Atomic Transparency of Bose Condensed Systems

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I. INTRODUCTION

- We have proposed (PRL 71, 2429 (1993)) an experiment to observe 'transparency' of a slab or droplet of superfluid helium to a helium atom incident on it.
- The origin of the predicted 'transparency' is the bose condensed character of the superfluid.
- To carry out the experiment we require

(1) Two parallel surfaces of superfluid separated by superfluid.

(2) Temperatures less than 0.4 ^{o} K.

(3) Maximum isolation of the superfluid from the thermal environment.

- To interpret the results we require a many body calculation of the transmission rate.
- Similar results can be expected in weakly coupled dilute Bose gases.

THE IDEA

One body density matrix:

$$\rho_1(\vec{r},\vec{r}\,') = N\int d\vec{r}_2 ... d\vec{r}_N \Phi^*(\vec{r},\vec{r}_2,...,\vec{r}_N)\Phi(\vec{r}\,',\vec{r}_2,...,\vec{r}_N)$$

ODLRO:

$$\lim_{|\vec{r}-\vec{r}'|\to\infty} \rho_1(\vec{r},\vec{r}') = \Psi_1^*(\vec{r})\Psi_1(\vec{r}\,')$$

For a uniform fluid in the absence of flow

$$\Psi_1(\vec{r}) = \sqrt{n_0\rho}$$

while in the presence of superfluid flow

$$\Psi_1(\vec{r}) = \sqrt{n_0\rho}e^{iS(\vec{r})}$$

and

$$\vec{v}_s = \hbar\nabla S/m$$

Rewrite

$$\rho_1(\vec{r},\vec{r'}) \equiv$$
$$= N < \Phi_N \mid \psi^\dagger(\vec{r})\psi(\vec{r'}) \mid \Phi_N >$$

In other words, consider the wave function

$$\psi^\dagger(\vec{r'}) \mid \Phi_N >$$

with one more particle at $\vec{r'}$. The existence of ODLRO says that this overlaps (at order $O(1/V)$) the wave function

$$\psi^\dagger(\vec{r}) \mid \Phi_N >$$

The wave function with one more particle at a far away point $\vec{r'}$.

This suggests a many body version of a tunneling experiment.

THE PROPOSED EXPERIMENT

- Pulses of very low energy ($\approx 1\,^o$ K) helium atoms are to be shot at a suspended film or droplet of superfluid helium.
- Helium atoms are expected to quickly emerge from the opposite side of the droplet after the pulse hits the droplet.
- Two processes contribute to this outgoing pulse: (i) Helium $\rightarrow$ roton $\rightarrow$ Helium ; (ii) Helium $\rightarrow$ condensate $\rightarrow$ Helium .

ENERGY CONSIDERATIONS

- To be more precise, the process of interest can be described as:

  One helium atom at z$\rightarrow -\infty$ with energy ϵ_k

  $+$

  N particle ground state of superfluid

  $\rightarrow$

  N+1 particle ground state of superfluid

  $\rightarrow$

  One helium atom at z$\rightarrow +\infty$ with energy ϵ_k

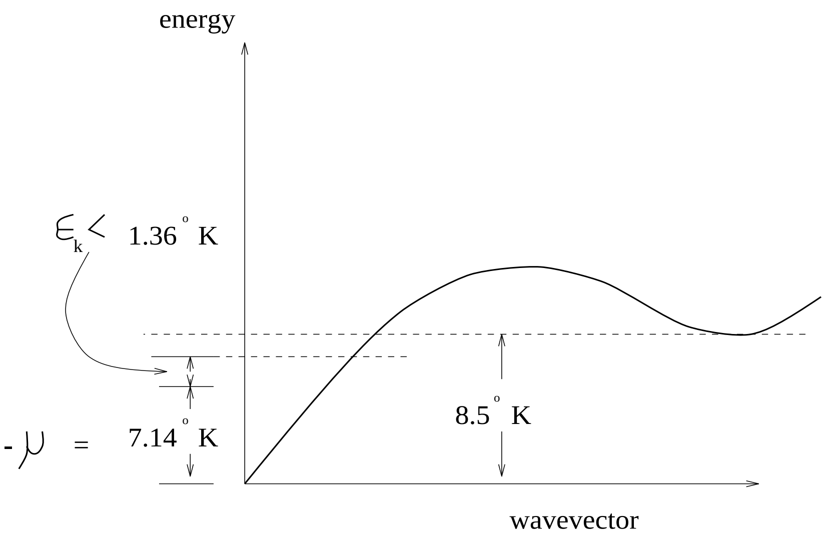
  $+$

  N particle ground state of superfluid

- Energies:

  Initial state: $\epsilon_k + E_N$

  intermediate state: E_{N+1}



Difference: Initial State - Intermediate State= $\delta E = E_N - E_{N+1} + \epsilon_k = -\mu + \epsilon_k$.

- For bosons $\mu < 0$. For free particles $\epsilon_k > 0$
- Conclusion: The process is virtual and the uncertainty principle gives a time delay $\delta t \approx \hbar/\delta E$.

