Swimming and crawling at microscopic scales: lessons from 1D model systems

Antonio DeSimone

SISSA

International School for Advanced Studies, Trieste, Italy

http://people.sissa.it/~desimone

Drawing from **joint work** with **many** people, see list of references at the end.

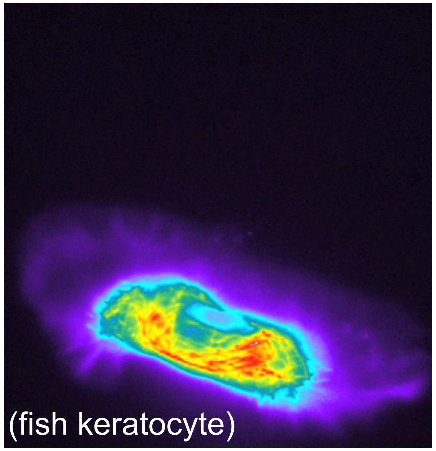
Focused Working Group on “Self-Propelled Micro-Objects”, KITP Santa Barbara, 5.3.2014

Motility at the micron scale

Motility is the capability of exhibiting directed, purposeful movement.

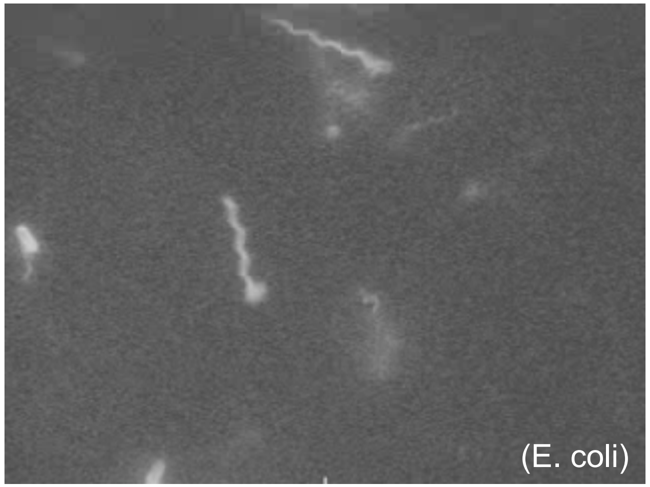
Motile cells: fascinating examples of motility at microscopic scales (1-50 μm)

Metastatic tumor cells
(**crawling** on a solid)


(fish keratocyte)

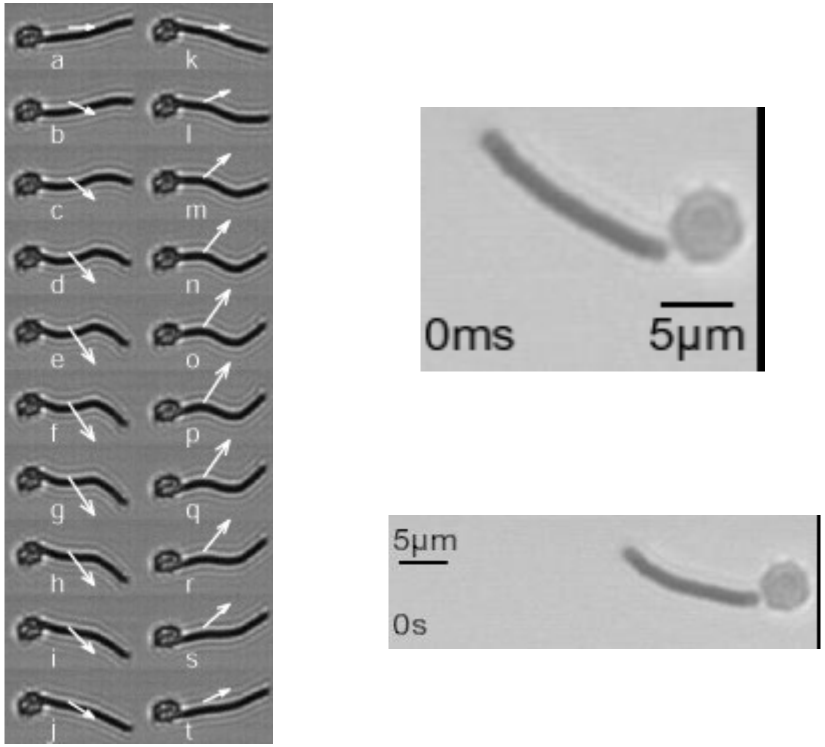
Sperm cells
(**swimming** in a fluid)

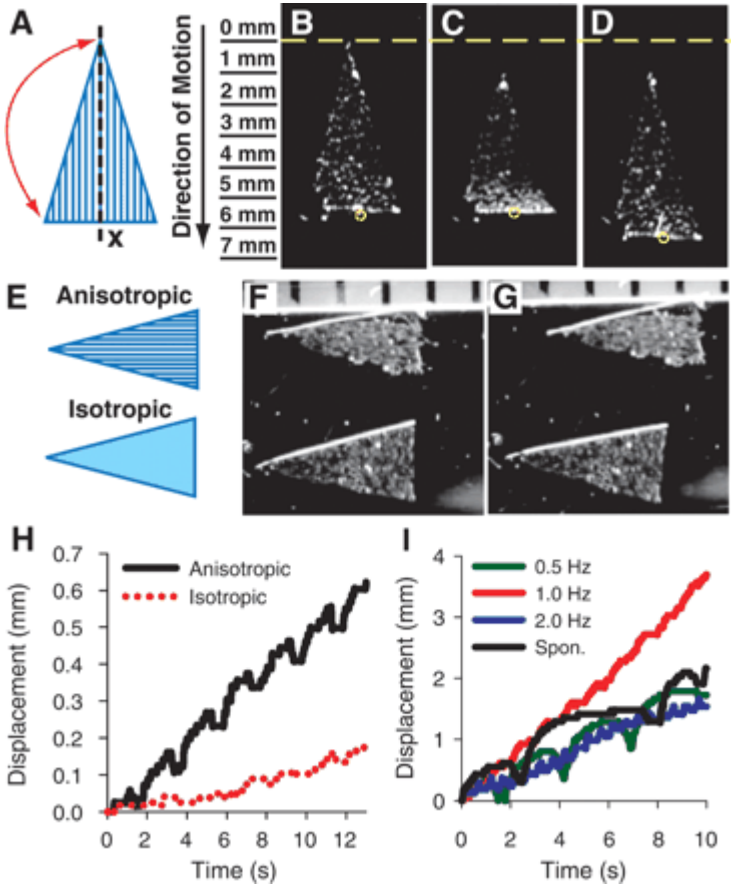
Bacteria
(**swimming** in a fluid)


