Vibrated granular rods and active matter

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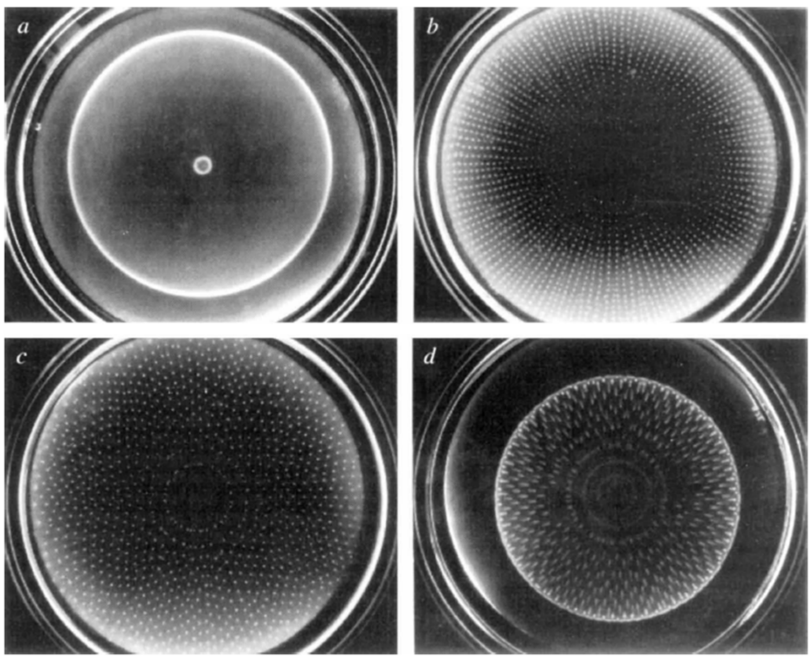
Cooperative behavior

Couzin: Anchovies

Guardian: Flocks of starlings

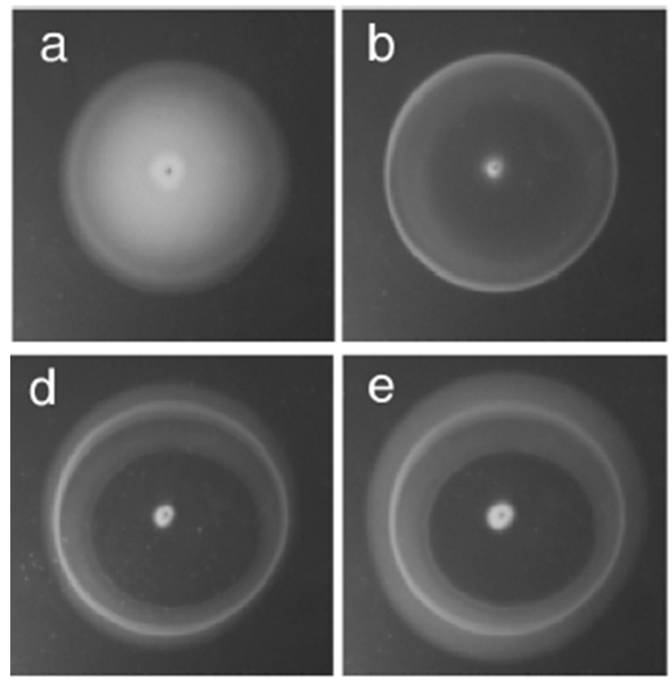
Pattern formation in bacterial colonies

- Budrene & Berg, Nature (1995)



- Chemotaxis, aspartate

- Delprato et al, PRL (2001)

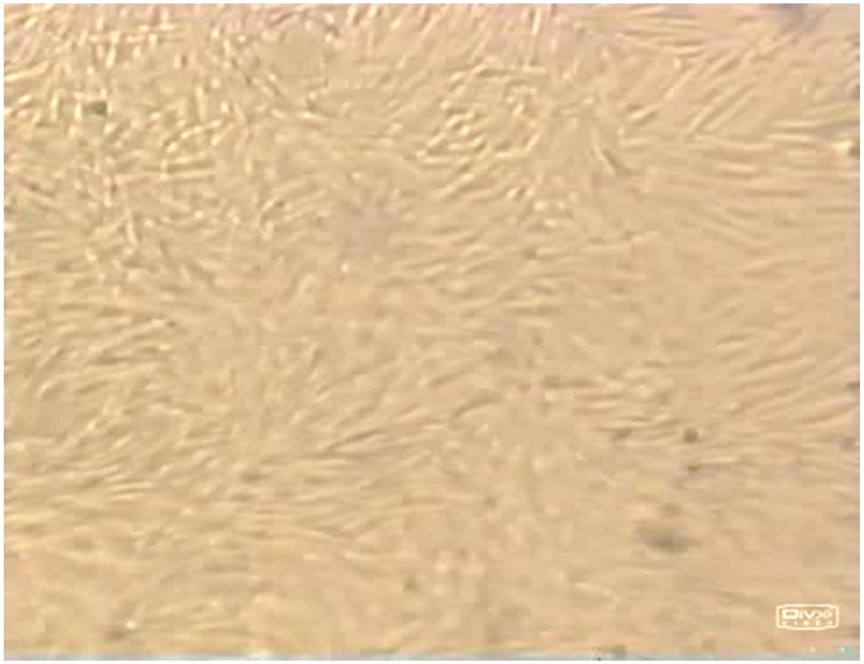


- UV radiation

Background

Bacillus subtilis colony in a peptone and agar rich thin layer
9 μm long rod shaped, driven by flagella, change direction by tumbling

Low agar (viscosity)

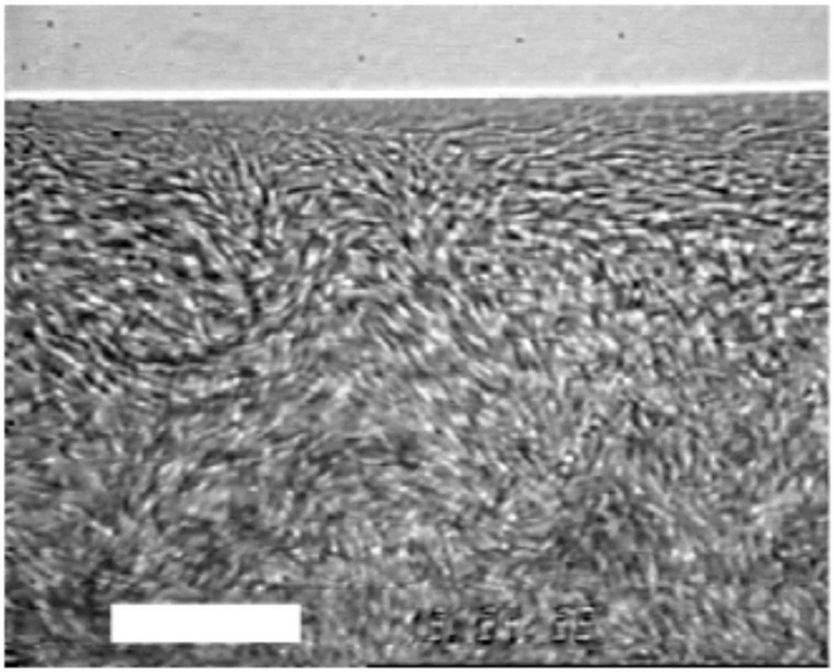


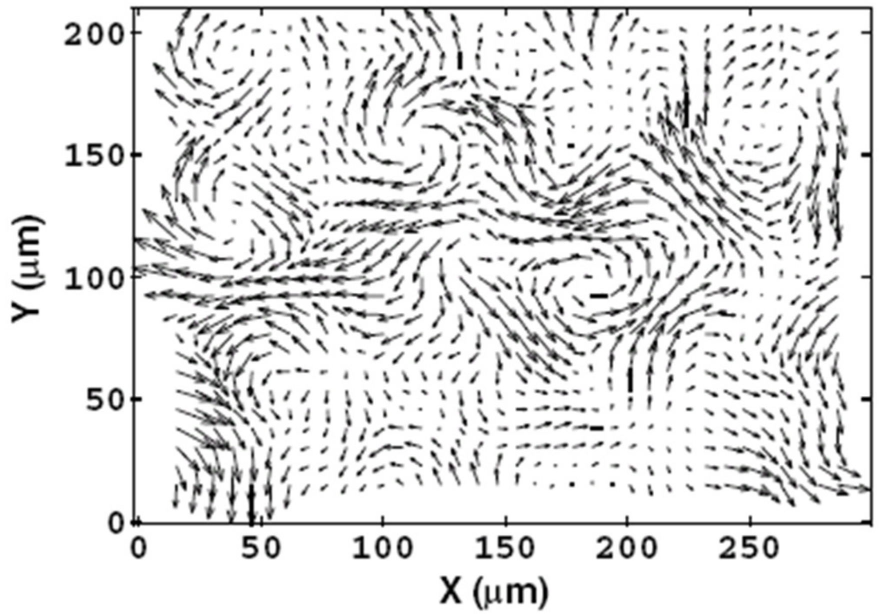
High agar (visco-elastic)

A. Delprato, A. Samadani & A.K. (2001)

Large-Scale Coherence in Bacterial Dynamics

- Dombrowski, Goldstein, Kessler, et al, PRL (2004)





- bacterial "turbulence"

Elephant seal colonies

San Simone, CA

Motivation

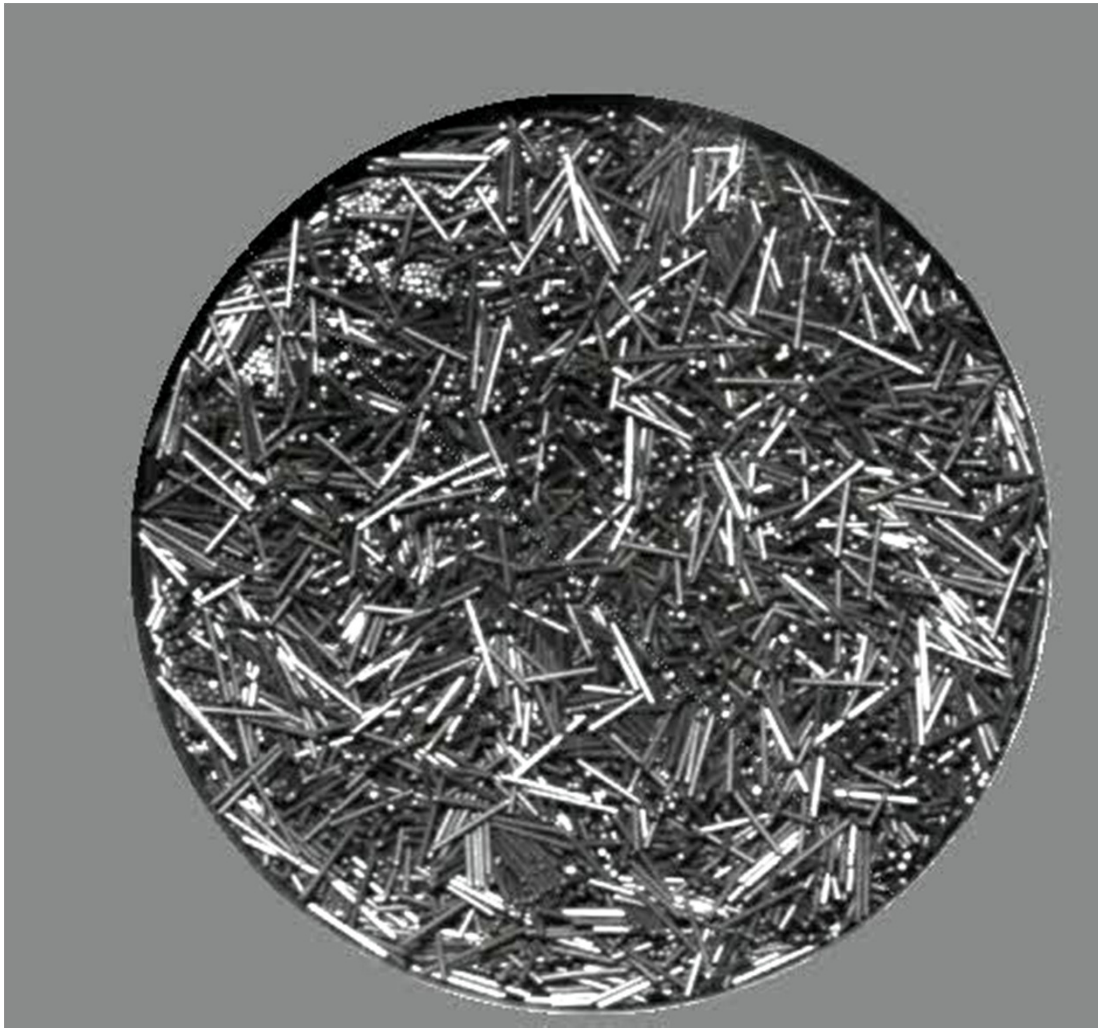
Examine the structure and dynamics of athermal apolar or polar, rigid or flexible rods which interact only during contact

Dynamics
- random walk, directed random walk
- how does the diffusion scale with rod length, density
- convection, vortices
- ratchet motion

Structure
• can the configurations be described by polymer models
• are ordering transitions observed with density

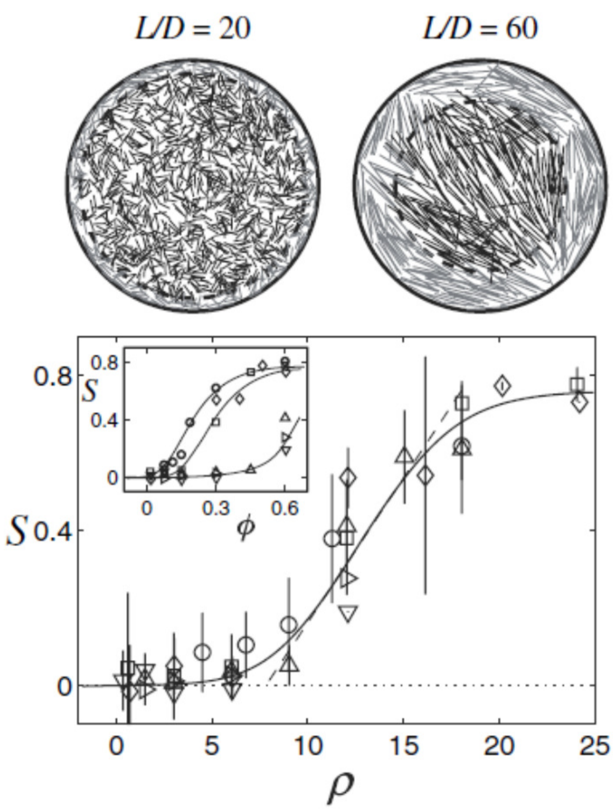
Active particle hydrodynamics?

