The Bardeen-Petterson Effect in MHD

with Kareem Sorathia and John Hawley

Disks and Internal Angular Momentum Axis Often Uncorrelated

- Accretion onto a protostellar binary
- Stellar mass transfer onto a spinning black hole
- Accretion of interstellar matter onto an AGN
- Tidal disruption of a star by a massive black hole
- Circumbinary disk around a binary black hole

(4 angular momenta to align!)

Tilts Make Torques

Rotating solitary mass: $\mathbf{T} = 2(G/c^2)\mathbf{J} \times \mathbf{L}/r^3$

$$\Omega_{\rm prec} \sim (c/r_g)(r/r_g)^{-3}$$

Binary: $\mathbf{T} = -(3\eta/4)(\mathbf{J}/M) \times \mathbf{L}/r^2$

What happens at small radii, where the precession grows faster and faster, and the inter-ring twists greater and greater?

Presumption: “Friction” Aligns Inner Rings

- **Bardeen & Petterson (1975)**
- Hatchett, Begelman & Sarazin (1981)
- Papaloizou & Pringle (1983)
- Kumar & Pringle (1985)
- Pringle (1992)
- Papaloizou & Lin (1995)
- Scheuer & Feiler (1996)
- Ivanov & Illarionov (1997)
- Ogilvie (1999, 2000)
- Nelson & Papaloizou (2000)
- Lubow, Ogilvie & Pringle (2002)
- Lodato & Pringle (2006, 2007)
- Martin, Pringle & Tout (2010)
- Lodato & Price (2010)
- Nixon, King, Price & Frank (2012)
- ……………

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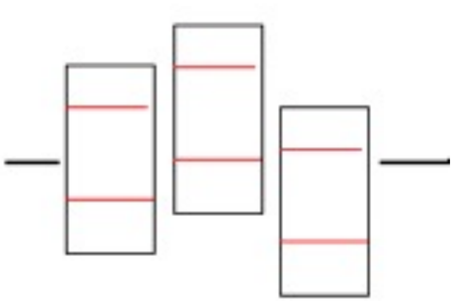
Although dissipation may be a consequence, alignment requires torque, yet *local* precessional torque **cannot** align!

Radial Transport of Misaligned Angular Momentum



Offset in orbital plane creates offset in vertical hydrostatic pressure gradient:

Radial pressure gradients!

Radial pressure gradients drive radial flows

Radial flows mix angular momentum

Useful measure: $\hat{\psi} \equiv |d\hat{l}/d\ln r|/(h/r)$— criterion for order unity radial pressure contrast

Questions

- What controls how far and fast the radial flows travel?
- If the inner disc aligns and the outer disc doesn’t, where is the transition? Is it gradual or sudden?
- Where is the source of aligning torque?

Traditional Answer: α-ology

(estab. 1973)

α-ology, Part 1

Flat disks need internal stress to accrete

Dimensional analysis yields a provisional answer:
$T_{r\phi} \sim \alpha\, p$

Makes no statement about mechanism, only magnitude

Powerful *ansatz*—permits complete solution for surface density profile of a time-steady disk

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Frequently imagined—aphysically—as a “viscosity”

α-ology, Part 2

•Assumption by nearly (but see Fragile et al.) all previous work:

"α viscosity" exists, and acts isotropically within tilted discs

•Such a viscosity curbs the vertical shear in the radial flows

•Result: effective angular momentum diffusion coefficient $\alpha_2 \sim \alpha^{-1}$

Modern Answer: MHD Turbulence

(estab. 1991)

Working with MHD Turbulence
(see also Fragile, this afternoon)

Nonlinear turbulence demands numerical treatment

This problem is expensive: intrinsically global, fully 3-d (no geometric symmetries), when h/r << 1, high resolution required to describe MHD turbulence

