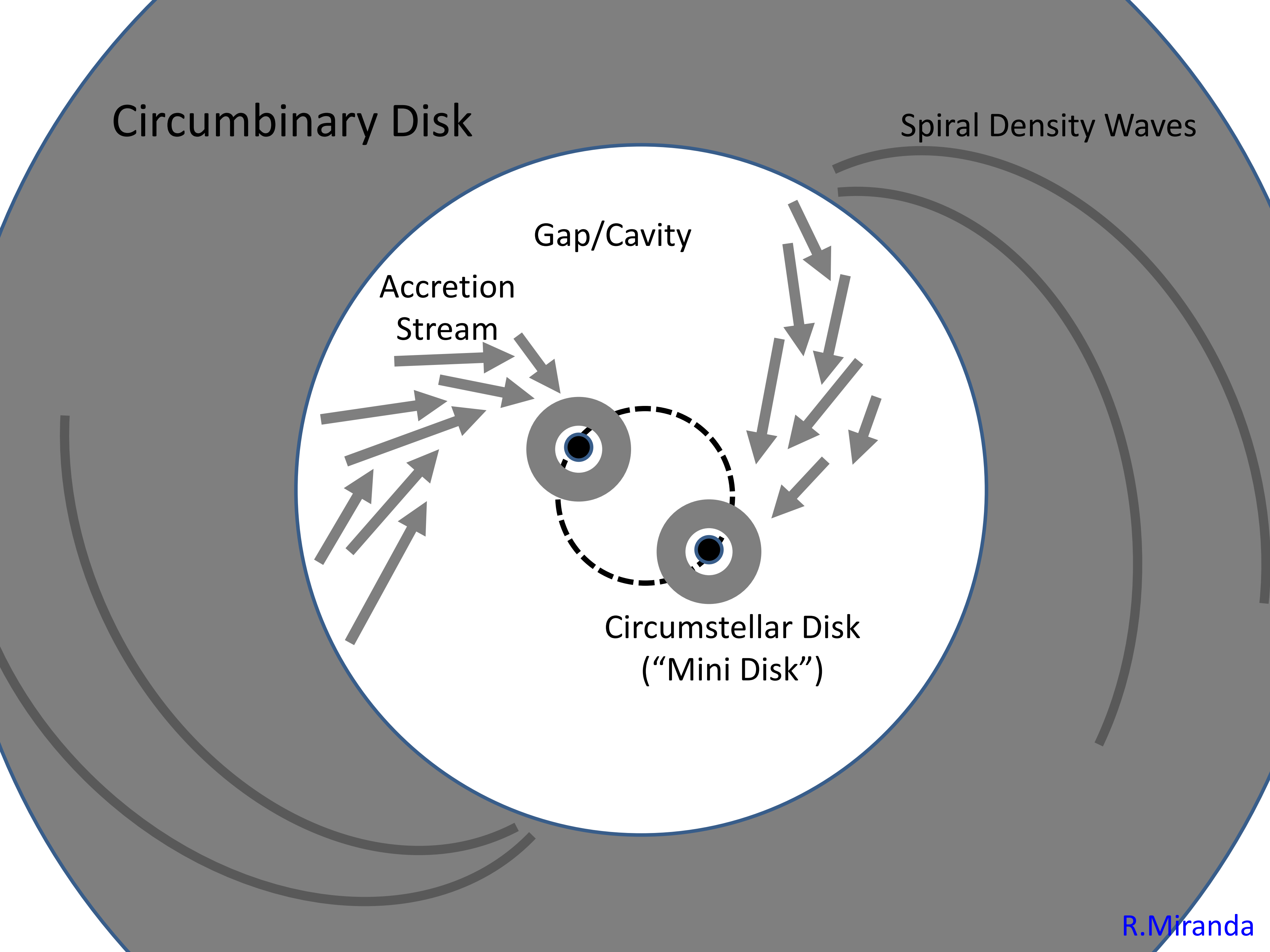
Disks Around Binaries
Binaries (& Binary Formation) in Big Disks

Dong Lai

Cornell University

"Building Bridges: A Unified Pictures of Stellar and BH Binary", KITP, 3/16/2022

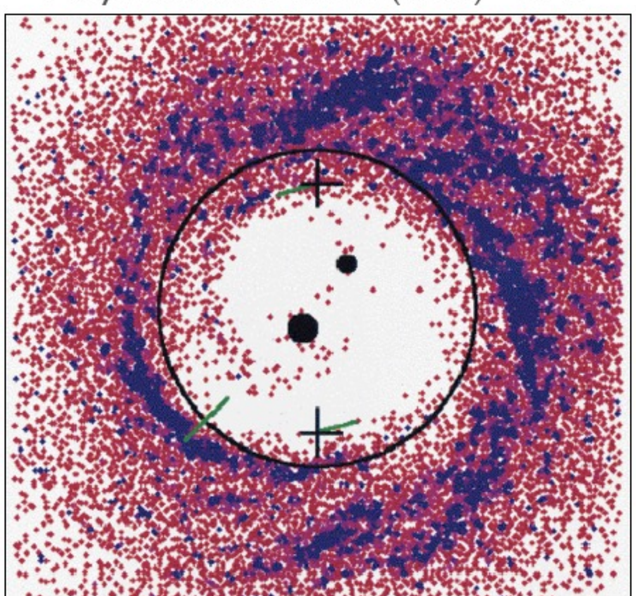

Circumbinary Disk
Spiral Density Waves
Gap/Cavity
Accretion
Stream
Circumstellar Disk
("Mini Disk")
R.Miranda

Simulations of Circumbinary Accretion

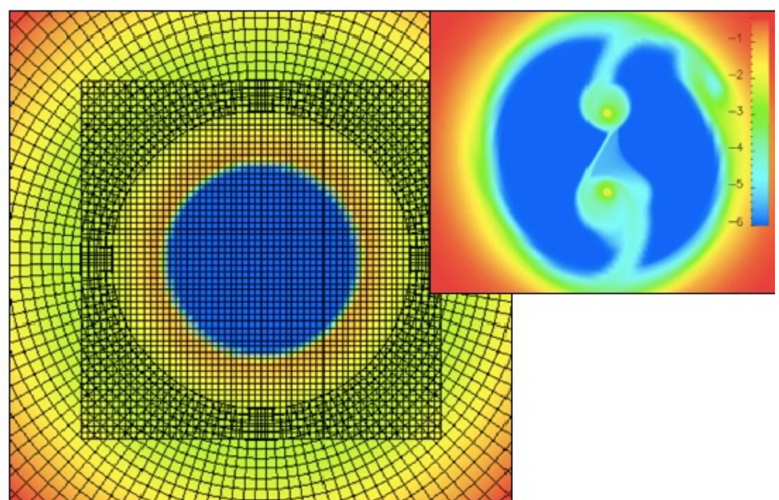
Artymowicz & Lubow 1996; Günther & Kley 02; MacFadyen & Milosavljević 08; Cuadra et al.09; Hanawa et al. 10; de Val-Borro et al. 11; Roedig et al. 12; Noble et al.12; Shi et al. 12; D'Orazio et al. 13; Pelupessy & Portegies-Zwart 13; Farris et al. 14; Shi & Krolik 15; Lines et al. 15; O'Ozario et al. 16; Ragusa et al. 16, Munoz & Lai 2016; Miranda, Munoz & Lai 2017; Tang et al. 17; Bowen et al.17,19; Munoz, Miranda, Lai 2019; Moody, Shi & Stone 19; Munoz, Lai et al.2020; Duffell et al.20; Tiede et al. 20; Heath & Nixon 20; D'Orazio & Duffell 21; Zrake et al.21; Penzlin et al.22; Siwek et al.22...

Some pioneering works:

Artymowicz & Lubow (1996) – SPH



Günther & Kley (2002) – Hybrid grid



Simulations of Circumbinary Accretion

Artymowicz & Lubow 1996; Günther & Kley 02; MacFadyen & Milosavljević 08; Cuadra et al.09; Hanawa et al. 10; de Val-Borro et al. 11; Roedig et al. 12; Noble et al.12; Shi et al. 12; D'Orazio et al. 13; Pelupessy & Portegies-Zwart 13; Farris et al. 14; Shi & Krolik 15; Lines et al. 15; O'Ozario et al. 16; Ragusa et al. 16, Munoz & Lai 2016; Miranda, Munoz & Lai 2017; Tang et al. 17; Bowen et al.17,19; Munoz, Miranda, Lai 2019; Moody, Shi & Stone 19; Munoz, Lai et al.2020; Duffell et al.20; Tiede et al. 20; Heath & Nixon 20; D'Orazio & Duffell 21; Zrake et al.21; Penzlin et al.22; Siwek et al.22...

Many simulations excised the inner "cavity"

Some cover the whole domain: Circumbinary disk → stream → circumsingle disks:

Using finite-volume moving mesh codes:

- DISCO: Farris, Duffell, MacFadyen, Haiman 2014...
- AREPO: resolve accretion onto individual body to $0.02a_b$
  (Munoz & Lai 2016; Munoz, Miranda & Lai 2019; Munoz, Lai et al 2020...)
- ATHENA++ (Moody, Shi & Stone 2019)

Summary of Key Dynamical Results

- Short-term variabilities
- Long-term variabilities
- Angular momentum transfer and binary evolution

Our works:

-- Solve viscous hydrodynamic equations in 2D

-- alpha viscosity, (locally) isothermal sound speed

$$\text{Disk } H/r \sim 0.1, \ \alpha = 0.05 - 0.1 \ (\text{down to } 0.01)$$

Diego Munoz

(Harvard PhD'13->Cornell -> Northwestern)

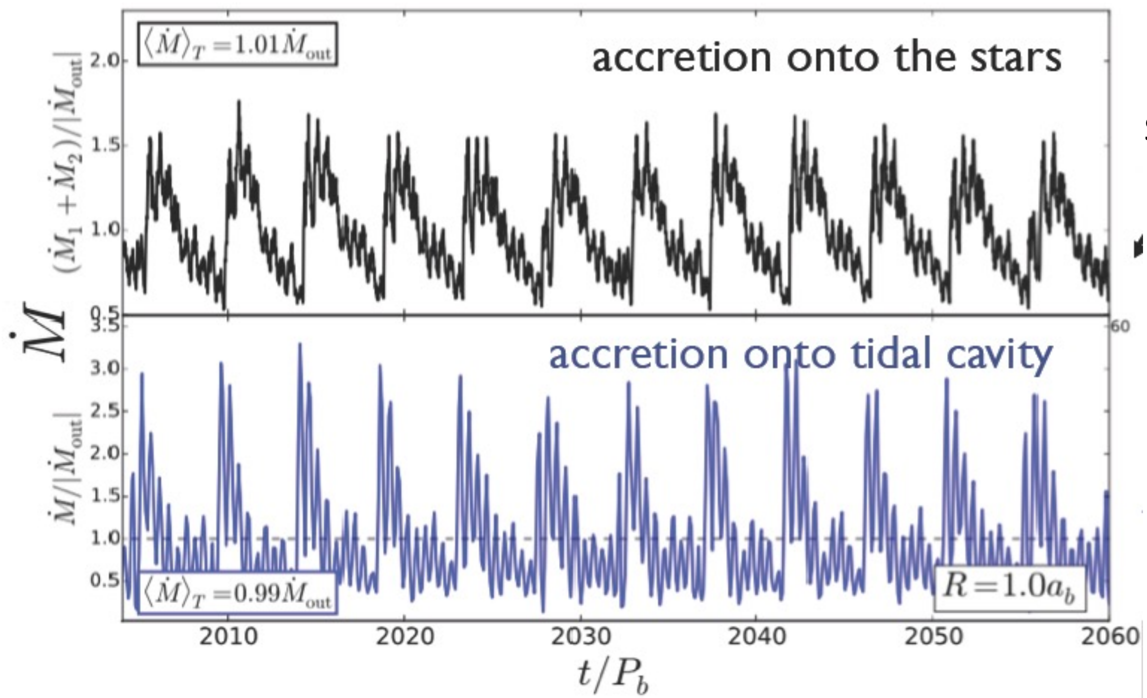
Ryan Miranda

(Cornell Ph.D.17 → IAS →industry)

Short-term (~P_b) Accretion Variabilities

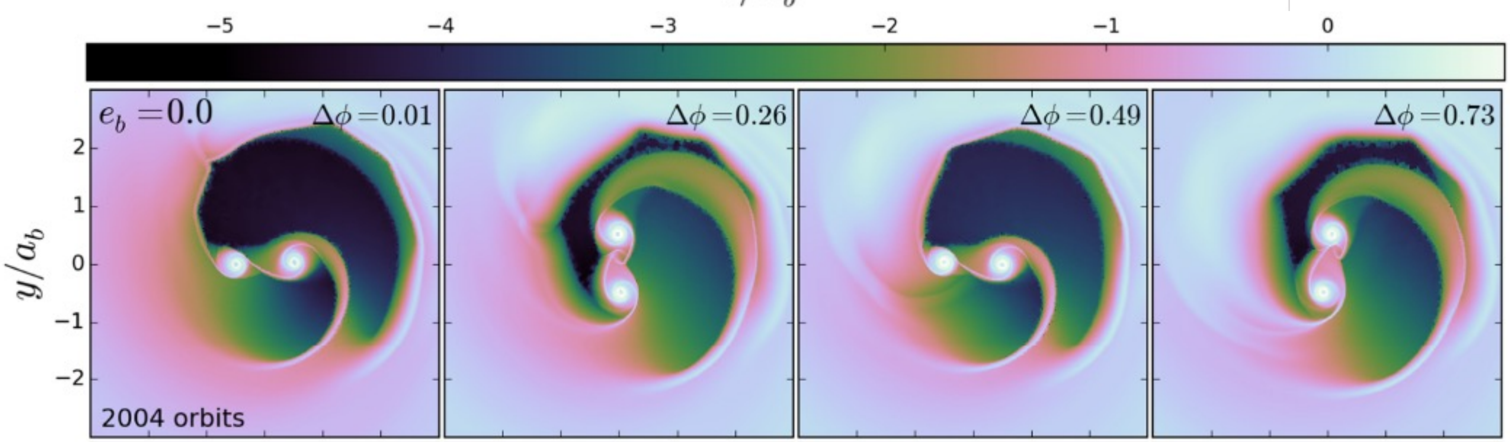
For $e_b \lesssim 0.05$: $\dot{M}(= \dot{M}_1 + \dot{M}_2)$ varies at $\sim 5P_b$ (Kepler period at r_{in} ~ $3a_b$)

e_b=0



Known from MacFadyen & Milosavljevic 08, Shi et al.12, D'Orazio et al.13, Farris et al.14

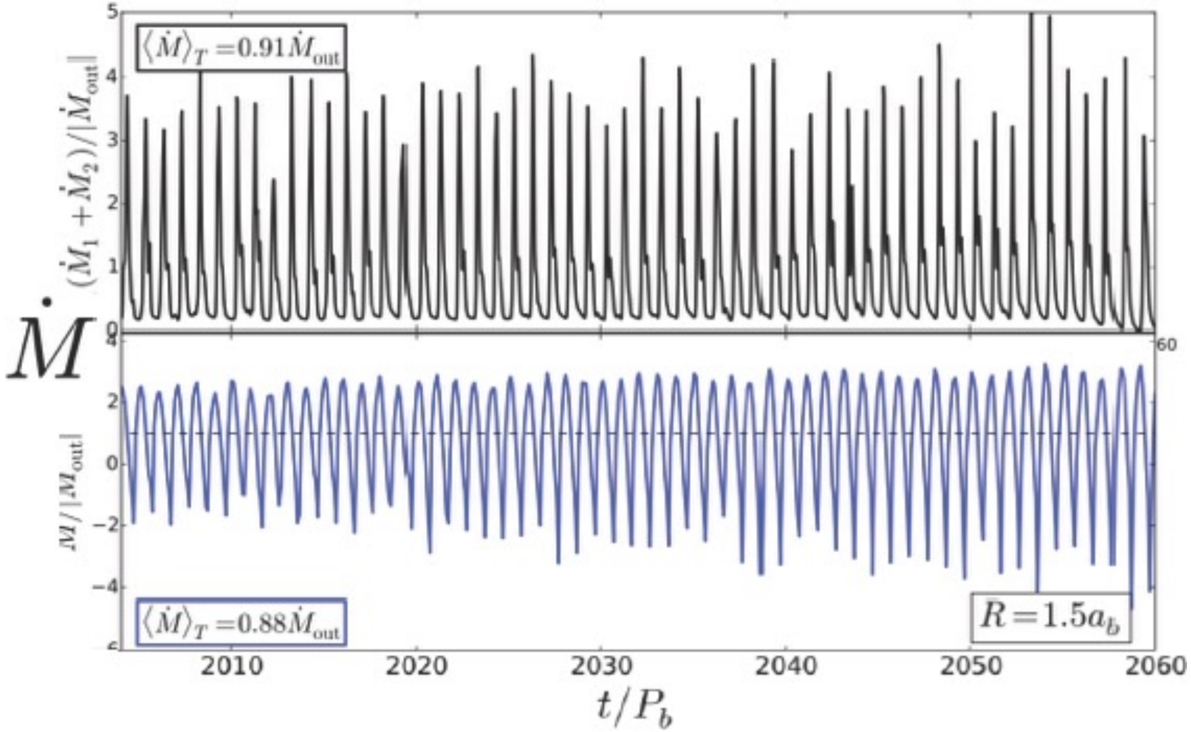
Munoz & DL 16



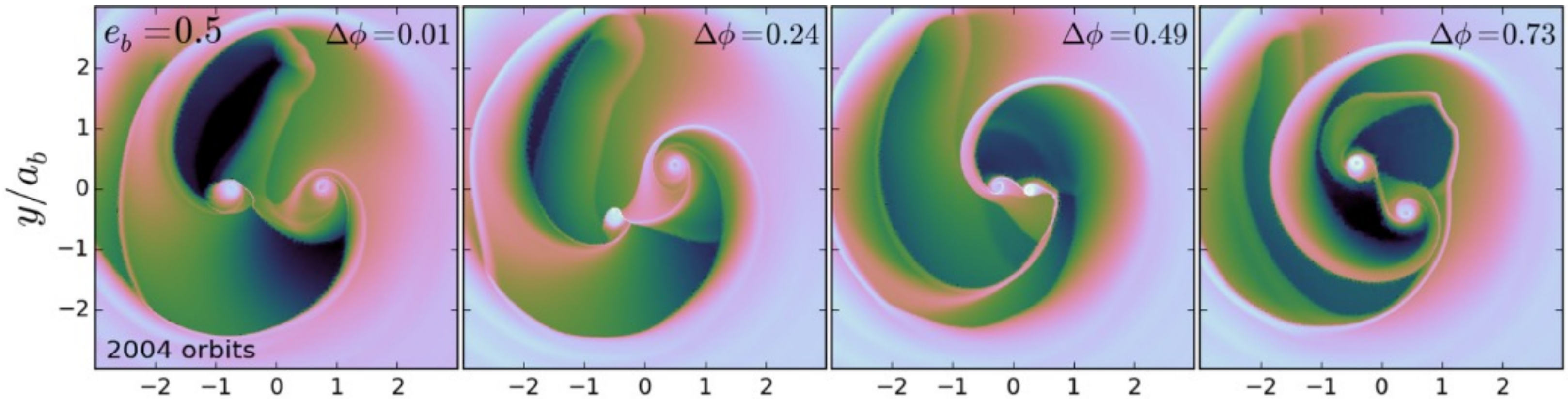
Short-term (~P_b) Accretion Variabilities