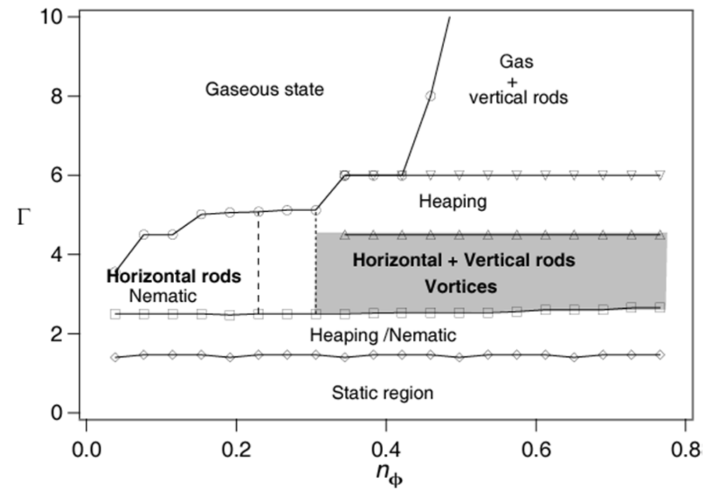
Self-organized vortex patterns with rods



D. Blair, T. Neicu, & A.K., PRE (2003)

Phase diagram

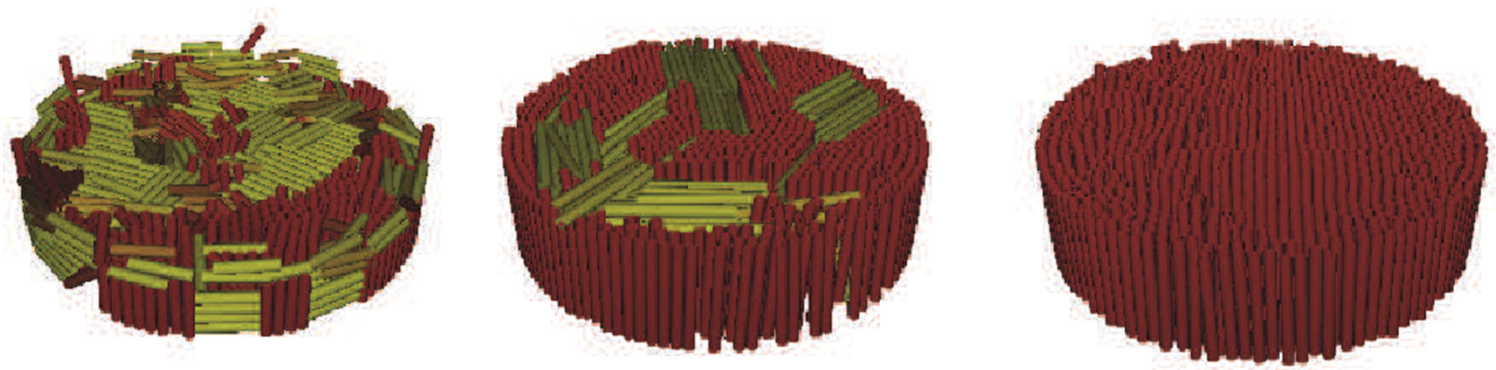




- Isotropic-nematic transition studied in a monolayer by Galanis et al, PRL (2006) – consistent with Onsager's mean field model but at somewhat lower density

- Vortex motion observed only when rods are tilted with respect to horizontal

Why are rods vertically aligned?



$A_r = 5$ $A_r = 10$ $A_r = 15$

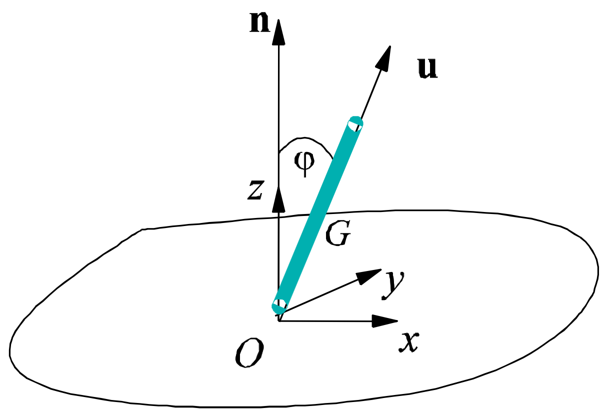
- X-ray tomography shows that rods are increasingly vertically oriented with rod aspect ratios – V. Yadav and A.K., PRE (2013)

Void filling mechanism:

- A vertically aligned rod can fall into a smaller void than a horizontal one.

- A vertically aligned rod in the center of a vertical pack cannot easily hop out and become horizontal.

Why do rods move?



Collision of a rod and the plate

$m\ dc = dP$

$I\ d\omega = -\ l/2\ u \times dP$

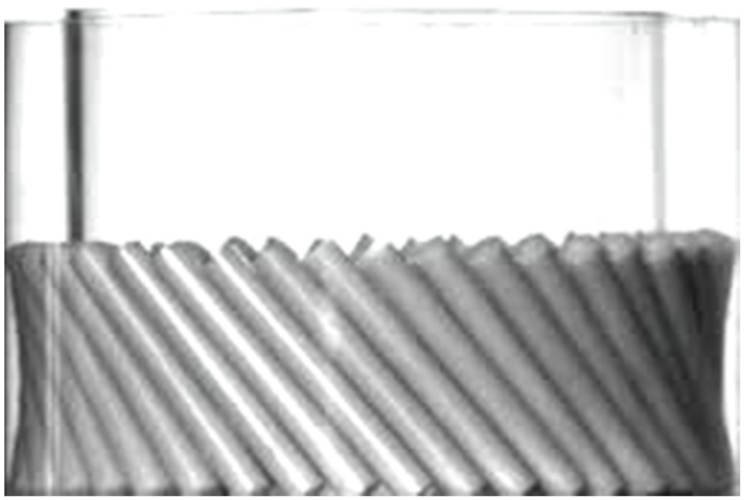
Apply Newton's law at contact point to relate velocity before and after collision

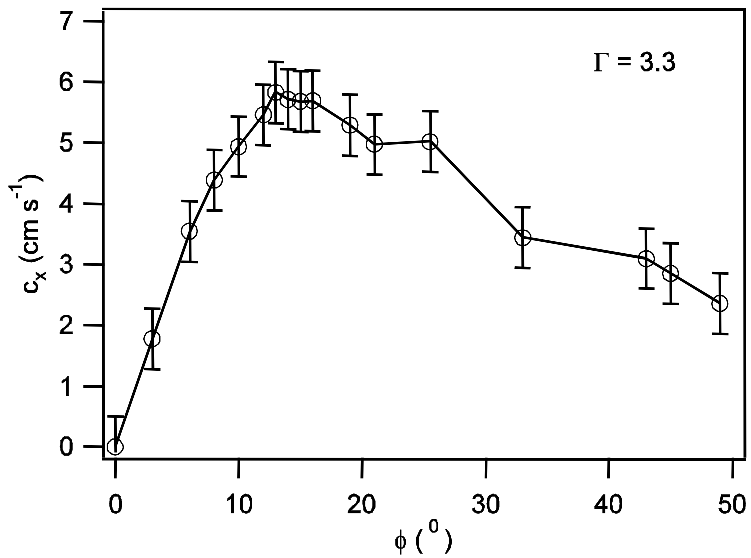
During collision, three possible scenarios:

- slip
- slip-stick
- slip reversal

geospaceplay.com

Collective dynamics of rigid rods





Slip-Stick regime

$$c'_x = \frac{6(1+\epsilon)\sin\phi\cos\phi}{1-\epsilon+6\sin^2\phi}V_z$$

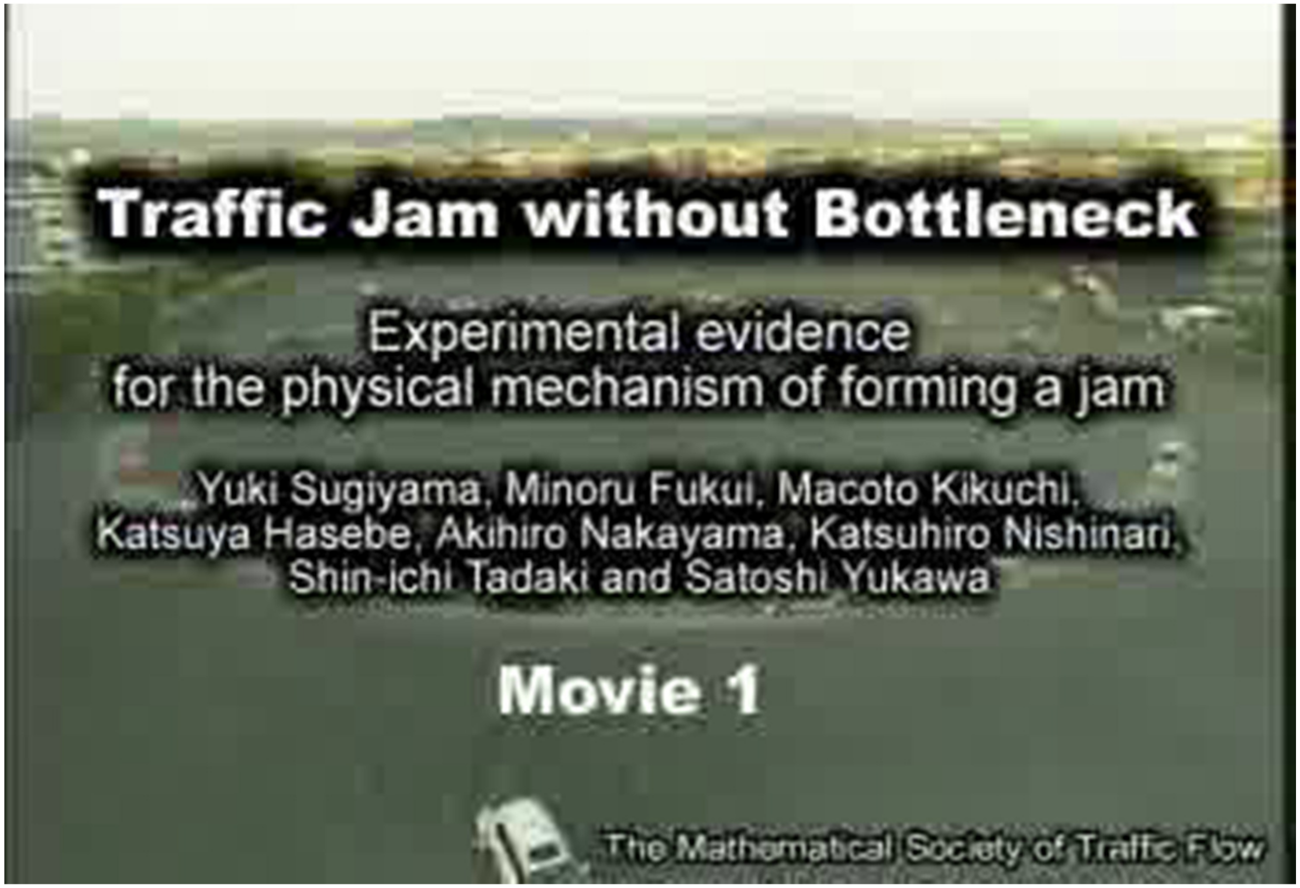
Slip Reversal regime

$$c'_x = \frac{(1+\epsilon)[\mu(1+3\cos^2\phi)+3\sin\phi\cos\phi]}{1-\epsilon+6\sin^2\phi}V_z$$

where, $V_z = V_0/2$

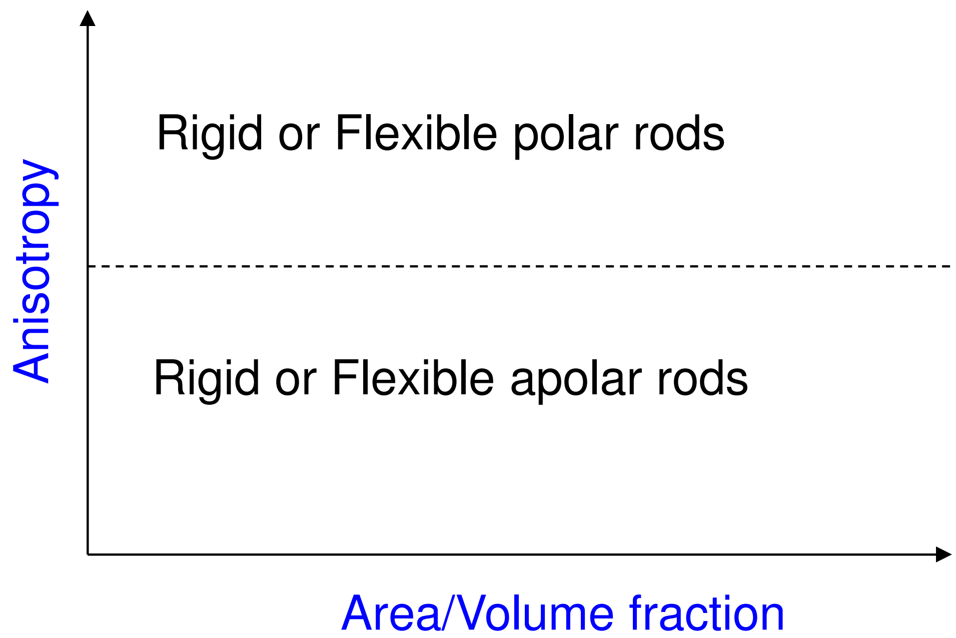
- D. Volson, A.K., L. Tsimring, PRE (2004)

Traffic flow



Sugiyama *et al.*, NJP 2008

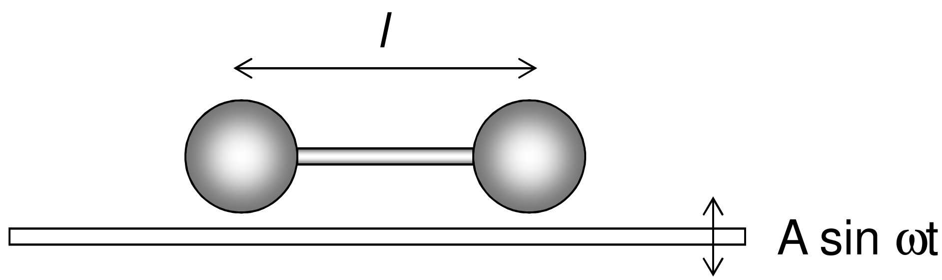
Polar and apolar self-propelled particles



- Flexible rods – beaded chain with and without a head
- Rigid rods - Rods, dimers, Robo-bug

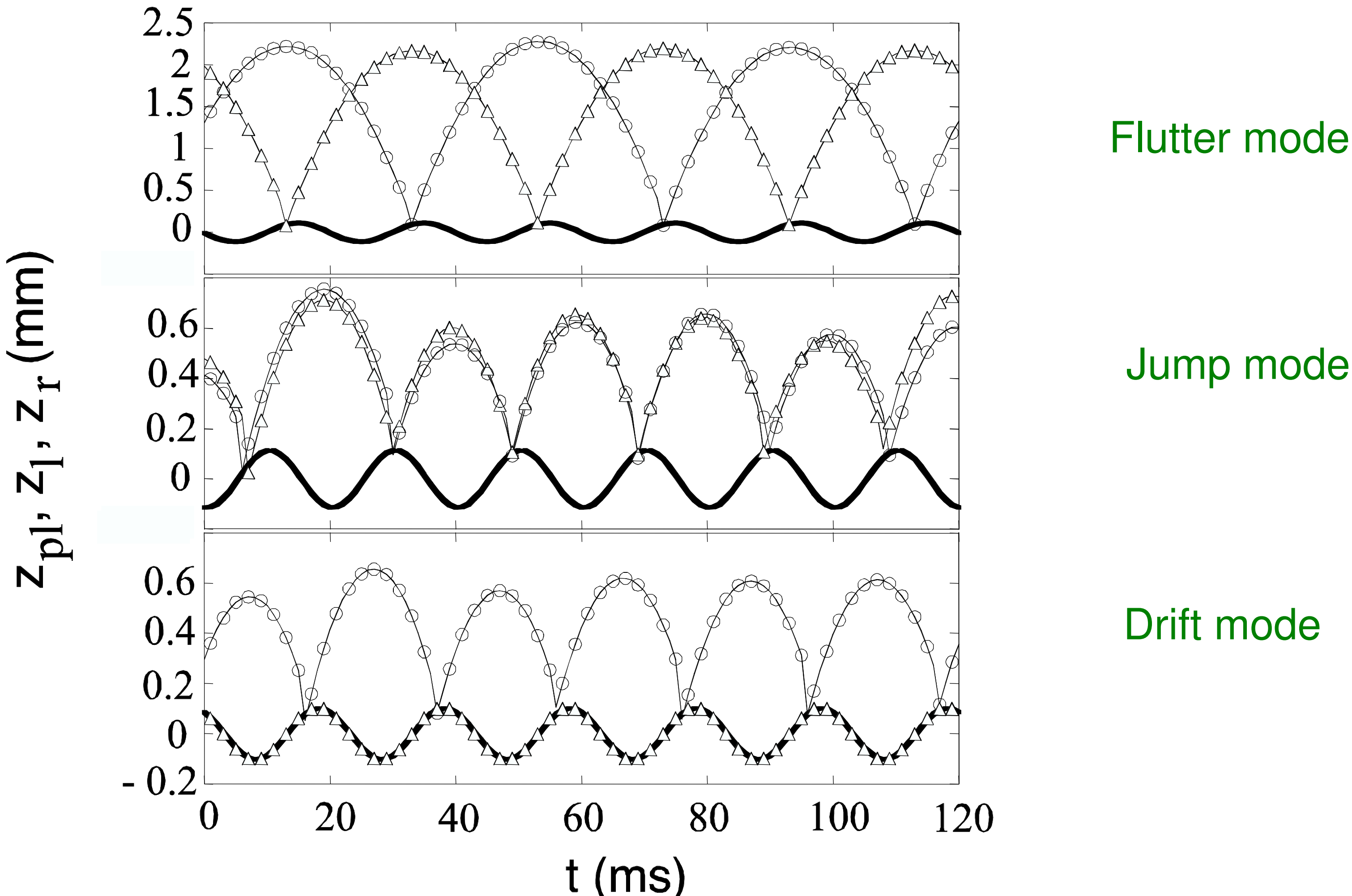
Motion of a non-spherical particle on a vibrated plate

