Elasticity-based mechanism for collective motion in natural and artificial swarms

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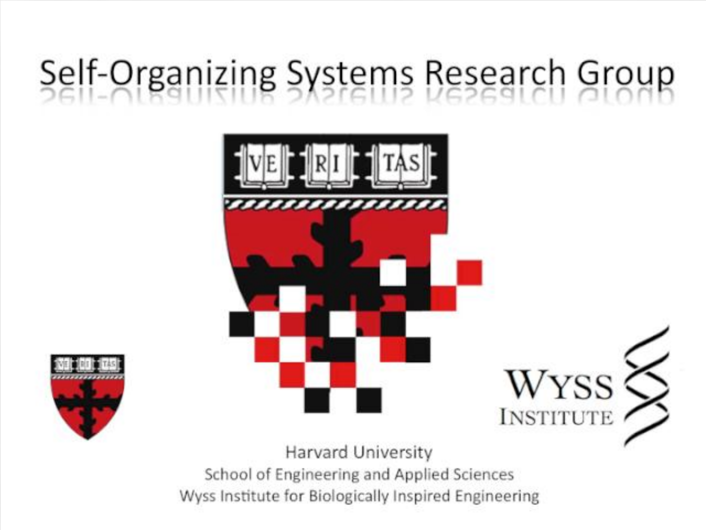
Background: Biological motivation

- Collective motion is observed in diverse animal species: From bacteria to humans
- Fish schools & bird flocks can have from a few to several thousand individuals
- Locust swarms can contain 10^9 insects traveling thousands of kilometers

from: *Collective Minds* (documentary by Jacob Kneser)

Background: Robotics

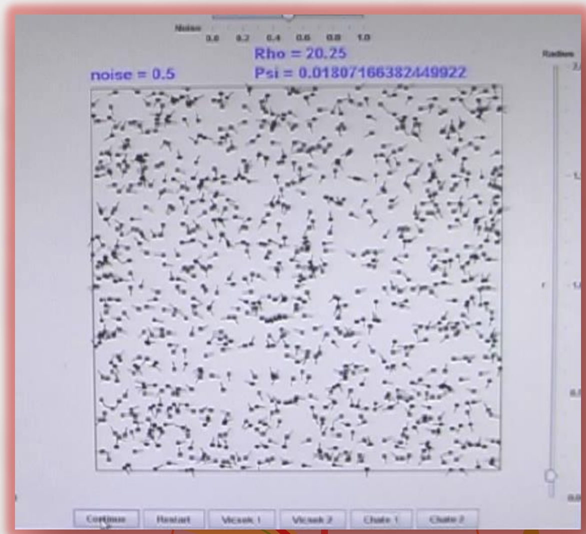
- Groups of robots effective for:
  - Deploying sensor networks
  - Parallel tasks
  - Micro-robotics
- Control algorithms for groups of autonomous robots must be:
  - **Decentralized, Scalable, Robust**
- Additional constrains
  - Small processing power & bandwidth
  - Limited communication (nearby neighbors, line-of-sight, direct contact)



Background: Random thoughts

- My first works on Collective Motion (before it was cool):
  - *Phase transitions in self-driven many-particle systems and related non-equilibrium models: A network approach.*
    M. Aldana and C. Huepe – *J. Stat. Phys.* 112 (1-2), 135-153 (**2003**).
  - *Intermittency and clustering in a system of self-driven particles*
    C. Huepe & M. Aldana – *Phys. Rev. Lett.* 92 (16)168701 (**2004**).
- Current research on active matter (non extensive list):
  - Hydrodynamic descriptions
    - self-propelled agents on fluids or fluids of self-propelled agents
    - fluids with *aligning spins* or *self-energized advection*
    - active complex fluids, - etc.
  - Agent-based
    - explicit/implicit alignment,
    - escape-pursuit, - etc.
- Most fascinating for me: how active systems self-organize

- **Motivation**
  - Non-equilibrium self-organizing statistical system
  - Energy injected at smallest scales



- **The Vicsek model** (1995 results)

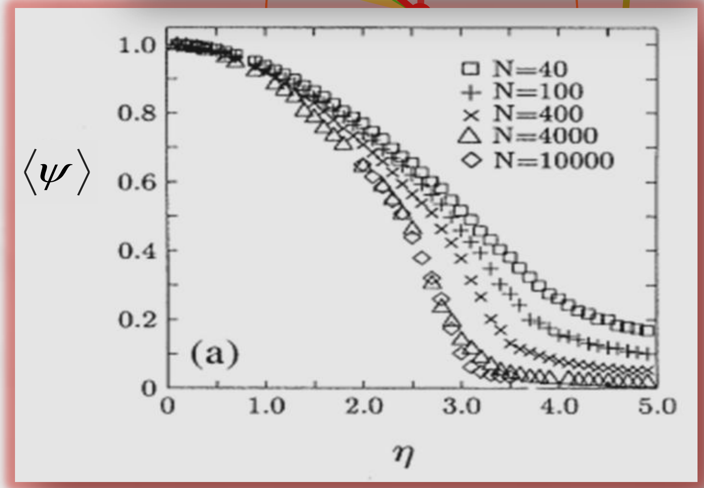
$$\theta_i(t+\Delta t) = \mathrm{Ang}\left\{\sum_{Z_i(t)} \vec{\mathrm{Vel}}\left[\theta_j(t)\right]\right\} + \eta\, \xi_i(t)$$

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t) + \vec{\mathrm{Vel}}\left[\theta_i(t)\right]\,\Delta t.$$

- **Order parameter** (alignment/magnetization)

$$\psi(t) = \frac{1}{N s}\left|\sum_{i=1}^{N} \vec{\mathrm{Vel}}[\theta_i(t)]\right|$$

- **Main Result**
  - "Novel type" of **phase transition**



Dr. Yael Katz

Prof. Kolbjørn Tunstrøm

Prof. Christos Ioannou

Prof. Iain Couzin

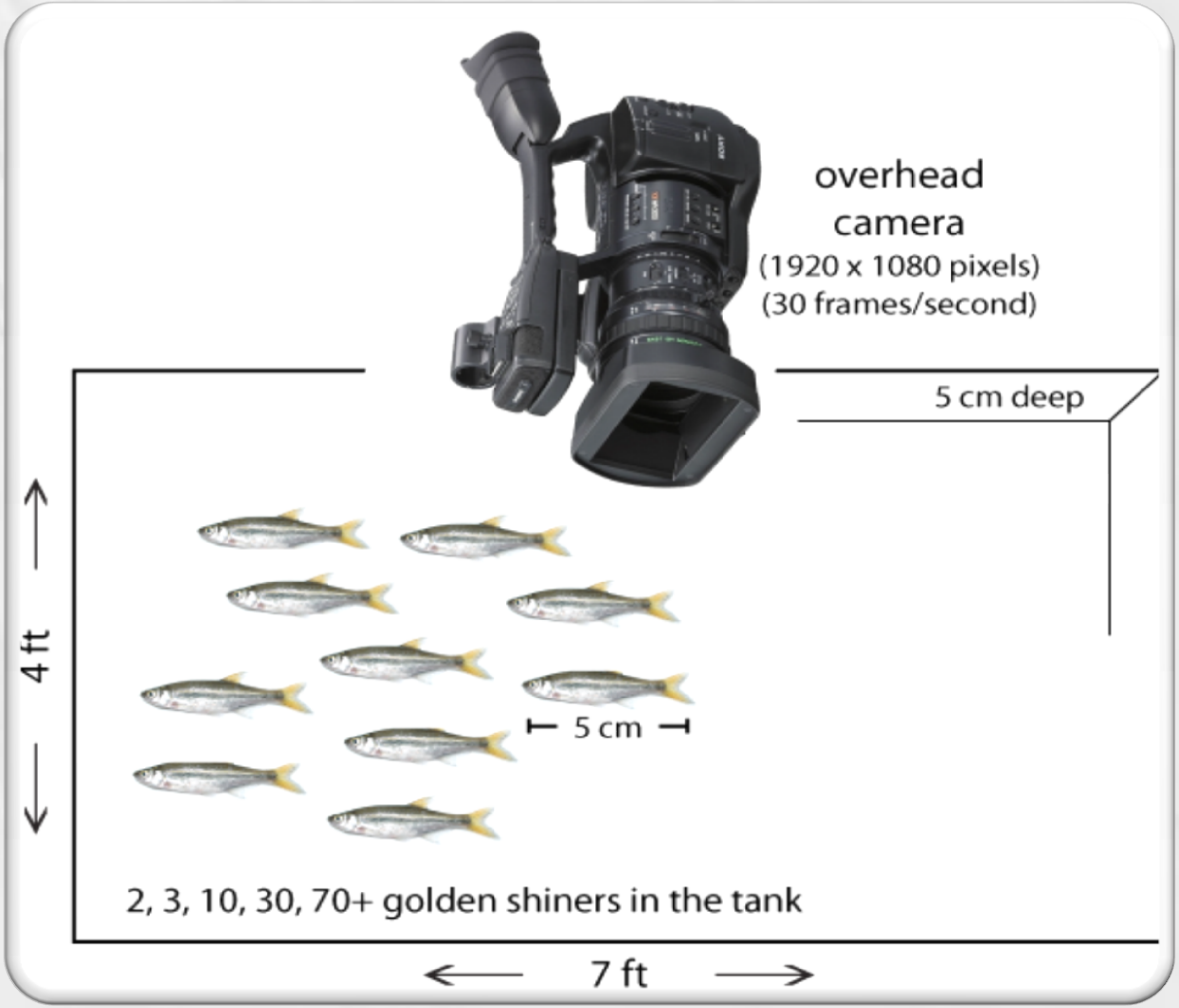
❑ Data-driven analysis of fish schooling experiments

(Inspiration for this work)

