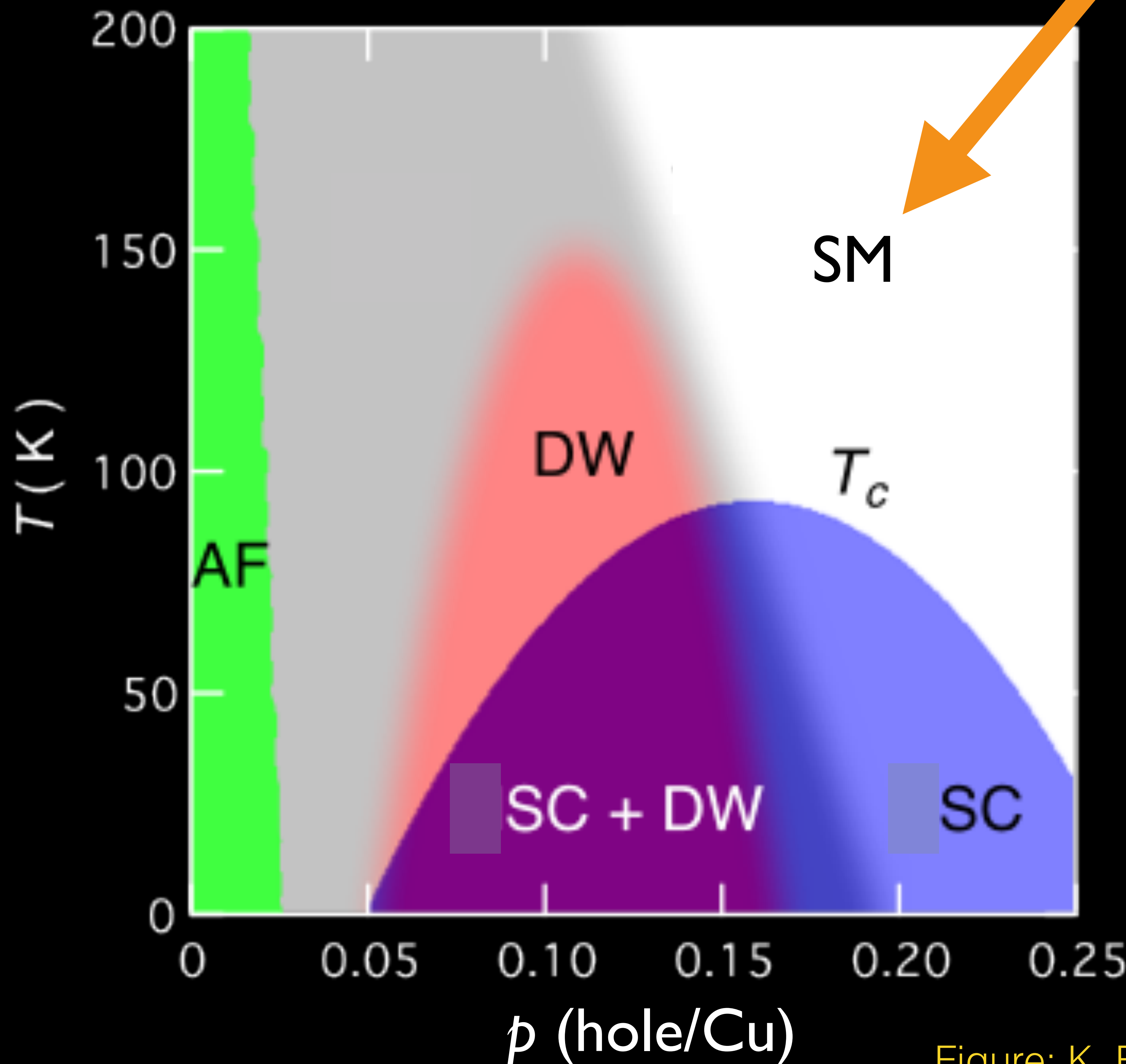
Resistivity $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Quantum matter without quasiparticles

Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties

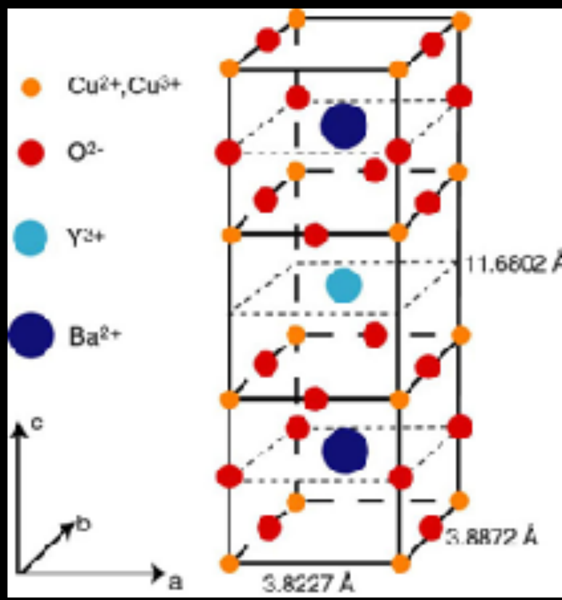


Figure: K. Fujita and J. C. Seamus Davis

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

Quantum matter <u>with</u> quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order $\hbar E_F/(k_B T)^2$ as $T \to 0$, where E_F is the Fermi energy.

Quantum matter <u>without</u> quasiparticles:

- No quasiparticle decomposition of low-lying states
- Rapid thermalization

Quantum Ising models

Qubits with states $|\uparrow\rangle_i$, $|\downarrow\rangle_i$, on the sites, i, of a regular lattice.

$$\sigma^z |\uparrow\rangle = |\uparrow\rangle \quad , \quad \sigma^z |\downarrow\rangle = -|\downarrow\rangle$$
$$\sigma^x |\uparrow\rangle = |\downarrow\rangle \quad , \quad \sigma^x |\downarrow\rangle = |\uparrow\rangle$$

$$H = -J \left(\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + g \sum_i \sigma_i^x \right)$$

For $g = 0$, ground state is a ferromagnet:

$$|G\rangle = |\cdots \uparrow\uparrow\uparrow\uparrow\uparrow \cdots\rangle \quad \text{or} \quad |\cdots \downarrow\downarrow\downarrow\downarrow\downarrow \cdots\rangle$$

For $g \gg 1$, unique ‘paramagnetic’ ground state:

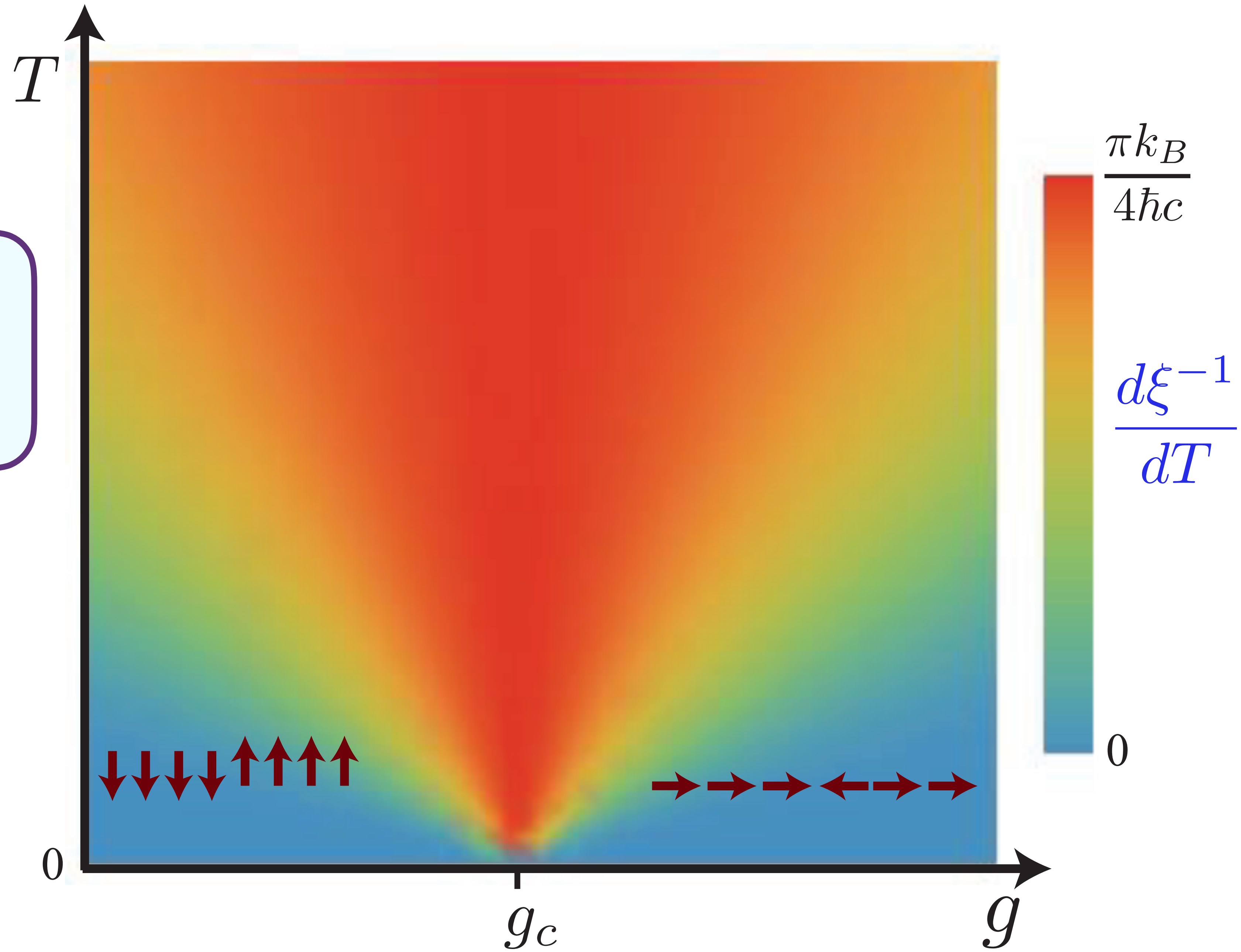
$$|G\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle$$

where

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right) \quad , \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle - |\downarrow\rangle \right)$$