- Stephane Dorbolo



- Complex modes observed depending on the relative phase of motion of the two particles on the plate

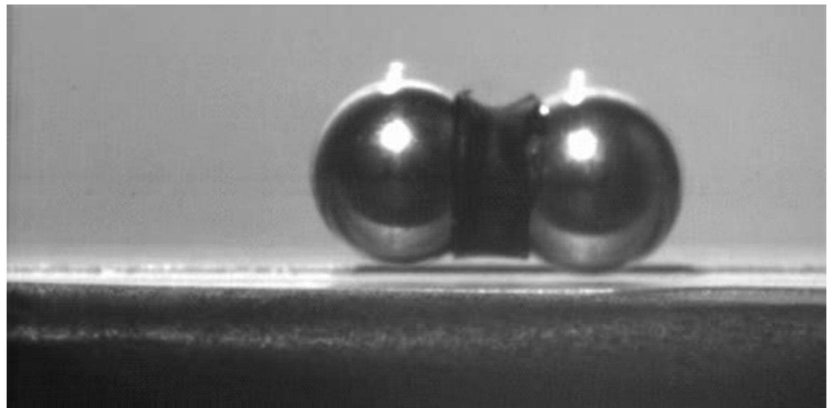
Evolution of the lowest excited modes



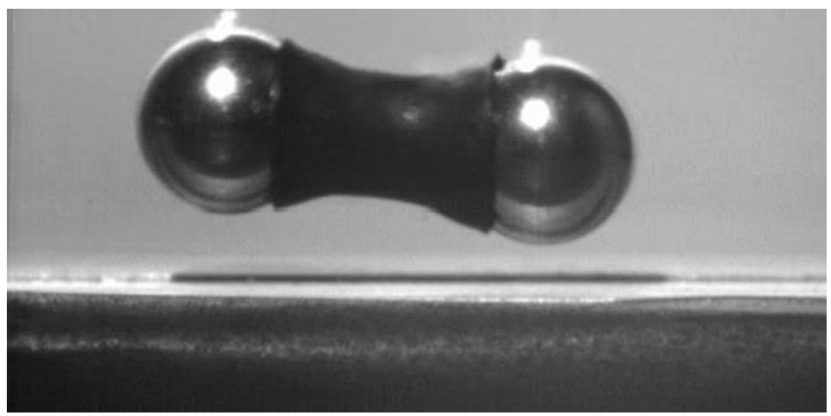
(A_r = 3.8, f = 50 Hz, Γ = 0.9)

Dynamics of a bouncing dimer

Flutter mode



Jump mode



Drift mode in a symmetric dimer on a vibrated plate

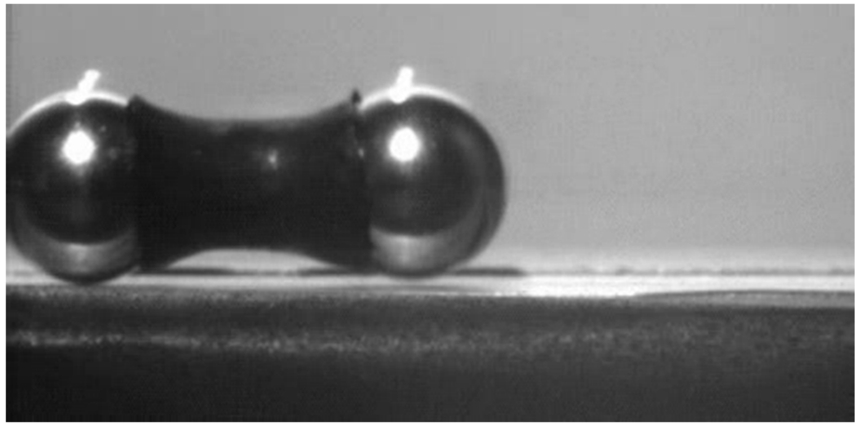
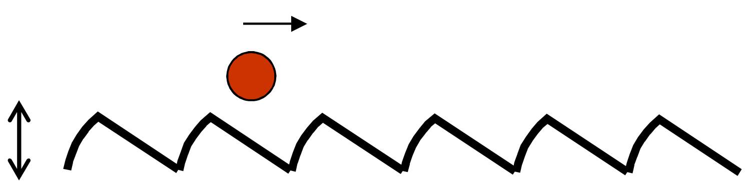


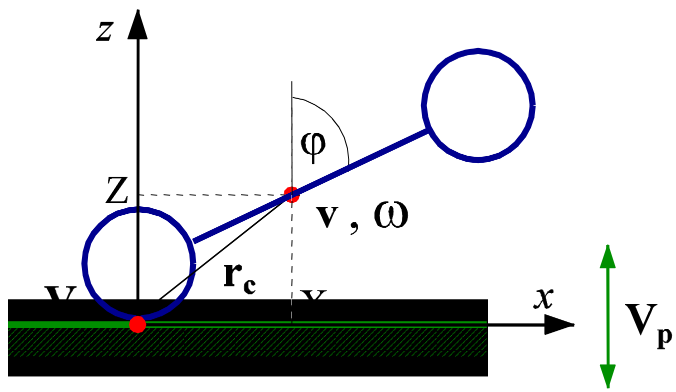
Plate surface is smooth and therefore our situation is different from previous examples of ratchets



e.g. Derenyi *et al*, Chaos (1998)

S. Dorbolo, D. Volfson, L. Tsimring, A.K., PRL (2005)

Collision of the dimer with the plate



Newton's laws:

$$m\dot{\mathbf{v}} = \sum_c \mathbf{F}^c + m\mathbf{g}, \qquad I\dot{\omega} = \sum_c \mathbf{r}_c \times \mathbf{F}^c \qquad (1)$$

m: mass   *I*: moment of inertia

Force at contact points $F^c = (F_x^c, 0, F_z^c)$

During slip: $F_x^c = -\mathrm{sgn}\, U_c \mu F_z^c$

Stick-slip: $|F_x^c| = \mu_s F_z^c$

μ_s depends on the contact time

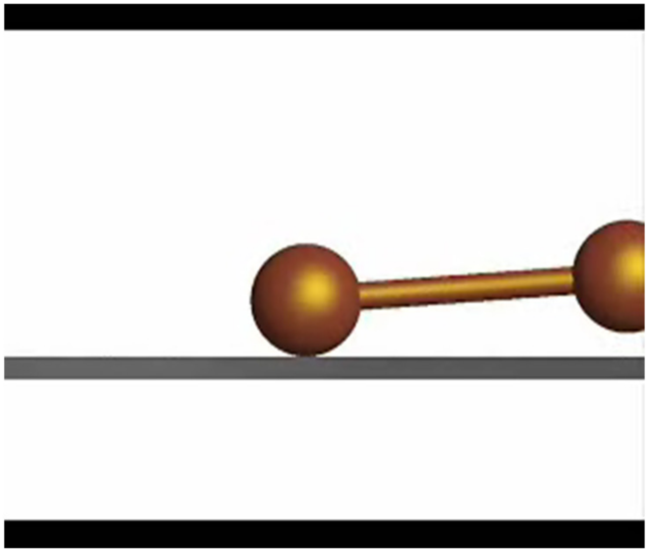
Single collision
1: Continuous slide
2: Slip-stick
3: Slip reversal

Double collision
1: Double slide
2: Double slip-stick
3. Double slip reversal

1: Rolling without slip
2: Rolling with slip

Need to consider all possible collisions to obtain correct dynamics

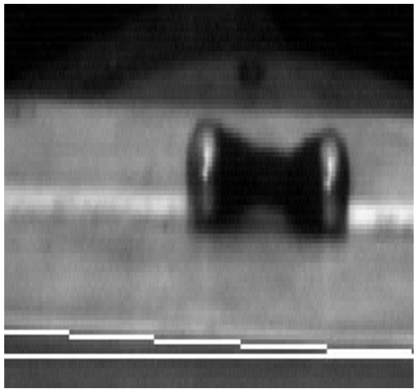
Event driven simulation based on single and double collision rules

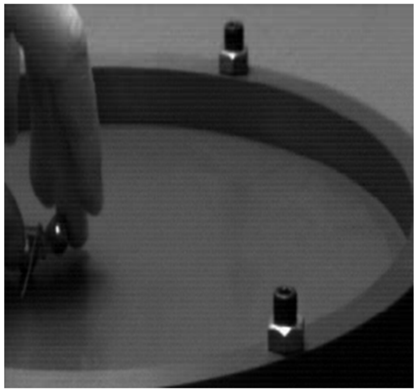


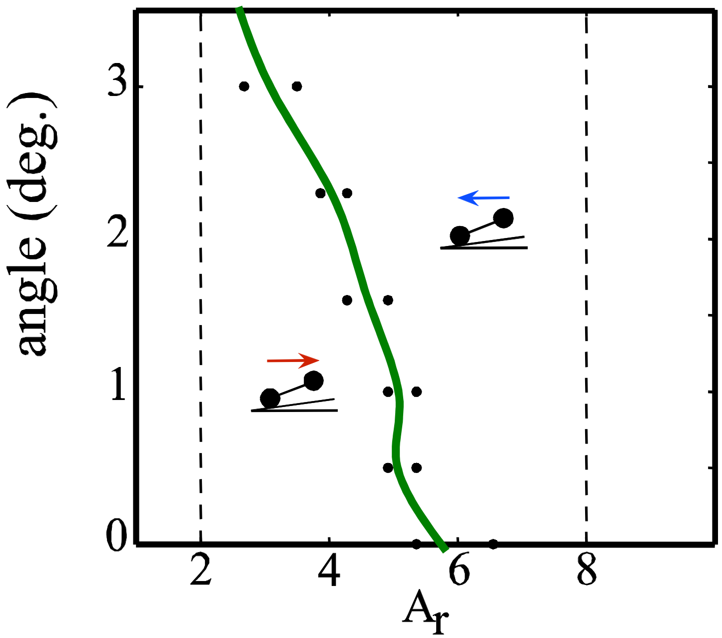
$\Gamma = 0.9$, $f = 25$ Hz, $A_r = 3.8$

The friction coefficients and inelasticity parameters used were directly measured from the experiment

A dimer can climb an oscillating hill



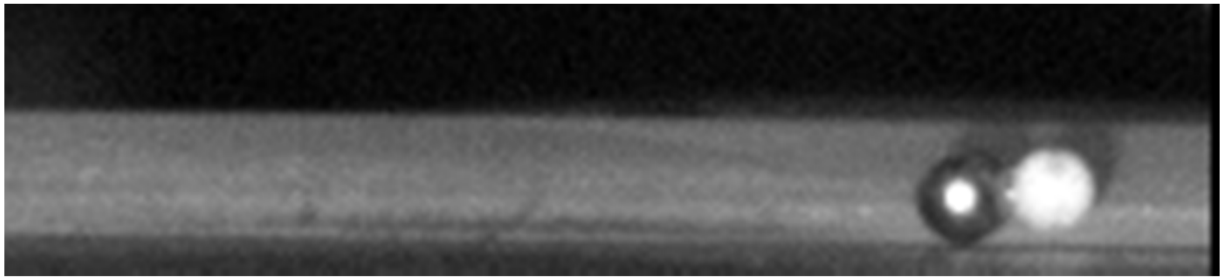




S. Dorbolo, D. Volfson, L. Tsimring, A.K., PRL (2005)

Asymmetric Dimer Ratchets: Robo-Bug

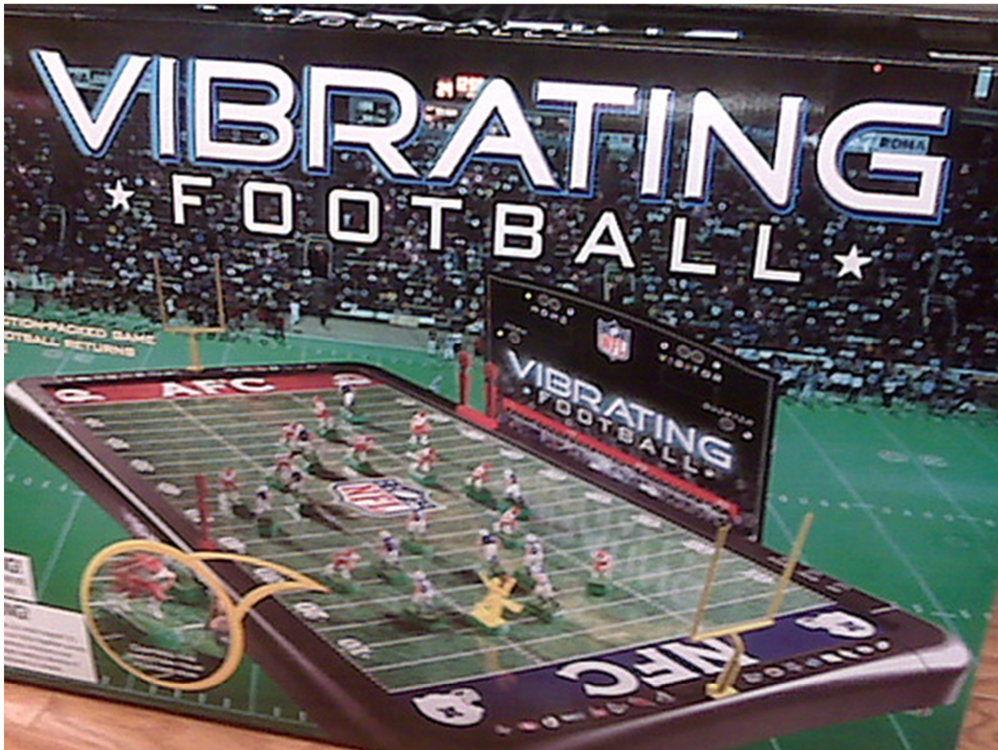
"Simplest" examples of noise driven motors

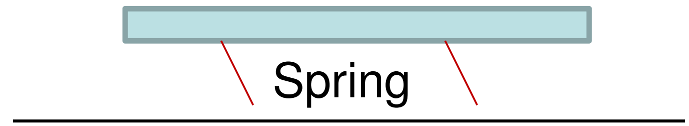


Steel-Glass beads

- Symmetry breaking leads to directed motion

Also: Yamada, Hondou, Sano (2003)

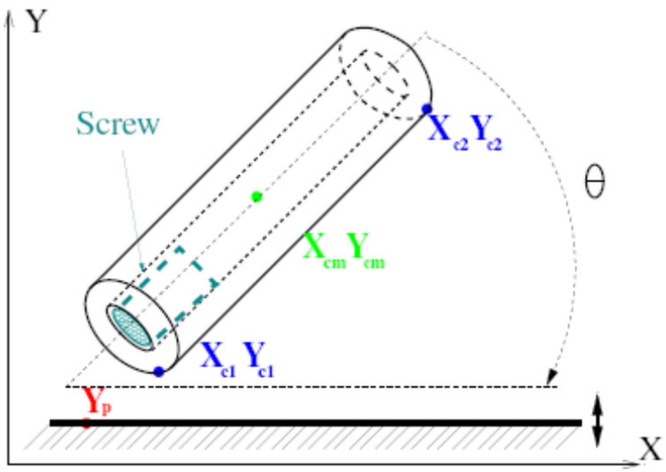




Popular toy in1960s

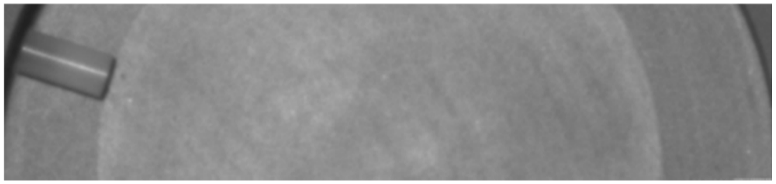
Polar Rods

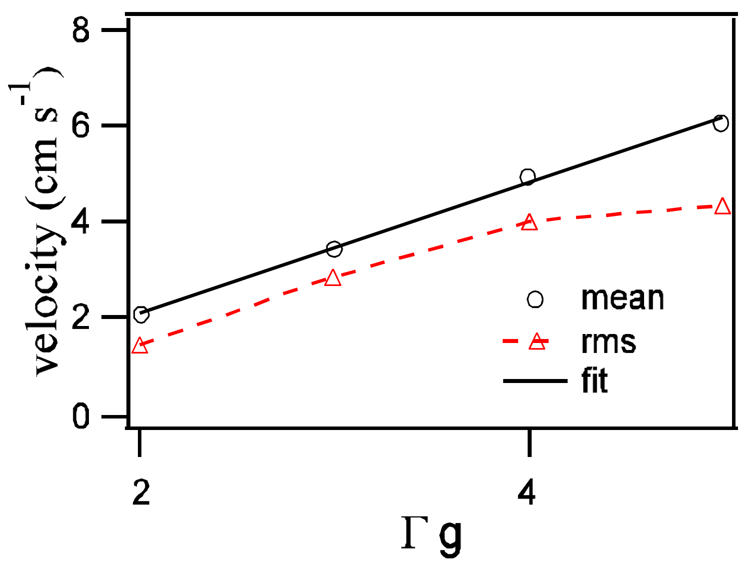
- Geoffroy Lumay



Nylon cylinder: $l_{cyl} = 9.5$ mm, $d_{cyl} = 4.76$ mm, $m_{cyl} = 0.143$ g.

