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Synchronization and LC order in soft active matter

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Plan

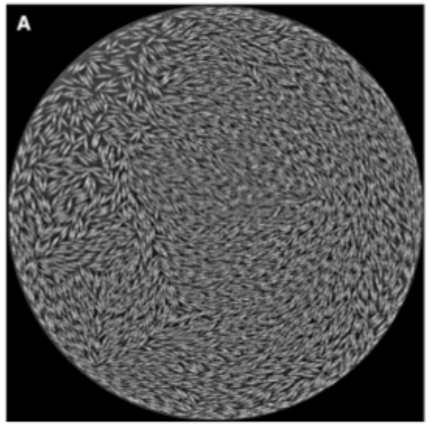
1. Introduction …
2. Focus on active elements …
3. Cycles, synchronization …
4. Dynamic active particle suspensions (time)
5. Outlook

active matter ?

Collection of interacting self- driven units with emergent behavior at large scales

Energy input that maintains the system out of equilibrium is on each unit, not at boundaries

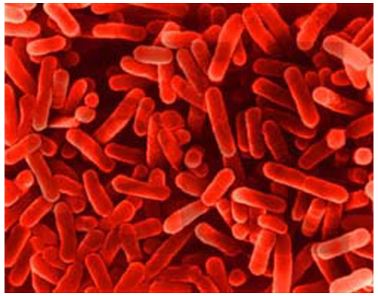
Robust behavior in the presence of noise (generally nonthermal)



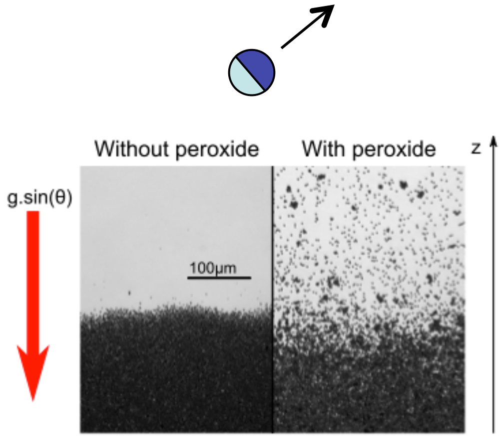
Orientable units → states with orientational order

(soft) active matter?

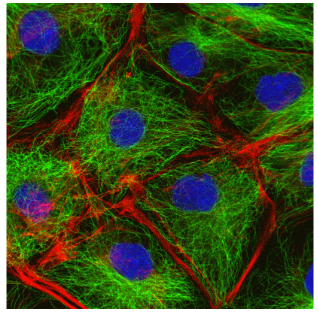
swimming bacteria, janus particles, cell cytoskeleton



Legionella



Active suspension



Tubulin - green,
Actin -red,
Nucleus- blue

Active elements – complex fluid constituents
- polymers, colloids, membranes, …

Weak interactions ~ $k_B T$, slow dynamics, fluctuations, …

+

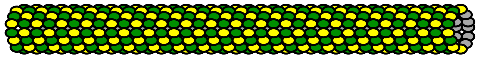
Life - an internal source of energy

Cycles in the Cytoskeleton

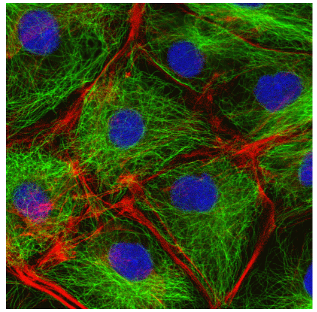
- **Filaments/motors**
- Cytoskeletal filaments

F-actin

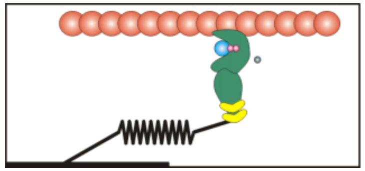
7 nm



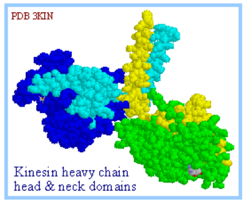
microtubule

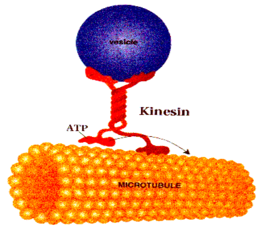


- F-actin and Myosin use ATP to turn chemical energy into mechanical work (duty cycle)

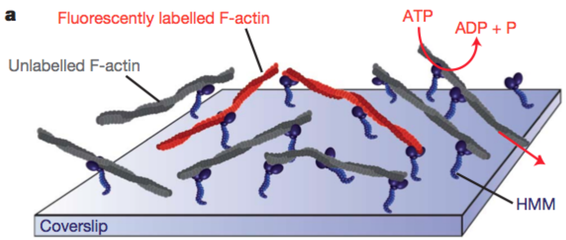


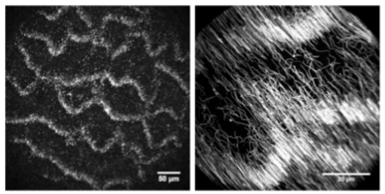
- Kinesin ‘walks’ along Microtubules using ATP as fuel

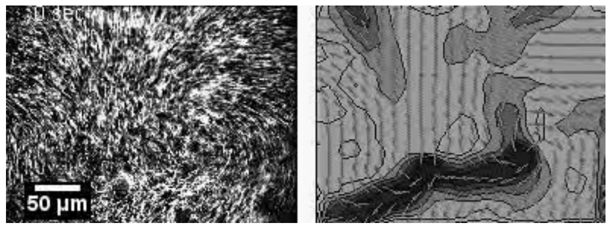




- Collective behaviour of fiaments/motors







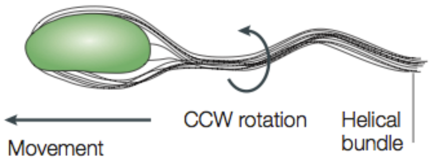
Schaller et al, (2010, 2013)

Strokes of micron sized swimmers

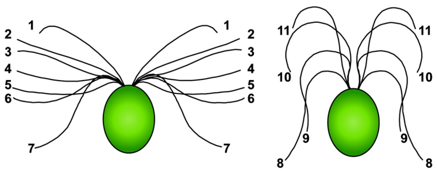
Bacteria

Algae

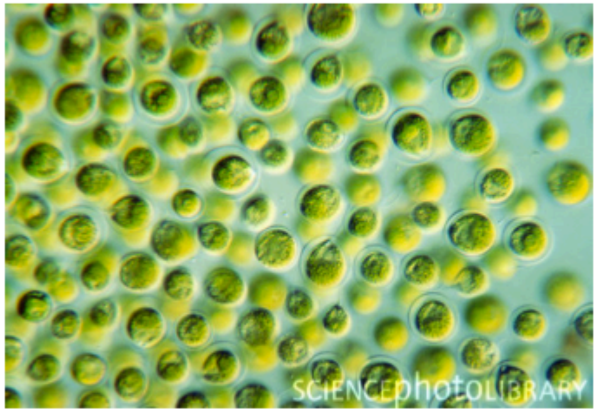
Micron size objects

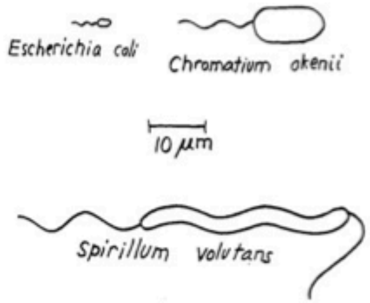


Butler & Camilli, (2005)



Weibel et al, PNAS (2005)

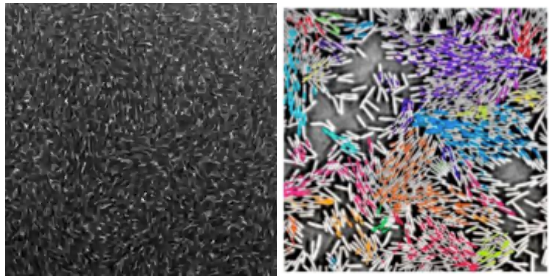




Lighthill, SIAM review (1976)

Collective behaviour of bacillus subtilis

Zhang et al, (2010)