Publications:

- *Collective states, multistability and transitional behavior in schooling fish*, PLoS Computational Biology Feb. 2013 [9(2), e1002915]
- *Inferring the structure and dynamics of interactions in schooling fish,* PNAS July 2011 [doi:10.1073/pnas.1107583108]

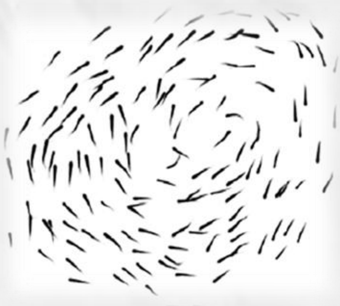
- Experimental System



- 1000 fish dynamics

Polarized state

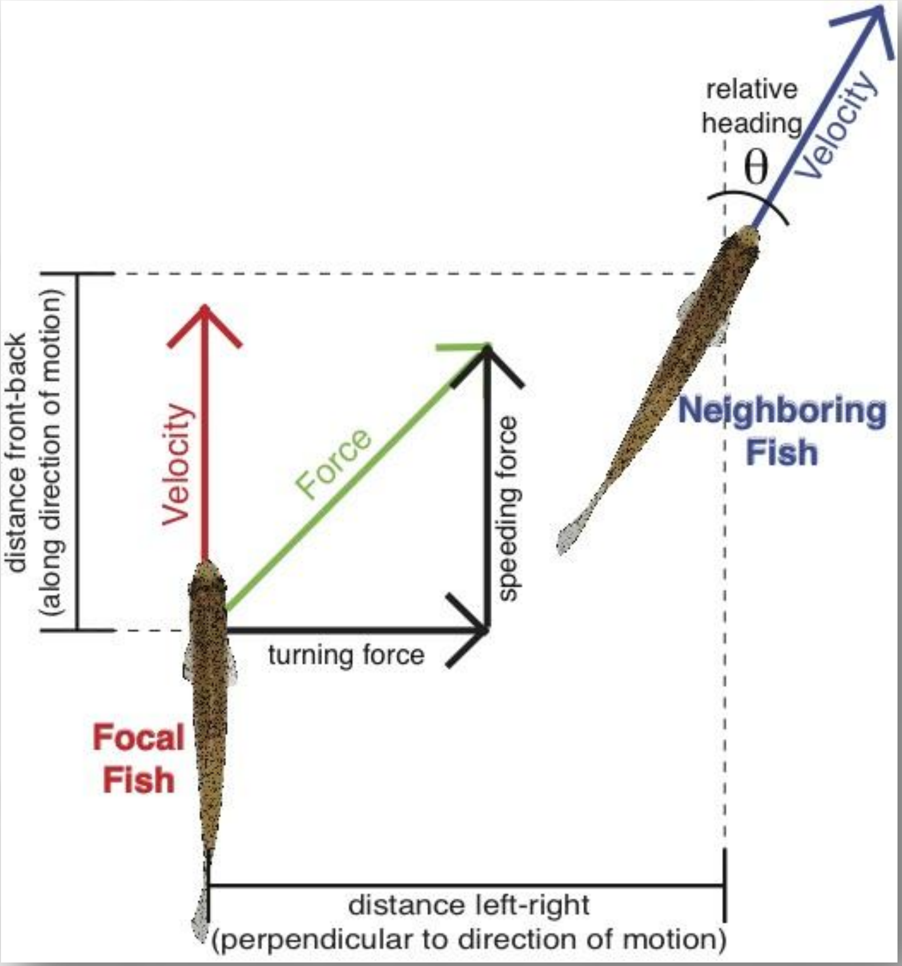
Rotating state



Swarming state

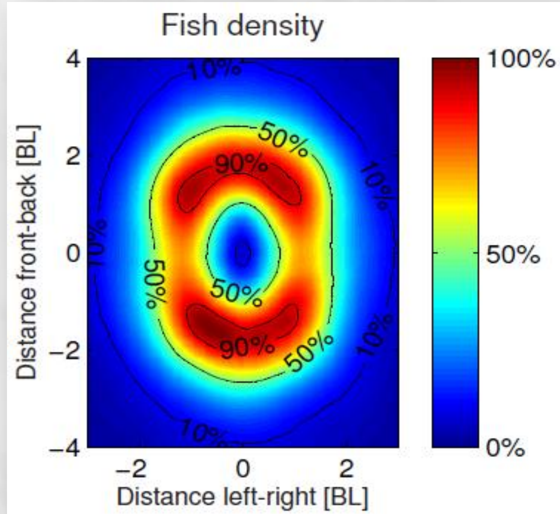
The two-fish system

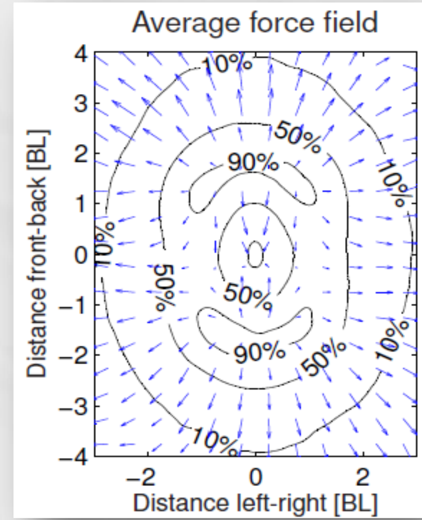
- **Effective-force approach**
  - Measure mean *effective (social) forces* on 2-fish & 3-fish systems
  - Use large dataset: 14 experiments of 56 minutes each
- **Goals**
  - *"Model-free"* approach on clear mathematical grounds
  - Gain intuition over possible non-standard interactions



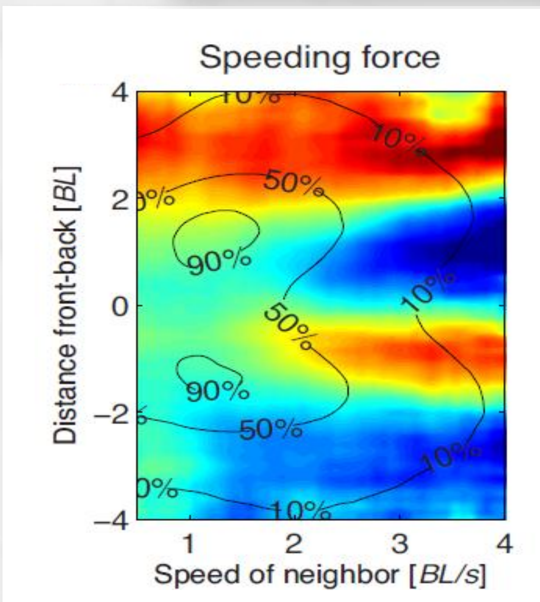
The two-fish system

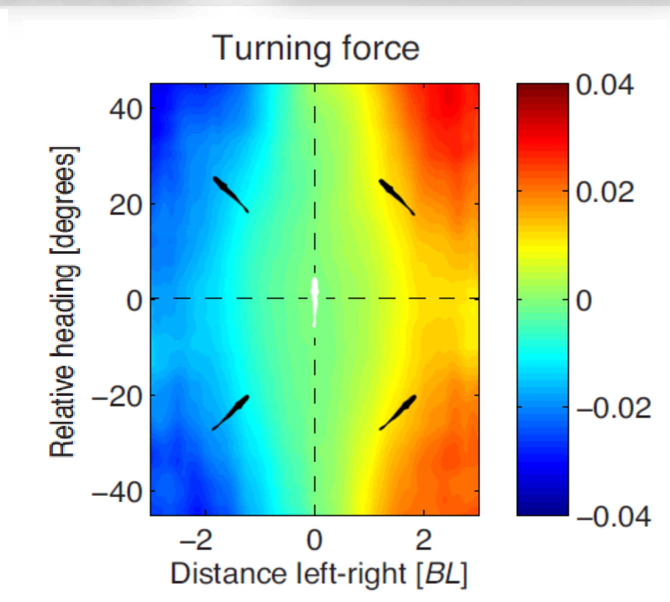
- Mean **effective forces** (social) on 2-fish & 3-fish systems
- **14** experiments of **56** minutes each





• Zero force ⇔ high density





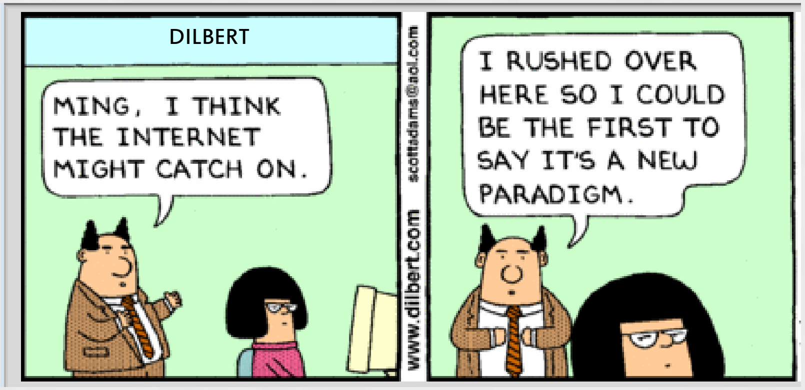
• Higher speed ⇒ larger front-back distance

• Higher left-right distance ⇒ larger turning force

❑ Elasticity-based mechanism for collective motion in active solids

Dr. Eliseo Ferrante

Prof. Ali-Emre Turgut



Publications:

- *Elasticity-Based Mechanism for the Collective Motion of Self-Propelled Particles with Springlike Interactions: A Model System for Natural and Artificial Swarms.* **Phys. Rev. Lett., 111, 268302 (2013)**
- *Collective motion dynamics of active solids and active crystals.* **New J. Phys. 15 095011 (2013)**