Metalic screw: $l_{screw} = 4$ mm , $d_{screw} = 2.5$ mm , $m_{screw} = 0.077$ g.



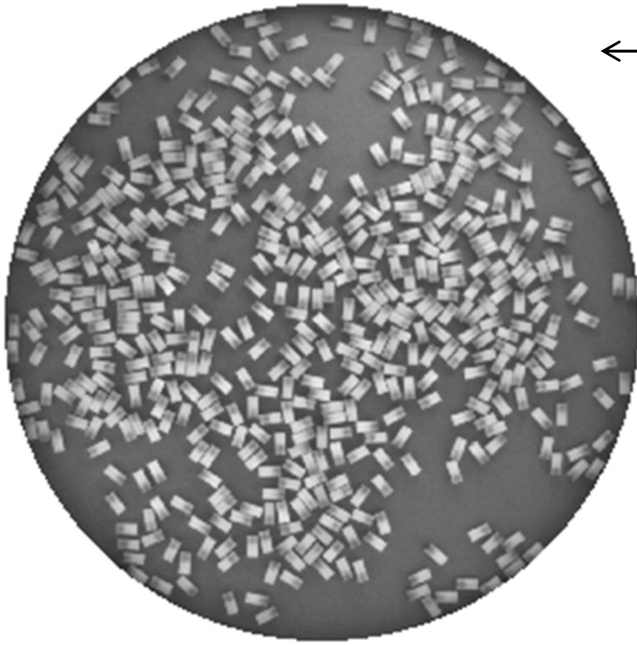


Cooperative dynamics with polar rods

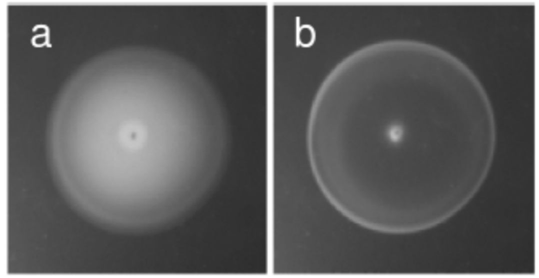
$\phi \sim 0.3, \Gamma = 2$

- Particles migrate to the boundary on a flat bed under low noise conditions

Cooperative dynamics with polar rods



$\longleftrightarrow$



B. subtilis

$\phi \sim 0.3, \Gamma = 2$

- Particles migrate to the boundary on a flat bed under low noise conditions
- Not chemotaxis

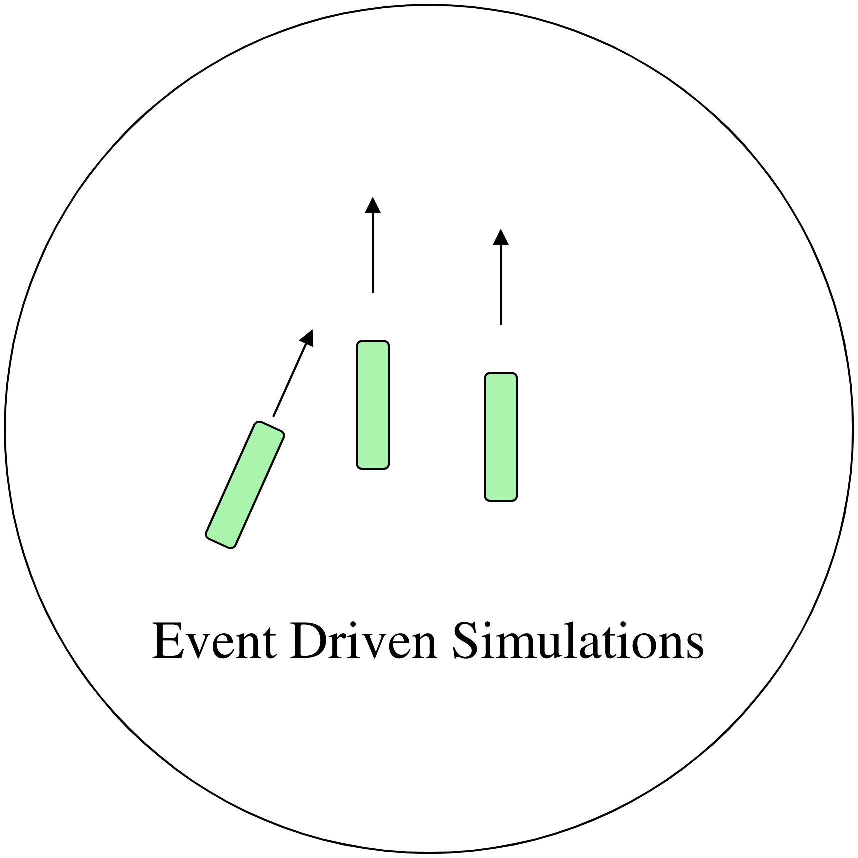
Alignment at boundaries observed in bacterial colonies?

Bacillus subtilis colony

Healthy colony (no UV)

Event driven simulation model

Dmitri Volfson and Lev Tsimring



Simulation of polar rods moving on a substrate inside a circular boundary

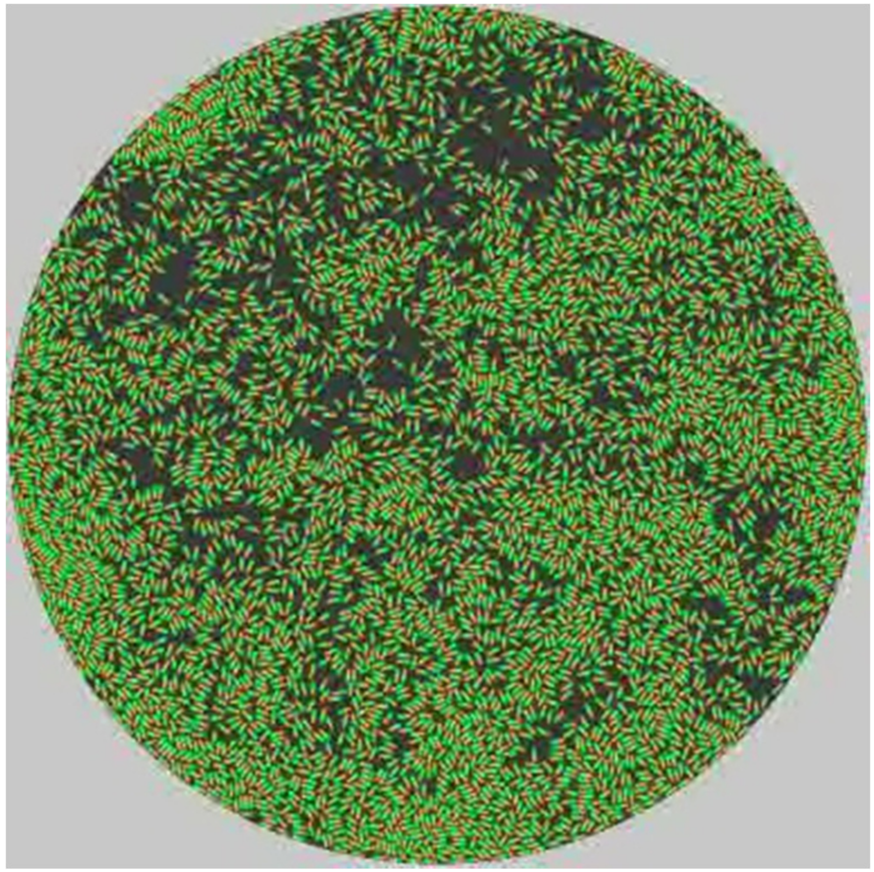
$R=60$, $d_r = 0.732$, $L_r/d_r = 3.5$ (from tip to tip)

$\mu_{rr} = \mu_{rw} = 0.3$, $\varepsilon_{rr} = \varepsilon_{rw} = 0.9$, $C_v damp = 0.5$

Also: Peruani et al, PRE (2006)

Event driven simulation model

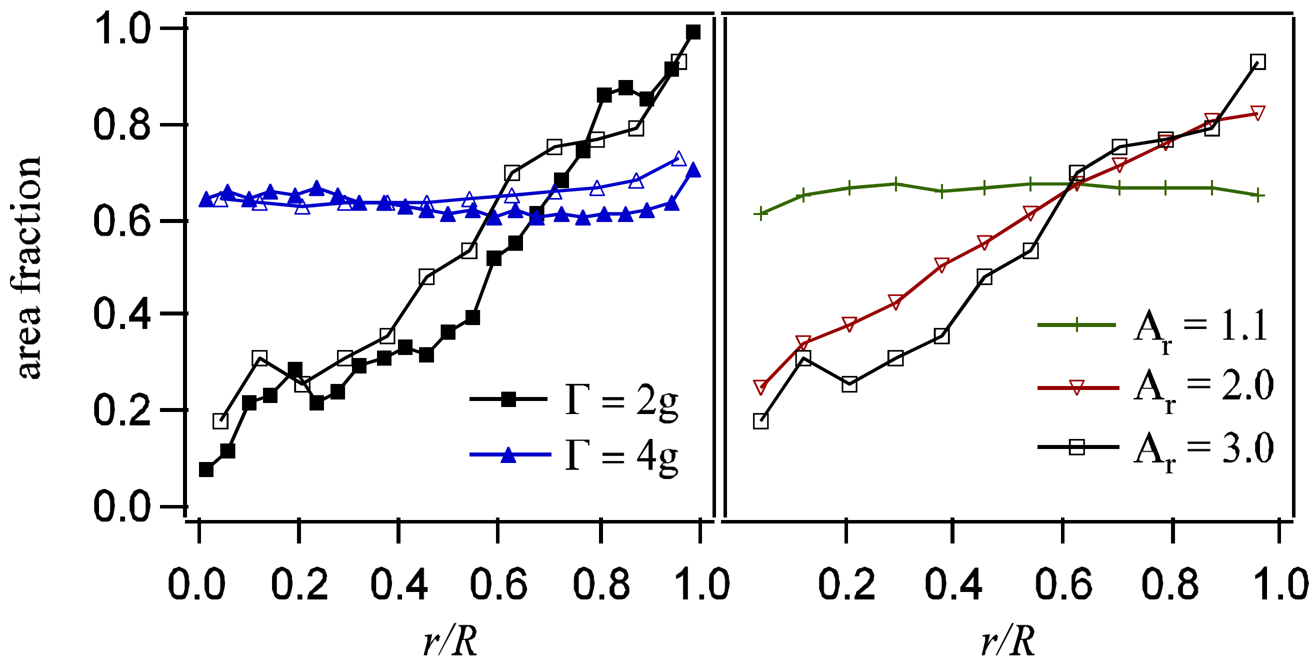
Dmitri Volfson and Lev Tsimring



R=60, d_r = 0.732, L_r/d_r = 3.5 (from tip to tip)

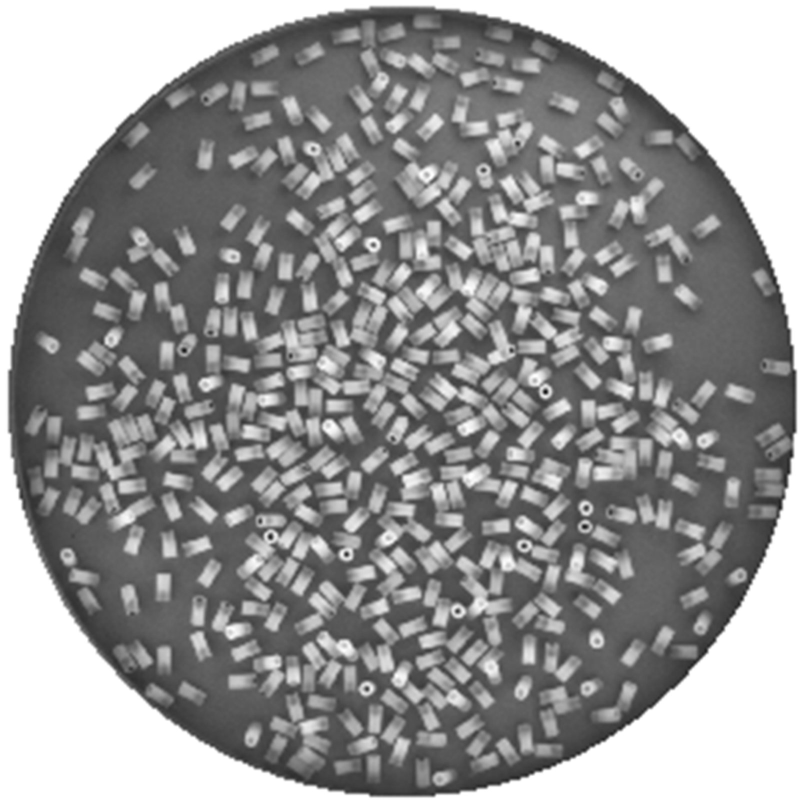
$\mu_{rr} = \mu_{rw} = 0.3$, $\varepsilon_{rr} = \varepsilon_{rw} = 0.9$, $C_v$damp = 0.5