For $e_b \gtrsim 0.05$: $\dot{M} = \dot{M}_1 + \dot{M}_2$ varies at $\simeq P_b$

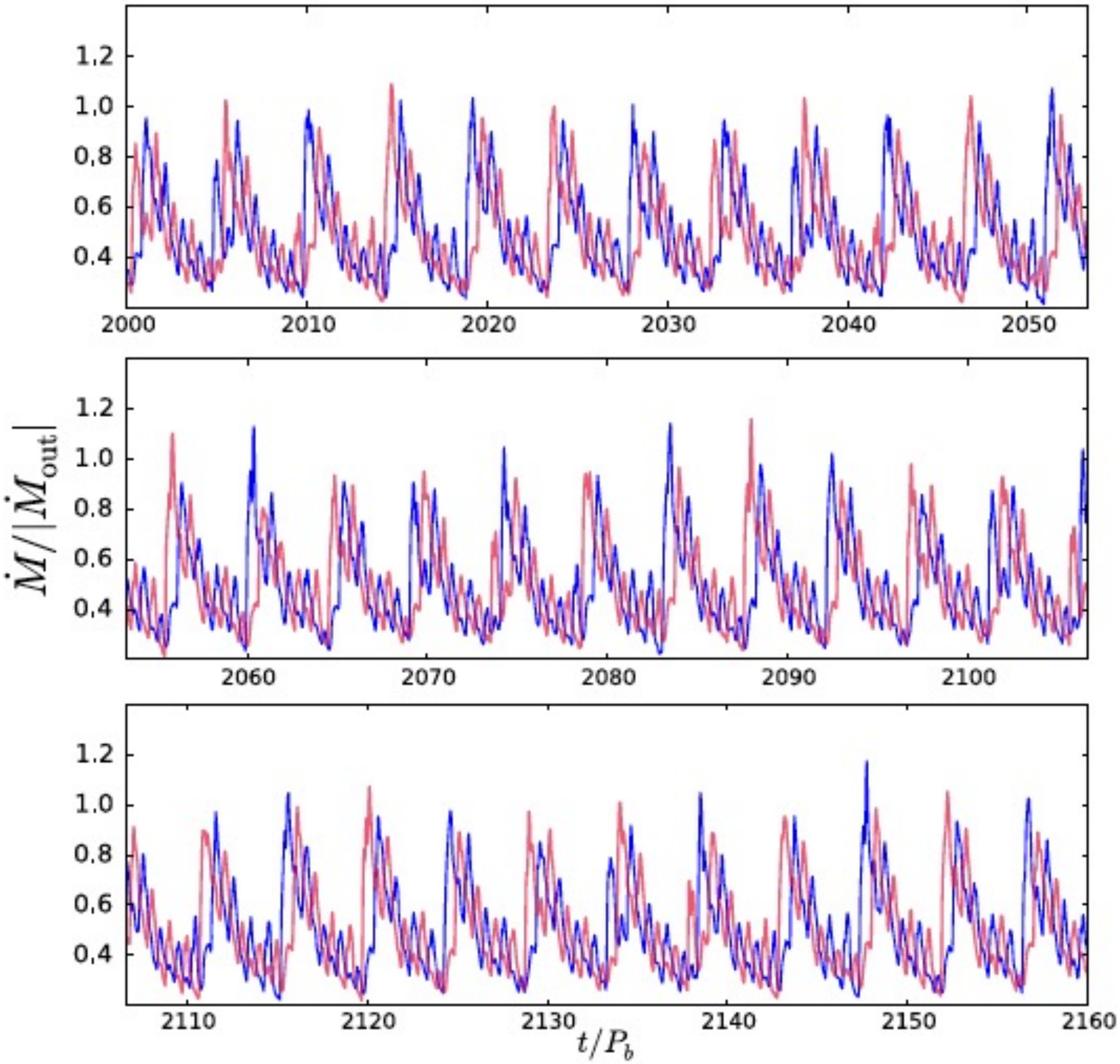
e_b=0.5



Munoz & DL 16



Long-Term Variability:

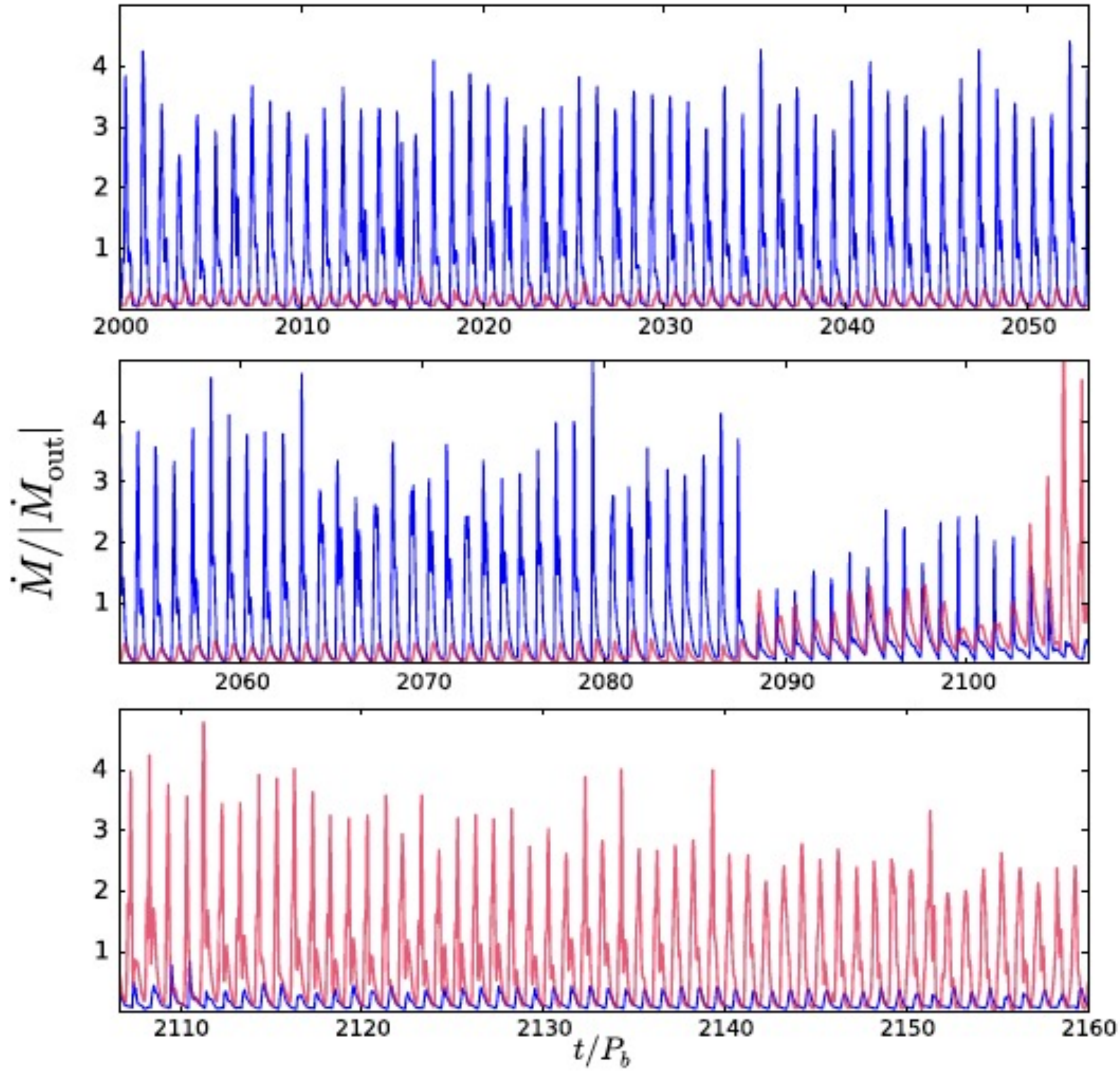


e_b=0

q_b=1

$$\dot{M}_1 \simeq \dot{M}_2$$

Long-Term Variability: Symmetry Breaking



e_b=0.5

q_b=1

Switch between

$\dot{M}_1 \gtrsim 20\dot{M}_2$

and

$\dot{M}_2 \gtrsim 20\dot{M}_1$

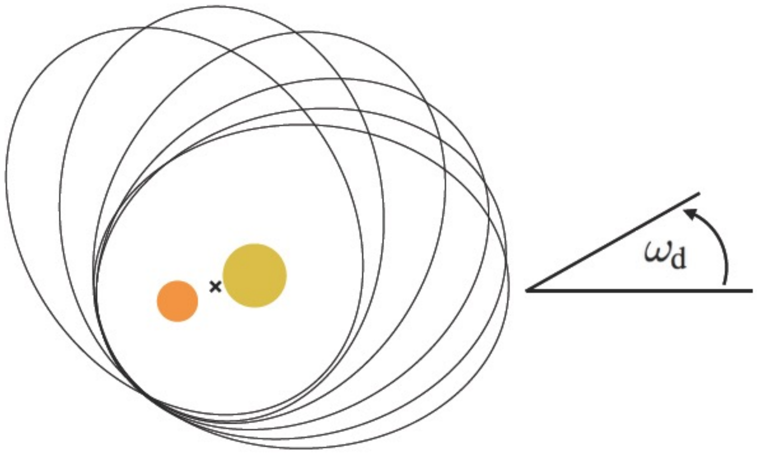
every ~200 P_b

Munoz & Lai 2016

Single AGN with binary BHs ?

Apsidal precession of eccentric disk around the binary

$$\dot{\omega}_d \simeq \frac{3\Omega_b}{4}\frac{q_b}{(1+q_b)^2}\left(1+\frac{3}{2}e_b^2\right)\left(\frac{a_b}{R}\right)^{7/2}$$
$$\sim 0.006\,\Omega_b\left(\frac{3a_b}{R}\right)^{7/2},$$



Precession period 200-300 P_b

Theory of eccentric disks around binary: see Miranda, Munoz & Lai 2017
Munoz & Lithwick (2020)
Wang HY, Bai, Lai (2022)

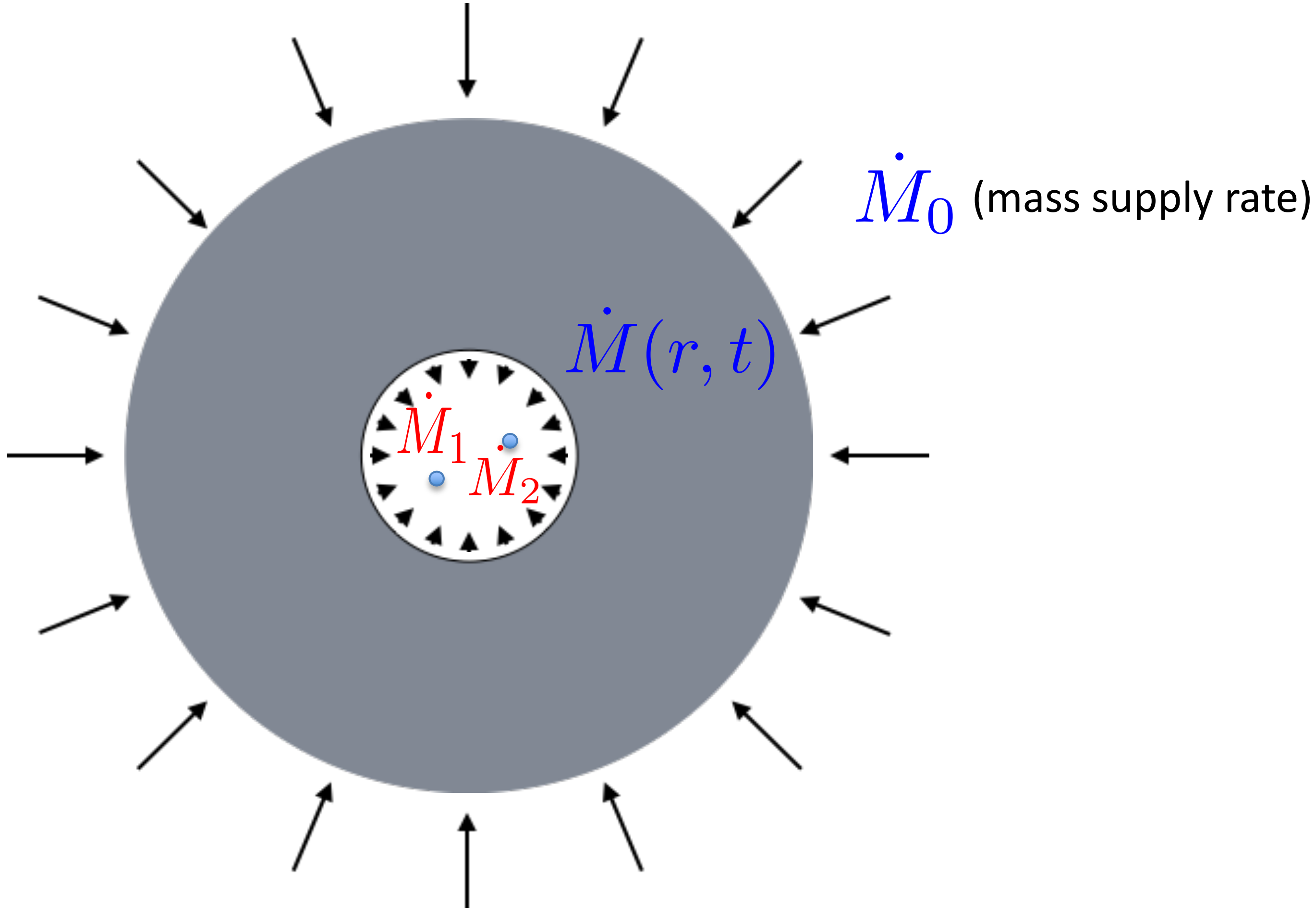
Angular Momentum Transfer to Binary and Long-term Orbital Evolution

Many claims of orbital decays (1980s-2017):

Suppressed accretion onto binary (?), binary loses AM through outer Lindblad torque …

First indication of orbital expansion: Miranda, Munoz & Lai 2017
(using PLUTO, excised cavity)

But see Matthew Bate's talk (in star formation) context on Monday



$\dot{M}(r,t)$, $\dot{M}_1$, $\dot{M}_2$ are highly variable

Quasi-Steady State: $\langle \dot{M}(r,t) \rangle = \langle \dot{M}_1 \rangle + \langle \dot{M}_2 \rangle = \dot{M}_0$

Angular Momentum Current (Transfer Rate) in CBD

$$\dot{J}(r,t) = \dot{J}_{\rm adv} - \dot{J}_{\rm visc} - T_{\rm grav}^{>r}$$

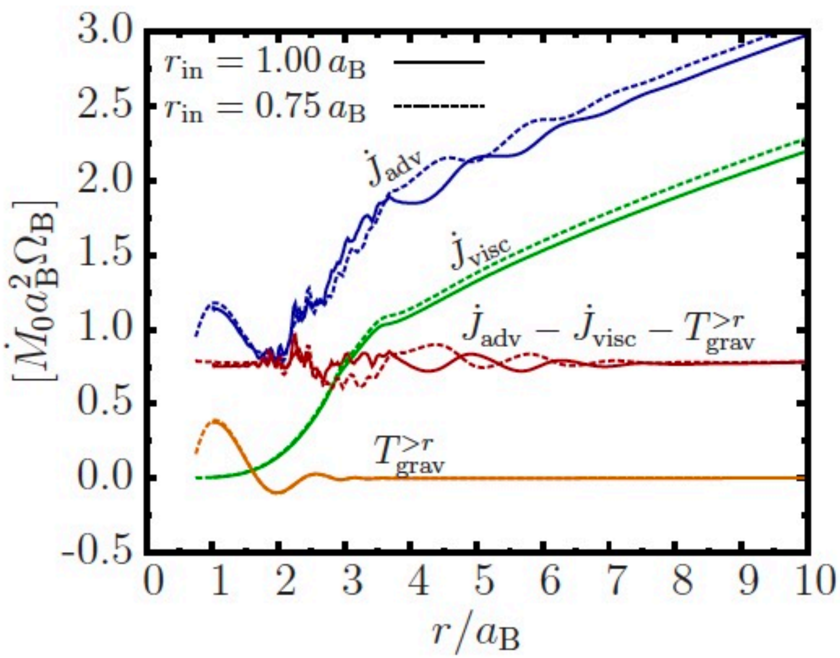
$$\dot{J}_{\rm adv} = -\oint r^2 \Sigma u_r u_\phi \mathrm{d}\phi$$

$$\dot{J}_{\rm visc} = -\oint r^3 \nu \Sigma \left[\frac{\partial}{\partial r}\left(\frac{u_\phi}{r}\right) + \frac{1}{r^2}\frac{\partial u_{\rm r}}{\partial \phi}\right] \mathrm{d}\phi$$

$$T_{\rm grav}^{>r} = \int_r^{r_{\rm out}} \frac{\mathrm{d}T_{\rm grav}}{\mathrm{d}r}\mathrm{d}r, \qquad \frac{\mathrm{d}T_{\rm grav}}{\mathrm{d}r} = -\oint r\Sigma \frac{\partial \Phi}{\partial \phi}\mathrm{d}\phi$$

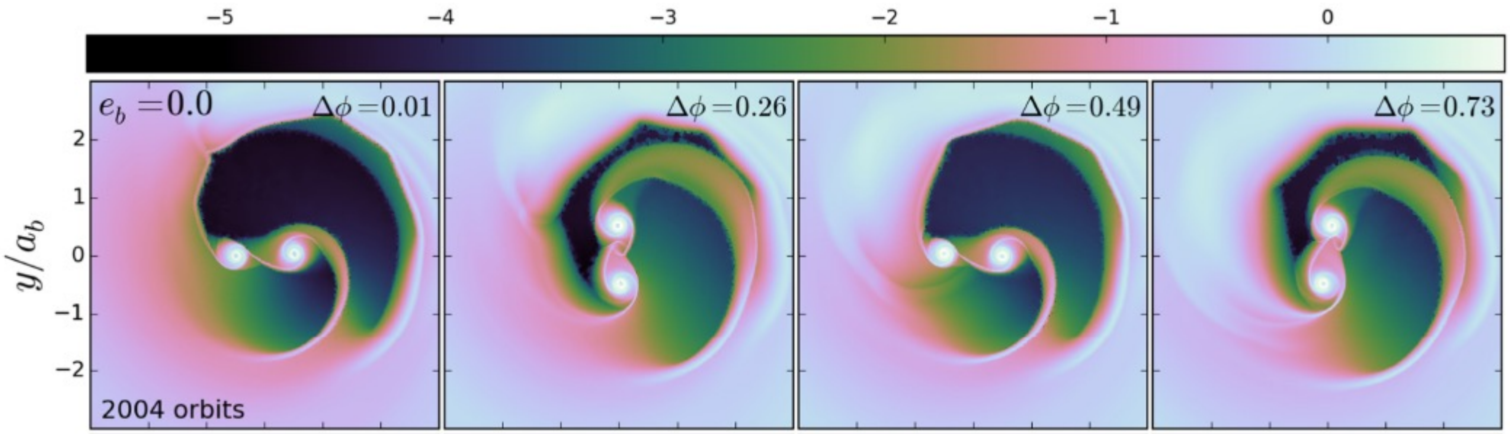
Miranda, Munoz & Lai (2017) found:

$$\langle \dot{J} \rangle = \mathrm{const} \simeq (0.7 a_{\rm B}^2 \Omega_{\rm B}) \langle \dot{M} \rangle$$