(E. coli)

(bull sperm cell)


Bio-inspired artefacts: swimming micro-robots





red blood cell + flexible magnetic filament

J. Bibette, H. Stone et al.: Nature (2005)

polymer film + muscle cells

G. Whitesides et al., Science (2007)

Reynolds number

$$Re = \frac{VL\rho}{\eta}$$

Velocity (typical order of magnitude) V

Diameter (typical size) L

Mass density of the fluid ρ

Viscosity of the fluid η

For water at room temperature $\rho/\eta = 10^6\ (m^2 s^{-1})^{-1}$.

Orders of magnitude for swimmers:

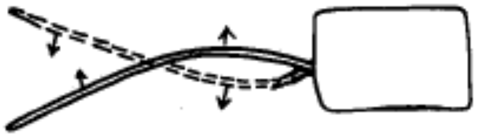
Men, dolphins, sharks:	$L=1m$, $V=1\text{-}10\ ms^{-1}$	$Re=10^6\text{-}10^7$
Bacteria:	$L=1\times10^{-6}m$, $V=1\text{-}10\times10^{-6}\ ms^{-1}$	$Re=10^{-6}\text{-}10^{-5}$

Life at low Reynolds numbers (E. Purcell)

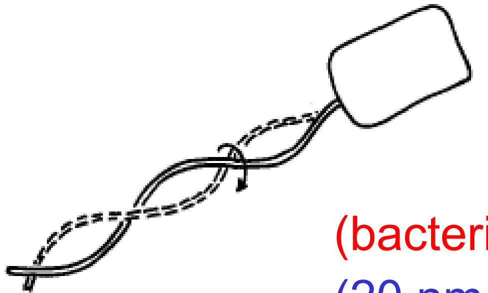
in a flow regime obeying **Stokes equations**, whatever forward motion will be produced by closing the valves, it will be exactly canceled by a backward motion upon reopening them

Micro-motility as a struggle against the scallop theorem

Nature:



(eukariotic flagella, cilia)
(bending of 9+2 μ-tubule bundles)

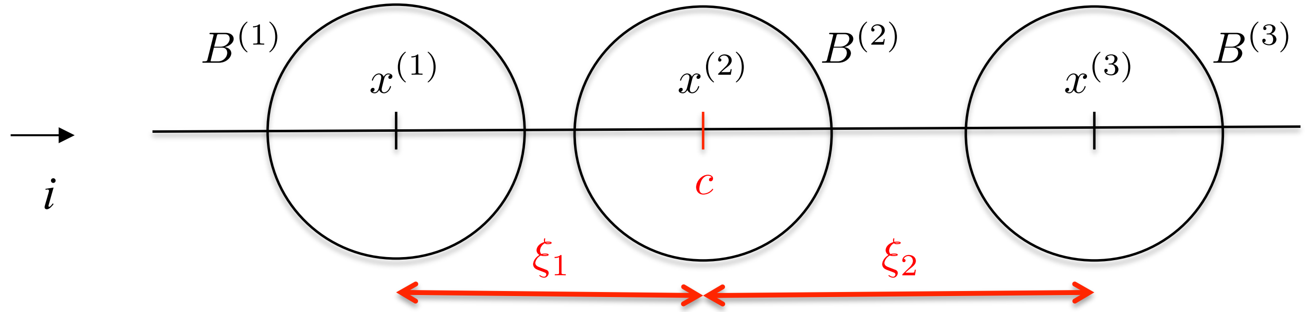


(bacterial flagella)
(20 nm rotary motor)

Design:

3-sphere swimmer (Najafi and Golestanian, 2004)

Positional change from shape change



$$\dot{x}^{(1)} = \dot{c} - \dot{\xi}_1$$
$$\dot{x}^{(2)} = \dot{c}$$
$$\dot{x}^{(3)} = \dot{c} + \dot{\xi}_2$$

$$0 = F = i \cdot \int_{\partial\Omega} \sigma_{visc}(x)\, n(x)\, dA$$

$$= \varphi_1(\xi, \cancel{c})\, \dot{\xi}_1 + \varphi_2(\xi, \cancel{c})\, \dot{\xi}_2 + \underline{\varphi_3(\xi, \cancel{c})}\, \dot{c}$$

by transl. inv.

<0 (nonzero)

Then $\dot{c}$ is uniquely determined and depends linearly on $\dot{\xi}_1$ and $\dot{\xi}_2$

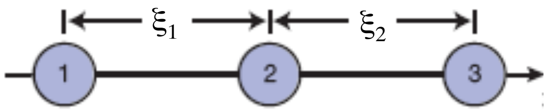
$$\frac{dc}{dt} = V_1(\xi)\frac{d\xi_1}{dt} + V_2(\xi)\frac{d\xi_2}{dt}$$

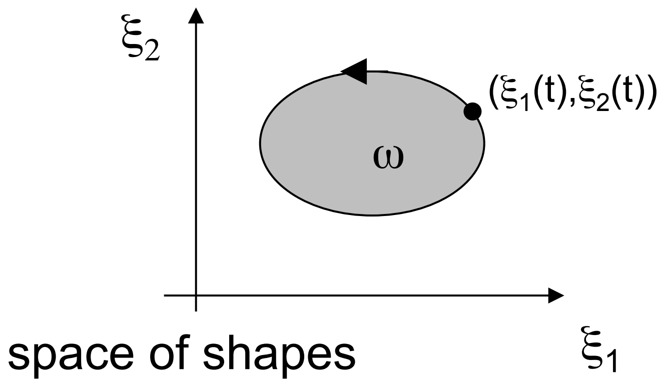
non-linear ODE if **non**-trivial depend. on ξ !