Cooperative Behavior



Boundary aggregation vanishes for small aspect ratio

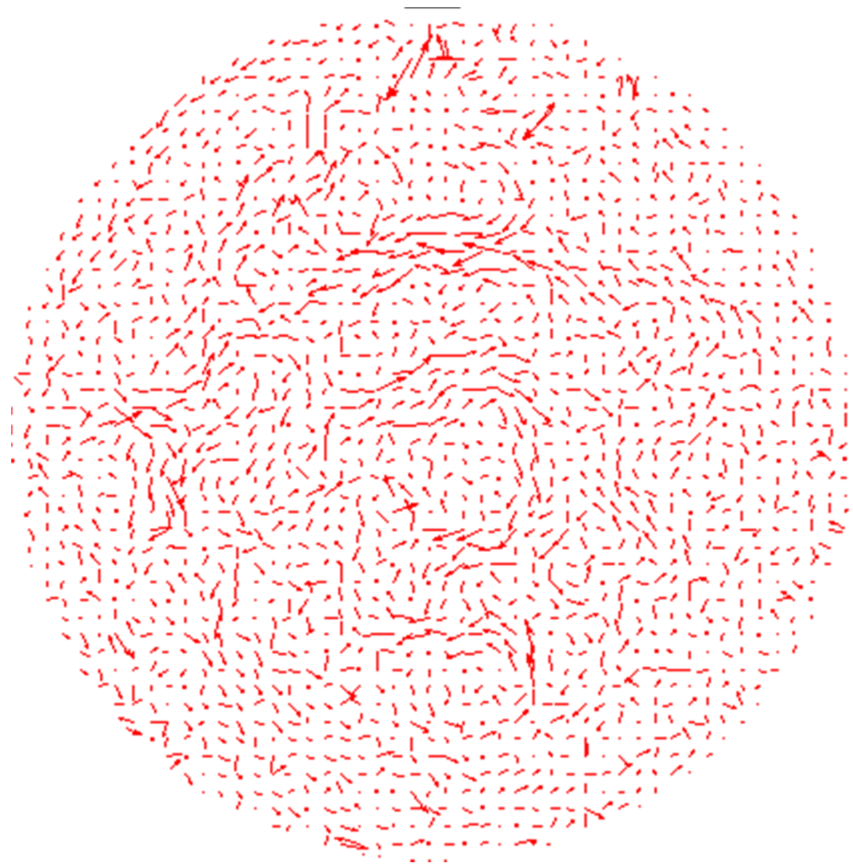
Cooperative dynamics with polar rods



$\phi \sim 0.3, \Gamma = 4$

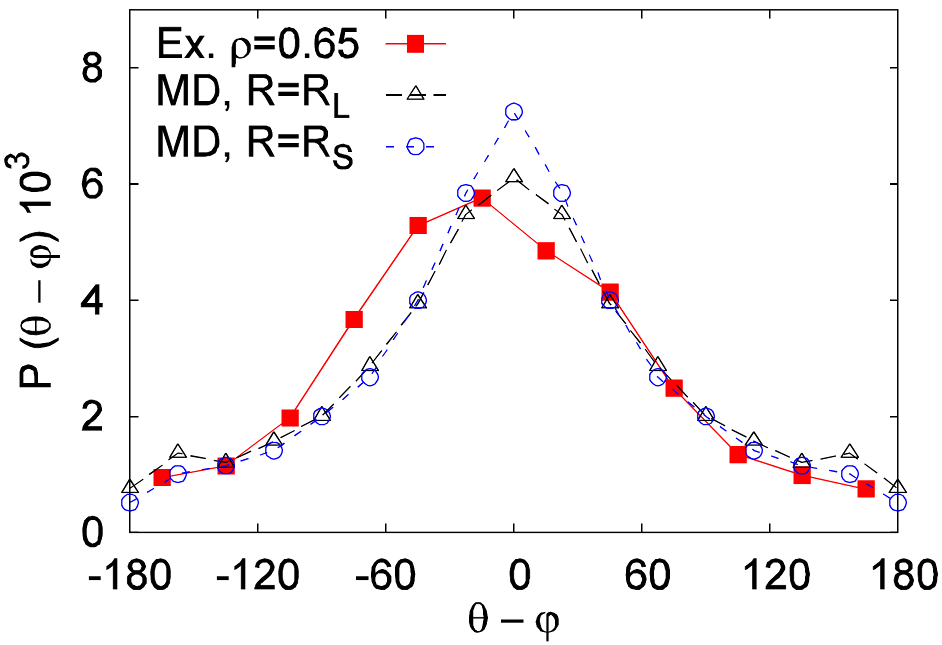
- Particles are uniformly distributed at higher excitation

Velocity field of the polar rods



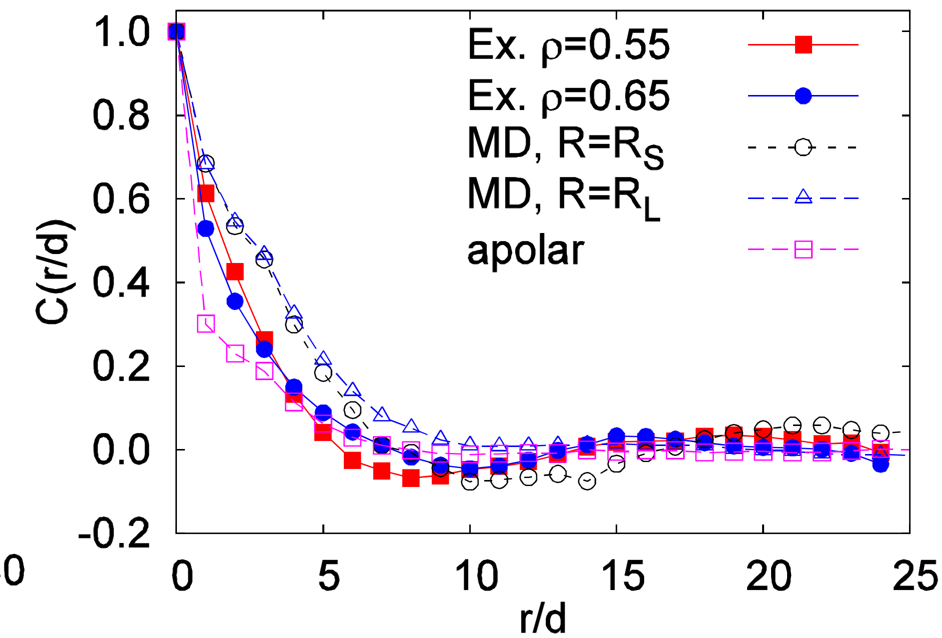
$\Gamma = 3$, $\tau = 5s$

Velocity–rod director correlation



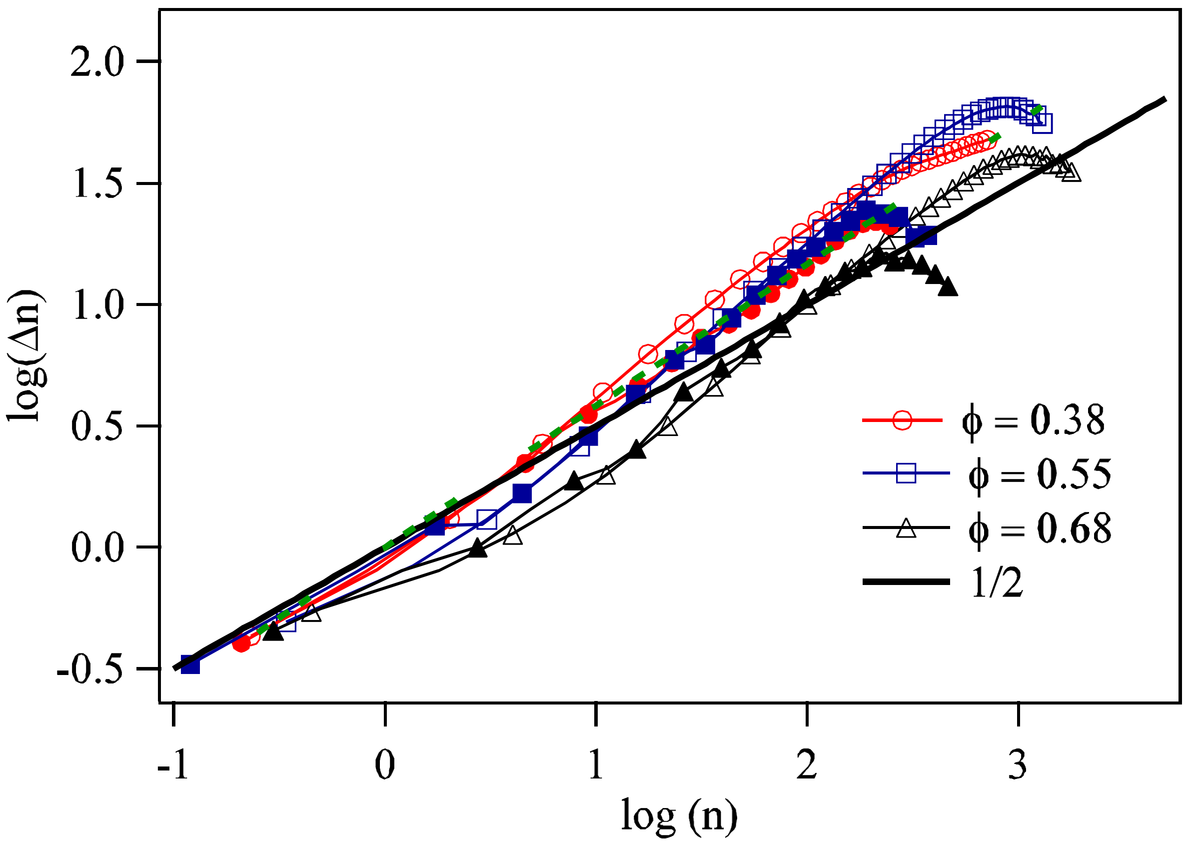
• Velocity strongly correlated with director even in presence of rod-rod collisions

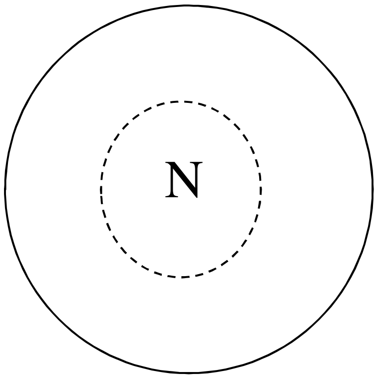
Spatial velocity correlation



• Correlation length is small and therefore system is in disordered state

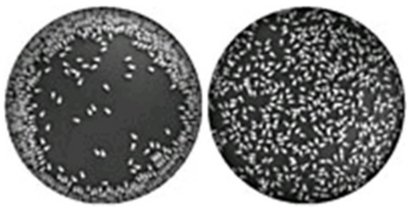
Particle number fluctuations





- Tu & Toner (1997), Toner predict greater than $N^{½}$ fluctuations for ordered polar self-propelled particles

- Fit gives an exponent close to 2/3



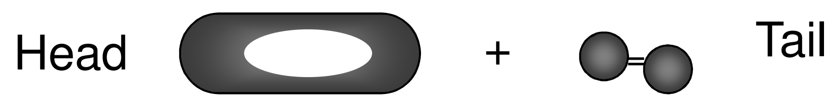
Swarming and swirling in self-propelled granular rods, A. K., G. Lumay, D. Volfson, and L. Tsimring, PRL (2008)

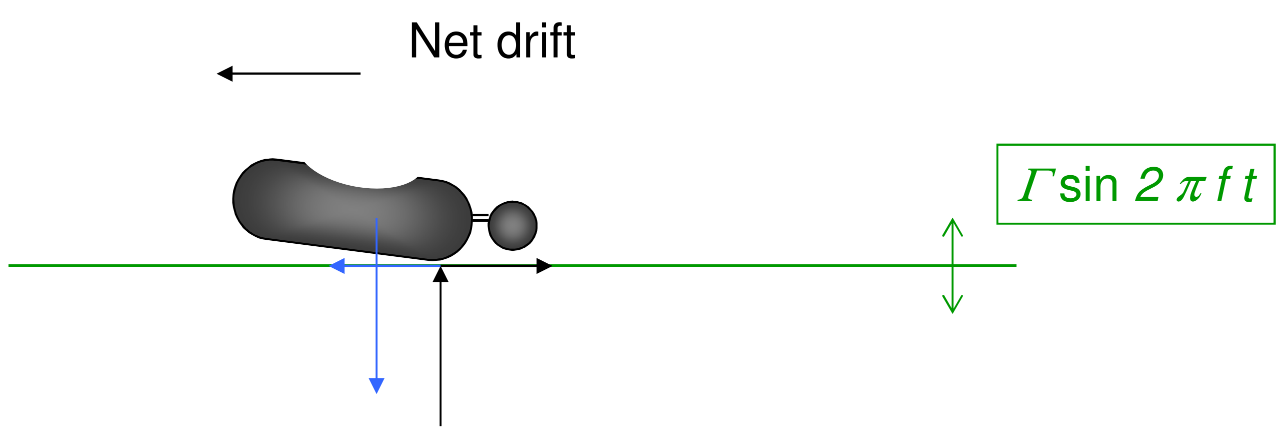
- Rigid polar rods are trapped at the boundary under low noise conditions

- Incipient clustering observed due to interplay between directed motion and particle shape even in disordered regime

Diffusion of flexible self-propelled polar particle

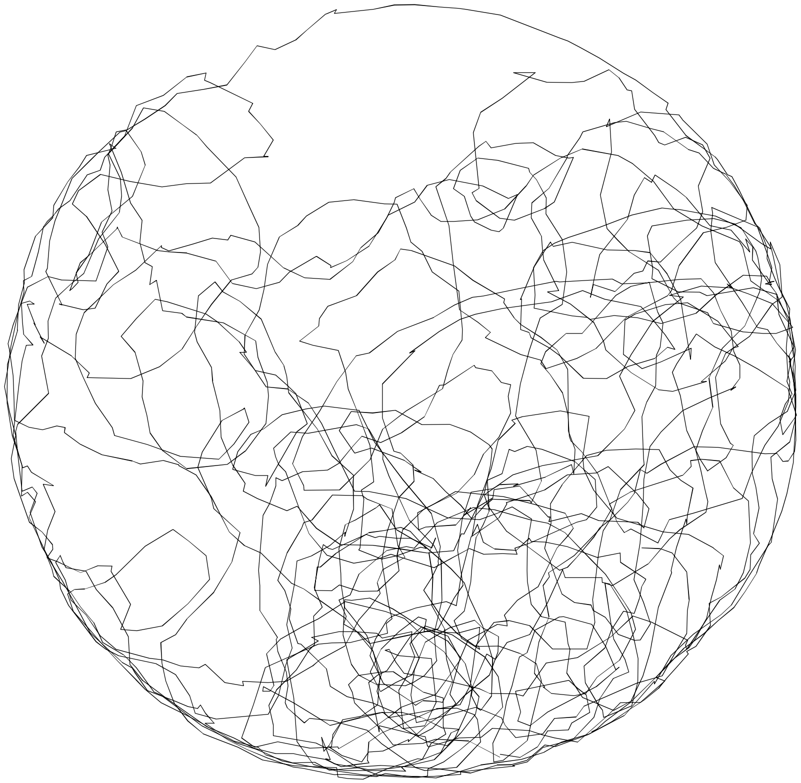
Construction of a particle with a head and a flexible tail





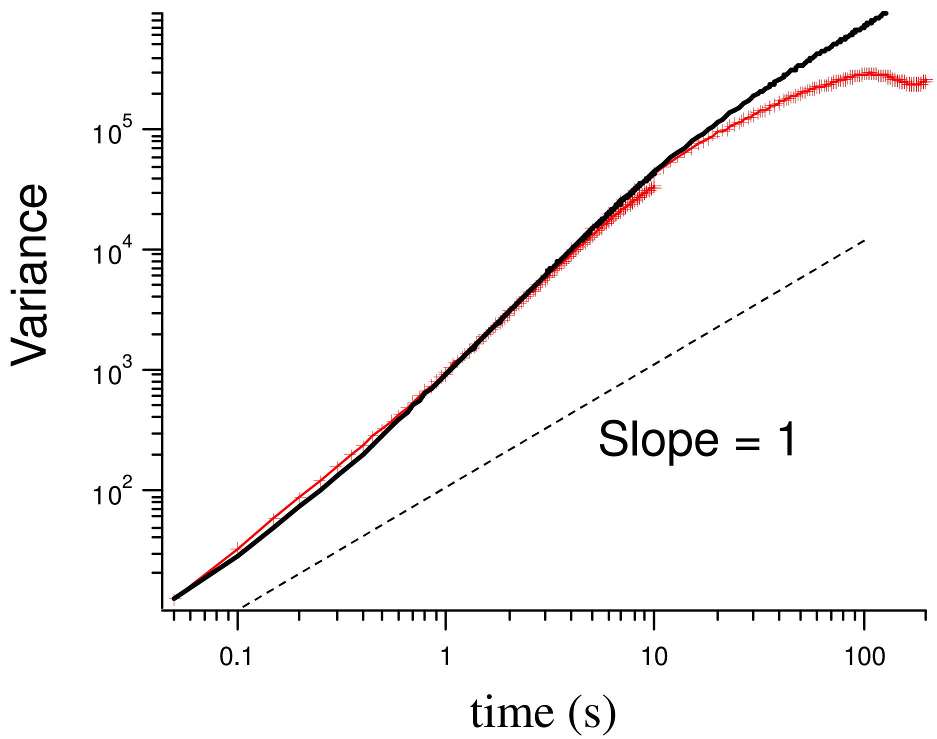
Two types of surfaces:
- Sand-blasted surface with 50 μm roughness
- Layer of 1 mm steel beads glued on vibrated surface

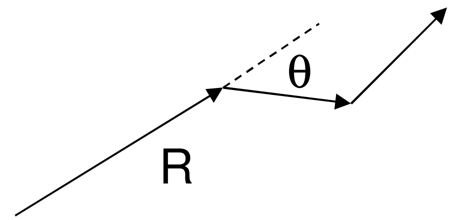
Trajectory on a smooth substrate



- Motion of the SPP over 2.5s (left), 2000s (right).
- Confinement becomes important over long times.