Definitive proof requires simulation of the cavity

Munoz & Lai 2016
Munoz et al. 2019



AREPO

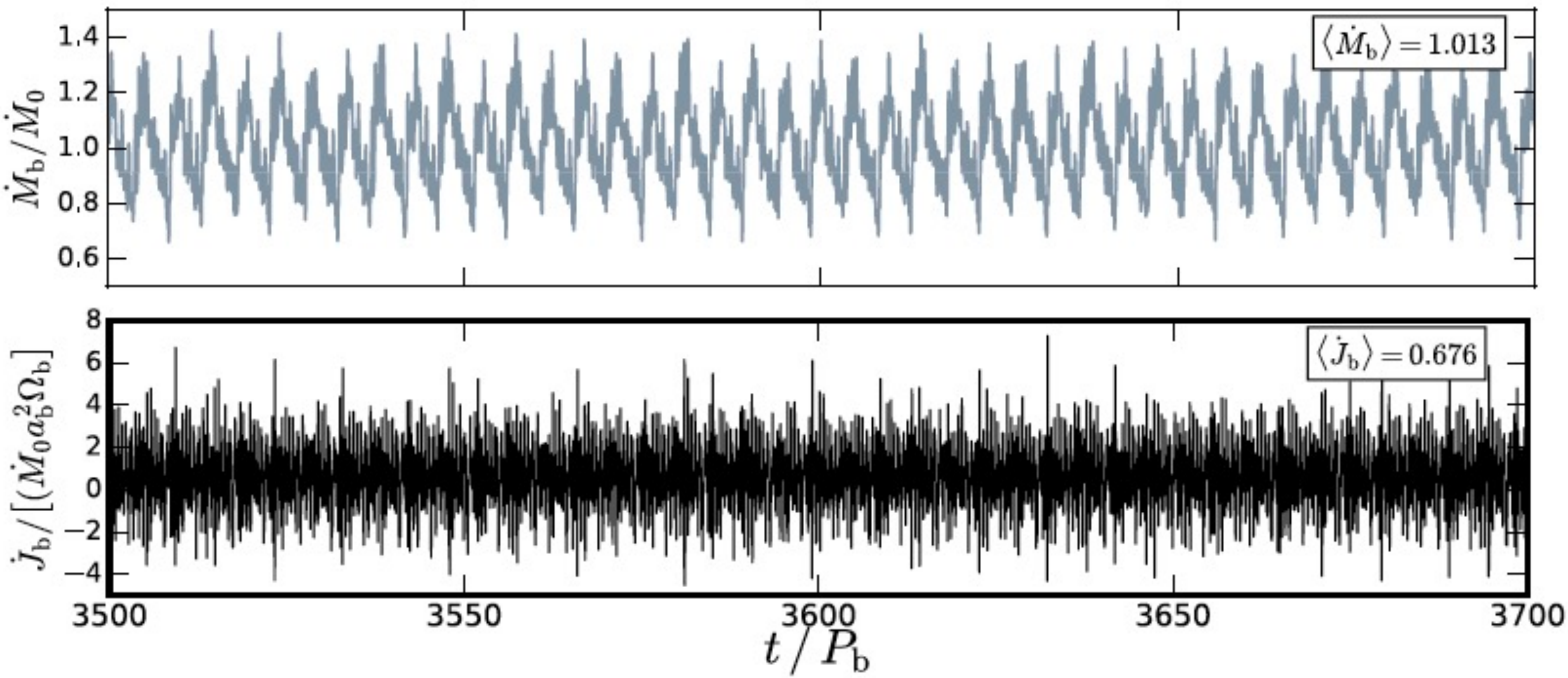
Direct computation of torque on the binary

Gravitational torque from all gas + Accretion torque

$$\dot{J}_b = (\dot{L}_b)_{\rm grav} + (\dot{L}_b)_{\rm acc} + (\dot{S}_1)_{\rm acc} + (\dot{S}_2)_{\rm acc}$$

NOTE: Use "passive binary" with prescribed motion; no "Newton's 3rd law" problem.
NOTE: I now think we should have added torque from pressure (see Rixin Li & Lai 2022 for method)

Direct computation of torque on the binary



$$l_0 \equiv \frac{\langle \dot{J}_b \rangle}{\langle \dot{M}_b \rangle} = 0.68 a_b^2 \Omega_b$$

$e_b = 0$

Angular momentum transfer to the binary per unit accreted mass

Recap: Although the accretion flow is highly dynamical, the system reaches quasi-steady state:

$$\langle \dot{M}(r,t) \rangle = \langle \dot{M}_1 \rangle + \langle \dot{M}_2 \rangle = \dot{M}_0$$

$$\langle \dot{J}_b \rangle \simeq \langle \dot{J}_{\rm disk}(r,t) \rangle = {\rm const}$$

Angular momentum transferred to the binary per unit accreted mass:

$$l_0 \equiv \frac{\langle \dot{J}_b \rangle}{\langle \dot{M}_b \rangle} = 0.68 a_b^2 \Omega_b$$

Munoz, Miranda & DL 2019

Confirmed by Moody, Shi & Stone 2019 (ATHENA++)
Duffell et al. (2020), ….

Implication of $\dot{J}_B > 0$:

For $q = 1$, $e_B = 0$ binary:

$$\dot{J}_B = \dot{M}_B l_0 \qquad l_0 \simeq 0.68\, l_B \qquad \text{where } l_B = a_B^2 \Omega_B$$

$$\rightarrow \quad \frac{\dot{a}_\mathrm{B}}{a_\mathrm{B}} = 8\left(\frac{l_0}{l_\mathrm{B}} - \frac{3}{8}\right)\frac{\dot{M}_\mathrm{B}}{M_\mathrm{B}}$$

Binaries can expand due to circumbinary accretion !

For e_B=0: $$\frac{\dot{a}_B}{a_B} \simeq 2.68 \frac{\dot{M}_B}{M_B}$$

Munoz, Miranda & Lai 2019

Eccentric Binaries

To obtain $\dot{a}_b$ and $\dot{e}_b$, we need $\dot{J}_b$ and $\dot{E}_b$

$$\mathcal{E}_b \equiv \frac{1}{2}\dot{\mathbf{r}}_b^2 - \frac{GM_b}{r_b} \qquad \text{where } \mathbf{r}_b = \mathbf{r}_1 - \mathbf{r}_2,\ M_b = M_1 + M_2$$

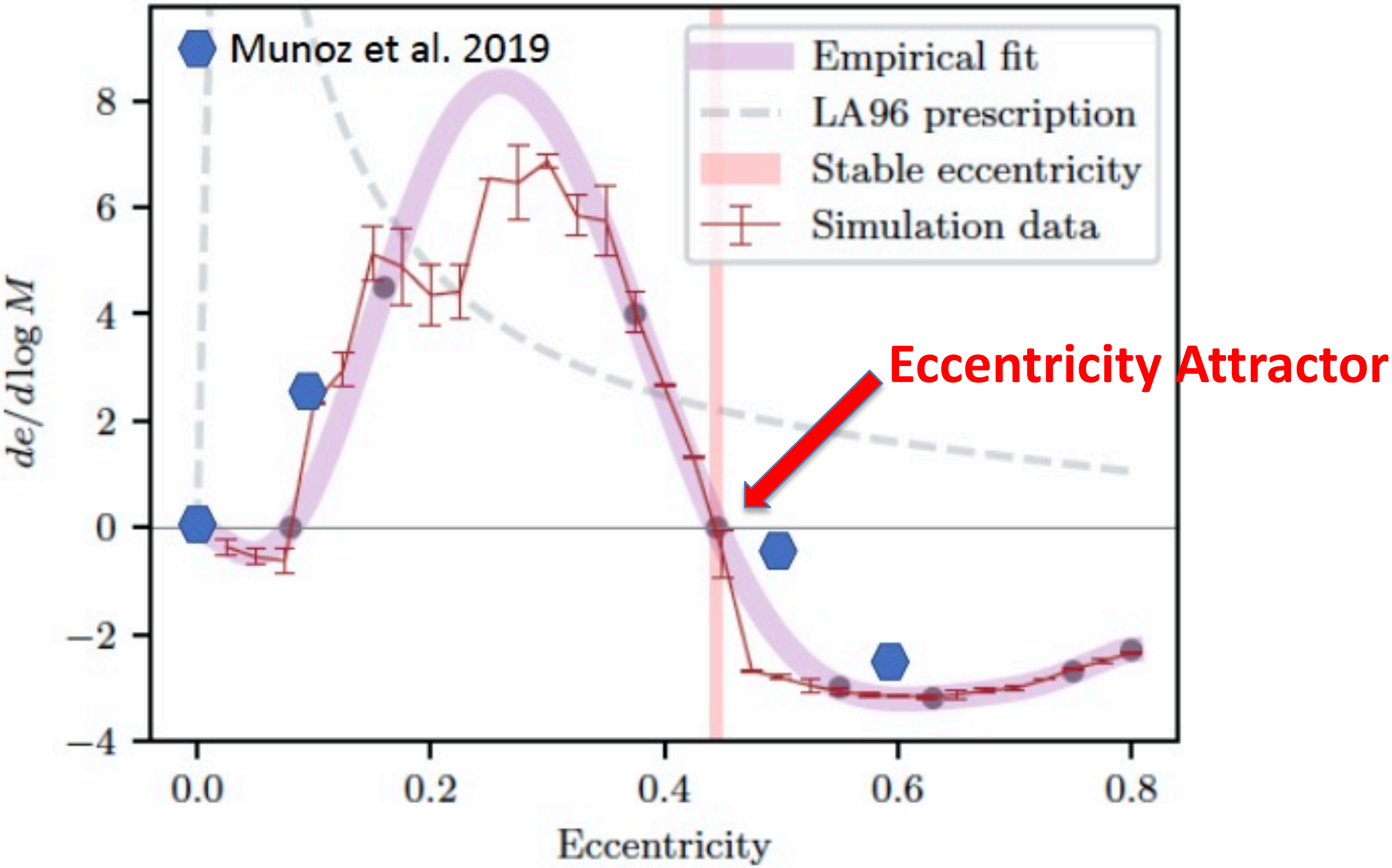
$$\frac{d\mathcal{E}_b}{dt} = -\frac{G\dot{M}_b}{r_b} + \dot{\mathbf{r}}_b \cdot (\mathbf{f}_1 - \mathbf{f}_2)$$

$$\mathbf{f}_1 = (\text{force/mass on } M_1) = \mathbf{f}_{1,\text{gravity}} + \mathbf{f}_{2,\text{accretion}}$$

Munoz et al. 2019

e_b	$\dot{J}_b$ $[\dot{M}_b a_b^2 \Omega_b]$	$\dot{a}_b/a_b$ $[\dot{M}_b/M_b]$	$\dot{e}_b$ $[\dot{M}_b/M_b]$
0	0.68	2.2	0.0
0.1	0.43	0.75	2.4
0.5	0.78	0.95	-0.20
0.6	0.81	0.47	-2.34