– Motivation

- Consider limit opposite to Vicsek model
  - No explicit aligning interaction
    - Interactions based on relative positions (not relative orientations)
  - Fixed interacting neighbors (Following Mermin-Wagner theorem condition)

- Active Elastic Sheet model
  - Only attraction-repulsion forces
  - Spring-mass model of elastic membrane
  - Overdamped & self-propelled dynamics

– Toy model with only attraction-repulsion interactions:

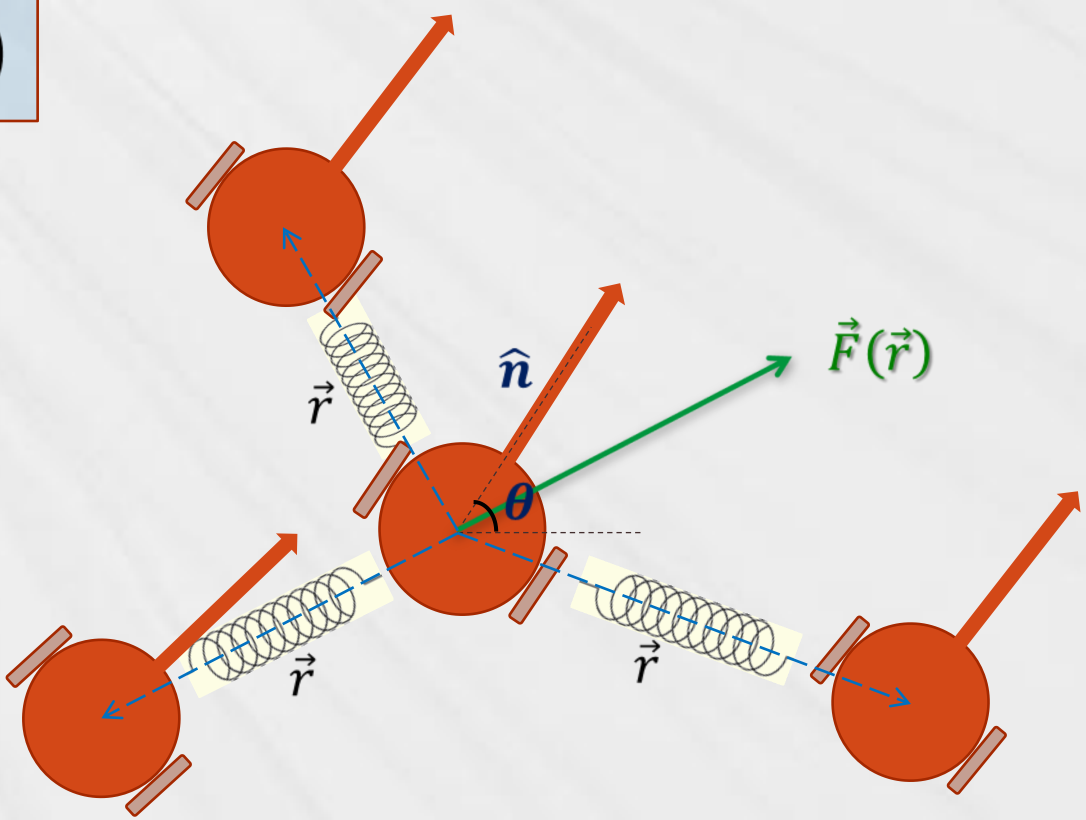
Active Elastic Sheet model

- Forward speed:

$$v = v_0 + \alpha(\vec{F} \cdot \hat{n})$$

- Turning rate:

$$\dot{\theta} = \beta(\vec{F} \cdot \hat{n}_\perp)$$



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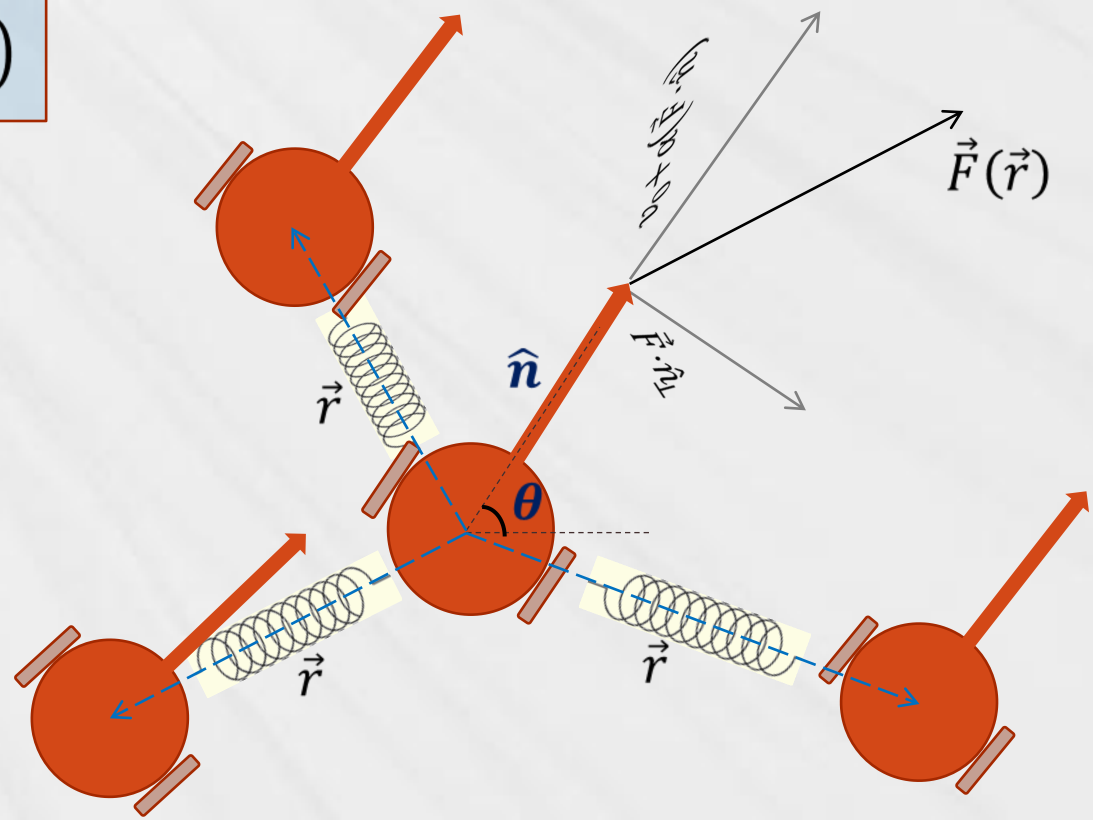


Active Elastic Sheet model

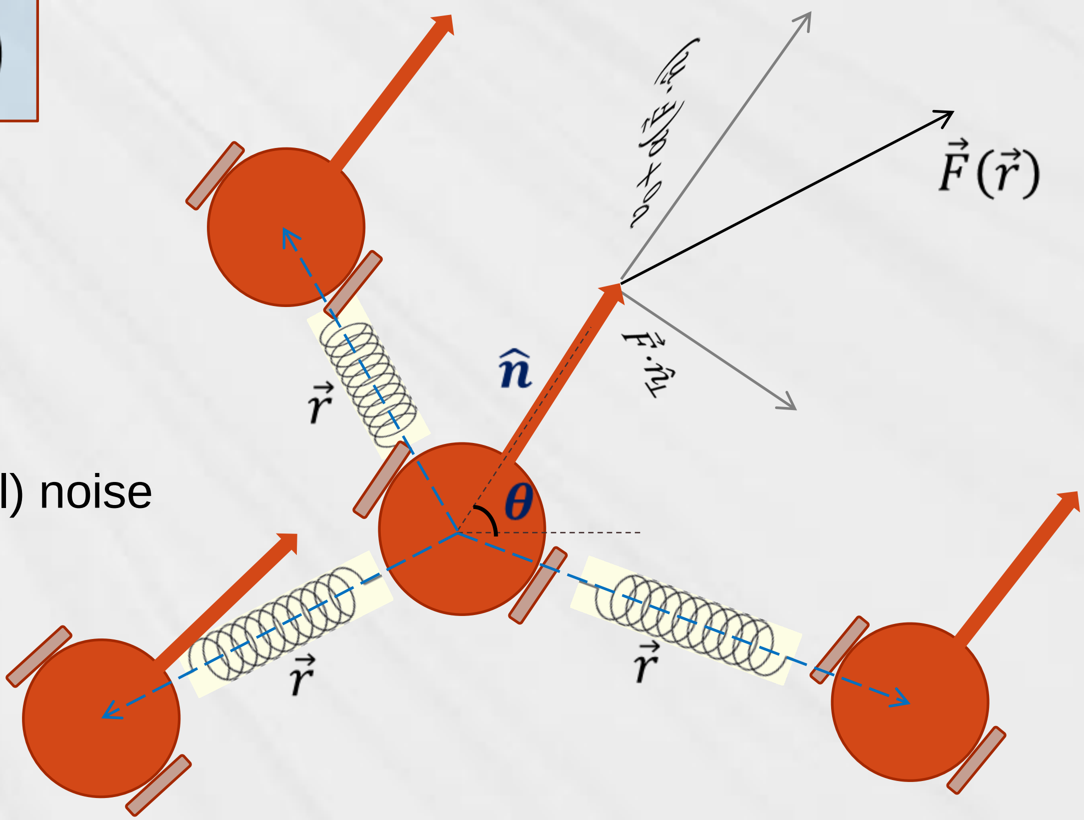
- Forward speed:

$$v = v_0 + \alpha(\vec{F} \cdot \hat{n})$$

- Turning rate:

$$\dot{\theta} = \beta(\vec{F} \cdot \hat{n}_\perp)$$

  - Sensing (vectorial) noise
- Actuation (angular) noise



– Toy model with only attraction-repulsion interactions:

