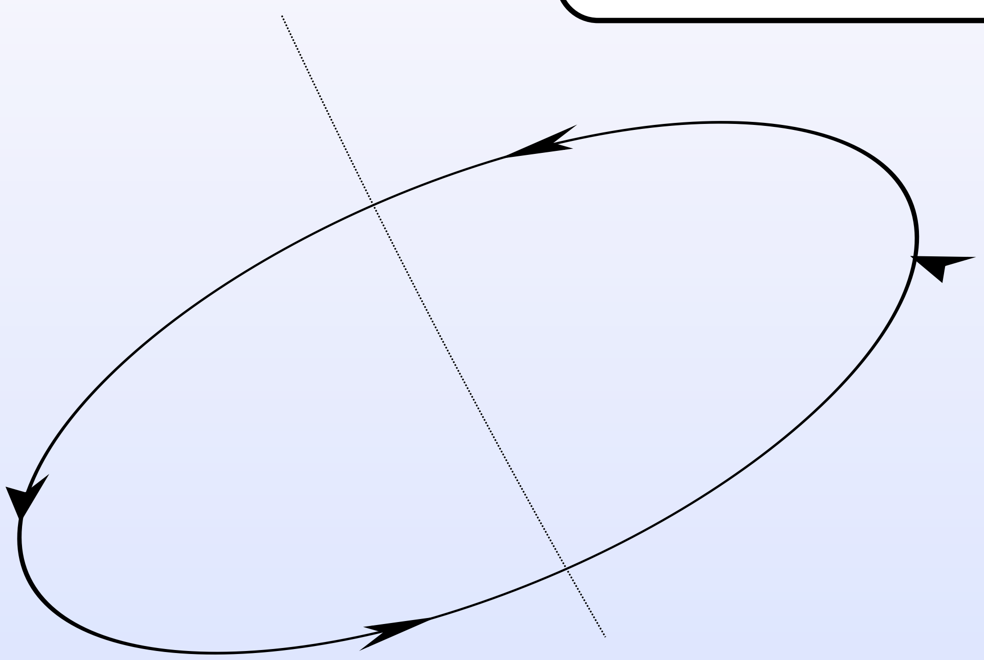
THE AUSTRALIAN NATIONAL UNIVERSITY

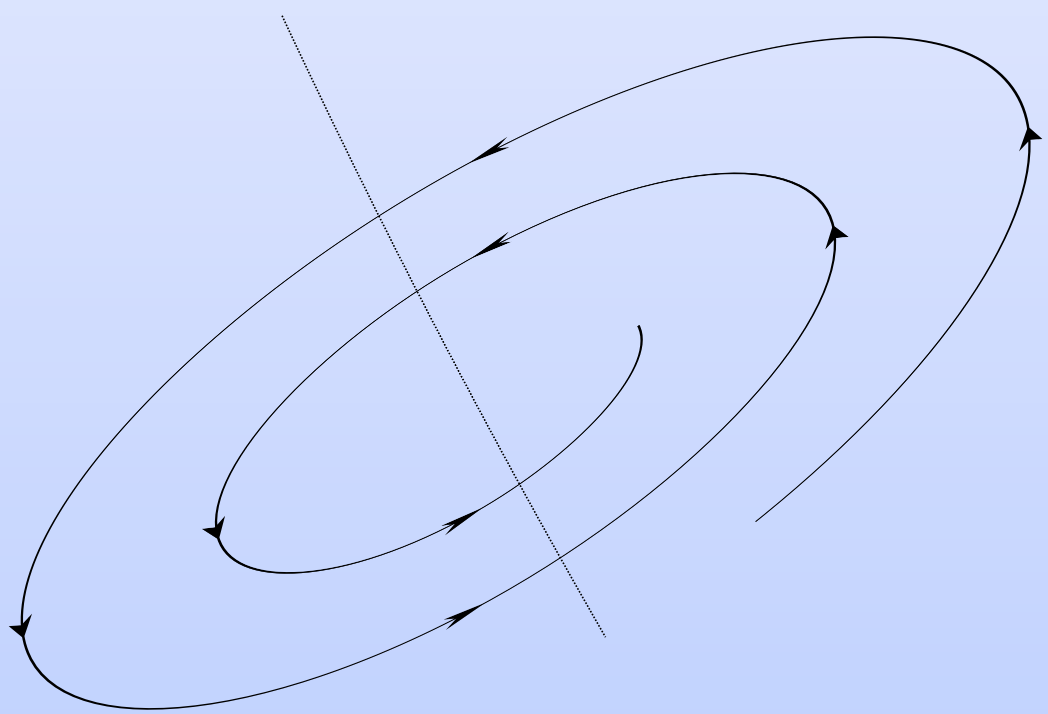
Global Simulations of Magnetised Accretion Disk Turbulence

Ross Parkin & Geoff Bicknell

Rule number one



No angular momentum loss
= test particle orbits forever



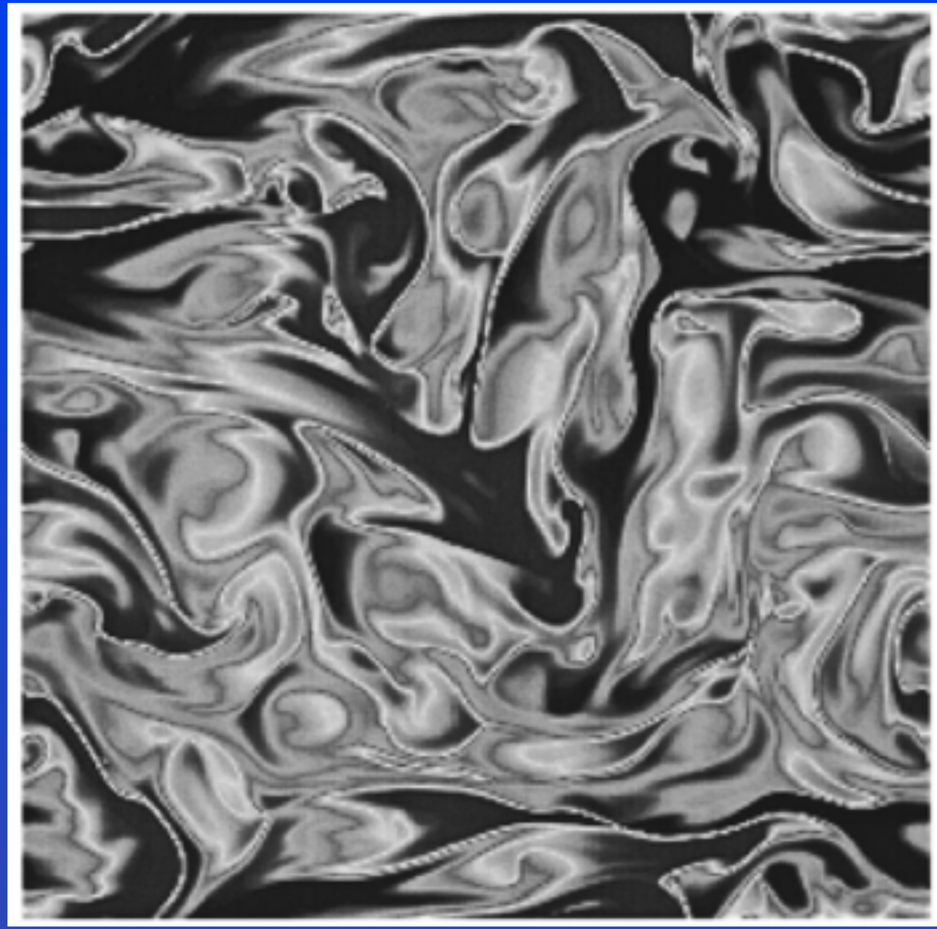
Angular momentum loss
= test particle falls on to
central object

Magnetorotational turbulence

- Angular momentum transport can be facilitated by magnetorotational instability driven turbulence

- Sustained turbulence demonstrated in unstratified shearing boxes (Hawley+ 1995)

- However, stress decreases with increasing resolution in zero-net flux simulations (Fromang & Papaloizou 2007)