Eccentric Binaries



Zrake et al. 2021
See also D'Orazio & Duffell 2021

Unequal-mass binaries

$q = M_2/M_1 < 1$

$e_b = 0$ Munoz, Lai, Kratter, Miranda 2020

See also Duffell+2020

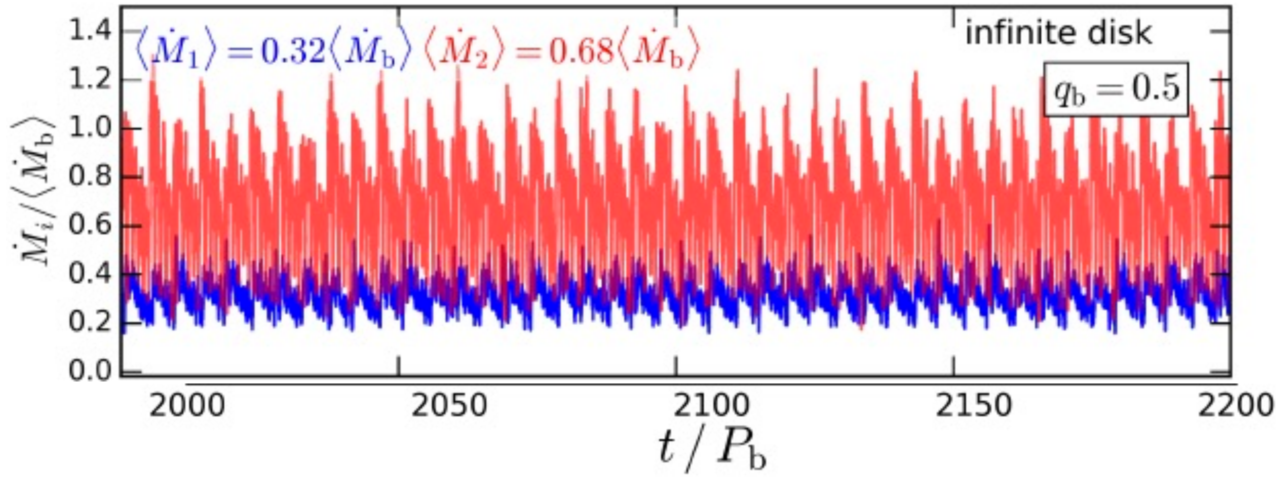
Unequal-mass binaries

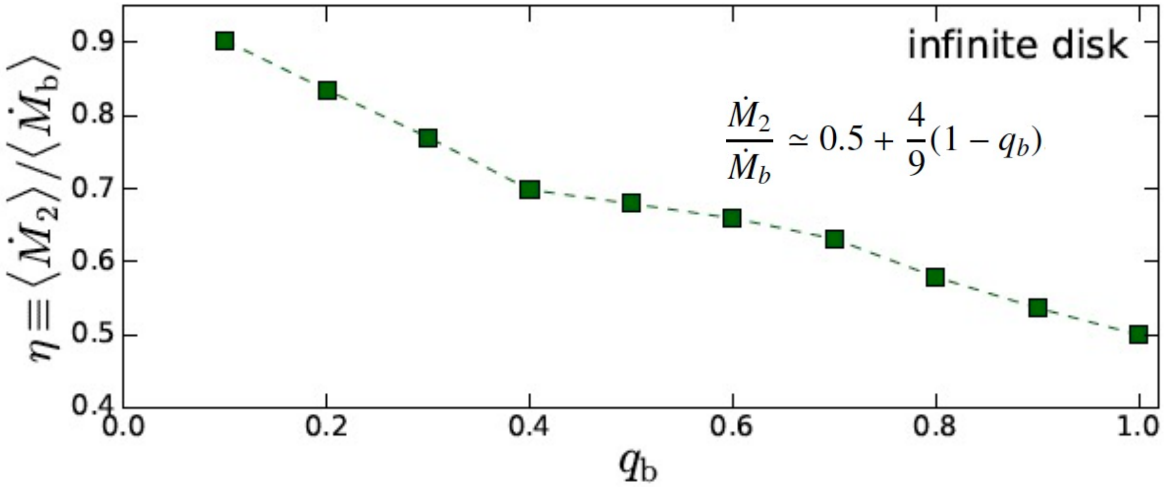
$q = M_2/M_1 < 1$

$e_b = 0$ Munoz, Lai, Kratter, Miranda 2020

-- Low-mass component accretes more

See also Bate+2000; Farris+2014





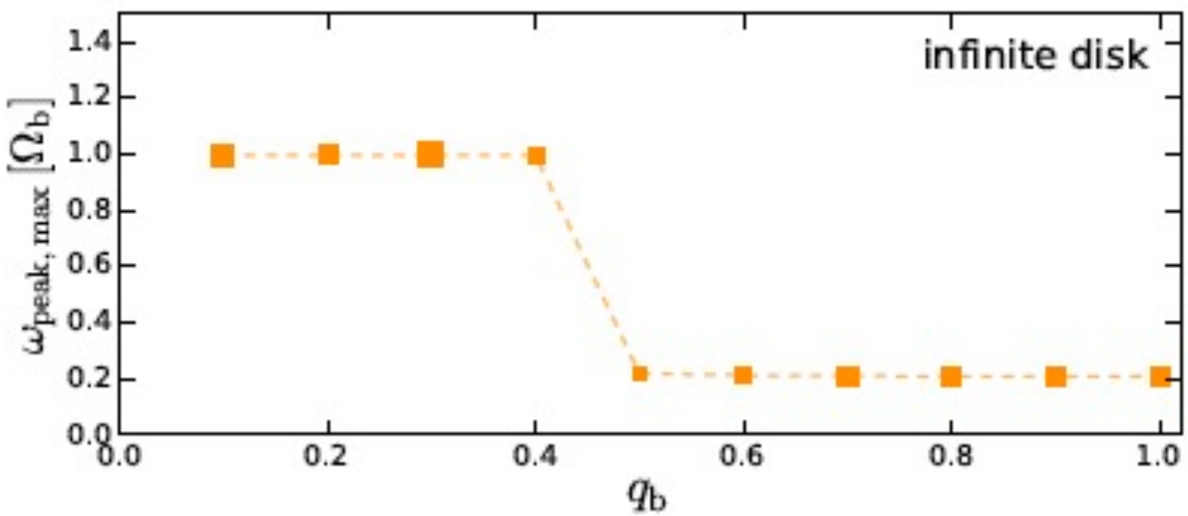
Unequal-mass binaries

$q = M_2/M_1 < 1$

$e_b = 0$

Munoz, DL +2020

-- Dominant variability frequency



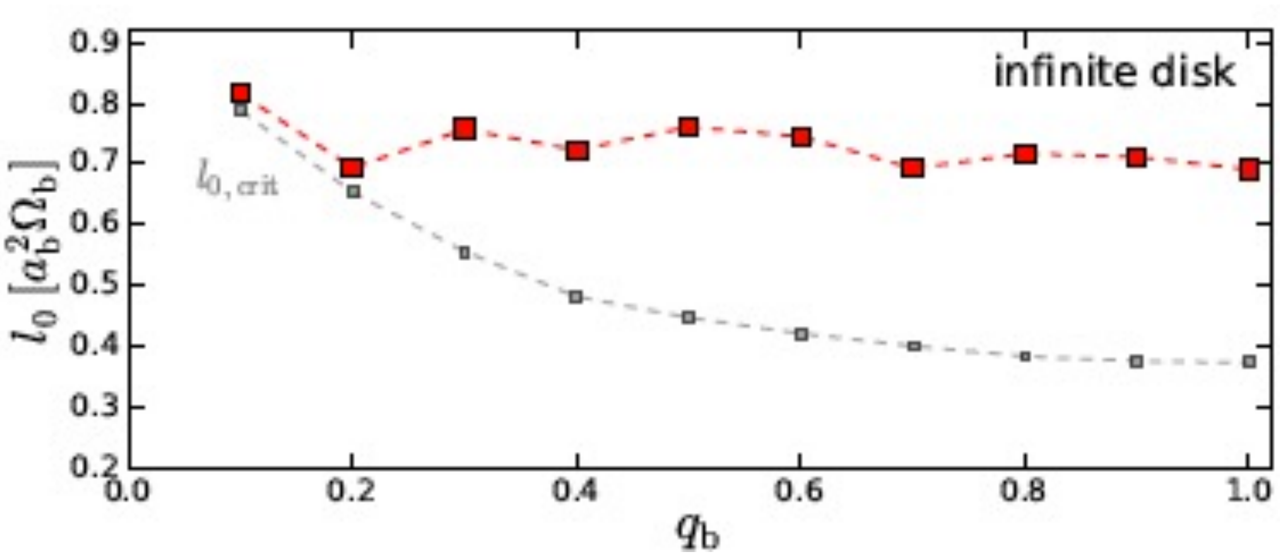
Unequal-mass binaries

$q = M_2/M_1 < 1$

$e_b = 0$

Munoz, DL +2020

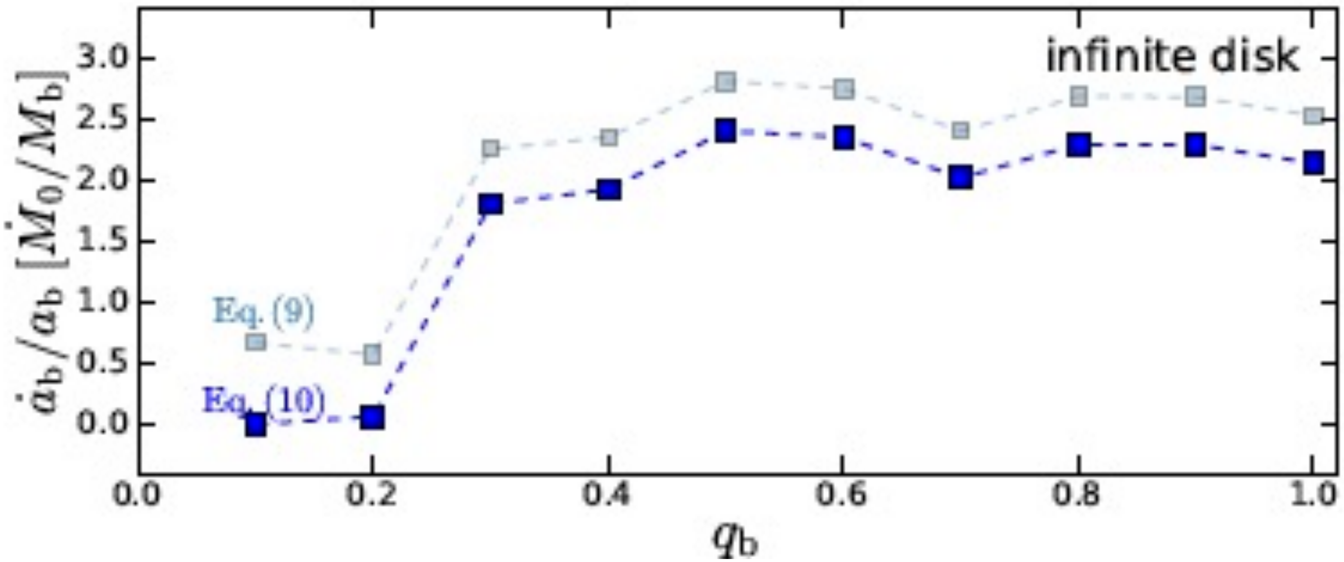
-- Angular momentum transfer



Unequal-mass binaries

$q = M_2/M_1 < 1$
$e_b = 0$

-- Orbit evolution



Munoz, DL +2020

See also Duffell et al. 2020: $\dot{a}_b < 0 \text{ for } q_b \lesssim 0.05$

Unequal-mass, eccentric binaries:

see M.Siwek, Weinberger, Munoz, Hernquist, arXiv:2203.02514

Recap:

In quasi-steady state, comparable-mass binary can expand while accreting from CBD

Is binary decay possible ?

e.g. Supermassive BH Binaries, final pc problem
e.g. Formation of close (AU) stellar binaries?

Is binary decay possible ?

e.g. Supermassive BH Binaries, final pc problem
e.g. Formation of close (AU) stellar binaries?

Yes/maybe…

e.g. Thin (low-viscosity) disks

“steady-state”? finite torus = mass-fed disk? Pressure?

Chris Tiede’s talk on Tuesday
See Penzlin, Kley et al. 2022

e.g. Large (locally) massive disk:

$$\Sigma \pi a_b^2 \gtrsim M_2$$

Likely what is happening for young star binaries (Maxwell Moe)

e.g. Gas could get ejected in outflow (?)…

Caveats of 2D viscous hydro simulations:

Equation of state/cooling (Haiyang Wang, Bai, Lai 2022 in prep)
B fields, turbulence…..

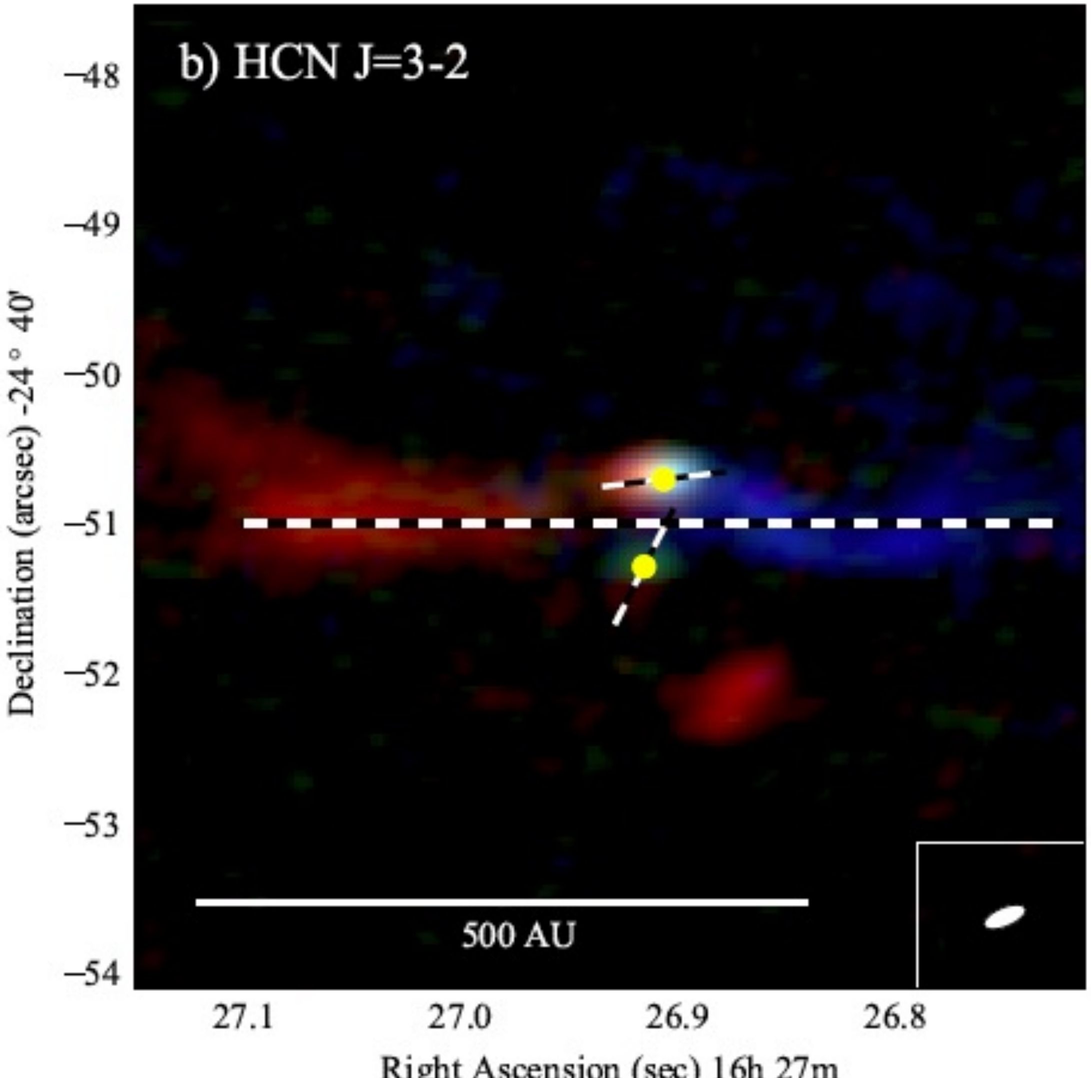
So far: Co-planar disks

What about misaligned disks ?

See Steve Lubow's talk (next)

Observations:

An example of Misaligned circumbinary disk



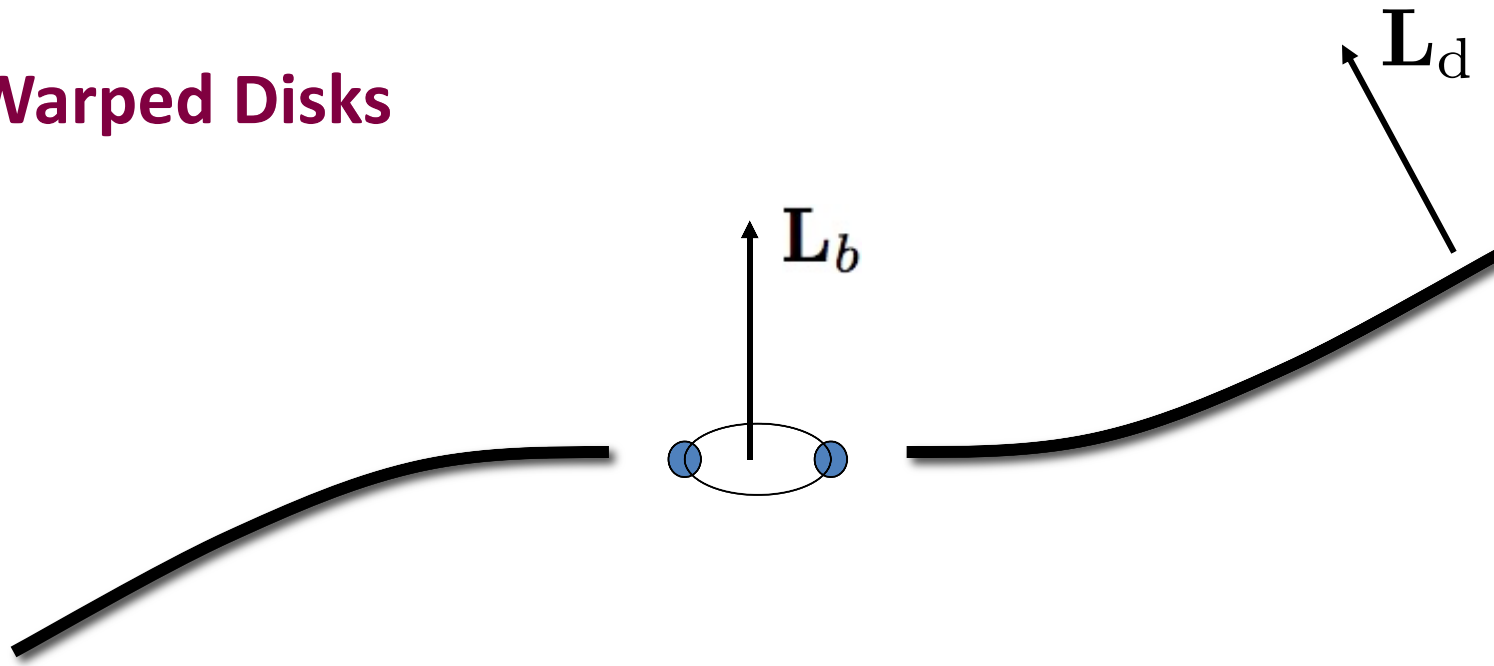
IRS 43

ALMA

a_b ~ 74 au, three disks

Brinch et al. 2016

Warped Disks

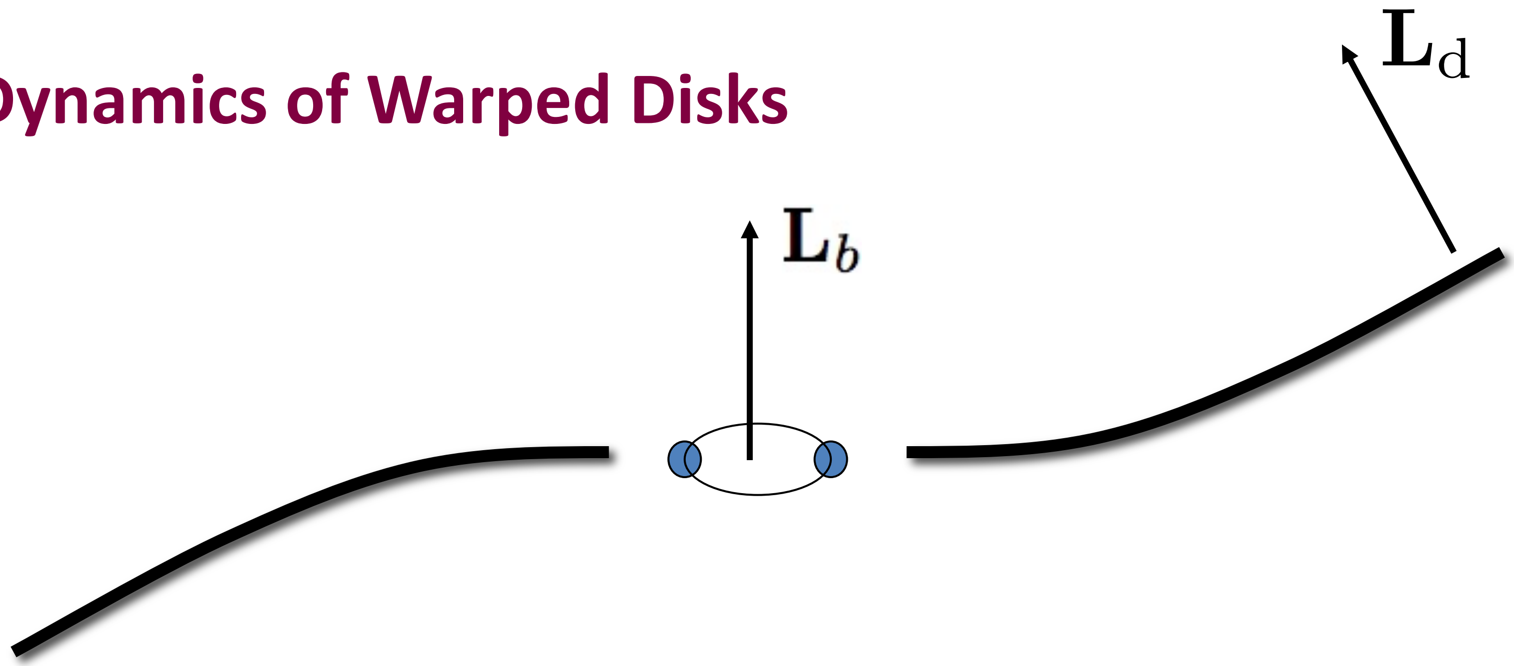


Torque from binary on disk => disk (ring) nodal precession

$$\Omega_p(r) \simeq \frac{3\mu}{4M_t}\left(\frac{a}{r}\right)^2 \Omega(r)$$

Differential precession + internal fluid stress ==> warped/twisted disk

Dynamics of Warped Disks



Warp + Viscosity ➔ Dissipation ➔ Align $\mathbf{L_b}$ and $\mathbf{L_d}$

$$\frac{\partial \hat{\mathbf{l}}}{\partial \ln r} \sim \frac{\alpha}{c_s^2} \mathbf{T}_{\rm ext} \qquad |\mathbf{T}_{\rm ext}| \sim r^2 \Omega\, \omega_{\rm ext}, \quad \omega_{\rm ext} = \Omega_{\rm prec}$$

$$\left| \frac{d\hat{\mathbf{l}}}{dt} \right|_{\rm visc} \sim \left\langle \left(\frac{\alpha}{c_s^2} \right) \frac{\mathbf{T}_{\rm ext}^2}{r^2 \Omega} \right\rangle \sim \left\langle \frac{\alpha}{c_s^2} (r^2 \Omega) \omega_{\rm ext}^2 \right\rangle$$

Typical alignment time can be short (~ precession period)

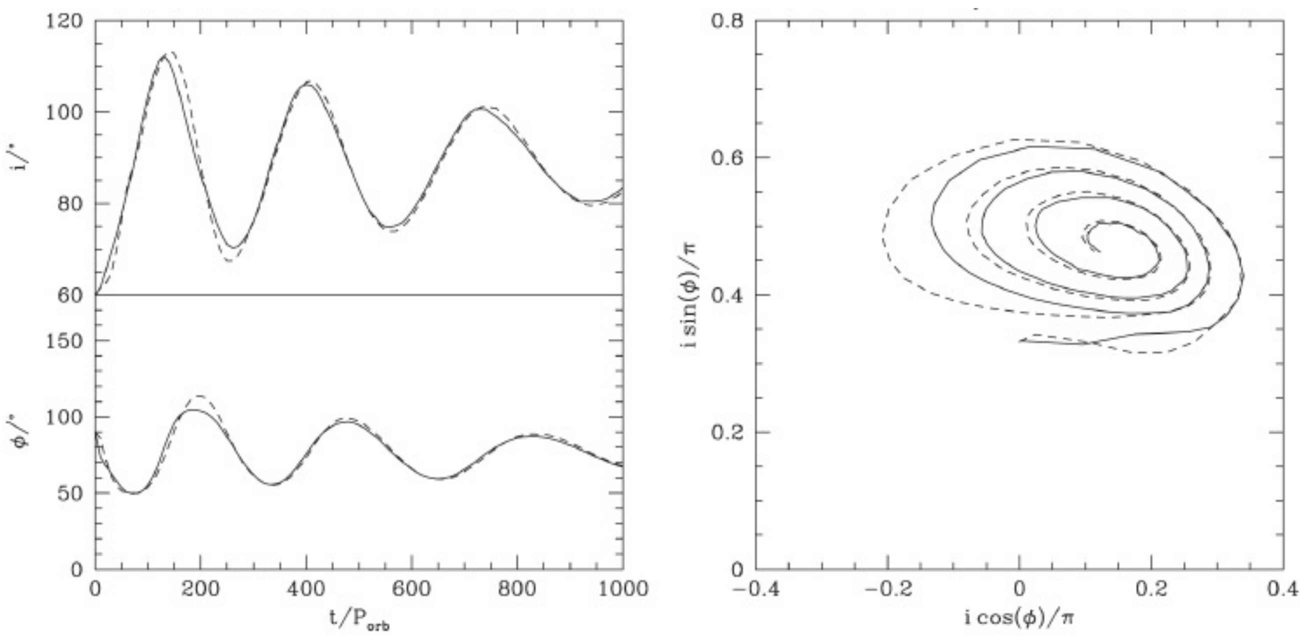
Foucart & DL 2014
Zanazzi & DL 2018

Surprise: Disk around eccentric binary may evolve toward polar alignment

Martin & Lubow (2017): viscous hydro simulation using SPH

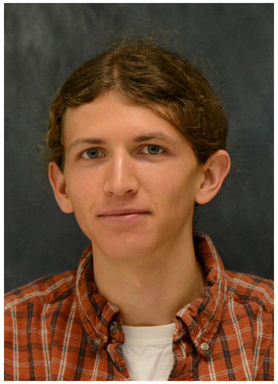
Initial disk-binary inclination $I(0) = 60^\circ$

Binary eccentricity $e_b = 0.5$.