Active Elastic Sheet model

- Linear spring-like forces over each agent:

$$\vec{F} = \sum_{i \in S} \vec{f}_i + D_r\, \hat{\xi}_r = -\sum_{i \in S} k\,(|\vec{r}_i| - l)\,\frac{\vec{r}_i}{|\vec{r}_i|} + D_r\, \hat{\xi}_r$$

S : set of all agents connected to focal agent

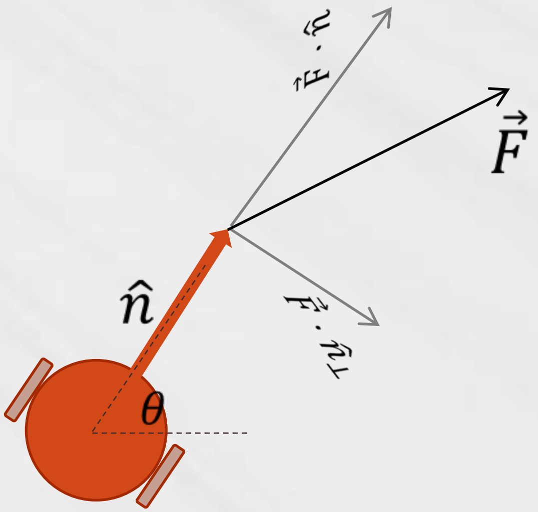
$\vec{r}_i$: position of i with respect to focal agent

k & l : spring constants & natural lengths

- Equations of motion for focal agent

Forward speed : $v = v_0 + \alpha(\vec{F} \cdot \hat{n})$

Turning rate : $\dot{\theta} = \beta(\vec{F} \cdot \hat{n}_\perp) + D_\theta\, \xi_\theta$



v_0: preferred speed

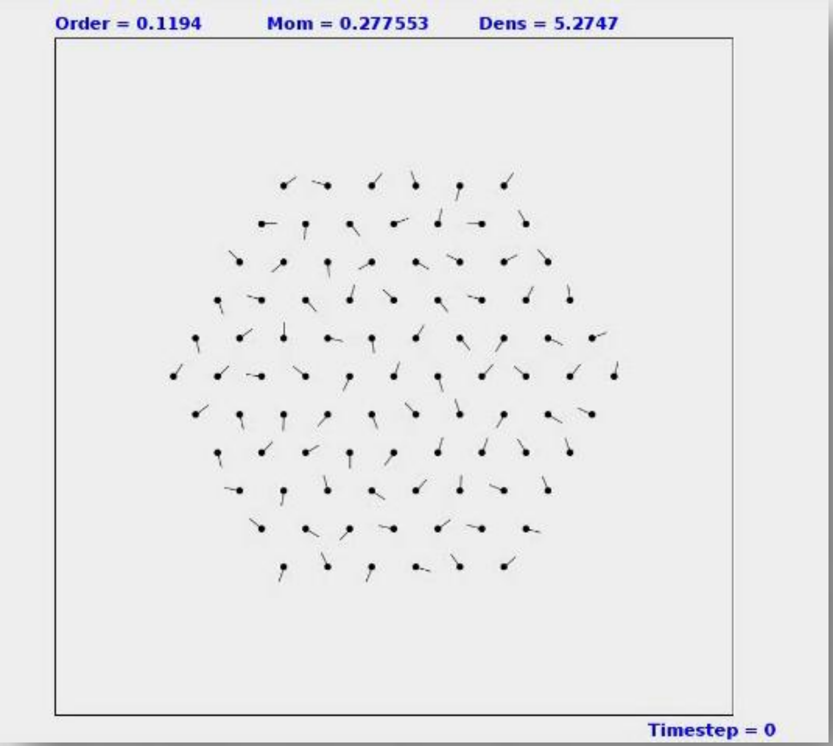
α, β : inverse damping coefficients

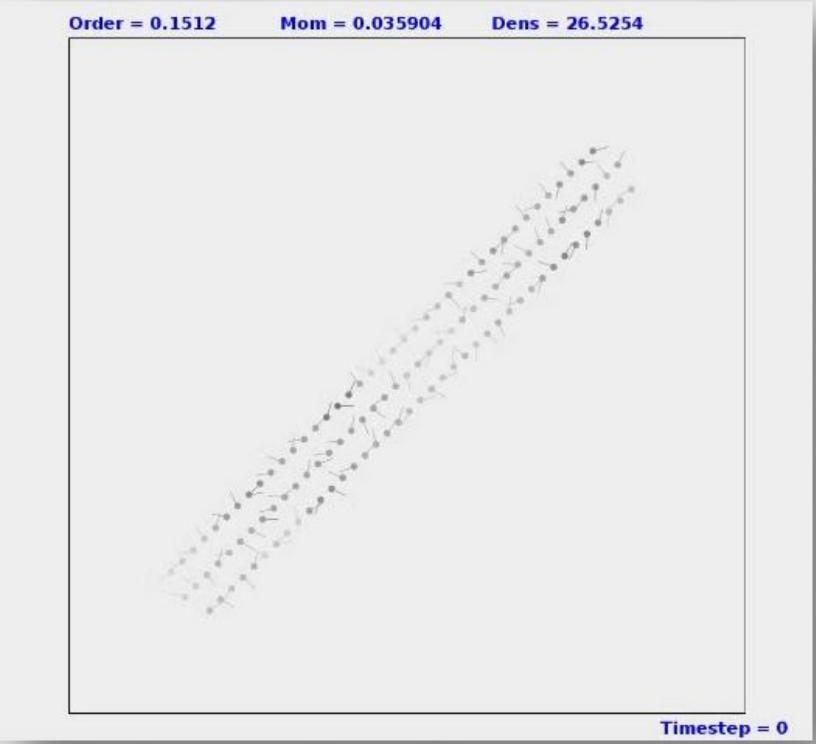
$\hat{n}$: unit heading vector

ξ : random variables

– Simulation dynamics

- *Spring-mass* model of elastic sheet
- Active crystals:

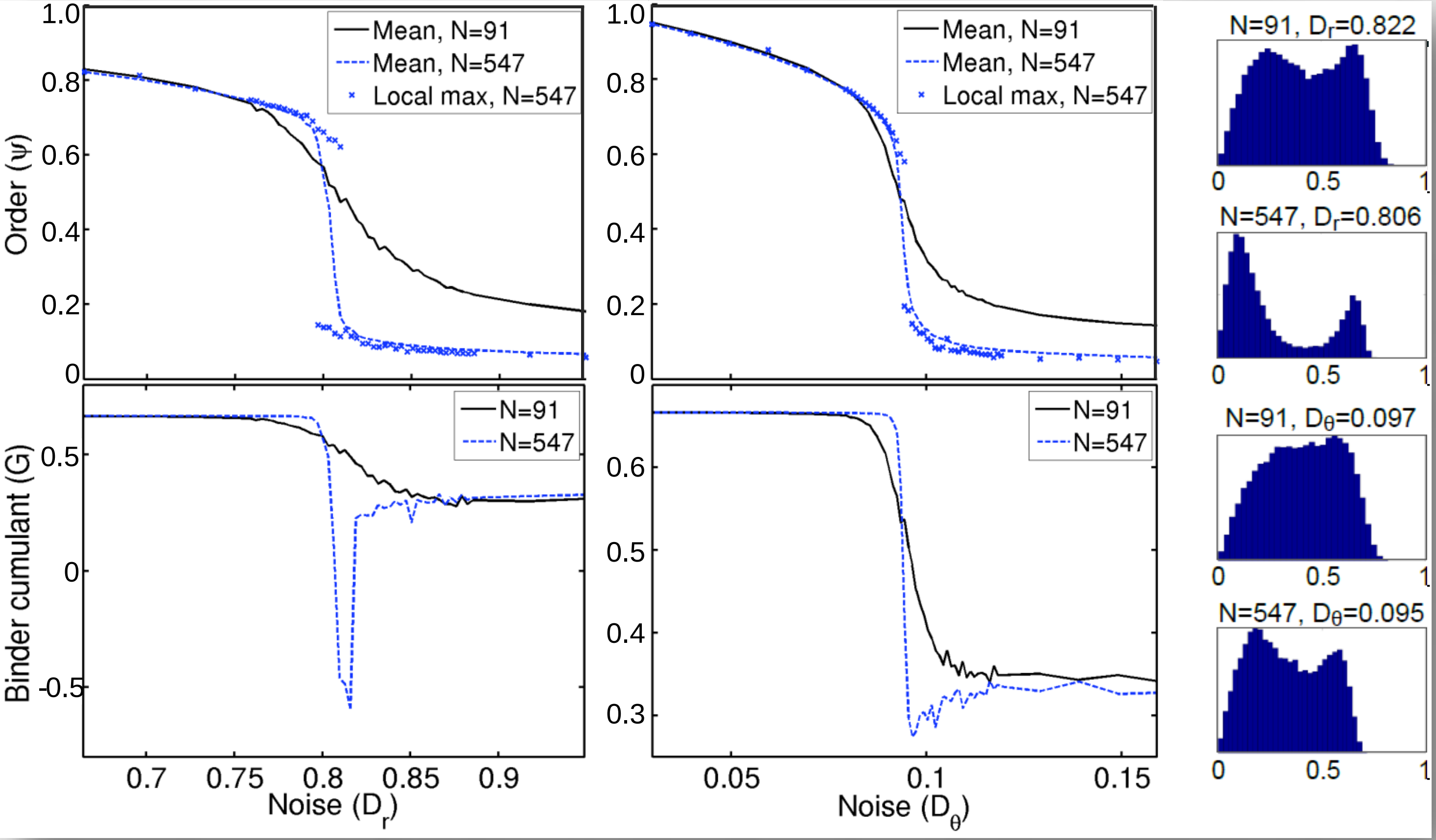




- **Rotational** ordered state (metastable)
- **Translational** ordered state (preferred heading direction)