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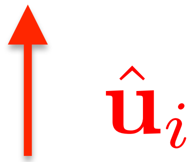
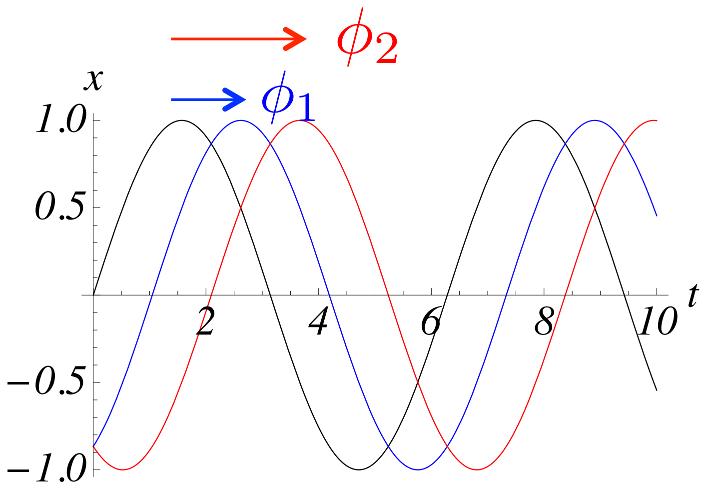
Phenomenology of active units

Polar object



Cyclic behaviour



Phase variable

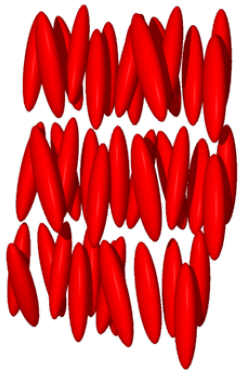
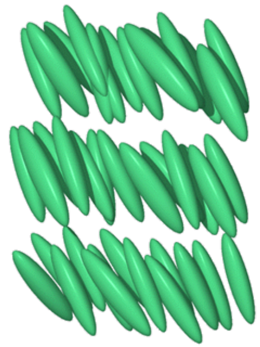
$$\phi_i \in [0, 2\pi]$$

Liquid crystals

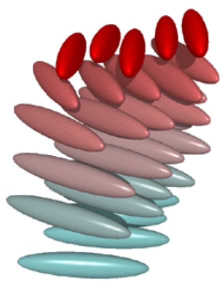
Anisotropic objects (mesogens) -> "between" liquid & crystal

- Increase density, decrease T



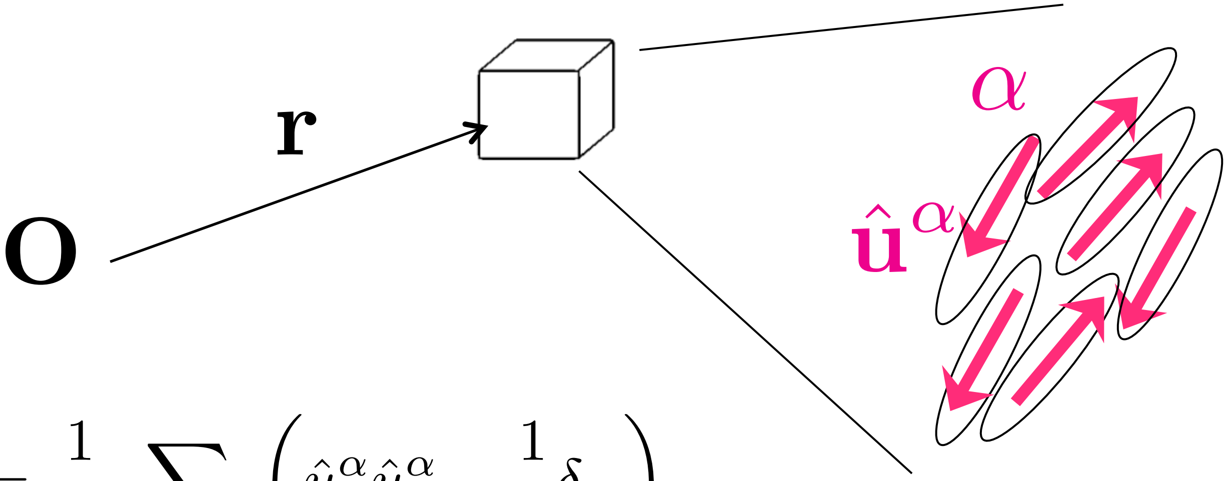



Mesophase	isotropic	nematic	smectic A	smectic C
Broken Symmetry	none	rotational	translational + rotational	trans. + rot.

Other phases : columnar, cholesteric

Order parameters



Nematic (uniaxial)

$$Q_{ij}(\mathbf{r}) = \frac{1}{N} \sum_{\alpha \in \mathrm{box}} \left(\hat{u}_i^{\alpha} \hat{u}_j^{\alpha} - \frac{1}{2}\delta_{ij} \right)$$

Nematic order

Traceless, symmetric tensor

$$Q_{ij} = Q_0 \left(\hat{n}_i \hat{n}_j - \frac{1}{3}\delta_{ij} \right)$$

director:

$\hat{\mathbf{n}} \leftrightarrow -\hat{\mathbf{n}}$

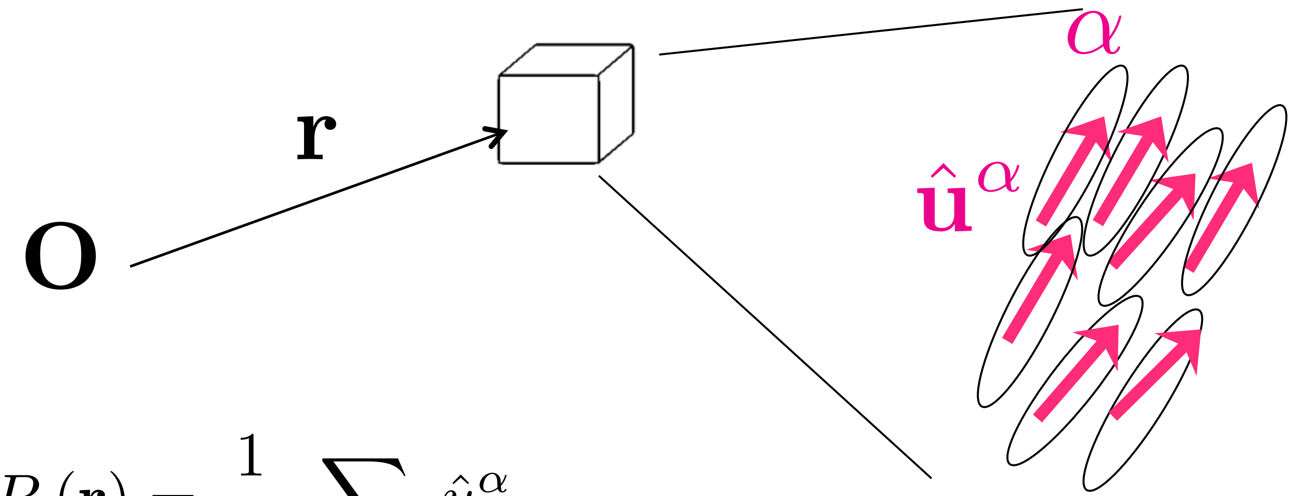
Isotropic

$Q_0 = 0$

Nematic

$Q_0 \neq 0$

Order parameters



Polar mesophase

$$P_i(\mathbf{r}) = \frac{1}{N} \sum_{\alpha \in \text{box}} \hat{u}_i^\alpha$$

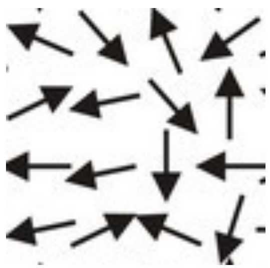
director

Vector $P_i = P_0 \hat{p}_i$

$\hat{\mathbf{p}} \neq -\hat{\mathbf{p}}$

Isotropic

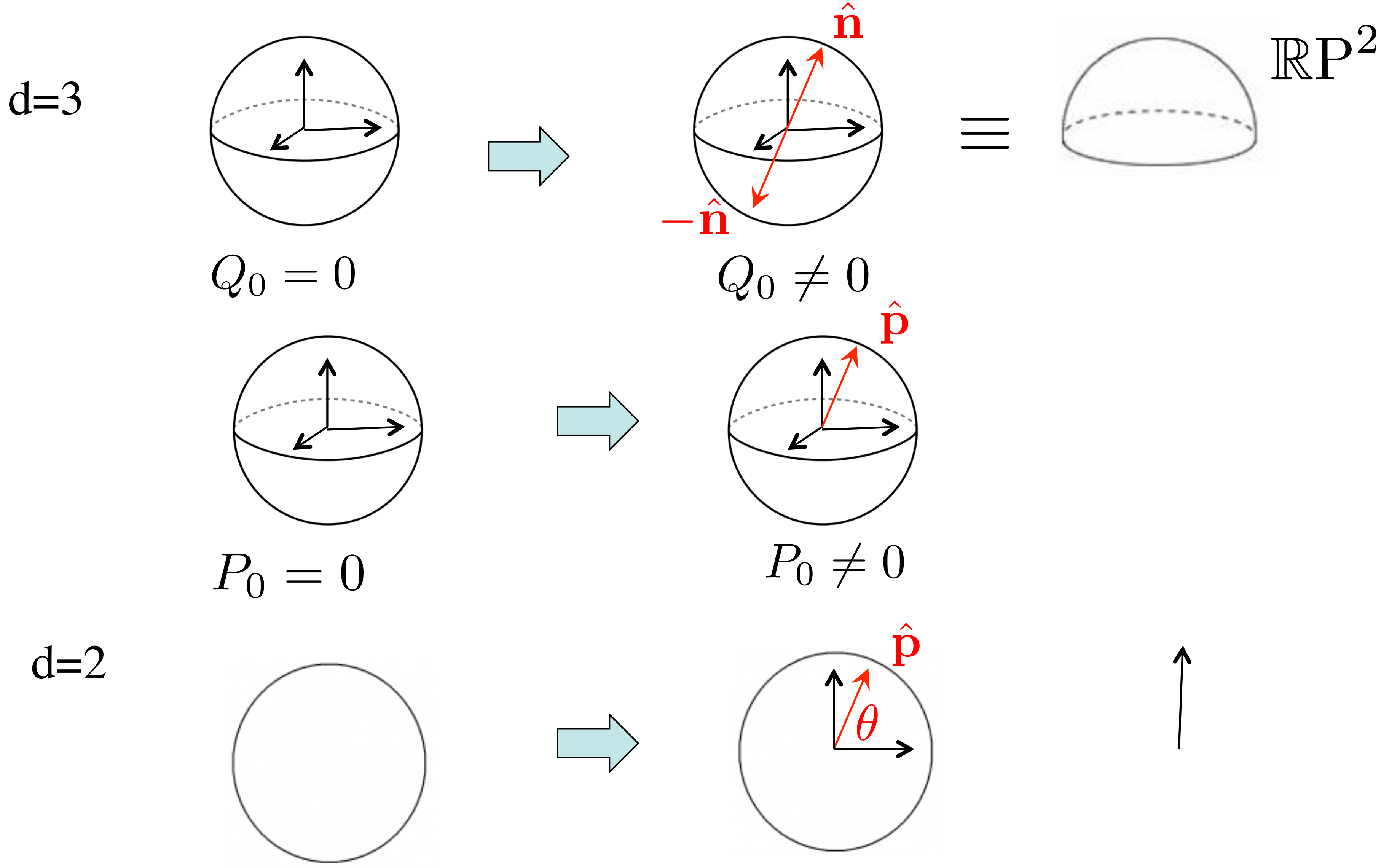
$P_0 = 0$



Polar

$P_0 \neq 0$

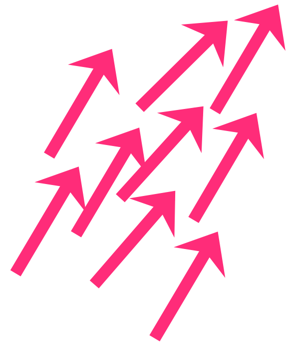
Broken continuous symmetries



modelling collective behaviour

Identify large scale degrees of freedom

Generic models of possible behaviours



Bottom-up modelling

Properties of ingredients

Macroscopic properties

simplifications required to
make process tractable

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Programme ?

Simple models for micron sized self propelled objects (SPO) – obtain design principles

↓

Simplified physical theory that links microscopic properties of SPO to macroscopic behaviour

↓

Understand macroscopic mechanical properties (initially qualitative -> eventually quantitative)

Micro-hydrodynamics

$\mathrm{Re} = \rho V L / \eta \ll 1$

Micron sized objects in fluid

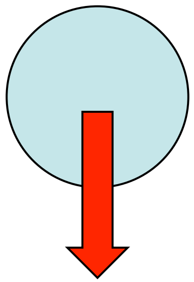
$$\rho\,(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = \eta\nabla^2\mathbf{v}(\mathbf{r},t) - \nabla p \qquad \nabla\cdot\mathbf{v} = 0$$

$$\nabla\cdot\boldsymbol{\sigma} = 0 \qquad \sigma_{ij} = \eta(\partial_i v_j + \partial_j v_i) - p\delta_{ij}$$

$\rho=1000$ kg m^{-3}, $V=10^{-6}$ m s^{-1}
$L= 10^{-6}$ m , $\eta=10^{-3}$ Pa s
$\mathrm{Re}=10^{-6}$

Cf: Sedimenting Colloid

$$-\vec{F} = \int_S d\vec{S}\cdot\boldsymbol{\sigma}$$



Nonslip BC

$$\vec{F} = mg\hat{z}$$

Total force on colloid from fluid balances external force

Neutrally buoyant swimming = no external forces

– zero total force from fluid !!

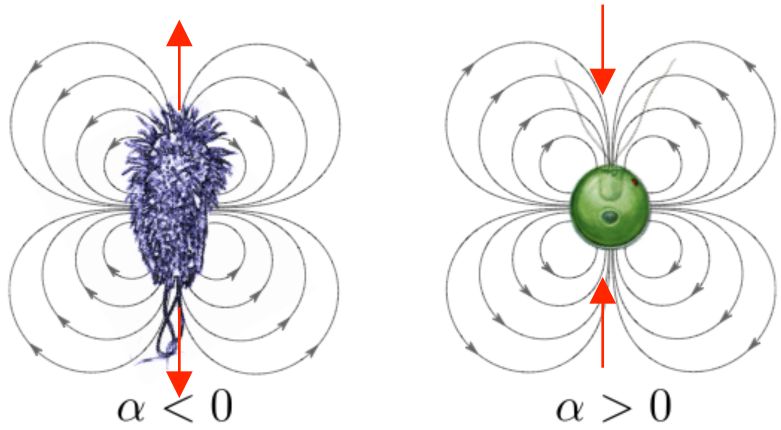
$$\vec{F} = 0$$

Phenomenology of active units

Contractile/Extensile

Sign of force dipole

- $\alpha < 0$ / $\alpha > 0$
- **Pushers/Pullers**
- **E-Coli/Chlamydomonas**



We can classify/understand some aspects of the collective behaviour in terms of these force dipoles.

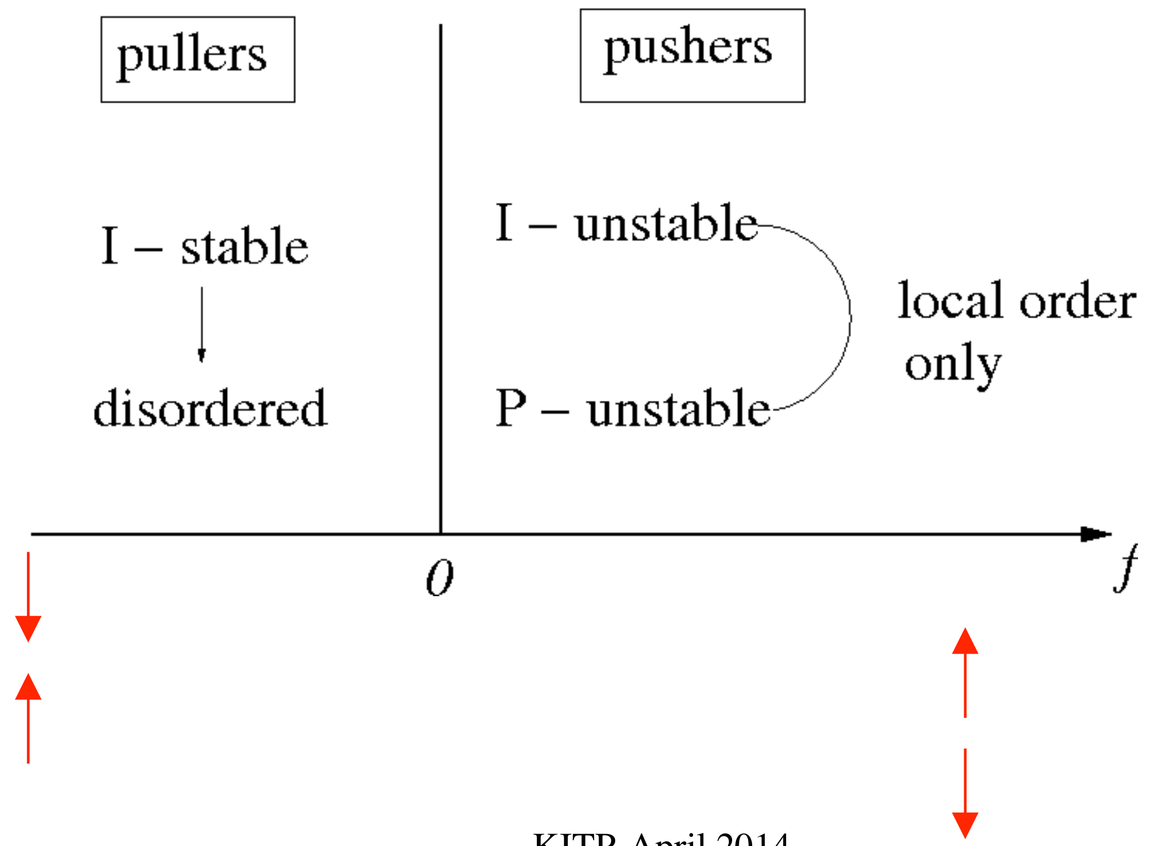
Pushers : regions of orientational order, disordered on large lengthscales

Pullers : disordered on all lengthscales

Phenomenology of behaviour

Contractile/Extensile

Sign of force dipole



What about the duty cycle ?