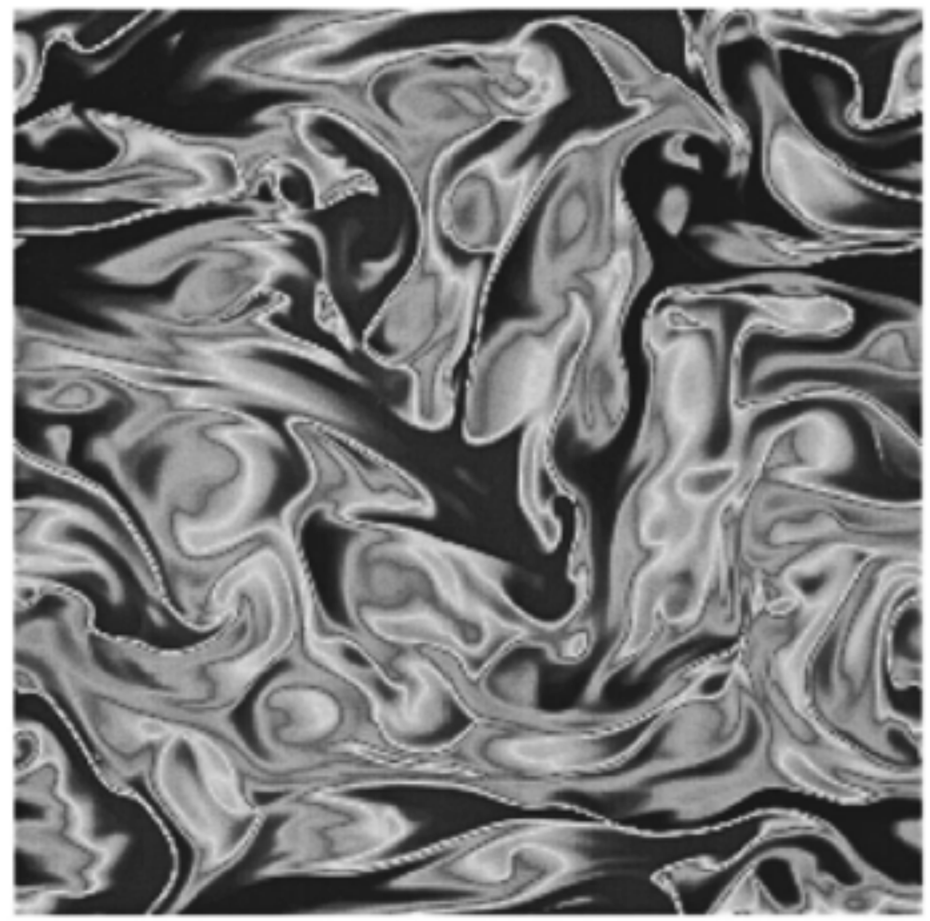


Balbus & Hawley (1998)

THE AUSTRALIAN NATIONAL UNIVERSITY

Magnetorotational turbulence

- Angular momentum transport can be facilitated by magnetorotational instability driven turbulence
- Sustained turbulence demonstrated in unstratified shearing boxes (Hawley+ 1995)
- However, stress decreases with increasing resolution in zero-net flux simulations (Fromang & Papaloizou 2007)



Balbus & Hawley (1998)

$$W_{R\phi} = \rho \delta v_{\mathrm{R}} \delta v_{\phi} - B_{\mathrm{R}} B_{\phi}$$

$$\alpha = \frac{W_{R\phi}}{P}$$

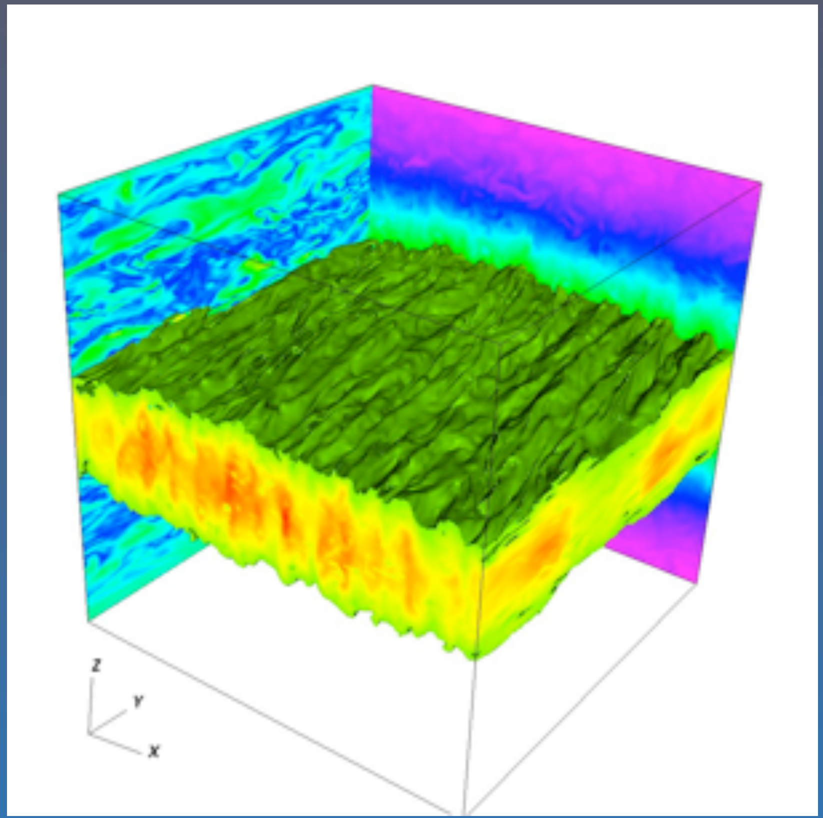
Model	Box size	Resolution	Run time (in orbits)	α_{Rey}	α_{Max}	α
FS64	$(H, 2\pi H, H)$	(64, 200, 64)	300	1.8×10^{-3}	4.2×10^{-3}	5.9×10^{-3}
STD64	$(H, \pi H, H)$	(64, 100, 64)	1000	9.4×10^{-4}	3.2×10^{-3}	4.1×10^{-3}
STD128	$(H, \pi H, H)$	(128, 200, 128)	250	5.0×10^{-4}	1.7×10^{-3}	2.2×10^{-3}
STD256	$(H, \pi H, H)$	(256, 400, 256)	105	2.4×10^{-4}	8.1×10^{-4}	1.1×10^{-3}

ANU
THE AUSTRALIAN NATIONAL UNIVERSITY

Stratified models show convergence

- Sustained turbulence demonstrated in stratified shearing boxes (e.g. Brandenburg+1995, Stone+1996, Davis+ 2010, Shi+2010, Simon +2011, Oishi & MacLow 2011)