Comparison with persistent walk model





$\langle R(t)^2\rangle = \langle R_m{}^2\rangle\, t + 2.0\, \langle R_m\rangle^2\;\; c/(1 - c) * (t - (1 - c^t)/(1.0 - c))$

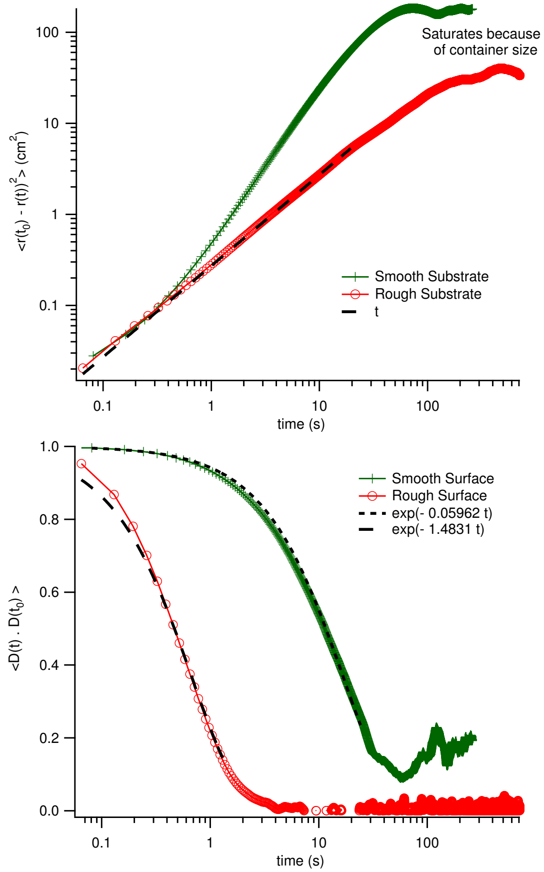
$\langle R_m{}^2\rangle$: variance after 1 step

$\langle R_m\rangle$: mean displacement after 1 step

$$c = \int_{-\pi}^{\pi} p(\theta)\cos\theta d\theta$$

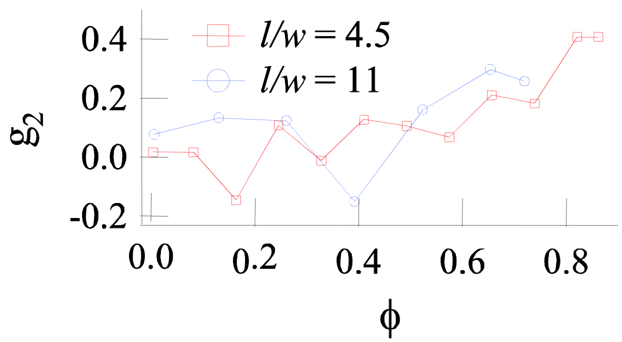
Kareiva & Shigesada (1983)

Rough versus smooth substrate

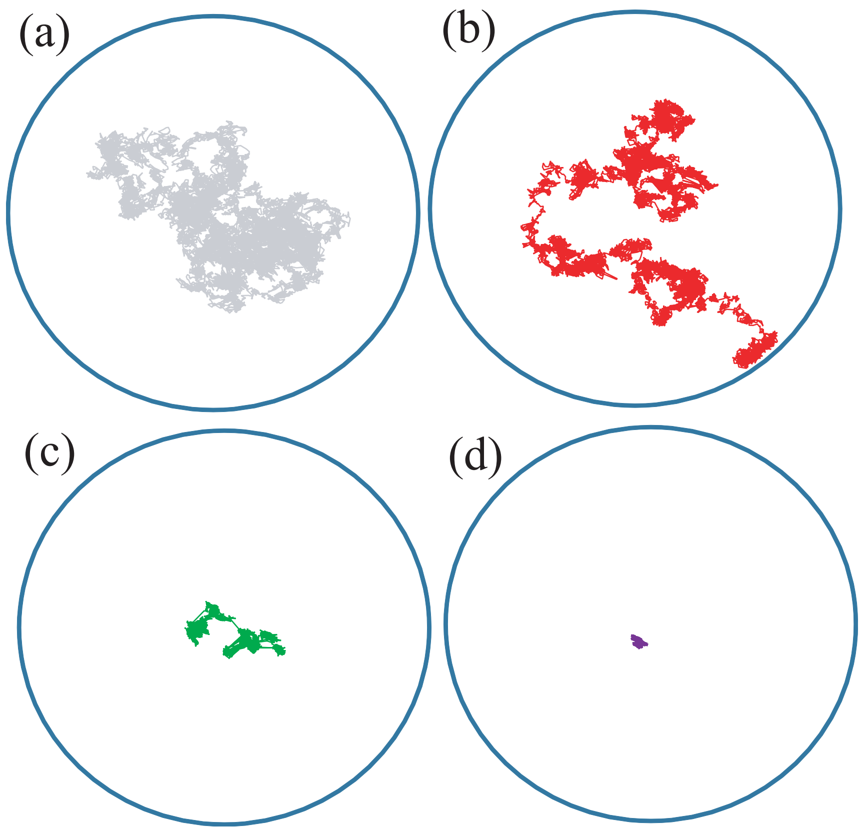


• Motion on a rough substrate quickly becomes diffusive, but on smooth substrate motion appears super-diffusive because of persistent nature of motion

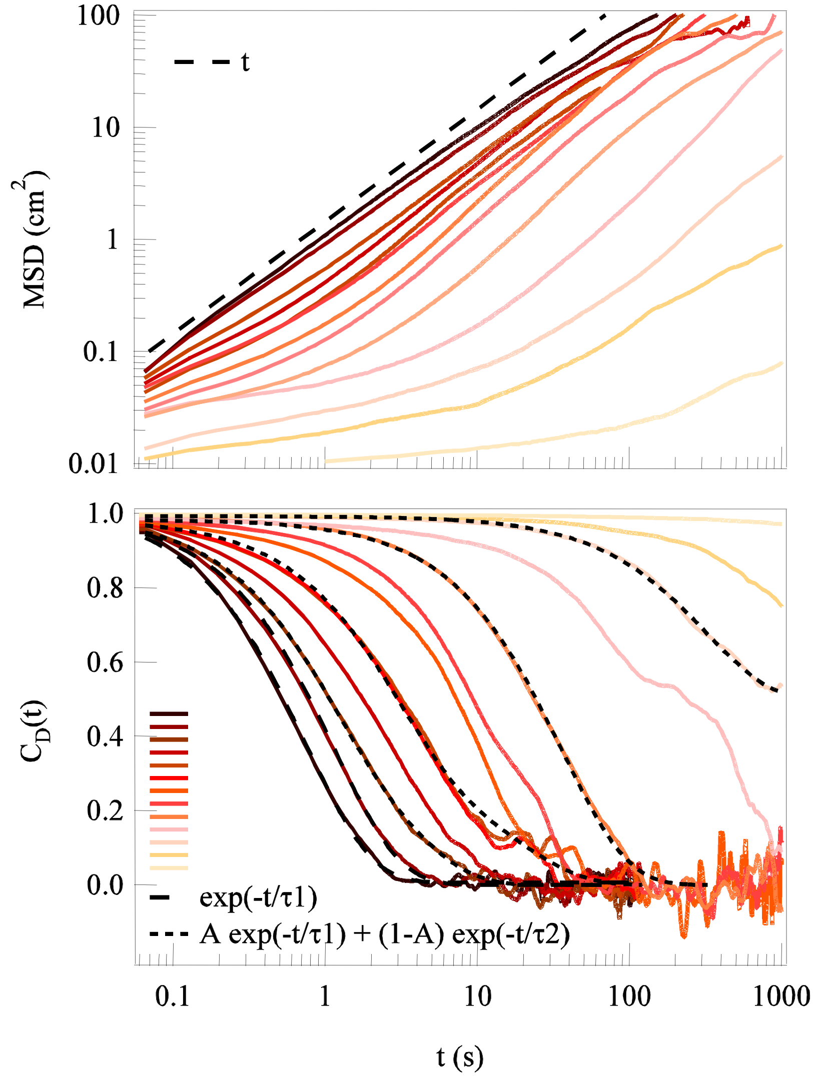
Orientation order grows with area fraction



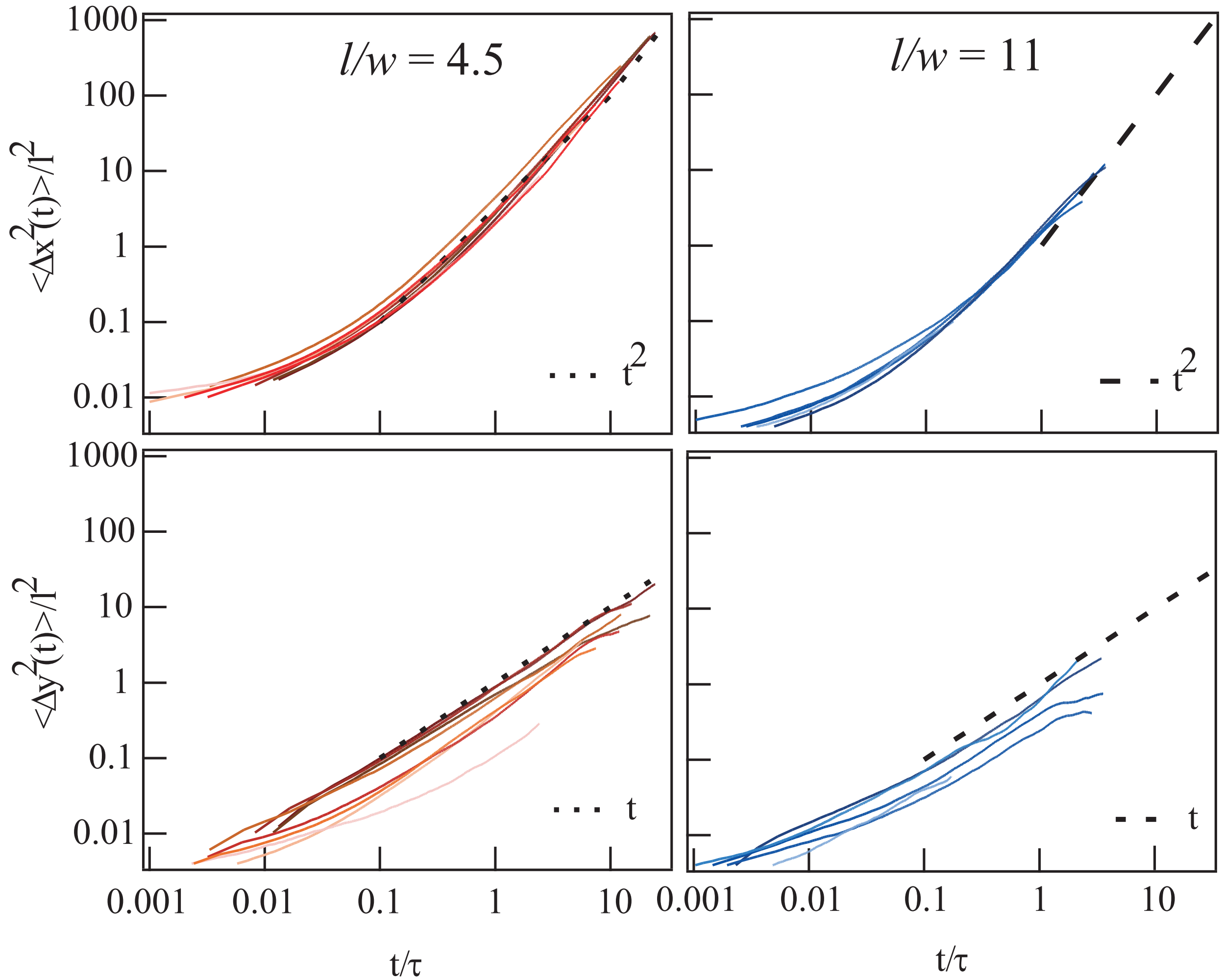
Diffusion of SPR



• SPR are tracked using a tracer technique over long time. Diffusion decreases with area fraction till finally arrest is observed



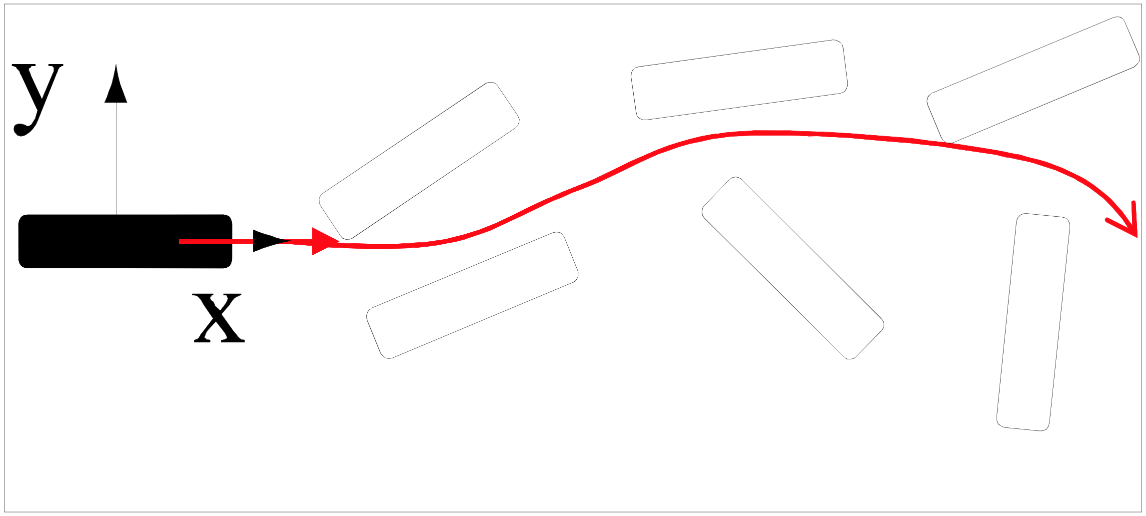
• Mean square displacements and velocity auto-correlation decrease systematically with time in the lab frame of reference



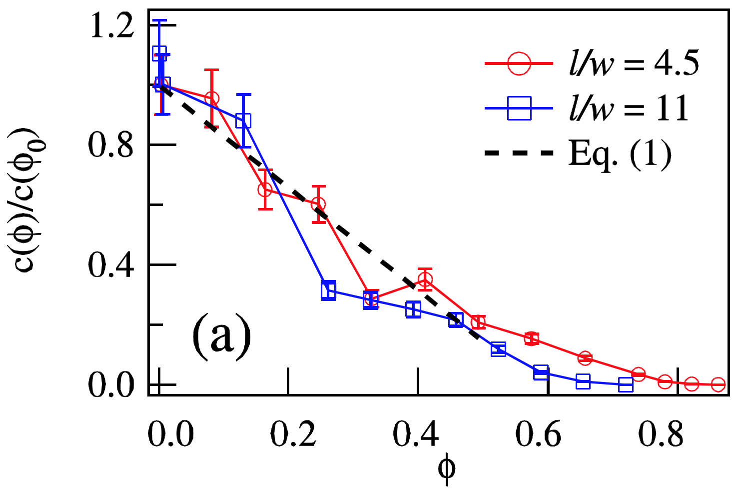
• Mean square displacements <u>in the body frame of reference</u> are observed to scale in the parallel and perpendicular direction with the time scale τ needed to travel a body length l/c.