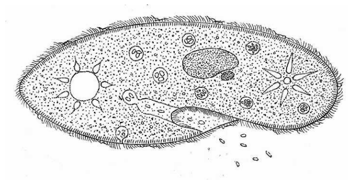
Q: Does this internal phase variable have consequences on the behaviour of active matter?

Phase differences -> possibility of synchronization

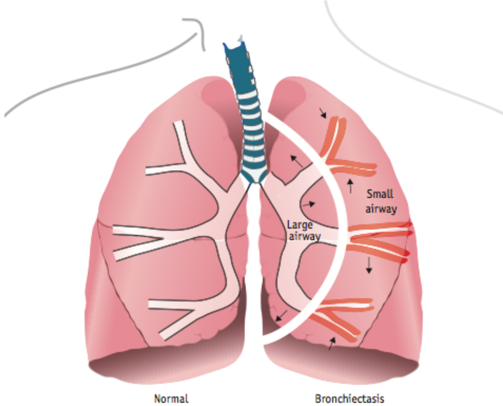
Place where this happens is collective behaviour of cilia

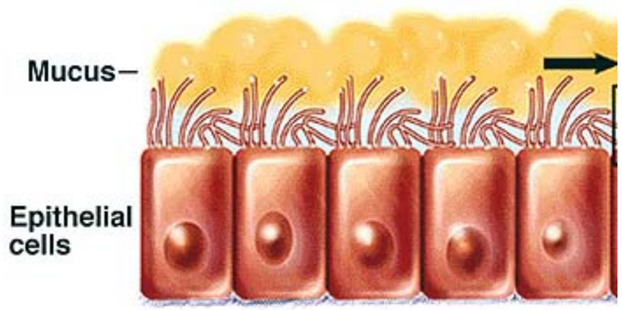
Collective behaviour of cilia

Ciliated organisms – paramecium (100 μm , 10 lengths/s)



Collective transport by cilia – mucociliary tract





Can the cilia transport material on scales >> size ?

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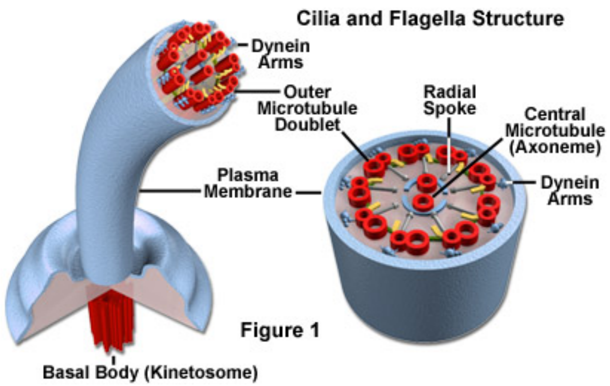
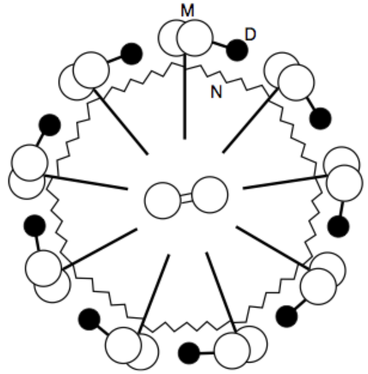
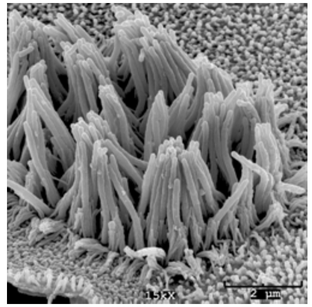
Structure of axonemic cilia

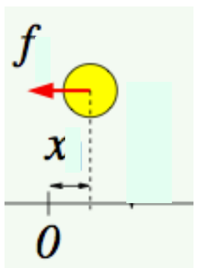
Microtubules (M), Dynein (D), Nexin spring (N)







Generic simplified model – single object subject to time varying force



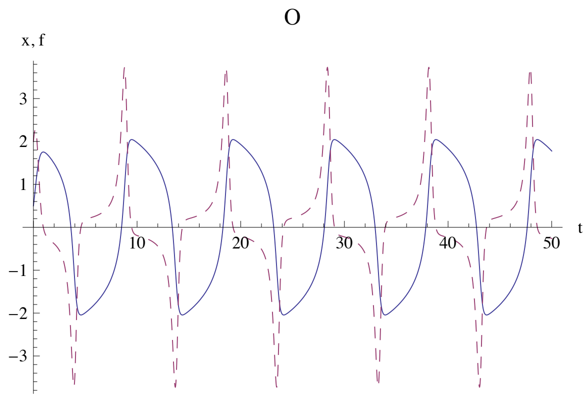
$$\dot{x} = \frac{f}{\gamma} \qquad \gamma = 6\pi\eta a \quad \textbf{sphere radius, } a$$

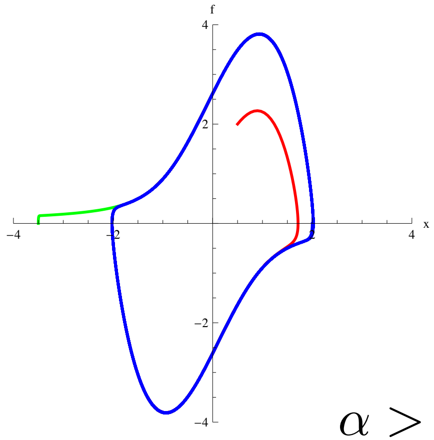
$$\dot{f} = -\frac{k}{\tau}x + \mu\frac{f}{\gamma}\left(1 - \sigma x^2\right) + \alpha\, x^3$$

Viscosity dominated dynamics

Oscillator

Viscosity dominated dynamics -> spontaneous oscillations





Limit cycle

$$\alpha > 0 \Rightarrow \frac{d\omega}{dR} < 0$$

$\alpha \neq 0$ **Nonisochronous oscillations**

On limit cycle, frequency of oscillations is amplitude dependent

$$\dot{x} = \frac{f}{\gamma}$$

$$\dot{f} = -\frac{k}{\tau}x + \mu\frac{f}{\gamma}(1-\sigma x^2) + \alpha x^3$$

Amplitude tightly constrained to limit cycle, Phase varies more freely

$$\omega = \omega(\mu, k, \tau, \sigma, \alpha)$$

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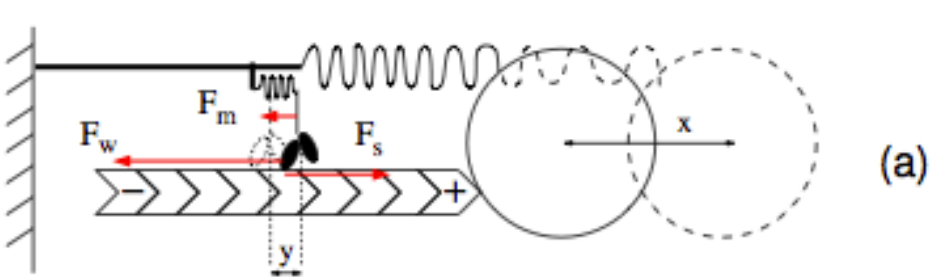
Filaments/motors -> oscillator