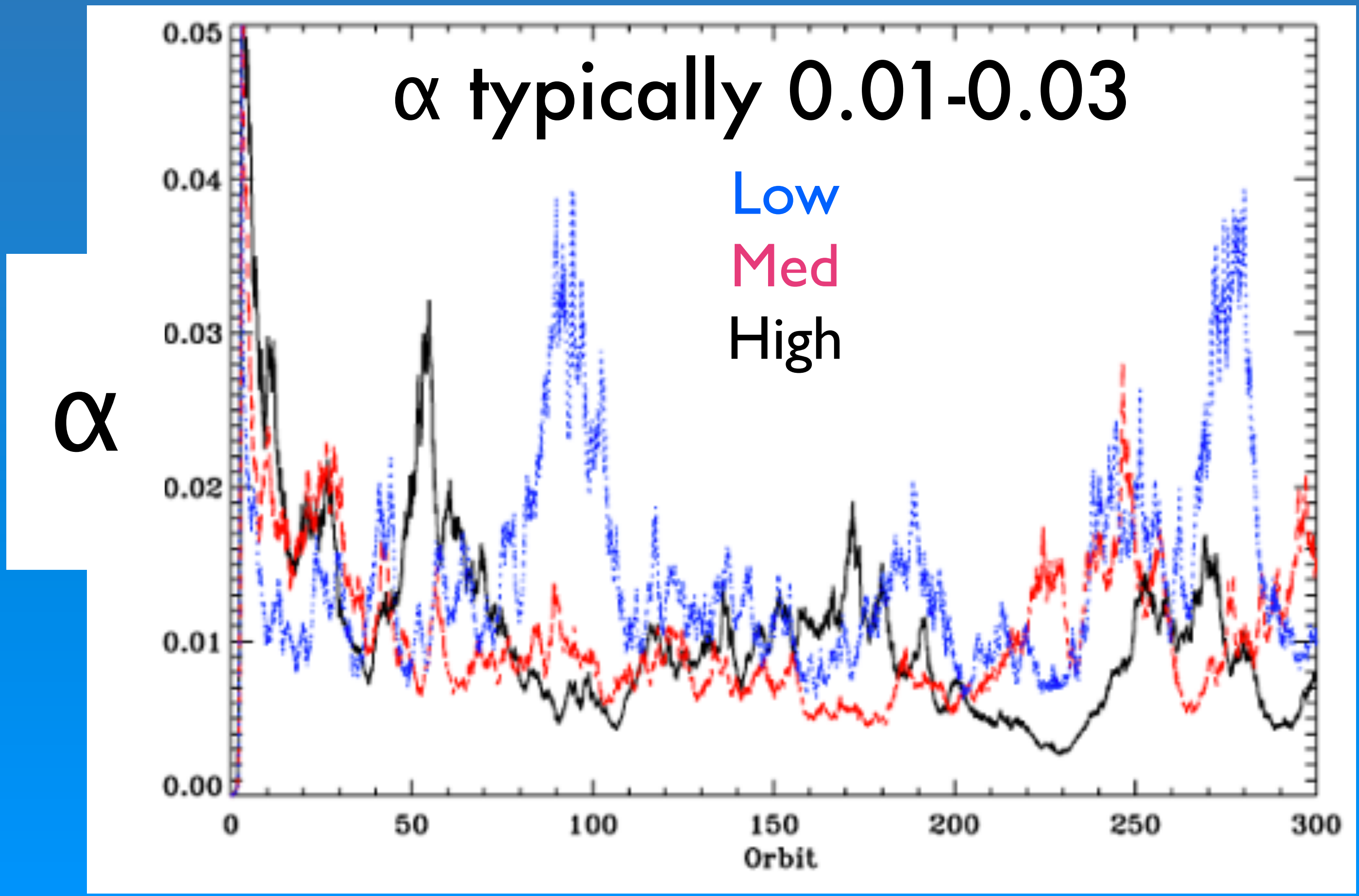
- Results converge with increasing resolution when the vertical component of gravity is included (ie stratification).



Davis+ (2010)

$$W_{R\phi} = \rho \delta v_{\mathrm{R}} \delta v_{\phi} - B_{\mathrm{R}} B_{\phi}$$

$$\rightarrow \quad \alpha = \frac{W_{R\phi}}{P}$$



THE AUSTRALIAN NATIONAL UNIVERSITY

Main questions

THE AUSTRALIAN NATIONAL UNIVERSITY

Main questions

- Will global stratified disk models converge?

THE AUSTRALIAN NATIONAL UNIVERSITY

Main questions

- Will global stratified disk models converge?
- If so, at what resolution, and with what value for the turbulent stresses (α)?

THE AUSTRALIAN NATIONAL UNIVERSITY

Main questions

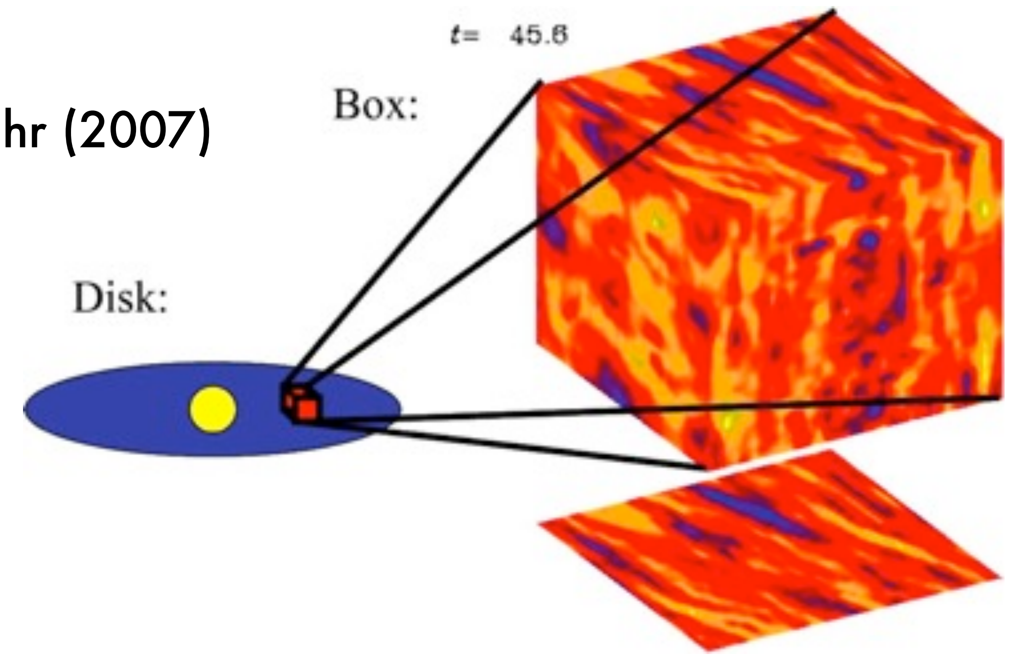
- Will global stratified disk models converge?
- If so, at what resolution, and with what value for the turbulent stresses (α)?
- How do global models and shearing-boxes differ in terms of magnetic energy production and large scale field development?

THE AUSTRALIAN NATIONAL UNIVERSITY

Shearing-box

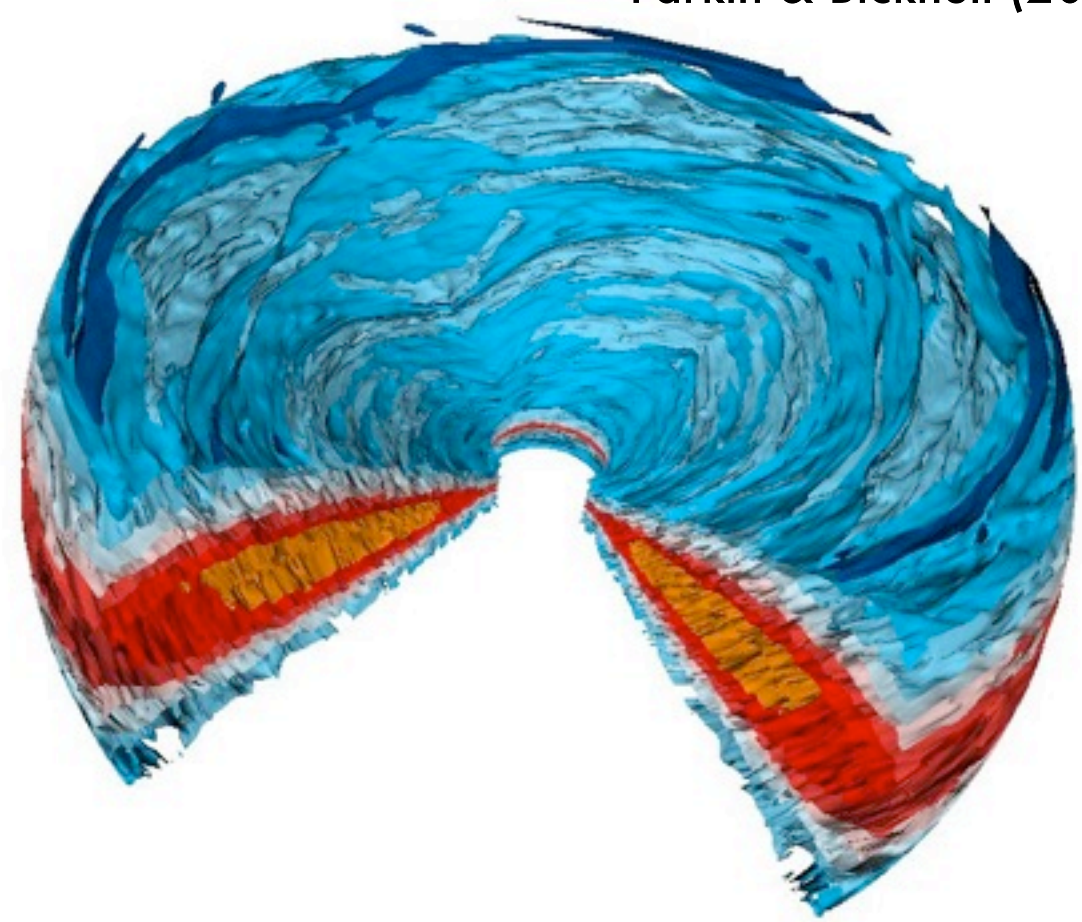
Global simulations

Klahr (2007)