- The figure also illustrates the energetic constraints on a competing process in which the intermediate state is one of the quasi-particle phonon roton spectrum of states:

  If $\epsilon_k < 1.36^o$ K, then only energy conserving excitation of a phonon state which is damped due to anomalous dispersion is possible.

  If $\epsilon_k > 1.36^o$K then excitations with a large density of states at the roton minimum can be in an energy conserving intermediate state.

  Virtual excitation of the phonon-roton spectrum is always possible. We have not discussed ripplons. At normal incidence, single ripplon creation is forbidden by momentum conservation.

UNCERTAINTY PRINCIPLE ESTIMATE OF THE TIME DELAY

Difference: Initial State - Intermediate State= $\delta E = E_N - E_{N+1} + \epsilon_k = -\mu + \epsilon_k$.

- For bosons $\mu < 0$. For free particles $\epsilon_k > 0$
- Conclusion: The process is virtual and the uncertainty principle gives a time delay $\delta t \approx \hbar/\delta E$.
- For helium four $\mu = -7.14^o$ K . For the planned experiment (see below) $\epsilon_k \approx 1.5^o K$ giving $\delta t \approx 10^{-12}$ sec independent of sample thickness of helium.
- For thicknesses larger than ≈ 2mm retardation effects must enter leading to a time delay thickness/c .

III. MANY BODY CALCULATIONS OF THE TIME DELAY

Review of Simplified Analysis (PRL 71, 2429 (1993)

- The coupling of the incoming and outgoing particles to the fluid is described by a kind of transfer Hamiltonian,

$$H' = \sum_{\vec{k}} (T^L_{\vec{k}} b_{\vec{k},L} a^\dagger_0 + T^R_{\vec{k}} b^\dagger_{\vec{k},R} a_0 + h.c.) \qquad (1)$$

  $b^\dagger_{\vec{k},L,R}$ create particles coming in from the left and leaving to the right respectively and

  $a^\dagger_0$ create particles in the fluid condensate.

- For process (ii) (Helium $\rightarrow$ condensate $\rightarrow$ Helium) the Fermi Golden rule to second order in H gives a rate of

$$\frac{2\pi}{\hbar} N_0^2 \sum_{\vec{k}'} \frac{|T^L_{\vec{k}} T^R_{\vec{k}'}|^2}{(\epsilon_{\vec{k}} - E_{N+1} + E_N)^2} \delta(\epsilon_{\vec{k}} - \epsilon_{\vec{k}'}) \qquad (2)$$

  E_N is the fluid ground state energy when the fluid has N particles and N_0 is the number of particles in the condensate (assumed $>> 1$).

- Note the factor N_0^2 which arises from stimulated absorption and emission to and from the condensate.

- T will scale as

$$T = t\sqrt{\frac{\lambda A}{V}}\sqrt{\frac{\lambda A}{L_{outside} A}} = \frac{t\lambda}{\sqrt{L_{outside}}}\sqrt{\frac{A}{V}} \quad (3)$$

  - ⋄ A is the cross-sectional area of the incoming beam of helium atoms,
  - ⋄ λ is a length of the order of the width of the interface,
  - ⋄ V is the entire volume of the liquid helium sample
  - ⋄ $L_{outside}$ is the length normal to the interfaces of the the cylindrical volume in which the wavefunctions of the incoming and outgoing beams are normalized.
  - ⋄ t is a matrix element with the dimensions of energy and independent of the macroscopic dimensions..

- Conclusion:

$$\frac{condensate\ mediated\ transmission\ probability}{roton/phonon\ mediated\ transmission\ probability} \propto A^2$$

Calculation of the Matrix Element (P. T. Boinske and J. W. Halley)

- Incoming wave: phase shifted wave incident on a fixed step function density distribution and interacting with it with a 6-12 potential integrated over the distribution. (6-12 potential from Hirschfelder, Curtiss and Bird)
- Condensate wave function $\sqrt{\rho(r)n_O(r)}$ where ρ and n_O are taken from Krotschek, Phys. Rev. B 32, 5713(1985)
- Result: The perturbation theory predicts one outgoing particle for every incoming particle when

$$\left[\frac{A}{\pi}\right]^{1/2} \approx 10\mathring{A}$$

- For areas larger than this, the lowest order calculation IS NOT ADEQUATE.

MANY BODY VARIATIONAL FORMULATION (PRL 79, 3930 (1997)

- Since we are not dealing with the ground state in this scattering problem the variational problem must be carefully formulated.
- We minimize

$$G = \frac{<\Psi_{N+1} \mid (H - E_N - \epsilon_{\vec{k}})^2 \mid \Psi_{N+1}>}{(E_n + \epsilon_{\vec{k}})^2}$$

$$+ \int dz W(z) \left(\frac{<\Psi_{N+1} \mid J(z) \mid \Psi_{N+1}>}{J_0} - 1 \right)^2$$