Force balance $\dot{x} = \frac{F}{\gamma}$

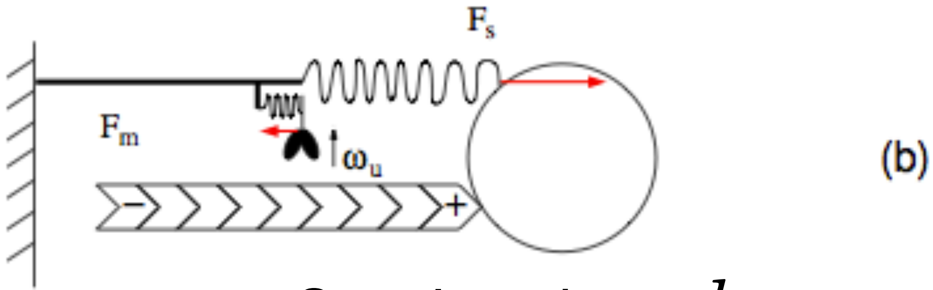
$$F = F_w + F_m + F_s$$



Binding, Unbinding rate

$$F_m = -\kappa y \qquad F_s = -Kx - \Lambda x^3$$

$$\omega_b < \gamma/K, \gamma/\kappa < \omega_u$$

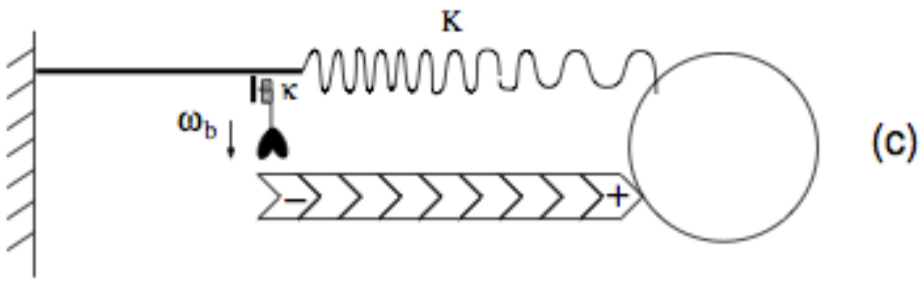


Parameters

Step length = d

$$\frac{k}{\tau} \sim \omega_u^0 \frac{d^2 \kappa K}{k_B T} \quad , \quad \mu \sim (\kappa - K)$$

$$\sigma \sim \frac{3\Lambda}{\kappa - K} \quad , \quad \alpha \sim -\frac{d\kappa}{k_B T}\Lambda$$



Hydrodynamic interactions

- On lengthscales large compared to object, micron sized object as collection of point forces on fluid

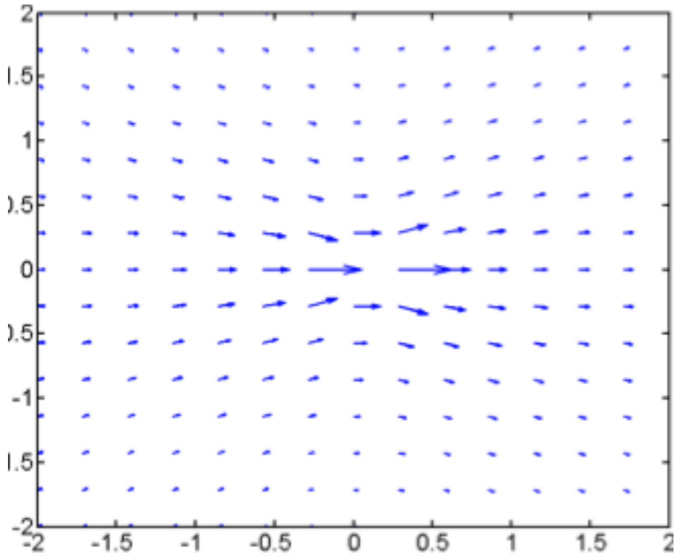
$$\eta\nabla^2\mathbf{v} - \nabla p = \mathbf{g}(\mathbf{r}) \quad ; \quad \nabla\cdot\mathbf{v} \qquad \mathbf{g}(\mathbf{r}) = \mathbf{f}\delta(\mathbf{r})$$

- Green's function

$$\mathbf{v}(\mathbf{r}) = \mathbf{H}(\mathbf{r})\cdot\mathbf{f}$$

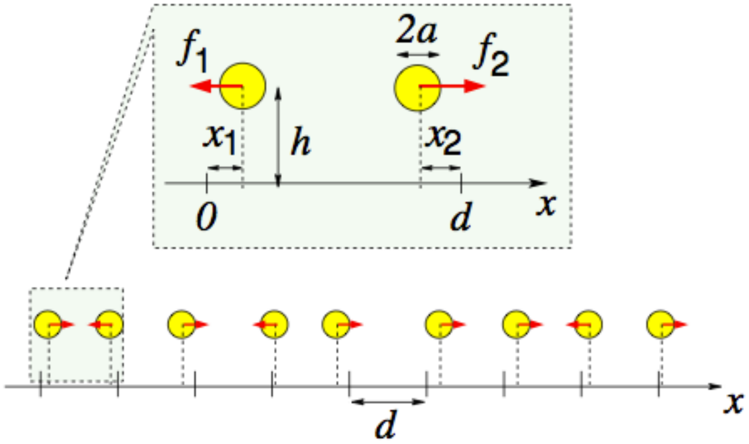
$$\mathbf{H}(\mathbf{r}) = \frac{1}{8\pi\eta r}\left(\mathbf{I} + \hat{\mathbf{r}}\,\hat{\mathbf{r}}\right)$$

Oseen tensor



Two interacting oscillators

$\alpha \neq 0$ **means amplitude-dependent frequency of oscillations**



Hydrodynamic interactions -

$$r = d + x_2 - x_1$$

Fast variable : amplitude

Slow variable : phase

$$\dot{x}_1 = \frac{1}{\gamma}\left(f_1 + H(r) f_2\right);$$

$$\dot{x}_2 = \frac{1}{\gamma}\left(f_2 + H(r) f_1\right),$$

$$\dot{f}_i = -\frac{k}{\tau} x_i + \mu \frac{f_i}{\gamma}\left(1 - \sigma x_i^2\right) + \alpha x_i^3$$

$$H(r) \sim \frac{1}{r}$$ **bulk**

$$H(r) \sim \frac{h^2}{r^3}$$ **near wall**

Average over fast modes -> effective equation for slow modes

Hydrodynamic Synchronization

Weakly interacting limit

$$d \gg x_2 - x_1 \qquad a/d \ll \mu/k, \alpha\tau/k\sigma$$

$$x_k = \frac{1}{2}\left(A_k e^{i\omega t} + A_k^* e^{-i\omega t}\right) \qquad k = 1, 2$$

Average over fast oscillations ⇨ $\bar{A}_k = R_k e^{i\phi_k}$

Phase difference $\psi = \phi_2 - \phi_1$ $\quad \dot{\psi} = f(\cos\psi)\sin\psi$

Enhanced when $\alpha \neq 0$

In phase - $\alpha > 0$

Antiphase - $\alpha < 0$

Collective dynamics ?

Much slower if $\alpha = 0$

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Tutorial example: Brownian motion

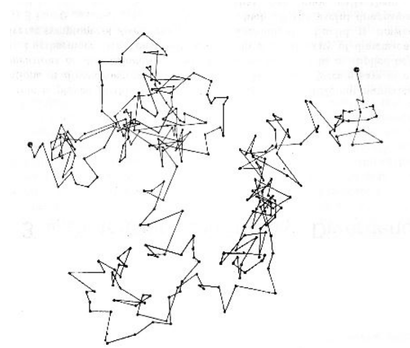
Langevin equation in over-damped limit (no inertia)

$$\frac{d}{dt}\mathbf{r}(t) = \boxed{\mathbf{v}\left(\mathbf{r}(t)\right)} + \boxed{\mathbf{u}(t)}$$

deterministic (e.g. from potential)

Random (thermal fluctuations)

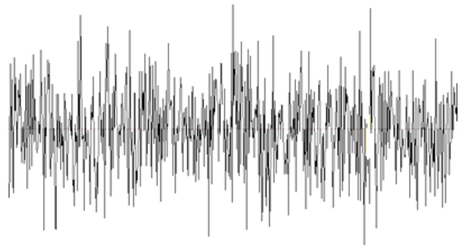
$$\mathbf{v}(\mathbf{r}) = -\frac{1}{\zeta}\boldsymbol{\nabla} V(\mathbf{r}) \quad ,$$



$$\langle \mathbf{u}(t) \rangle = \mathbf{0}$$

Gaussian white noise

$$\langle u_i(t) u_j(t') \rangle = 2D\, \delta_{ij}\, \delta(t - t')$$



Probability density of finding particle in vol. $d^d x$ around $\mathbf{x}$

$$P(\mathbf{x}, t) = \langle \delta\left(\mathbf{x} - \mathbf{r}(t)\right) \rangle$$

$\langle .. \rangle \equiv$ average over noise

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From stochastic trajectory to probability