See also: Brandenburg et al. 1995; Stone et al. 1996; Miller & Stone 2000; Fleming et al. 2000; Brandenburg 2005; Johansen et al. 2009; Gressel 2010; Shi et al. 2010; Davis et al. 2010; Simon et al. 2011; Guan & Gammie 2011; Oishi & Mac Low 2011; Simon et al. 2012

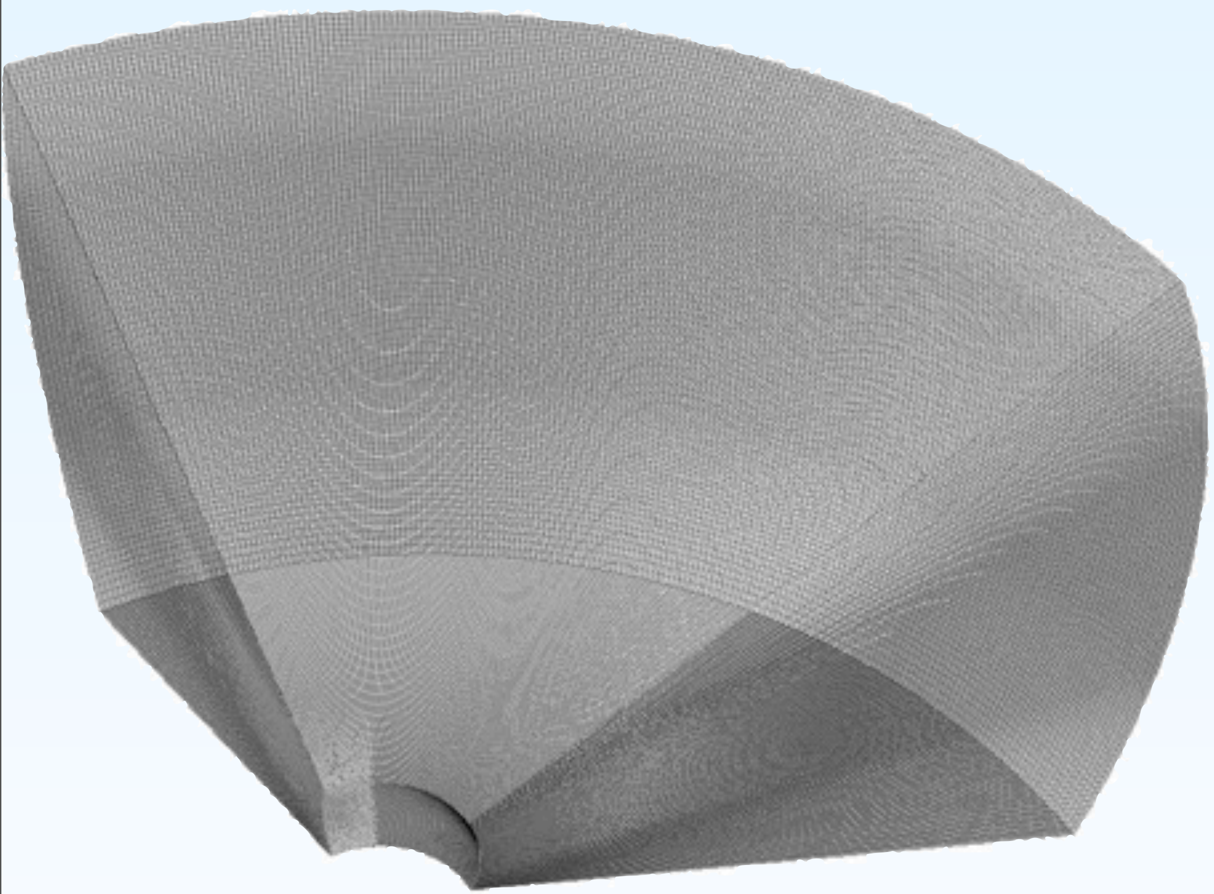
Parkin & Bicknell (2013a,b)