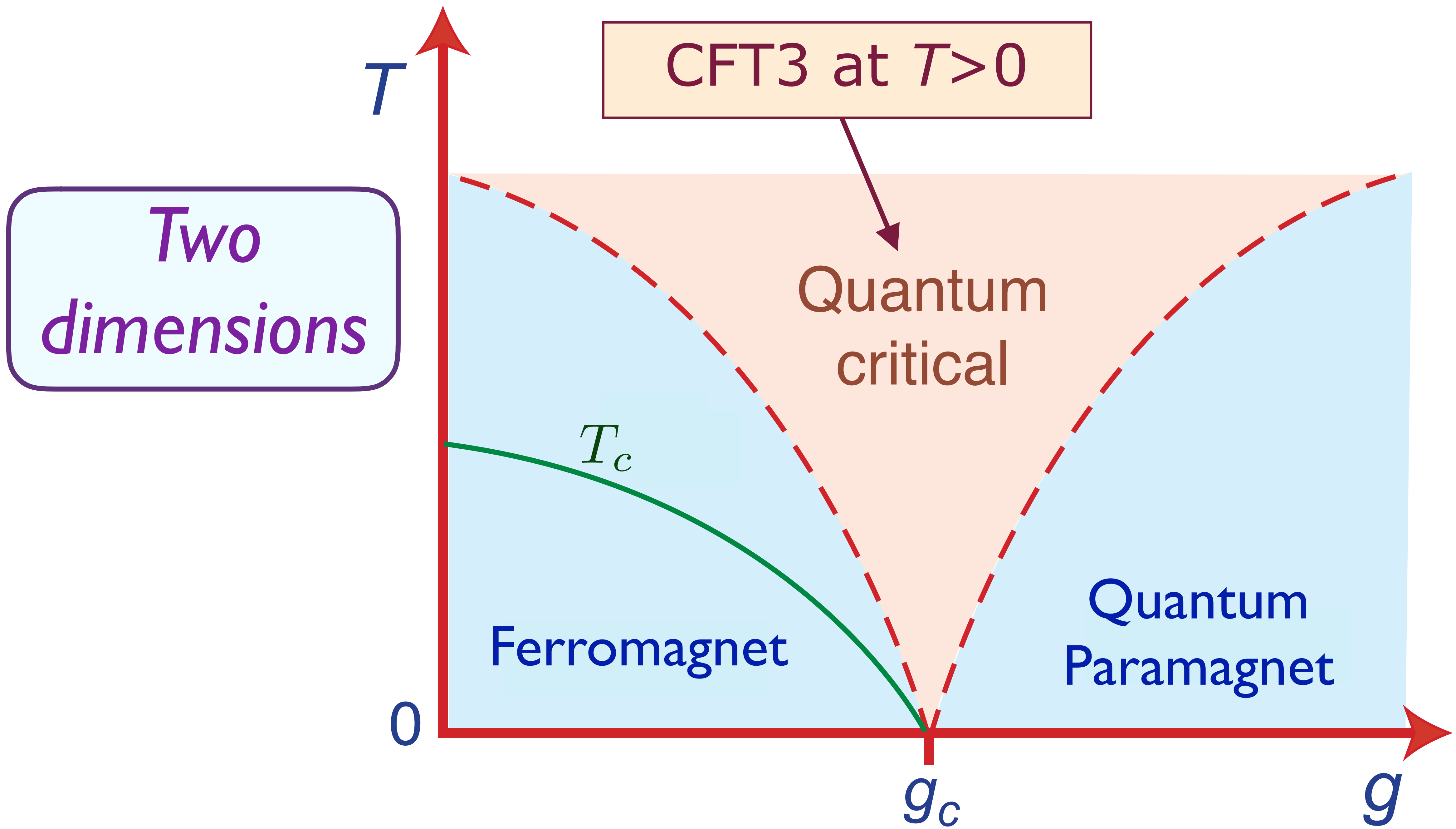
Quantum Ising models

One dimension



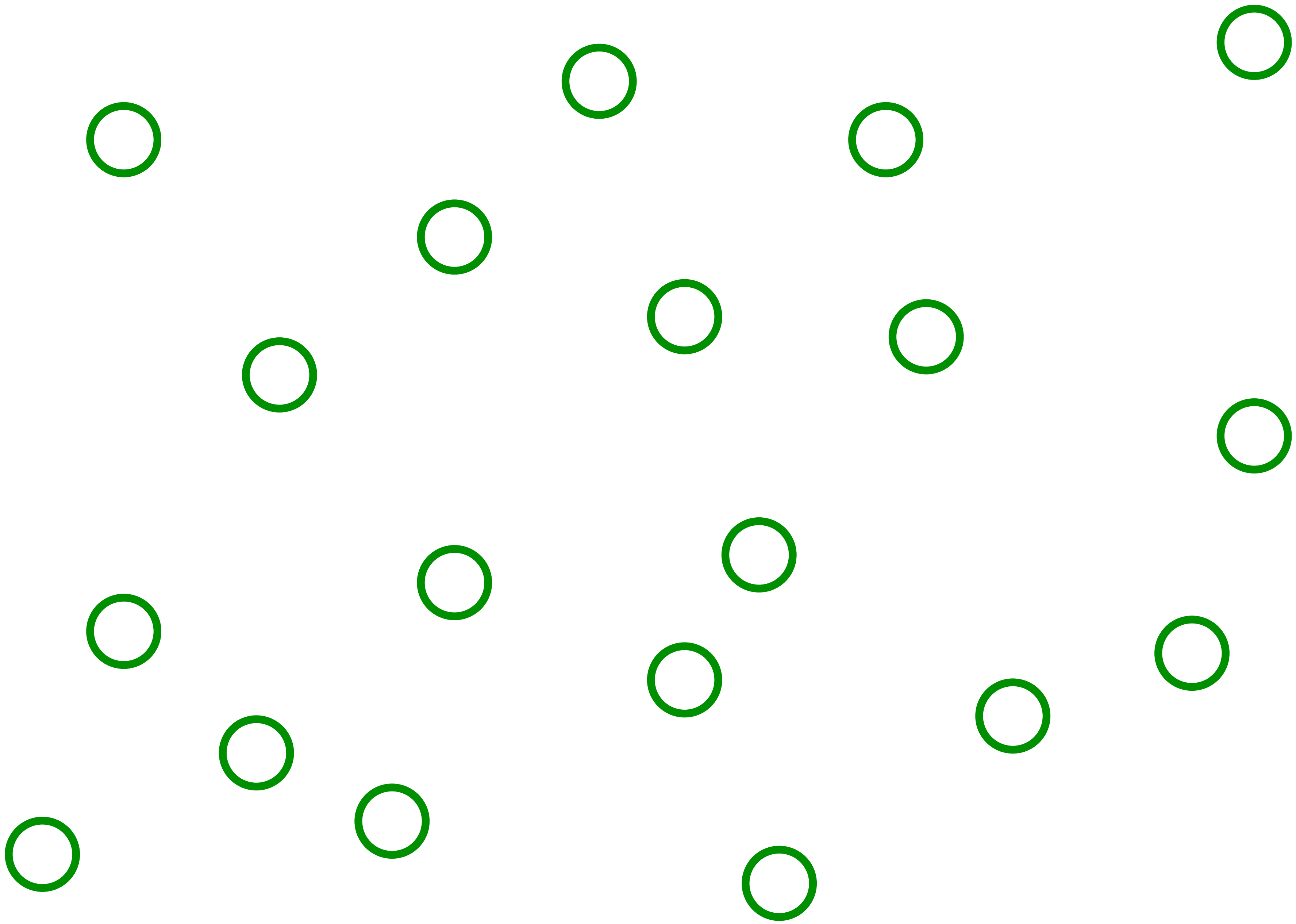
- In one dimension, quasiparticles exist even at the quantum critical point: there is a non-local transformations from the qubits to a system of free fermions.

Quantum Ising models



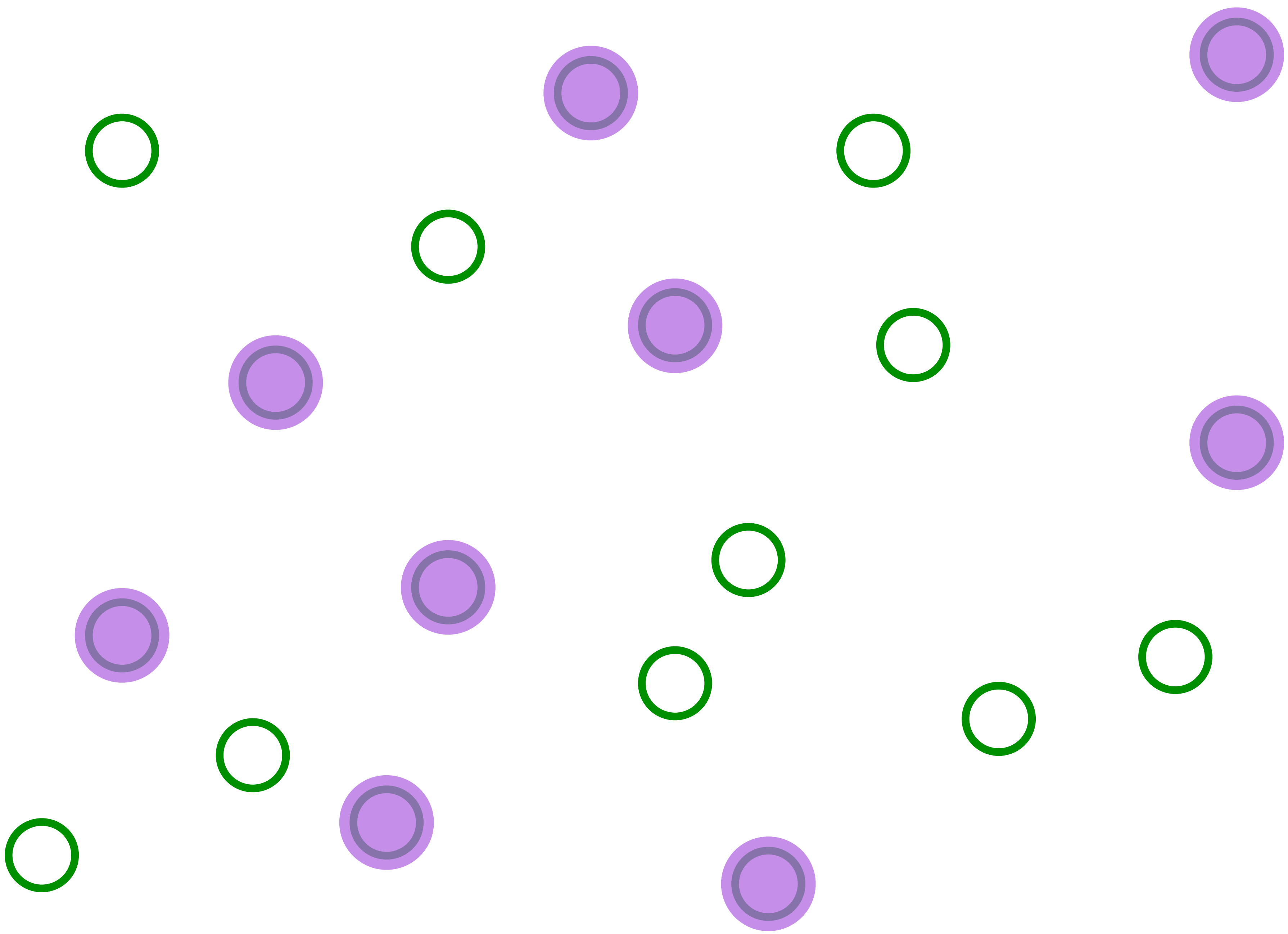
- In two dimensions, the "quantum critical" region provides us the first example of a system without a quasiparticle description. This is described by a strongly-coupled conformal field theory (CFT) in 2+1 dimensions, and dynamic properties cannot be computed accurately.