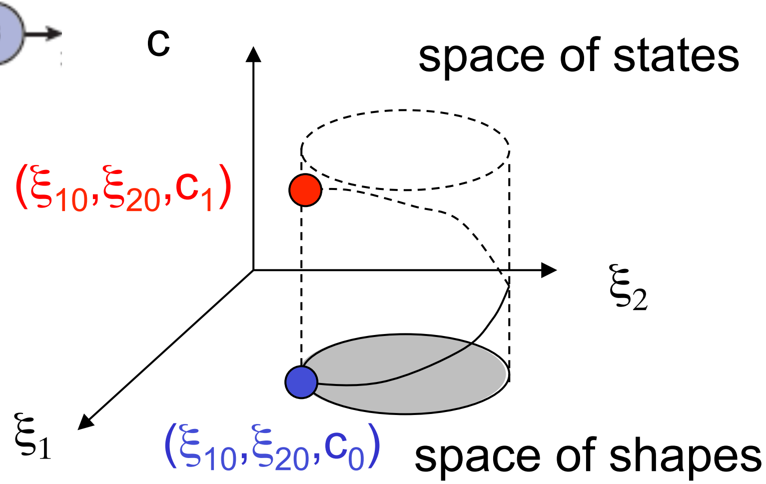
$$V_i(\xi) = -\varphi_i(\xi) / \varphi_{n+1}(\xi)$$

Swimming with one formula

$$\Delta c = \int_0^T \dot{c}\ dt = \int_0^T \left(V_1 \frac{d\xi_1}{dt} + V_2 \frac{d\xi_2}{dt}\right) dt = \int_\omega \mathrm{curl}_\xi V(\xi_1, \xi_2) d\xi_1 d\xi_2$$







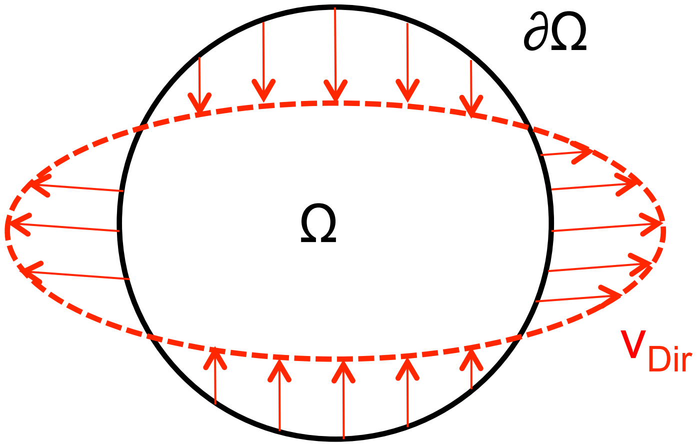
V(ξ) summarizes hydrodynamic interactions and swimming capabilities of a swimmer

Need to solve infinitely many outer Stokes problems to compute V(ξ) !!!

F. Alouges, ADS, A. Lefebvre: J Nonlinear Science **18**, 277-302
Lie-bracket generating or totally non-holonomic affine control system

Motility = Action/Reaction
(theory=eq of motion of fluid + eq of motion of swimmer)

1. Find u,p in the surrounding fluid induced by swimmer's shape changes:



$$\begin{cases} -\eta\Delta u + \nabla p = 0 \\ \operatorname{div} u = 0 \end{cases} \quad \text{on } \mathbb{R}^3 \setminus \Omega$$

$u = v_{Dir}$ on $\partial\Omega$

$u \to 0$ at infinity.

$\sigma_{visc}\, n = DN_{\Omega}[v_{Dir}]$ stress, viscous "reactive" force p.u. area on $\partial\Omega$

Not all of v_{Dir} is a-priori known (only shape is given, not position)

2. Find swimmer's translational and rotational velocities by solving 6 ODEs:

Total (viscous) force = 0 Total (viscous) torque = 0

From equations of motion to non-holonomic constraints

- Viscous forces, reaction to shape change by surroundings (c position, ξ shape):

$$F(c, \xi; \dot{c}, \dot{\xi})$$

- Equation of motion of the swimmer (no inertia, no other forces but viscous drag):

$$\cancel{ma} = F(c, \xi; \dot{c}, \dot{\xi}) + \cancel{F_{\text{gravity}}} + \cancel{F_{\text{other ext}}}$$

0=

- c constrained to ξ:

$$0 = F(c, \xi; \dot{c}, \dot{\xi})$$

net motion from cyclic shape change = constraint must be non-holonomic

- Viscous forces linear in velocity, but shape-dependence introduces the nonlinearity:

$$0 = F = F_c(\xi)\,\dot{c} + \sum_i F_i(\xi)\,\dot{\xi}_i$$

Crawling motility

- From swimming motility….

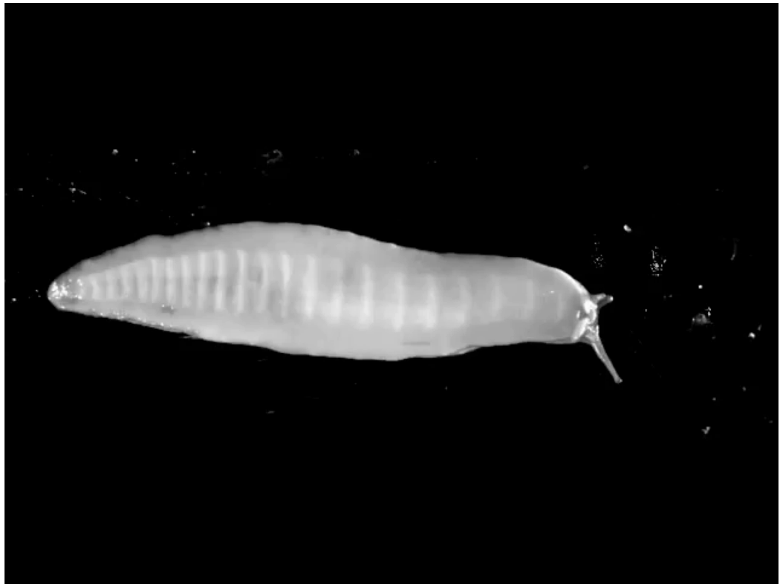
The scallop theorem

- ….to motility by crawling

The snail theorem ?

Is nonlinear rheology of mucus **necessary** for locomotion ?