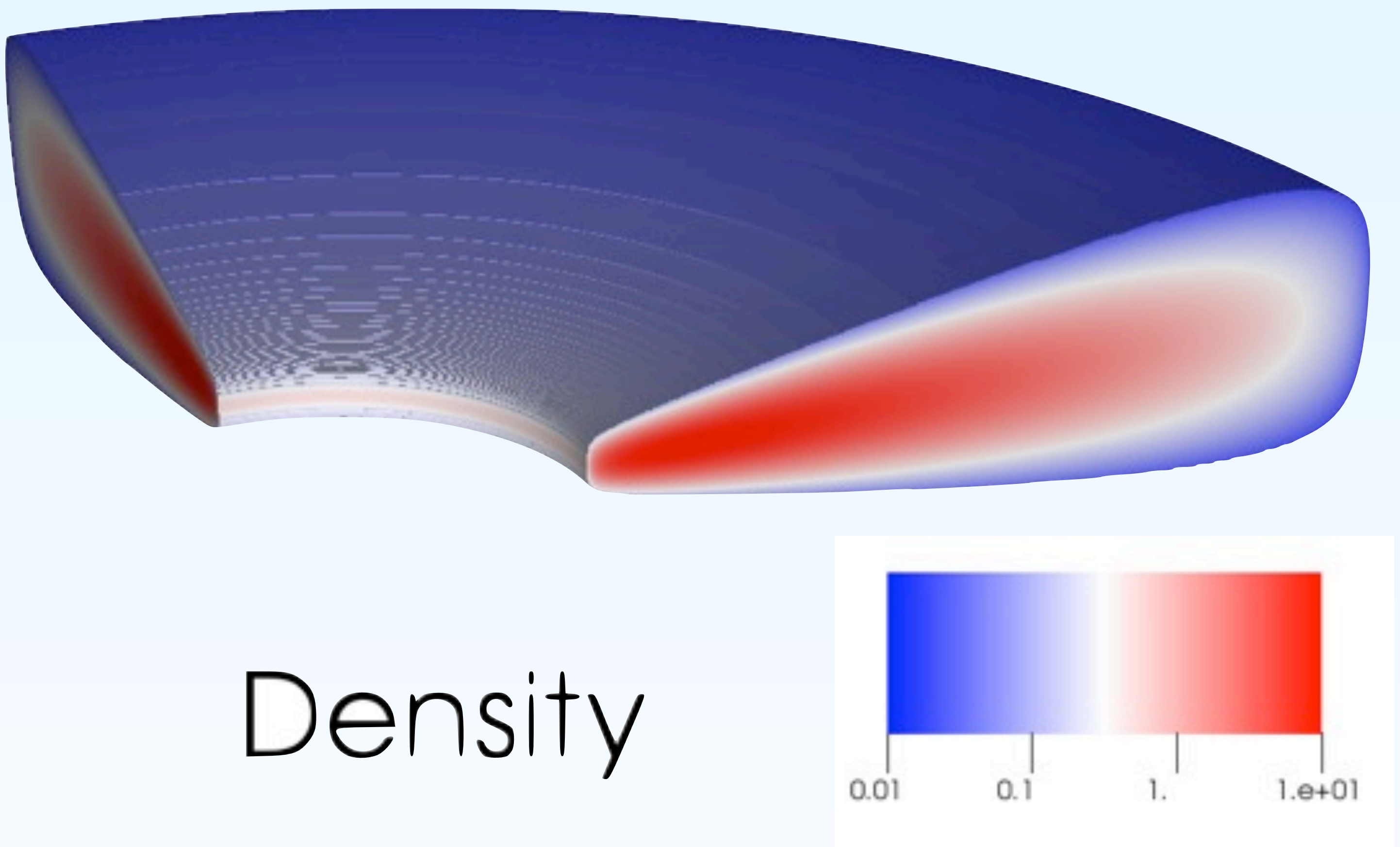


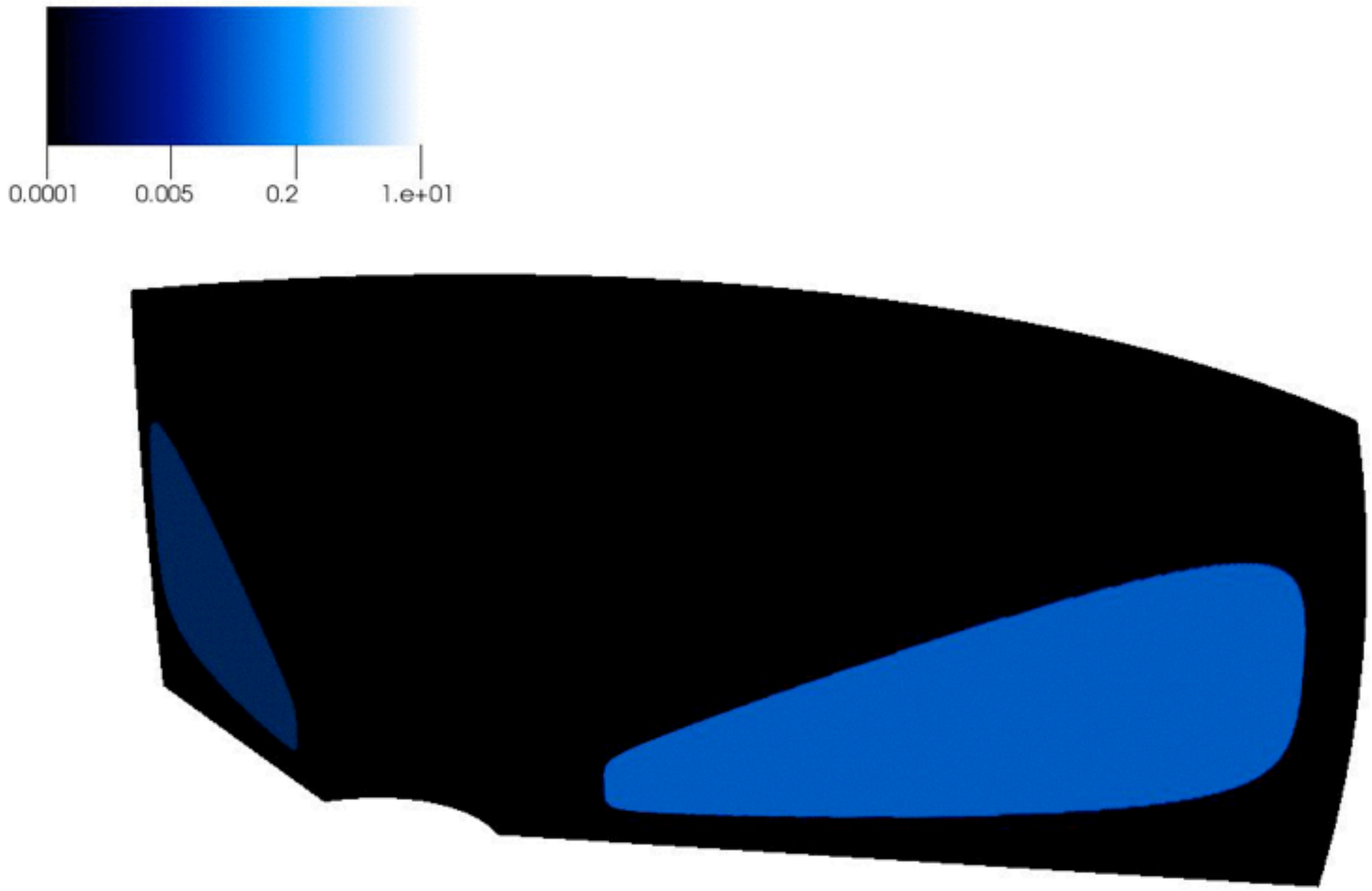
See also: Hawley 2000; Hawley & Krolik 2001; Arlt & Rudiger 2001; Fromang & Nelson 2006, 2009, Fragile et al. 2007, 2009; Beckwith et al. 2008, 2011; Lyra et al. 2008; Reynolds & Fabian 2008; Sorathia et al. 2010; O'Neill et al. 2011; Flock et al. 2011, 2012a,b; Noble et al. 2009, 2010, 2011; Hawley et al. 2011, 2013; McKinney et al. 2012, Dexter et al. 2013

Global Simulations

- Equilibrium magnetized disk with purely toroidal field (Parkin & Bicknell 2013, Apj, 763, 99)
- No unphysical periodic boundaries
- Performed on a 3D spherical grid using the PLUTO MHD code (Mignone+2007)

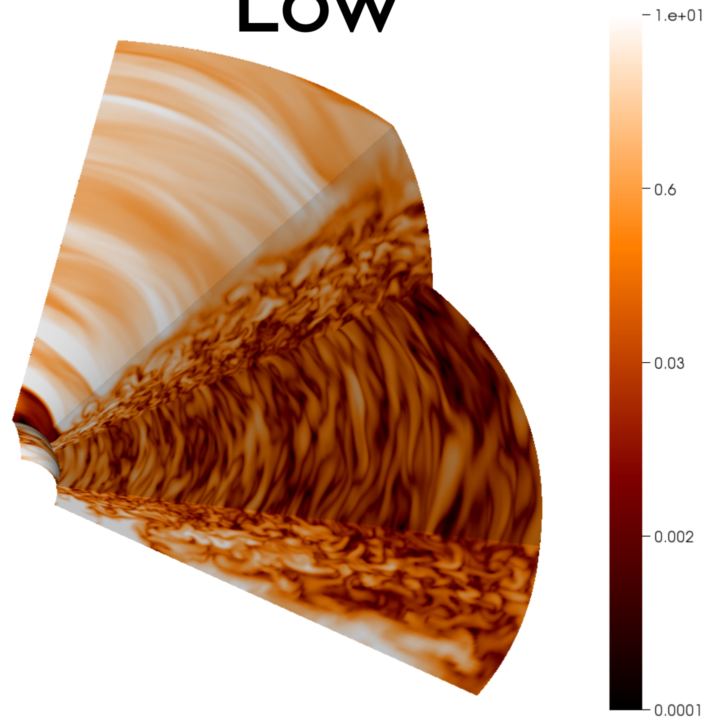




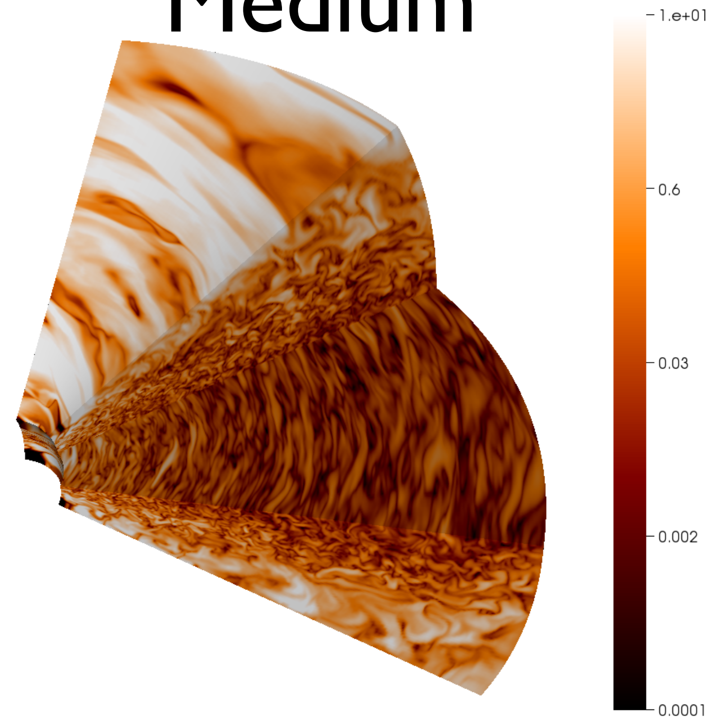


(Magnetic pressure)/(Gas pressure)