A simple model of a metal with quasiparticles



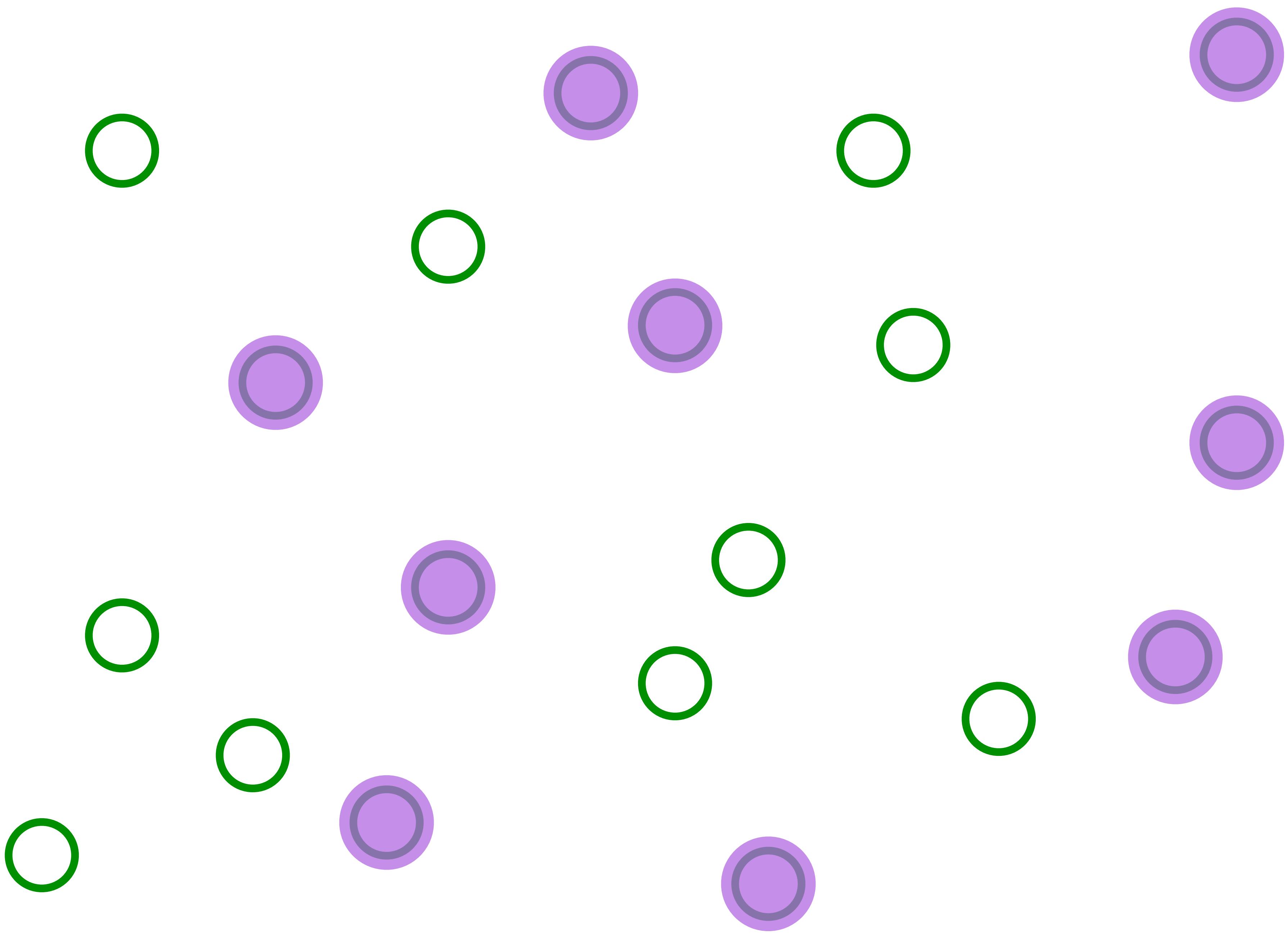
Pick a set of random positions

A simple model of a metal with quasiparticles



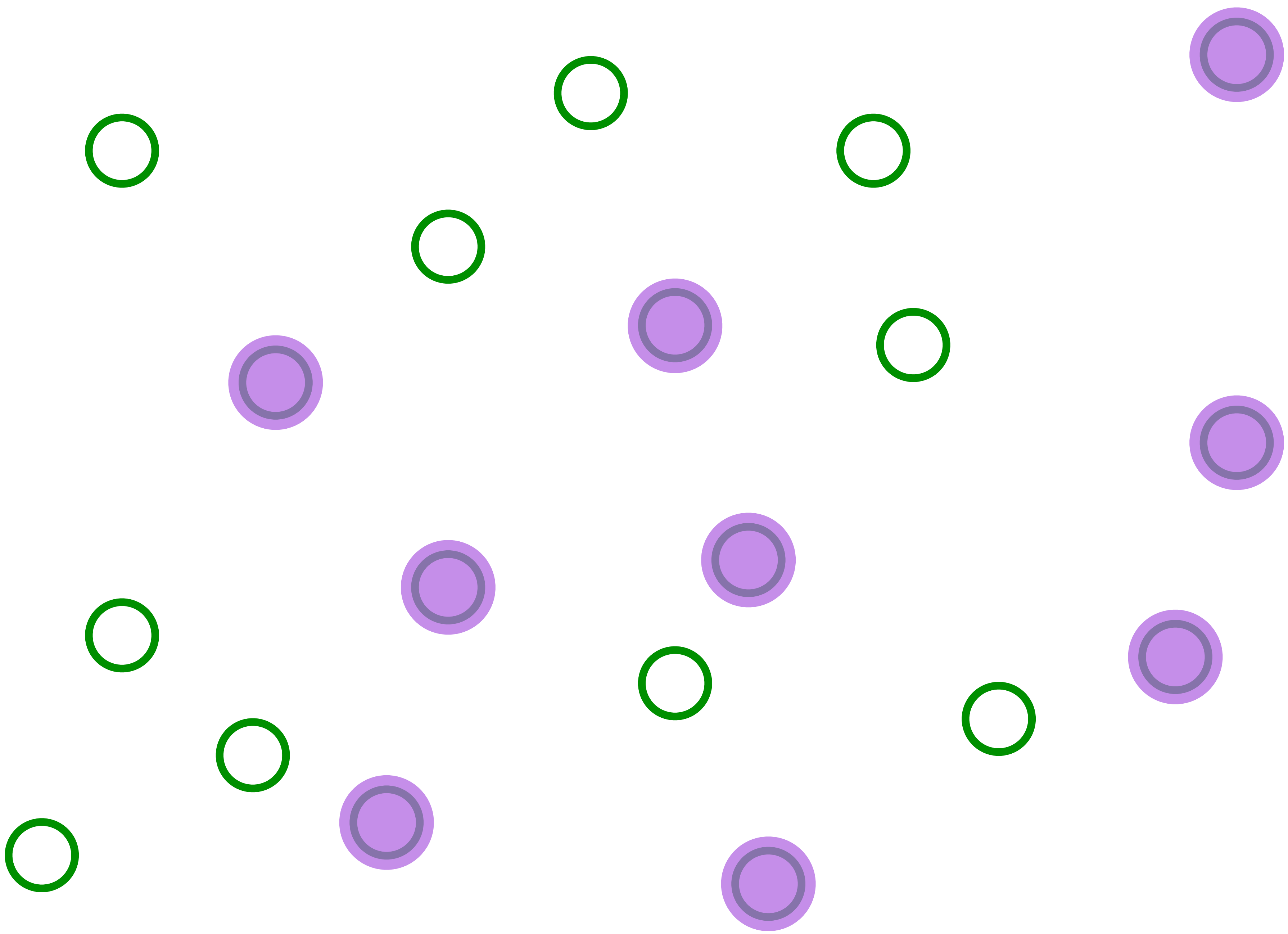
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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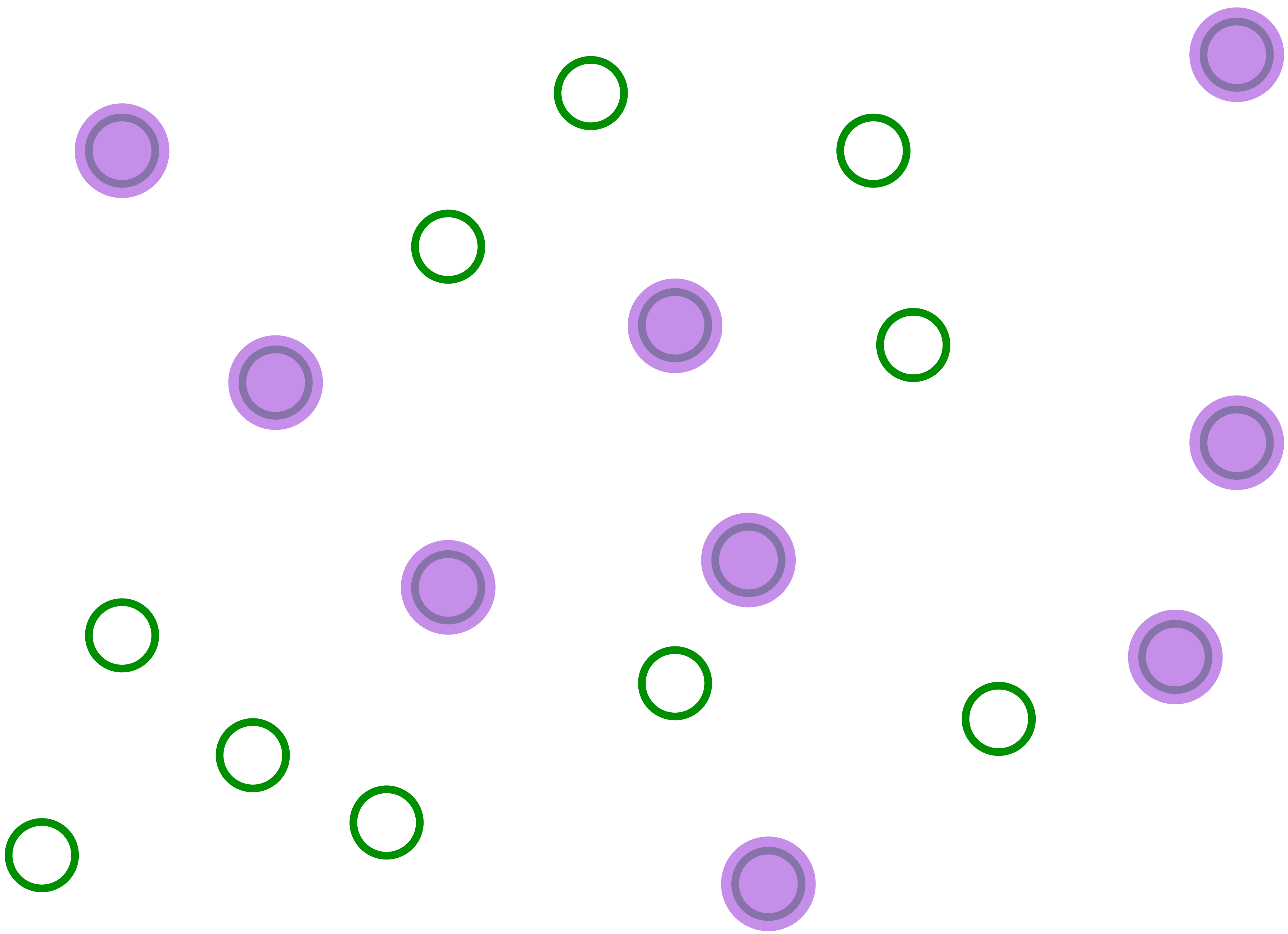

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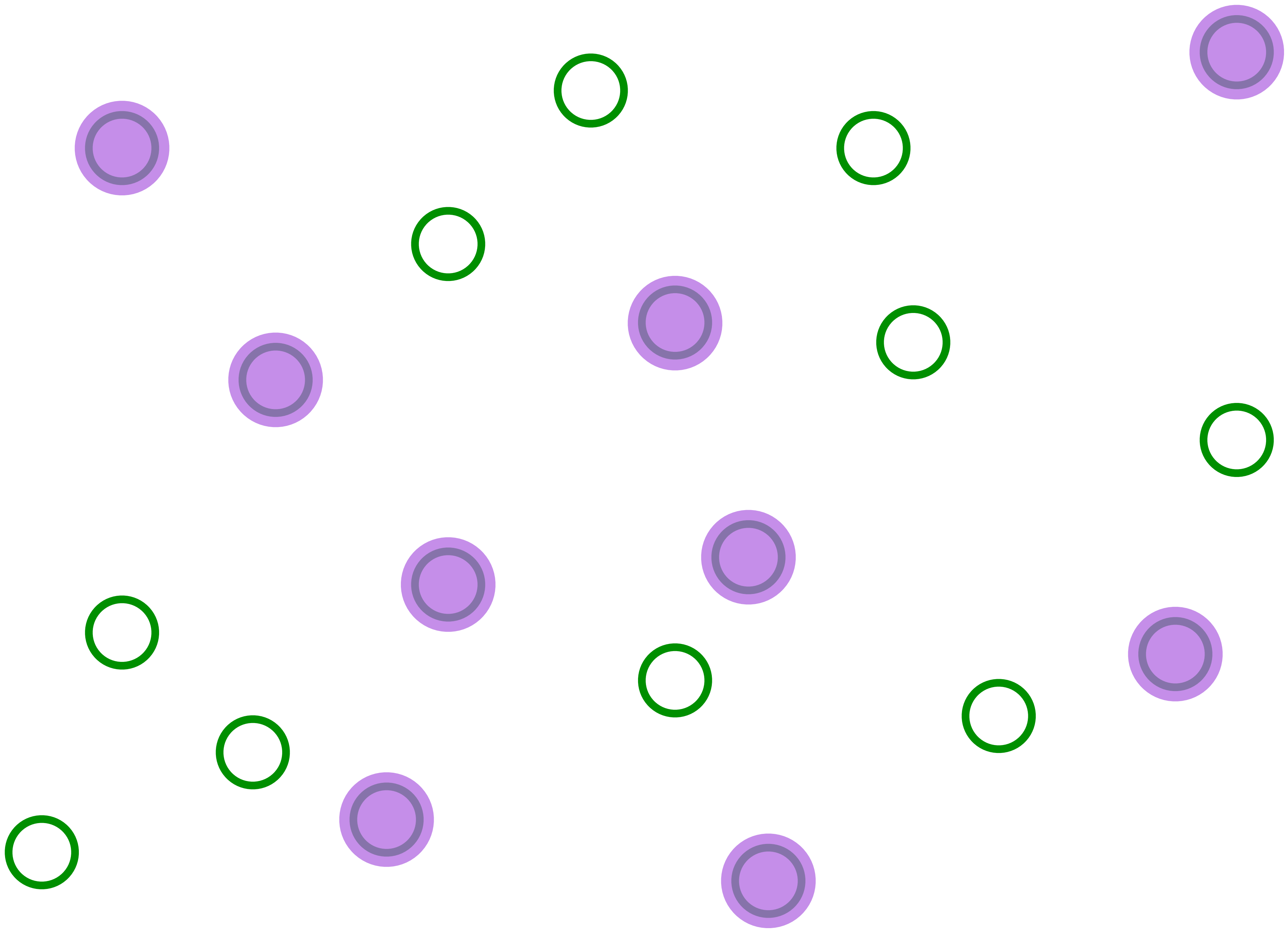