Strategy: Newtonian + 1PN to save cycles

make $\Omega_{prec}/\Omega_{orb} \sim 0.1$ to save cycles

choose $h/r \sim 0.1$

use savings to fund resolution

still 1.3M processor-hours

Simulation Parameters and Procedure

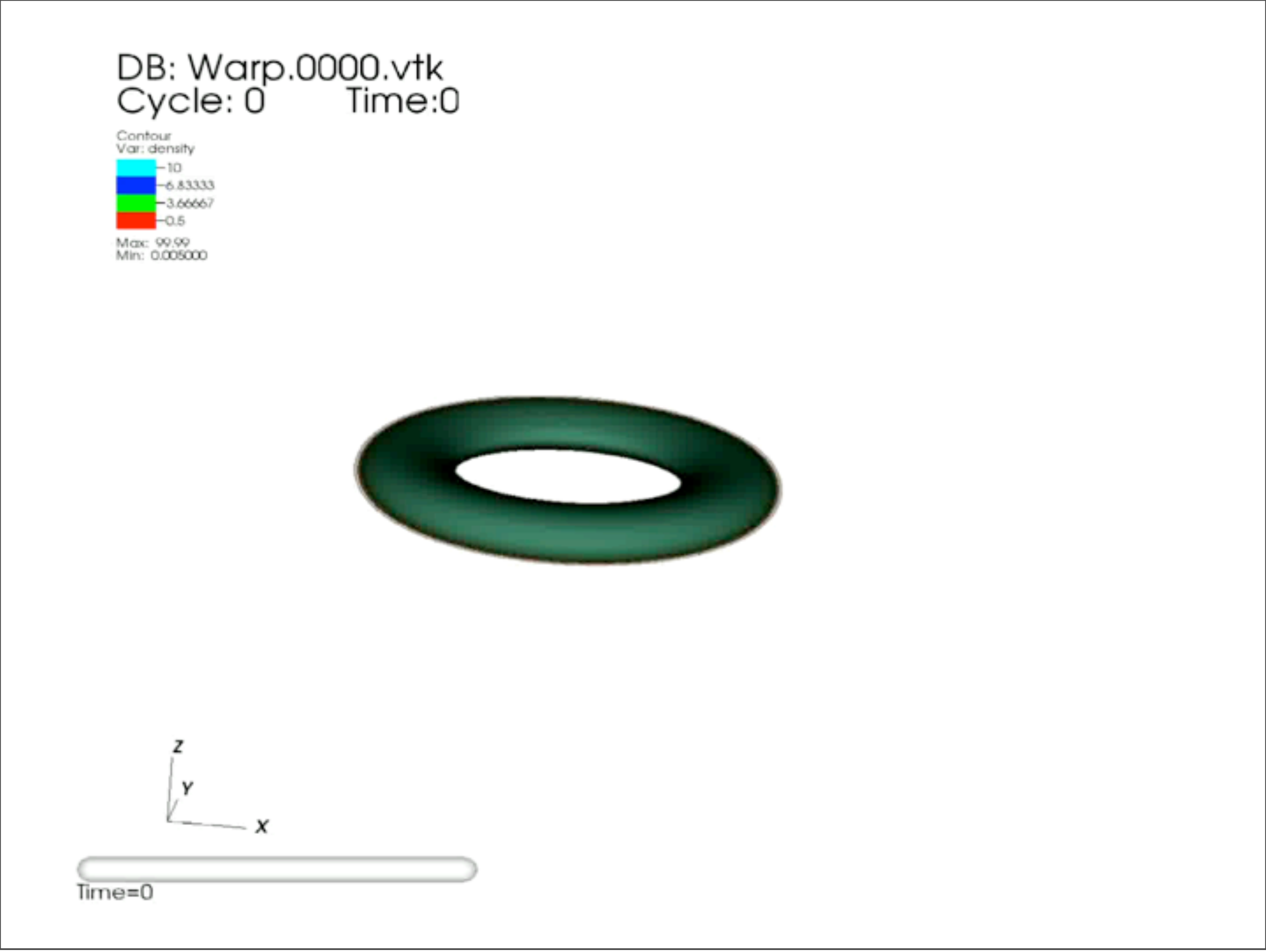
Initial disc: hydrostatic torus with h/r ~ 0.1, weak dipolar magnetic field; surface density profile peaks in middle (r=10)