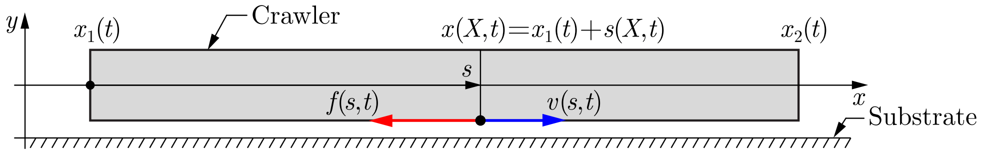
“1D” biological crawlers

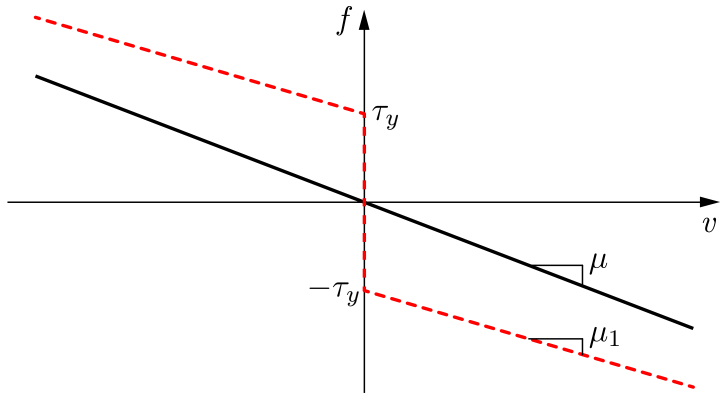


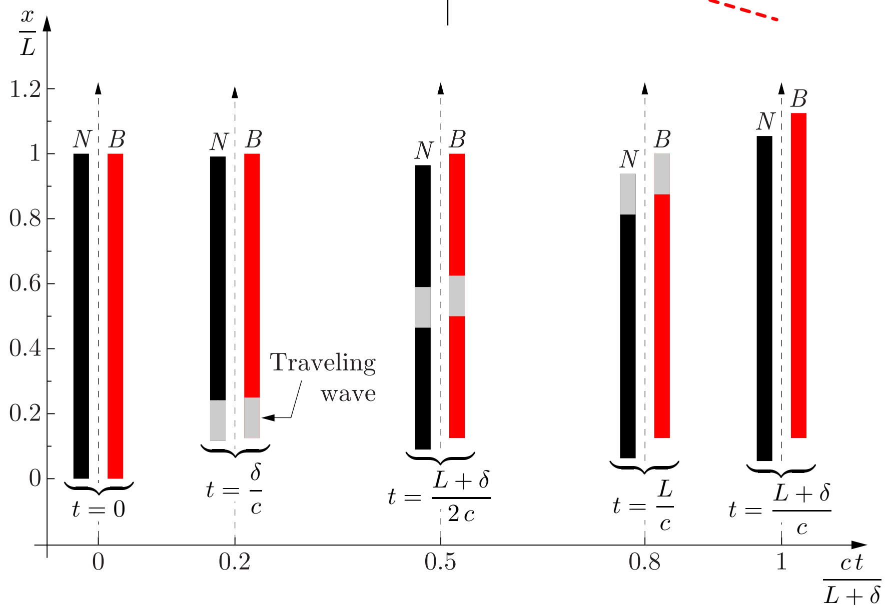
Locomotion by propagation of traveling “contraction” waves

Crawlers on fluid layers: contraction waves

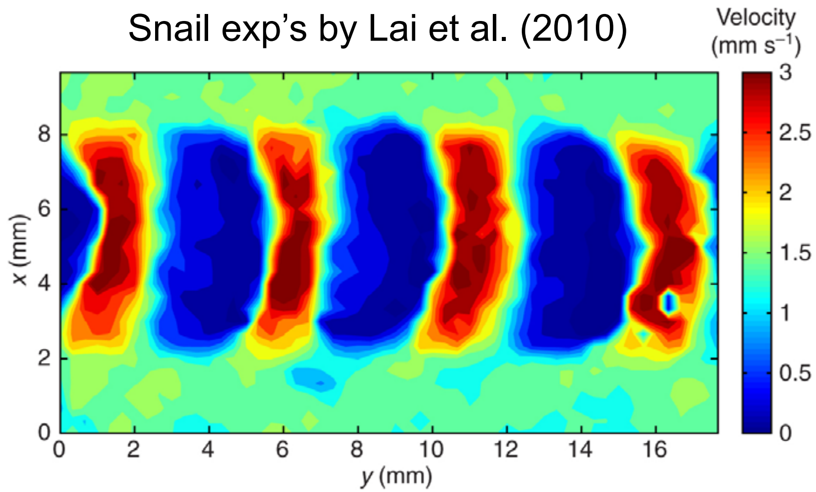
ADS, F. Guarnieri, G. Noselli, A. Tatone: Int J Nonlinear Mechanics **56**, 142-147 (2013)