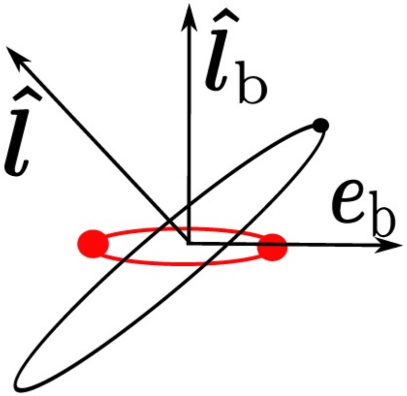


Theoretical Calculation of Polar Alignment of Disks Around Eccentric Binaries

Zanazzi & Lai 2018

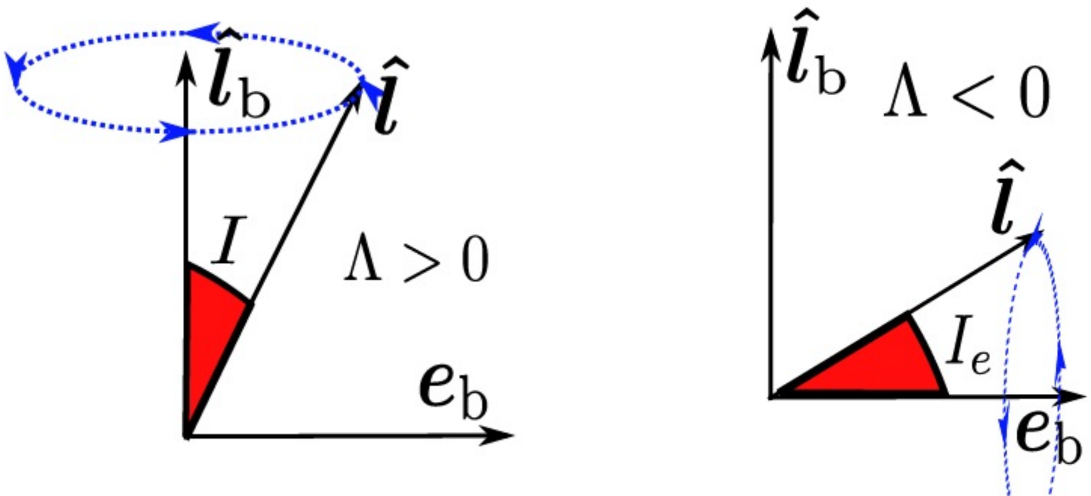


J.J. Zanazzi
(Cornell Ph.D.18
→CITA→Berkeley)

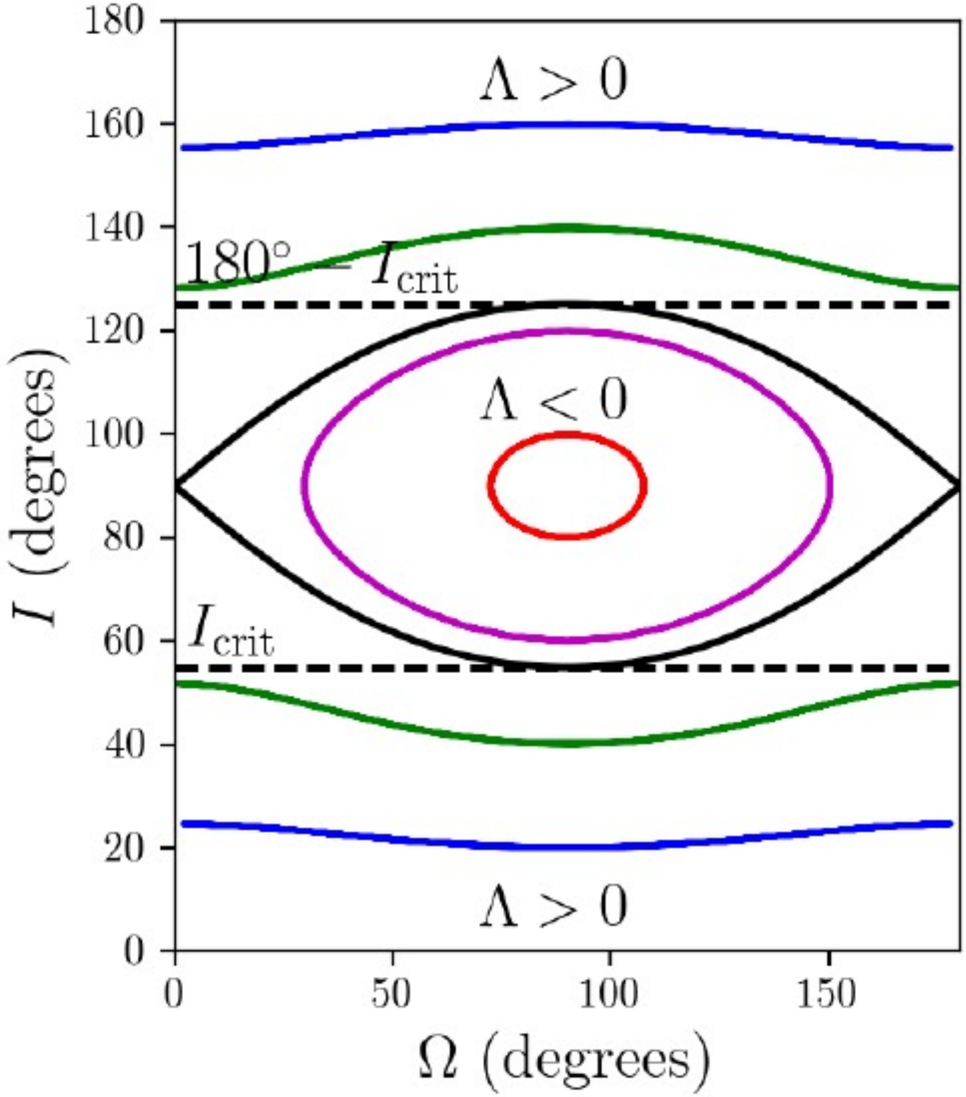


Test particle around eccentric binary has two "masters"

$$\Lambda = (1 - e_b^2)(\hat{l}\cdot\hat{l}_b)^2 - 5(\hat{l}\cdot e_b)^2$$



$$\Lambda = (1 - e_{\rm b}^2)(\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{l}}_{\rm b})^2 - 5(\hat{\boldsymbol{l}}\cdot\boldsymbol{e}_{\rm b})^2$$



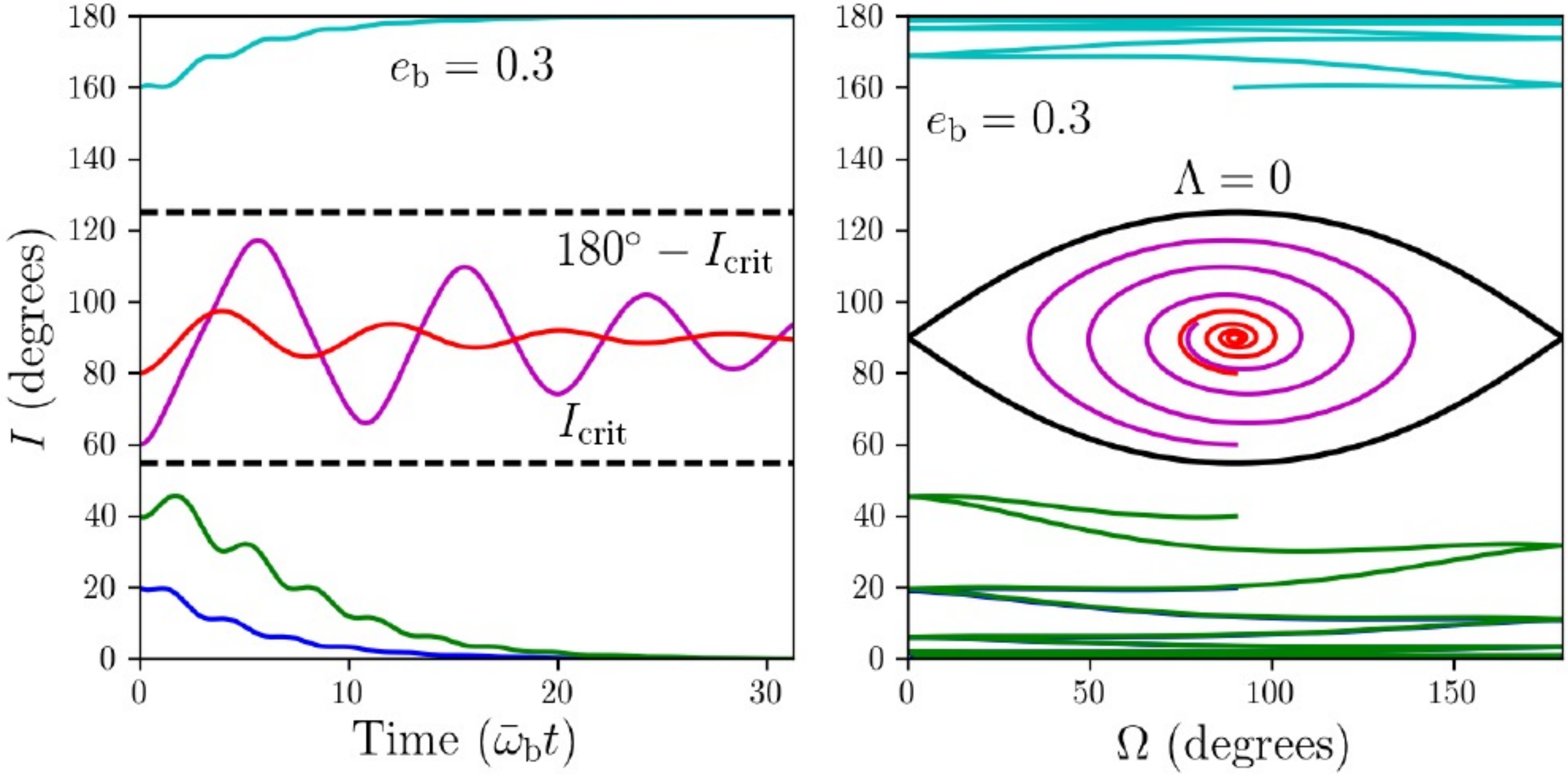
For $\hat{l}$ to precess around $\hat{e}_b$, require $\sin I > \sin I_{\rm crib}$

$$I_{\rm crit} = \cos^{-1}\sqrt{\frac{5e_{\rm b}^2}{1+4e_{\rm b}^2}}$$

Zanazzi & DL 2018

Warped viscous disk around eccentric binary

Evolve towards either align (anti-align) or polar align with the binary

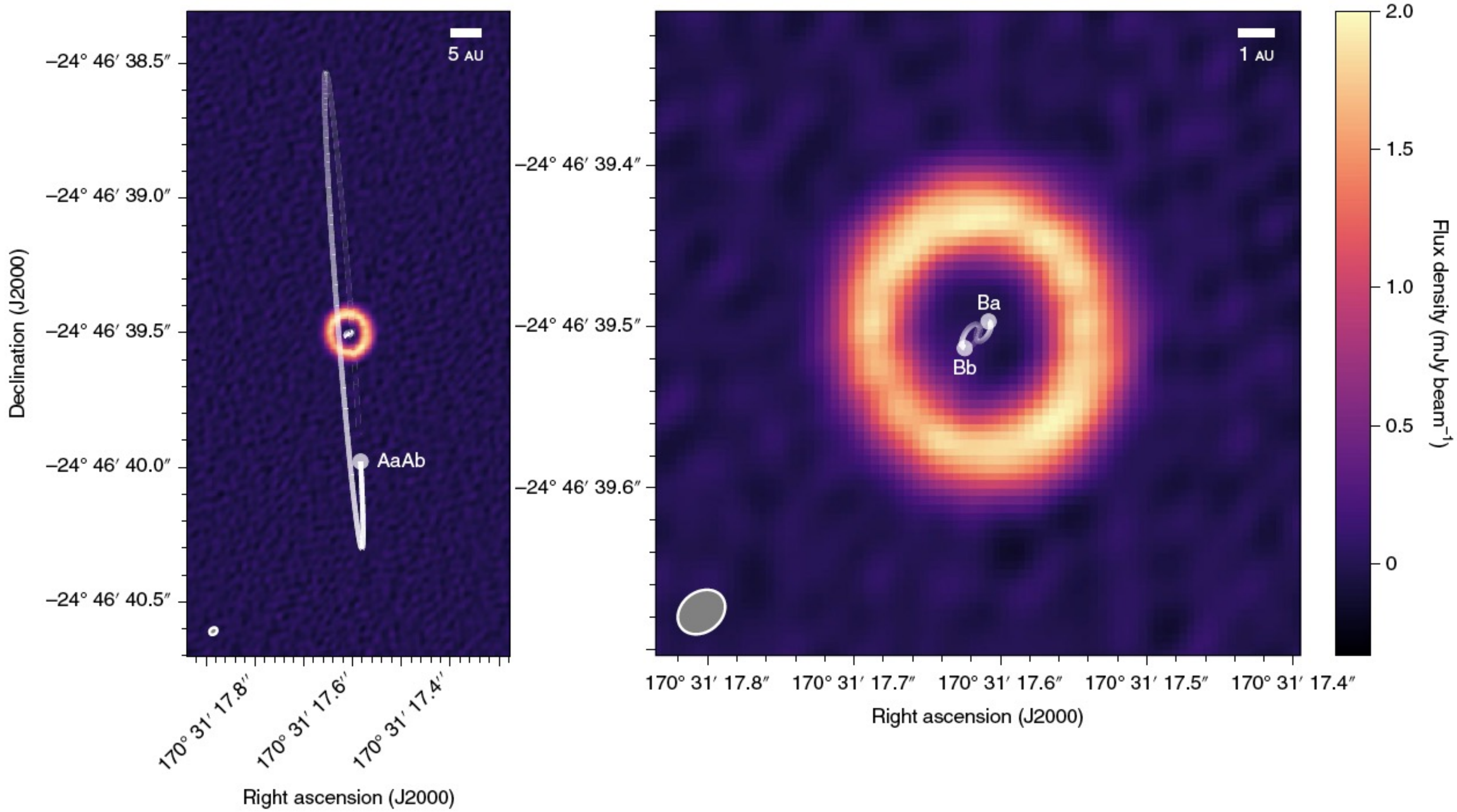


Zanazzi & DL 2018

LETTERS
https://doi.org/10.1038/s41550-018-0667-x
nature astronomy
Corrected: Publisher Correction

A circumbinary protoplanetary disk in a polar configuration

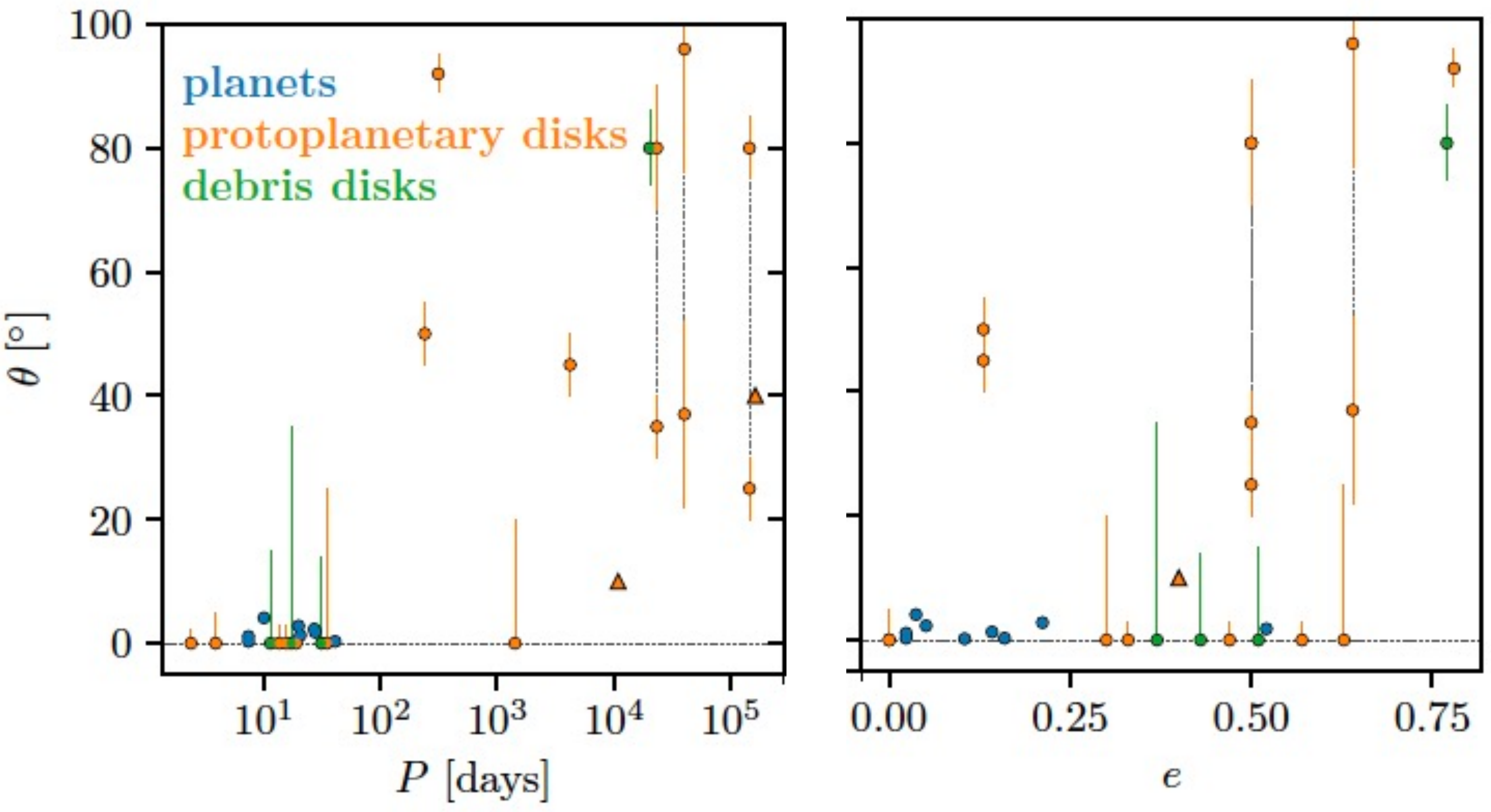
Grant M. Kennedy[1,2]*, Luca Matrà[3], Stefano Facchini[4,5], Julien Milli[6], Olja Panić[7], Daniel Price[8,9], David J. Wilner[3], Mark C. Wyatt[10] and Ben M. Yelverton[10]