The variational function:

$$\Psi_{N+1} = \sum_s \left[f(z_s) e^{ikz_s} + r(z_s) e^{-ikz_s} \right] e^{-(\frac{1}{2}) \sum_{j \neq s} u_2(j,s)}$$

$$\times \Psi_N^{(o)} (\vec{r}_1 - \vec{r}_{cm,s}, \ldots \vec{r}_{s-1} - \vec{r}_{cm,s}, \vec{r}_{s+1} - \vec{r}_{cm,s}, \ldots, \vec{r}_{N+1} - \vec{r}_{cm,s})$$

$$+ \lambda e^{ik' z_{cm}} \Psi_{N+1}^{(o)} (\vec{r}_1 - \vec{r}_{cm}, \ldots \vec{r}_{N+1} - \vec{r}_{cm})$$

in which

$$f(z_s) = \frac{1}{e^{b'(z_s - z_{cm,s} + a'/2)} + 1} + \frac{T}{e^{b'(-z_s + z_{cm,s} + a'/2)} + 1}$$

$$r(z_s) = \frac{R}{e^{b'(z_s - z_{cm,s} + a'/2)} + 1}$$

and $z_{cm} = \frac{1}{N+1} \sum_s z_s$, $\mid R \mid^2 = 1 - \mid T \mid^2$
and $z_{cm,s} = \frac{1}{N} \sum_{s' \neq s} z_{s'}$.

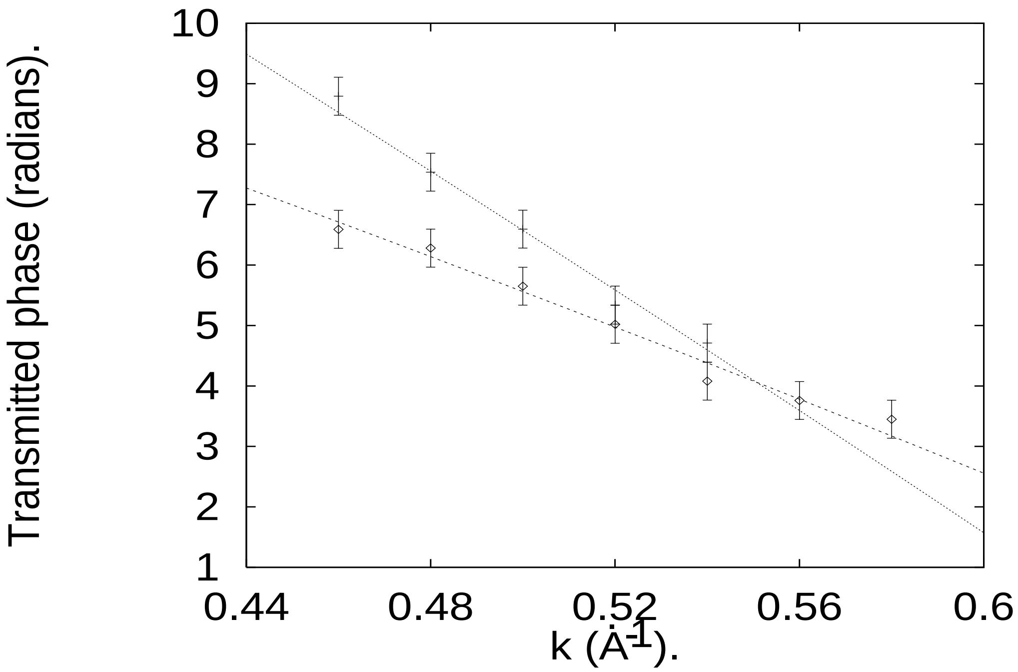
Some Details:

- The N body and $N+1$ body wave functions are confined by a harmonic potential. This is both physically realistic and computationally convenient.
- These many body wave functions are defined by a Jastrow function with one body factors u_1 defining the surface structure and two body factors u_2 to take account of correlations. We are using a bulk form for u_2 taken from MacMillan and a form for u_1 taken from earlier work on slabs by Krotschek and collaborators (E. Krotschek, G.-X. Qian and W. Kohn, Phys. Rev. B32, 5713 (1985))
- Calculations are done for 66 particles. Periodic boundary conditions in the directions parallel to the slab. Slab is 12.2 Å by 12.2 Å by about 20 Å.
- The variational parameters are ϕ,ϕ ′(the phases of T and R),$\mid T \mid$, a', b' , and k'.

- The variational procedure is to rewrite the variational function a form which separates out the factors depending on the parameters ϕ,ϕ $'$ and $\mid T \mid$. For fixed a',b' and k' one then calculates the coefficients in the last equation as many body integrals by Monte Carlo methods and then finds the extrema with respect to ϕ,ϕ $'$,$\mid T \mid$, and k' analytically at that point a',b'. Finally, one finds the global minimum by repeating the procedure for different a',b'
- The procedure turns out to be sensitive to the weighting of the current and energy in the variational function. We show some results below.

- We show results below for two slabs slab 20 Å thick (value of a) with 66 particles and 40 Åwith 132 particle by 20Åx 20 Å in cross section (periodically continued). The calculations required 20 cpu hours per k point on the JPL Galaxy Cray for the 20 Åthick slab .
- We obtain the delay time for transmission between wavepacket incidence on the slab and condensate mediated re-emission from the wave vector dependence of the transmited phase ϕ_k using the relation $t = \frac{(a + \frac{d\phi}{dk})}{v_g}$ where v_g is the group velocity of the incident wavepacket.