run without torque for 15 orbits at r=10 to let MHD turbulence saturate

Torques: 12° (~2h/r) tilt, $\Omega_{orb}/\Omega_{prec}$ = 15 in mid-disk

Duration: 15 mid-disk orbits

Hydro foil: match h(r), Σ(r) to MHD disk at t=15, run with same torques for 15 mid-disk orbits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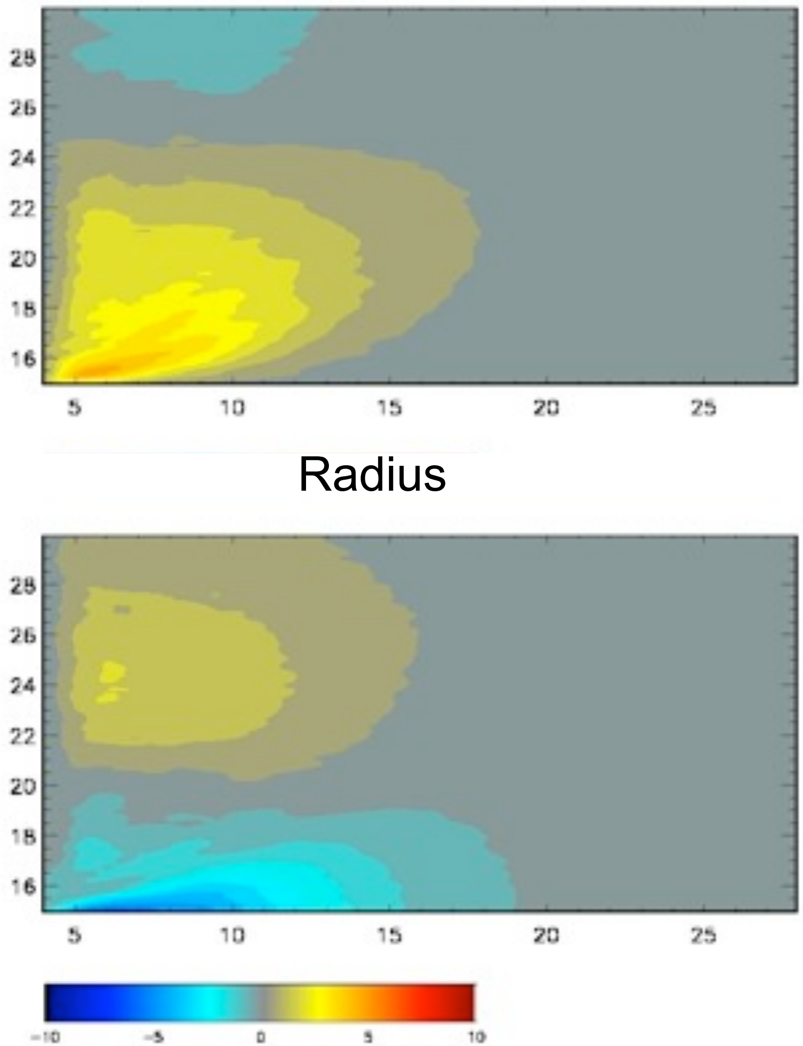
Cycle: 0 Time:0
Contour
Var: density
10
6.83333
3.66667
0.5
Max: 99.99
Min: 0.005000
Z
Y
X
Time=0