Dynamical equation for prob. density

$$\lim_{\Delta t\to 0}\frac{\partial}{\partial t}P(\mathbf{x},t)\equiv\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[P(\mathbf{x},t+\Delta t)-P(\mathbf{x},t)\right]$$

$$=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\left\langle\delta^3\left(\mathbf{x}-\mathbf{r}(t+\Delta t)\right)\right\rangle-\left\langle\delta^3\left(\mathbf{x}-\mathbf{r}(t)\right)\right\rangle\right]$$

$$=-\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_t^{t+\Delta t}dt_1\boldsymbol{\nabla}\cdot\left(\mathbf{v}(\mathbf{x})\,P(\mathbf{x},t_1)+\left\langle\mathbf{u}(t_1)\,\delta^3(\mathbf{x}-\mathbf{r}(t_1))\right\rangle\right)$$

$$\mathbf{r}(t+\Delta t)-\mathbf{r}(t)=\int_t^{t+\Delta t}dt_1\dot{\mathbf{r}}(t_1)$$

Current, $\mathbf{J}(\mathbf{x},t)$: $\quad\frac{\partial}{\partial t}P+\frac{\partial}{\partial x_i}J_i=0$

$$J_i(\mathbf{x},t)=v_i(\mathbf{x})\,P(\mathbf{x},t)-\lim_{\Delta t\to 0}\frac{1}{2\Delta t}\int_t^{t+\Delta t}dt_1\int_t^{t+\Delta t}dt_2\sum_{j=1}^{3}\underbrace{\langle u_i(t_1)u_j(t_2)\rangle}_{2\,D\,\delta_{ij}\,\delta(t_1-t_2)}\frac{\partial}{\partial x_j}P(\mathbf{x},t)$$

$$J_i(\mathbf{x},t)=v_i(\mathbf{x})\,P(\mathbf{x},t)-D\,\frac{\partial}{\partial x_i}P(\mathbf{x},t)$$

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Stochastic trajectory to probability II

N interacting particles + fluctuations

$$\frac{\partial \vec{x}_i}{\partial t} = \vec{V}_i\left(\{\vec{x}_1, \ldots, \vec{x}_N\}\right) + \vec{\xi}_i(t)$$

$$\langle \xi_{i\alpha}(t)\xi_{j\beta}(t')\rangle = 2D_{i\alpha}\delta_{ij}\delta_{\alpha\beta}\delta(t-t')$$

Up to pairwise interactions

$$\vec{V}_i\left(\{\vec{x}_1, \ldots, \vec{x}_N\}\right) = \vec{v}^{(1)}(\vec{x}_i) + \sum_{j \neq i} \vec{v}^{(2)}(\vec{x}_i, \vec{x}_j)$$

One particle density $\quad c_1(\vec{r}, t) = \sum_{i=1}^{N} \langle \delta\left(\vec{r} - \vec{x}_i(t)\right)\rangle$

Conservation law : $\quad \partial_t c_1 + \vec{\nabla} \cdot \vec{J}_1 = 0$

Stochastic trajectory to probability III

Current :

$$\vec{J}_1(\vec{r}_1,t) = \vec{v}^{(1)}(\vec{r}_1)c_1(\vec{r}_1,t) + \int d\vec{r}_2\, \vec{v}^{(2)}(\vec{r}_1,\vec{r}_2)c_2(\vec{r}_1,\vec{r}_2,t)$$

$$-D\vec{\nabla}c_1$$

2-particle density

$$\partial_t c_2 = F[c_2, c_3]\ldots$$

Closure problem (usual non-equil. Stat. mech. prob.)

Mean field approximation $c_2(\vec{r}_1,\vec{r}_2,t) = c_1(\vec{r}_1,t)c_1(\vec{r}_2,t)$

(ignore correlations)

Stochastic trajectory to probability IV

One particle density

$$c_1(\vec{r},t) = \sum_{i=1}^{N} \langle \delta\left(\vec{r} - \vec{x}_i(t)\right)\rangle$$

$$\partial_t c_1 + \vec{\nabla}\cdot\vec{J}_1 = 0$$

Conservation law :

Nonlinear current :

$$\begin{aligned}\vec{J}_1(\vec{r}_1,t) = & -D\,\vec{\nabla}c_1 + \vec{v}^{(1)}(\vec{r}_1)c_1(\vec{r}_1,t) \\ & + \int d\vec{r}_2\;\vec{v}^{(2)}(\vec{r}_1,\vec{r}_2)c_1(\vec{r}_1,t)\;c_1(\vec{r}_2,t)\end{aligned}$$

Interacting oscillators : $\vec{r}_i = \{\phi_i, y_i\}$

Velocities $\vec{v}^{(2)} = \{\dot{\phi}_1 - \dot{\phi}_2, 0\}$

Gradient operator $\vec{\nabla} = \{\partial_\phi, \partial_y\}$

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Coarse-graining the dynamics ..

Project on to moments (hydrodynamic modes) :

$$\rho(y,t) := \int_0^{2\pi} d\phi c(\phi,y,t);$$ Density

$$\Phi(y,t) := \int_0^{2\pi} d\phi e^{i\phi} c(\phi,y,t);$$ Phase coherence

Coarse-grain in space/time $a \ll d \ll h \ll r$

Global behaviour $$\partial_t \Phi^0 = -\left(D - \frac{\chi}{\beta}\frac{3ak}{4\omega\tau\gamma}\rho^0\right)\Phi^0 + \text{h.o.t}$$

$$\partial_t \rho^0 = 0 \qquad \chi = \frac{3}{8}\frac{\alpha}{\gamma\omega} \qquad \beta = \frac{\mu\sigma}{\gamma}$$

Synchronized state for $\alpha > 0$

Leoni, TBL (2012)

Coarse-graining the dynamics ..

Project on to moments (hydrodynamic modes) :

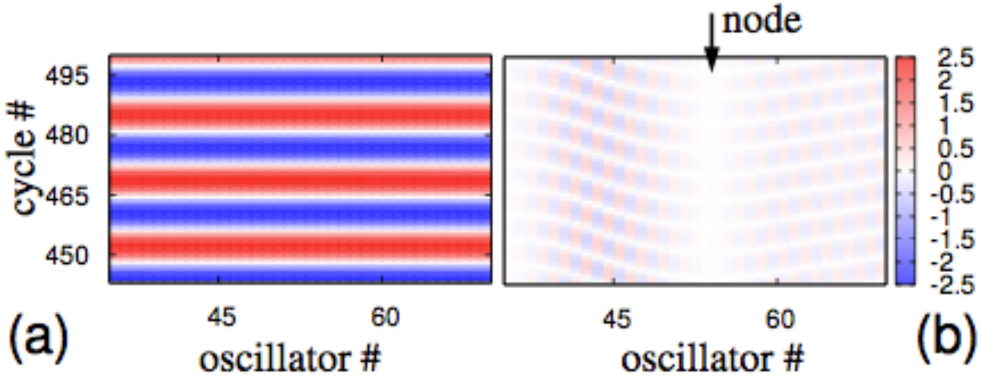
$$\rho(y,t) := \int_0^{2\pi} d\phi c(\phi, y, t);$$ Density

$$\Phi(y,t) := \int_0^{2\pi} d\phi e^{i\phi} c(\phi, y, t);$$ Phase coherence

Coarse-grain in space/time $a \ll d \ll h \ll r$

$\alpha > 0 \Rightarrow$ Globally inphase

$\alpha < 0 \Rightarrow$ Locally antiphase
+ long wavelength
phase modulations



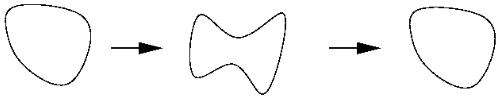
Leoni, TBL (2012)

Cyclic swimmers

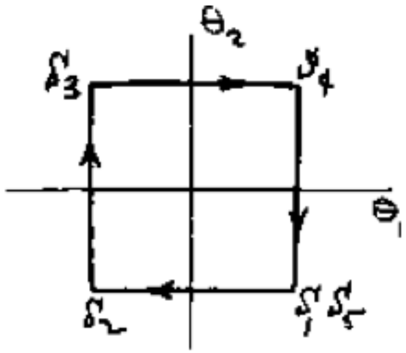
- (Re=0) Stokes equation is linear

$$\eta \nabla^2 \mathbf{v} - \nabla p = 0 \quad ; \quad \nabla \cdot \mathbf{v}$$

- Swimming is a periodic sequence of shape changes



- If e.g. forward stroke = backward stroke ➔ no net motion



$$0 = \int_S d\vec{S} \cdot \boldsymbol{\sigma}$$

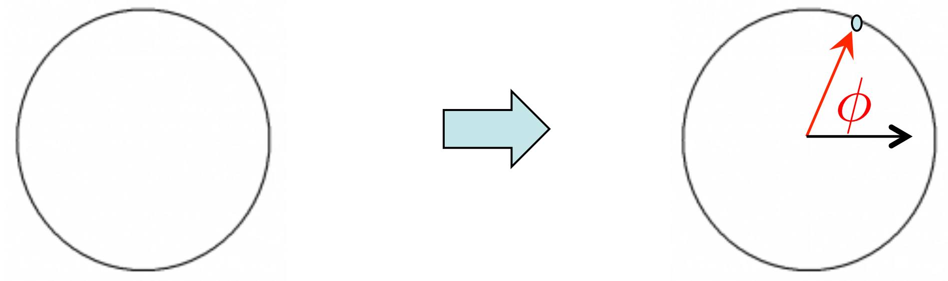
Force - free

Purcell (1977)