- If the delay time is independent of thickness, the negative slope $\frac{d\phi}{dk}$ must be twice as large if the sample thickness doubles. This expectation is born out by the calculations to within a few percent.
- The delay time calculated from the slope is a few picoseconds, consistent with the simple uncertainty principle arguments discussed above.



Optimum variational parameters sets for various wavevectors.

k($\mathring{A}^{-1}$)	G	$\vert T \vert$	ϕ
0.46	0.0867	0.65	6.59
0.48	0.0868	0.65	6.28
0.50	0.0862	0.55	5.65
0.52	0.0861	0.65	5.02
0.54	0.0864	0.55	4.08
0.56	0.0916	0.55	3.76
0.58	0.0975	0.55	3.45

III. WEAK COUPLING

Motivations:

- To clarify the physics in a controlled calculation.
- To predict the results of a possible experiment on dilute Bose condensed alkali metal gases.

Model:

- Confining well is a spherically symmetric square well of finite depth.
- Scattering length approximation for interactions.

Approximations:

- Gross Pitaevski equation for the condensate wave function.
- Bogoliubov equations for the bound and scattering states.

Gross Pitaevski equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_1 - \mu + N_0 g \mid \psi_0 \mid^2\right]\psi_0 = 0$$

Eigenvalue is chemical potential.

Bogoliubov equations. Write

$$\psi(\vec{r},t) = \psi_0(\vec{r},t) + \delta\psi(r,t)$$

$$\delta\psi(\vec{r},t) = \sum_l \left[U_l(\vec{r}) b_l e^{-i\omega_l t} + V_l(\vec{r}) b_l^\dagger e^{i\omega_l t}\right]$$

then

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_1 - \mu + 2N_0 g\psi_0^2 - \hbar\omega_l\right] U_l(\vec{r},t) + N_0 g\psi_0^2 V_l = 0$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_1 - \mu + 2N_0 g\psi_0^2 + \hbar\omega_l\right] V_l(\vec{r},t) + N_0 g\psi_0^2 U_l = 0$$

Solution of Bogoliubov Equations

$$\psi_0(r) = u_0(r)/r$$

$$U_l = Y_{lm}(\hat{r}) u_l(r)/r$$

$$V_l = Y_{lm}(\hat{r}) v_l(r)/r$$

$$V_1 = -V_0/(1. + e^{(r-a)/\sigma})$$

$$g = 4\pi a_s \hbar^2/2m$$

$$\hbar\omega_0 = \hbar^2/2ma^2$$

$$\frac{d^2 u_l}{dr^2} - \left[l(l+1)/r^2 + 8\pi N_0 a_s \psi_0^2 - \frac{\hbar\omega_l + \mu}{\hbar\omega_0 a^2} \right] u_l - 8\pi N_0 a_s \psi_0^2 v_l = 0$$

$$\frac{d^2 v_l}{dr^2} - \left[l(l+1)/r^2 + 8\pi N_0 a_s \psi_0^2 - \frac{-\hbar\omega_l + \mu}{\hbar\omega_0 a^2} \right] v_l - 8\pi N_0 a_s \psi_0^2 v_l = 0$$

Boundary conditions:

If $\hbar\omega_l > \mid \mu \mid$ (continuum):

$$u_l \rightarrow r j_l(kr + \delta_l(k)) \ \text{as r} \rightarrow \infty$$

$$v_l(r) \rightarrow constant \ \ x \ \ e^{-\kappa r} \ \text{as r} \rightarrow \infty$$

$k = \sqrt{\frac{2m(\hbar\omega_l + \mu)}{\hbar^2}}$

$\kappa = \sqrt{\frac{2m(\hbar\omega_l - \mu)}{\hbar^2}}$

If $\hbar\omega_l < \mid \mu \mid$ (bound states)

$$u_l(r) \rightarrow constant \ \ x \ \ e^{-\kappa' r} \ \text{as r} \rightarrow \infty$$

$$v_l(r) \rightarrow constant \ \ x \ \ e^{-\kappa r} \ \text{as r} \rightarrow \infty$$