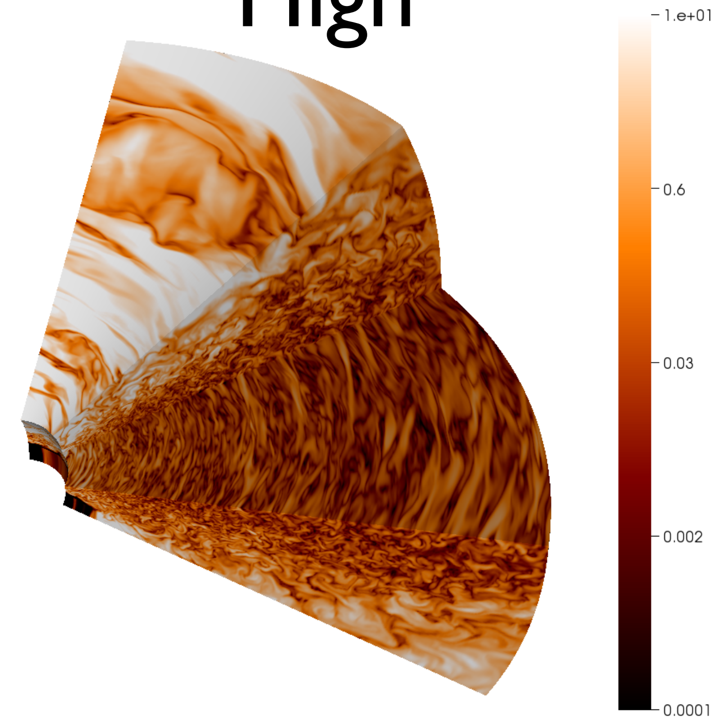
Low



Medium



High



340×112×128 cells

512×170×196 cells

768×256×256 cells

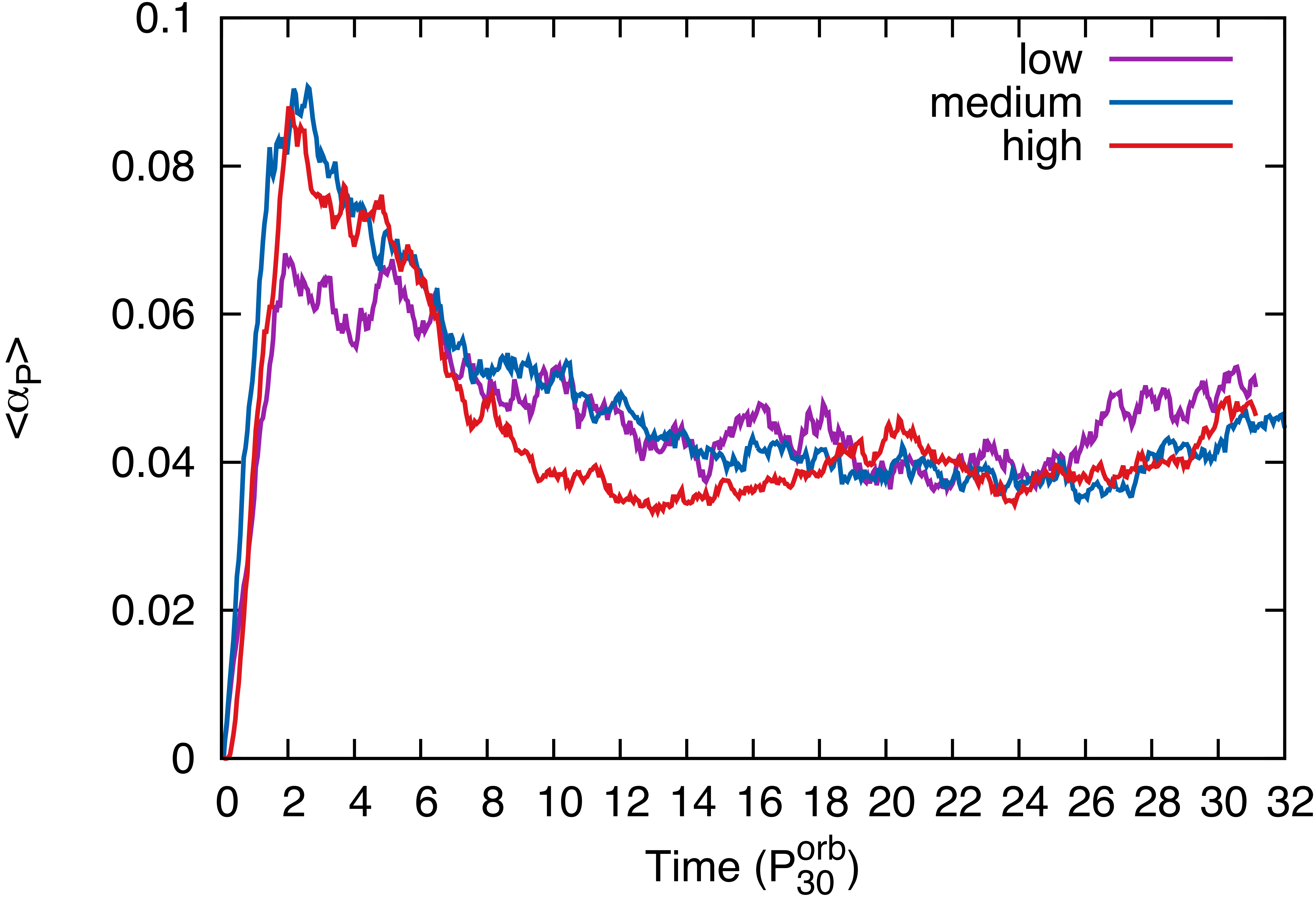
($n_r \times n_\theta \times n_\phi$)

Plots show (magnetic pressure)/(gas pressure)