A closer look at swimmer dynamics

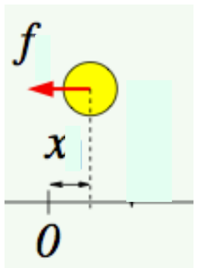
Swimming stroke = oscillation

Identical swimmers = all oscillations in phase



Another possible broken symmetry

Dissipative oscillator – single object subject to time varying force



$$\dot{x} = \frac{f}{\gamma} \qquad \gamma = 6\pi\eta a$$

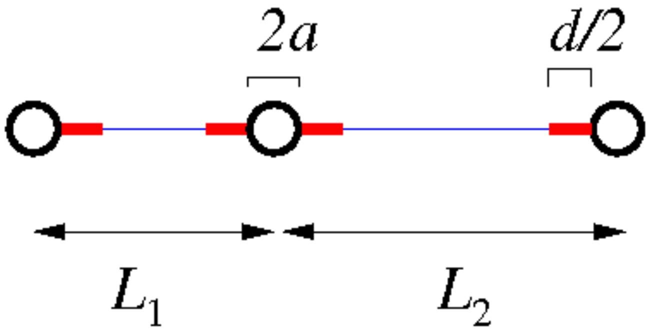
sphere radius, *a*

$$\dot{f} = -\frac{k}{\tau}x + \mu\frac{f}{\gamma}\left(1 - \sigma x^2\right) + \alpha\, x^3$$

Viscosity dominated dynamics -> spontaneous oscillations $\mu > 0$

An ‘Ising model’ of swimmers

3 bead model → $\hat{\mathbf{u}}$



$$L_i = l_i + d_i$$
$$d_1 = d\cos(\omega t)$$
$$d_2 = d\sin(\omega t)$$

Najafi-Golestanian, (2004)

$$\bar{v} \simeq \omega d^2 \left(\frac{1}{l_1^2} + \frac{1}{l_2^2} - \frac{1}{(l_1 + l_2)^2} \right) + O(d^3/l_i^3)$$

non-reciprocal internal motion

- Dynamic force density, with average

$$\bar{\mathbf{f}}(\mathbf{r}) = -f_d \bar{L} \hat{\mathbf{u}}\, \hat{u}_i \nabla_i \delta(\mathbf{r} - \mathbf{r}_0) + \frac{1}{2} f_q \bar{L}^2 \hat{\mathbf{u}}\, \hat{u}_i \hat{u}_j \nabla_i \nabla_j \delta(\mathbf{r} - \mathbf{r}_0) + \ldots$$

dipole — quadrupole

$$\bar{f}_d \sim \bar{f}_q \sim \gamma \omega d \frac{da}{\bar{L}^2}$$

$\kappa = l_2/l_1$ $\qquad \kappa = 1 \Rightarrow f_d = 0$

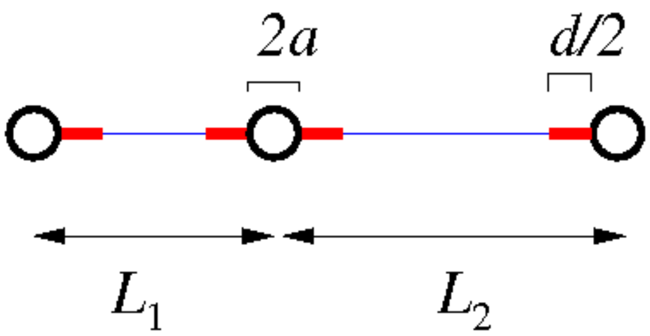
$\kappa < 1$ pushers

$\kappa > 1$ pullers

An ‘Ising model’ of swimmers

Modify → $\hat{\mathbf{u}}$

$$L_i = l_i + d_i$$



$$\dot{d}_1 = \frac{1}{\gamma} F_1 + \hat{\mathbf{u}} \cdot (\mathbf{v}(x_2) - \mathbf{v}(x_1))$$

$$\dot{F}_1 = -\frac{k}{\tau} d_1 + \mu(1 - \sigma d_1^2)\frac{F}{\gamma} + \alpha d_1^3$$

Arm 1 = Spontaneous oscillator

- **Arm 2 is slaved to arm 1**

$$d_1 \simeq d\cos(\omega t + \varphi)$$

$$d_2 \simeq d\sin(\omega t + \varphi)$$

$$\alpha > 0$$

- **Average over fast period -> Swimmer characterised by**

$$\mathbf{X} = (\hat{\mathbf{u}}, \mathbf{r}, \varphi)$$

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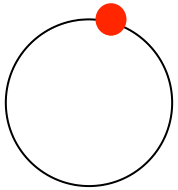
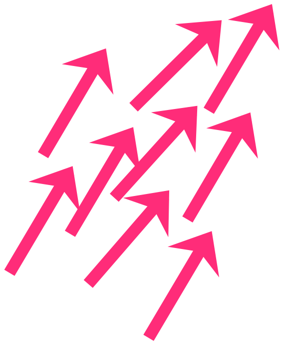
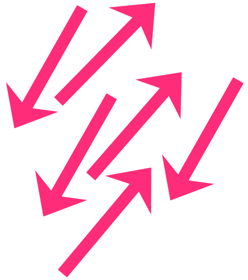
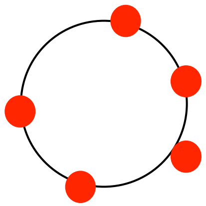
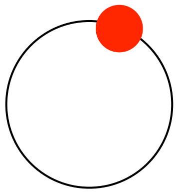
Capturing collective behaviour

Active element  $\hat{\mathbf{u}}_i$ 

Polar order 

Nematic order 

Phase incoherent 

Synchronized (phase coherent) 

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Stochastic trajectory to probability II

Nonlinear current :

$$\vec{J}_1(\vec{r}_1,t) = -D\vec{\nabla}c_1 + \vec{v}^{(1)}(\vec{r}_1)c_1(\vec{r}_1,t) + \int d\vec{r}_2\, \vec{v}^{(2)}(\vec{r}_1,\vec{r}_2)c_1(\vec{r}_1,t)\, c_1(\vec{r}_2,t)$$

$$\partial_t c_1 + \vec{\nabla}\cdot\vec{J}_1 = 0 \qquad D = D(\vec{r}_1)$$

Interacting objs : $\vec{r}_i = \{\mathbf{r}_i, \hat{\mathbf{u}}_i, \varphi_i\}$

Velocities : $\vec{v}^{(i)} = \{\mathbf{v}^{(i)}, \boldsymbol{\omega}^{(i)}, \varphi_i\}$

Gradient operator $\vec{\nabla} = \{\boldsymbol{\nabla}, \boldsymbol{\mathcal{R}}, \partial_\varphi\}$

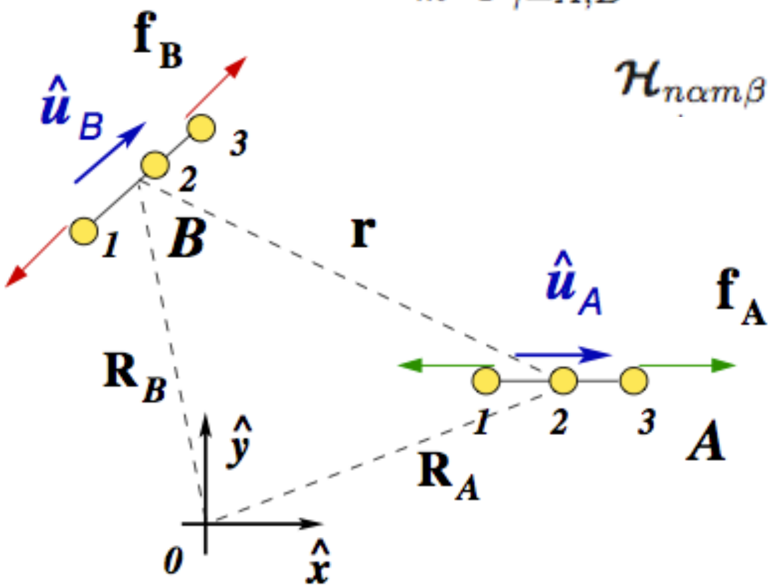
$$\boldsymbol{\nabla} = \frac{\partial}{\partial \mathbf{r}}$$

$$\boldsymbol{\mathcal{R}} = \hat{\mathbf{u}} \times \frac{\partial}{\partial \hat{\mathbf{u}}}$$