$\kappa' = \sqrt{\frac{2m(-\hbar\omega_l - \mu)}{\hbar^2}}$

at the origin, $u_l, v_l \propto r^{l+1}$

RESULTS

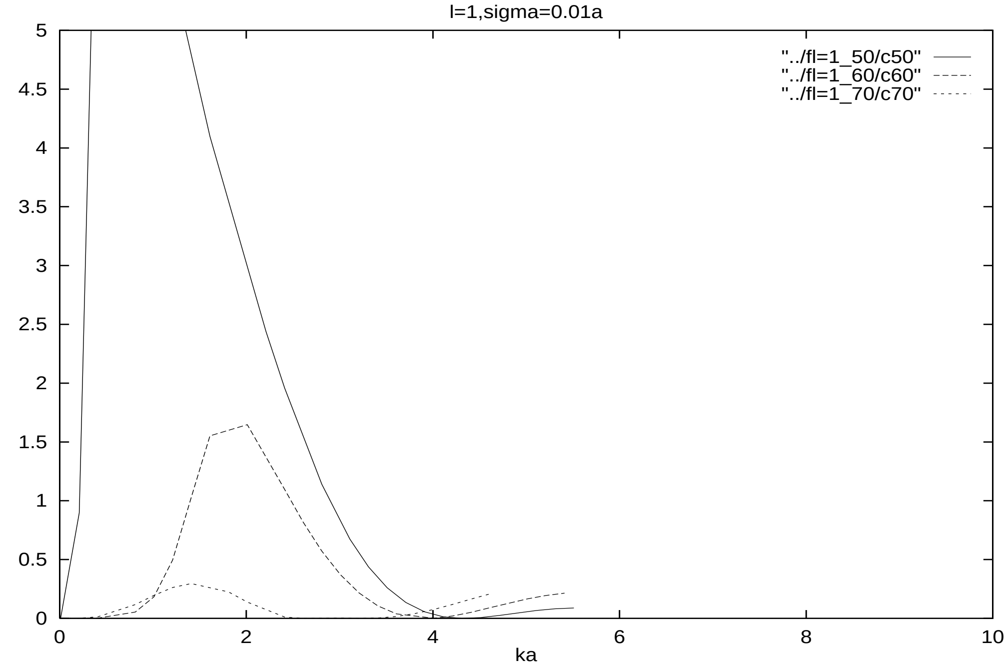
Phase shifts are determined from requirement that

$$u_k(r) \propto sin(kr - l\pi/2 + \delta_l(k)) \mathrm{as\,r} \to \infty$$

crosssections from

$$\sigma_l(k) = 4\pi(2l+1) sin^2 \delta_l(k)/k^2$$

We show results for a series of scattering lengths below. N=1000 , a = 14,000 Å, well depth = 38.44 $\hbar\omega_0$, m=85 amu


l=1,sigma=0.01a
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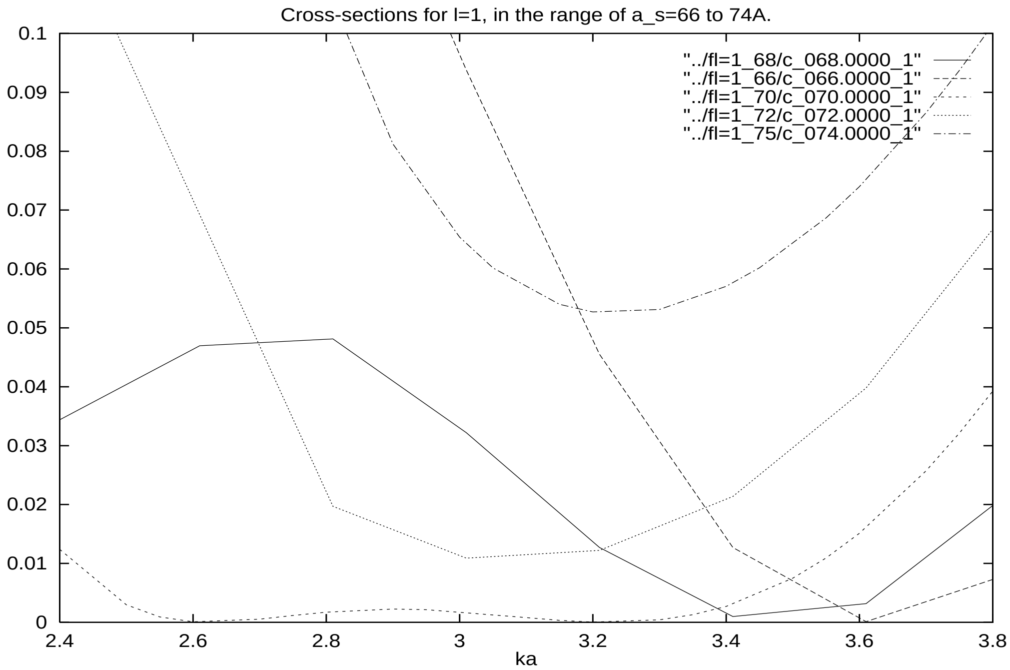
"../fl=1_60/c60"
"../fl=1_70/c70"
ka


Cross-sections for l=1, in the range of a_s=66 to 74A.
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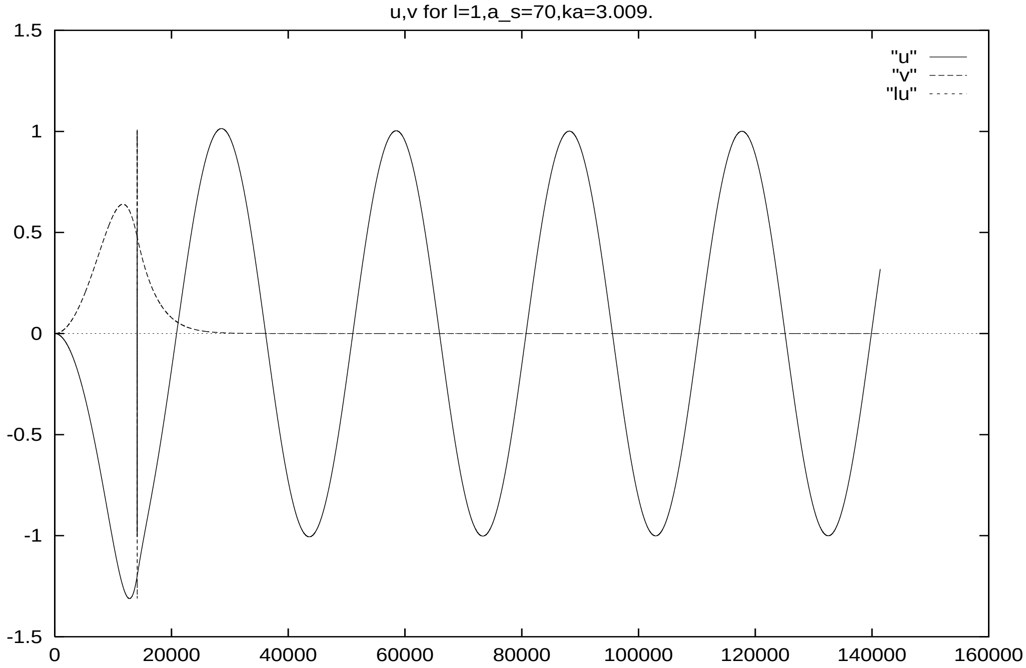
"../fl=1_66/c_066.0000_1"
"../fl=1_70/c_070.0000_1"
"../fl=1_72/c_072.0000_1"
"../fl=1_75/c_074.0000_1"
ka