- Ordered states develop below critical noise

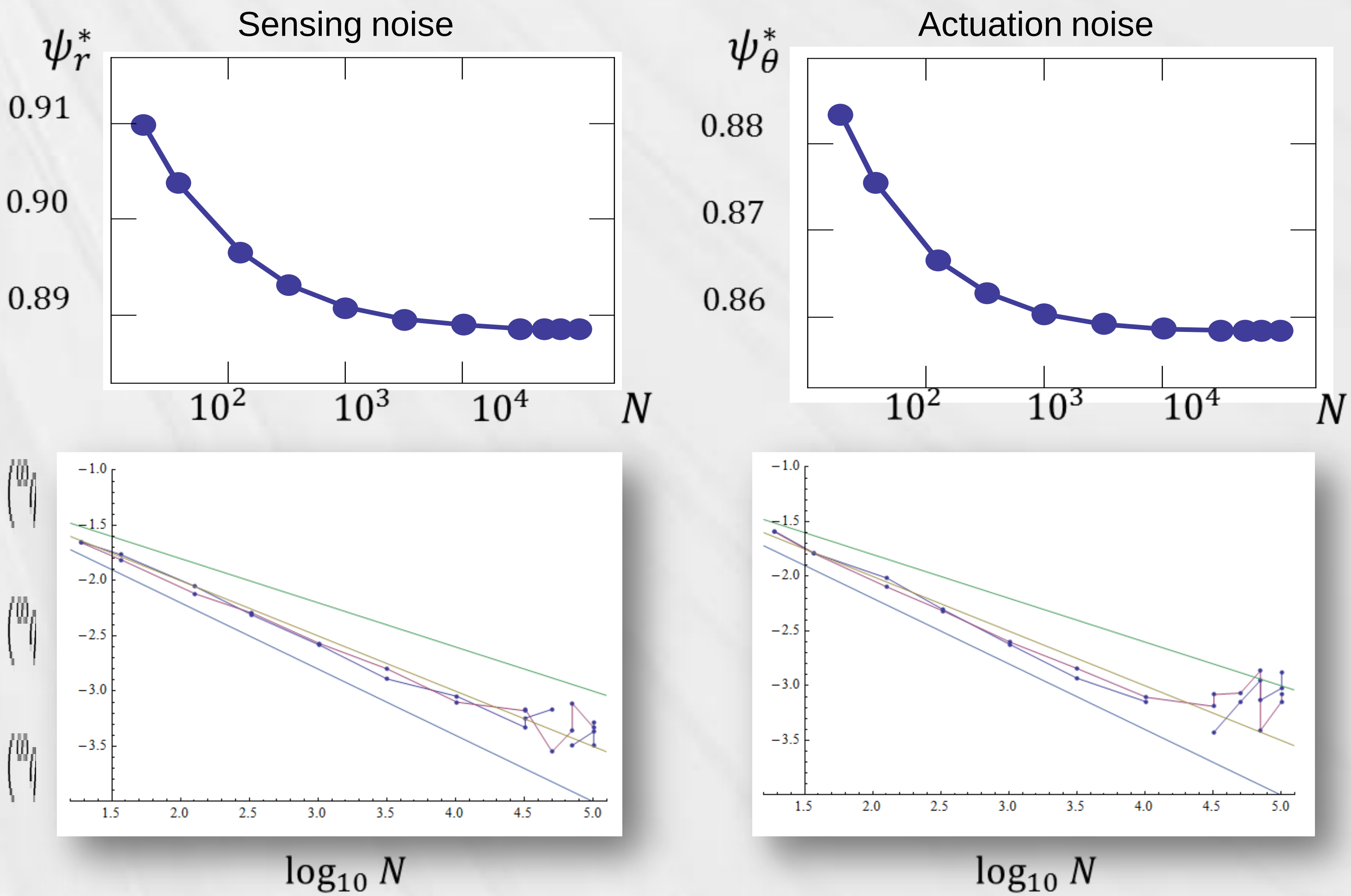
Phase transition



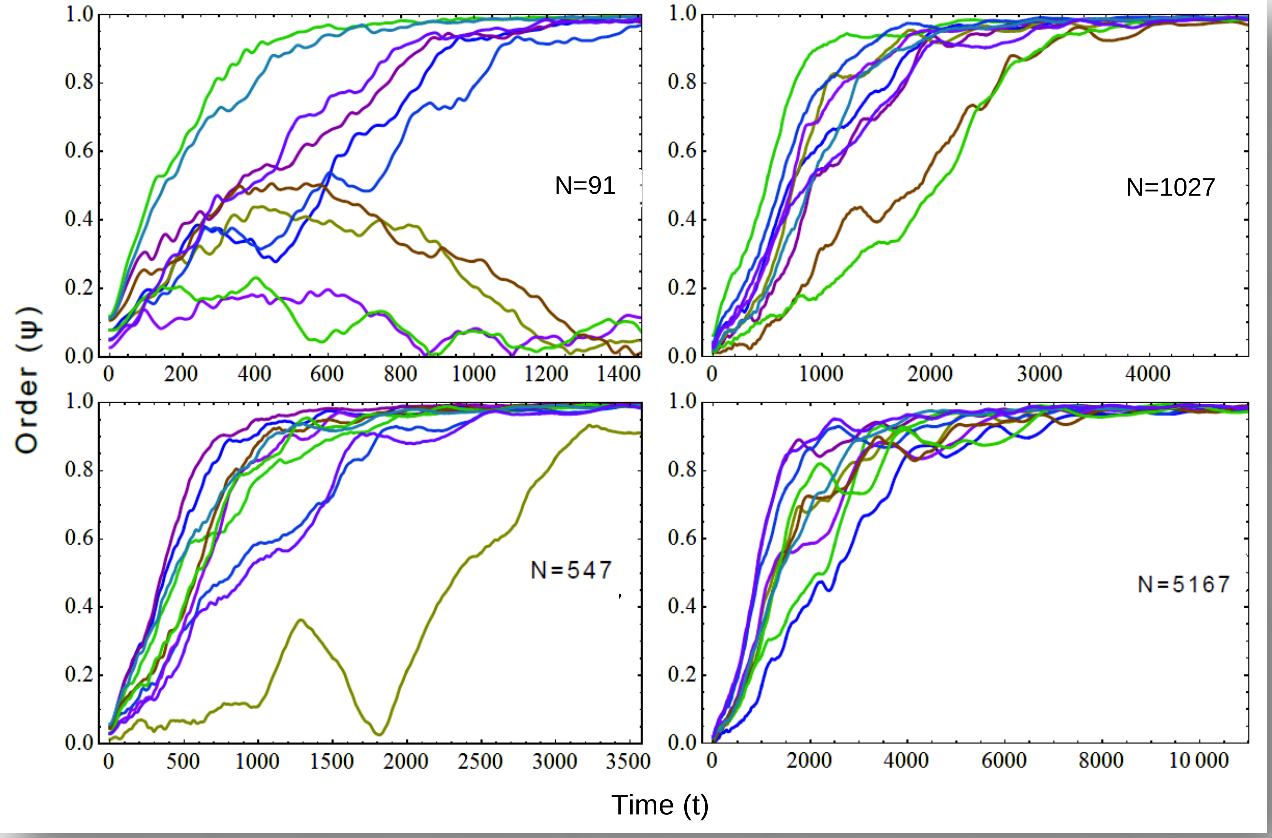
- Alignment order parameter, Binder cumulant, and bimodal distributions:

  → First order phase transition

- **Long-range order** verified through numerical finite scaling analysis
- Approximate scaling at ~60% of critical noise: $\psi = C\, N^{-1/2} + \psi_\infty$