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

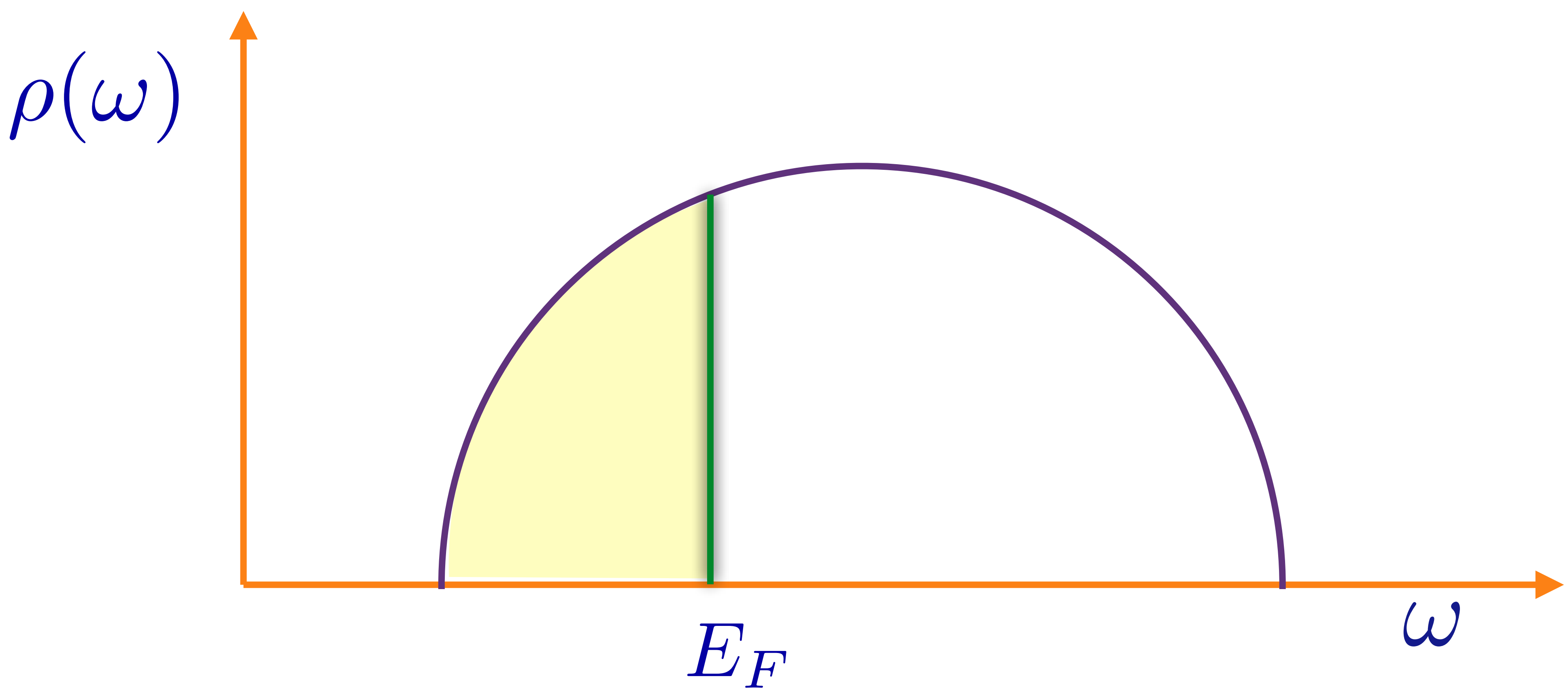
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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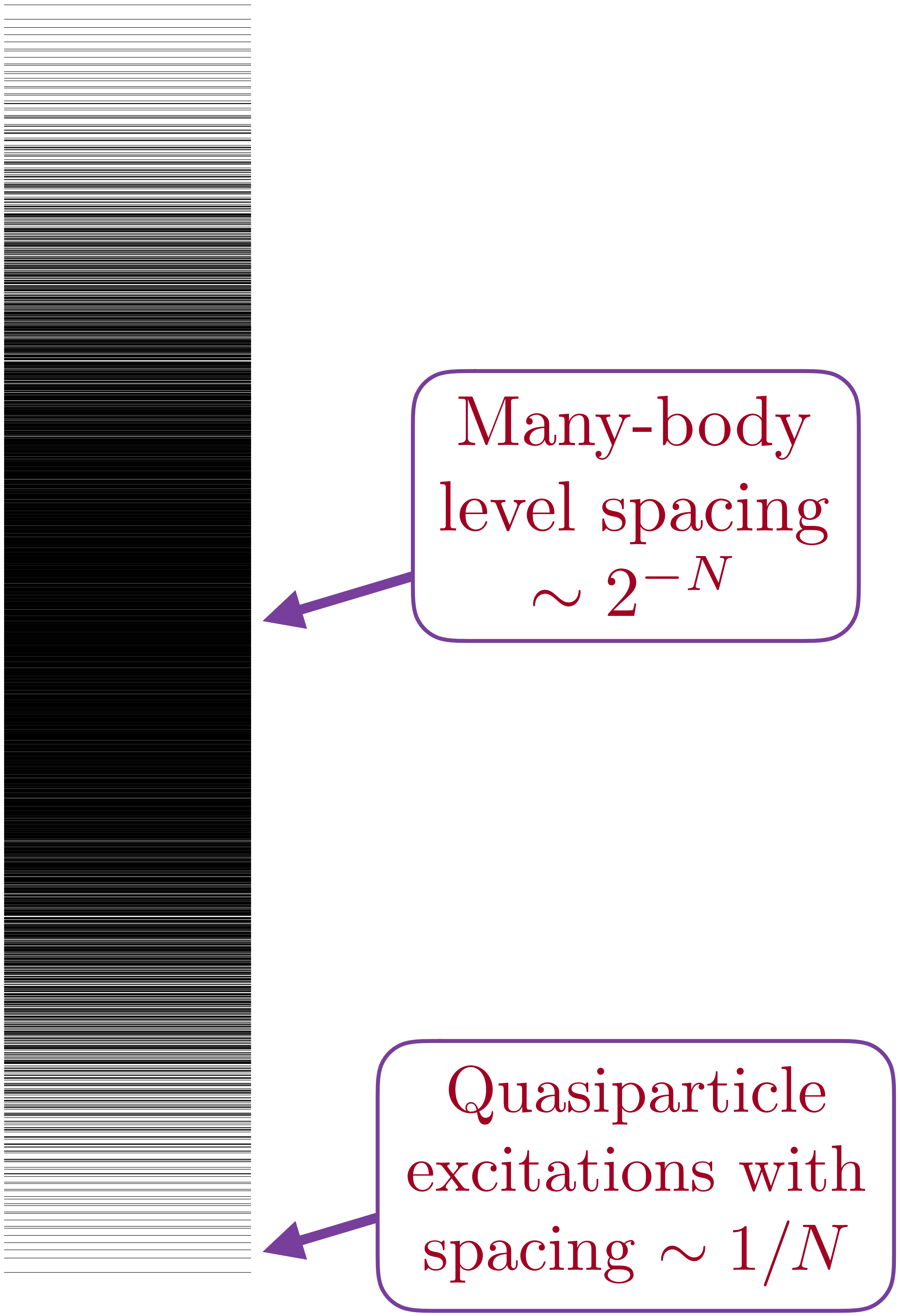
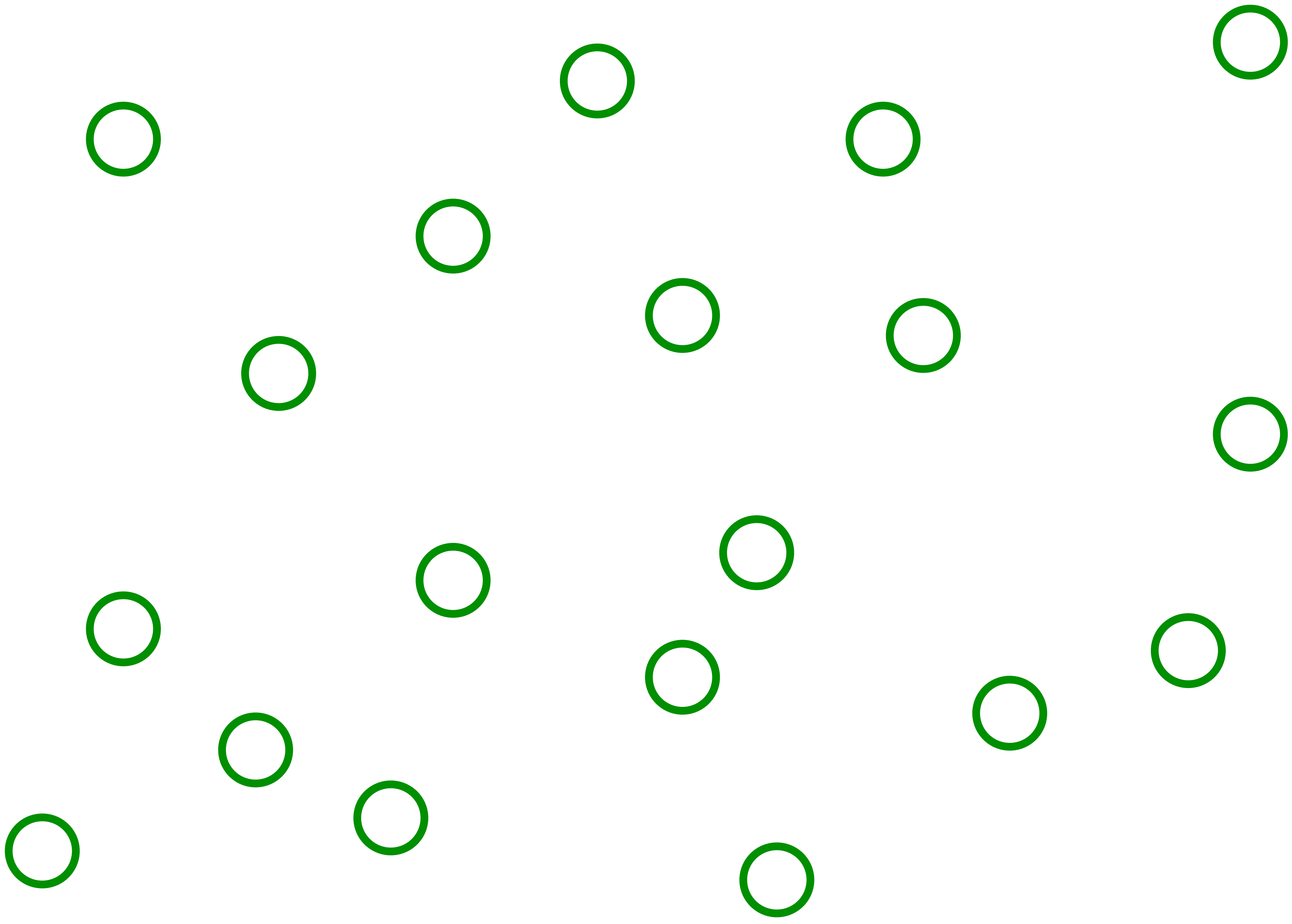


There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

The Sachdev-Ye-Kitaev (SYK) model



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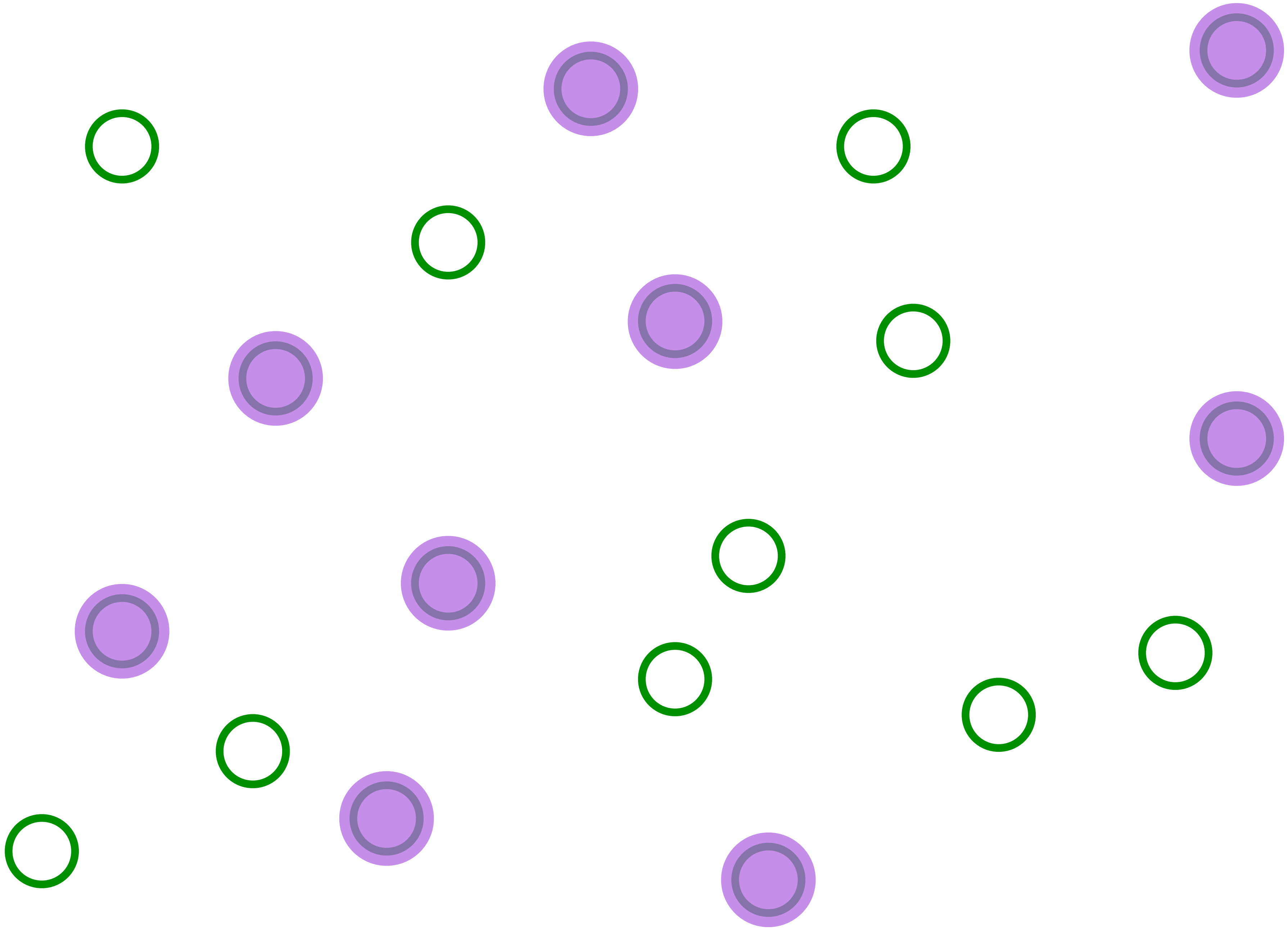
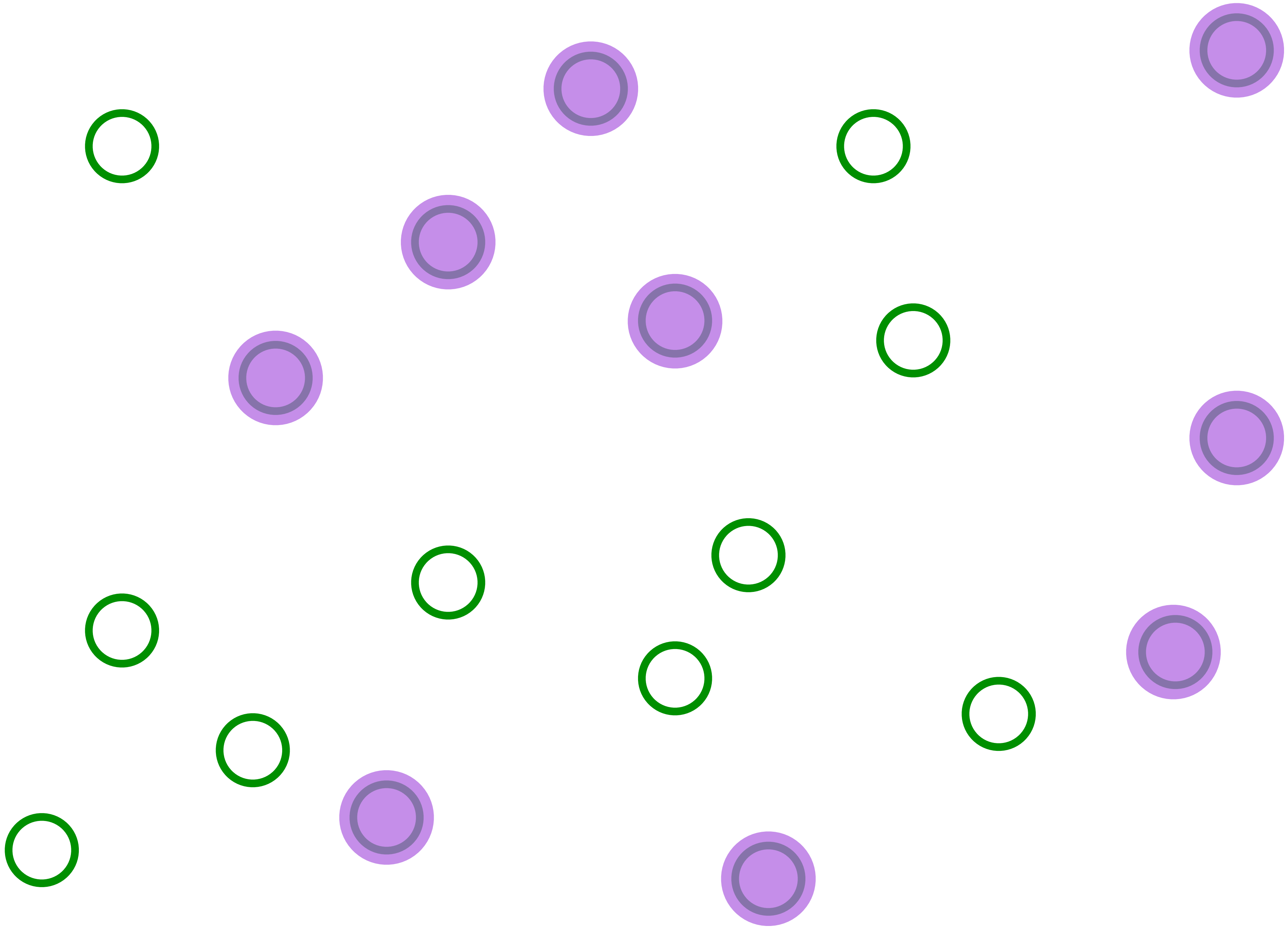


Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model



Entangle electrons pairwise randomly

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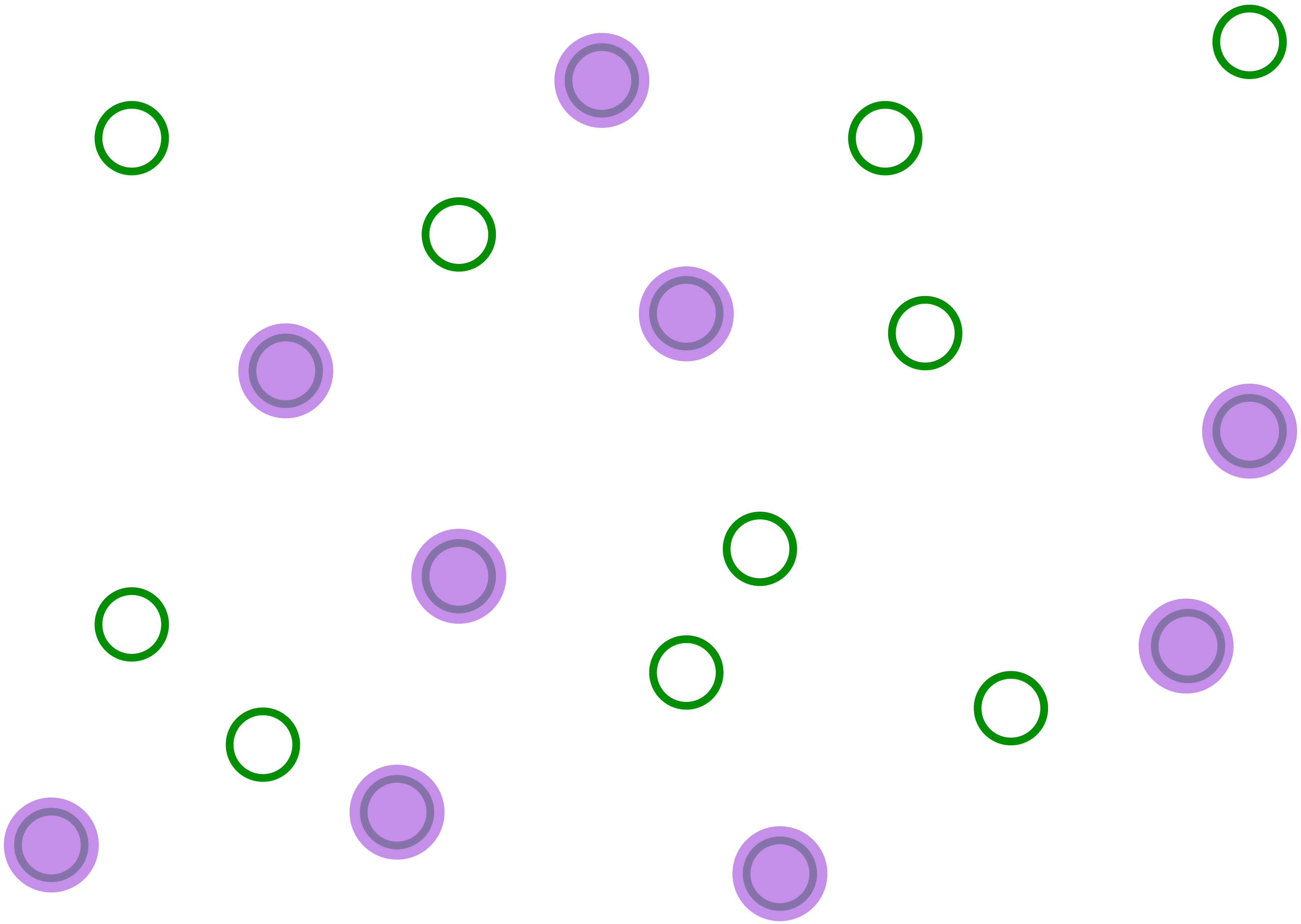

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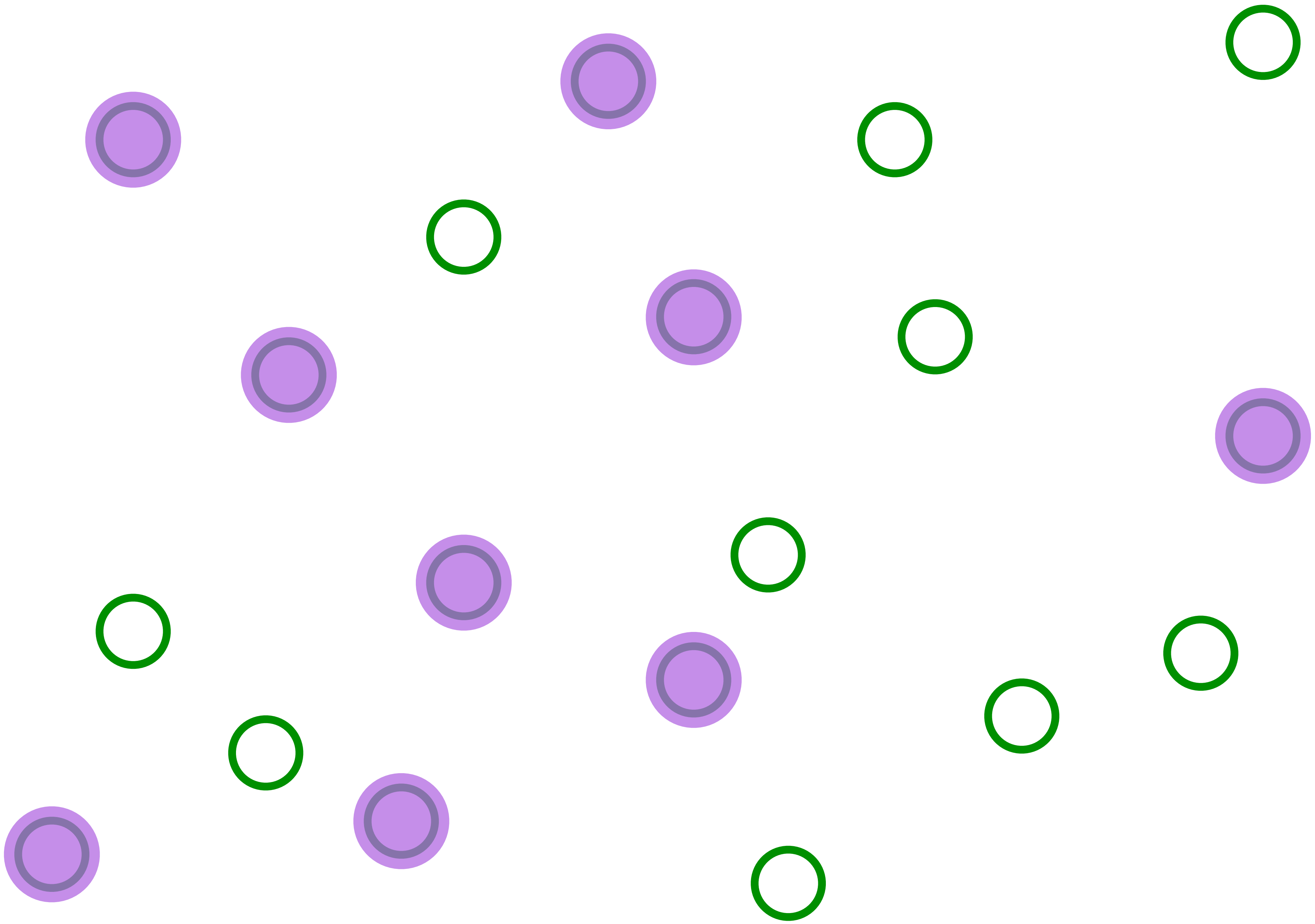

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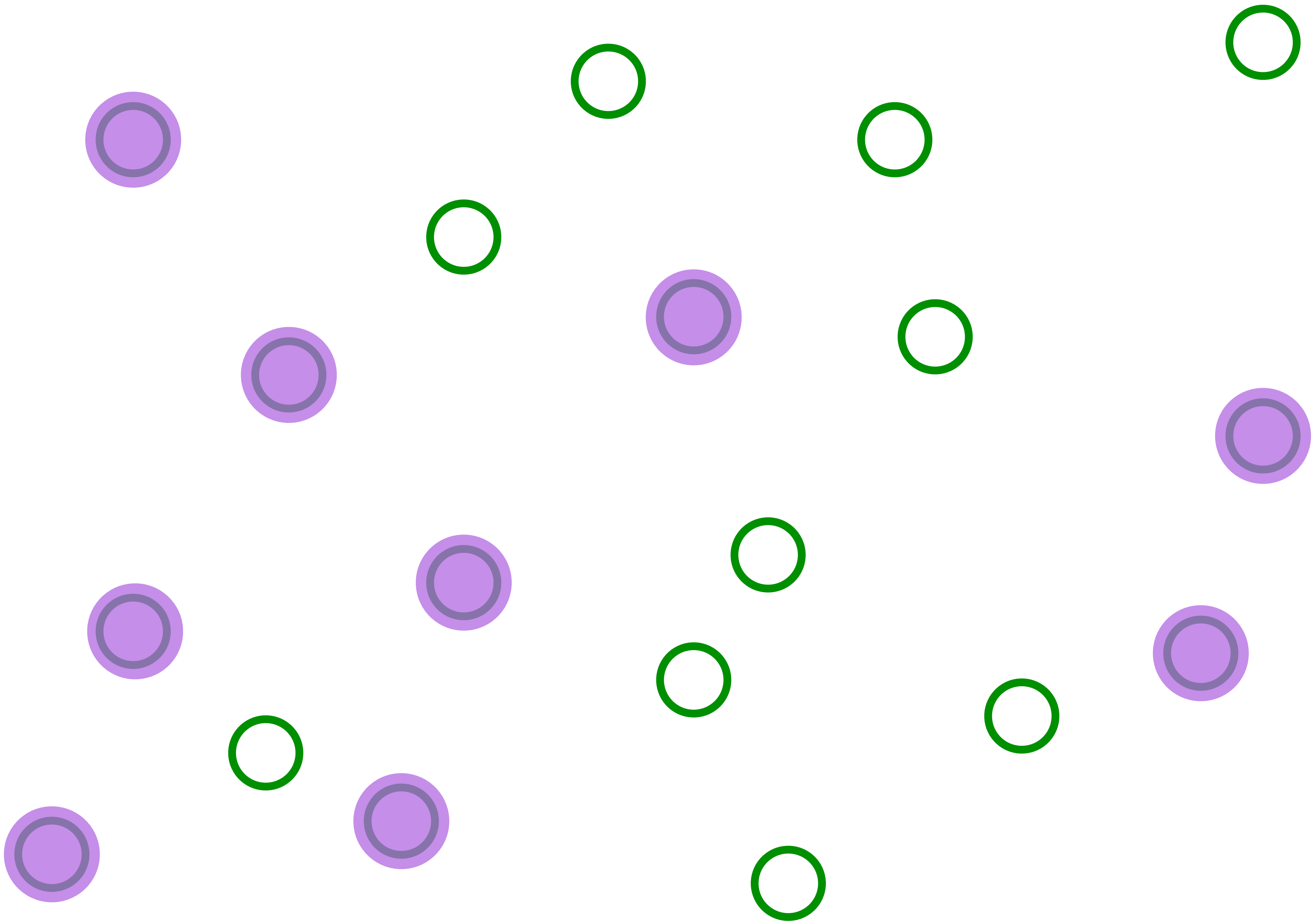

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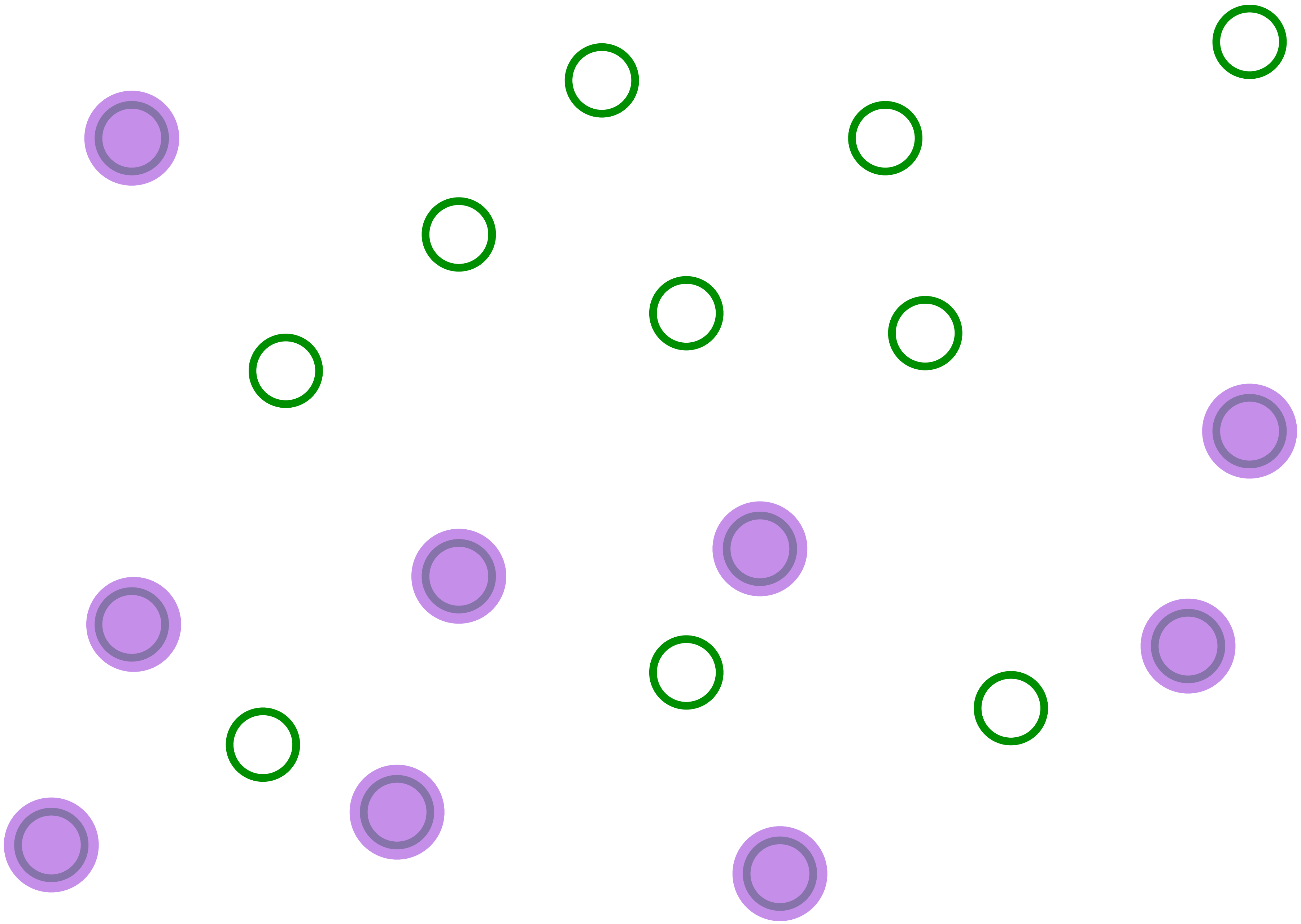