u,v for l=1,a_s=70,ka=3.009.
"u"
"v"
"lu"
1.5
1
0.5
0
-0.5
-1
-1.5
0
20000
40000
60000
80000
100000
120000
140000
160000

THEORETICAL SUMMARY

Strong coupling:

- Variational calculations are giving reasonable currents and energies and substantial transmission coefficients.
- The wave function needs further refinement to get better behavior of the current near the surfaces.
- We now have our first quantitative estimates of the time delay. They are consistent with the uncertainty principle argument and with the argument that the delay should be independent of sample thickness for small enough thickness.
- The way to add the roton mediated process to this calculation is quite clear and will be done next.

Weak coupling:

- We see an enhanced Ramsauer-Townsend resonance at selected energies and coupling constants which seems to correspond to the transparency expected on the basis of our heuristic arguments.
- We established that it is not present in the absence of exchange with the condensate through the Bogoliubov v_k .
- The coupling constant dependence is not fully understood.

IV. HELIUM EXPERIMENT :SUSPENDING A HELIUM SLAB

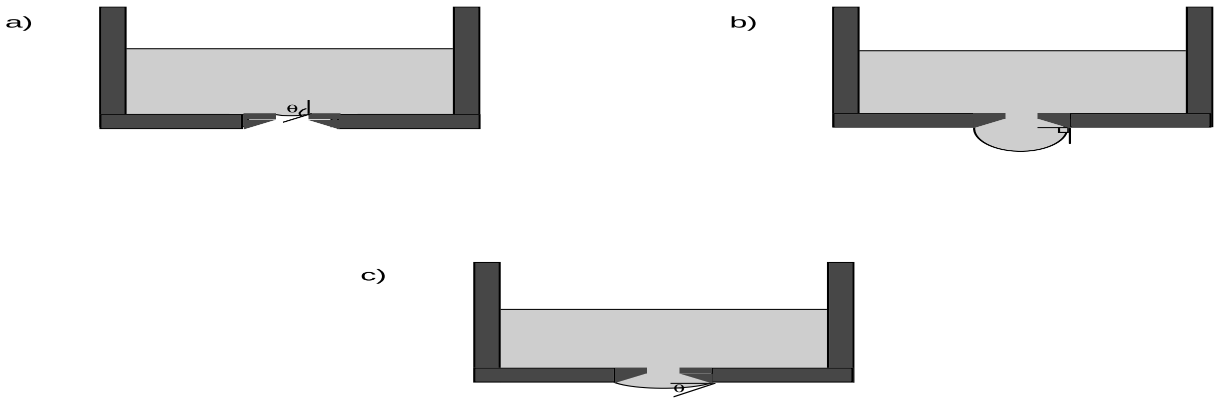
- Methods of achieving the required parallel interfaces include:
- Droplet suspension:
  * Magnetic suspension: Requires a large field gradient. (21 $(\text{Tesla})^2/\text{cm.}$) (See poster)
  * Electric suspension: Requires charging the droplet.
  * Microgravity: suspension becomes much easier.

OR

- Small, cesiated orfices: The surface tension allows suspension of 2mm of fluid above a fairly substantial hole ($\approx 100\mu$ under suitable conditions.) We report progress on this next (M. Williams, C. Giese and J. W. Halley, Physical Review B53, 6627 (1996))

EXPERIMENTAL DESIGN AND PREDICTED DEPTH

Conceptual design (showing the actual shape of the hole):



The hole is in a platinum disc, and is 70 μm at the top and $100\mu m$ at the bottom.

PREDICTION OF CRITICAL HEIGHT:

(hole of radius top radius r_t; bottom radius r_b; helium film depth h)

With the a hole of the indicated shape, we consider three possible modes of suspension as indicated.