Convergence dynamics



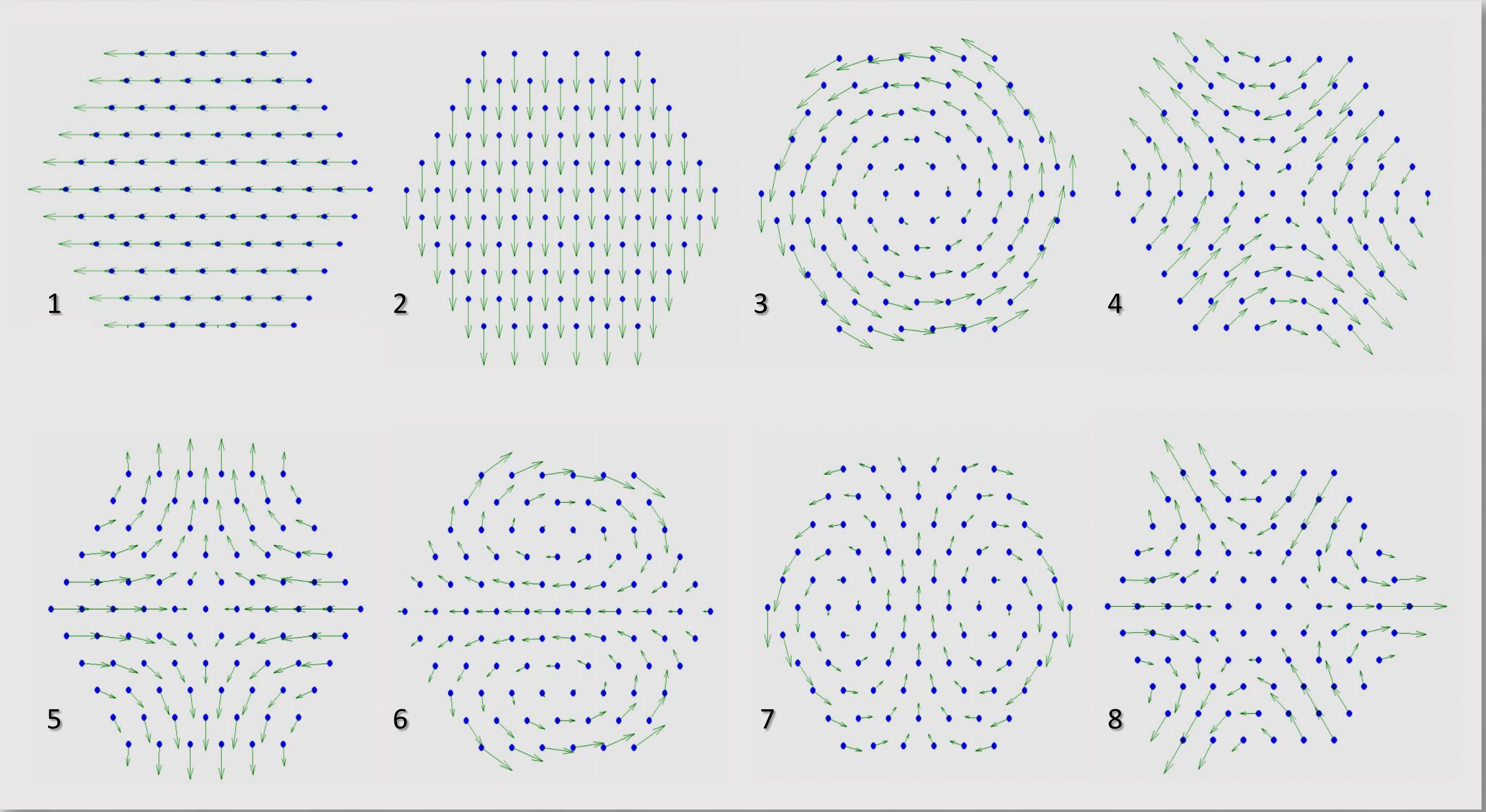
- Variability reflects **complex dynamical landscape**
- Very different amounts of time spent in **metastable states.**
- **Typical convergence time** grows here slower than $\sim N^{1/2}$

– Self-organizing Mechanism

- Why does the system self-organize into common heading direction?
  - Despite:
    - no ferromagnetic-like interaction
    - no clear consensus-based dynamics
    - fixed interacting neighbors
  - Elasticity-based mechanism?

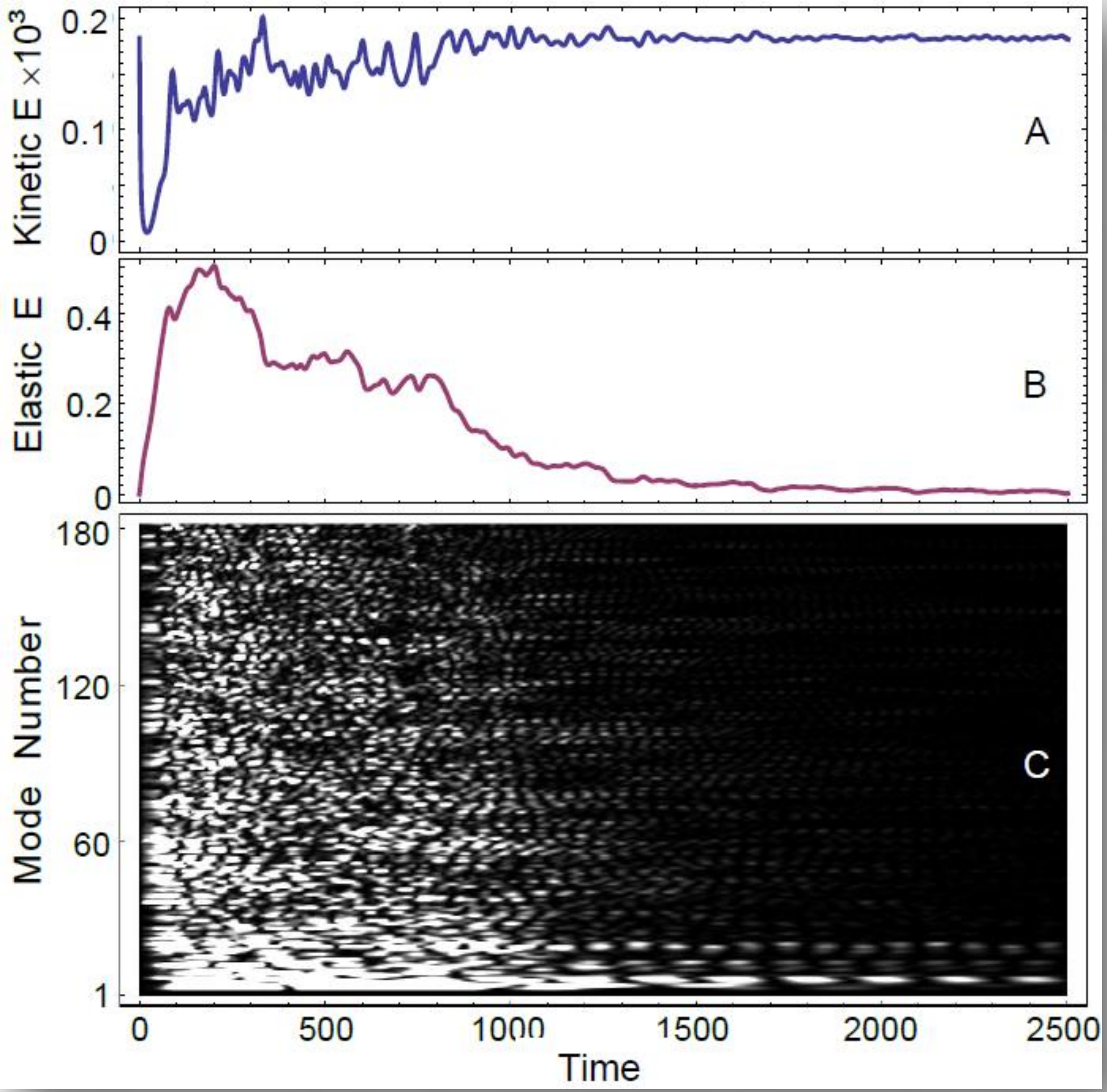
Unveiling the self-organizing mechanism

- We first decompose into normal elastic modes
  - E.g.: First 8 modes of hexagonal structure:



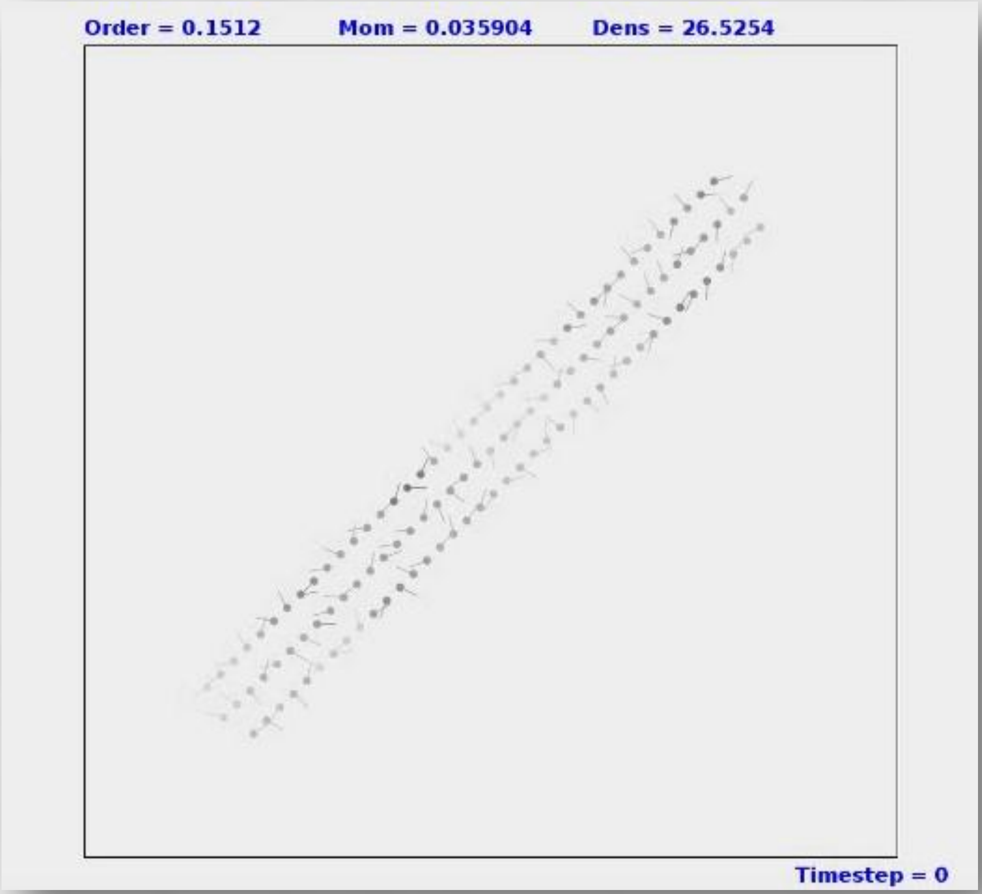
Spectral decomposition of elastic energy vs. time

- Self-propelled agents **steer away** from high-energy modes
- Energy **cascades** towards **low-energy** modes
- Agents **self-organize** into growing regions of coherent motion



Going back to see the modes

- Darker agents:
  higher local alignment
- Visible cascade of bending modes
- Preferred direction of self-organized motion