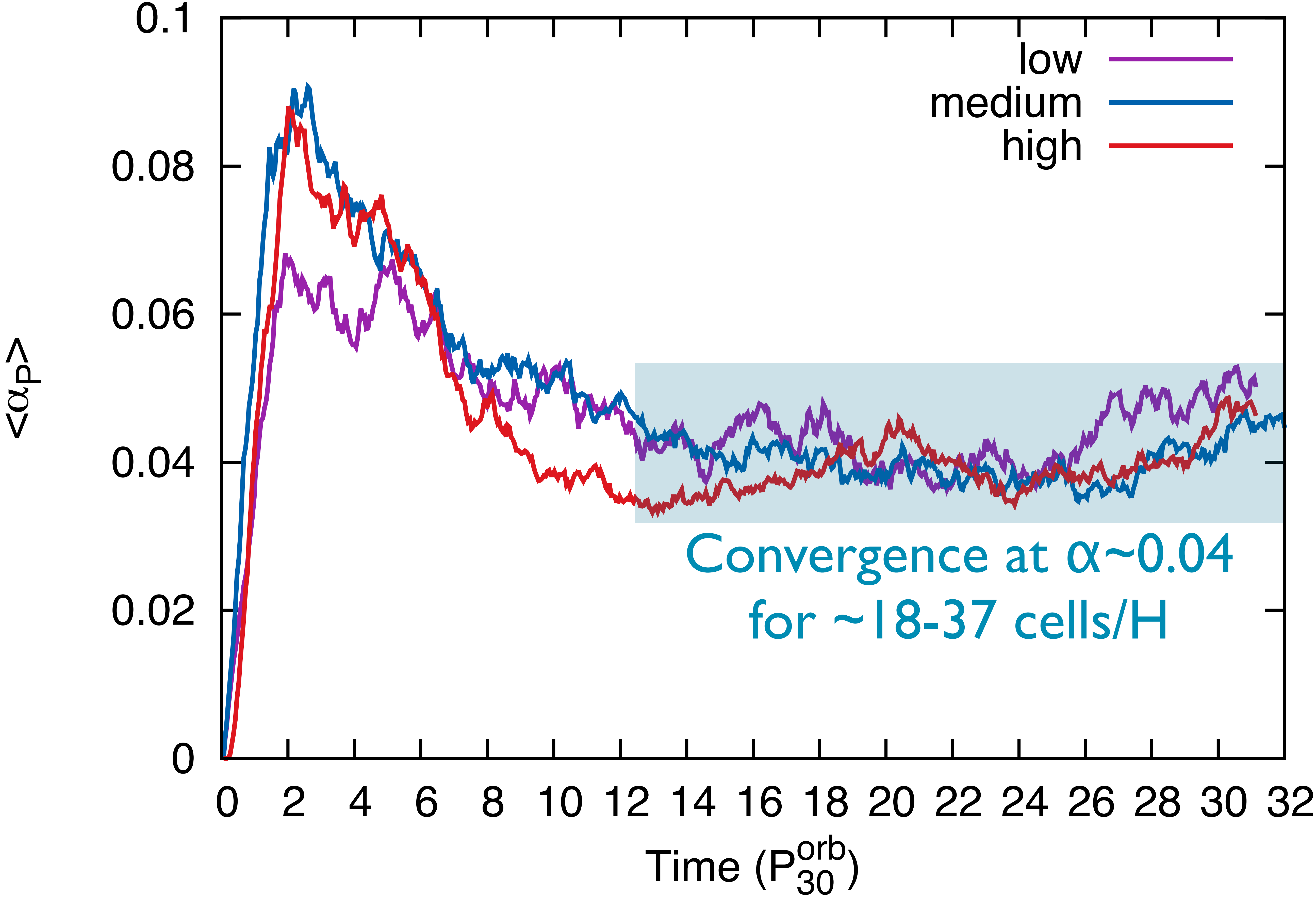
Model	Resolution (n/H) ($r \times \theta \times \phi$)
low	(9-36) × 18 × 8
medium	(12-51) × 27 × 13
high	(18-77) × 37 × 16

THE AUSTRALIAN NATIONAL UNIVERSITY

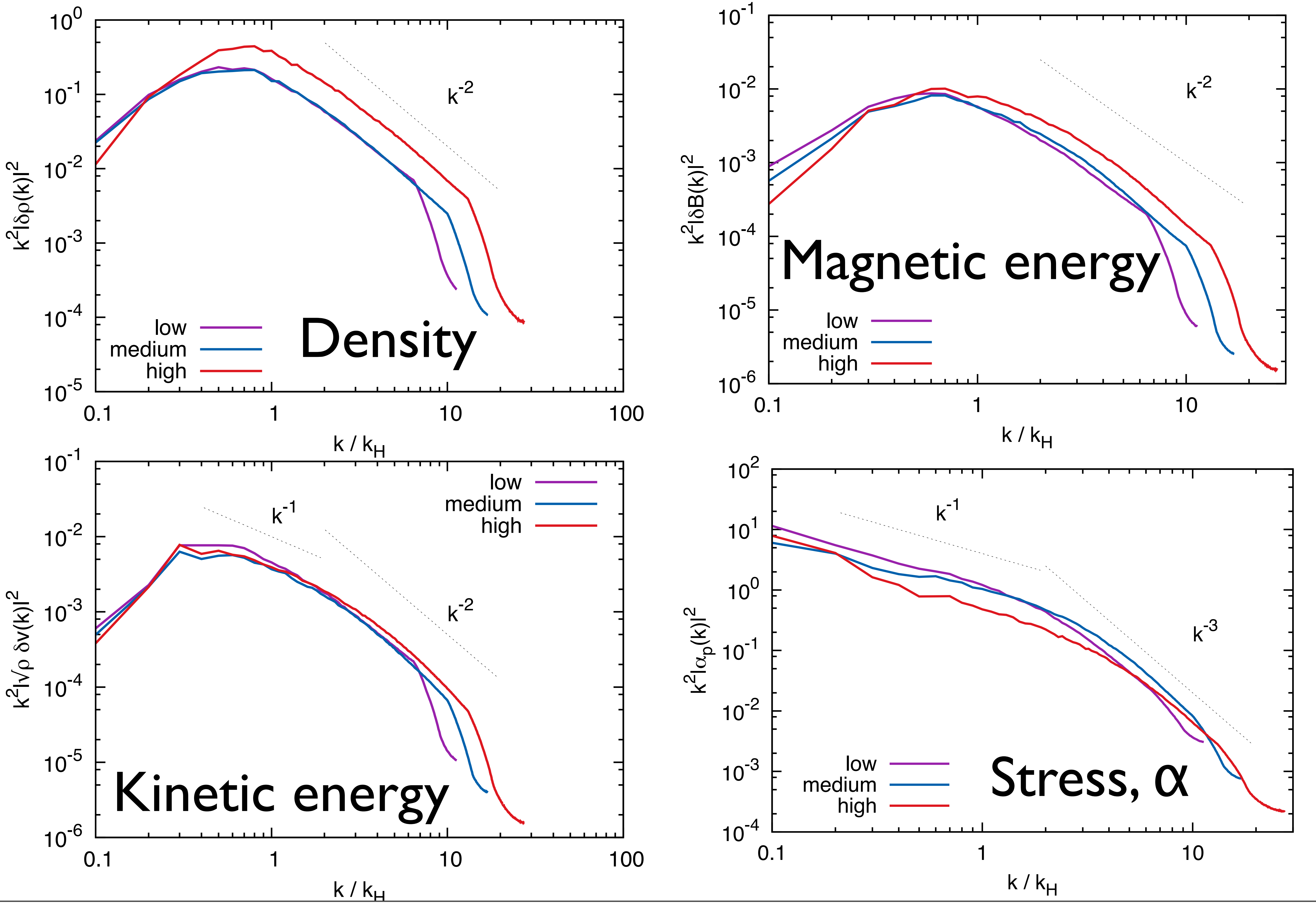
Convergence with resolution in global simulations



Convergence with resolution in global simulations

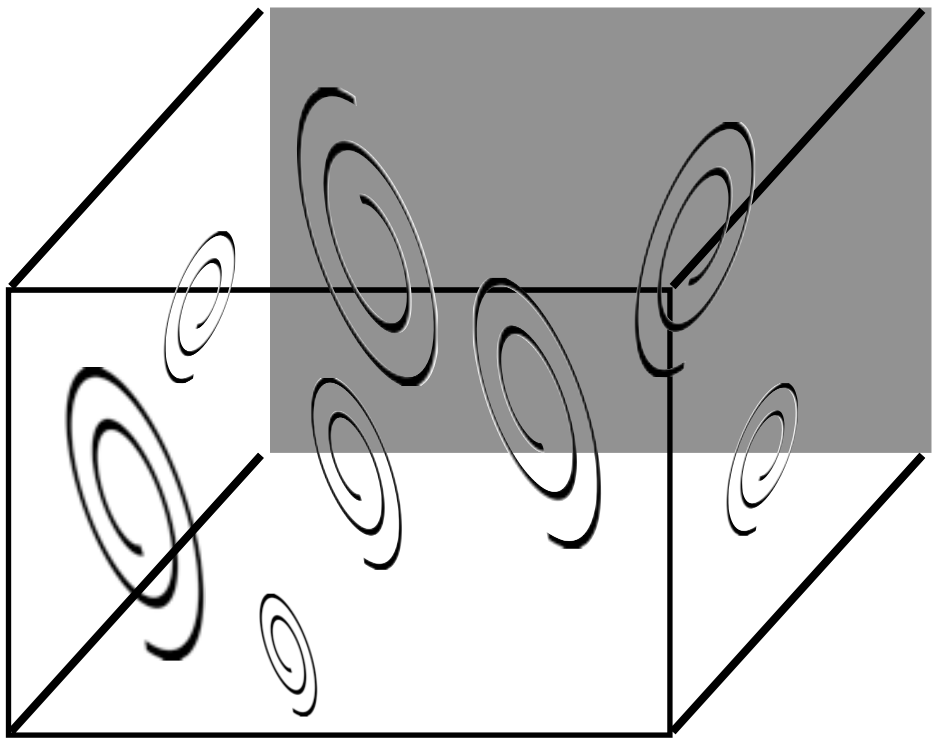


Power spectra



Magnetic energy maintenance

(Switch brain from real space to Fourier space)



Magnetic energy production must replenish energy on the largest scales.