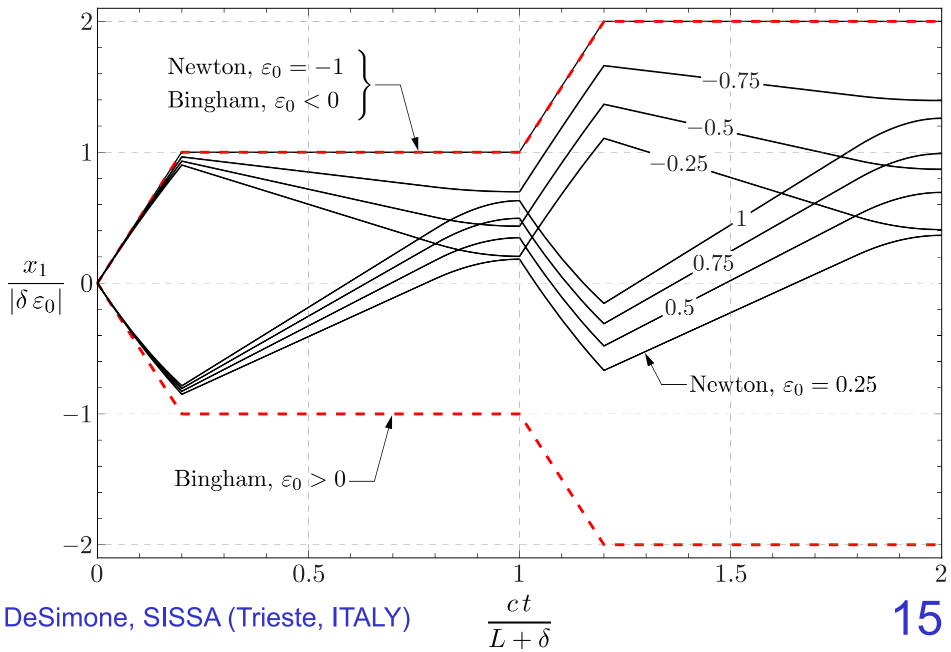




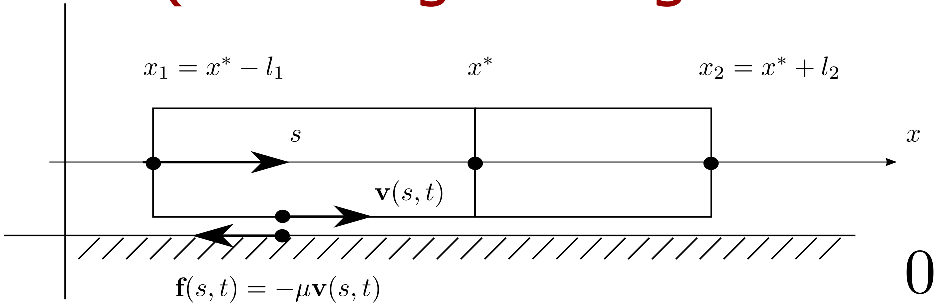


Snail exp's by Lai et al. (2010)





Discrete argument
(crawling analog of the three-sphere-swimmer)



$$0 = F = \int_0^{l_1+l_2} f(s)ds$$

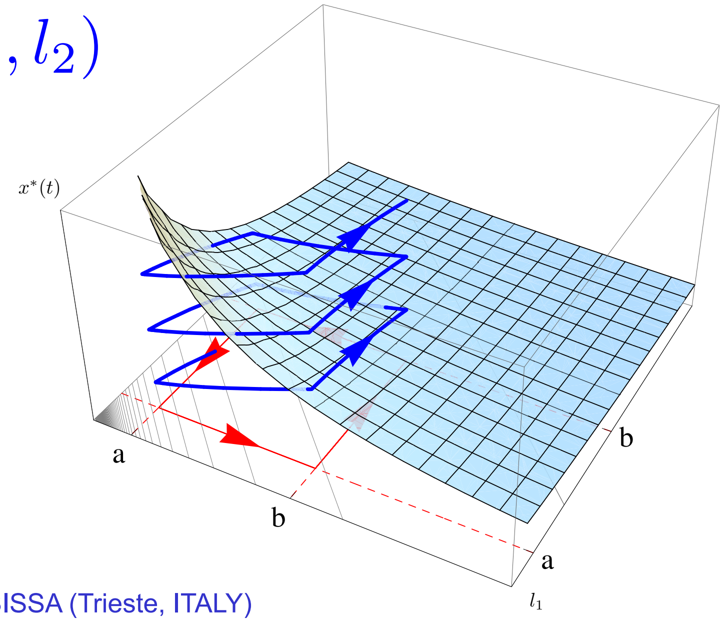
$$0 = -\mu\left((l_1 + l_2)\dot{x}^* - \frac{l_1}{2}\dot{l}_1 + \frac{l_2}{2}\dot{l}_1\right)$$

$$\dot{x}^\star = \frac{l_1}{2(l_1+l_2)}\dot{l}_1 - \frac{l_2}{2(l_1+l_2)}\dot{l}_2$$

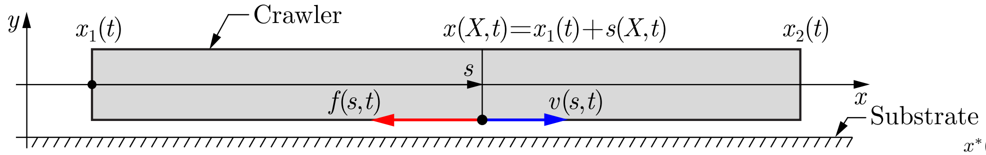
$V_1(l_1, l_2)$ $\quad$ $V_2(l_1, l_2)$

$$\Delta x^* = \int_{[a,b]^2} \operatorname{curl} \mathbf{V}(l_1, l_2)\, dl_1 dl_2$$

$$\operatorname{curl} \mathbf{V}(l_1, l_2) = \frac{1}{2}\frac{1}{l_1 + l_2}$$

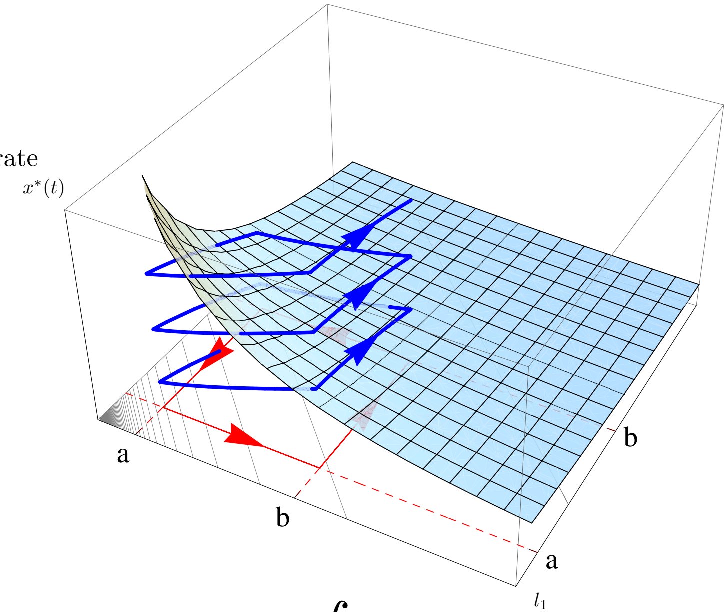


Oversimplifying the model ….