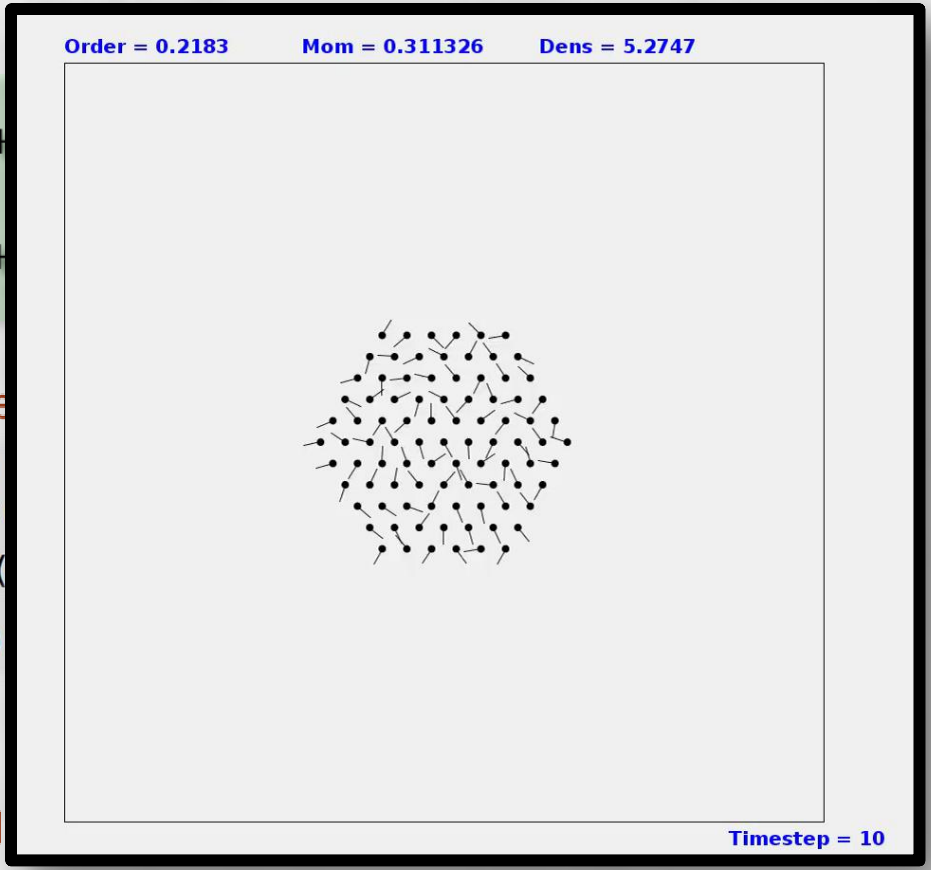
Linear stability analysis (no noise)

- Linear continuous approximation

$$\dot{u}_x = \alpha\, F_x$$
$$\dot{u}_y = v_0\, \phi$$
$$\dot{\phi} = \beta\, F_y$$

$F_x = (\lambda +$

$F_y = (\lambda +$



- Stability analysis in Fourier space

Stability matrix = $\begin{pmatrix} -k^2\,\alpha\,(\lambda + 3\,\mu) & -k^2\,\alpha \\ 0 & \\ -k^2\,\beta\,(\lambda+\mu) & -k^2\,\beta\,(\end{pmatrix}$

Characteristic polynomial: $\Lambda^3 + \alpha\,(\lambda + 3\,\mu)$

Routh's stability criterion: C0 C3 – C1 C2

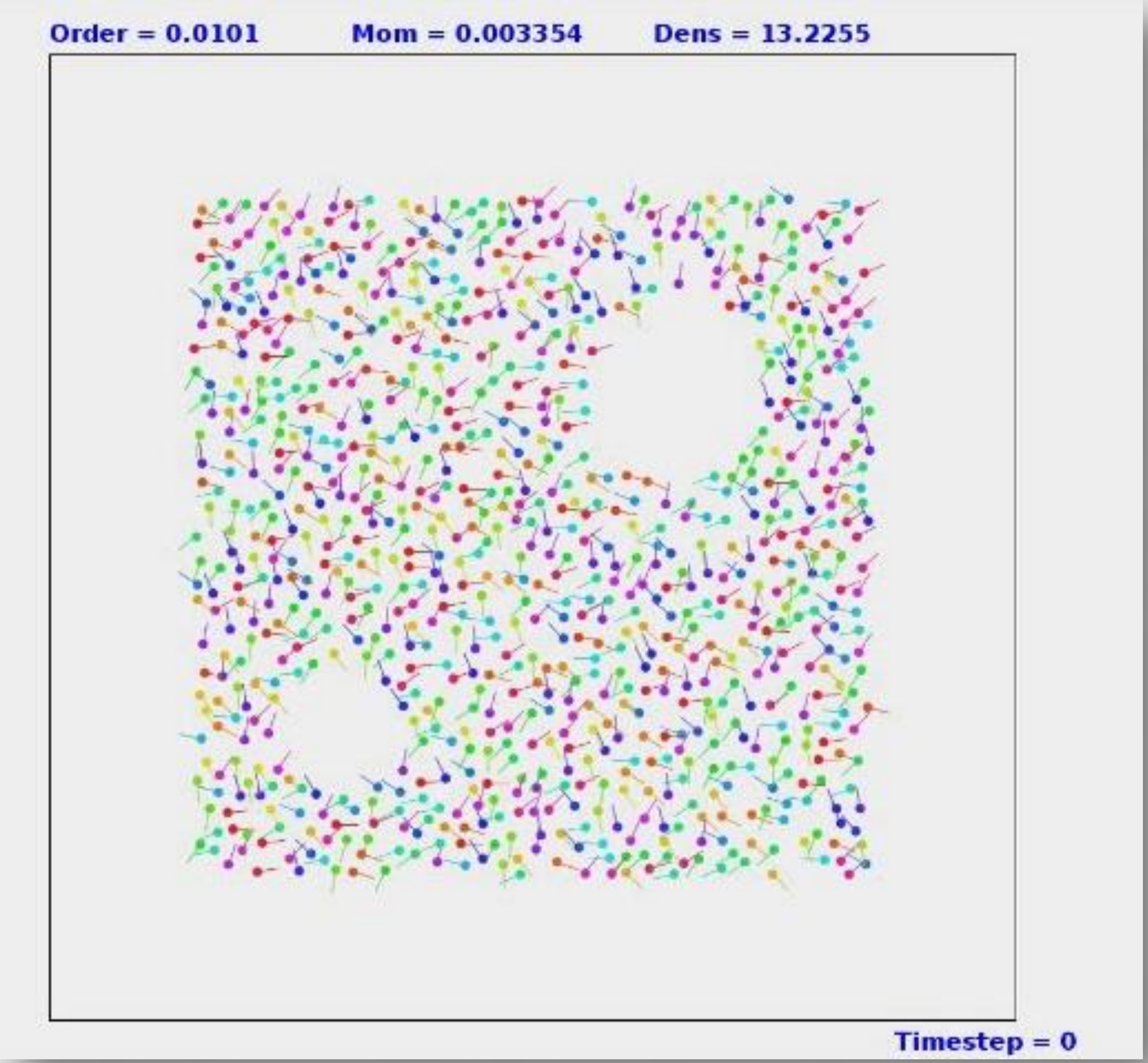
- Is the fixed speed case $(\alpha = 0)$ al

Characteristic polynomial: $\Lambda^3 + \beta\, v_0\, (\lambda + 3\mu)\, k^2 \Lambda = 0$

$$\Longrightarrow \boxed{\Lambda = 0} \text{ or } \boxed{\Lambda = \pm\sqrt{-k^2 v_0\, \beta\, \lambda - 3\, k^2 v_0 \beta\, \mu} \in i\,\mathbb{R}}$$

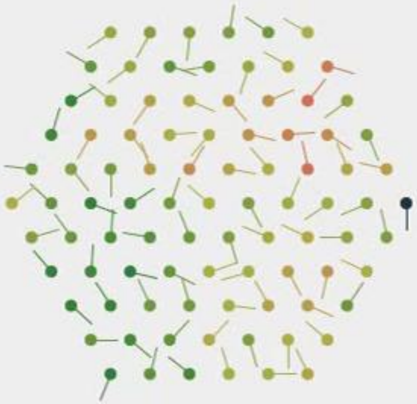
Active solids

- Active solid setup
  - Random positions
  - Neighbors connected by linear springs
- Agents here colored by angle
  - Coherent regions grow
  - Energy flows to lower modes