Basic Principles of Disk Alignment

I: Torque Delivered at Smallest Radius with Tilt and Mass

Time

G_x



Radius

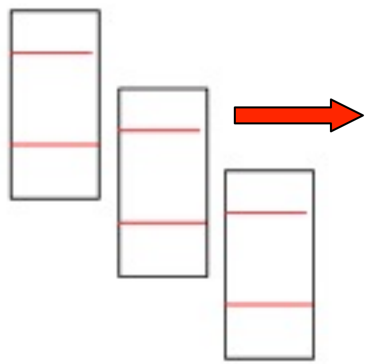
Time

G_y

II: Hydrodynamics Governs Radial Flows

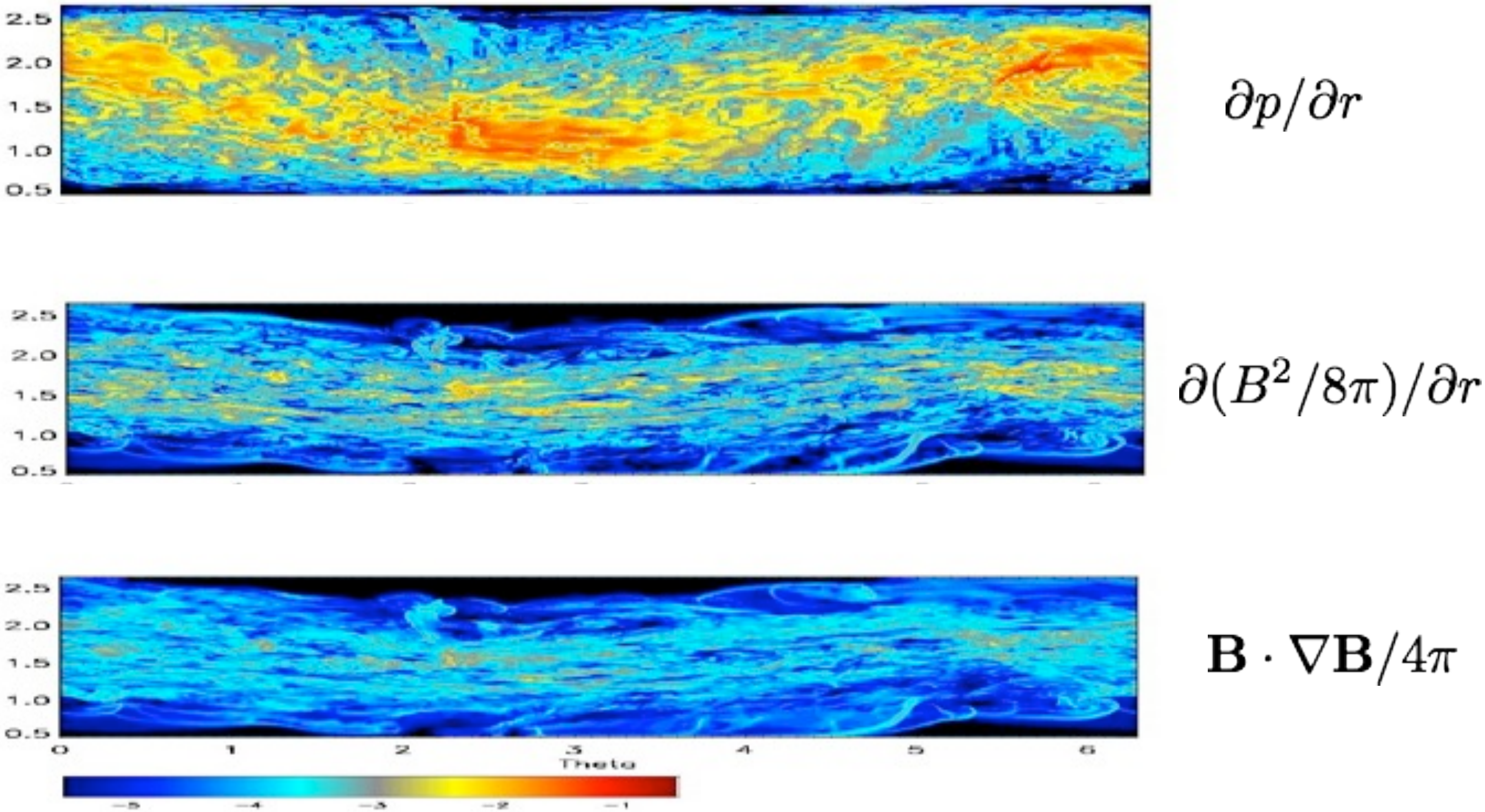
When $\hat{\psi} > 1$, $v_r > c_s$:
v_r regulated by acoustics, not viscosity

Only barrier to free expansion is orbital mechanics: $\Delta r \sim h$ when transonic



⇒ large Reynolds stress

Not MHD Forces



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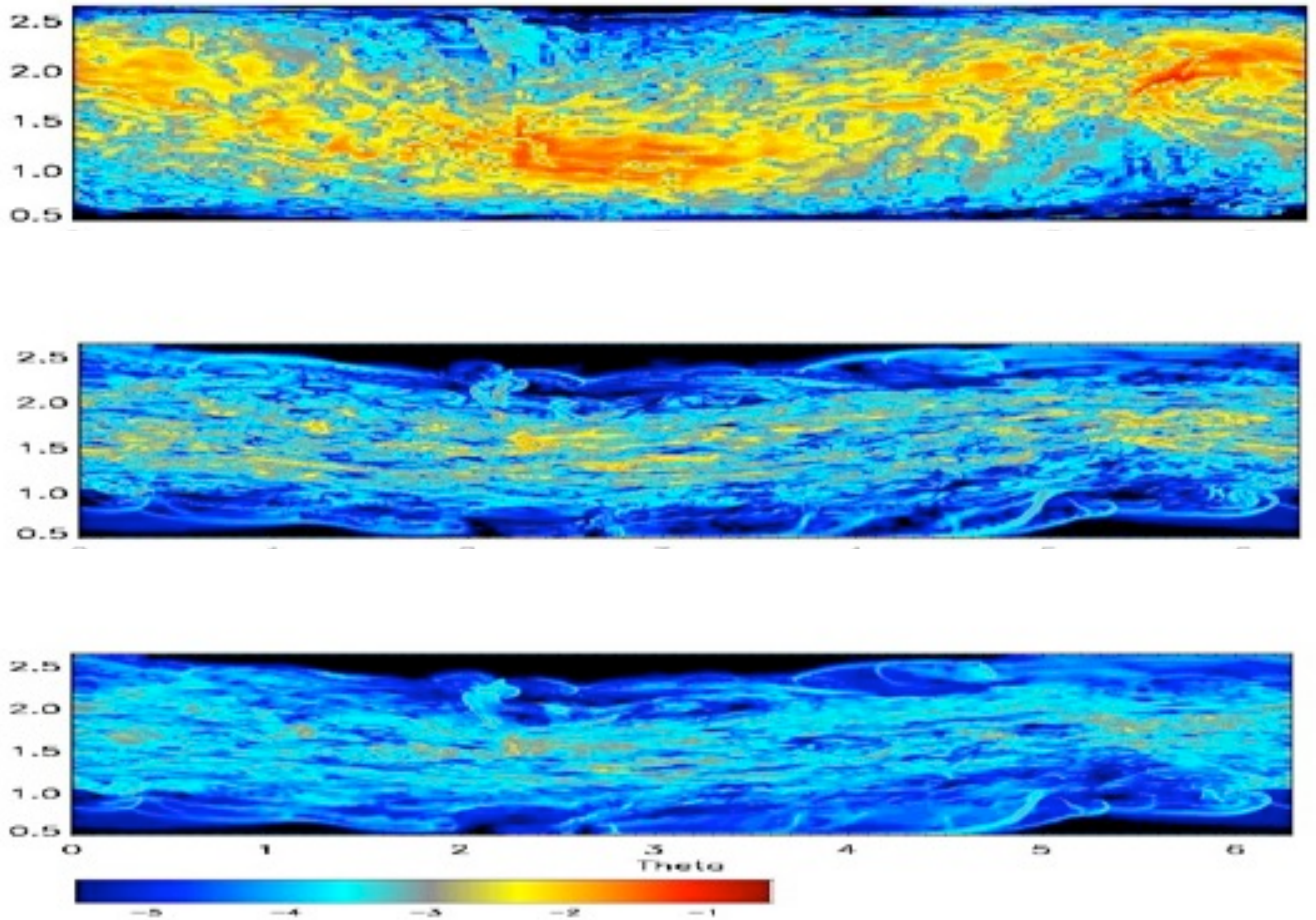