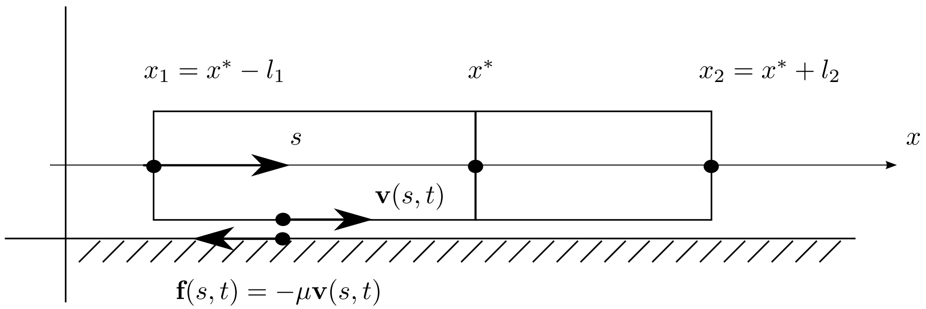
$$\dot{x}_1(t) = \frac{1}{l(t)} \int_0^L \dot{s}(X,t) s'(X,t) dX$$

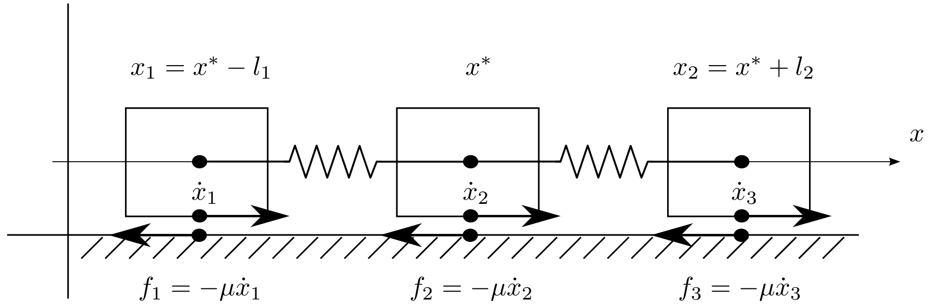
$$s' \sim (1+\epsilon) \qquad \dot{s}s' \sim \dot{s}$$



$$\Delta x^* = \int_{[a,b]^2} \text{curl}\,\mathbf{V}(l_1,l_2)\, dl_1 dl_2$$

$$\text{curl}\,\mathbf{V}(l_1,l_2) = \frac{1}{2}\,\frac{1}{l_1+l_2}$$





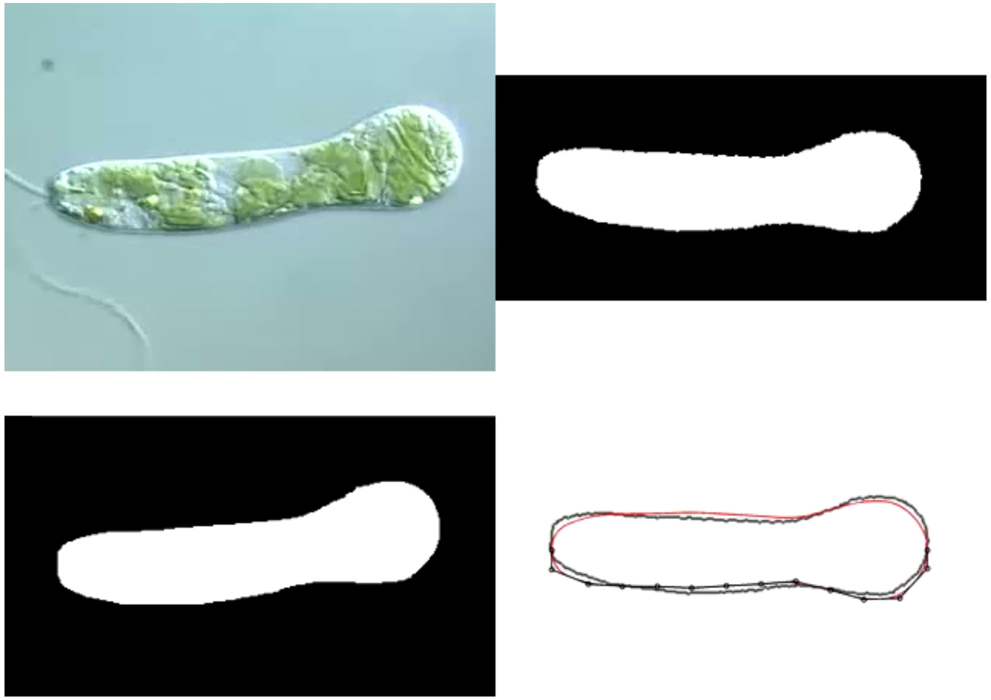
$$\dot{x}^* = \frac{1}{3}\dot{l}_1 - \frac{1}{3}\dot{l}_2$$

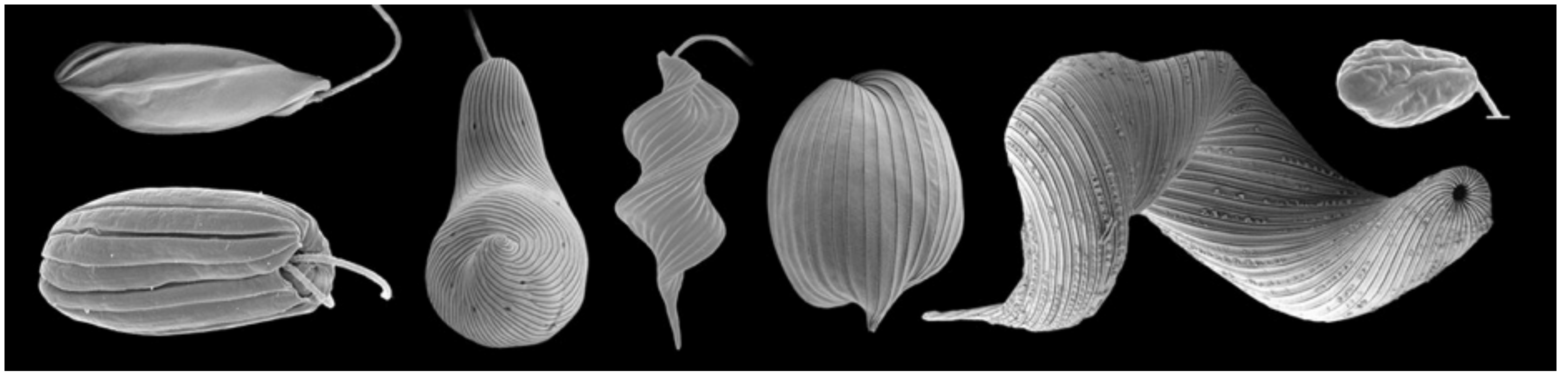
$$\text{curl}\,\mathbf{V}(l_1,l_2) = 0$$