The Astrophysical Journal, 883:22 (24pp), 2019 September 20
https://doi.org/10.3847/1538-4357/ab287b

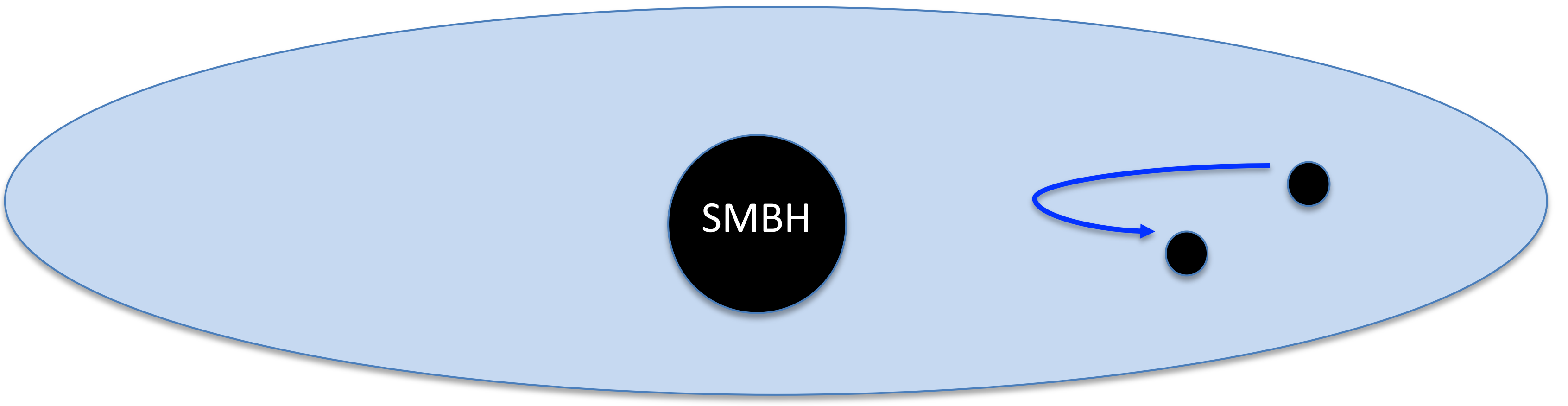
The Degree of Alignment between Circumbinary Disks and Their Binary Hosts

Ian Czekala[1,8], Eugene Chiang[1,2], Sean M. Andrews[3], Eric L. N. Jensen[4], Guillermo Torres[3], David J. Wilner[3], Keivan G. Stassun[5,6], and Bruce Macintosh[7]



Ian Czekala Disucssion Session this afternoon

Binary in a big disk

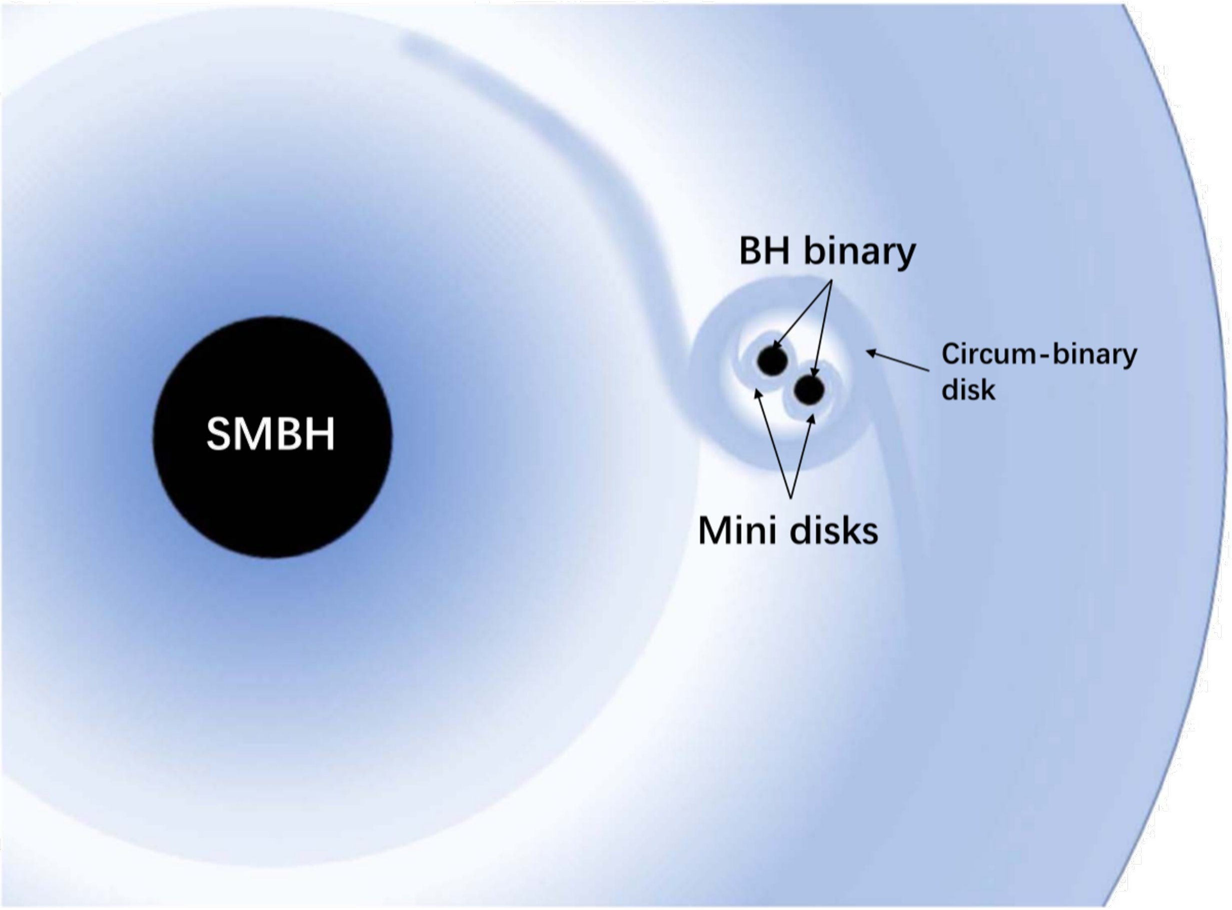


Binary BH mergers in AGN disks ?

McKernan+12, Bellovary+16, Bartos+17, Stone+17, McKernan+18, Secunda+18, Yang+19, Tagawa+20, etc

See Saavik Ford's talk on Tuesday

Is it like "Circumbinary Accretion"?

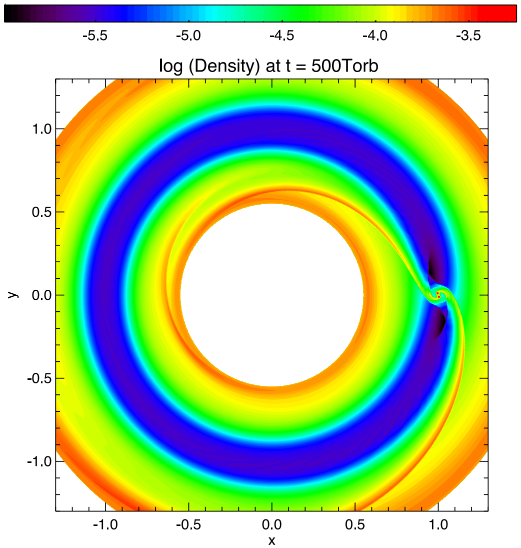


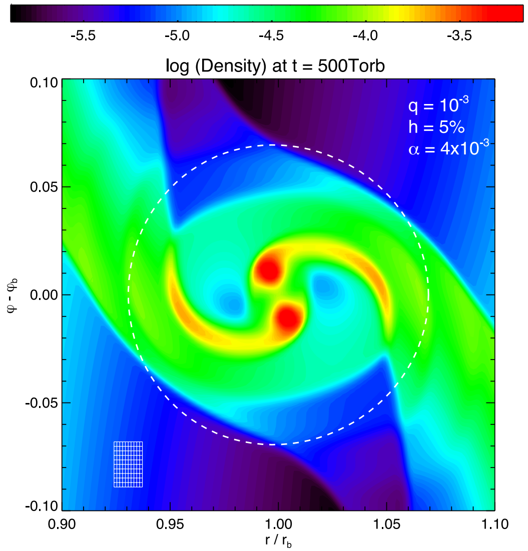
Picture from Li & Cheng (2019)

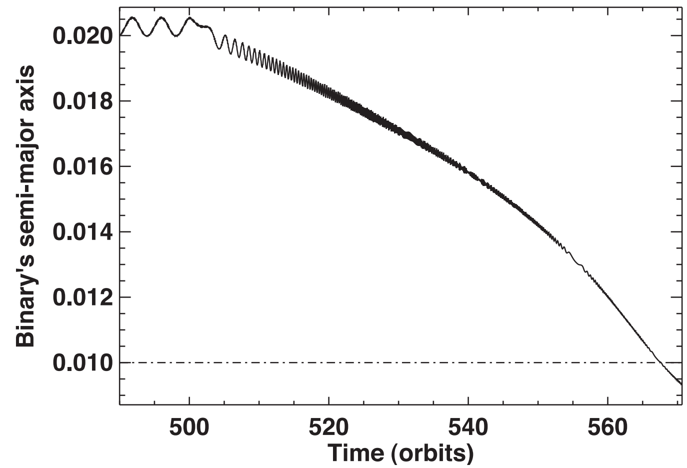
Not clear in general
What is Mdot?
How does binary evolve?

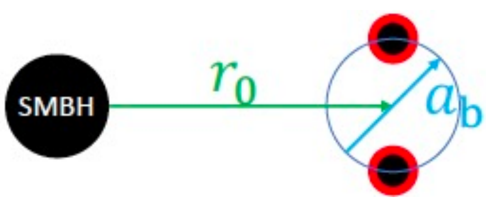
Simulations of binary in disk

Baruteau, Cuadra & Lin 2011









Global disk (FARGO):

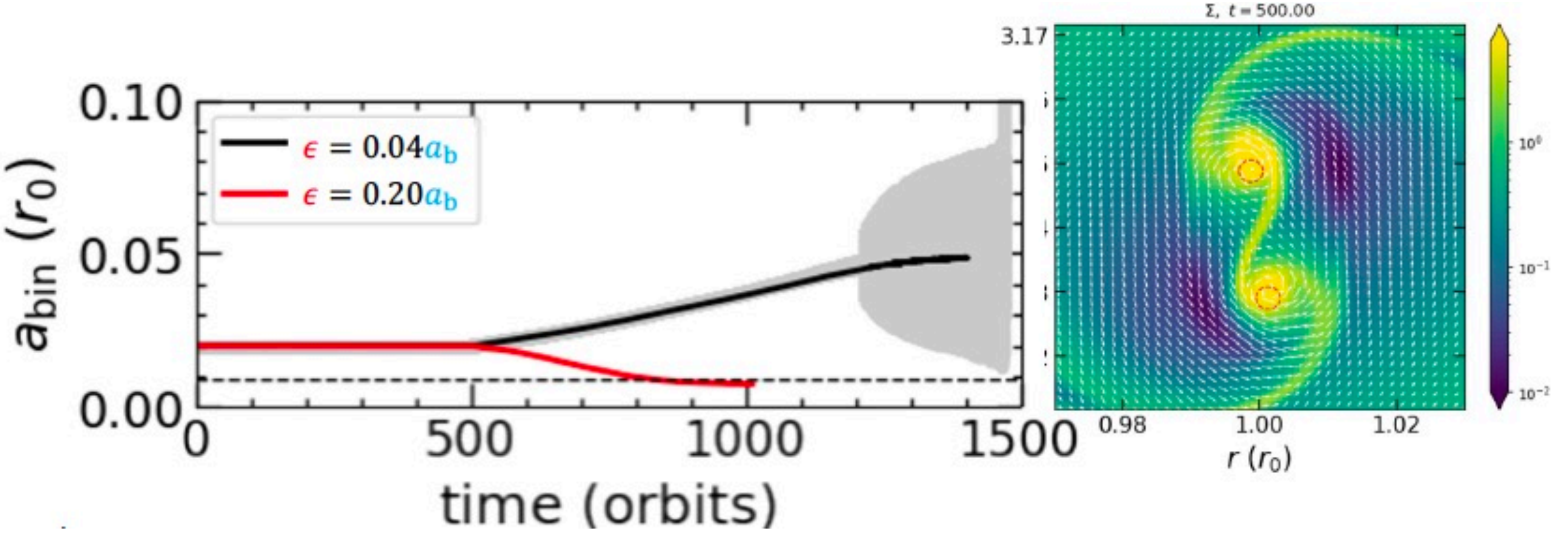
$$q = 10^{-3}$$

$$a_b = 0.04 r_0$$

➔ Orbital decay

Simulations of binary in disk

Y.Li… Hui Li… (LANL) 2021



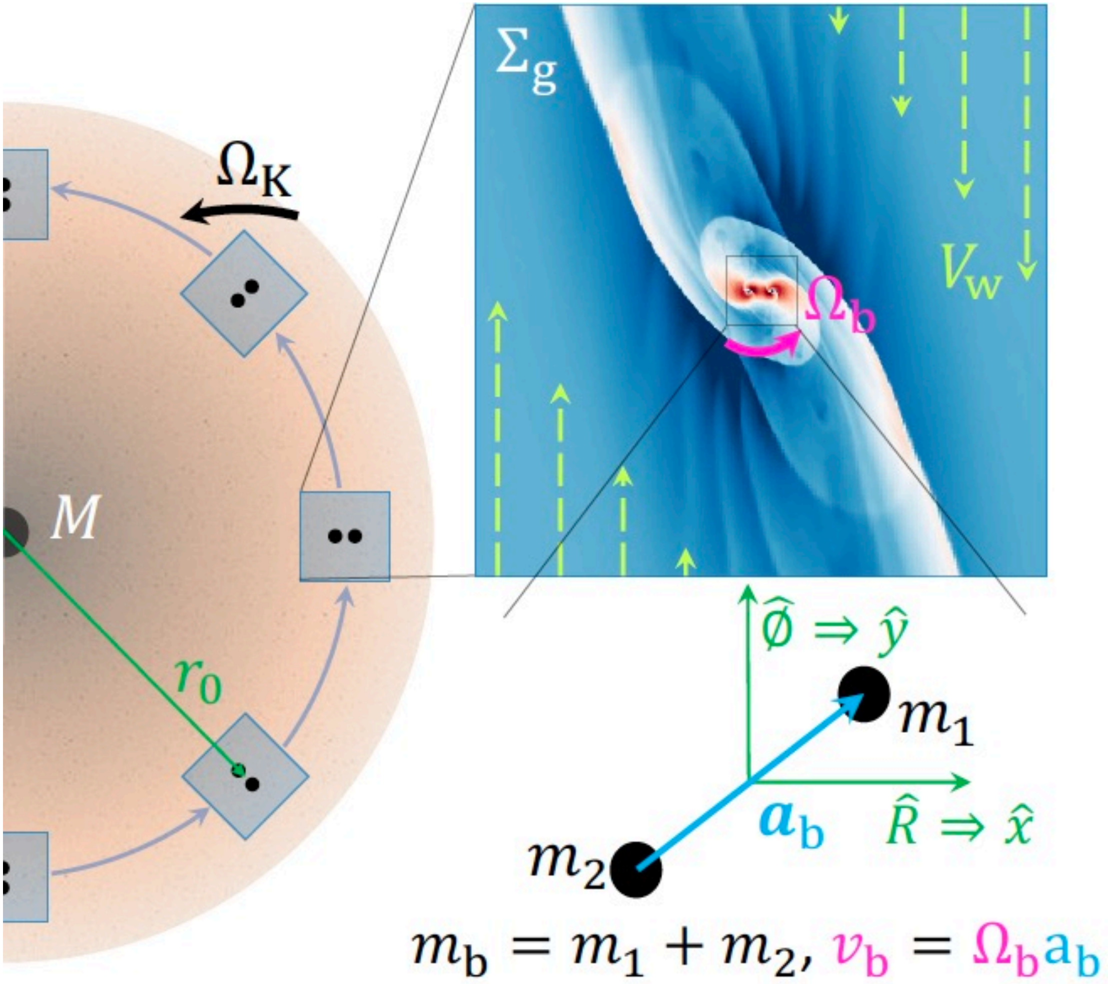
Orbit expands (if gravitational softening is small enough)
Orbit decays if inner disk is heated (Li …Hui Li.. 2021b)

See Dempsey's talk
Hui Li discussion session

Local simulations of binary in disk

R.Li & Lai, arXiv:2022.07633
Ri.Li & Lai, in prep

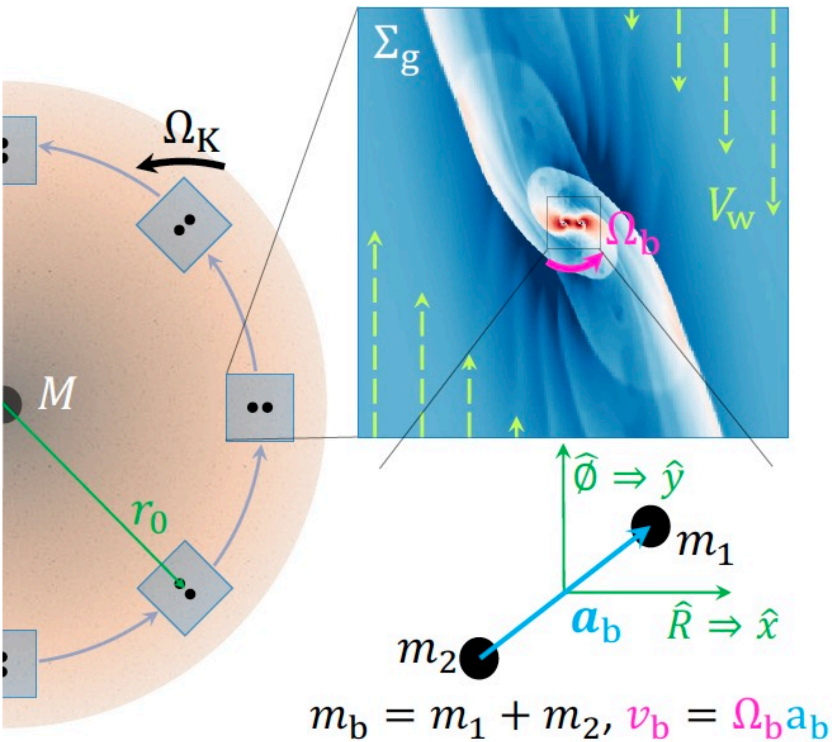
Dr. Rixin Li
(Cornell)



Local shearing box

(not "local wind tunnel box" used by Kaaz et al. 2021)