$\partial p/\partial r$

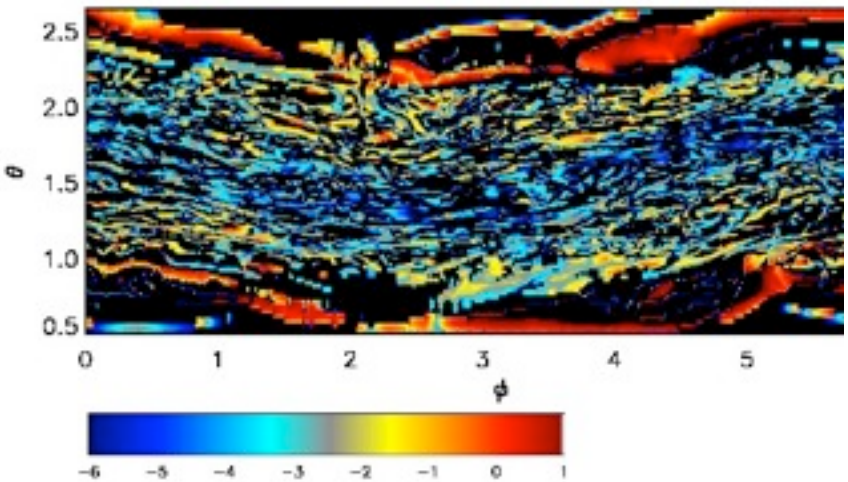
$\partial(B^2/8\pi)/\partial r$

$\mathbf{B}\cdot\nabla\mathbf{B}/4\pi$

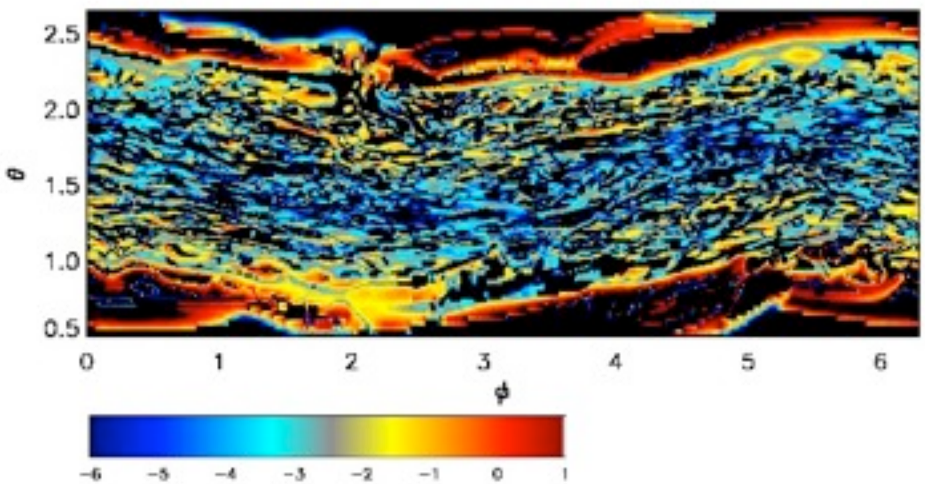
But MHD **is** important for creating turbulence

Nor “Isotropic Viscosity”

$$\alpha_* \equiv -\frac{B_r B_\theta/4\pi}{p(\partial v_r/\partial z)/\Omega}: \sim \pm 3 \times 10^{-5} \text{ in the disk body}$$



$\alpha_* > 0$



$\alpha_* < 0$

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We Should Have Known....