Interactions

Interactions between swimmers (no fluctuations)

$$\dot{\mathbf{x}}_n^{\alpha} = \sum_{m=1}^{3} \sum_{\gamma=A,B} \mathcal{H}_{n\alpha m\gamma} \cdot \mathbf{f}_m^{\gamma} \qquad \alpha,\gamma = A,B \text{ and } n,m = 1,2,3.$$



$$\mathcal{H}_{n\alpha m\beta} := \boldsymbol{H}(\mathbf{x}_n^{\alpha} - \mathbf{x}_m^{\beta}) \text{ for } \alpha \neq \beta$$

'velocities'

$$\frac{d}{dt}\hat{\boldsymbol{u}}^{\alpha} = \boldsymbol{\omega}^{\alpha} \times \hat{\boldsymbol{u}}^{\alpha}$$

interactions

$$\frac{d}{dt}\boldsymbol{R}^{\alpha} = v\,\hat{\boldsymbol{u}}^{\alpha} + \boldsymbol{V}^{\alpha}$$

- Take account of phase
- Interactions mediated by fluid (averaging over a cycle)

Leoni, TBL, (2010, 2014)

Length/timescales

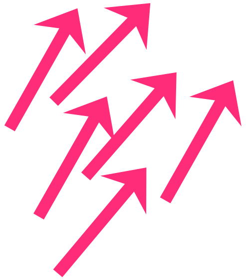
- Internal motions ~ 10 nm
- in ms timescales
- Stall force of ~ 10 pN
- Objects of length ~ 1 micron

↕ Separation of scales

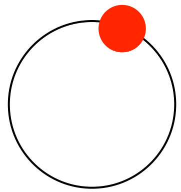
- Looking for phenomena
- on lengthscales > 100 microns
- Timescales of seconds-minutes

Projecting on to hydrodynamic variables

Polarized state



synchronized



Moment expansion of concentration + truncation :

$$\rho(\mathbf{r},t) = \int d\hat{\mathbf{u}}\, d\varphi\, c(\mathbf{r},\hat{\mathbf{u}},\varphi,t)$$

density - conserved quantity

$$\rho\mathbf{P}(\mathbf{r},t) = \int d\hat{\mathbf{u}}\, d\varphi\, \hat{\mathbf{u}}\, c(\mathbf{r},\hat{\mathbf{u}},\varphi,t)$$

$$\rho\Phi(\mathbf{r},t) = \int d\hat{\mathbf{u}}\, d\varphi\, e^{i\varphi}\, c(\mathbf{r},\hat{\mathbf{u}},\varphi,t)$$

} possible broken symmetries

Dynamics on length-scales much bigger than L

$\Rightarrow$ Gradient expansion

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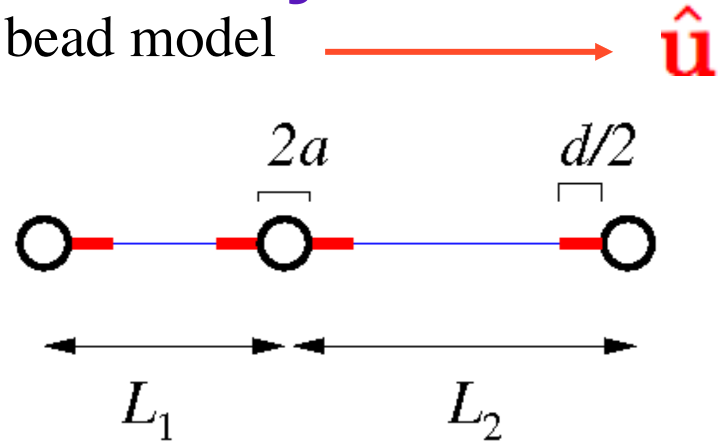
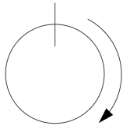
Synchronizable swimmers

3 bead model

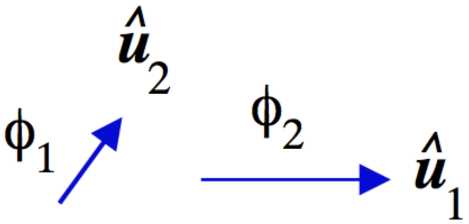


One arm spontaneous osciilator

Additional phase variable



Leoni, Liverpool, PRL, in press (2014)



Order parameters

Polarization $$\mathbf{P}(t) = \frac{1}{N}\sum_{n=1}^{N} \hat{\boldsymbol{u}}_n(t)$$

Phase coherence $$\boldsymbol{\Phi}(t) = \frac{1}{N}\sum_{n=1}^{N} e^{i\phi_n(t)}$$

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Synchronization & polar order

$$\mathbf{P} = P\,\hat{\mathbf{p}} \qquad \boldsymbol{\Phi} = M\,e^{iQ}$$

$$\partial_t P = (b + vM^2)P$$

$$\partial_t M = (\zeta + \xi P^2)M$$

$$b = -D_r + \rho(1-\kappa)\Gamma_b \qquad v = \frac{\Gamma_b \Gamma_v (\kappa - 1)}{\rho\,\Gamma_v - D_r}$$

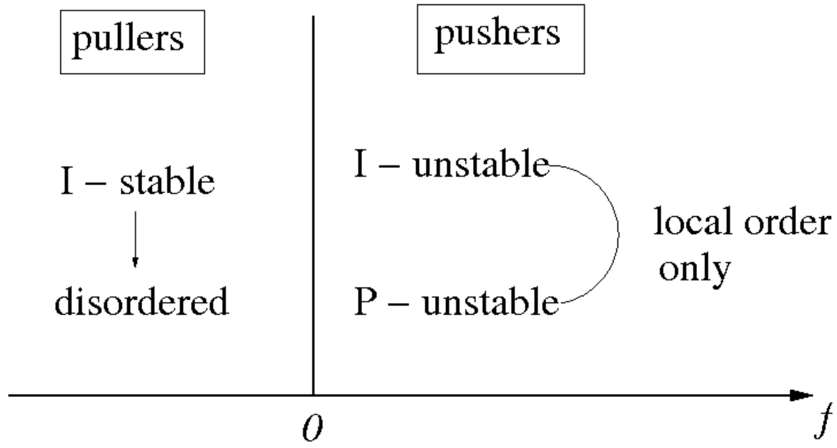
$$\zeta = \kappa(2\kappa + 1)\Gamma_\zeta \qquad \xi = \frac{v}{\Gamma_v}$$

Leoni, Liverpool, PRL, in press (2014)

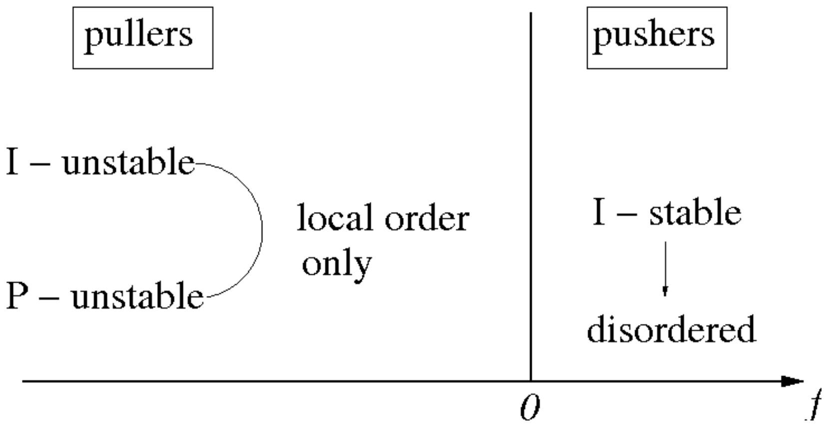
$\kappa < 1$ pushers

$\kappa > 1$ pullers

Phase-force dipole duality



Phase coherent



Phase incoherent

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Summary

Q: Does an internal phase variable have consequences on the behaviour of active matter?

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Summary

- **A: Yes !!**

Studied collective behaviour of simple model swimmer with a dynamic phase variable and simple force multipole to show this

These consequences are not dependent on specific model (can be obtained by phenomenology – writing down most general terms allowed by symmetry)

Leoni, Liverpool, PRE, (2012)

Furthauer, Ramaswamy , PRL, (2013)

Leoni, Liverpool, PRL, in press (2014)

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Summary

We focused on oscillators which synchronise in phase $\alpha > 0$

'Ferromagnets'

Very interesting to explore oscillators which synchronize out of phase

(metachronal waves, ….)

'antiferromagnets' ….