ATHENA
Mesh refinement
Resolution: a_b ~250 cells
zero softening in gravity



Length scales of the problem:

$$a_b, \quad R_B \sim \frac{Gm_b}{c_\infty^2}, \quad R_H \sim r_0 \left(\frac{m_b}{M}\right)^{1/3}, \quad H$$

Velocity scales of the problem:

$$v_b, \quad c_\infty, \quad V_{\rm shear}$$

➔ Dimensionless ratios:

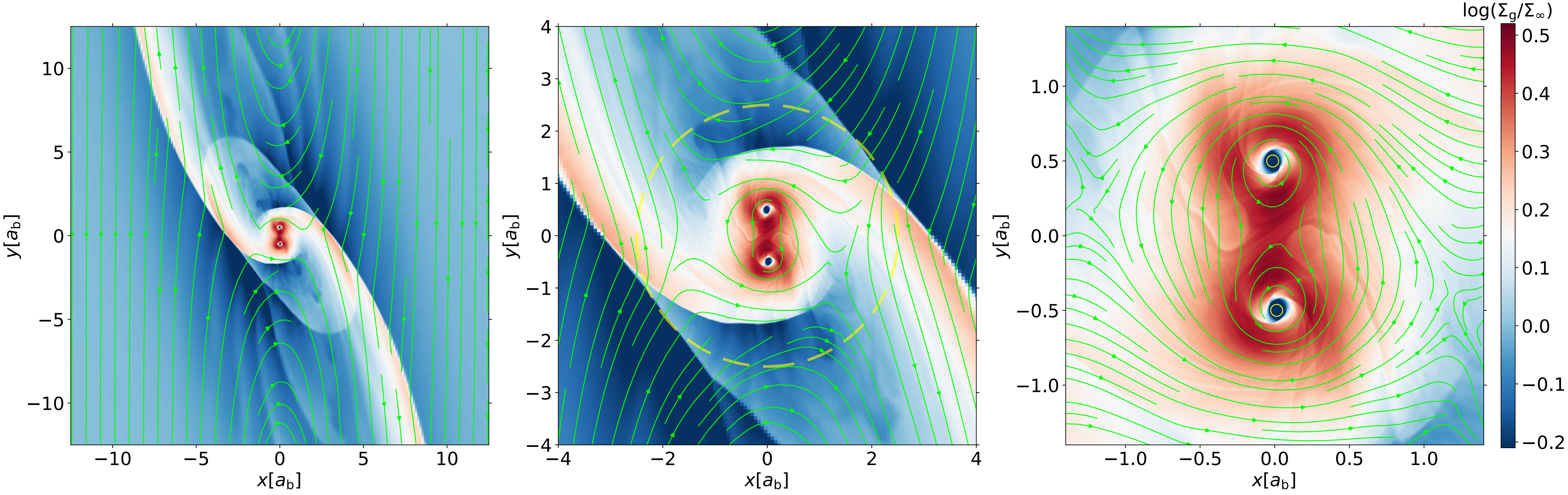
$$\frac{q}{h^3}, \quad \frac{R_H}{a_b} \equiv \lambda$$

where $q = m_b/M$, $h = H/r_0$

$$m_2/m_1, \quad e_b, \quad \text{EOS} \quad (\text{e.g. } \gamma \text{ law})$$

Example of flow structure

Pairs of bow shocks, spiral shocks



BH = absorbing sphere: sink radius: $r_{\rm sink} = 0.04\, a_{\rm b} \simeq 10$ cells ➔ $\dot{m}_b$

Force on each BH: from gravity + accretion + pressure

➔ Torque on binary, energy transfer rate ➔ $\dot{a}_b, \quad \dot{e}_b$

Some "Typical" Results:

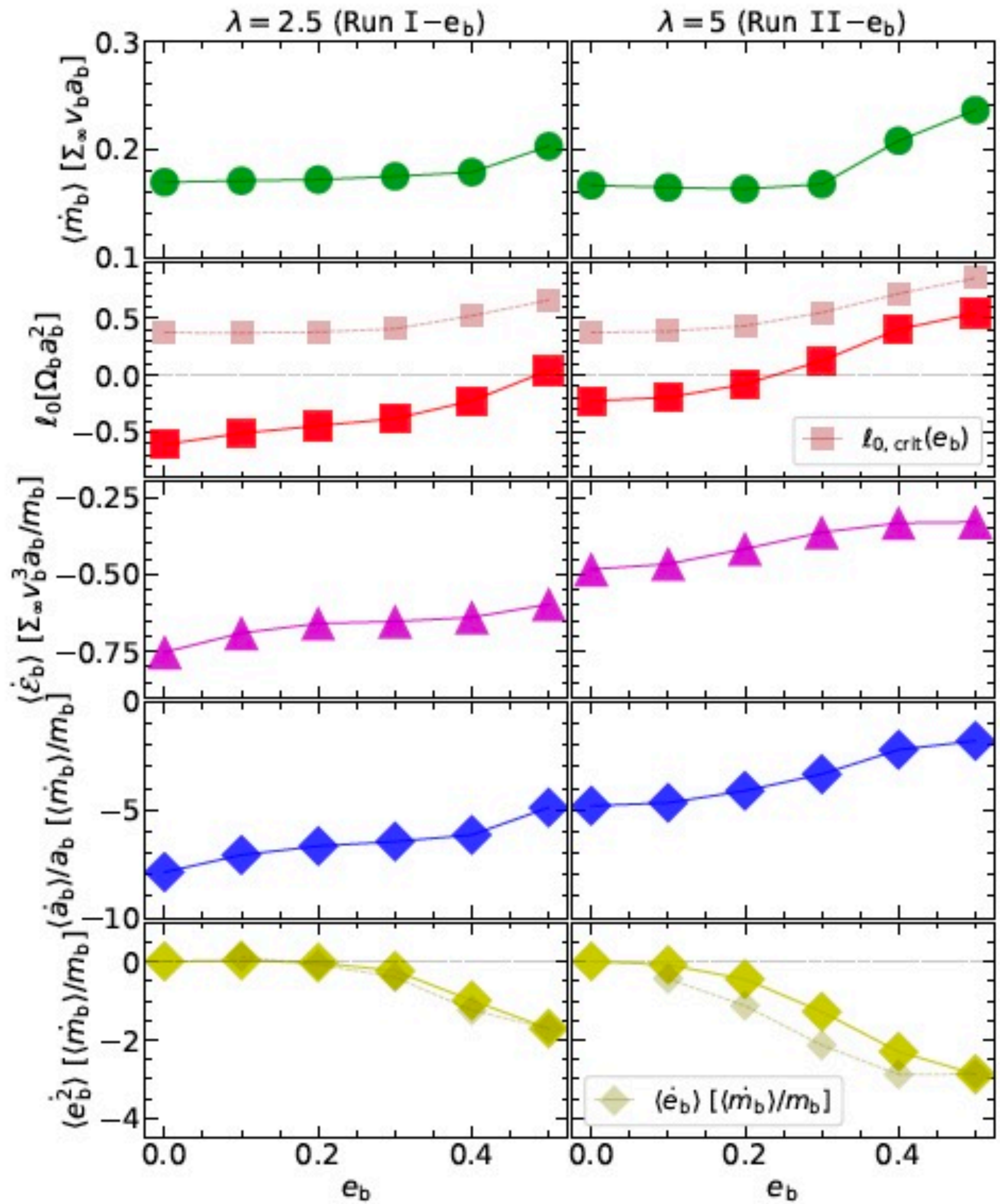
$\langle \dot{m}_b \rangle$: can be << Bondi-Hoyle-Lyttleton rate (even including shear)
depends on sink radius (for non-viscous flow)

$$\ell_0 = \frac{\langle \dot{L}_\mathrm{b} \rangle}{\langle \dot{m}_\mathrm{b} \rangle} = -0.23\ v_\mathrm{b} a_\mathrm{b}$$

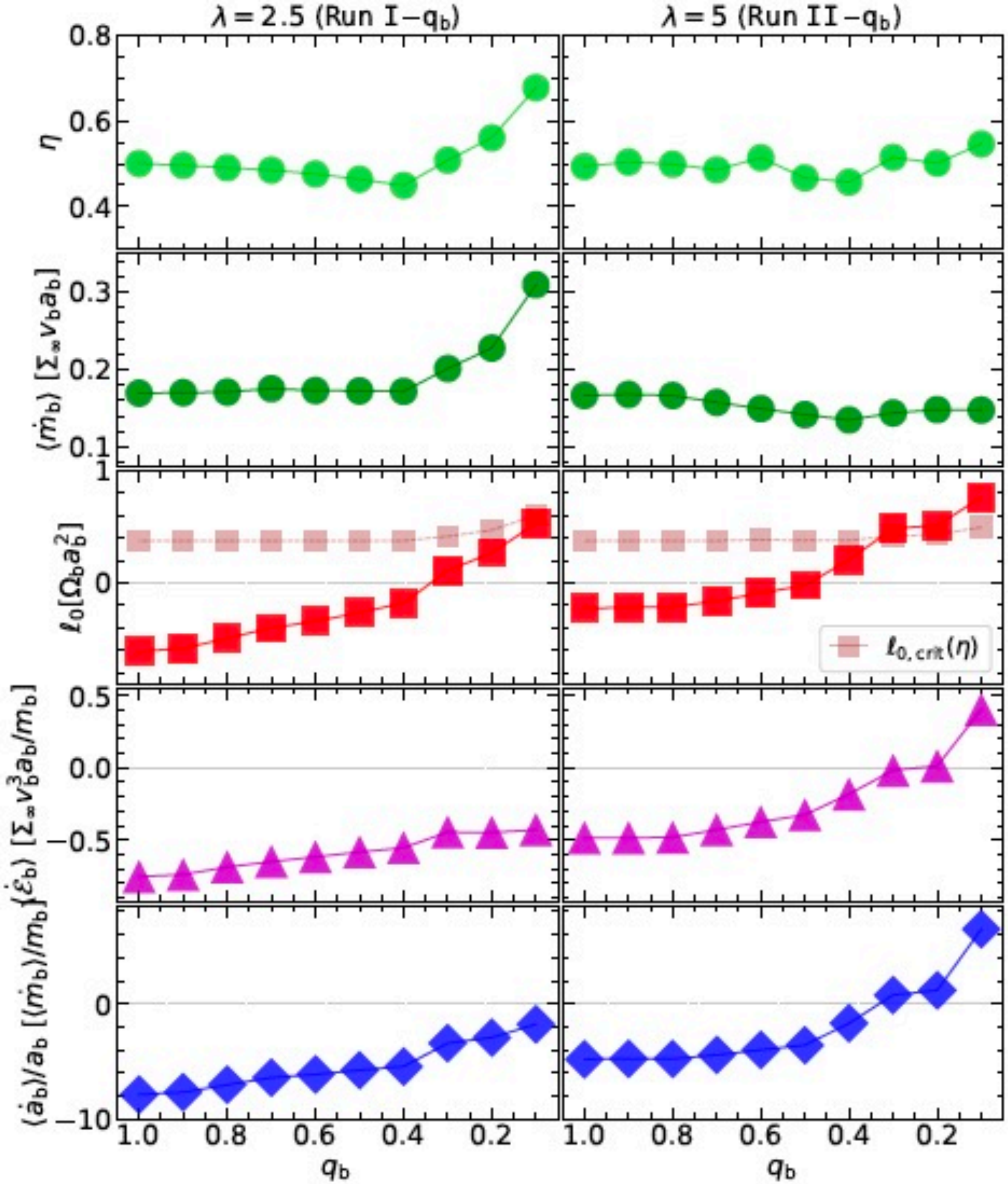
$$\frac{\langle \dot{a}_\mathrm{b} \rangle}{a_\mathrm{b}} = -4.81 \frac{\langle \dot{m}_\mathrm{b} \rangle}{m_\mathrm{b}}$$

$$m_2/m_1 = 1, \quad e_b = 0$$
$$q/h^3 \sim 1, \quad \lambda = 5, \quad \gamma = 5/3$$

Eccentric, equal-mass binaries



Circular, unequal-mass binaries

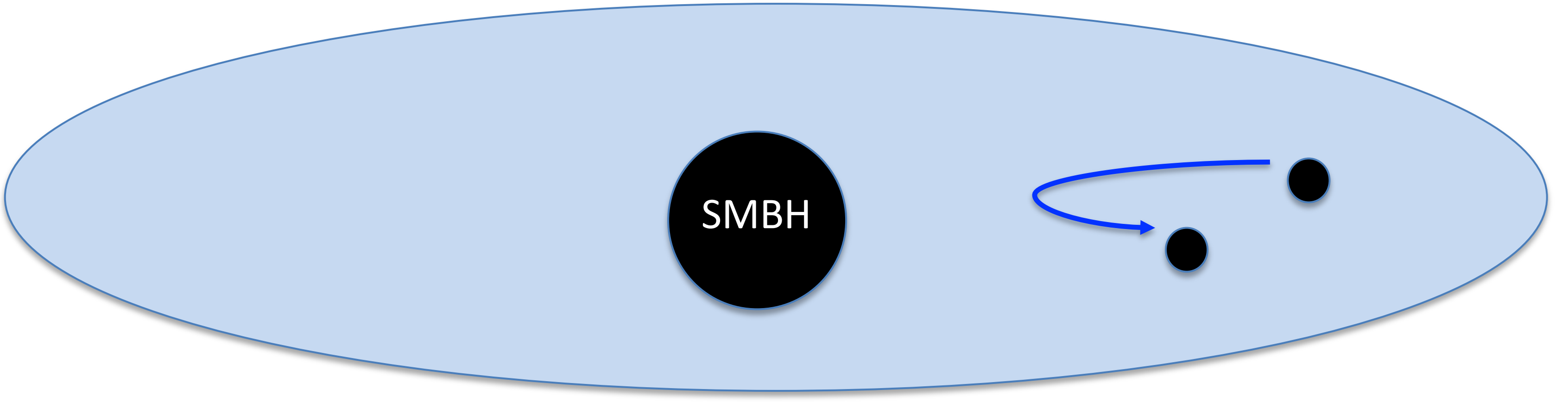


Summary II: Hydrodynamics of Binary Embedded in a Disk

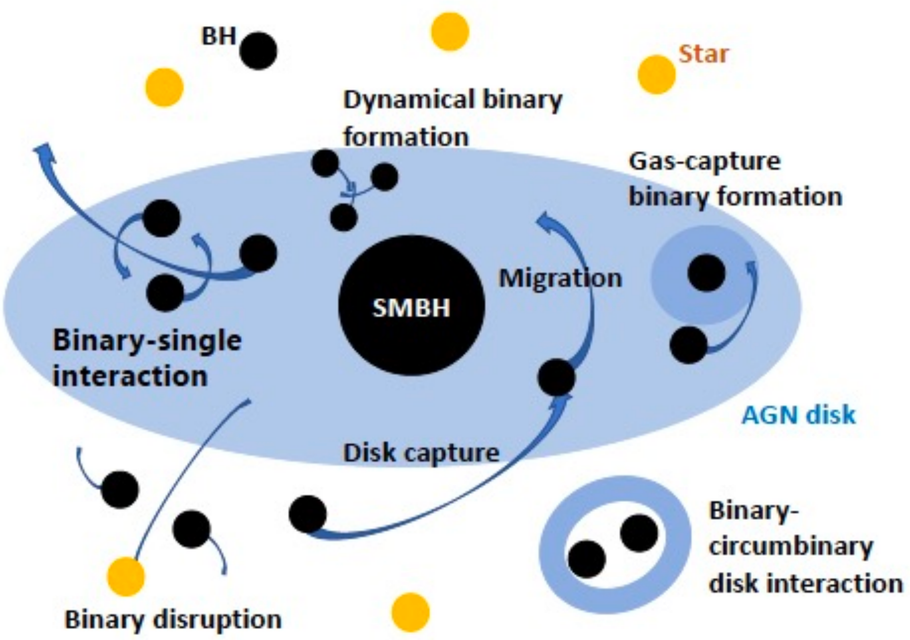
Many issues remain to be explored/understood

2D vs 3D
Local vs global
EOS, viscosity (→ magnetic field, turbulence)

Formation of Merging BH Binary in AGN Disk



Where do BH binaries in AGN disks come from?



Tagawa, Haiman, Kocsis 2020

See also Bartos+17; Stone+17; Secunda+18;….

1. Binaries form in disks via GI (~pc)
2. Binaries in nuclear clusters get captured in disks
3. Single BHs in AGN disks get captured in binaries

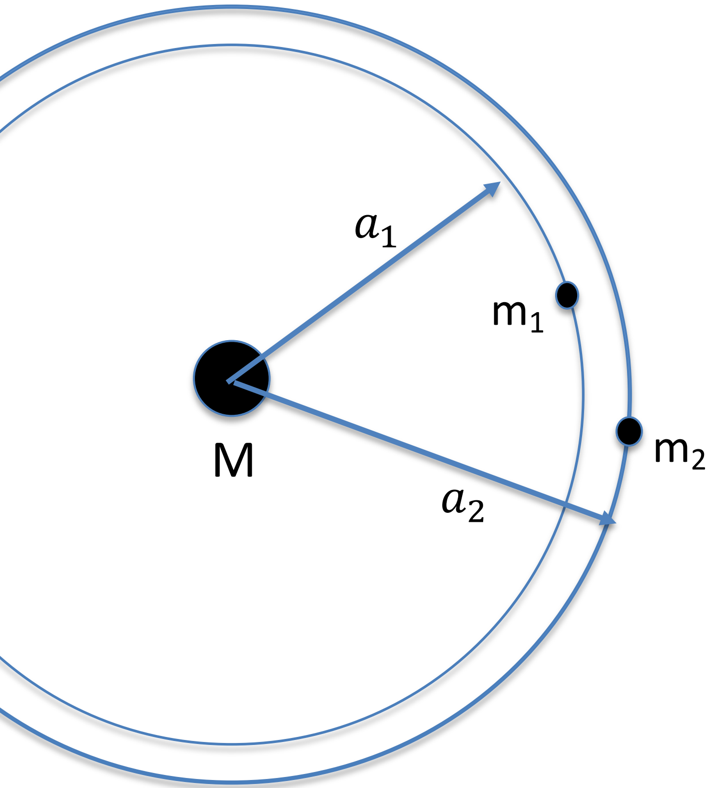
Long-Term Evolution of Tightly-Packed Stellar BHs in AGN Disks: Formation of Merging BH Binaries via Close Encounters

Jiaru Li, Dong Lai, Laetitia Rodet

arXiv:2203.05584

Jiaru Li
(Cornell Ph.D. 2023)

The Problem:



Two BHs (m_1, m_2) on closely-packed, nearly circular, nearly-coplanar orbits around a SMBH (M)
(e.g. brought together by migration in AGN disks)

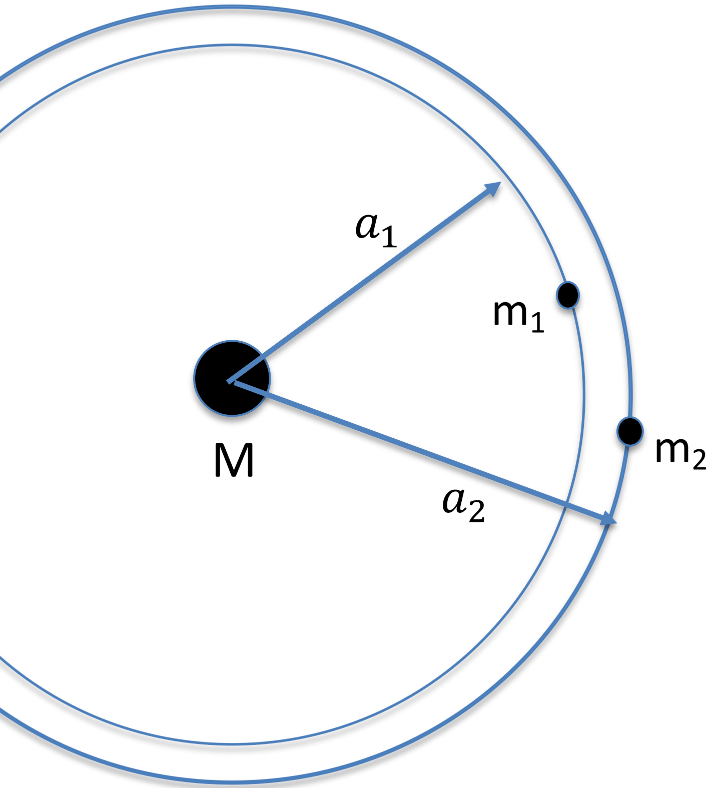
When $a_2 - a_1 \lesssim 2\sqrt{3}\, R_{\rm H}$

$$R_{\rm H} = a_1 \left(\frac{m_{12}}{3M}\right)^{1/3}, \quad m_{12} = m_1 + m_2$$

orbits are dynamically unstable.