$|B_\phi| > |B_r| > |B_z|$ in MRI turbulence

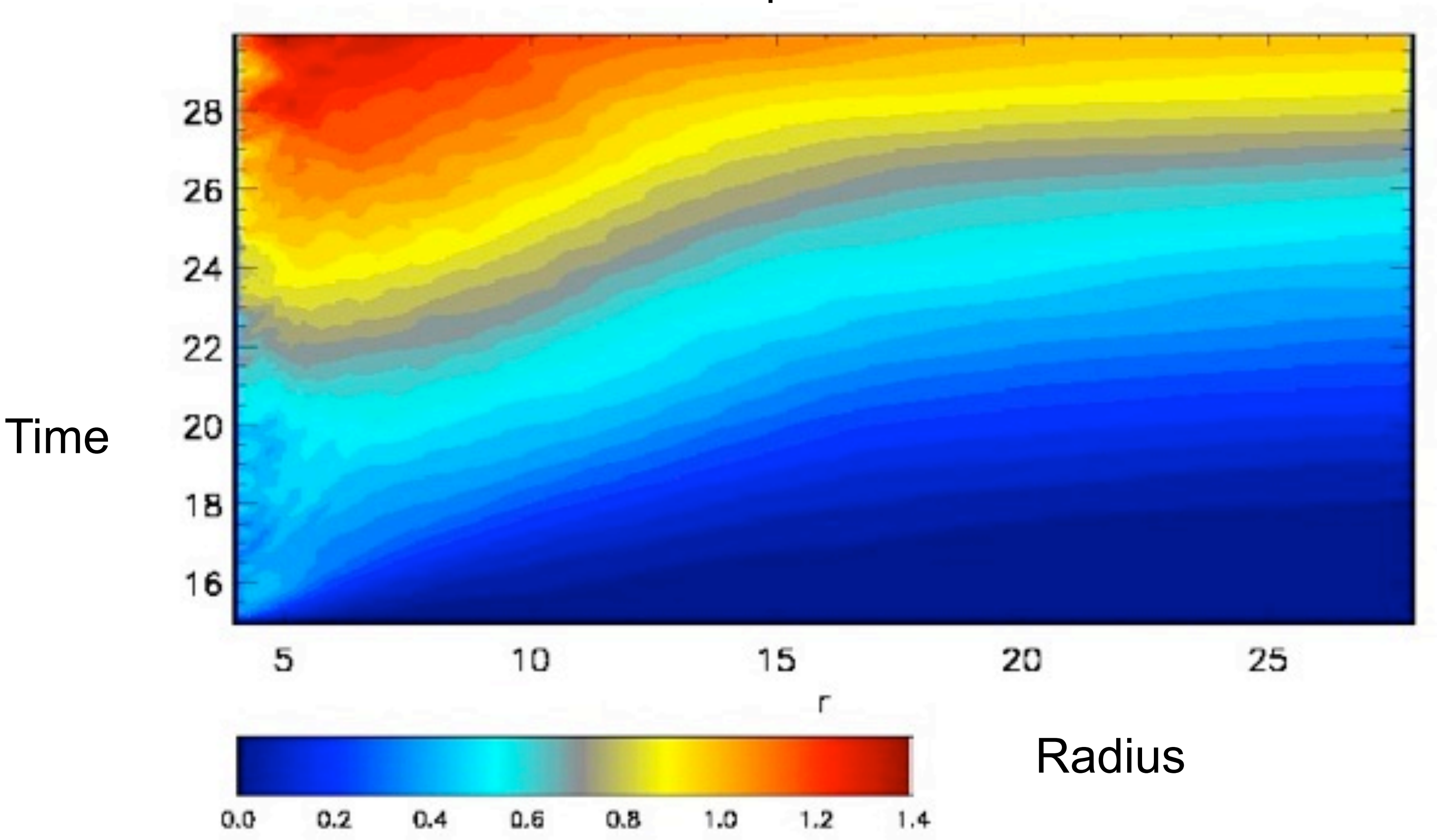
Both velocities and field turbulent, fluctuating