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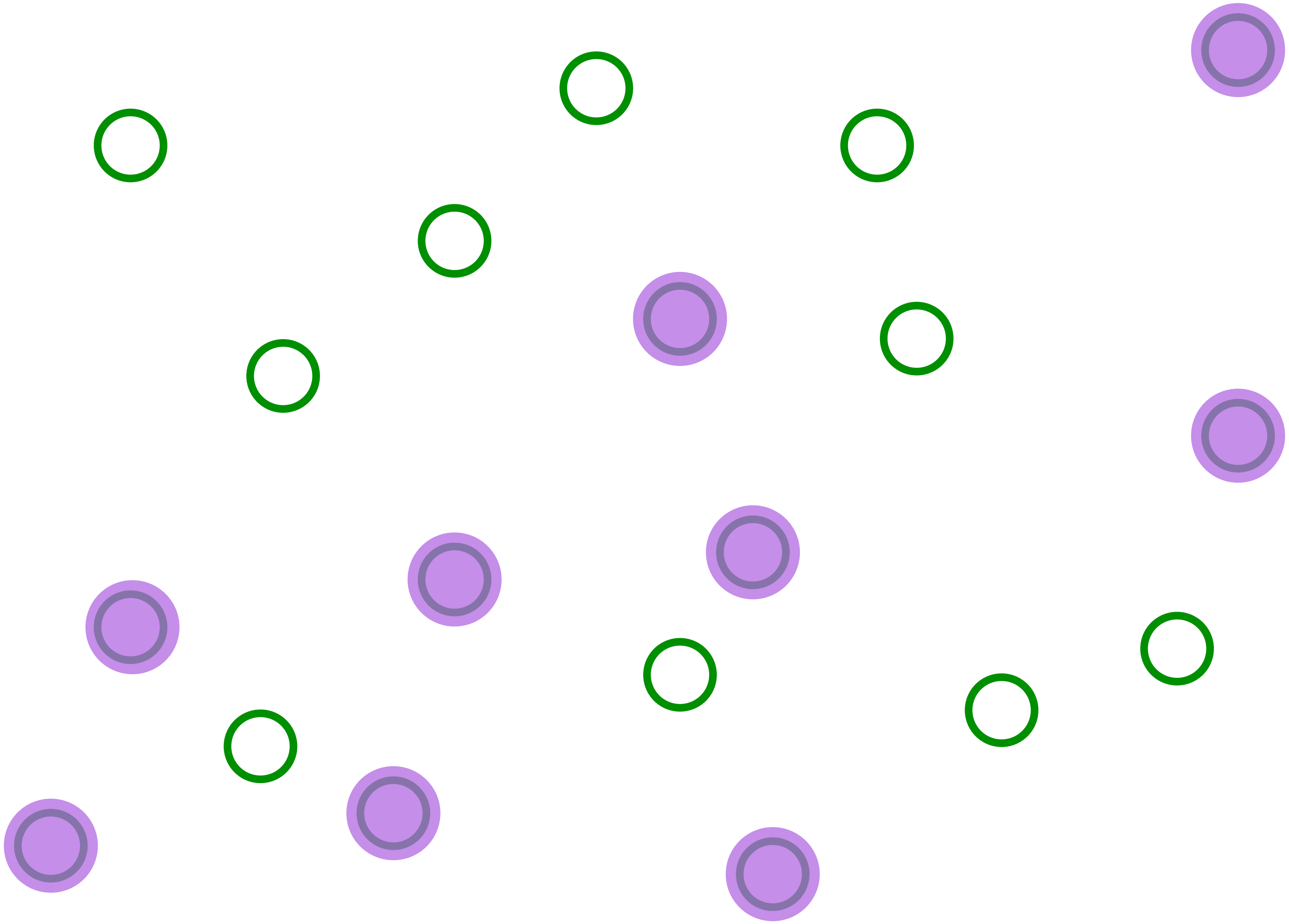

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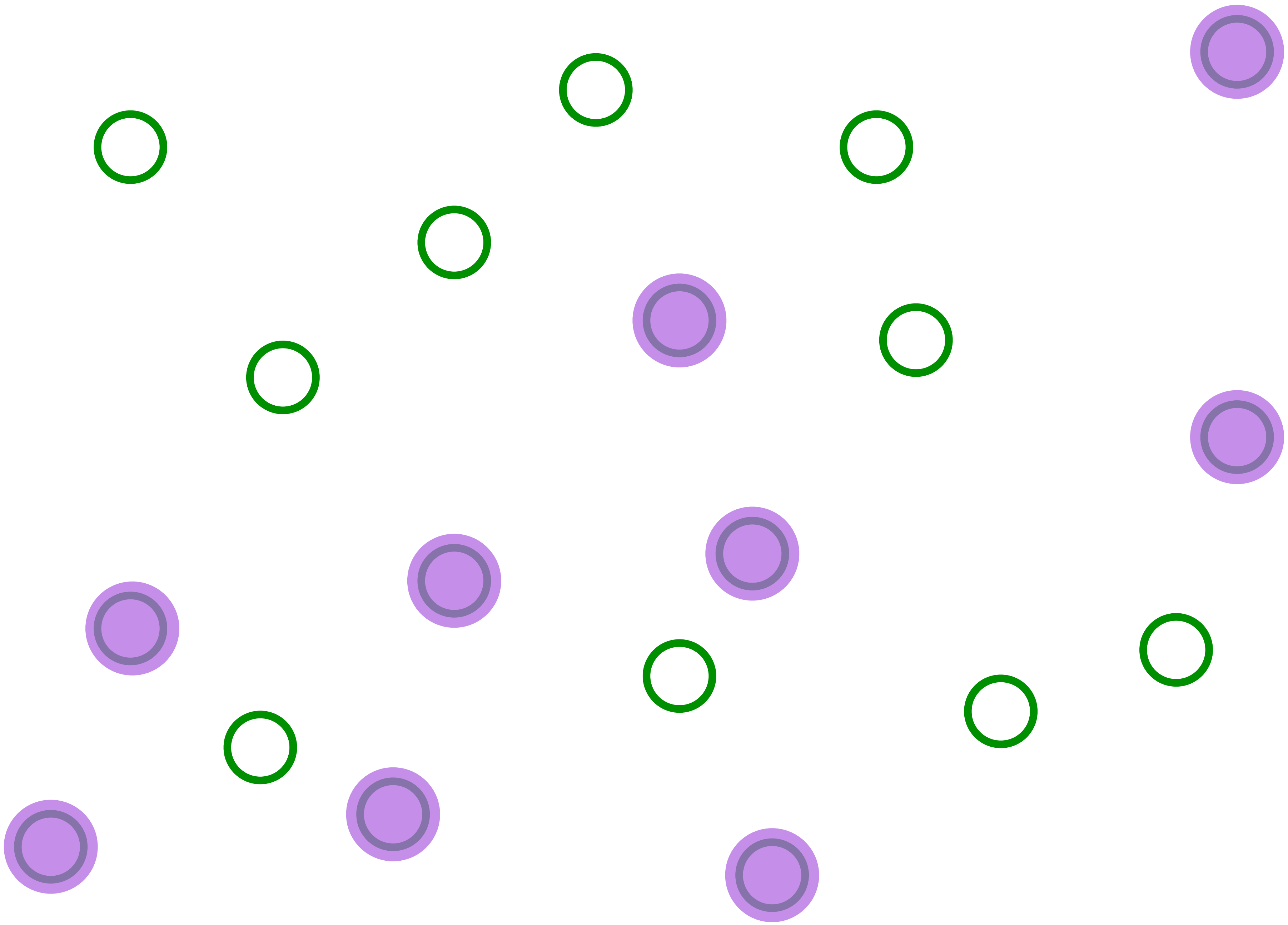


This describes both a strange metal and a black hole!

The Sachdev-Ye-Kitaev (SYK) model

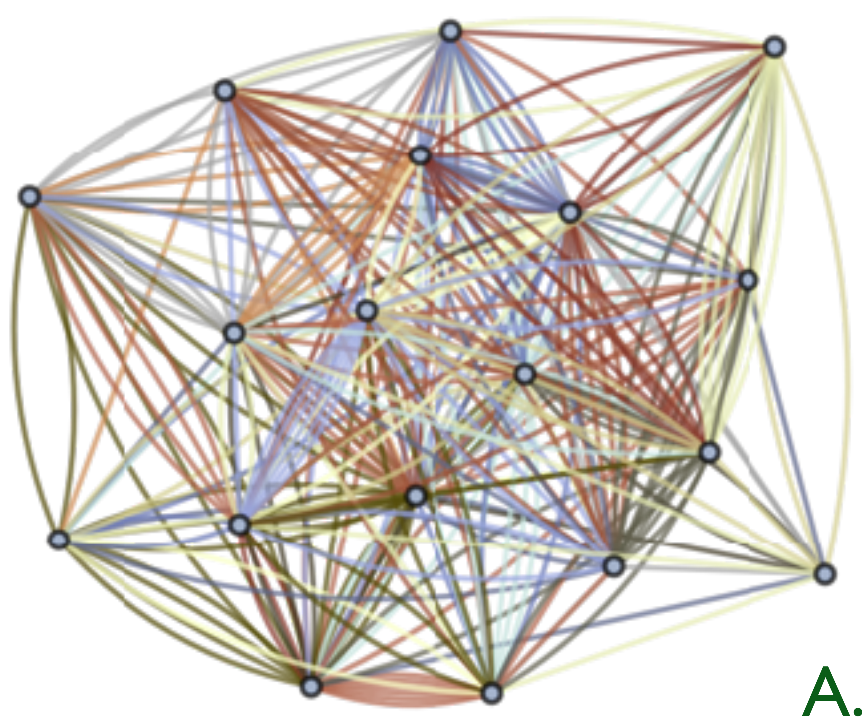
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

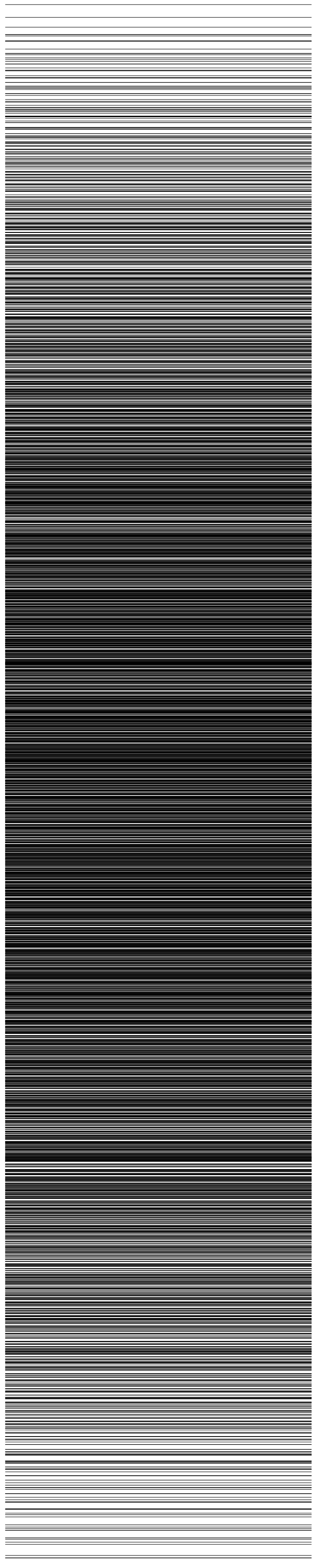
$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The Sachdev-Ye-Kitaev (SYK) model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

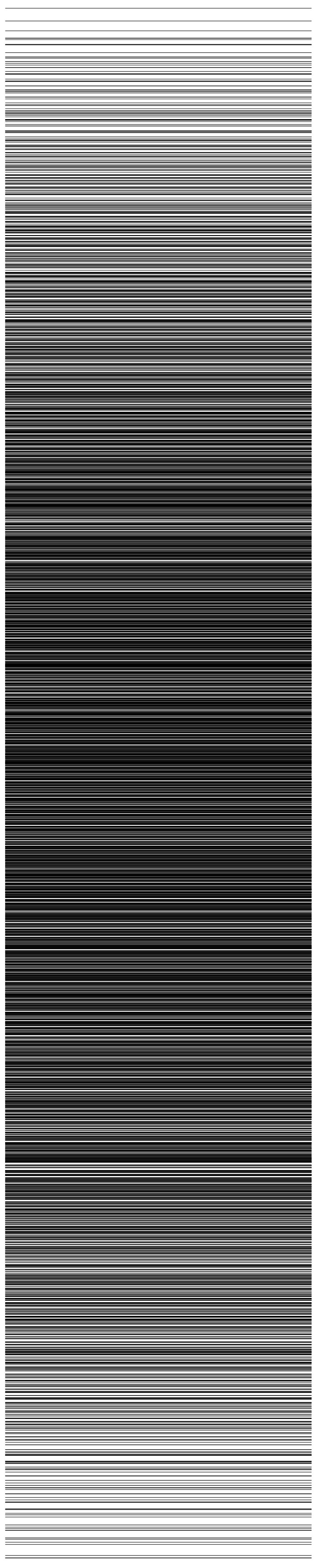
$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The Sachdev-Ye-Kitaev (SYK) model



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$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

$$< \ln 2$$

No quasiparticles !

$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha$

$+ \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

ev,

PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The Sachdev-Ye-Kitaev (SYK) model

- Low energy, many-body density of states
  $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)N\gamma})$ (sinh factor is for Majorana version)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

D. Stanford and E. Witten, 1703.04612

A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593

D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

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- Low temperature entropy $S = N s_0 + N\gamma T + \ldots$.

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

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- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ.