What happens to the two BHs?

Neglect gas effect for now…



Two planets in unstable orbits around a star:

Two outcomes:

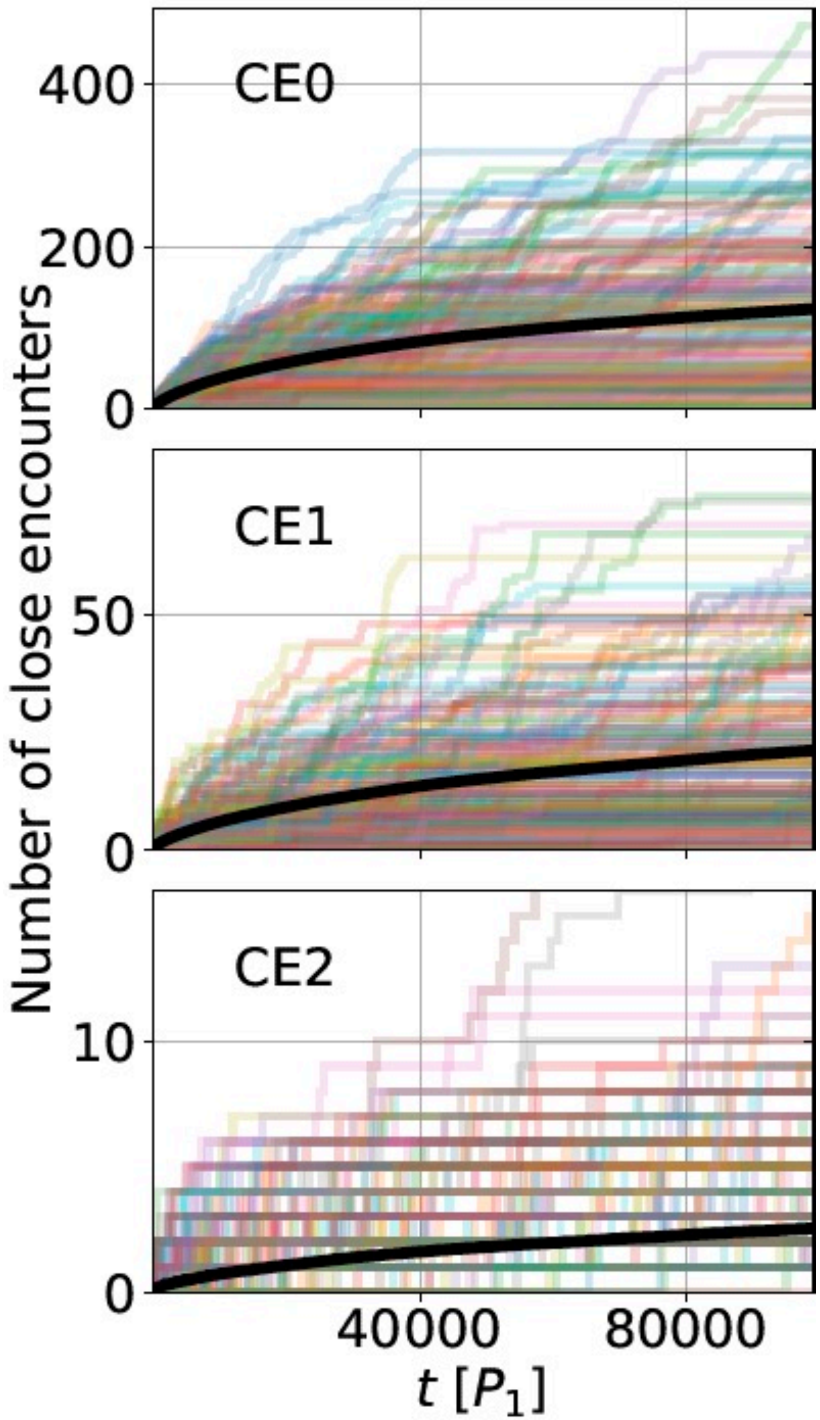
1. Ejection of lower-mass planet
2. Planet-planet collision

See Li, Lai, Anderson & Pu 2021 and refs therein

Two BHs in unstable orbits around a SMBH:

Since $M/m_{12} \sim 10^6 \gg 1$
ejection is not possible (takes many orbits > Hubble time)

➔ The two BHs undergo "chaotic" motion, experience recurring closer encounters (separation < R_H)

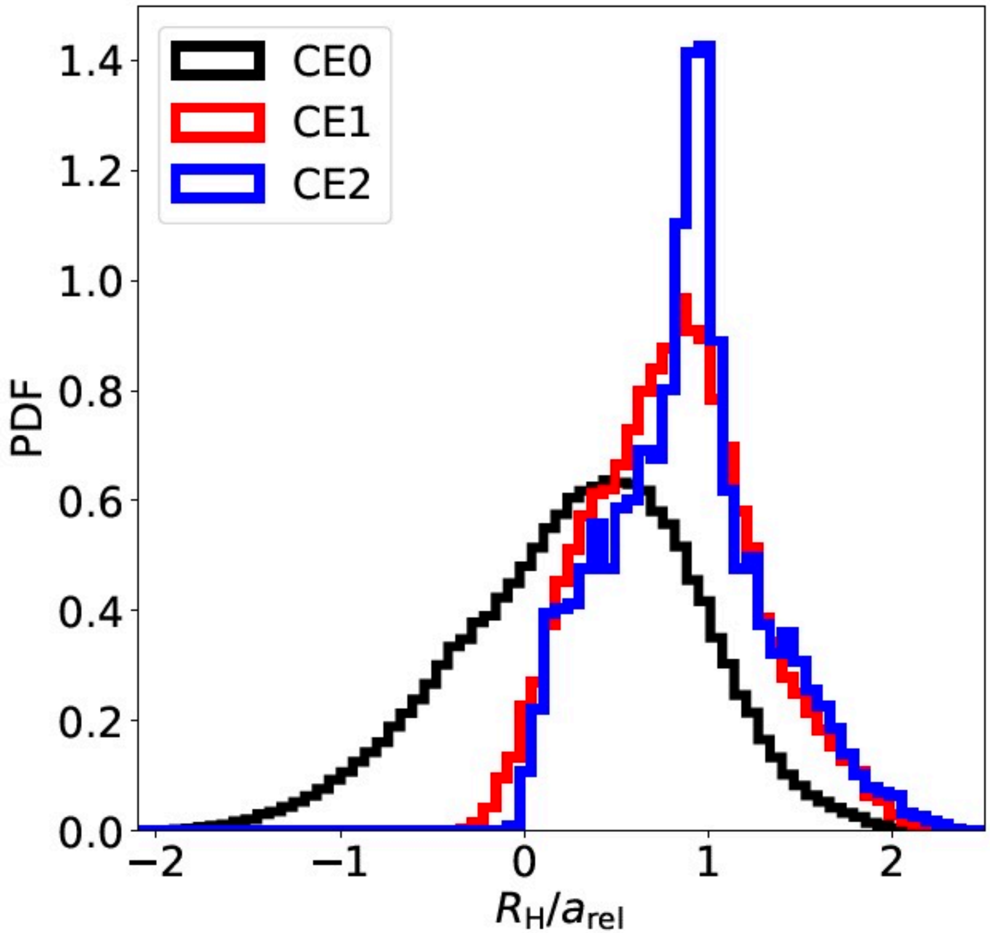


Close encounters with $r_{\rm rel} < R_{\rm H}$

Close encounters with $r_{\rm rel} < 0.1 R_{\rm H}$

Close encounters with $r_{\rm rel} < 10^{-2} R_{\rm H}$

During close encounters, the BH pairs are temporarily bound with $a_{\rm rel} \sim R_{\rm H}$



But they are all short-lived (destroyed by tide from SMBH in ~ one orbit)

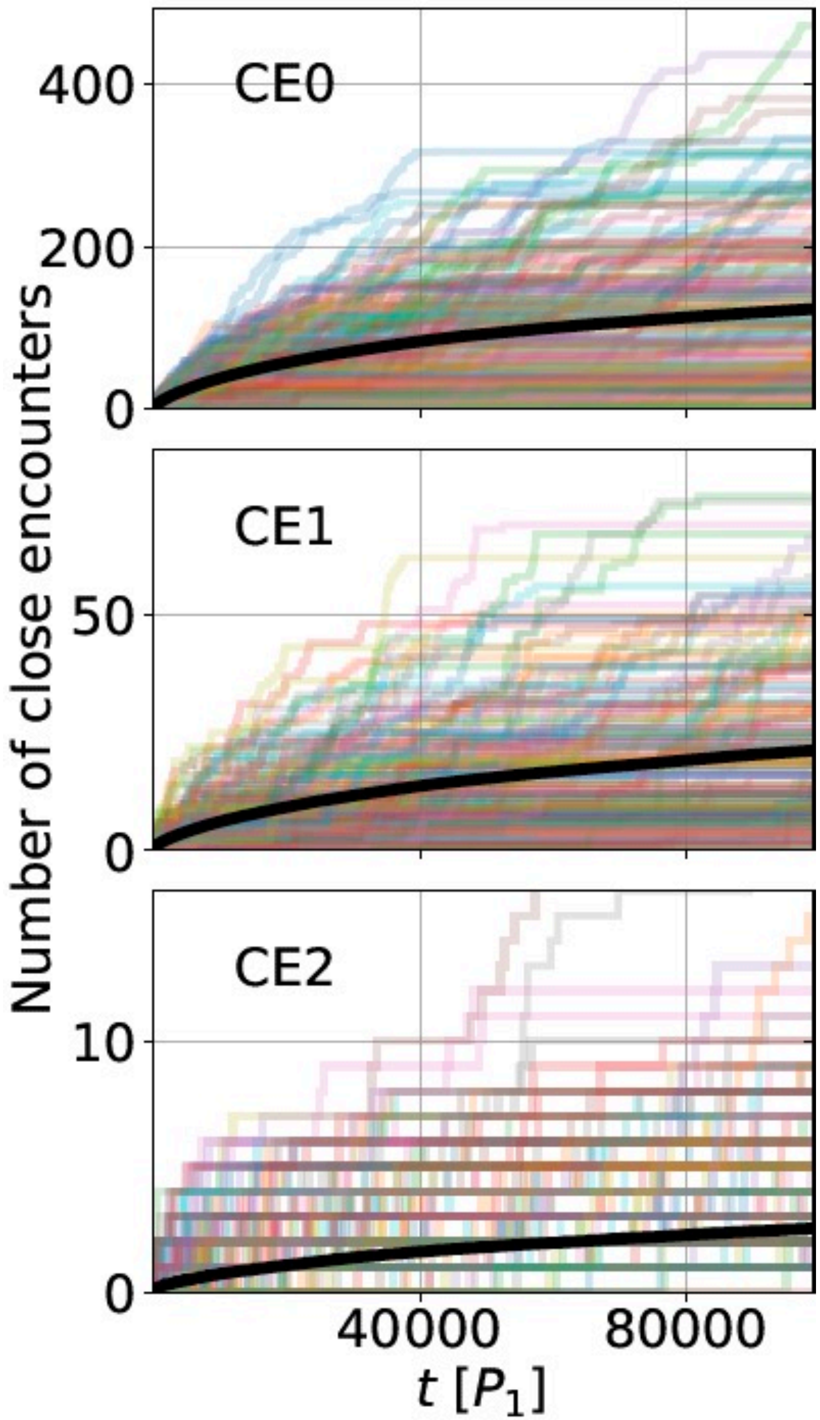
For VERY close encounter:

GW emission $$\Delta E_{\rm GW} \sim \frac{\mu^2 m_{12}^{5/2}}{r_{\rm rel}^{7/2}} \gtrsim \frac{G m_1 m_2}{R_{\rm H}}$$

$$\Rightarrow \quad \frac{r_{\rm rel}}{R_{\rm H}} \lesssim 10^{-4} \left(\frac{4\mu}{m_{12}}\right)^{2/7} \left(\frac{10^6 m_{12}}{M}\right)^{10/21} \left(\frac{a_1}{100M}\right)^{-5/7}$$

Capture radius for forming "permanent" binary due to GW bremsstrahlung

What is the cumulative capture rate (i.e. CE4 rate)?



Close encounters with
$r_{\rm rel} < R_{\rm H}$

Close encounters with
$r_{\rm rel} < 0.1R_{\rm H}$

Close encounters with
$r_{\rm rel} < 10^{-2}R_{\rm H}$

For a typical "SMBH + 2 BHs" system (in unstable orbits), what is the cumulative capture rate to form real bound binary?

$$l_{\rm rel} \simeq \sqrt{2 m_{12} r_{\rm rel}}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\,l_{\rm rel}} \propto l_{\rm rel} \quad \Rightarrow \quad P(< r_{\rm rel}) \propto r_{\rm rel}$$

$$\Longrightarrow \quad \langle N_{\rm cap}(t) \rangle \simeq 6 \times 10^{-5} \left(\frac{t}{P_1} \right)^{0.52} \left(\frac{r_{\rm cap}}{10^{-4} R_{\rm H}} \right)$$

It takes $10^8 P_1$ (on average) for two BHs to capture into bound merging binary

Captured BH binary as GW source

$$f_{\rm cap} \simeq (1.4\,{\rm Hz}) \left(\frac{4\mu}{m_{12}}\right)^{-3/7} \left(\frac{M}{10^8 M_\odot}\right)^{-2/7} \left(\frac{m_{12}}{100 M_\odot}\right)^{-5/7} \left(\frac{a_1}{100M}\right)^{-3/7}$$

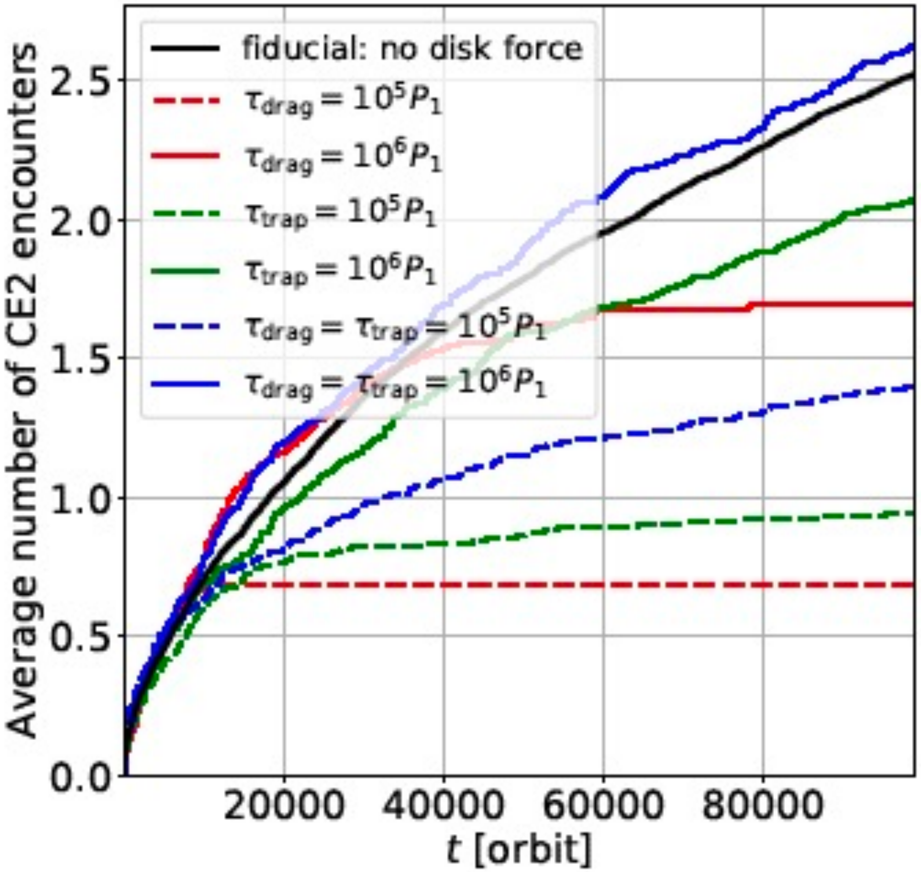
Once capture, it will take a few orbits to merge
it enters LIGO band with $e \gtrsim 0.5$

Tentative: Rate $\sim 1\,{\rm Gpc}^{-3}{\rm yr}^{-1}$ assuming each AGN has one BH pair trapped at 100M

What about the gas effect?

In N-body simulations, add

$$\boldsymbol{F}_{\text{drag}} = -\frac{\boldsymbol{v} - \boldsymbol{v}_{\text{K}}}{\tau_{\text{drag}}} \quad \text{and/or} \quad \boldsymbol{F}_{\text{trap}} = -\frac{\Omega_{\text{K},0}(r - r_0)}{\tau_{\text{trap}}}\hat{\boldsymbol{\theta}}$$



Gas does not increase the capture rate

Summary

Circumbinary Accretion:

-- short-term variabilities: ~ 5 P_b (for e_b~0) vs P_b (finite $e_{b,}$ or q<0.4)
-- Small-mass accretes more; symmetry breaking in accretion (q=1, finite e_b)
-- Binary can gain angular momentum and can expand (q>0.1); but thin disks?
-- Eccentricity attractor e_b~0.4

Hydrodynamics of binary in a big (AGN) disk:

-- Scaling parameters for simulations
-- Accretion can be strongly suppressed compared to Bondi
-- Orbital evolution (decay) always accompanied by accretion

Merging BH binary from closer encounters in AGN disks:

-- GW bremsstrahlung capture, very eccentric merger