From general concepts to biological detail: *metaboly* of euglenids

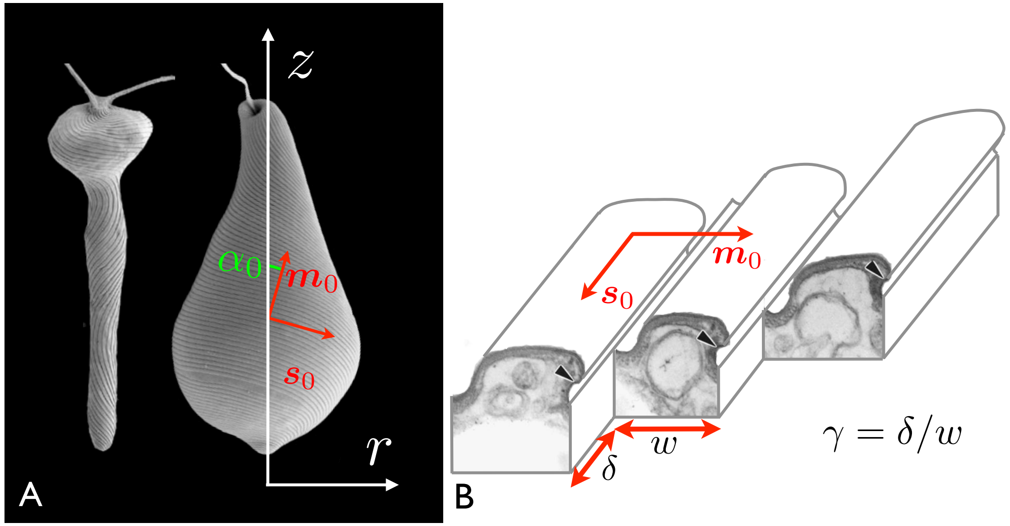
Euglenoid movement
(metaboly)

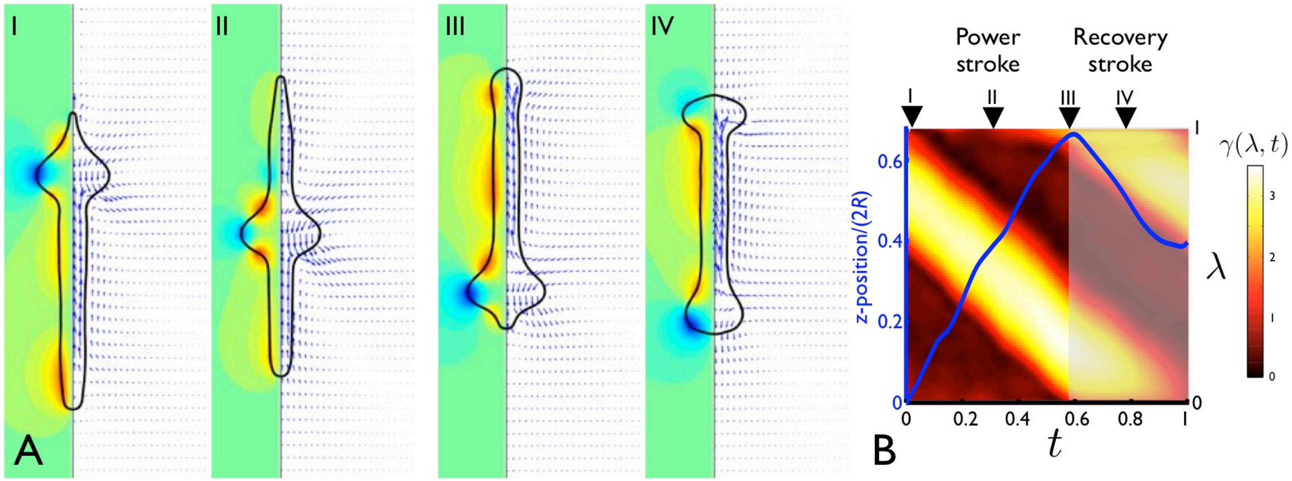
A swimmer with a twist





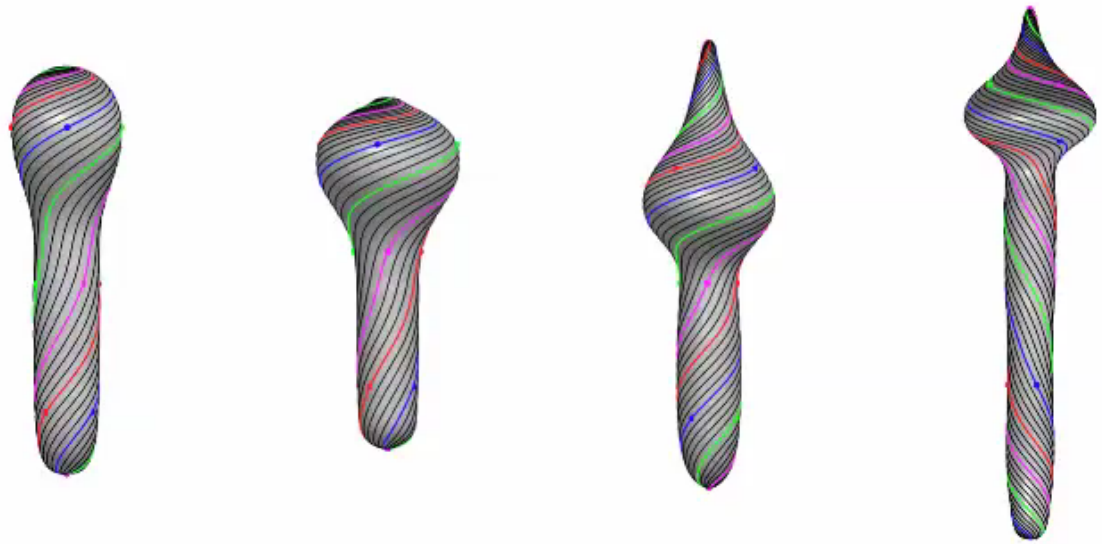
Reverse engineering the euglenoid movement