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- $T > 0$ Green's function has conformal invariance
  $G \sim (T / \sin(\pi k_B T \tau / \hbar))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The Sachdev-Ye-Kitaev (SYK) model

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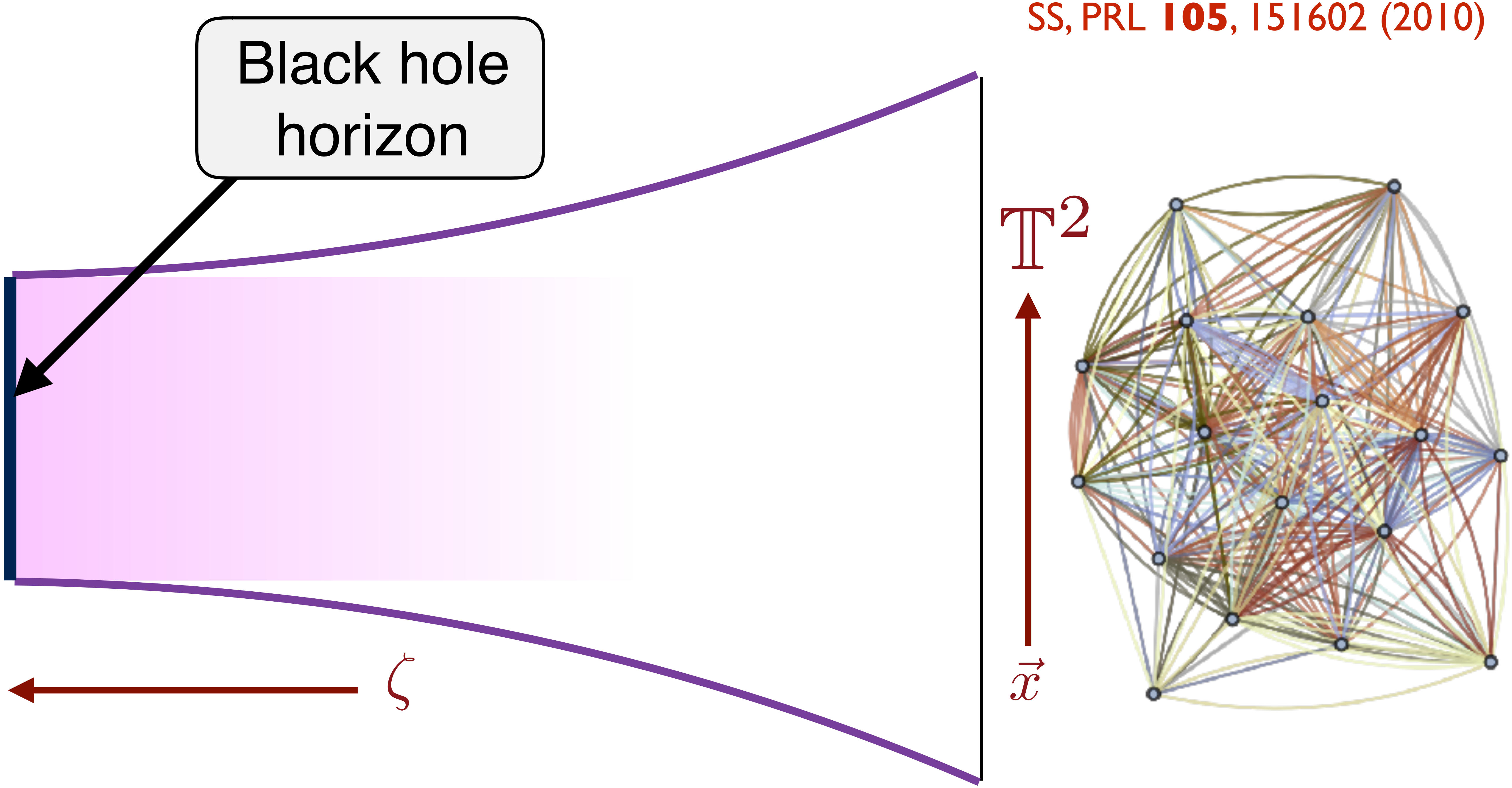
The SYK model is holographically dual to black holes with AdS2 horizons, which share the properties above

S. Sachdev, PRL **105**, 151602 (2010)

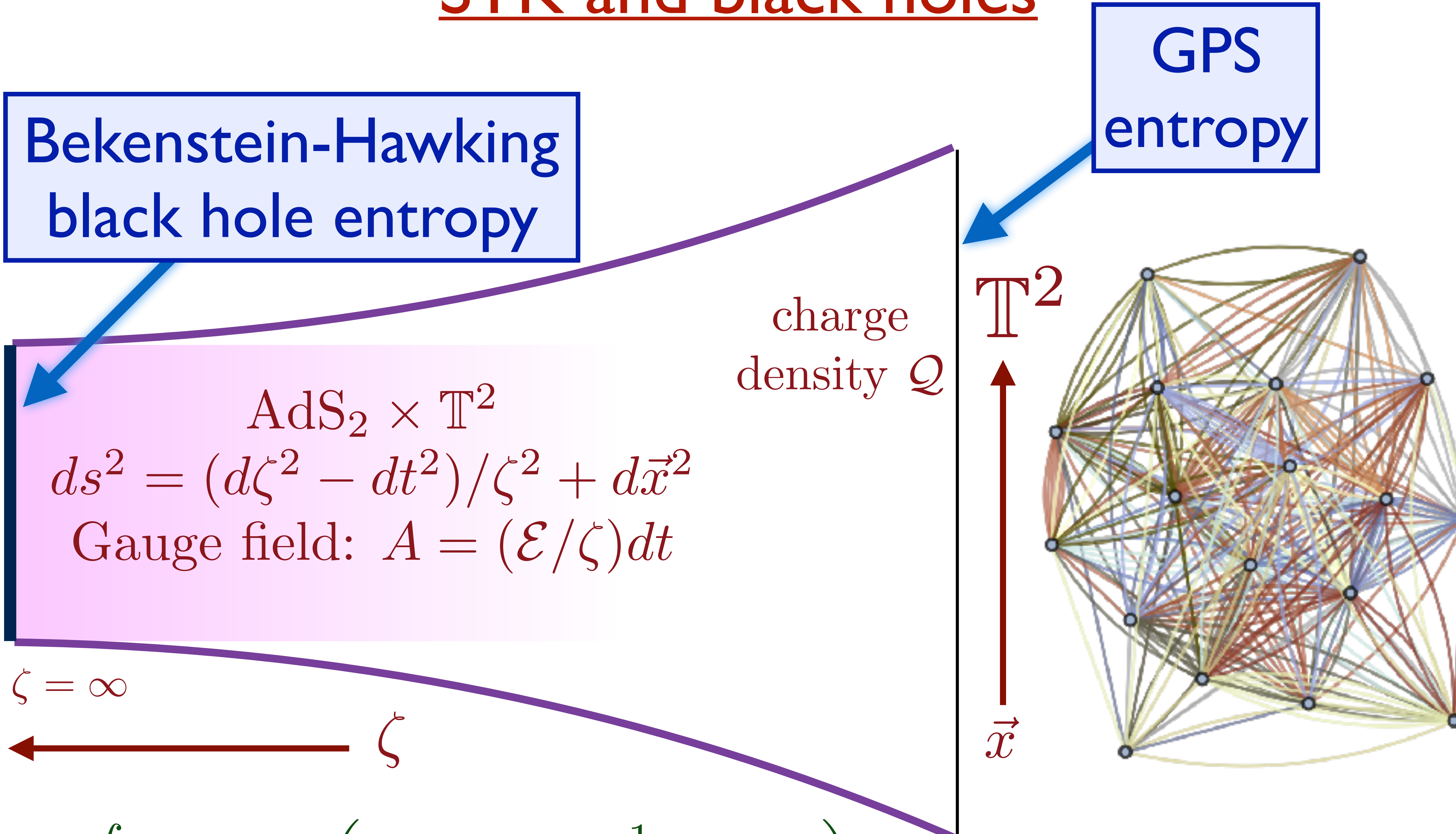
SYK and black holes

SS, PRL **105**, 151602 (2010)



The SYK model has "dual" description in which an extra spatial dimension, ζ, emerges. The curvature of this "emergent" spacetime is described by Einstein's theory of general relativity

SYK and black holes



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

SS, PRL **105**, 151602 (2010)

The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

SYK and black holes

Equilibrium dynamics described by a theory with SL(2,R) invariance, and effective Schwarzian action, $S[h(\tau)]$, of a time reparameterization $\tau \to h(\tau)$.

ζ

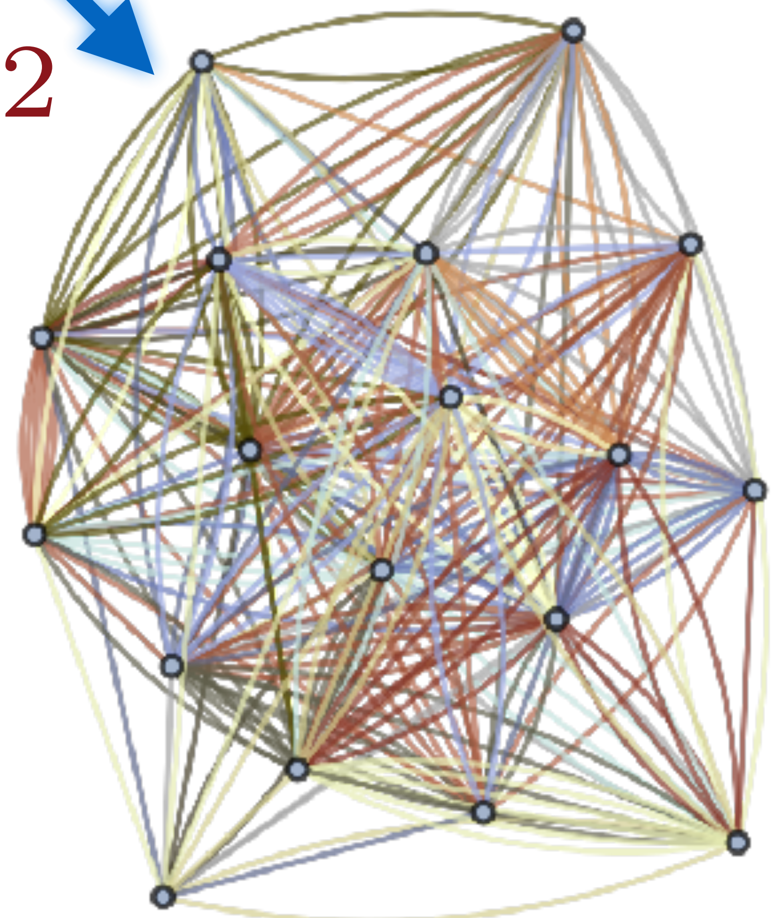
$\zeta = \infty$

$$\mathrm{AdS}_2 \times \mathbb{T}^2$$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

$\mathbb{T}^2$

$\vec{x}$



SL(2,R) is the isometry group of AdS_2: $ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Thermalization

- If we start the SYK model from a random initial state, it reaches thermal equilibrium in a time of order $\hbar/(k_B T)$. Note that this time is independent of the coupling energies in the Hamiltonian.

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Thermalization

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- If we perturb a black hole, its quasi-normal modes "ring", and decay to thermal equilibrium in a time of order $\hbar/(k_B T_H)$, where T_H is the Hawking temperature.

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- If we perturb a black hole, its quasi-normal modes "ring", and decay to thermal equilibrium in a time of order $\hbar/(k_B T_H)$, where T_H is the Hawking temperature.

- This relaxation/thermalization time, τ_φ, was conjectured to be the shortest possible among all many-body quantum systems

$$\tau_\varphi > C \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

Many-body quantum chaos

- In classical chaos, we measure the sensitivity of the position at time t, $q(t)$, to variations in the initial position, $q(0)$, *i.e.* we measure

$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\mathrm{P.B.}}\right)^2$$

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$$\left(\frac{\partial q(t)}{\partial q(0)}\right)^2 = \left(\{q(t), p(0)\}_{\text{P.B.}}\right)^2$$

- By analogy, we define τ_L as the <u>Lyapunov time</u> over which the wave-function of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x,t), \hat{B}(0,0)] \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left[t - \frac{|x|}{v_B}\right]\right),$$

where v_B is the 'butterfly velocity'. This time τ_L was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}.$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

Many-body quantum chaos

- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

S. Shenker and D. Stanford, 1306.0622
A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

Quantum matter without quasiparticles:

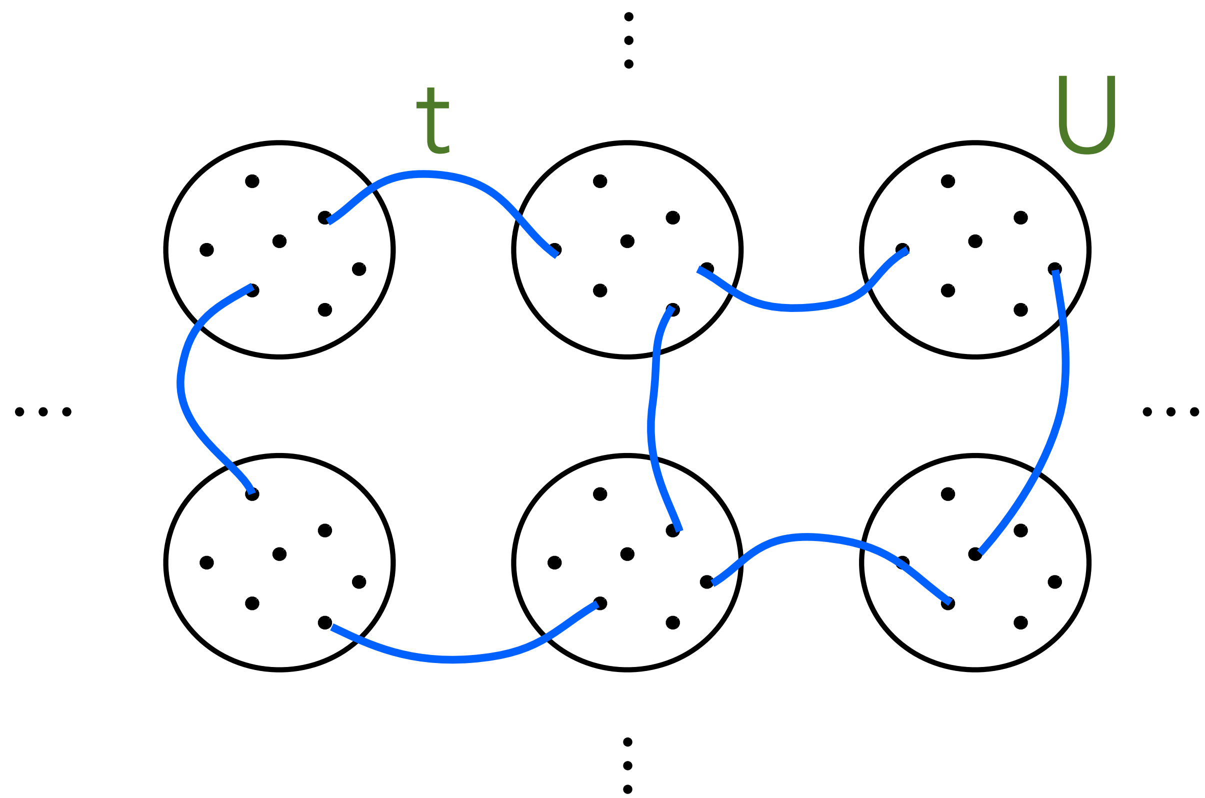
- No quasiparticle decomposition of low-lying states: $E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