SPR Tube model

Following Edwards and Evans (1981) for rigid rod but using the mean drift velocity of the rod:

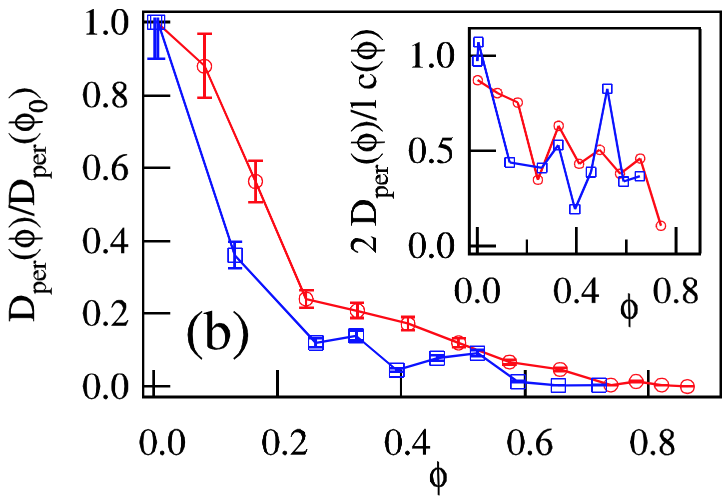


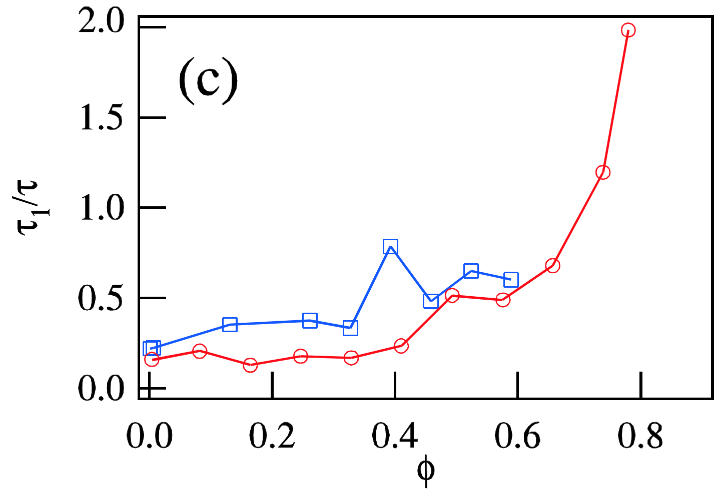
Body frame of reference



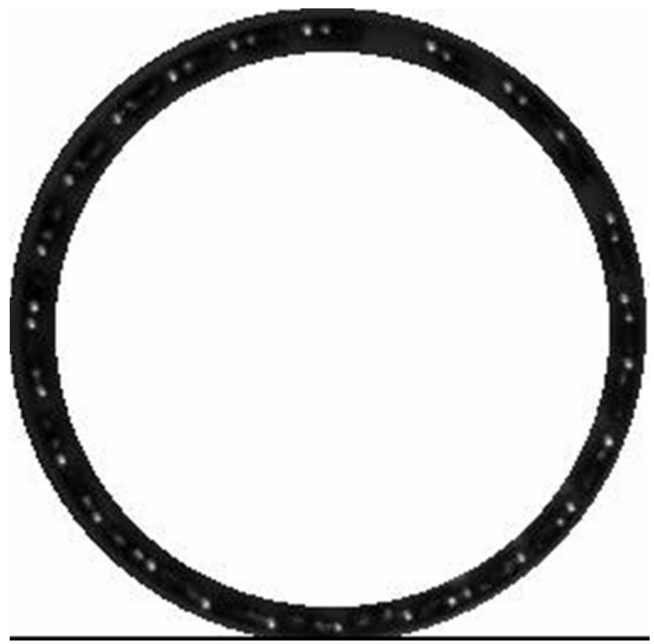
The decrease of mean speed with area fraction can be captured by a modified tube model of elongated rigid rods where τ replaces the diffusion time scale

$$c(\phi) = c(0)\ (1 - \alpha\ (1\ /\ \sqrt{2} + \sqrt{2}\ \ w/l)\ \phi$$





- A.K., PRL (2010)



Collisions with neighbors makes the diffusion non-trivial, observe density waves.

Apolar flexible chains

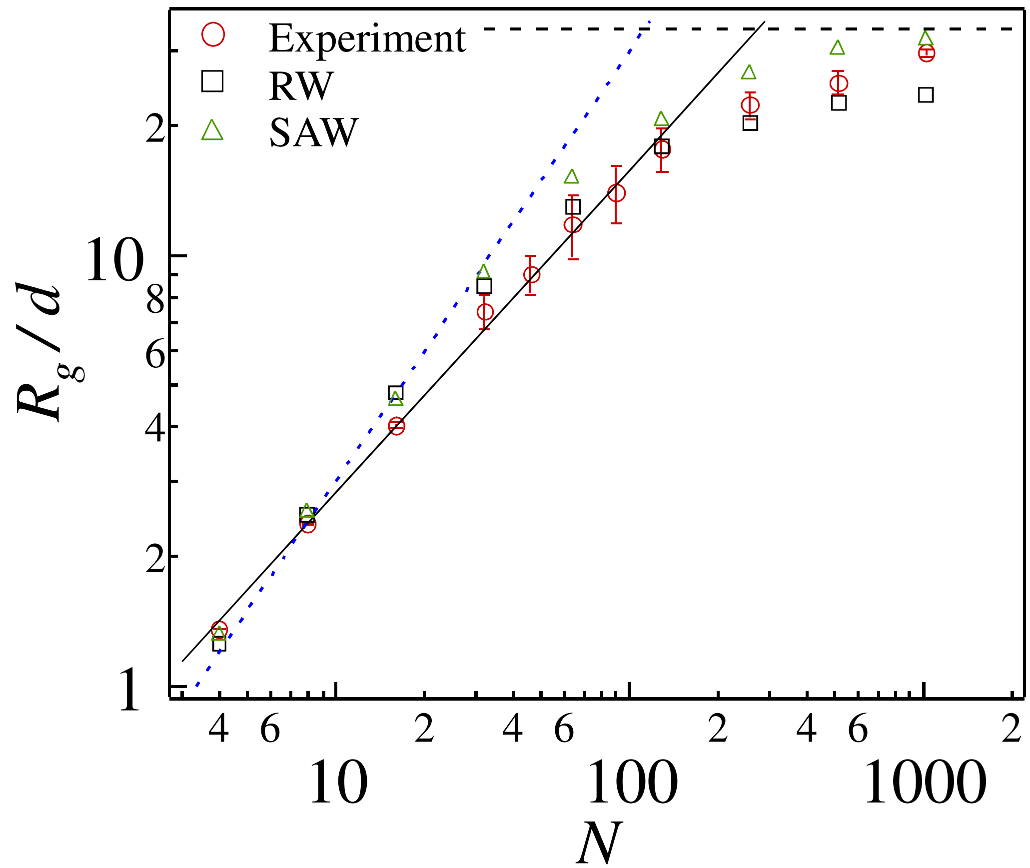
With Kevin Safford, Yacov Kantor and Mehran Kardar

N = 1024

Experimental parameters:
$\Gamma = 3g$, $f = 30$ Hz

Particle diameter d = 3.12 mm, connected by links 0 to 1.5 mm

Radius of Gyration



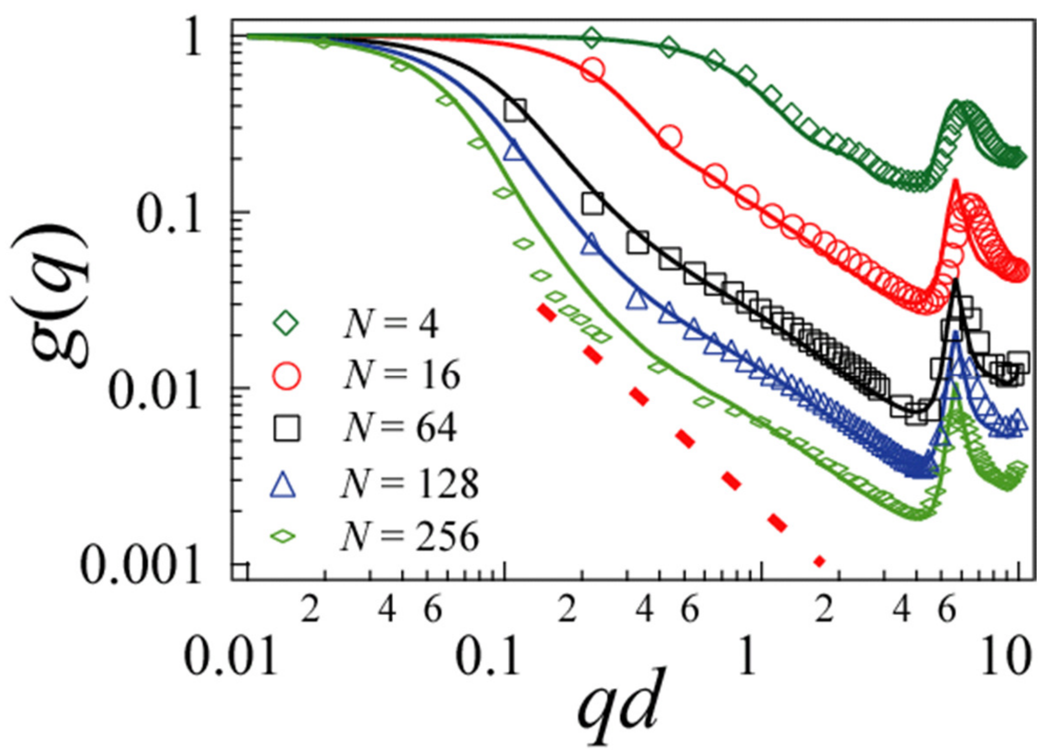
$$R_g^2 = \frac{1}{N}\sum_{n=1}^{N}\left\langle (R_n - R_{cm})^2 \right\rangle$$

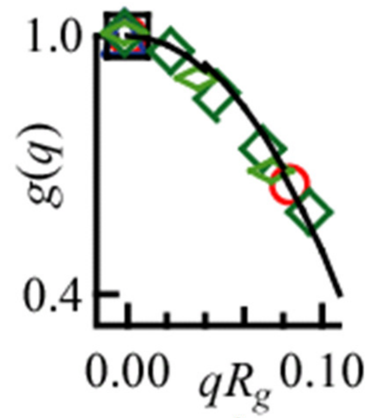
$$R_{cm} = \frac{1}{N}\sum_{n=1}^{N} R_n$$

- Random Walk and Self-Avoided Walks simulations performed by Yacov Kantor
- Walks have a persistence length and were confined to a circle

Pair correlation

$$g(q) = \langle \frac{1}{N^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \left\langle \exp[iq \cdot (R_m - R_n)] \right\rangle \rangle_\phi$$





$$g(q) = 1 - \frac{1}{2}(qR_g)^2$$

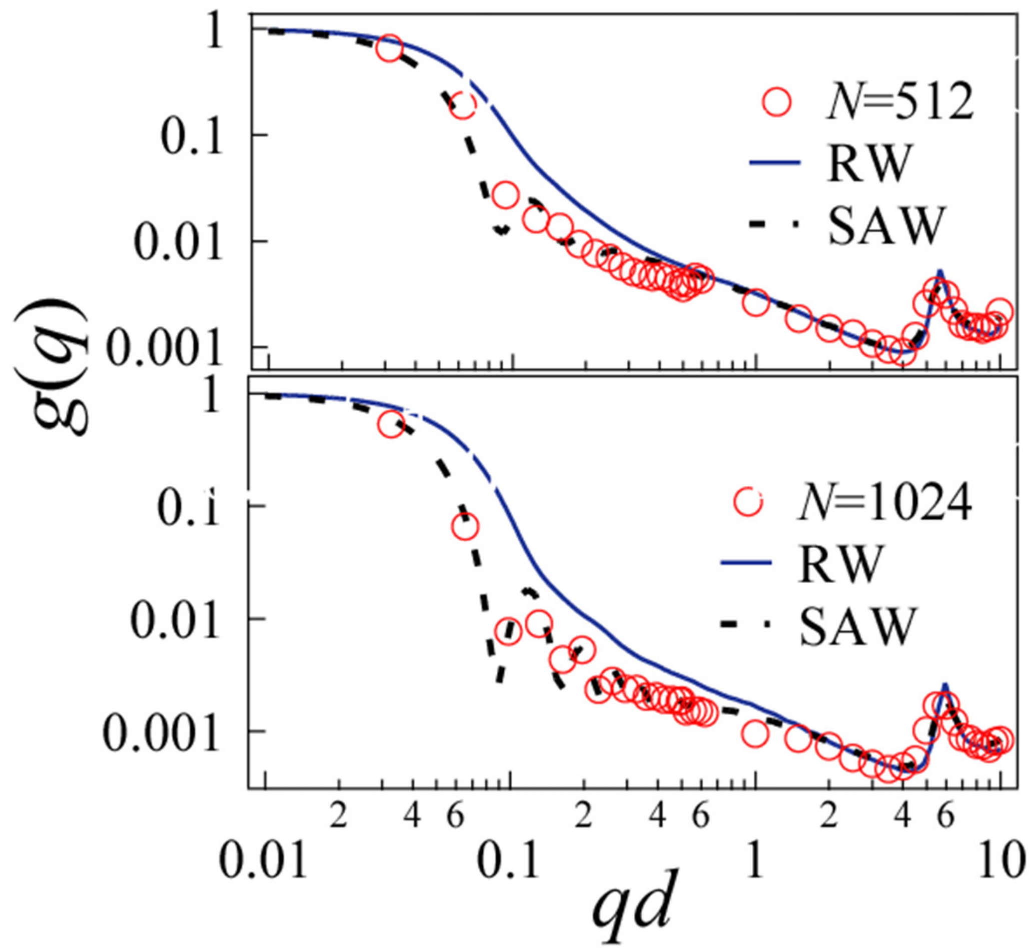
for $q\,R_g < 1$

Unconfined
SAW model

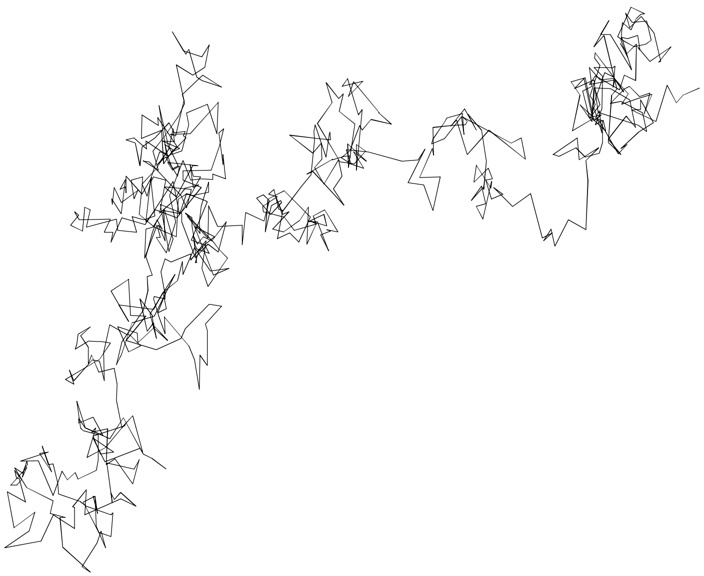
$$g(q) \propto (qR_g)^{-1/\nu} \quad \text{for } q\,R_g > 1$$

Doi, Edwards (1999)