To compute the critical depth in each case we use the Laplace equation

$$P_l = \rho g h = 2\sigma_{lg}/R$$

and the Young relation

$$\sigma_{sg} = \sigma_{lg} \cos\theta + \sigma_{sl}$$

Case a. $\left(\theta > \pi/2\right)$

$$h = \frac{2\sigma}{g\rho r_t} \mid \cos\theta \mid$$

Case b. $\left(\theta = \pi/2\right)$

$$h = 2\frac{\sigma_{lg}}{\rho g r_b}$$

Case c. $\left(\theta < \pi/2\right)$

$$h = 2\frac{\sigma_{lg}}{\rho g r_b} \mid \sin\theta \mid$$

We are certain that case c. is the relevant one, despite some some early reports of $\theta > \pi/2$.

PREDICTIONS OF THE CRITICAL HEIGHT.

We invert the Young equation

$$\sigma_{sg} = \sigma_{lg} \cos\theta + \sigma_{sl}$$

giving, since $\theta(T_W) = 0$ at the wetting temperature

$$\sigma_{sg} - \sigma_{sl} = \sigma_{lg}(T_W)$$

Assuming that $\sigma_{sg} - \sigma_{sl}$ is temperature independent

$$\cos\theta(T) = \frac{\sigma_{lg}(T_W)}{\sigma_{lg}(T)}$$

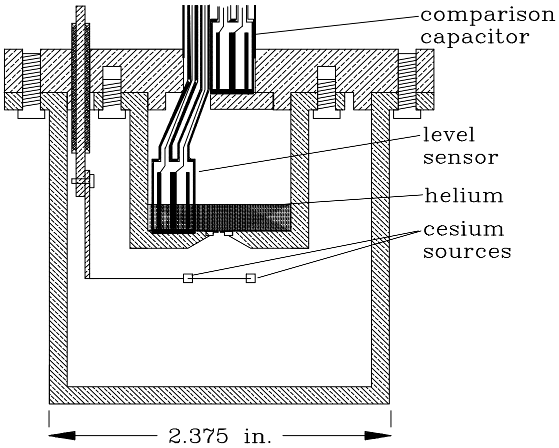
from which , for case c, we predict the critical height assuming $T_W = 2^o K$.

DETERMINATION OF THE CONTACT ANGLE FROM THE CRITICAL HEIGHT

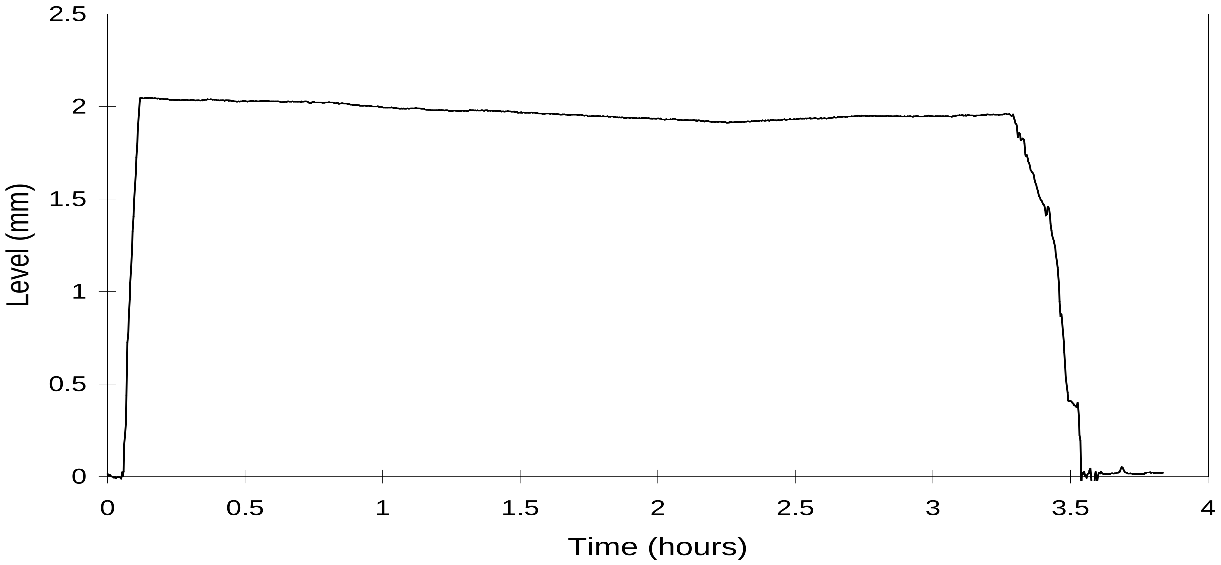
Obviously we can invert the above analysis to determine a contact angle if the critical height is determined experimentally (as we have done as described below). Specifically:

$$\theta = \sin^{-1}\left(\frac{\rho g h_C}{2\sigma_{lg}}\right)$$

EXPERIMENTAL DESIGN



- Levels measured by capacitance bridge.
- The capacitance bridge is accurate of 0.05mm in liquid depth.



Liquid level as a function of time in an experiment in which suspension was observed at a height of 2mm. From this we deduce a contact angle of $24.5\,^{o}\pm 1.3^{o}$ at 1.2 K in this experiment.

COMMENTS

- Suspension has been reproduced in many runs and is fully reproducible.
- The method can be used to measure the contact angle as a function of temperature.