Otherwise, the reservoir of energy at the largest scales will be drained by cascading of energy to smaller scales.

THE AUSTRALIAN NATIONAL UNIVERSITY

Convergence

Convergence

Convergence with increasing simulation resolution requires:

Convergence

Convergence with increasing simulation resolution requires:

- Convergence in magnetic energy input/injection on large scales

Convergence

Convergence with increasing simulation resolution requires:

- Convergence in magnetic energy input/injection on large scales
- Directly relates to the presence, or lack there-of, of magnetic field on large scales

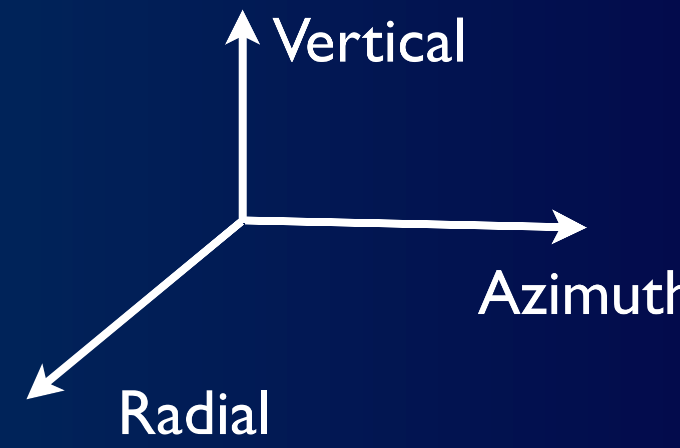
Convergence

Convergence with increasing simulation resolution requires:

- Convergence in magnetic energy input/injection on large scales
- Directly relates to the presence, or lack there-of, of magnetic field on large scales
- Can assess these factors using a volume integrated analysis of the magnetic energy and induction equations with appropriate boundary conditions - see Parkin & Bicknell, 2013, arXiv:1306.1084

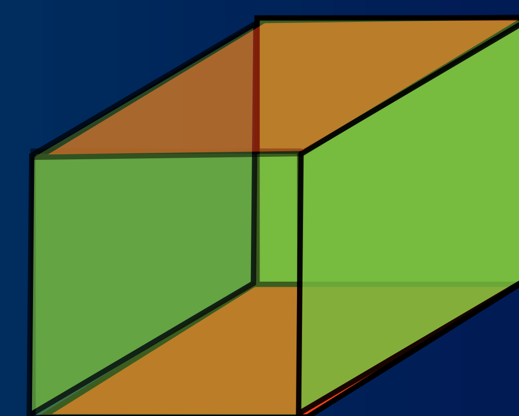
Boundary conditions for models

Periodic boundary   Open boundary

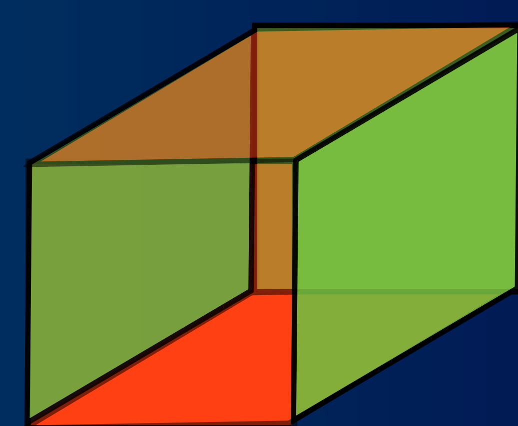


Stratified shearing box

(vertical surfaces are the disk-corona interface)



Global disk



Magnetic energy balance

The situation simplifies to,

$$\oint v_{\mathrm{i}} M_{\mathrm{ij}}^{\mathrm{B}} n_{\mathrm{j}} dS - \int v_{\mathrm{i}} \frac{\partial M_{\mathrm{ij}}^{\mathrm{B}}}{\partial x_{\mathrm{j}}} dV \approx \eta_{\mathrm{num}} \int |j|^2 dV.$$

, where

$$M_{\mathrm{ij}}^{\mathrm{B}} = B_{\mathrm{i}} B_{\mathrm{j}} - \delta_{\mathrm{ij}} u_{\mathrm{B}}$$

(Maxwell stress tensor)

Applying appropriate boundary conditions and preserving dominant terms, we have,

Shearing box

$$-q\Omega L_{\mathrm{x}} \int_{\mathrm{x}_1} B_{\mathrm{x}} B_{\mathrm{y}} dS_{\mathrm{x}} - \int v_{\mathrm{y}} B_{\mathrm{x}} \frac{\partial B_{\mathrm{y}}}{\partial x} dV \approx \eta_{\mathrm{num}} \int |j|^2 dV.$$

Global disk

$$-\int_{r_1} B_{\mathrm{r}} B_{\phi} v_{\phi} dS_{\mathrm{r}} - \int v_{\phi} B_{\mathrm{r}} \frac{\partial B_{\phi}}{\partial r} dV \simeq \eta_{\mathrm{num}} \int |j|^2 dV.$$

Magnetic energy balance

The situation simplifies to,

$$\oint v_{\mathrm{i}} M^{\mathrm{B}}_{\mathrm{ij}} n_{\mathrm{j}} dS - \int v_{\mathrm{i}} \frac{\partial M^{\mathrm{B}}_{\mathrm{ij}}}{\partial x_{\mathrm{j}}} dV \approx \eta_{\mathrm{num}} \int |j|^2 dV.$$

, where $M^{\mathrm{B}}_{\mathrm{ij}} = B_{\mathrm{i}} B_{\mathrm{j}} - \delta_{\mathrm{ij}} u_{\mathrm{B}}$

(Maxwell stress tensor)

Applying appropriate boundary conditions and preserving dominant terms, we have,

Shearing box

$$- q\Omega L_{\mathrm{x}} \int_{\mathrm{x}_1} B_{\mathrm{x}} B_{\mathrm{y}} dS_{\mathrm{x}} - \int v_{\mathrm{y}} B_{\mathrm{x}} \frac{\partial B_{\mathrm{y}}}{\partial x} dV \approx \eta_{\mathrm{num}} \int |j|^2 dV.$$

Terms on LHS are larger in a global model due to open radial boundaries and large scale radial gradients

Global disk

$$- \int_{r_1} B_{\mathrm{r}} B_{\phi} v_{\phi} dS_{\mathrm{r}} - \int v_{\phi} B_{\mathrm{r}} \frac{\partial B_{\phi}}{\partial r} dV \simeq \eta_{\mathrm{num}} \int |j|^2 dV.$$

Summary

THE AUSTRALIAN NATIONAL UNIVERSITY

Summary

- Our initially purely toroidal field global disk models converge with $\alpha \sim 0.04$, in agreement with Beckwith+(2011) and Hawley +(2013)

THE AUSTRALIAN NATIONAL UNIVERSITY

Summary

- Our initially purely toroidal field global disk models converge with $\alpha \sim 0.04$, in agreement with Beckwith+(2011) and Hawley +(2013)
- Convergence at lower resolution in a global model (~30 cells/ H) than in a local model (stratified shearing-box ~ 64-128 cells/H) (- see Davis+2010; Hawley+2011 and refs there-in)

THE AUSTRALIAN NATIONAL UNIVERSITY

Summary

- Our initially purely toroidal field global disk models converge with $\alpha \sim 0.04$, in agreement with Beckwith+(2011) and Hawley +(2013)
- Convergence at lower resolution in a global model (~30 cells/H) than in a local model (stratified shearing-box ~ 64-128 cells/H) (- see Davis+2010; Hawley+2011 and refs there-in)
- Arises because boundary conditions in global models allow more effective energy generation than in a shearing-box (Parkin & Bicknell, 2013, arXiv:1306.1084)

THE AUSTRALIAN NATIONAL UNIVERSITY