arXiv:1705.00117

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N.$$

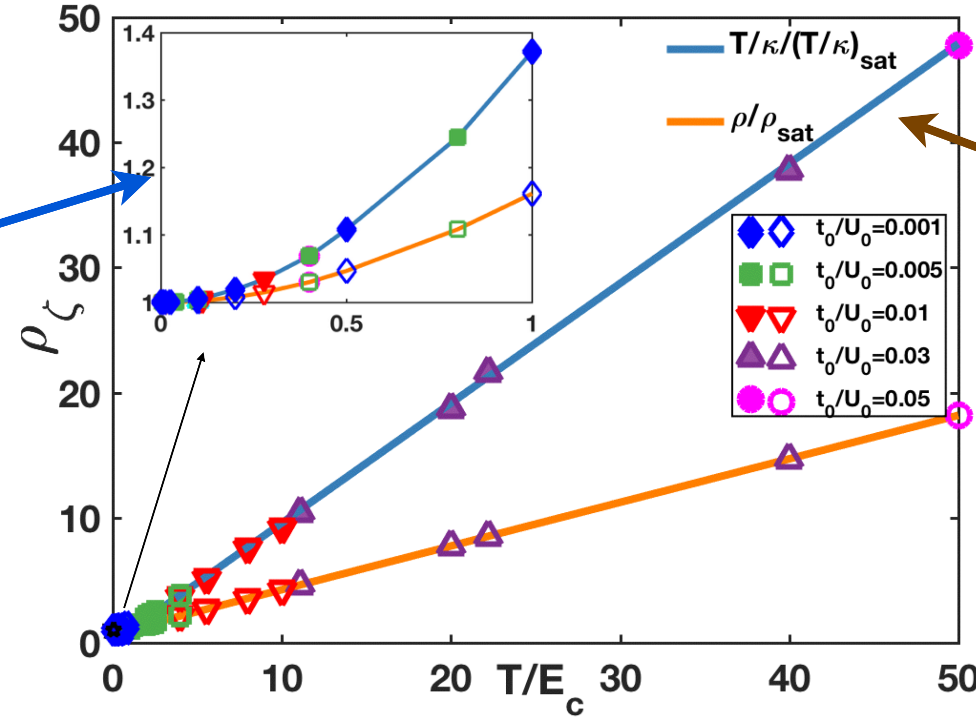
arXiv:1705.00117

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

Fermi liquid
$R=R_0+AT^2$
for $T \ll E_c$

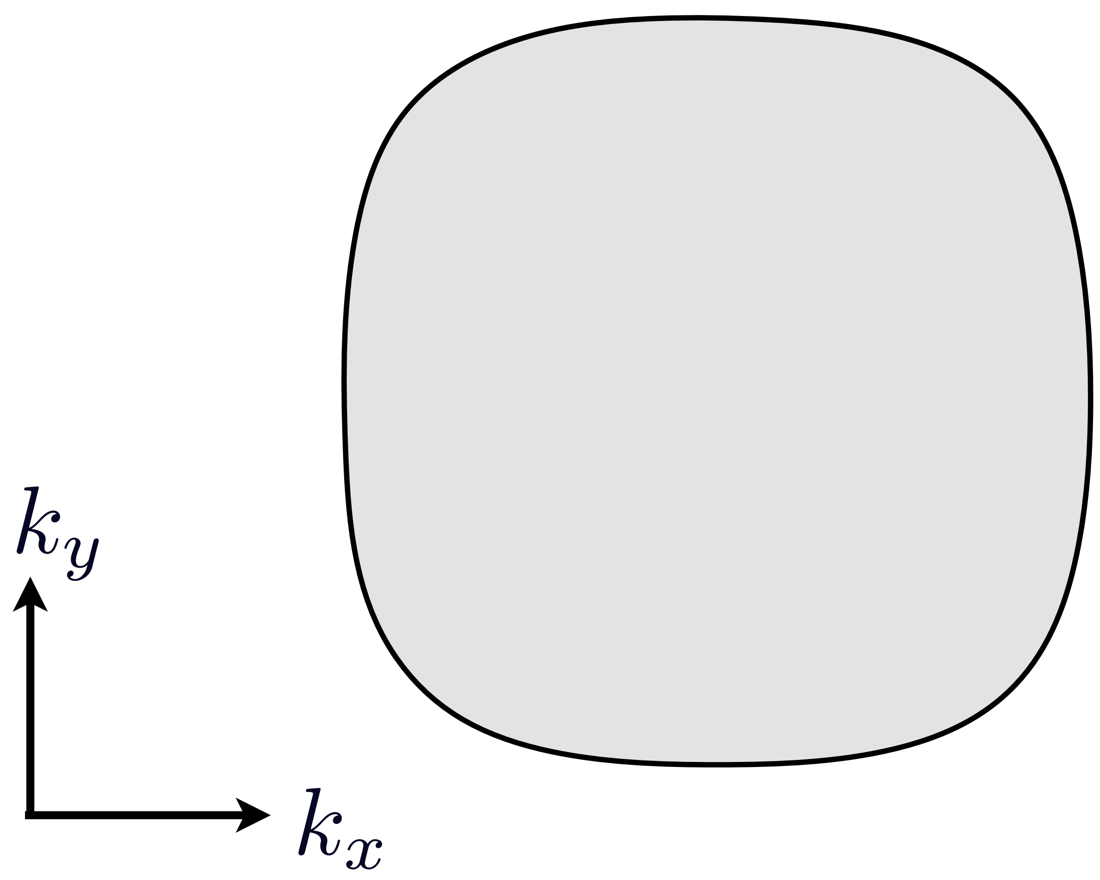


Linear in T for
$E_c \ll T \ll U$

Crossover from heavy FL to strange metal

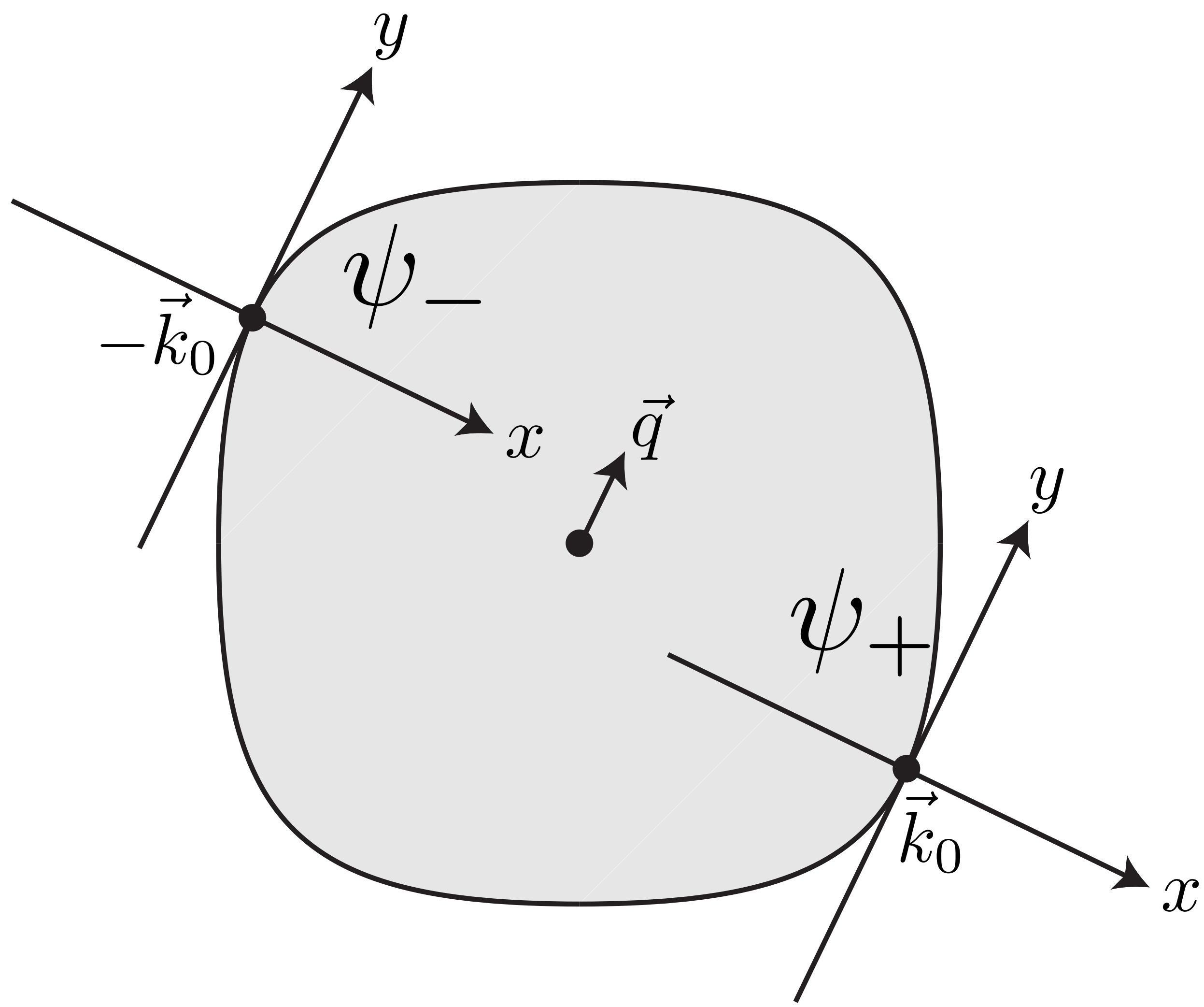
- Small coherence scale $E_c=t^2/U$
- Heavy mass γ~m*/m ~ U/t
- Small QP weight Z ~ t/U
- Kadowaki-Woods A/γ^2 = constant
- Linear in T resistivity and T/κ, $R \sim (h/e^2)(T/E_c)$
- Lorenz ratio crosses over from FL to NFL value

Fermi surface coupled to a gauge field



$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Fermi surface coupled to a gauge field



$$\mathcal{L}[\psi_\pm, a] =$$

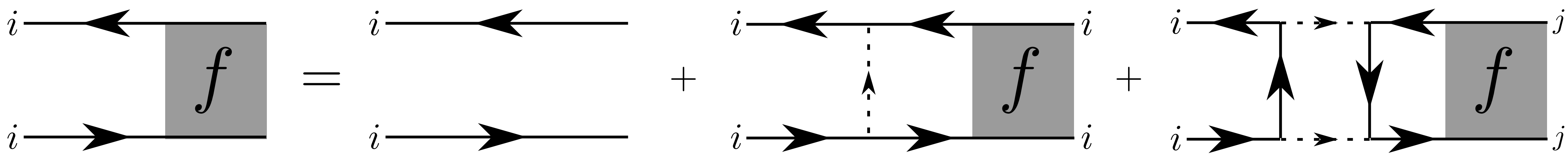
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



$$f(t) = \frac{1}{N^2}\theta(t)\sum_{i,j=1}^{N}\int d^2x \, \mathrm{Tr}\Big[e^{-\beta H/2}\{\psi_i(x,t),\psi_j^\dagger(0)\}$$

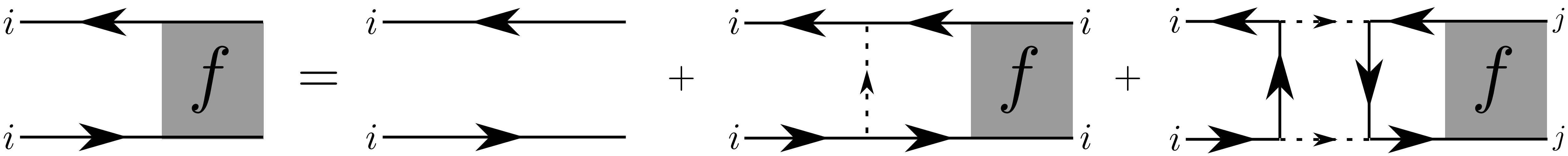
$$\times\, e^{-\beta H/2}\{\psi_i(x,t),\psi_j^\dagger(0)\}^\dagger\Big]$$

$$\sim \exp\Big((t - x/v_B)/\tau_L\Big)$$

A. A. Patel and S. Sachdev, PNAS **114**, 1844 (2017)

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



Strongly-coupled theory with no quasiparticles and fast scrambling:

$$\tau_L \approx \frac{\hbar}{2.48\, k_B T}$$

$$v_B \approx 4.1 \frac{N T^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$D_T = \frac{\text{thermal conductivity}}{\text{specific heat at fixed density}} \approx 0.42\, v_B^2 \tau_L$$

N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant. More generally, we find $D_T \sim v_B^2 \tau_L$ in a large number of holographic models

A. A. Patel and S. Sachdev, PNAS **114**, 1844 (2017); M. A. Blake, R. A. Davison, and S. Sachdev 1705.07896

Entangled quantum matter without quasiparticles

- Is there a connection between strange metals and black holes?
  Yes, the SYK model leads to an explicit duality mapping.

- Why do they have the same local equilibration time $\sim \hbar/(k_B T)$?
  Strange metals don't have quasiparticles and thermalize rapidly;
  General relativity leads to black hole quasi-normal modes,
  whose decay time $\sim \hbar/(k_B T_H)$,
  where T_H is the Hawking temperature".

- Theoretical predictions for strange metal transport in graphene agree well with experiments