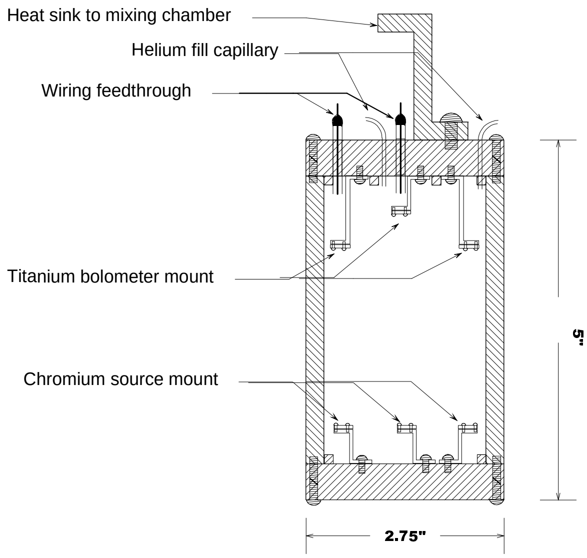
DESIGN OF A NEW TRANSMISSION EXPERIMENT

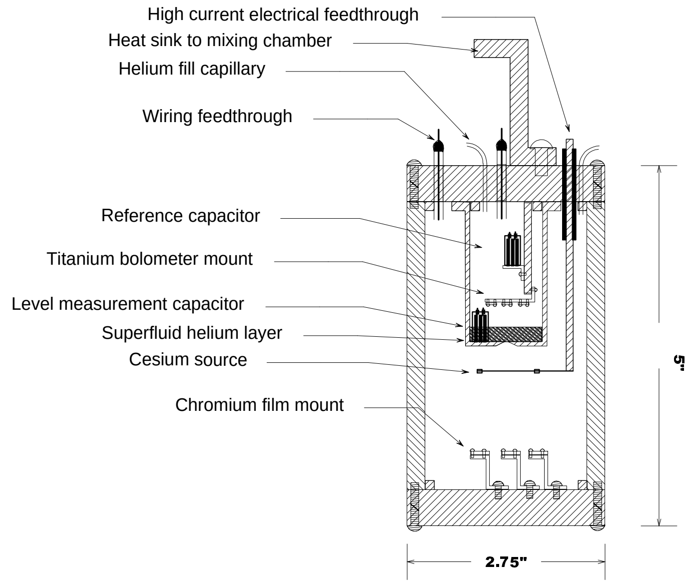
Design Criteria:

- Temperatures 0.1^o to $0.3\ ^o$K
- Time resolution to detect time delays due to quantum absorption and evaporation due to phonons.
  - Estimate of time delay:

$$\Delta t = 2mm/240m/s = 8.333 \times 10^{-6} sec$$

    Resolution of superconducting bolometers of 1 μsec have been reported J. of Physics C:Sol. St. Phys. 15, 1897(1982)). If this can be achieved (see below) s the delay can be measured.
- Pulsed sources giving energies in the range $0 < \epsilon_{\vec{k}} < 2.0^O K$.
- Facilities for study of the dependence on the direction of the incoming pulse.


Heat sink to mixing chamber
Helium fill capillary
Wiring feedthrough
Titanium bolometer mount
Chromium source mount
5"
2.75"


High current electrical feedthrough
Heat sink to mixing chamber
Helium fill capillary
Wiring feedthrough
Reference capacitor
Titanium bolometer mount
Level measurement capacitor
Superfluid helium layer
Cesium source
Chromium film mount
5"
2.75"

BOLOMETER DEVELOPMENT

- The particles are detected by thin film superconducting bolometers which are made by the group at Minnesota and are operated at their transition temperature. We experimented with zinc and titanium bolometers and are currently using titanium. Some data on resistivity of one of our titanium films is appended at the end of these notes.
- Recently we have begun experimental studies of pulsed beams (no liquid helium film) propagating in vacuum at .29 K.
- Detectors are placed at 3.5 cm and 5.5 cm from the source.

REMARKS ON THE BEAMS

- Time resolution is (barely) adequate.
- Energies may not be quite low enough to avoid roton creation.
- There is no evidence of collisional narrowing.
- There are mysteries concerning the multi-peak structure of the energy distribution at high power.

CONCLUSIONS

- The many body calculations are now under quite good control and are showing very substantial transmission coefficients and delay times consistent with uncertainty principle estimates and consistent with the argument that the time delay will be independent of sample thickness for thin samples.

- Weak coupling calculations on a spherically confined dilute Bose gas show a similar transparency effect

- Further improvements and refinements required in the theory include studies of the dependence of the results on direction of the incoming wave, on beam cross-sectional area and on adding effects of rotons and ripplons .

- It is possible to suspend a 'slab' of superfluid liquid helium across a hole in a horizontal plate if the surface of the plate and hole are covered with cesium. The depth of the fluid which can be suspended is inversely proportional to the diameter of the hole. We have suspended about 2mm of liquid helium in this way repeatedly and used the method to measure the contact angle as a function of temperature.
- The needed refrigerator has been acquired and installed, bolometers and sources have been developed and pulsed beams have been produced, detected and studied.
- We are hoping to carry out a first attempt at the transmission experiment in academic year 1997-98.