Magnetic stress reflects *strain*, not *shear*

III: Precession Phase Contrast Is Essential

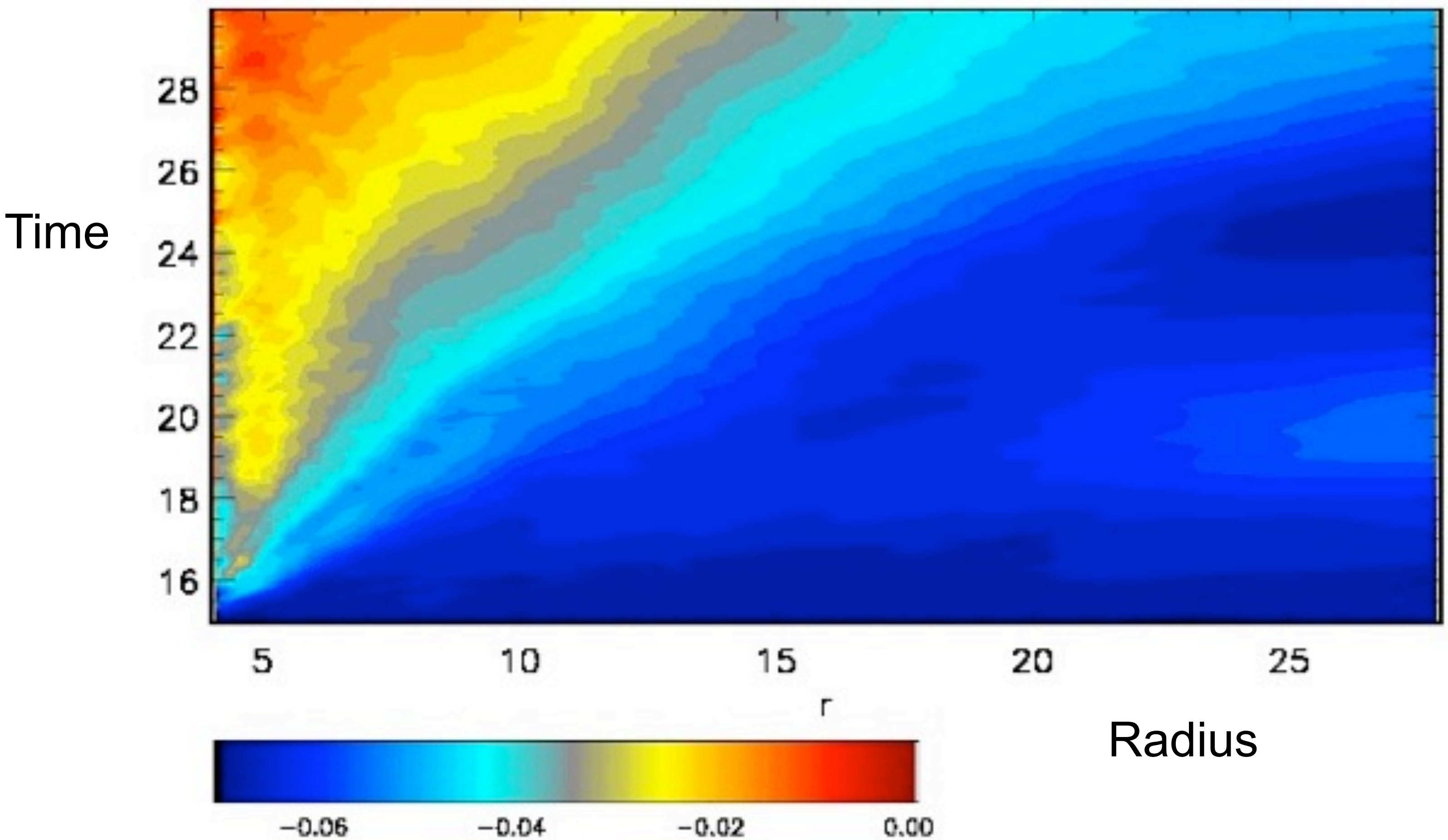
- MHD turbulence disrupts solid-body precession, creating radial precession phase shifts
- If r_{in} is $\pi/2$ ahead of r_{out} , $\mathbf{T}(r_{in}) = -|c|\mathbf{L}_{mis}(r_{out})$
- $|\mathbf{T}(r_{in})|/|\mathbf{T}(r_{out})| \sim (r_{out}/r_{in})^3 \, |\mathbf{L}_{mis}(r_{in})|/\mathbf{L}_{mis}(r_{out})|$
- Radial flows carry canceling angular momentum outward, producing progressive alignment