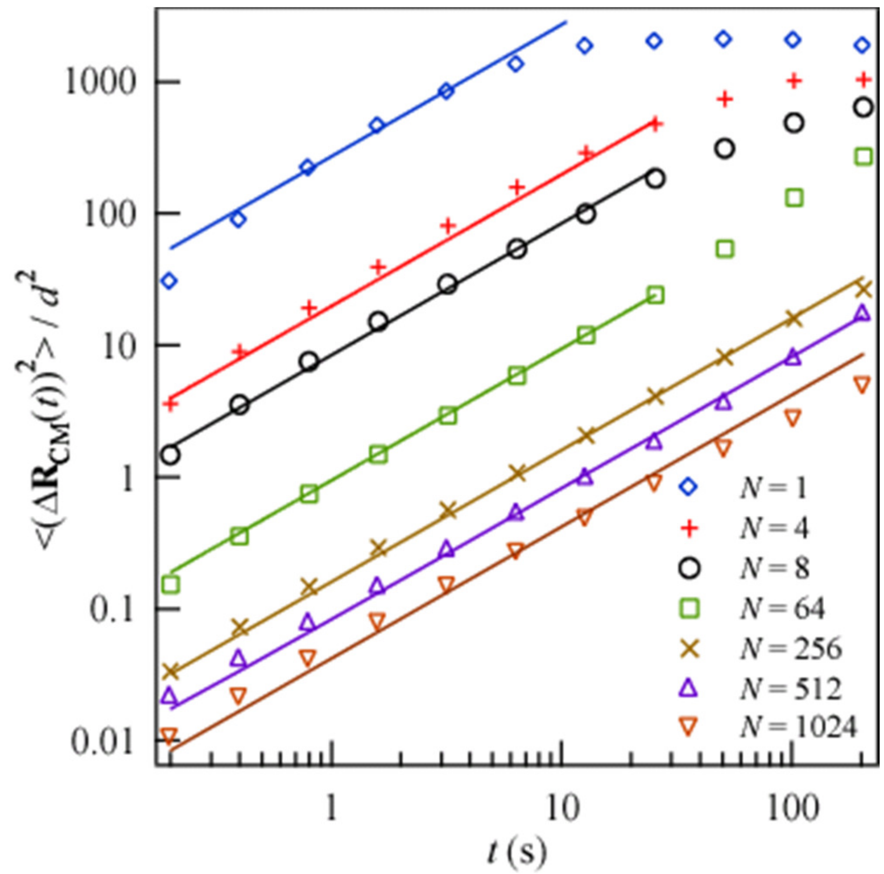
Comparison with a self avoided walk simulation



Dynamics of the chain



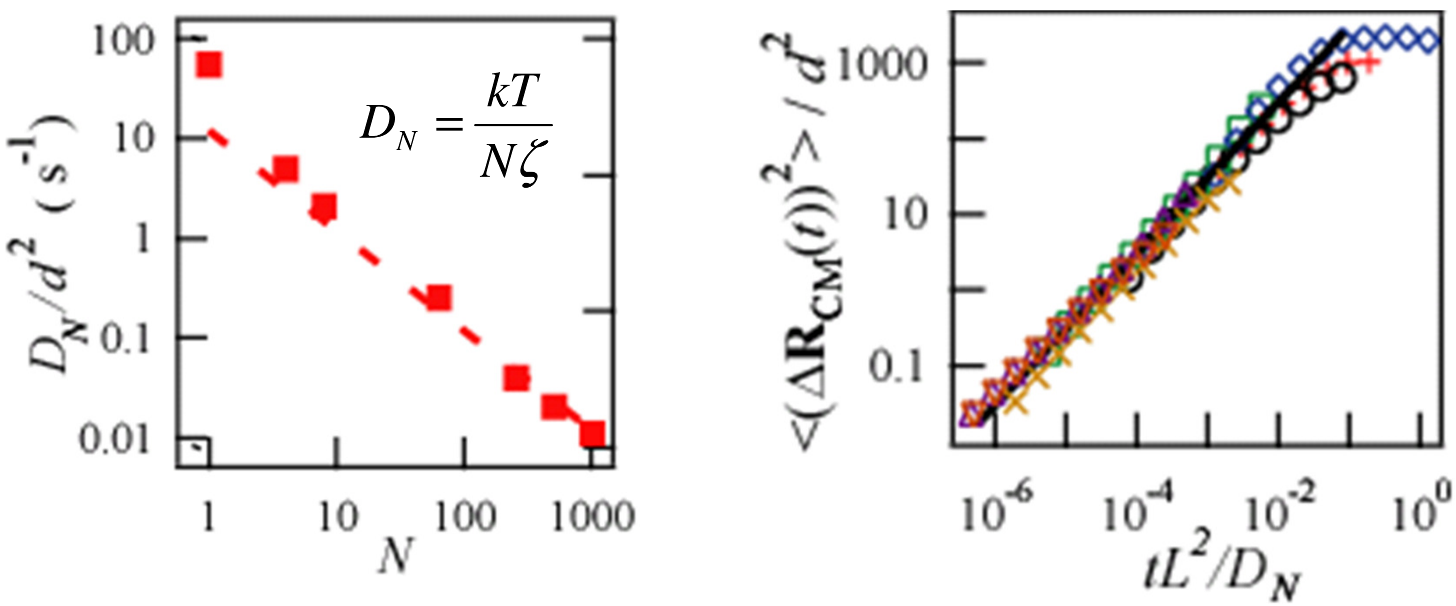
center of mass
of the chain



$$\left\langle (R_{cm}(t) - R_{cm}(0))^2 \right\rangle = 4Dt$$

Diffusion constant versus chain length

Rouse model - assumes that each monomer experiences a viscous drag proportional to its velocity



$kT \to \frac{1}{2} m\langle v^2\rangle \sim 0.5$ J, the granular temperature is constant

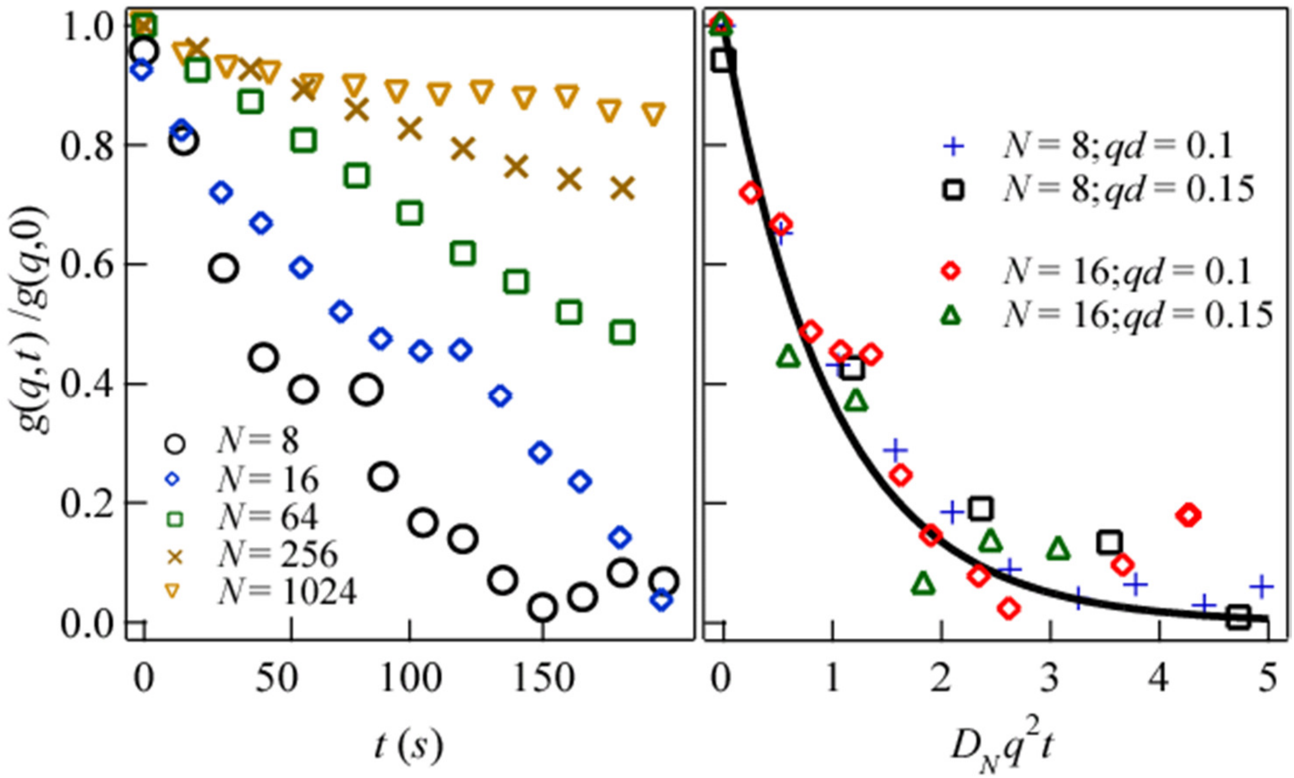
$\zeta = 2.87 \times 10^{-2}$ N-m^{-1}s, the drag coefficient

Thus, vibrated surface acts like a thermal fluid which gives and takes energy

Dynamic Structure

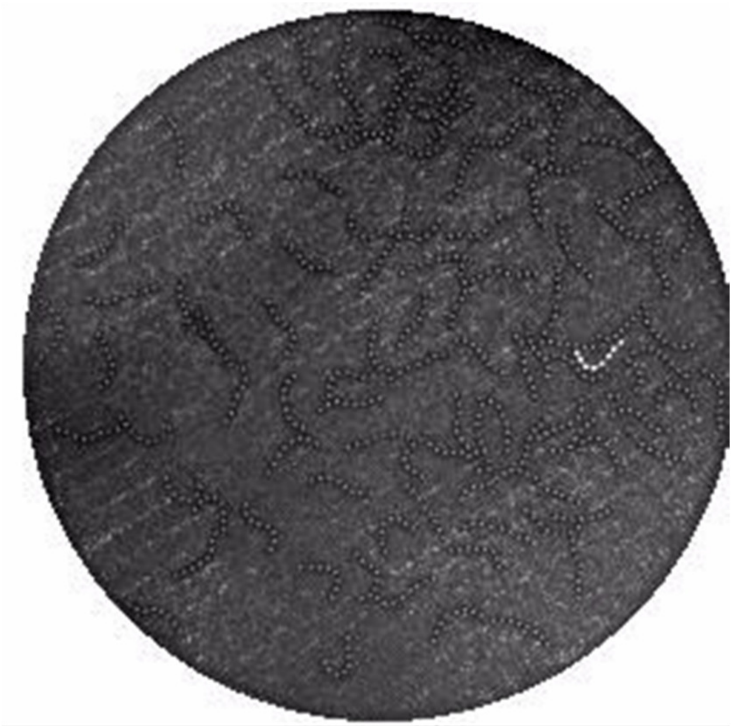
$$g(q,t) = \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} \langle e^{iq\cdot(R_m(0)-R_n(t))} \rangle$$

- In the limit of $q\,R_g \ll 1$, $r_n - r_m \gg R_g$ and t large, $g(q,t) \to N \exp(-k^2\, t/\, D)$



Structure and dynamics of vibrated granular chains: Comparison to equilibrium polymers, K. Safford, Y. Kantor, M. Kardar, & A. K., PRE (2009)

Diffusion of an apolar rod as a function of density



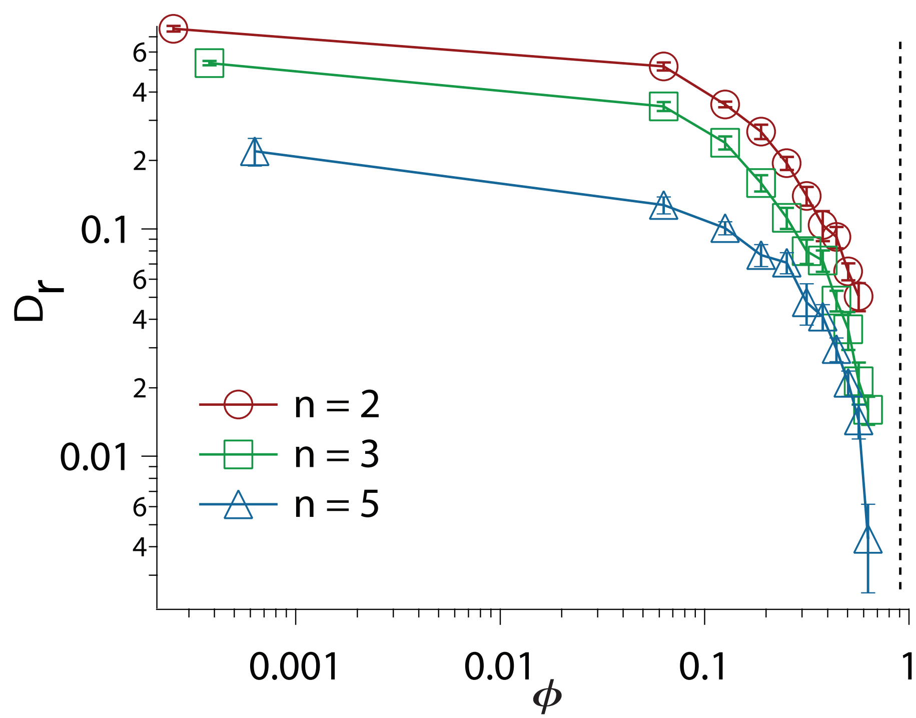
$\phi = 0.12$

$\phi = 0.48$

Chains of length N = 8

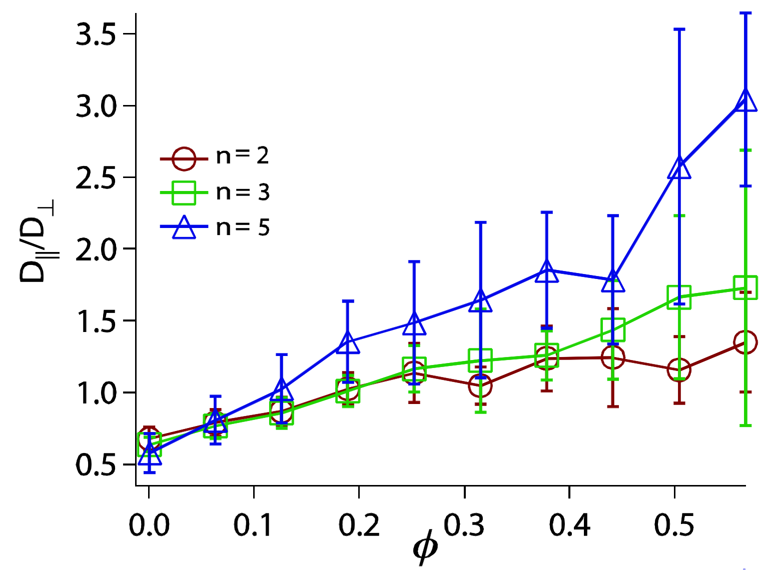
Area fraction $\phi = n\, N\, (d/D)^2$

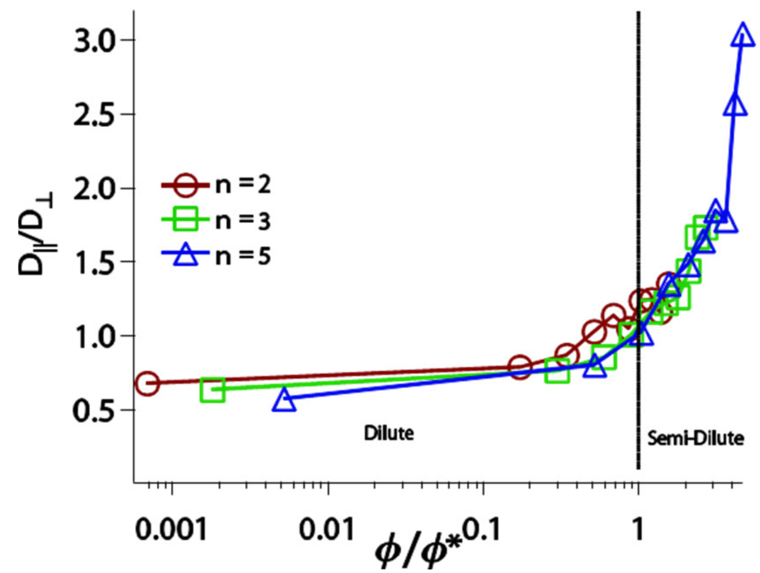
Rotational diffusion as a function of density



Y. Yadav and A.K., EPJE (2012)

Diffusion in the horizontal plane as a function of density





$\phi^* = n/A_r^2$

- Diffusion becomes anisotropic
- Diffusion decreases to zero well before maximum packing

- Y. Yadav and A.K., EPJE (2012)

Conclusions

- Self-propelled particles can be constructed by using asymmetric mass distributions, motion described by persistent random walk models
- Show novel aggregation patterns such as swarming ring without any potential attractant

- Polymer models give good description of configurations and dynamics of vibrated granular chains
- Diffusion decreases in perpendicular direction as chain concentration is increased

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Tel-Aviv
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http://physics.clarku.edu/~akudrolli