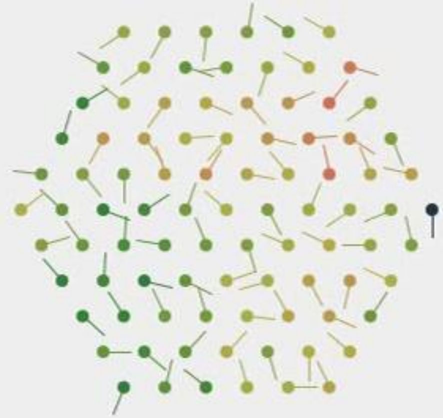


Parameter exploration

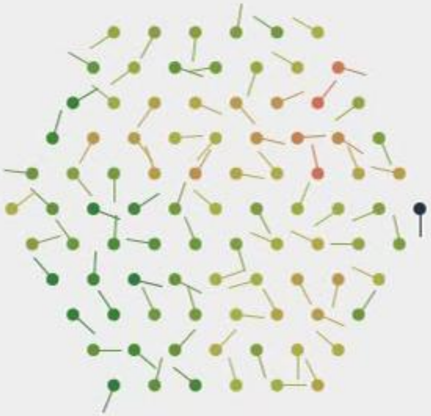
Hi α | Low v_0



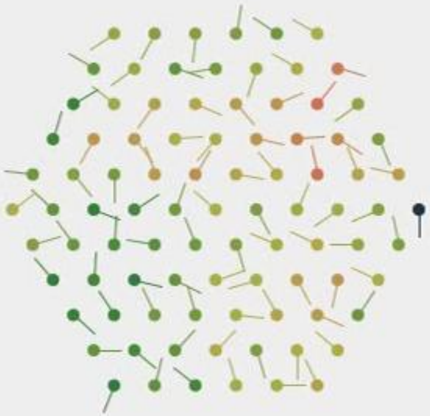
Low α



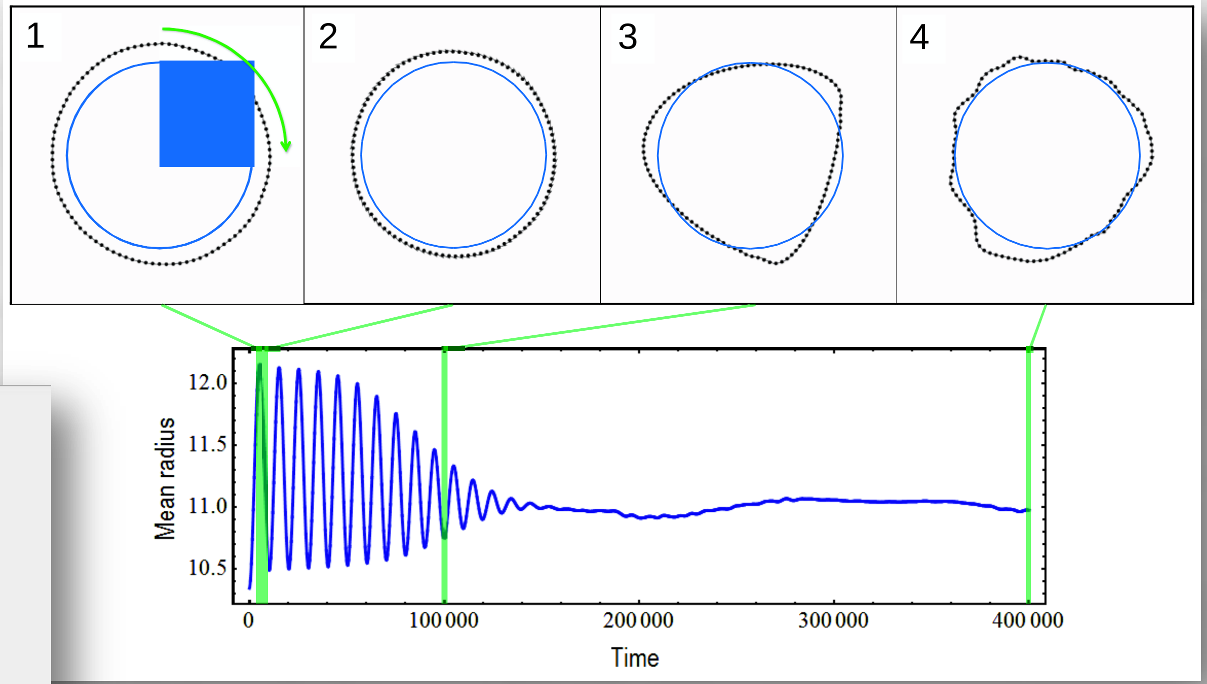
Hi β

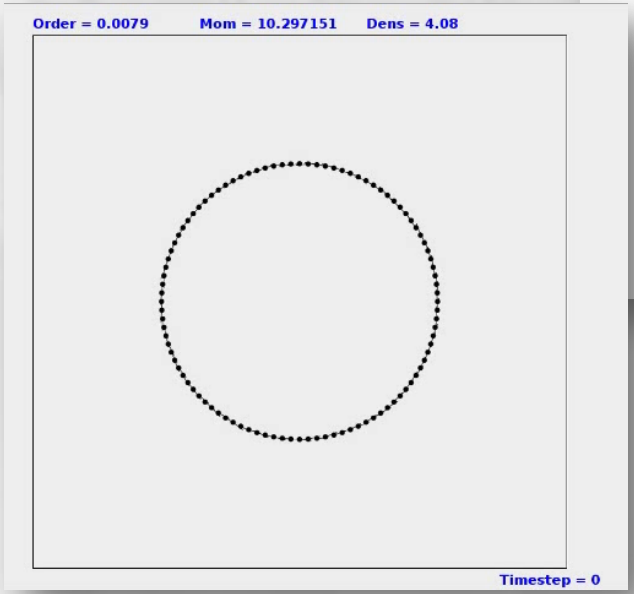


Low β



Ring dynamics

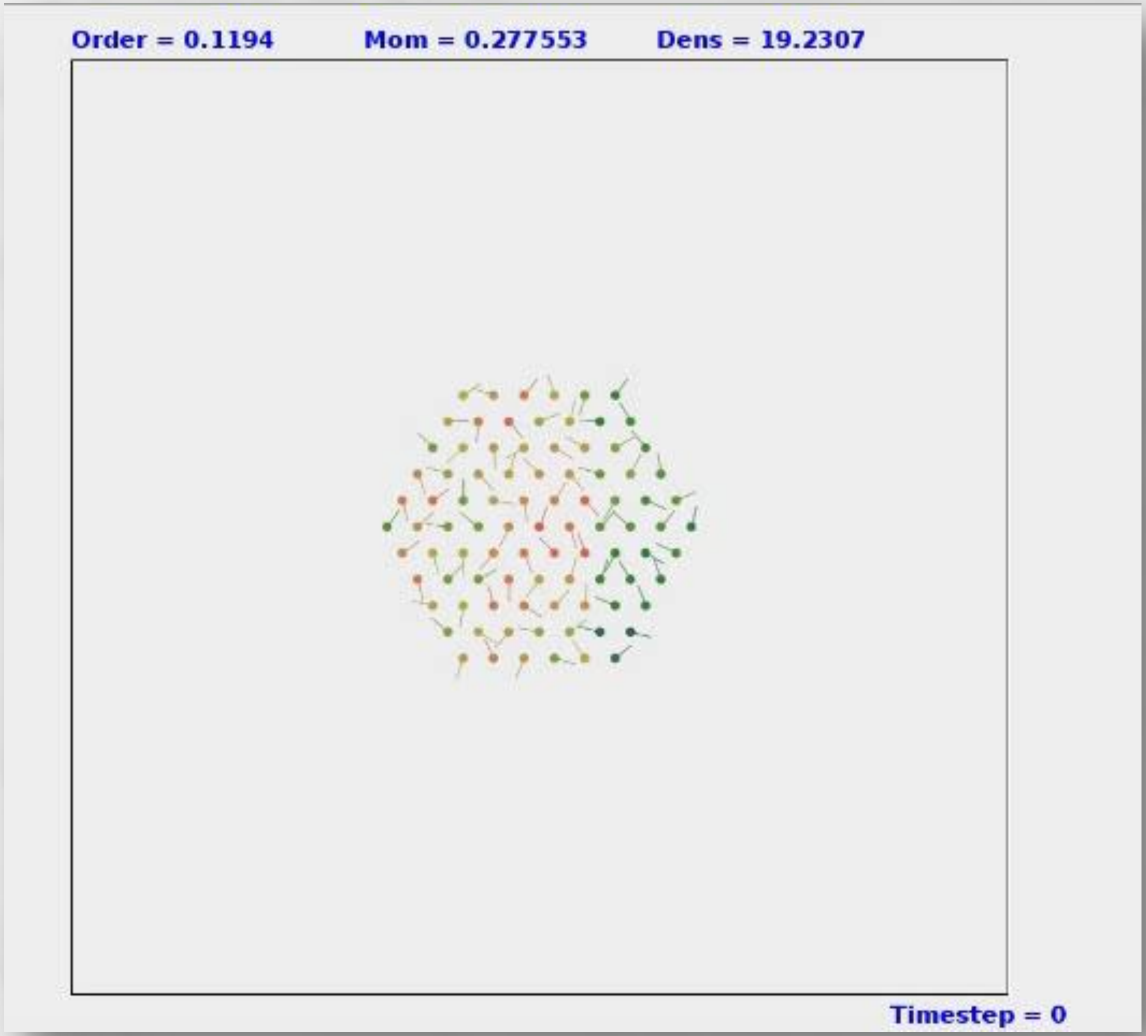




Heterogeneous self-propulsion speeds

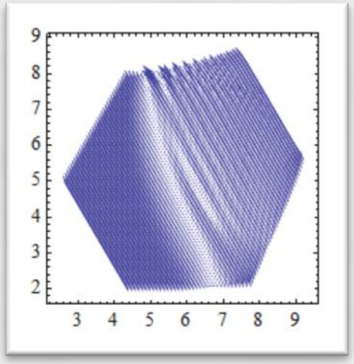
- Randomly assigned self-propulsion speeds

- System self-organizes in much longer time

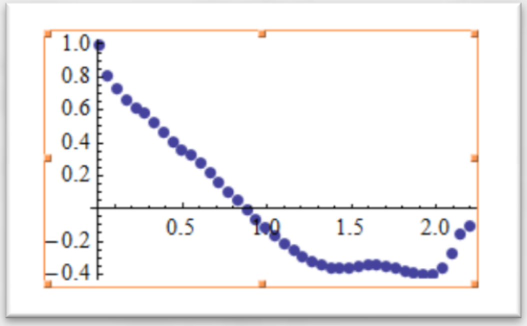


Final teaser – work in progress… lots to do…

- Dispersion relation
  (with Naomi Oppenheimer – U of C)



- Alternative explanation for flocking scale-free correlations



- Other elasticity-based self-organizing systems?
- Give me agents with different α, β, v0, k, l0
  and I'll build you a bicycle… or an 'amoeba'

Recap & *The End*

- **Active elastic membrane** (crystal/solid structure)
- **No alignment** interactions required
- **No mixing** required
- **Elasticity-based** mechanism for self-organization
- Rich **idealized limit-case** model system

... Fin