ϕ_{prec}/π



Near solid-body precession,
but with a near-constant twist

$$\beta_{\rm align}/\pi$$



IV: Alignment Front Speed Regulated by Torque Supply and Demand

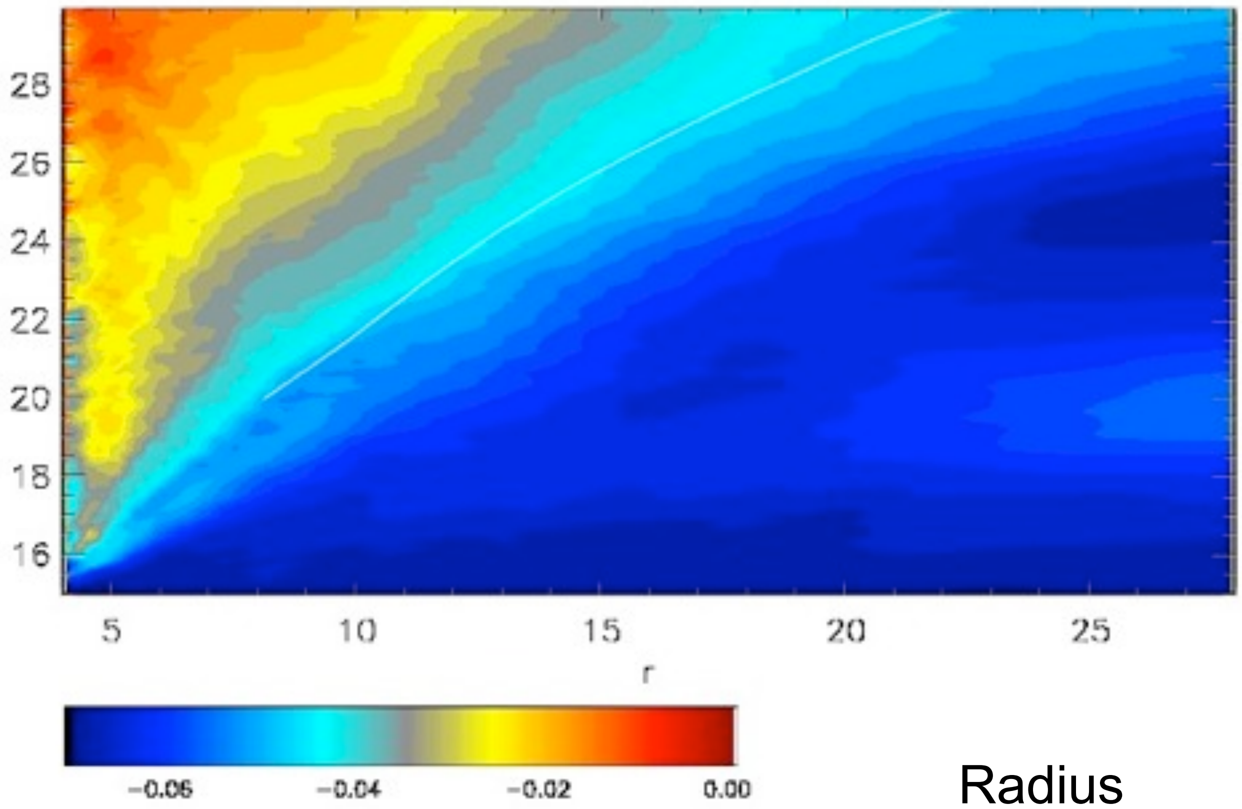
Model:

$$v_f = \langle \cos\gamma \rangle \frac{G(< r_f)}{dL_\perp(r_f)/dr}$$

Empirically, $\langle \cos\gamma \rangle \simeq 0.5$

β/π

Time



Radius

Location of the Transition Radius

Mass accretion and radial flows carry misaligned angular momentum inward; both are relatively rapid at large r, and the former near the ISCO.

The alignment front stops where $v_f = v_{in}$:

$$R_T/r_g = \left[2\langle\cos\gamma\rangle a_* \Omega(R_T) t_{\rm in} \int_0^1 dx\, x^{-3/2} \frac{\sin\beta(x)}{\sin\beta(R_T)} \frac{\Sigma(x)}{\Sigma(R_T)}\right]^{2/3}$$

$$x \equiv r/R_T$$

$$t_{\rm in} \gtrsim (R_T/h)^2 \Omega_T^{-1} ?$$

Summary

It is now possible to describe precessing disks with **physical** internal stresses

Radial flows transport angular momentum capable of alignment due to radial precession phase gradient maintained by MHD turbulence

Text

Alignment controlled by balance between torque and misaligned angular momentum inflow; mixing flows ~transonic

Implications for alignment between binary orbital plane and circumbinary disk, as well as between black hole spins and the binary orbital plane