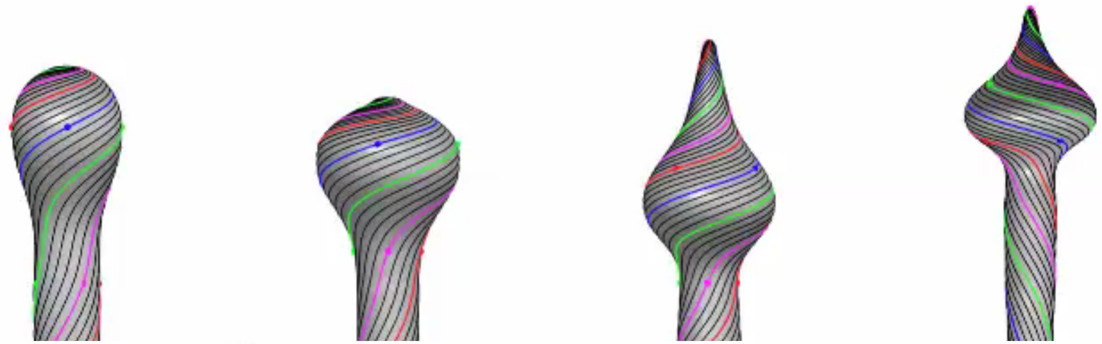


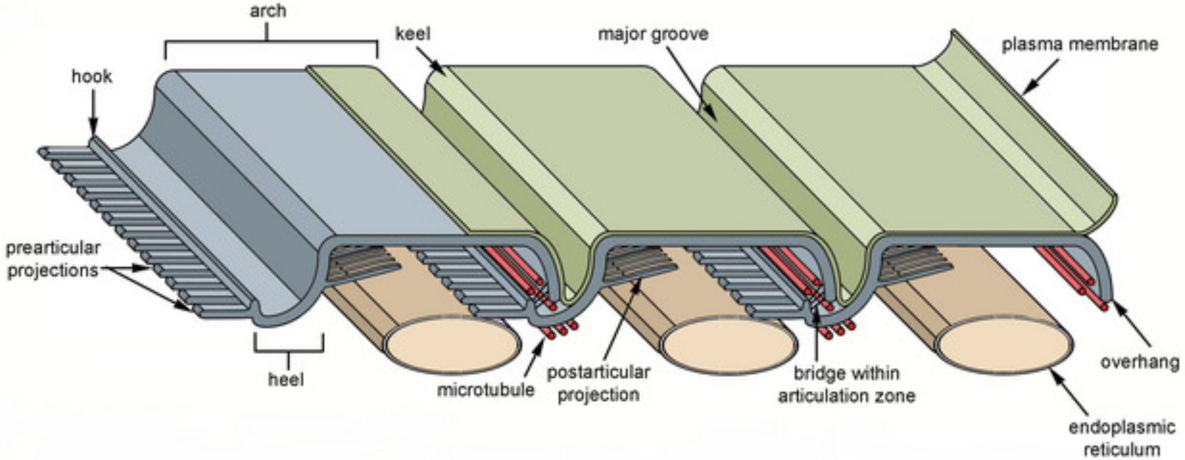
M. Arroyo, D. Milan, L. Heltai, ADS, PNAS **109**, 1784-1789 (2012)

Proposed mechanism: traveling shear wave



Proposed mechanism: traveling shear wave





Thanks

to many former and current students and collaborators:

F. Alouges, A. Lefebvre, B. Merlet, L. Heltai;

M. Morandotti, G. Dal Maso;

L. Cardamone, A. Laio, V. Torre, R. Shahapure;

M. Arroyo, D. Milan;

L. Giraldi, M. Zoppello;

G. Cicconofri;

G. Noselli, A. Tatone, F. Guarnieri;

………………………..

and to you for your kind attention!

References on swimming motility

- F. Alouges, A. DeSimone, A. Lefebvre: Optimal strokes of low Reynolds number swimmers: an example. Journal of Nonlinear Science 18, 277-302 (2008).
- F. Alouges, A. DeSimone, A. Lefebvre: Biological fluid dynamics. Springer Encyclopedia of Complexity and Systems Science (2009).
- F. Alouges, A. DeSimone, A. Lefebvre: Optimal strokes of axisymmetric microswimmers. European Physical Journal E 28, 279–284 (2009).
- F. Alouges, A. DeSimone, L. Heltai: Numerical strategies for stroke optimization at low Re numbers. M3AS 21, 361-397 (2011).
- F. Alouges, A. DeSimone, L. Heltai, A. Lefebvre, B. Merlet: Optimally swimming stokesian robots. Discrete and Continuous Dynamical Systems B, in press (2013).
- F. Alouges, A. DeSimone, L. Giraldi, M. Zoppello: Self-propulsion of slender micro-swimmers by curvature control: N-link swimmers. Int. J. Nonlinear Mechanics **56**, 132-141 (2013).

- G. Dal Maso, A. DeSimone, M. Morandotti: An existence and uniqueness result for the self-propelled motion of micro-swimmers. SIAM J. Math. Analysis **43**, 1345-1368 (2011).

- A. DeSimone, F. Alouges, L. Heltai, A. Lefebvre: Computing optimal strokes for low Reynolds number swimmers. In: “Natural Locomotion in Fluids and on Surfaces”, IMA Volumes, Springer Verlag (2012).
- M. Arroyo, L. Heltai, D. Milan, A. DeSimone: Reverse engineering the euglenoid movement. PNAS **109**, 1784-1789 (2012).

References on crawling motility

- L. Cardamone, A. Laio, R. Shahapure, V. Torre, A. DeSimone: Cytoskeletal actin networks in motile cells are critically self-organized systems synchronized by mechanical interactions. PNAS **108**, 13978–13983 (2011).

- A. DeSimone, A. Tatone: Crawling motility through the analysis of model locomotors: two case studies. Eur. Phys. Journal E **35**, 85–92 (2012).

- A. DeSimone, F. Guarnieri, G. Noselli, A. Tatone: Crawlers in viscous environments: linear vs nonlinear rheology. Int. J. Nonlinear Mechanics **56**, 142-147 (2013).

- P. Gidoni, G. Noselli, A. DeSimone: Crawlers on directional surfaces. Int. J. Nonlinear Mechanics **61**, 65-73 (2014).